Economic Growth and Polluting Resources: market equilibrium and optimal policies

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Abstract

We study the decentralized market equilibrium and the optimum in an economy which uses polluting resources in the production process and where the Government conducts environmental policy. Two forms of endogenous technical change are considered: pollution-reducing knowledge in the final-goods sector and horizontal innovation in the resources sector. We show that a growing economy may also have a cleaner environment if it adequately allocates its resources evenly among the two alternative technologies. Moreover, if two policy tools are used, the negative economic effects of a higher emission tax may be offset by a subsidy to final consumption.

JEL Classification: O13, O31,O44, Q58.

Keywords: Natural resources; Pollution; Economic growth; Environmental policy.

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1 Introduction

The standard economic growth literature dealing with natural resources has often focused on the conditions of growth under scarcity, but has ignored a key aspect: resource use generates pollution (e.g., Barbier, 1999; Garg and Sweeney, 1978;Scholz and Ziemes, 1999; Grimaud and Rougé, 2003). The combustion of fossil fuels and mineral resources is in fact responsible for a large share of anthropogenic Greenhouse Gas (GHG) emissions. Environmental policies have been conducted worldwide in an attempt to reduce pollution and mitigate environmental issues such as climate change (e.g., Halicioglu, 2009; Soytas and Sari, 2009; Sadorsky, 2009).

However, if production, which generates pollution, consumes natural resources, two major questions arise: how does environmental policy affect both economic growth (and consumption levels) and what are the properties of the equilibrium path?

The approach to this problem has differed from study to study. Authors who consider pollution often consider polluting resources as necessary, but non-essential to production, as they may be substituted by non-polluting resources or innovations (e.g., Gradus and Smulders, 1993; Grimaud and Rougé, 2003). Authors who have found compatibility between a cleaner environment and economic growth generally consider resource scarcity (e.g., Grimaud and Rougé, 2005; Schou, 2002, 2000). As Schou (2002) points out, the properties of natural resources greatly influence the predictions of growth models: (i) if resources are scarce, in the long run the increasing need to save resources will necessarily reduce pollution; (ii) the source of pollution (resource stock) is fixed.

In our endogenous growth model we ignore resource scarcity to avoid this problem. Implicitly we are assuming that the economy can extract as much as it needs to satisfy production. We explore a new path which aims to make economic growth and a cleaner environment compatible.

Natural resources literature has considered the role of innovation in overcoming resource scarcities, but innovation is usually modeled as exogenous. On the other hand, endogenous growth theory has often ignored this question and the contribution of natural resources to growth (Barbier,1999).

We consider that technical change assumes two forms: final-goods producers run research activities to produce emission-reducing knowledge and resource firms run R&D to increase the variety of usable natural resources.

The general set up of our model follows Grimaud and Tournemaine (2007). Nevertheless, we depart from their model in several aspects. First, in their article, growth is sustained by human-capital accumulation and no natural resources are considered. We adapt the final-goods production function by using only natural resources. Consequently, emissions will be generated by the consumption of natural resources. In fact, it is the combustion of fossil fuels and mineral resources that generates most emissions/pollution in the production process. Another major difference between our model and that of Grimaud and Tournemaine (2007) is that we assume a natural resources/R&D sector with horizontal innovation.

Throughout time, scientists have found ways to use resources that were not usable before. For instance, Uranium was not particularly useful before the development of nuclear fission technology. These innovations increased the variety of usable natural resources. This type of differentiation is in line with Barro and Sala-i-Martin (2004, Ch. 6), for example. It implies that when new varieties of natural resources are discovered or made usable, old ones do not become obsolete.

Finally, we depart from Grimaud and Tournemaine (2007) by assuming that final goods producers generate knowledge by investing a given amount of their own product instead of human capital; i.e. our model is lab-equipment and not knowledge driven (e.g., Rivera-Batiz and Romer, 1991). Our model shows that a balanced allocation of resources among the two alternative technological options is crucial for positive economic growth. Furthermore, if the efficiency of knowledge to reduce pollution is sufficiently high, a cleaner environment will be compatible with a growing economy.

We also explore the policy implications when the Government has two policy tools: a tax on emissions and a subsidy for final consumption. We show that if the efficiency of pollution-reducing knowledge and the environmental tax are sufficiently high, both the tax and the subsidy will reduce steady-state emissions. A higher subsidy stimulates final-goods demand but also stimulates the investment on pollution-reducing knowledge. The first effect increases emissions, whereas the second decreases them. For a sufficiently high efficiency of knowledge to reduce pollution, the second effect will dominate.

Furthermore, the subsidy may offset, at least partially, the negative effect that a higher tax would have on the steady state level of output, the stock of pollution-reducing knowledge and the number of resource varieties. Finally, we derive the centralized optimum of the economy and determine the conditions to impose on public policies in order to achieve an optimal equilibrium.

The remainder of the paper is organized as follows. Section 2 presents the set-up of the model. Section 3 shows the market equilibrium conditions in the balanced growth path, highlighting its major properties. Section 4 sums up the environmental policy implications. Section 5 characterizes the optimum. Finally, Section 6 concludes the paper.

2. Set-up of the model

We consider a model in continuous time with differentiated final consumption goods, and a natural resource sector.

2.1. Consumers

There is a mass [0,1] of identical individuals who own the assets of the economy. We assume no population growth so that all aggregate variables can be interpreted as per capita quantities. The individuals value a clean environment, that is, their utility increases with consumption and decreases with pollution.

Preferences are represented as follows:

$$U = \int_0^\infty \left\{ \ln \left[\sum_{n=1}^{N} c(n,t)^{\mu} \right]^{\frac{1}{\mu}} - \omega ln E(t) \right\} . e^{-\rho t} dt \quad (1)$$

where: n is the sector in which the final good is produced ($n = 1, \dots, N$), each with Q_n identical firms, ($q_n = 1, \dots, Q_n$); $c(n,t) = \sum_{q_n=1}^{Q_n} c(q_n, t)$ is per-capita purchase of the differentiated final good produced in sector n at time t; $0 < \mu < 1$ represents the elasticity of consumption; $\omega > 0$ is a parameter reflecting how strong the preference for a clean environment is; $0 < \rho < 1$ is the rate of time preference; and $E(t) = \sum_{n=1}^{N} \sum_{q_n=1}^{Q_n} E(q_n, t)$ is the total flow of polluting emissions.

This type of specification is in line with Schou (2002) and Grimaud and Tournemaine (2007), among others. Other articles consider that emissions also affect productivity negatively (e.g., Bovenberg and Smulders, 1995; Schou, 2000). We have chosen the specification where emissions only affect utility for simplicity's sake and also because this specification is the subject of a great part of the literature (e.g., Grimaud and Rougé, 2005; Gradus and Smulders, 1993).

2.2 Final-goods producers

The differentiated consumption goods ($Y(n,t)$) are produced by an exogenous number of sectors ($n = 1, \dots, N$), each one comprising Q_n identical firms ($q_n = 1, \dots, Q_n$). Final goods are sold in imperfectly competitive markets and are produced by using natural resources, R, which generate polluting emissions, E:

$$Y(q_n, t) = A \sum_{j=1}^{J} R(j, q_n, t) \tag{2}$$

where: $Y(q_n, t)$ is the output of firm q_n; $R(j, q_n, t)$ is the amount of the jth type of natural resources used; J is the number of usable varieties of natural resources;[1] parameter $A > 0$ represents the overall productivity or efficiency of the economy.

Additionally, final-goods producers run indoor research activities to generate pollution-reducing knowledge, Z. As in Grimaud and Tournemaine (2007), the stock of knowledge at each t is composed of a continuum of pieces of knowledge. A piece of knowledge is an indivisible, infinitely-lived, differentiated, public good. In this specific case it refers to techniques which allow less pollution for a given level of resources consumed (for instance, new production processes or procedures).

Each firm spends $\zeta(q_n, t)$ units of $Y(q_n, t)$ to produce new pieces of knowledge. $Z(q_n, t)$ is the stock of knowledge produced by firm q_n until the moment t.[2] New pieces of knowledge are produced with the technology:

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1 For simplicity, we do not consider the labor market since our main purpose is to analyze natural resources. However, the results would be the same if we assumed a given labor supply equal to population and no population growth. Furthermore, we abstract from capital accumulation (as, e.g., Grimaud and Tournemaine, 2007; Schou, 2002).

