An endogenous timing model with heterogeneous beliefs *

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March 9, 2012

Abstract

This paper analyzes how decisions about the timing of entry in markets are affected by firms' beliefs about the state of the world. A second contribution of the paper is to study whether firms have incentives to become optimistic. To do that the paper considers an endogenous timing model with incomplete information about demand. We show that with Bayesian firms there exists a unique perfect Bayesian equilibrium where firms with optimistic beliefs produce in the first period while firms with pessimistic beliefs only produce in the second period. We also find that that when firms could choose to be overconfident they choose not to be. Nevertheless, they are weakly better by having that option, despite not using it.

Keywords: Optimism, Strategic Delegation, Endogenous Timing

JEL Classification: C72, D03, D21, D43, D82, L10, L20

*We are particularly thankful to Luis Santos-Pinto because the paper builds on some earlier joint work and some of the main ideas of this paper have been proposed by him. We are also grateful to Eddie Dekel, Marciano Siniscalchi, William Rogerson and Marco Ottaviani for useful comments and discussions

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1 Introduction

The failure of firms and congestion in markets is often associated with firms' optimism and lack of information at the time of entry. This idea is strengthened by results of some experiments[1] that found that optimism and imperfect information can lead to excessive and too early market entry. These findings suggest that firms' perceptions and beliefs about the state of the world affect their entry decisions and competition behavior in markets. Empirical evidence also shows that executives are particular prone to display optimism[2] and that affects their decisions. In particular, Malmendier and Tate (2003, 2005a, 2005b) have shown that executives' optimism affects the firms investment decisions and the cash flows sensitivity. Glaser, Schafers and Weber (2008), using data from Germany, show that firms with optimistic managers invest more. In particular, CEO optimism explains larger capital expenditures while optimism of all managers increases the probability of acquisition (but CEO optimism alone does not). Therefore, there is a need to understand the effects of optimism in market decisions.

Our main research question is how heterogeneous beliefs affect the timing of market entry. We also analyze possible sources of heterogeneous beliefs and how the outcomes of the model differ in each of these explanations. The explanations analyzed rely on the idea that firms can choose to be optimistic (or pessimistic). Therefore, our paper also answers how optimism (or pessimism) affects entry and whether firms have incentives to become optimistic.

In our framework one can see the firm's problem as the decision of an entrepreneur or a manager that chooses all the firm's actions. Therefore, our paper is related with the literature on strategic delegation since our model enables to evaluate the benefits and losses of hiring managers with different beliefs about the state of the world or managers that are intrinsically more optimistic or pessimistic than others. Given the importance of this literature, without loss of generality, we will interpret the results as obtained from a model where firm's decision are made by a manager that maximizes firm value.[3]

[1] See Camerer and Lovallo (1999) and Brocas and Carrillo (1999)

[2] See for example Langer (1975), Larwood and Whittaker (1977), Weinstein (1980), March and Shapira (1987).

[3] We could also have interpreted our results as obtained from a model where decisions are made by internal organization structures where a specific behavior or beliefs emerge as dominants. In any case, since we are interested in firm behavior in markets rather than on the internal organization of firms, the specific decision process within the firm is not relevant as long as the decision maker maximizes the firm value. Therefore, we assume the inexistence of Principal-Agent problems within the firm.

The proposed framework extends the endogenous timing model of Hamilton and Slutsky (1990) where two players compete in quantities and must decide whether to enter the market at date 1 or at date 2. Our model departs from the standard framework by assuming that firms have incomplete information about demand and by modeling the source of heterogeneous beliefs.

We pursue our analysis in a sequential way. In a first step, we consider that firms are completely uniformed about demand. They only know that demand can be high or low. Firms are Bayesian and so they have subjective beliefs about the value of demand. We allow for the possibility of firms having different beliefs and so they might "agree to disagree".

In a second step, we analyze why firms have different beliefs. We do so by adding a stage previous to the entry decisions. In that stage firms can take actions that affect their posterior beliefs. We consider two different frameworks to model this extra stage.

The first one builds in the model proposed by Brunnermeier and Parker (2005) and it assumes that firms can choose subjective beliefs. These beliefs might differ from the objective beliefs (priors) and they will be used in the entry game. In this model agents choose beliefs in order to maximize happiness and so firms have subjective beliefs that are optimal but might be incorrect. This is a depart from the usual rational expectations assumption. We call it the Optimal Expectations model.

The second framework extends the model proposed in Benabou and Tirole (2002). We assume there exists a period 0 where firms receive a signal about the true demand. Firms have access to a mechanism that allows them to forget the received signal. We call it the Overconfidence model. These two frameworks enable to view the model as a model where firms could choose to be optimistic or pessimistic.

The paper shows that with Bayesian firms there exists an unique perfect Bayesian equilibrium. In that equilibrium the firms with more positive beliefs about the state of the world produce in the first period while firms with more negative beliefs about the state of the world only produce in the second period. Therefore, the proposed model is consistent with the empirical evidence of excessive entry and earlier entry of firms with more optimistic managers or entrepreneurs. These results could be extended to an environment where firms with more negative beliefs about the state of the world are defined as firms that follow a maxmin rule. We show that, under the suitable assumptions, this equilibrium fits with an equilibrium derived from an Optimal Expectations Model.

We also find that when firms receive a signal about the true value of demand, but one of the firms has access to a mechanism that allows to forget that signal, equilibrium outcomes satisfying forward induction are such that in the equilibrium path the firm with the mechanism to forget does not use it. The firm that could be optimistic chooses not to be. Nevertheless, the firm with the mechanism always moves first, and so is weakly better by having the mechanism, despite not using it. One interpretation of this result is that firms with the possibility of hiring or delegating firm's decisions to an optimistic manager are better than firms without this possibility, even if this delegation never occurs.

Our paper is essentially related with three topics of economic literature: endogenous timing decisions, optimism and strategic delegation. In classical industrial organization firms may play simultaneous or sequentially, but the choice of the game played by firms is normally taken as an assumption and not as a choice of firms. Endogenous timing models go beyond this weakness of classical models by endogeneizing the decision of playing a simultaneous or a sequential game. The seminal paper in the endogenous timing literature is Hamilton and Slutsky (1990)[4]. The authors propose a model with two players who must decide a quantity to be produced in one of two periods, before the market clears. If a player commits to a quantity in the first period, she acts as a leader but she does not know the other player's decision. If a player waits until the second period to commit, then she observes the action of the other player in the first period.

Endogenous timing models have been explored by many papers in recent years. The literature has tried to find and establish conditions which might lead firms to play either a sequential-move Stackelberg game or a simultaneous-move Cournot game. Some examples are Branco (2008), van Damme and Hurkens (1999, 2004) and Normann (2002). van Damme and Hurkens (1999, 2004) analyze endogenous timing in a duopoly model where firms have different marginal costs and compete in quantities (1999)/prices (2004). They find that, with risk dominance considerations, the efficient firm moves first, while the inefficient firm waits until the second period either for quantity or price competition. The model with different marginal costs is extended by Branco (2008) who considers that firms are privately informed about their costs. He finds that in the informative perfect Bayesian equilibrium, a firm with a low cost produces in the first period, while

[4] Despite we are considering Hamilton and Slutsky (1990) as the seminal paper in endogenous timing literature, we should also mention Gal-Or (1987) which analyzes first versus second mover advantages.

a firm with a high cost produces in the second period, after learning the other firm's decision in the first period. Another important paper (and closely related to ours) that extends the literature on endogenous timing models is Normann (2002). Normann analyzes the Hamilton and Slutsky's endogenous timing model with action commitment in a duopoly with incomplete information, in which one firm knows the state of the demand while the other remains uninformed. He finds that the Cournot equilibrium in the first period and the Stackelberg equilibria with either the informed or the uniformed firm as Stackelberg leader emerge as outcome of that game. Our paper endogeneizes the difference in the information structure and refines the results of Normann.

Our paper is also related with another important topic in economics: optimism. Optimism is something natural to human behavior and has been identified as a fundamental human impulse. Hence, we should expect economic decisions and interactions to be affected by it. In fact Heifetz, Shannon and Spiegel (2007) show that in a large class of strategic interactions the equilibrium payoffs of optimists may be higher. This happens because the optimism of one of the players leads the adversary to change equilibrium behavior, possibly to the benefit of the optimistic player. This paper proposes that optimism may appear as tendency which takes over. Consequences of optimism have been recently formalized, among others[5], by Benabou and Tirole (2002) and Brunnermeier and Parker (2005). We use the work of these authors and apply it in the information structure of an endogenous entry decision game.

Benabou and Tirole (2002) propose a general economic model to explain why people value their self-image and how they seek to forget or preserve it through a variety of seemingly irrational behaviors. The basic idea behind the model is that individuals can affect the probability of remembering information, particularly they have "costly" mechanisms that allow them to forget bad signals and recall good news, whenever that is optimal. The ideas of selective memory or awareness management are extended by Benabou (2009) to develop a general model of groupthinking. This model tries to understand how wishful thinking and reality denial spread through organizations and markets.

Brunnermeier and Parker (2005) propose a structural model where agents can choose their beliefs in order to maximize their happiness[6]. In particular, their model assumes that before

[5] See Kyle and Wang (1997), Benabou and Tirole (1999,2006), Odean (1999), Barber and Odean (2001), Scheinkman and Xiong (2003) and Grubb (2008)

[6] Brunnermeier and Parker (2005) define happiness as the sum of the actual and future flow utilities.

choosing their actions, agents choose the beliefs that maximize their lifetime happiness and these are the beliefs used to choose subsequent actions. Therefore, in this model agents have subjective beliefs that are optimal but might be incorrect. The model allows to predictions that are opposite to two classical assumptions in economic literature: the share of a common prior by agents and the rational expectations assumption.

The interaction between the previous topics was explored by Pires and Santos-Pinto (2008). Their paper considers an endogenous timing model with two firms where one is optimistic about its costs. They found that for moderate levels of optimism there is a unique cost-dependent equilibrium where the optimistic player has a higher ex-ante probability of being the leader than the rational player. In this equilibrium optimism reduces the profits of the rational player but can increase the profits of the optimistic player, provided that cost asymmetries are small.

Also closely related to our paper, De Meza and Southey (1996) propose a model with optimistic entrepreneurs to rationalize some of the stylized facts of small-scale businesses. In their model most of the facts characterizing small-scale businesses, including high failure rates, reliance on bank credit rather than equity finance and credit rationing, can be explained by a tendency for those who are excessively optimistic to dominate new entrants.

The seminal literature on strategic delegation (Vickers, 1985) analyzed how firms can strategically distort their managers' compensation contract away from profit maximization. Recently, some authors have analyzed the benefits and losses of employing managers with irrational behavior or different beliefs about the state of the world. Eichberger, Kelsey and Schipper (2005) find that under ambiguity optimistic or pessimistic responses to ambiguity affect behavior. Englmaier (2007) shows that it may be optimal for a firm to employ an optimistic manager because that can serve as credible commitment to get a competitive edge over the competitors.

The remainder of the paper is organized as follows. In section 2 we present the basic model. Section 3 describes and analyzes the model with unknown demand and Bayesian firm. Sections 4 and 5 provide an explanation to the heterogeneous beliefs that firms might have when the entry game starts. In section 4 we offer an explanation to the heterogeneous beliefs by extending the model proposed by Brunnermeier and Parker (2005) while in section 5 we extend the model by assuming that firms receive a signal about demand but could forget it. Section 6 concludes the paper. All proofs are in the appendix.

2 Model

We consider two firms which produce a homogeneous good. Firms produce with zero marginal costs and have no fixed costs. The price of the good is given by $P(q_A + q_B) \equiv \max\{\theta^* - q_A - q_B, 0\}$ where $\theta^* \in \{1, \theta\}$ and $\theta > 1$. Firms maximize expected utility and are Bayesians according to Savage (1954) axiomatic foundations, that is, they make their choice using subjective probability distributions. There are only two subjective probability distributions: one with all mass in 1 and the other with all mass in θ. We denote a firm with the latter subjective probability by High Belief firm (H) and a firm with the former subjective probability by Low Belief firm (L). Each type of firm is uniquely defined by their subjective probability distribution on θ^*, so the types of firms are $\tau_i \in \{H, L\}$.

The paper considers an endogenous timing model with action commitment as in Hamilton and Slutsky (1990). Firms must decide a quantity to be produced at one of two dates. In the first period firms decide simultaneously whether to produce or not. If a firm does not produce at date 1, it must decide its level of production at date 2. Finally, at date 3, given the production decisions, the market clears.