2 Several models consider research conducted using only labor (e.g., Grimaud and Rougé, 2003; Grimaud and Rougé, 2005; Schou, 2002). In line with lab-equipment growth models (e.g., Rivera-Batiz and Romer, 1991), we modify this view by considering firms spending a given amount of resources to conduct research.

$$\dot{Z}(q_n, t) = \delta\zeta(q_n, t) \tag{3}$$

where $\delta > 0$ is a productivity parameter. The more the firm spends on research activities, the more knowledge it will generate. A similar knowledge accumulation function may be found in, for example, Buonanno *et al.*, (2003) and Goulder and Schneider, (1999).

Knowledge is used to reduce pollution (e.g., Bovenberg and Smulders, 1995; Grimaud and Rougé, 2003; Grimaud and Tournemaine, 2007). The flow of emissions is as follows:

$$E(q_n, t) = \sum_{j=1}^{J} R(j, q_n, t)\, Z(t)^{-\beta} \tag{4}$$

where $\beta > 0$ measures the efficiency of knowledge to reduce pollution. Departing from Grimaud and Tournemaine (2007), we assume that emissions increase with consumption of natural resources (as, e.g., Bovenberg and Smulders, 1995; Schou, 2002), the combustion of fossil fuels or waste generation from a factory. We treat emissions as a flow instead of a stock. Even though many environmental issues last for several decades, considering pollution as a flow simplifies the analysis and yields similar results to treating it as a stock (e.g., Stokey, 1998; Gradus and Smulders, 1993).[3]

2.3. Resource sector

For the innovation in the resource sector we follow the prominent works by Romer (1990), Rivera-Batiz and Romer (1991), Grossman and Helpman (1991, Ch. 3) and Barro and Sala-i-Martin (2004, Ch. 6), among others. Indeed, in the specification of this sector, we consider horizontal R&D, which, in turn, results in technical progress. The specification reflects that, over the years, scientists have found ways to use resources which were not useful before. Since there are no quality improvements, no innovation

[3] For a deeper discussion of this issue see, e.g., Grimaud and Tournemaine (2007) and (Grimaud and Rougé, 2005).

ever becomes obsolete: firms that become producers remain leaders from then on without supporting further R&D, since they are granted a patent that lasts forever.

Thus, monopolistic firms sell natural resources to final-goods producers and at the same time conduct R&D activities in order to increase the number/variety of usable resources. Let us assume that the production of natural resources can be interpreted as the extraction and refinery of oil or the treatment of wood to make it usable by final good producers, for instance. R&D activities open up new possibilities of resource use, making it possible to use a resource that was not usable before. This endogenous technical progress is reflected in an increase of J.

2.4. Government

We start by analyzing the decentralized or market equilibrium with Government intervention. The public nature of knowledge and pollution and the imperfect competition in final good and resource markets creates distortions in the decentralized equilibrium. To deal with these distortions we assume, following Grimaud and Tournemaine (2007), two policy instruments: a tax on emissions,$\tau(t)$, and a subsidy to final-goods purchase, $\sigma(q_n, t)$.

3. Equilibrium

This section aims to present the decentralized long-run equilibrium conditions. The balanced growth path or steady state is characterized by constant growth rates of all variables and clearance in all markets. In what follows we describe the characteristics of the symmetric long-run equilibrium and the behavior of the agents in more detail.

3.1. Symmetric equilibrium and steady-state

In the symmetric equilibrium all firms in the final-goods sector produce the same, use the same amount of natural resources, generate the same amount of knowledge, emit the same amount of pollution, charge the same price and make the same profit. Similarly, all firms in the resource sector produce the same amount of resources, charge the same price and make the same profit. These results are proved in the deduction of the market equilibrium.

These conditions may be summarized as follows. In the symmetric equilibrium,

$$R(t) = R(j, q_n,, t)J(t)QN;\ Y(t) = Y(q_n, t)QN;\ \ Z(t) = Z(q_n, t)QN;\ \ \zeta(t) = \zeta(q_n, t)QN;\ \ E(t) = E(q_n, t)QN, \forall n;\ p(n, t) = p(t),\ \pi(q_n, t) = \pi(t),\ \vartheta(t) = \vartheta(q_n, t)QN,\ \ \Theta(t) = \Theta(q_n, t)QN,\ \forall n;\ \ \psi(j, t) = \psi(t),\ \Lambda(j, t) = \Lambda(t), \forall j.$$

where: $p(n, t)$is the price of the final good of sector n; $\pi(q_n, t)$ is the profit of firm q_n; $\vartheta(q_n, t)$ is the firm's q_n WTP to use a piece of knowledge at time t; $\Theta(q_n, t)$ is the value of a piece of knowledge to firm q_n; $\psi(j, t)$ is the price of resource variety j; and $\Lambda(j, t)$ is the profit of the monopolist of resource variety j. In equilibrium, all output is used for consumption, investment in knowledge in the final-good sector or investment in new resource varieties:

$$Y(t) = C(t) + \zeta(t) + \eta_R\dot{J}(t) + J(t)R(j, t) \tag{5}$$

where: $R(j, t) = \sum_{n=1}^{N} \sum_{q_n=1}^{Q_n} R(j, q_n, t)$. As in Grimaud and Tournemaine (2007), the existence of a steady state requires the term $\tau(t)Z(t)^{-\beta}$ to be constant over time. The government will choose a growth path for the tax on pollution so that $g_\tau = \beta g_Z$ at any *t*. Hence, $\tau(t)Z(t)^{-\beta} = \tau(0)Z(0)^{-\beta}$ where $\tau(0)$ and $Z(0)$ are the initial values for tax and knowledge. We will designate $x = \tau(0)Z(0)^{-\beta}$, which represents environmental policy. In the same way, the subsidy for the demand for the differentiated good (equal for all goods and constant overtime in steady-state equilibrium) is: $\sigma(n, t) = \sigma, \forall t$ (e.g., Grimaud and Rougé, 2003). The government budget constraint is balanced at each moment: environmental policy and subsidies are financed by a lump-sum transfer from individuals, $T(t)$.

Since knowledge is a public good, some difficulties arise in a decentralized equilibrium. Imperfect competition in the final-goods market solves this problem. Other authors, for instance, Grimaud and Rougé (2005), consider public funded R&D. In our model, the final good market has Cournot competition and free entry. Firms sell the differentiated goods in an imperfectly competitive market at a price greater than the marginal cost of production, and gain resources to buy knowledge. Nevertheless, since there is free entry into the market, final profits are zero.

Knowledge is traded using bilateral contracts between inventors and users, and sellers extract the entire willingness to pay (WTP) for knowledge from all buyers. This means there are no problems of verification, exclusion and information. Firm q_n profit without payment of knowledge is denoted by $\tilde{\pi}(q_n, t)$. As referred, $\mathcal{v}(q_n, t) = \frac{\partial \tilde{\pi}(q_n, t)}{\partial Z(t)}$ is the firm's WTP to use a piece of knowledge at time t. The price paid to use a piece of knowledge from t to infinity is $\Theta(q_n, t) = \int_t^{\infty} \mathcal{v}(q_n, s) e^{-\int_t^s r(u)du} ds$, where $r(u)$ denotes the interest rate. The value of a piece of knowledge; that is, the value perceived by a firm for the sale of a piece of knowledge is $\Theta(t) = \int_t^{\infty} \mathcal{v}(s) e^{-\int_t^s r(u)du} ds$, where $\Theta(t) = \sum_{n=1}^{N} \sum_{q_n=1}^{Q_n} \Theta(q_n, t)$ and $\mathcal{v}(t) = \sum_{n=1}^{N} \sum_{q_n=1}^{Q_n} \mathcal{v}(q_n, t)$. Differentiating the expression of $\Theta(t)$ with respect to time we obtain:

$$r(t) = \frac{\mathcal{v}(t)}{\Theta(t)} + g_{\Theta(t)} \tag{6}$$

where g_x is the growth rate of any variable x.