For a firm there is a clear trade-off between the timing decisions. By producing in the first period a firm gets the possibility of becoming the leader and, in that way, influence the other firm's decision. However, by choosing the first period to produce, a firm cannot observe the other firm's decision and obtain information from that decision. Furthermore, there is the risk that the opponent also produces at date 1. On the other hand, by waiting until the second period a firm cannot influence the opponent's decision but it will have more information when it takes its decision since it can observe the quantity chosen by the rival, or the rival's decision to wait.

Summing up,the timing of the model is:

Period -1: Nature draws θ

Period 0: Each firm receives information about θ and takes an action with effect in the posterior beliefs

Period 1: Firms decide whether to commit to a particular quantity or to wait

Period 2: A firm which has not produced at date 1 decides its quantity at date 2.

Period 3: Market clears

3 Bayesian framework

In this section we consider that the game starts in period 1. Firms make their choice under "complete ignorance", that is, they only know that $\theta^* \in \{1, \theta\}$. We assume that firms maximize expected utility and they are bayesian, i.e., they have a subjective probability distribution P_i on θ^*. There are only two subjective probability distributions: one with all mass in 1 and the other with all mass in θ. We assume firms might "agree to disagree", that is, firms might have different subjective beliefs and they are aware of that. Both types of firm have a common prior about the type of the other firm. Let λ be the belief each firm has that the other firm is a High Belief firm.

In this section a pure strategy for each firm is a choice of a production period $\bar{t}_i \in \{1, 2\}$ and a set of functions $\vartheta_i : \{(\bar{t}_i = 1, \bar{t}_j = 1), (1, 2), (2, 1) \times \mathbb{R}^+, (2, 2)\} \rightarrow \mathbb{R}^+$ which is firm's quantity choice as a function of production periods, and q_j if it is a Stackelberg follower. We assume that given the decisions to produce in period 1 or 2, firms will not mix over outputs[7]. Let $\mu_i^t(.|\theta^*)$ be the beliefs of firm i in period t conditional on θ^* and observed variables. The beliefs in period 1 are defined as $\nu_i^1(\theta^*, \bar{t}_j, \vartheta_j) = P_i(\theta^*) \times \mu_i^1(\bar{t}_j, \vartheta_j|\theta^*)$ while the beliefs in period 2 for a player that chooses to wait in period 1 are $\nu_i^2(\theta^*, \vartheta_j|\bar{t}_j = 2) = P_i(\theta^*) \times \mu_i^2(\vartheta_j|\bar{t}_j = 2, \theta^*)$ if $\bar{t}_j = 2$ and $\nu_i^2(\theta^*|\bar{t}_j = 1) = P_i(\theta^*)$ if $\bar{t}_j = 1$. According to our definitions

$$\begin{aligned} P_i(\theta^* = 1) &= 1 \text{ if } \tau_i = L \\ P_i(\theta^* = \theta) &= 1 \text{ if } \tau_i = H \end{aligned}$$

In proposition 1, we will show that there exists a perfect bayesian equilibrium where the High Belief firm produces in the first period while the Low Belief firm produces in the second period. Next, in proposition 2 it is shown that the previous equilibrium is the unique perfect bayesian equilibrium in pure strategies.

Proposition 1: *If $\lambda \geq \lambda^*(\theta)$, then there exists an equilibrium with beliefs*

$$\mu_i^1\left(\bar{t}_j = 1, \vartheta_j = \frac{\theta - \frac{1-\lambda}{2}}{2\lambda + 1}\right) = \lambda,$$

[7] Given this definition a mixed strategy is a randomization over production periods.

$$\mu_i^1\left(\bar{t}_j = 2, \vartheta_j = \frac{1 - q_{1i}}{2}\right) = 1 - \lambda$$

and

$$\mu_i^2\left(\vartheta_j = \frac{1}{3}|\bar{t}_j = 2\right) = 1$$

for $i, j = 1, 2$ *in which the firms will have the following strategies:*

1. *If firm is* High Belief *then* :

   (a) *It produces* $q_H = \frac{\theta - \frac{1-\lambda}{2}}{2\lambda+1}$ *at date 1;*

   (b) *If it did not produced at date 1 and the other firm produced* $\bar{q}$ *at date 1, it would produce at date 2 according to* $q_H = \frac{\theta - \bar{q}}{2}$*;*

   (c) *If both firms did not produced at date 1, then it would produce* $q_H = \frac{3\theta-1}{6}$ *at date 2;*

2. *If firm is* Low Belief t*hen*

   (a) *It produces at date 2;*

   (b) *If it were to produce at date 1, it would produce* $q_L = \frac{1+4\lambda-2\lambda\theta+\lambda^2}{6\lambda+4\lambda^2+2}$*;*

   (c) *If the other firm produces* $\bar{q}$ *at date 1, it will produce at date 2 according to* $q_L = \frac{1-\bar{q}}{2}$*;*

   (d) *If both firms do not produced at date 1, then it will produce* $q_L = \frac{1}{3}$ *at date 2;*

Proposition 2: *The equilibrium described in proposition 1 is the unique Perfect Bayesian equilibrium in pure strategies*

Propositions 1 and 2 imply that when λ is sufficiently high, in a Perfect Bayesian equilibrium, a High Belief firm produces in the first period while a Low Belief firm produces in the second period. These results are consistent with the evidence found by some experiences that more optimistic players move first. High Belief firms produce in the first period because their expected gain from the first-mover advantage is higher than the expected loss of a Stackelberg war. On the other hand, for Low Belief firms the inverse applies. This difference is explained by the different priors about market size. Note that the first-mover gain and the loss from Stackelberg war are proportional to the market size. Thus, differences in beliefs about the market size lead to different expected gains and losses.

Results described in proposition 1 and 2 are conditional on a sufficiently high value of λ. The intuition is the following. Suppose λ is low, that is, for each firm the belief that the other firm is a Low Belief firm is high. In this case, a Low Belief firm has a large incentive to deviate from the strategy proposed in proposition 1, because if it deviates, there is a high probability of becoming a Stackelberg leader instead of playing a Cournot game. Thus, there exists a high probability of achieving higher profits through the first mover advantage. On the other hand, the possible loss associated with a Stackelberg war has a low probability. Therefore, with a low λ the gains from deviating are higher than the losses.

As shown in the appendix the cutoff value of λ that enables the proposed strategy to be an equilibrium is increasing with θ. The basic idea is that the quantity produced by a High Belief firm in first period is increasing in θ. Therefore, if a Low Belief firm deviates and produces in the first period, the cost of a Stackelberg war will be higher for it. Thus, a higher θ reduces the Low Belief firms' incentive to deviate. The table in the appendix shows that θ does not need to be very large in order to obtain an equilibrium for a large range of values of λ. For example, if the priors of both types of firms differ in 25% then there exists an equilibrium for values of λ higher than 0.17. So, we think that the prediction that an High Belief firm moves first is robust. In the appendix we also show that this result is generalizable for other non linear demand specifications.

Corollary 1 *If $\lambda \geq \lambda^* (\theta)$ and firms have different beliefs, then a firm with high beliefs becomes Stackelberg leader.*

All in all, the results in proposition 1 and 2 suggest that in a world where managers have different beliefs about the value of demand, firms with managers with a more positive view of the world will enter earlier in the market, as long as the market players have a sufficiently high belief that the other competitor has a positive view of the world. Firms with a manager with a high belief about demand produce a larger quantity than firms with a manager with a low belief about demand. The larger production is explained by two reasons: (i) the expectation of a greater demand and (ii) the earlier entry and consequent Stackelberg leadership advantage. These results are in line with some of the findings in Malmendier and Tate (2008) and Glaser, Schafers and Weber (2008) where it is shown that more optimistic managers invest more.

3.1 Robustness of results to beliefs specification

In this subsection we analyze the sensitivity of the results to beliefs specification.

Let a Low Belief firm to be defined in the same way. The previous results can be extended to a model where the High Belief firm is defined as a firm with a positive prior of state θ, but not necessarily with all mass at θ. For example, let p be the subjective probability that state is θ and $1-p$ the subjective probability that state is 1, with $p>0$. So, the mean belief of a High Belief firm is $\bar{\theta}_H = E_O(\theta^*) = p(\theta-1)+1$. Replacing θ by $\bar{\theta}_H$ in Proposition 1 and 2 and respective proofs, they can be applied to this new framework.

The last example shows that our results about the timing of entry can be extended to more generic models where firms have beliefs that are nondegenerate distributions on θ^*. For example, another possible generalization of the standard framework is to define the Low Belief type as the type with the lowest mean belief and the High Belief type as the type with the highest mean belief. From this generalization we obtain the following corollary from proposition 1 and 2:

Corollary 2 *If the distribution characterizing the beliefs of one type of firm first order stochastically dominates the distribution of the other type of firm, then in a Perfect Bayesian Equilibrium the latter type produces in the second period while the former produces in the first period.*

Finally, let us relax the assumption that the two types of firms are Bayesian. Suppose, one type is Bayesian with a prior $\{1-p,p\}$ on $\{1,\theta\}$ where $p>1$. The other type follows a maxmin criterion. It is easy to check that the problem of the type that follows a maxmin criterion is equal to the problem of a Bayesian firm with a subjective probability with all mass in 1. So, one more time the results of proposition 1 and 2 can be extended to this framework (given the definition of High Belief firm we only need to replace θ by $\bar{\theta}$) where the Low Belief firm is the firm following the maxmin criterion.

The previous examples show that the model might be extended to a more broad class of situations. Therefore we have confidence that the results are applicable in many situations. The model does not crucially depend on the extreme assumptions that we made on the beliefs.

3.2 Welfare Analysis

Welfare analysis in this model is ambiguous, since we assume that the true state of demand and firms' beliefs are unknown. Nevertheless, there are some comments that can be made.

We take the endogenous timing model with known demand of Hamilton and Slutsky (1990) as benchmark. It has three pure strategies equilibria: one Cournot equilibrium in the first period and two Stackelberg equilibria. The equilibrium outcome of our model with unknown demand depends on firms' beliefs and so we can have three different situations: one High Belief and one Low Belief firm, two High Belief firms, two Low Belief firms. We will present the welfare analysis for the three cases.

Case 1: one High Belief and one Low Belief firm

If the true value of θ^* is θ, then the High Belief firm is weakly better while the Low Belief firm is strictly worse than in the model with known demand. On the other hand, if the true value of θ^* is 1, then the Low Belief firm is also strictly worse but the result for the High Belief firm is ambiguous.

The intuition for these results is the following. When the Low Belief firm does not know the demand, it loses any possible first move advantage that it may have in the model with complete information. Furthermore, because the other firm is High Belief then it will overproduce and so the Low Belief firm has to decrease its production to avoid a large price decrease. Despite the reduction of the produced quantity by the Low Belief firm, the prices will decrease and so Low Belief firm's profits will be lower.

Concerning the High Belief firm we have that profits can increase or decrease. There are two effects with opposite directions. On one hand, in the model with unknown demand the High Belief firm has surely a first mover advantage. On the other hand, when θ^* is 1 the firm does an optimization mistake which implies losses. When the firm's belief coincides with the true value, then a more favorable view of the world increases the Stackelberg leader gain and so the High Belief firm is better. On the other hand, if firm's beliefs differs from the true value, the High Belief firm will have a loss due to the bias in judgement. Nevertheless, if the bias is not too high, the High Belief firm remains better because the strategic advantage is higher than the loss due to optimization mistake. On the other hand, the Low Belief firm is always worse off when the other firm is a High Belief firm.

Summing up, as pointed by Heifetz, Shannon and Spiegel (2007), being recognized as a firm with High Beliefs always gives rise to a strategic advantage.

Case 2: two Low Belief firms

When the two firms are Low Belief and they play according to the strategies described in proposition 1, the differences in firms' profits depend on the equilibrium outcome in the standard case and the true demand parameter. If $\theta^* = 1$, then profits with unknown demand are equal to Cournot profits in the standard case. If competition is *à la* Stackelberg with complete information, then the leader (with complete information) is worse with unknown demand and the follower is better.