3.2. Agents' behaviour

3.2.1. Individuals

The representative individual maximizes his/her utility Eq(1), subject to the budget constraint $\dot{B(t)} = r(t)B(t) - \sum_{n=1}^{N}\big(1-\sigma(n,t)\big)p(n,t)c(n,t) - T(t)$, and chooses plans for consumption of each differentiated final good, $c(n,t)$, and wealth, $B(t)$.

The current value Hamiltonian for this problem is:

$$CVH = \left(\ln\left([\sum_{n=1}^{N} c(n,t)^{\mu}]^{1/\mu}\right) - \varepsilon lnE(t)\right) \\ + \lambda(t)\left[r(t)B(t) - \sum_{n=1}^{N}\big(1-\sigma(n,t)\big)p(n,t)c(n,t) - T(t)\right]$$

where $\lambda(t)$ is the dynamic multiplier of the number of resource varieties. The first order conditions are: $\frac{\partial CVH}{\partial c(n,t)} = 0$; $\frac{\partial CVH}{\partial B(t)} = -\dot{\lambda}(t) + \lambda(t)\rho$. The transversality condition is: $\lim_{t\to\infty}\lambda(t)B(t)e^{-\rho t} = 0$.

The first order conditions gives the following conditions

$$c(n,t)^{\mu-1} = \lambda(t)[1-\sigma(n,t)]p(n,t)\sum_{n=1}^{N} c(n,t)^{\mu} \tag{7}$$

$$\frac{\dot{\lambda(t)}}{\lambda(t)} = \rho - r(t) \tag{8}$$

Using Eq (7), one derives the inverse demand function for each differentiated good:

$$p(n,t) = \frac{D(t)^{1-\mu}[c(n,t)]^{\mu-1}}{[1-\sigma(t)]} \tag{9}$$

where $D(t) = \sum_{k=1}^{N}\big(1-\sigma(k,t)\big)p(k,t)c(k,t) / \sum_{k=1}^{N}[\big(1-\sigma(k,t)\big)p(k,t)]^{\mu/(\mu-1)}$.

In equilibrium, consumption is a given proportion of output: $c(n,t) = \gamma Y(n,t)$. Since $c(n,t) = \sum_{q_n=1}^{Q_n} c(q_n,t)$, Eq (9) may be written as:

$$p(n,t) = \frac{D(t)^{1-\mu}\left[\sum_{q_n}^{Q_n}\gamma Y(q_n,t)\right]^{\mu-1}}{[1-\sigma(n,t)]} \tag{10}$$

Combining Eq (7) and (8) one derives the usual Keynes-Ramsey rule:

$$r(t) = (1-\mu)g_{c(n,t)} + g_{\Omega(t)} + g_{(1-\sigma(t))} + g_{p(n,t)} + \rho \tag{11}$$

where $\Omega(t) = \sum_{n=1}^{N} c(n,t)^{\mu}$. In the stationary equilibrium $g_{\Omega(t)} = \mu g_{c(n,t)}$. This condition summarizes the consumers' decisions. Individuals postpone consumption if the rate of return compensates for the pure rate of time preference and the change in the marginal value of consumption (e.g., Bovenberg and Smulders, 1995).

3.2.2. Final-goods sector

Firms in the final goods sector buy natural resources to produce differentiated final goods and sell them in an imperfectly competitive market. At the same time, they generate knowledge to reduce emissions.

Each firm maximizes:

$$\tilde{\pi}(q_n,t) = p(n,t)Y(q_n,t) - \sum_{j=1}^{J} \psi(j,t)R(j,q_n,t) - \tau(t)p(n,t)E(q_n,t) + \Theta(t)\dot{Z}(q_n,t) - \zeta(q_n,t)$$

subject to the production technology, Eq (2), the accumulation of knowledge, Eq (3), the flow of emissions, Eq (4), and the inverse demand function, Eq (10). In the symmetric equilibrium, all resources will be used in the same amount and have the same price. Furthermore, $c(n,t) = \gamma Y(n,t)$. After substitutions, we have:

$$\tilde{\pi}(q_n,t) = Y(q_n,t)\left\{\frac{D(t)^{1-\mu}}{\big(1-\sigma(n,t)\big)}\left(\sum_{q_n}^{Q_n} \gamma Y(q_n,t)\right)^{\mu-1}\left[1-\frac{\tau(t)Z(t)^{-\beta}}{A}\right] - \frac{\psi(j,t)}{A}\right\} + \Theta(t)\delta\zeta(q_n,t) - \zeta(q_n,t) \tag{12}$$

From the first order conditions $\frac{\partial\pi(q_n,t)}{\partial Y(q_n,t)} = 0$; $\frac{\partial\pi(q_n,t)}{\partial\zeta(q_n,t)} = 0$, we have respectively:

$$Y(q_n,t) = \frac{D(t)}{Q(t)\gamma}\left[\frac{\left(A-\tau(t)Z(t)^{-\beta}\right)\left(1+\frac{(\mu-1)Y(q_n,t)}{Y(n,t)}\right)}{\psi(j,t)(1-\sigma(n,t))}\right]^{\frac{1}{1-\mu}} \tag{13}$$

And

$$\Theta(t)\delta = 1 \Leftrightarrow \Theta(t) = \Theta = \frac{1}{\delta} \tag{14}$$

Eq (13) is the final-goods production of each firm. Higher taxes on emissions and higher resource prices yield lower production. At the same time, a higher subsidy to final consumption will stimulate production. From Eq (14) one gets $g_{\Theta(t)} = 0$, which means that the value of a piece of knowledge is constant over time.

The WTP, at time t, to use a piece of knowledge, $v(q_n, t) = \frac{\partial \tilde{\pi}(q_n,t)}{\partial Z(q_n,t)}$, is:

$$v(q_n, t) = \frac{\beta}{A} \frac{D(t)^{1-\mu}}{\left(1 - \sigma(n,t)\right)} [\gamma Y(n,t)]^{\mu-1} \tau(t) Z(t)^{-\beta-1} Y(q_n, t) \tag{15}$$

where $Y(n,t) = \sum_{q_n}^{Q_n} Y(q_n, t)$. A higher tax rate leads to a higher WTP for knowledge.

The free entry condition on the final goods market, $\pi(q_n, t) = \tilde{\pi}(q_n, t) - \Theta(q_n, t)\dot{Z}(t) = 0, \forall q_n$ implies:

$$\zeta(q_n, t) = \frac{1}{\mu A}\left(\frac{1}{1 + \frac{\mu - 1}{Q(t)}} - 1\right) Y(q_n, t) \tag{16}$$

Let $\alpha(t)$ be the share of total output invested in knowledge in each t, which, in steady-state, is constant: $\alpha(t) = \alpha = \frac{\zeta(q_n,t)}{Y(q_n,t)}$. In equilibrium, the expression for Q is:

$$Q(t) = Q = \frac{(1-\mu)(1+\alpha\mu A)}{\alpha\mu A} \tag{17}$$

From where $g_Q = 0$.

In equilibrium, Eqs (10) and (13) give:

$$p(n,t) = p = \frac{1}{\mu(A - x)\left(1 + \frac{\mu - 1}{Q}\right)} \tag{18}$$

3.2.3. Natural-resource sector

In this sector there is no perfect competition. Following Barro and Sala-i-Martin (2004, Ch. 6) there is horizontal R&D, and thus technical progress is reflected in an expansion of the number of varieties of natural resources that can be used.[4]

Once a new variety has been invented, by devoting resources to R&D, the firm receives a perpetual monopoly rent. This allows the firm to sell its product at a price which maximizes its profits. Each firm faces a two step decision process. Initially, it decides whether to devote resources to discovering a new variety of natural resources and to making it usable. It will expend that amount if the net present value of future expected profits covers the amount spent up front. Secondly, the inventors determine the optimal price at which to sell the newly invented resources.

The process is solved backwards. Firstly, the optimal price is derived, assuming that the new resource variety has already been discovered. Secondly, the present value of profits is calculated and compared with R&D costs. If the latter are lower, the firm will undertake R&D expenditures. Stage 2: the present value of the returns for discovering the jth natural resource variety is given by:

$$V(t) = \int_t^{\infty} \Lambda(j, \vartheta) e^{-\bar{r}(t,\vartheta).(\vartheta - t)} d\vartheta \tag{19}$$

where $\Lambda(\mathrm{j}, \vartheta)$ is the profit flow at date ϑ, and $\bar{\mathrm{r}}(\mathrm{t}, \vartheta) \equiv [1/(\vartheta - \mathrm{t})]$. The average interest rate between t and ϑ is given as $\int_t^{\vartheta} r(\omega) d(\omega)$. In equilibrium, the interest rate, r, is constant, and the present value factor simplifies to $e^{-r.(\vartheta - t)}$. The flow of profits for the firm equals revenues less production costs. We assume, as usual (e.g., Barro and Sala-i-Martin, 2004, Ch. 6) that once discovered, the new variety of natural resource of type j

[4] Hence, we assume that the firm invents the new variety and becomes the monopolist of that variety by selling it to final goods producers. Similar results would be obtained if we assumed that those activities were developed in two different types of firms and that there was a royalty shared among them.

costs one unit of Y to produce. This corresponds to the assumption that the marginal and average cost of production is a constant, normalized to 1.

Stage 1: The firms have to decide whether to invest in R&D or not. We assume, as in Barro and Sala-i-Martin, (2004, Ch. 6), that the cost of discovering a new variety of natural resources is fixed, being $\eta_R$units of Y. Hence, in equilibrium, profit of R&D activities equals costs:

$$V(t) = \eta_R, \forall t \tag{20}$$

Differentiating Eq (19) with respect to time, we have:

$$r(t) = \frac{\Lambda(\mathrm{j},\mathrm{t})}{V(t)} + \frac{\dot{V(t)}}{V(t)} \tag{21}$$

The profit flow is given by:

$$\Lambda(j,t) = [\psi(j,t) - 1].R(j,t) \tag{22}$$

where $R(j,t) = NQ(t)R(j,q_n,t)$. Using Eq (2) and (13), one obtains the maximization problem:

$$\max \Lambda(j,t) = [\psi(j,t) - 1]\frac{NQD(t)}{J(t)A}\left[\frac{\left(A - \tau(t)Z(t)^{-\beta}\right)\left(1 + \frac{(\mu-1)Y(q_n,t)}{Y(n,t)}\right)}{\psi(j,t)(1-\sigma)}\right]^{\frac{1}{1-\mu}}$$

The first order condition is $\frac{\partial\Lambda(j,t)}{\partial\psi(j,t)} = 0$, which after some manipulation gives the equilibrium price condition:

$$\psi(j,t) = \psi = \frac{1}{\mu} > 1 \tag{23}$$

From where $g_\psi = 0$. The price is the same for all natural resources j. This monopoly price, ψ, is a mark-up over the marginal cost of production, which is 1.

Replacing the expression of ψ with the demand for R gives:

$$R(j,q_n,t)=R(q_n,t)=\frac{D(t)}{Q\gamma J(t)A}\left[\frac{\mu(A-x)\left(1+\frac{\mu-1}{Q}\right)}{(1-\sigma)}\right]^{\frac{1}{1-\mu}} \quad (24)$$

which is constant across j. Other things being equal, the larger the number of varieties available, the smaller is the demand for each resource variety. Note that even though the demand is equal for all final good firms, we have decided to maintain the index q_n to keep the interpretation of the variables easier. Replacing Eq (24) in (2) taking into consideration that, in equilibrium, $Y(q_n,t)=AJ(t)R(q_n,t)$, gives:

$$Y(q_n,t)=\frac{D(t)}{Q\gamma}\left[\frac{\mu(A-x)\left(1+\frac{\mu-1}{Q}\right)}{(1-\sigma)}\right]^{\frac{1}{1-\mu}} \quad (25)$$

The direct analysis of Eq (25) shows that an increase in the rate of subsidy stimulates final demand and leads to a higher equilibrium output. These effects are similar to the ones verified for the demand for natural resource since that demand is driven by final output demand.

Eq (22), (23) and (24) give the profit for each firm, which is equal for all firms:

$$\Lambda(j,t)=\Lambda(\,t)=\left[\frac{(1-\mu)(1-s)}{\mu}\right]NQ\frac{D(t)}{Q\gamma J(t)A}\left[\frac{\mu(A-x)\left(1+\frac{\mu-1}{Q}\right)}{(1-\sigma)}\right]^{\frac{1}{1-\mu}} \quad (26)$$

3.2.4. Financial market

The financial model is perfectly competitive (e.g., Grimaud and Rougé, 2005, 2003; Grimaud and Tournemaine, 2007; Grimaud, 1999). From Eq (6), (14), (15) and knowing that $v(t)=\sum_{n=1}^{N}\sum_{q_n=1}^{Q_n}v(q_n,t)$, one derives the interest rate in the final good sectors:

$$r(t)=\frac{\beta Y(q_n,t)QNx}{\mu A\delta(A-x)\left(1+\frac{\mu-1}{Q}\right)Z(t)} \quad (27)$$

Using Eq (2), (20), (21) and (26) one obtains the interest rate of the resources sector:

$$r(t) = \frac{\Lambda(t)}{\eta_R} = \frac{(1-\mu)}{\mu\eta_R}\frac{Y(q_n, t)QN}{AJ(t)} \tag{28}$$

Eq (28) may be interpreted as the technology curve and summarizes the decisions made on the production side. Since the financial market is perfectly competitive, and there is only one interest rate in the economy, Eq (27) and (28) give the condition:

$$Z(t) = \frac{\beta x \eta_R}{\delta(A-x)\left(1+\frac{\mu-1}{Q}\right)(1-\mu)} J(t) \tag{29}$$

which implies that $g_{J(t)} = g_{Z(t)}$. Eq (29) shows that, in each *t*, the number of resource varieties in the economy and general knowledge grow in the same proportion. The higher the cost of discovering new varieties, the lower the proportion $\frac{J(t)}{Z(t)}$ will be. Additionally, in order to guarantee that the proportion is positive, the following parameter condition must hold:

$$(A-x)\left(1+\frac{\mu-1}{Q}\right) > 0$$

since $\left(1+\frac{\mu-1}{Q}\right) = \frac{1}{\alpha A\mu+1} > 0$ one has that $(A-x) > 0$.

In equilibrium, Eq (16) implies that $g_{\zeta(q_n,t)} = g_{Y(q_n,t)}$ and Eq (27) implies that $g_{Y(q_n,t)} = g_{Z(t)} = \frac{\delta\zeta(q_n,t)QN}{Z(t)}$. By combining these results with Eq (11), (17), (25), (29) and (30) one obtains the equilibrium solution for the model. The results are summarized below.

Number of firms in each sector:

$$Q = \frac{(1-\mu)(1+\alpha\mu A)}{\alpha\mu A}$$

Quantities:

$$Y(q_n, t) = \frac{D(t)}{Q\gamma}\left[\frac{\mu(A-x)\left(1+\frac{\mu-1}{Q}\right)}{(1-\sigma)}\right]^{\frac{1}{1-\mu}}$$

$$J(t) = \left[\frac{1}{\mu} - \frac{\alpha\delta^2(A-x)\left(1+\frac{\mu-1}{Q}\right)}{\beta x}\right]\frac{(1-\mu)Y(q_n,t)QN}{\rho\eta_R} \tag{30}$$

$$Z(t) = \frac{J(t)\eta_R\beta x}{(A-x)\left(1+\frac{\mu-1}{Q}\right)\delta(1-\mu)} \tag{31}$$

$$R(j,q_n,t) = \frac{Y(q_n,t)}{J(t)A} \tag{32}$$

$$E(t) = \frac{QNY(q_n,t)}{AZ(t)^{\beta}} \tag{33}$$

Prices:

$$\psi = \frac{1}{\mu}$$

$$p = \frac{1}{\mu(A-x)(1+\frac{\mu-1}{Q})}$$

$$r(t) = \frac{(1-\mu)Y(q_n,t)QN}{\mu\eta_R AJ(t)}$$

$$\Theta = \frac{1}{\delta}$$

$$v(t) = r(t)\,\delta$$

Growth rates:

$$g_Y = g_Z = g_J = g_\zeta = \frac{(1-\mu)Y(q_n,t)QN}{\mu\eta_R AJ(t)} - \rho \tag{34}$$

$$g_p = g_\psi = g_r = g_R = g_Q = 0 \tag{35}$$

$$g_E = (1-\beta)g_Y$$

3.3. Properties of the steady-state equilibrium path

The steady state conditions show some equilibrium characteristics and imply some parameter restrictions. Firstly, we may see that all prices are constant in steady-state. From Eq (25), we see that since $(A-x) > 0$, for output to be positive, one needs to impose $0 < \sigma < 1$. Eq (30) implies that, for $J(t)$ to be a positive number, the following condition must hold:

$$\alpha < \frac{\beta x}{\mu[\delta^2(A-x) - \beta Ax]}$$

That is, there is an upper bound for the share in investment in pollution-reducing knowledge. Eq (34) may be developed to give:

$$g = \rho \left[\frac{\beta x(1 + A\alpha\mu)}{Ax\beta(1 + A\alpha\mu) - A\alpha\mu\delta^2(A - x)} - 1 \right]$$