When $\theta^* = \theta$ and there is Cournot competition with complete information, then firms are better with unknown demand if and only if $\theta \in [1, 2]$. On the other hand, if firm is a Stackelberg leader with unknown demand, then it is always worse with incomplete information while if it is a Stackelberg follower with unknown demand, it is better if $\theta \in \left(1, \frac{4\sqrt{2}+8}{3}\right)$

Case 3: two High Belief firms

Assume firms play according to the strategies described in proposition 1 but now let the two firms be High Belief. When $\theta^* = \theta$, firms are better with unknown demand if and only if they are a Stackelberg follower in the complete information equilibrium and $\lambda < 0.671\,57$ and $\theta \in \left(1, \bar{\theta}\right]$ or $\lambda > 0.671\,57$ and $\theta > \bar{\theta}$ where

$$\bar{\theta} = \frac{1}{4\lambda^2 - 28\lambda + 17}\left(2\sqrt{2}\sqrt{\left(-2\lambda^2 + \lambda + 1\right)^2} - 20\lambda + 8\lambda^2 + 12\right)$$

Otherwise, firms are worse with unknown demand.

When $\theta^* = 1$, firms are better with unknown demand if and only if they are Stackelberg follower in the complete information equilibrium and $\theta \in \left(1, \frac{1}{8}\left((2\lambda + 1)\sqrt{2} + 6\right)\right)$. Otherwise, firms are worse with unknown demand.

4 Optimal Expectations

The goal of the next two sections is to provide an explanation for the heterogeneous beliefs that firms may have when the entry game starts. In this section we propose an extension of the model proposed by Brunnermeier and Parker (2005), which provides a motivation for firms´ heterogeneous beliefs.

Suppose that in period 0 both firms have a common prior ρ on $\theta^* \in \{1, \theta\}$ with

$$\theta < \frac{1}{18\lambda}\bar{\theta}_R\left(24\lambda + 3 + 9\lambda^2 + (1 + 2\lambda)\sqrt{3}\sqrt{2\lambda + 7\lambda^2 + 3}\right)$$

where $\bar{\theta}_R$ is the expectation of θ^* given the common prior, i.e.,

$$\bar{\theta}_R = \rho(1) + \theta\rho(\theta)$$

Assume that there are two types of firms. One type maximizes expected utility using the objective beliefs, the rational type (R). The second type maximizes expected utility using the "optimal"[8] beliefs that maximize its well being[9], the Optimal Expectations type (OE). We restrict the subjective beliefs that a OE firm can choose to be the objective beliefs and the beliefs that give probability 1 to state $\theta^* = \theta$ [10], that is, $\hat{\pi} \in \{\rho, \bar{\pi}\}$ where $\bar{\pi}(\theta) = 1$.

We assume each firm does not know the type of the other, and that firms assign a probability λ to the possibility that the other firm is of OE type. We also assume that a OE firm knows that the beliefs of a R firm are the objective beliefs π. Thus, if a OE firm chooses subjective beliefs that differ from the objective beliefs, then firms will "agree to disagree".

In this model a pure strategy for each firm is a choice of a production period $\bar{t}_i \in \{1, 2\}$ and a set of functions $\vartheta_i : \{(\bar{t}_i = 1, \bar{t}_j = 1), (1, 2), (2, 1) \times \mathbb{R}^+, (2, 2)\} \rightarrow \mathbb{R}^+$ which is firm's quantity choice as a function of production periods, and q_j if firm is a Stackelberg follower. The pure strategy of a OE firm also includes the choice of the subjective beliefs given the objective beliefs.

We assume that given the decisions to produce in period 1 or 2, firms will not mix over outputs[11]. The beliefs in period 1 are defined as $\nu_i^1(\theta^*, \bar{t}_j, \vartheta_j) = P_i(\theta^*) \times \mu_i^1(\bar{t}_j, \vartheta_j|\theta)$ while the beliefs in period 2 for a player that chooses to wait in period 1 are $\nu_i^2(\theta^*, \vartheta_j|\bar{t}_j = 2) = P_i(\theta^*) \times \mu_i^2(\vartheta_j|\bar{t}_j = 2, \theta)$ if $\bar{t}_j = 2$ and $\nu_i^2(\theta^*|\bar{t}_j = 1) = P_i(\theta^*)$ if $\bar{t}_j = 1$.

In proposition 3, we show that there exists a perfect Bayesian equilibrium where a OE firm chooses to be optimistic and produces in the first period while a R firm produces in the second period.

[8] We consider the definition of "optimal" beliefs proposed in Brunnermeier and Parker (2005)

[9] As in Brunnermeier and Parker (2005), we define well being as the expected time-average of the happiness of the firm/manager.

[10] Notice that if we allow the subjective beliefs to be chosen over a continuous choice set, then our problem does not have solution. However, we could relax the assumption of a choice set only with two elements, because that assumption only simplifies the algebra.

A possible motivation to our assumption is to suppose that players know that only two possible distributions of states could exist

[11] Given this definition a mixed strategy is a randomization over production periods.

Proposition 3: *If* $\lambda \geq \lambda^*(\theta)$, *then there exists a subgame perfect equilibrium with beliefs*

$$\mu_i^1\left(\bar{t}_j = 1, \vartheta_j = \frac{\theta - \frac{1-\lambda}{2}}{2\lambda + 1}\right) = \lambda,$$

$$\mu_i^1\left(\bar{t}_j = 2, \vartheta_j = \frac{1 - q_{1i}}{2}\right) = 1 - \lambda$$

and

$$\mu_i^2\left(\vartheta_j = \frac{1}{3}|\bar{t}_j = 2\right) = 1$$

for $i, j = 1, 2$ *in which firms have the following strategies:*

1. If firm is OE :

(a) It chooses $\hat{\pi} = \bar{\pi}$

(b) It produces $q_{OE} = \frac{\theta - \bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}$ *at date 1;*

(c) If it did not produced at date 1 and the other firm produced $\bar{q}$ *at date 1, it would produce at date 2 according to* $q_{OE} = \frac{\theta - \bar{q}}{2}$*;*

(d) If both firms did not produced at date 1, then it would produce $q_{OE} = \frac{3\theta - \bar{\theta}_R}{6}$ *at date 2;*

2. If firm is R

(a) It produces at date 2;

(b) If it were to produce at date 1, it would produce $q_R = \frac{\bar{\theta}_R\left(4\lambda+\lambda^2+1\right)-2\theta\lambda}{6\lambda+4\lambda^2+2}$*;*

(c) If the other firm produces $\bar{q}$ *at date 1, it will produce at date 2 according to* $q_R = \frac{\bar{\theta}_R - q}{2}$*, at date 2;*

(d) If both firms do not produced at date 1, then it will produce $q_R = \frac{\bar{\theta}_R}{3}$ *at date 2;*

Proposition 3 is analog to proposition 1. The main message behind this proposition is that the equilibrium derived in section 3 fits under the suitable assumptions with an equilibrium derived from an Optimal Expectation Model. This extension to the initial model also has the attractive feature of allowing to identify a high belief firm with an optimist firm. Here optimism is a consequence of the possibility of choosing optimal beliefs instead of objective beliefs.

Finally, notice that Proposition 3 only claims existence. We are not claiming uniqueness.

5 Endogeneizing overconfidence

In this section we extend the model by assuming that in period 0 each firm receives a signal about the demand parameter θ^*. The signal may be either high or low, which can be interpreted, respectively, as no news and bad news. Bad news, $\sigma^i = \sigma^i_L$, are received with probability $1-p$ and no news at all, $\sigma^i = \sigma^i_H$,with probability p. We assume that the signal completely reveals the true value of demand, that is,

$$\begin{aligned} P\left(\theta^* = 1 | \sigma^i = \sigma^i_L\right) &= 1 \\ P\left(\theta^* = \theta | \sigma^i = \sigma^i_H\right) &= 1 \end{aligned}$$

As in Benabou and Tirole (2002), we consider that firms have access to a costly mechanism which enables to forget a signal. Let $\delta \equiv P\left[\hat{\sigma} = \sigma_L | \sigma = \sigma_L\right]$ denote the probability that bad news are remembered accurately and $M(\delta)$ denote the memory cost. We will assume that the mechanism is such that $\delta \in \{0,1\}$, i.e. firms can choose for bad news to be perfectly recalled, or completely forgotten. Furthermore, $M(0) > 0$ and $M(1) = 0$. We assume that there are two types of mechanisms, particularly, $M(\delta) \in \left\{M^L(\delta), M^H(\delta)\right\}$ where $M^L(0) < M^H(0) = \infty$. and $M^L(0) < \left(\frac{2[p\theta+(1-p)]-2\theta^*}{3}\right)^2 - \frac{\theta^*}{16}$. So, with a mechanism of type $M^H(\delta)$ a firm cannot forget. The upper bound in the mechanism for M^L guarantees that the threat of using the mechanism is always credible.

The existence of the two mechanisms can be interpreted as the existence of different managers. There are firms with optimistic managers, and these have low cost of forgetting, and there are firms with rational managers, for whom it is impossible to forget. In the former case ignoring bad news is an option, whereas in the latter it is not.

We are going to assume that each firm's mechanism is public information. Furthermore, given our goal, we consider that each firm has a different type of mechanism and so we denote the firm with high mechanism by HM and the other firm by LM. Thus, in contrast with the previous section, here each firm knows the type of the other firm and, in each game there are always two different types of firms.

One can interpret the low mechanism firm's problem in period 0, when δ is chosen, as the choice of the game to play. In this particular case, one of the games is the standard game proposed by Hamilton and Slutsky (1990) while the other is the game with one uniformed firm and one informed firm.

In period 1 firms should take into account the reliability of their information. Therefore, when they do not recall any bad signal they should take into account the possibility that they could have forgotten it. Using Bayes rule the reliability of a "no recollection" signal is given by

$$r^* \equiv P\left[\sigma = \sigma_H | \hat{\sigma} = \sigma_H, \delta^*\right] = \frac{p}{p + (1-p)(1-\delta^*)}$$

We assume that firms that choose to forget can learn the true demand in the second period if the informed firm choice of period of production or quantity produced in first period is state dependent.

In this game a pure strategy for firm HM is a function $\chi_{HM} : \theta \times \delta \rightarrow \{1, 2\}$ which is the choice of a production period as function of the signal received in period 0 and the memory awareness of the other firm and a set of functions $\vartheta_{HM} : \{(\bar{t}_{HM} = 1, \bar{t}_{LM} = 1), (1, 2), (2, 1) \times \mathbb{R}^+, (2, 2)\} \times \theta \times \delta \rightarrow \mathbb{R}^+$ which is firm's quantity choice as a function of production periods, and q_{LM} if it is a Stackelberg follower. On the other hand a pure strategy for firm LM is a function $\Delta : \theta \rightarrow \{0, 1\}$ which is firm's choice of δ as function of the signal received in period 0, a function $\chi_{LM} : \{(\delta = 0), (\delta = 1) \times \theta\} \rightarrow \{1, 2\}$ which is the choice of a production period as function of degree of memory awareness and of the signal received in period 0 if $\delta = 1$ and a set of functions $\vartheta_{LM} : \{(\bar{t}_{LM} = 1, \bar{t}_{HM} = 1), (1, 2), (2, 1) \times \mathbb{R}^+, (2, 2)\} \times \theta \times \delta \rightarrow \mathbb{R}^+$ which is firm's quantity choice as a function of production periods, and q_H if it is a Stackelberg follower.

In this application we use Subgame Perfect Nash Equilibrium as the equilibrium concept. We are also going to use the Forward Induction refinement[12] and the equilibrium refinement D1 (Cho and Kreps, 1987).

Notice that in the subgame originated by $\delta^* = 1$ we are in the standard game proposed by Hamilton and Slutsky (1990) and so we know that the possible outcomes of the game are playing Cournot in the first period, or having either of the firms as a Stackelberg leader. In order to derive the equilibrium we need to understand what happens in the subgame created by the action $\delta^* = 0$.

Suppose the firm with low mechanism chooses $\delta^* = 0$ in period 0. Hence, in period 1 a firm

[12] For a discussion of this refinement see Govindan and Wilson (2009)

with low mechanism only recalls a high signal and so its beliefs are given by

$$\begin{aligned} P_{LM}\left[\sigma=\sigma_H|\hat{\sigma}_i=\sigma_H,\delta_i^*\right] &= p \\ P_{LM}\left[\sigma=\sigma_L|\hat{\sigma}_i=\sigma_H,\delta_i^*\right] &= (1-p)\,. \end{aligned}$$

On the other hand, for a firm with a high mechanism the beliefs are

$$\begin{aligned} P_{HM}\left[\hat{\sigma}_j=\sigma_H,\sigma=\sigma_L|\hat{\sigma}_i=\sigma_L,\delta_i^*\right] &= 1 \\ P_{HM}\left[\sigma=\sigma_H|\hat{\sigma}_i=\sigma_H,\delta_i^*\right] &= 1. \end{aligned}$$