The equilibrium economic growth rate may be positive or negative. This result is in line with Grimaud and Rougé (2003) and Schou, 2000) for instance. For the growth rate to be positive one has the condition:

$$\alpha > \frac{\beta x(A - 1)}{\mu A[\beta x(1 - A) + \delta^2(A - x)]}$$

We also know that $0 < \alpha < 1$. For $\frac{\beta x(A-1)}{\mu A[\beta x(1-A)+\delta^2(A-x)]} > 0$ one has the conditions $A > 1$ and $\beta < \frac{x(A-1)}{\delta^2(A-x)}$. For $\frac{\beta x(A-1)}{\mu A[\beta x(1-A)+\delta^2(A-x)]} < 1$ one has the condition $\beta < \frac{A\mu\delta^2(A-x)}{x(A-1)(1+A\mu)}$. Hence, under the conditions $A > 1$, $\beta < \frac{x(A-1)}{\delta^2(A-x)}$ and $\beta < \frac{A\mu\delta^2(A-x)}{x(A-1)(1+A\mu)}$ there is an interval for α such that the economy's growth rate and the number of varieties of resources are positive:

$$\frac{\beta x(A - 1)}{\mu A[x\beta(1 - A) + \delta^2(A - x)]} < \alpha < \frac{\beta x}{\mu[\delta^2(A - x) - \beta Ax]}$$

This result reflects the need for the economy to allocate its resources evenly among the two competing technological alternatives: pollution-reducing knowledge and resource varieties. If the economy assigned too many resources to knowledge, there would be no discovery of new resources varieties. However, if the economy assigned too many resources to the resource sector, the investment in knowledge would be insufficient and economic growth would be negative. A balanced distribution of resources will have a positive effect on growth. The growth rate of emissions will be negative if economic growth is negative ($g_Y < 0$) or if $\beta > 1$; i.e., if the efficiency of knowledge to reduce pollution is suitably high, a cleaner environment is compatible with a growing economy.

4. Policy effects

Now, we analyze how a change in the subsidy to final goods and in the emission tax affects the major variables of the model. Results are summarized in Tables 1 and 2.

4.1. Subsidy

Table 1[5]- Effects of a change in the subsidy to final goods consumption

$\frac{\partial Y(q_n,t)}{\partial \sigma} > 0$	$\frac{\partial Z(t)}{\partial \sigma} > 0$	$\frac{\partial J(t)}{\partial \sigma} > 0$	$\frac{\partial R(j,q_n,t)}{\partial \sigma} = 0$	$\frac{\partial E}{\partial \sigma} < 0$ $if\, \beta > 1$
$\frac{\partial p}{\partial \sigma} = 0$	$\frac{\partial r}{\partial \sigma} = 0$	$\frac{\partial \psi}{\partial \sigma} = 0$	$\frac{\partial g_y}{\partial \sigma} = 0$	$\frac{\partial g_E}{\partial \sigma} = 0$

As noted above, a higher subsidy to final goods purchase will increase the equilibrium output of each firm. Nevertheless, the subsidy does not affect the final goods price, Eq (18). If $J(t)$ is positive; i.e. if $\alpha < \frac{\beta x}{\mu[\delta^2(A-x)-\beta x A]}$, the positive effect on demand will affect technical change. The mechanism is as follows: a higher subsidy stimulates demand; in order to produce final-goods firms will have to buy and consume more natural resources; this, in turn, will have two effects - on the one hand, a higher consumption of resources will make firms pollute more and consequently pay more tax; to minimize this payment, final good producers will invest in pollution-reducing knowledge, thus, a higher subsidy will also lead to a higher stock of knowledge in steady-state, Eq (31); on the other hand, a higher demand for resources will lead to a higher number of resource varieties in equilibrium, Eq (30), because it will give resource firms an extra incentive to invest in research activities.

The effect of a higher subsidy on emissions will depend on two opposite effects. On the one hand, a higher subsidy stimulates demand and increases output. This increase in output generates more emissions. On the other hand, a higher subsidy will also increase the stock of pollution-reducing knowledge, which, in turn, decreases

[5] We omitted from the table the condition $\alpha < \frac{\beta x}{\mu[\delta^2(A-x)-\beta A x]}$, since this is a necessary condition for $J(t) > 0$, that is for the economic meaning of our model.

emissions. If the first effect dominates, emissions will increase, whereas if the second effect is stronger emissions will decrease. We conclude that for a sufficiently high efficiency of knowledge to reduce emissions ($\beta > 1$), the second effect will dominate and emissions will decrease.

Nevertheless, a higher subsidy will have no effects on the equilibrium level of each resource variety. This happens because the positive effect through a higher output will be totally offset by the increase in the number of varieties. In fact, the higher the number of varieties, *ceteris paribus*, the lower the level of each variety sold, Eq (32). For the same reason, the subsidy will also have no effect on the equilibrium growth rate of the economy. Finally, one may see that the subsidy will only have real effects on this economy since none of the prices is affected.

4.2. Emissions tax

Table 2- Effects of a change in the emissions tax

$\frac{\partial Y(q_n,t)}{\partial \tau} < 0$	$\frac{\partial Z(t)}{\partial \tau} < 0$ $If\, A > \frac{1}{\alpha(1-\mu)}$	$\frac{\partial J(t)}{\partial \tau} < 0$ $If\, A > \frac{1}{\alpha(1-\mu)}$	$\frac{\partial R(j,q_n,t)}{\partial \tau} < 0$ $If\, A > \frac{1}{\alpha(1-\mu)}$	$\frac{\partial E}{\partial \tau} < 0$ $If\, \beta > 1\ and\ x > \frac{A[\alpha(\beta-1)\delta^2\mu+\beta^2(1-\mu)(1+A\alpha\mu)]}{(\beta-1)[\beta+\alpha(A\beta+\delta^2)\mu]}$
$\frac{\partial p}{\partial \tau} > 0$	$\frac{\partial r}{\partial \tau} < 0$	$\frac{\partial \psi}{\partial \tau} = 0$	$\frac{\partial g_y}{\partial \tau} < 0$	$\frac{\partial g_E}{\partial \tau} < 0$

A higher tax on emissions will always lead to lower output of each final good firm. This happens because a higher tax will increase final goods price which, in turn, will decrease demand. If $J(t)$ is positive, a higher tax leads to a lower stock of pollution-reducing knowledge and a lower number of resource varieties if $A > \frac{1}{\alpha(1-\mu)}$.

There is a strong connection between the two alternative technical options. A higher number of resource varieties used will generate more pollution (since the amount of each variety remains unchanged), and this will require more investment in pollution-reducing knowledge. This makes $J(t)$ and $Z(t)$ complementary instead of substitutable.