In the game that follows from $\delta^*=0$, firms with high mechanism can be seen as informed firms while the firms with low mechanism can be seen as uniformed firms. Therefore, we have the same framework of Normann (2002) and by lemma 1 in that paper we know that in a pure-strategy equilibrium that satisfies the equilibrium refinement D1 (Cho and Kreps, 1987), all types of informed firm choose the same production period. Therefore, we are going to focus our attention in pure strategy equilibria where the order of move is given by the type of mechanism. In the next two lemmas we will restrict the number of equilibria. In proposition 4 we describe the equilibrium outcomes for the complete game (a complete description of the strategy profiles that are SPNE is given in the appendix)

Lemma 3 *(Adapted from Normann, 2002) If $\delta^*=0$ then both firms choosing period 2 is not an equilibrium*

Lemma 4 *If $\delta^*=0$, then in equilibrium the firm with a low mechanism never chooses to produce in period 2. Thus, there is no separating equlibrium with $\delta^*=0$ where firms with a low mechanism choose to produce in period 2.*

Proof. Suppose that there exists a separating equilibrium with $\delta^*=0$ where firms with low mechanism choose to produce in period 2. Now, suppose a firm with low mechanism deviates and chooses $\delta^*=1$. Thus, the firm will be informed about the value of demand. If the firm keeps producing in period 2, then the quantity produced by both firms remains the same, but the firm with low mechanism reduces its cost by not using the mechanism to forget. So, there is a gain from deviating and so we found a profitable deviation. Therefore, there is not any separating equilibrium where firms with low mechanism choose to produce in period 2 ■

In Proposition 4, rather than presenting the equilibrium strategies, we present the equilibrium outcomes. We denote the roles that firms take. C stands for Cournot competitor, L for Stackelberg leader and F for Stackelberg follower. We also specify which would be the equilibrium outcome if a different forgetting action would have been taken. In this way we can highlight the alternative for the LM firm.

Proposition 4: *The subgame perfect Nash equilibria of the game are characterized by the following outcomes*

1. $\delta = 0$, (C, C) *if* $\delta = 0$ *and* (C, C) *if* $\delta = 1$
2. $\delta = 0$, (C, C) *if* $\delta = 0$ *and* (F, L) *if* $\delta = 1$
3. $\delta = 0$, (L, F) *if* $\delta = 0$ *and* (C, C) *if* $\delta = 1$
4. $\delta = 0$, (L, F) *if* $\delta = 0$ *and* (F, L) *if* $\delta = 1$
5. $\delta = 1$, (L, F) *if* $\delta = 0$ *and* (L, F) *if* $\delta = 1$
6. $\delta = 1$, (C, C) *if* $\delta = 0$ *and* (L, F) *if* $\delta = 1$
7. $\delta = 1$, (F, L) *if* $\delta = 0$ *and* (L, F) *if* $\delta = 1$
8. $\delta = 1$, (F, L) *if* $\delta = 0$ *and* (C, C) *if* $\delta = 1$
9. $\delta = 1$, (F, L) *if* $\delta = 0$ *and* (F, L) *if* $\delta = 1$

The previous proposition shows that in this game there are several Subgame Perfect Nash Equilibria. In some of these equilibria the firm with low mechanism chooses to forget the signal.

Given the multiplicity of equilibria might be useful to understand which equilibria survive when we introduce some refinements. We use the Forward Induction refinement. Proposition 5 characterizes the outcomes of the Subgame Perfect Nash Equilibria that survive Forward Induction.

Proposition 5: *The outcomes*

$$\{\delta = 1, (L, F) \text{ if } \delta = 0 \text{ and } (L, F) \text{ if } \delta = 1\}$$

and

$$\{\delta = 1, (C, C) \text{ if } \delta = 0 \text{ and } (L, F) \text{ if } \delta = 1\}$$

are the unique SPNE that satisfy the forward induction refinement.

Proposition 5 implies that outcomes satisfying forward induction are such that in equilibrium the low mechanism firm does not use the mechanism, that is, the firm that could be optimist chooses not to be and does not forget any bad news. Nevertheless, the low mechanism firm always becomes a Stackelberg leader. Therefore, the low mechanism firm is weakly better by having the mechanism, despite not using it. Thus, the main insight of this result is that firms do not want to be optimist but they want to have the possibility to be.

The intuition for the result is the following. The memory awareness mechanism is a kind of signalling mechanism since it enables the firm with this mechanism to be leader. Since the other firm does not want to start a Stackelberg war, it chooses to be a follower because it knows that the firm with the mechanism will be the leader.

One interpretation of the latter results is that a firm with an optimistic manager who forgets bad news gets a strategic advantage. Even if this firm does not delegate anything to the optimistic manager, the possibility of doing so creates a competitive advantage with regard to the firms without that possibility. Hence the existence of this choice weakly improves the outcome of the firm.

6 Conclusion

In this paper we propose a framework that rationalizes the earlier entry of firms with more optimistic managers.

To do that we extend the endogenous timing model proposed by Hamilton and Slutsky (1990). We depart from the basic model by assuming imperfect information about demand and heterogenous beliefs about the true value of demand. In the unique Perfect Bayesian equilibria of the model, firms with more optimistic beliefs produce in the first period while firms with more pessimistic beliefs only produce in the second period. Therefore, if we interpret firms beliefs as the beliefs of their managers, we get that firms with more optimistic managers enter earlier.

In an extension of our model, we find that if one firm has an optimistic manager and the other firm has a rational manager, the firm with the optimistic manager does not delegate anything to the optimistic manager but this possibility generates a competitive advantage for the firm.

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8 Appendix

Proof of Proposition 1:

Our proof's strategy is based in Branco (2008) and Pires and Santos-Pinto (2008).

The first step of the proof is to show that strategies are sequential rational given beliefs. Thus, we start by finding the optimal level of production for each type of firm in each contingency, assuming that each firm takes the strategy of the other firm as given.