As in the case of the subsidy, a higher tax will have two opposite effects on steady-state emissions. On the one hand, it will decrease output and consequently emissions. On the other hand, if $A > \frac{1}{\alpha(1-\mu)}$, it will decrease the steady-state stock of pollution-reducing knowledge, which, in turn, increases emissions. If the first effect dominates, emissions will decrease, whereas if the second effect is stronger emissions will increase. We conclude that if $\alpha < \frac{\beta x}{\mu[\delta^2(A-x)-\beta Ax]}$, $\beta > 1$, and $x > \frac{A[\alpha(\beta-1)\delta^2\mu+\beta^2(1-\mu)(1+A\alpha\mu)]}{(\beta-1)[\beta+\alpha(A\beta+\delta^2)\mu]}$, i.e., if $J(t)$ is positive, the efficiency of knowledge to reduce emissions is sufficiently high and the environmental policy is significantly high, then a higher tax on emissions will decrease emissions (the first effect dominates).

A higher tax will decrease the demand for each variety of resources. Nevertheless, each variety is sold at the same price as before. A higher tax will also lead to a lower steady-state interest rate, that is, the return on investment will be lower. In turn, this will determine a lower growth rate of the economy and a lower growth rate of emissions.

5. Optimum

In the optimum, the number of firms in each sector, Q, is given. The social planner will maximize aggregate utility, Eq (1), subject to the aggregate production process of all differentiated goods, $Y(t) = \sum_{n=1}^{N}\sum_{q_n=1}^{Q_n} Y(q_n, t) = A\sum_{j=1}^{J} R(j,t) = AJ(t)R(j,t)$, the aggregate production process of knowledge, $\dot{Z}(t) = \delta\zeta(t)$, where $\zeta(t) = \sum_{n=1}^{N}\sum_{q_n=1}^{Q_n}\zeta(q_n, t))$, the flow of emissions, E(t)=$\sum_{n=1}^{N}\sum_{q_n=1}^{Q_n} E(q_n, t) = \sum_{j=1}^{J} R(j,t)\,Z(t)^{-\beta} = J(t)R(j,t)Z(t)^{-\beta}$, and the general equilibrium constraint, Eq (5). The current value Hamiltonian of the problem is:

$$CVH = \ln\left(N^{\frac{1-\mu}{\mu}} c(t)\right) - \omega\, lnJ(t)R(j,t)Z(t)^{-\beta} + \nu(t)\frac{1}{\eta_R}[AJ(t)R(j,t) - c(t) - \zeta(t) - J(t)R(j,t)] + \xi(t)\delta\zeta(t) \quad (36)$$

where $\nu(t)$and $\xi(t)$ are co-estate variables. The first order conditions are: $\frac{\partial CVH}{\partial c(t)} = 0$; $\frac{\partial CVH}{\partial \zeta(t)} = 0$; $\frac{\partial CVH}{\partial J(t)} = -\dot{\nu}(t) + \rho\nu(t)$; $\frac{\partial CVH}{\partial Z(t)} = -\dot{\xi}(t) + \rho\xi(t)$. The transversality conditions are: $\lim_{t\to\infty} \nu(t)J(t)e^{-\rho t} = 0$ and $\lim_{t\to\infty} \xi(t)Z(t)e^{-\rho t} = 0$.

The first order conditions give respectively:

$$\nu(t) = \frac{\eta_R}{c(t)} \quad (37)$$

$$\xi(t) = \frac{\nu(t)}{\delta\eta_R} \quad (38)$$

$$\frac{\nu(t)AR(j,t)}{\eta_R} - \frac{\nu(t)R(j,t)}{\eta_R} - \frac{\omega}{J(t)} = -\dot{\nu}(t) + \rho\nu(t) \quad (39)$$

$$\frac{\omega\beta}{Z(t)} = -\dot{\xi}(t) + \rho\xi(t) \quad (40)$$

Eq (37)-(40) and (5) give the general conditions of the centralized optimum:

$$J(t) = \frac{\left(1 - \frac{1}{A} - \omega\gamma\right)}{\eta_R\rho\left(\frac{\omega\gamma\beta}{\omega\gamma\beta - \alpha}\right)} Y(t)$$

$$Z(t) = \frac{\delta(\omega\gamma\beta - \alpha)}{\rho} Y(t)$$

$$R(j,t) = \left[\frac{(\frac{\alpha}{A} + \alpha\omega\gamma - \alpha\omega\beta\gamma + \omega\beta\gamma - \omega\beta\gamma^2 - \alpha)}{(\omega\beta\gamma - \alpha)(1 - \frac{1}{A} - \omega\gamma)}\right]\rho\eta_R$$

The complete deduction of these conditions is in Appendix B.

A specific environmental policy may guarantee that the decentralized equilibrium is optimal. The derivation of the conditions for that to be verified is also in Appendix B. If:

$$\tau(0) = \frac{\mu\alpha\delta^2 A\left(1+\frac{\mu-1}{Q}\right)[\alpha(A-1)+\omega\beta\gamma]}{\beta\alpha A+\mu\alpha\delta^2\left(1+\frac{\mu-1}{Q}\right)[\alpha(A-1)+\omega\beta\gamma]} Z(0)^{\beta} \tag{41}$$

and σ is the implicit solution of the equation

$$(1-\sigma) = \frac{x}{A} + \frac{\beta x\rho\eta_R\gamma\left[\mu(A-x)(1+\frac{\mu-1}{Q})\right]^{\frac{\mu}{\mu-1}}}{\left[\beta x-\mu\alpha\delta^2(A-x)\left(1+\frac{\mu-1}{Q}\right)\right](1-\mu)AD(t)N}(1-\sigma)^{\frac{1}{1-\mu}}$$

where

$$x = \frac{\mu\delta A\left(1+\frac{\mu-1}{Q}\right)[\omega\beta\gamma+\alpha(\delta-1)]}{\beta+\mu\delta\left(1+\frac{\mu-1}{Q}\right)[\omega\beta\gamma+\alpha(\delta-1)]}$$

the decentralized growth rate will be optimal.

6. Conclusions

In this paper we have considered a simple endogenous growth model with horizontal innovation in the natural resource sector. Firstly, we studied the decentralized steady-state equilibrium growth path, analyzing the conditions under which growth is positive. We highlighted some of the most important features of the steady-state equilibrium. Finally, we analyzed the effects of environmental policy on the most crucial variables and their growth rates.

We showed that, if the allocation of resources among the two technological options in the economy is balanced, a cleaner environment is compatible with a growing economy. Our model has two policy tools which can be jointly used: a tax on emissions and a subsidy to final goods consumption. Moreover, the revenues from the emission tax can be used totally or partially to finance the subsidy. By analyzing the results obtained for the variations of the subsidy and of the emission tax we derived some policy implications.

If the efficiency of knowledge to reduce pollution and the environmental policy are sufficiently high, a higher tax will decrease steady-state emissions. Additionally, a higher subsidy will also help to reduce steady-state emissions if the efficiency of knowledge to reduce pollution is sufficiently high. This happens because a higher subsidy stimulates both final goods demand and the investment in pollution-reducing knowledge. The first effect increases emissions while the second decreases them. For a suitably high efficiency of knowledge to reduce pollution, the second effect will dominate.

Our analysis shows that if the Government imposes a higher tax on emissions the negative effect on the output level may be offset, at least partially, by a higher subsidy to final good consumption. Under certain conditions, $A > \frac{1}{\alpha(1-\mu)}$, the same applies to the stock of knowledge and the number of resource varieties in steady-state. Nevertheless, the economy will always grow more slowly if a higher tax is imposed. At the same time, emissions will also grow more slowly.

We have also derived the conditions to impose on the policy tools such that the equilibrium is optimal.

We intend to conduct further research with this model, considering optimality conditions and the steady-state optimal path and comparing it with the equilibrium. The transitional dynamics of the model would also be an interesting subject of future research. Other interesting extensions would include a modification in the resource sector which takes into account both polluting and non-polluting resources, for example.

Appendix A

Environmental Policy effects

We analyze the signs of the derivatives of the main variables with respect to the subsidy and the emission tax.

Subsidy

$$\frac{\partial Y(q_n,t)}{\partial\sigma}=\frac{AD(t)\alpha\left[\frac{(A-x)\mu}{(1+A\alpha\mu)(1-\sigma)}\right]^{1+\frac{1}{1-\mu}}}{(A-x)\gamma(\mu-1)^2}>0\ (since A>x)$$

$$\frac{\partial J(Y(q_n,t),t)}{\partial\sigma}=\underbrace{\frac{\partial J(Y(q_n,t),t)}{\partial Y(q_n,t)}}_{>0\ if\ \alpha<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}}*\underbrace{\frac{\partial Y(q_n,t)}{\partial\sigma}}_{>0}>0\ if\ \alpha<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}$$

$$\frac{\partial \mathrm{Z}(J(t),\mathrm{t})}{\partial\sigma}=\underbrace{\frac{\partial \mathrm{Z}(J(t),\mathrm{t})}{\partial \mathrm{J}(\mathrm{t})}}_{>0}*\underbrace{\frac{\partial J(Y(q_n,t),t)}{\partial\sigma}}_{>0\ if\ \alpha<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}}>0\ if\ \alpha<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}$$

$$\frac{\partial E(Y(q_n,t),Z(t))}{\partial\sigma}$$

$$=\underbrace{\frac{\partial E(Y(q_n,t),Z(t))}{\partial Y(q_n,t)}}_{>0}*\underbrace{\frac{\partial Y(q_n,t)}{\partial\sigma}}_{>0}+\underbrace{\frac{\partial E(Y(q_n,t),Z(t))}{\partial Z(t)}}_{<0}$$

$$*\underbrace{\frac{\partial \mathrm{Z}(J(t),\mathrm{t})}{\partial\sigma}}_{>0\ if\ \alpha<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}}<0\ if\ \alpha$$

$$<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}\ and\ \frac{\partial E(Y(q_n,t),Z(t))}{\partial Z(t)}*\frac{\partial \mathrm{Z}(J(t),\mathrm{t})}{\partial\sigma}$$

$$>\frac{\partial E(Y(q_n,t),Z(t))}{\partial Y(q_n,t)}*\frac{\partial Y(q_n,t)}{\partial\sigma}$$

To see what the conditions on the parameters that we need to impose are, we compute the derivative:

$$\frac{\partial E(t)}{\partial \sigma}$$

$$= \frac{(A-x)(\beta-1)\delta\mu\rho\left[\frac{D(t)N[-A\alpha\delta^2\mu + x(\beta+\alpha(A\beta+\delta^2)\mu)]\left[\frac{(x-A)\mu}{(1+A\alpha\mu)(\sigma-1)}\right]^{\frac{1}{1-\mu}}}{(A-x)\gamma\delta\mu\rho}\right]^{1-\beta}}{A(1-\mu)[-A\alpha\delta^2\mu + x(\beta+\alpha(A\beta+\delta^2)\mu)](\sigma-1)}$$

$$< 0 \; if \; \beta > 1 \; and \; \alpha < \frac{\beta x}{\mu[\delta^2(A-x) - A\beta x]}$$

Emission tax

Since in steady-state $\tau(t)Z(t)^{-\beta}$ is constant over time and equal to $\tau(0)Z(0)^{-\beta}$ (x), we perform the derivatives with respect to x knowing that it will have the same sign as the derivative with respect to $\tau(0)$.

$$\frac{\partial Y(q_n,t)}{\partial x} = -\frac{AD(t)\alpha\mu\left[\frac{(A-x)\mu}{(1+A\alpha\mu)(1-\sigma)}\right]^{\frac{1}{1-\mu}}}{(A-x)\gamma(\mu-1)^2(1+a\alpha\mu)} < 0$$

$$\frac{\partial J(Y(q_n,t),t)}{\partial x} = \underbrace{\frac{\partial J(t)}{\partial x}}_{<0 \; if \; A > \frac{1}{\alpha(1-\mu)}} + \underbrace{\frac{\partial J(Y(q_n,t),t)}{\partial Y(q_n,t)}}_{>0 \; if \; \alpha < \frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}} * \underbrace{\frac{\partial Y(q_n,t)}{\partial x}}_{<0} < 0 \; if \; \alpha$$

$$< \frac{\beta x}{\mu[\delta^2(A-x) - A\beta x]} \; and \; A > \frac{1}{\alpha(1-\mu)}$$

$$\frac{\partial Z(J(t),t)}{\partial x} = \underbrace{\frac{\partial Z(t)}{\partial x}}_{<0 \; if \; A > \frac{1}{\alpha(1-\mu)}} + \underbrace{\frac{\partial Z(J(t),t)}{\partial J(t)}}_{>0} * \underbrace{\frac{\partial J(Y(q_n,t),t)}{\partial x}}_{<0 \; if \; A > \frac{1}{\alpha(1-\mu)} \; and \; \alpha < \frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}} < 0 \; if \alpha$$

$$< \frac{\beta x}{\mu[\delta^2(A-x) - A\beta x]} \; and \; A > \frac{1}{\alpha(1-\mu)}$$

Thus, we may write:

$$R(j,q_n,t) = \frac{Y(q_n,t)}{J(t)A} = \frac{\rho\eta_R}{\left[\frac{1}{\mu} - \frac{\alpha\delta^2(A-x)\left(1+\frac{\mu-1}{Q}\right)}{\beta x}\right](1-\mu)QNA}$$

From where:

$$\frac{\partial R(t)}{\partial x} = \frac{\alpha\delta^2\left(1+\frac{\mu-1}{Q}\right)\beta\eta\mu^2\rho}{NQ(\mu-1)\left[A\alpha\delta^2\left(1+\frac{\mu-1}{Q}\right)\mu - x(\beta+\alpha\delta^2\left(1+\frac{\mu-1}{Q}\right)\mu)\right]^2} < 0$$

$$\frac{\partial E(Y(q_n,t),Z(t))}{\partial x}$$

$$= \underbrace{\frac{\partial E(Y(q_n,t),Z(t))}{\partial Y(q_n,t)}}_{>0} * \underbrace{\frac{\partial Y(q_n,t)}{\partial x}}_{<0} + \underbrace{\frac{\partial E(Y(q_n,t),Z(t))}{\partial Z(t)}}_{<0}$$

$$* \underbrace{\frac{\partial Z(J(t),t)}{\partial x}}_{<0\ if\ A>\frac{1}{\alpha(1-\mu)}\ and\ \alpha<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}} > 0\ if\ \frac{\partial E(Y(q_n,t),Z(t))}{\partial Z(t)} * \frac{\partial Z(J(t),t)}{\partial x}$$

$$> \frac{\partial E(Y(q_n,t),Z(t))}{\partial Y(q_n,t)} * \frac{\partial Y(q_n,t)}{\partial x}$$

We compute:

$$\frac{\partial E}{\partial x} = \left\{ D(t)^2N^2\left(\underbrace{\underbrace{x(\beta-1)\left(\beta+\alpha\left(A\beta+\delta^2\right)\mu\right)}_{>0\ if\ \beta>1} + A\left(\underbrace{\alpha\delta^2\mu(1-\beta)+\beta^2(\mu-1)(1+A\alpha\mu)}_{<0}\right)}_{<0\ if x>\frac{A\left[\alpha(\beta-1)\delta^2\mu+\beta^2(1-\mu)(1+A\alpha\mu)\right]}{(\beta-1)\left[\beta+\alpha(A\beta+\delta^2)\mu\right]}}\right)\right.$$

$$* \underbrace{\left(\frac{D(t)N\left(-A\alpha\delta^2\mu + x\left(\beta+\alpha\left(A\beta+\delta^2\right)\mu\right)\right)\left[\frac{(x-A)\mu}{(1+a\alpha\mu)(\sigma-1)}\right]^{\frac{1}{1-\mu}}}{(A-x)\gamma\delta\mu\rho}\right)^{-1-\beta}}_{>0\ if\ \alpha<\frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}}$$

$$\left. * \underbrace{\left[\frac{(x-A)\mu}{(1+a\alpha\mu)(\sigma-1)}\right]^{\frac{2}{1-\mu}}}_{>0}\right\} * \underbrace{\frac{1}{(A(A-g)^2\gamma^2\delta(1-\mu)\mu\rho)}}_{>0} < 0\ if\ \beta > 1, \alpha$$

$$< \frac{\beta x}{\mu[\delta^2(A-x)-A\beta x]}\ and\ x > \frac{A\left[\alpha(\beta-1)\delta^2\mu+\beta^2(1-\mu)(1+a\alpha\mu)\right]}{(\beta-1)\left[\beta+\alpha(a\beta+\delta^2)\mu\right]}$$

$$\frac{\partial p}{\partial x} = \frac{1+A\alpha\mu}{(A-x)^2\mu} > 0$$

Since $r(t) = \frac{(1-\mu)Y(q_n,t)QN}{\mu\eta_R AJ(t)} = \frac{(1-\mu)QNR(j,q_n,t)}{\mu\eta_R}$:

$$\frac{\partial r\left(R(j,q_n,t)\right)}{\partial x} = \underbrace{\frac{\partial r(t)}{\partial R(j,q_n,t)}}_{>0} * \underbrace{\frac{\partial R(j,q_n,t)}{\partial x}}_{<0} < 0$$

In the same way, since $g_Y = r(t) - \rho$

$$\frac{\partial g_Y}{\partial x} = \underbrace{\frac{\partial r(t)}{\partial x}}_{<0} < 0$$

$$\frac{\partial g_E}{\partial x} = \underbrace{\frac{\partial g_E}{\partial g_y}}_{1-\beta \to >0\, if\ \beta>1} * \underbrace{\frac{\partial g_y}{\partial x}}_{<0} < 0\ if \beta > 1$$

Appendix B

To find the path for aggregate consumption ($c(t) = \sum_{n=1}^{N} c(n,t) = Nc(n,t)$) and the current value Hamiltonian of the social planner problem, we may manipulate Eq (1).

$$U = \int_0^\infty \left\{ \ln\left(\left[\sum_{n=1}^{N} c(n,t)^{\mu}\right]^{\frac{1}{\mu}} \right) - \omega ln(E(t)) \right\} . e^{-\rho t} dt$$

$$= \int_0^\infty \left\{ \ln\left(N^{\frac{1-\mu}{\mu}} c(t) \right) - \omega ln(E(t)) \right\} . e^{-\rho t} dt$$

The deduction of the first order conditions is straightforward. From Eq (37) one has $g_v = -g_c$. From Eq (38), one gets $g_\xi = g_v$. Dividing Eq (39) for $v(t)$ and using $g_v = -g_c$ we have:

$$\frac{AR(j,t)}{\eta_R} - \frac{R(j,t)}{\eta_R} - \frac{\omega}{J(t)v(t)} = g_c + \rho \tag{42}$$

Using Eq (37), the condition $c(t) = \gamma Y(t)$ and the aggregate production function, one has:

$$\frac{Y(t)}{J(t)\eta_R} - \frac{Y(t)}{AJ(t)\eta_R} - \frac{\omega\gamma Y(t)}{J(t)\eta_R} = g_c + \rho$$

which may be written as:

$$\frac{Y(t)}{J(t)\eta_R}\left(1-\frac{1}{A}-\omega\gamma\right)=g_c+\rho \tag{43}$$

From where $g_Y = g_J$.

Dividing Eq (40) for $\xi(t)$ and using $g_\xi = g_v = -g_c$ we have:

$$\frac{\omega\beta}{Z(t)\xi(t)}=g_c+\rho \tag{44}$$

Using Eq (37) and (38):

$$\frac{\omega\beta\delta\gamma Y(t)}{Z(t)}=g_c+\rho \tag{45}$$

From where $g_Y = g_Z$.

From Eq (43) and (45) one gets the relationship:

$$Z(t)=J(t)\frac{\eta_R\omega\beta\delta\gamma}{\left(1-\frac{1}{A}-\omega\gamma\right)} \tag{46}$$

From the aggregate production process of knowledge:

$$g_Z=\frac{\delta\zeta(t)}{Z(t)} \tag{47}$$

From where $g_\zeta = g_Z$.

Using Eq (45) and (47) and knowing that $\zeta(t) = \alpha Y(t)$ and $g_Y = g_Z$, one has:

$$g_c=\frac{\rho\alpha}{\omega\beta\gamma-\alpha} \tag{48}$$

The optimum will be characterized by:

$$J(t)=\frac{\left(1-\frac{1}{A}-\omega\gamma\right)}{\eta_R\rho\left(\frac{\omega\gamma\beta}{\omega\gamma\beta-\alpha}\right)}Y(t)$$

$$Z(t)=\frac{\delta(\omega\gamma\beta-\alpha)}{\rho}Y(t)$$

Using Eq (5), one has:

$$R(j,t)=\left[\frac{(\frac{\alpha}{A}+\alpha\omega\gamma-\alpha\omega\beta\gamma+\omega\beta\gamma-\omega\beta\gamma^2-\alpha)}{(\omega\beta\gamma-\alpha)(1-\frac{1}{A}-\omega\gamma)}\right]\rho\eta_R$$

In the optimum the growth rate of the economy is given by $g_c = \frac{\rho\alpha}{\omega\beta\gamma-\alpha}$. Eq (30) and (34) give the decentralized growth rate $g_c = \frac{\rho}{A}\left[\frac{\beta x}{\beta x-\mu\alpha\delta^2(A-x)\left(1+\frac{\mu-1}{Q}\right)} - 1\right]$. The condition for these growth rates to be equal is:

$$x = \frac{\mu\alpha\delta^2 A\left(1+\frac{\mu-1}{Q}\right)[\alpha(A-1)+\omega\beta\gamma]}{\beta\alpha A + \mu\alpha\delta^2\left(1+\frac{\mu-1}{Q}\right)[\alpha(A-1)+\omega\beta\gamma]}$$

We can also consider that $\frac{J(t)}{Y(t)}$ and $\frac{Z(t)}{J(t)}$ are equal in equilibrium and in the optimum. The condition for that to be true is:

$$x = \frac{\mu\delta A\left(1+\frac{\mu-1}{Q}\right)[\omega\beta\gamma+\alpha(\delta-1)]}{\beta + \mu\delta\left(1+\frac{\mu-1}{Q}\right)[\omega\beta\gamma+\alpha(\delta-1)]}$$

Since we have two policy instruments, we may impose those two conditions. The subsidy for the demand of differentiated goods is such that the price effectively paid by individuals to consume a unit of the differentiated good equals its marginal cost of production; that is,

$$(1-\sigma)p = \frac{\tau(0)Z(0)^{-\beta}p}{A} + \frac{\psi}{J(t)A} \tag{49}$$

where:

$$p = \frac{1}{\mu(A-x)(1+\frac{\mu-1}{Q})}$$

$$J(t) = \left[\frac{1}{\mu} - \frac{\alpha\delta^2(A-x)\left(1+\frac{\mu-1}{Q}\right)}{\beta x}\right]\frac{(1-\mu)Y(t)}{\rho\eta_R}$$

$$Y(q_n,t) = \frac{D(t)}{Q\gamma}\left[\frac{\mu(A-x)\left(1+\frac{\mu-1}{Q}\right)}{(1-\sigma)}\right]^{\frac{1}{1-\mu}}$$

Eq (49), may be written as:

$$(1-\sigma)\frac{1}{\mu(A-x)(1+\frac{\mu-1}{Q})}$$

$$=\frac{x}{A}\frac{1}{\mu(A-x)(1+\frac{\mu-1}{Q})}$$

$$+\frac{1}{\left\{\left[\frac{1}{\mu}-\frac{\alpha\delta^2(A-x)\left(1+\frac{\mu-1}{Q}\right)}{\beta x}\right]\frac{(1-\mu)Y(t)}{\rho\eta_R}\right\}\mu A}$$

$$\Leftrightarrow$$

$$(1-\sigma)=\frac{x}{A}+\frac{\beta x\rho\eta_R\gamma\left[\mu(A-x)(1+\frac{\mu-1}{Q})\right]^{\frac{\mu}{\mu-1}}}{\left[\beta x-\mu\alpha\delta^2(A-x)\left(1+\frac{\mu-1}{Q}\right)\right](1-\mu)AD(t)N}(1-\sigma)^{\frac{1}{1-\mu}}$$

σ will be the implicit solution of the equation where

$$x=\frac{\mu\delta A\left(1+\frac{\mu-1}{Q}\right)[\omega\beta\gamma+\alpha(\delta-1)]}{\beta+\mu\delta\left(1+\frac{\mu-1}{Q}\right)[\omega\beta\gamma+\alpha(\delta-1)]}$$

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