1) Consider first the problem of a High Belief firm

i) If it produces in the first period, it may be that the other firm also has a high belief and will also produce at first period or else it will produce at the second period, if it is a low belief firm. In this case the problem of an High Belief firm is

$$\max_{q_H} \lambda \left(\theta - q_H - \frac{\theta - \frac{1-\lambda}{2}}{2\lambda + 1} \right) q_H + (1 - \lambda) \left(\theta - q_H - \frac{1}{2} + \frac{1}{2} q_H \right) q_H$$

The solution to the problem is

$$q_H = \frac{\theta - (1 - \lambda) \frac{1}{2}}{2\lambda + 1}$$

ii) If it produces in the second period and the other firm produced a quantity q in the first period, then it must choose the quantity that solves the problem

$$\max_{q_H} (\theta - q_H - q) q_H$$

Therefore the choice of the firm is

$$q_H = \frac{\theta - q}{2}$$

iii) If it produces at period 2, knowing that the other firm has not produced yet: then it infers that the other firm is a Low Belief firm and so it will produce 1/3 ; thus the High Belief firm must produce a quantity that solves the following problem:

$$\max_{q_H} \left(\theta - q_H - \frac{1}{3} \right) q_H,$$

which leads to production of:

$$q_H = \frac{3\theta - 1}{6}.$$

2. Consider the problem of a Low Belief firm

i) If it produces in the first period, it may be that the other firm is an High Belief firm and will also produce at first period or else it will produce at the second period, if it is a Low Belief firm. In this case the problem is

$$\max_{q_L} \lambda \left(1 - q_L - \frac{\theta - (1-\lambda)\frac{1}{2}}{2\lambda + 1}\right) q_L + (1-\lambda)\left(1 - q_L - \frac{1 - q_L}{2}\right) q_L.$$

The solution to this problem is:

$$\begin{aligned} q_L &= \frac{4\lambda + \lambda^2 - 2\theta\lambda + 1}{6\lambda + 4\lambda^2 + 2} \\ &= \frac{(1 + \lambda(2 - \theta))\frac{1}{(1+\lambda)} - (1-\lambda)\frac{1}{2}}{2\lambda + 1} \end{aligned}$$

(ii) If it produces at date 2, knowing that the other firm has produced the quantity q at period 1: then it must produce the quantity that solves the following problem:

$$\max_{q_L} (1 - q - q_L)\, q_L,$$

which leads to the production of:

$$q_L = \frac{1 - q}{2}.$$

(iii) If it produces at period 2, knowing that the other firm has not produced at date 1: then it infers that the other firm is also a Low Belief firm and so it will produce 1/3 at date 2; thus, it must produce a quantity that solves the following problem:

$$\max_{q_L} \left(1 - q_L - \frac{1}{3}\right) q_L,$$

which leads to the production of:

$$q_L = \frac{1}{3}.$$

The optimal moment for production is determined by looking at the associated expected profits:

1. Consider the problem of an High Belief firm

a) If the High Belief firm produces in the first period its expected profit will be:

$$\begin{aligned} E\left(\pi_H^1\right) &= \lambda\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\left(\theta-\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)-\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\right) \\ &\quad+(1-\lambda)\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\left(\theta-\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)-\left(\frac{1}{2}-\frac{1}{2}\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\right)\right) \\ &= \frac{1+\lambda}{2}\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)^2 \end{aligned}$$

b) If the High Belief firm produces in the second period its expected profit will be

$$\begin{aligned} E\left(\pi_H^2\right) &= \lambda\left(\frac{\theta}{2}-\frac{1}{2}\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\right)\left(\theta-\left(\frac{\theta}{2}-\frac{1}{2}\left(\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\right)-\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right) \\ &\quad+(1-\lambda)\left(\frac{3\theta-1}{6}\right)\left(\theta-\frac{3\theta-1}{6}-\frac{1}{3}\right) \\ &= \frac{\lambda}{4}\left(\theta-\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)^2+(1-\lambda)\left(\frac{3\theta-1}{6}\right)^2 \end{aligned}$$

Thus, the difference between the payoffs is

$$E\left(\pi_H^1\right)-E\left(\pi_H^2\right)=\frac{1}{144}\frac{1-\lambda}{(2\lambda+1)^2}\left((6\theta-4)^2+\lambda(\lambda+1)(24\theta-25)-2\right)$$

and so the payoff of following the strategy and produce in period 1 is higher for all values of λ and θ.

2. Consider now the Low Belief firm problem

a) If the Low Belief firm produces in the first period its expected profit will be:

$$
\begin{aligned}
E\left(\pi_L^1\right) &= \lambda\frac{4\lambda+\lambda^2-2\theta\lambda+1}{6\lambda+4\lambda^2+2}\left(1-\frac{4\lambda+\lambda^2-2\theta\lambda+1}{6\lambda+4\lambda^2+2}-\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right) \\
&+(1-\lambda)\frac{4\lambda+\lambda^2-2\theta\lambda+1}{6\lambda+4\lambda^2+2}\left(1-\frac{4\lambda+\lambda^2-2\theta\lambda+1}{6\lambda+4\lambda^2+2}-\left(\frac{1}{2}-\frac{1}{2}\frac{4\lambda+\lambda^2-2\theta\lambda+1}{6\lambda+4\lambda^2+2}\right)\right) \\
&= \frac{1}{8(\lambda+1)(2\lambda+1)^2}\left(4\lambda+\lambda^2-2\theta\lambda+1\right)^2
\end{aligned}
$$

b) If it produces in the second period its expected profit will be

$$
\begin{aligned}
E\left(\pi_L^2\right) &= \lambda\left(\frac{1}{2}-\frac{1}{2}\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\left(1-\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}-\left(\frac{1}{2}-\frac{1}{2}\frac{\theta-(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\right) \\
&+(1-\lambda)\frac{1}{3}\left(1-\frac{1}{3}-\frac{1}{3}\right) \\
&= \frac{\lambda}{4(2\lambda+1)^2}\left(\frac{3\lambda+3-2\theta}{2}\right)^2+\frac{1-\lambda}{9}
\end{aligned}
$$

So, the difference between the payoffs is

$$
E\left(\pi_L^1\right)-E\left(\pi_L^2\right)=\frac{1}{144}\frac{\lambda-1}{(\lambda+1)(2\lambda+1)^2}\Delta(\lambda)
$$

where

$$
\Delta(\lambda,\theta)=36\theta\lambda(\theta+\lambda-1)+\lambda\left(\lambda^2-34\lambda-1\right)-2
$$

Let $\lambda^*(\theta)\equiv\inf\{\lambda\in[0,1]:\Delta(\lambda,\theta)\geq 0\}$. We can show that $\lambda^*(\theta)$ is non-empty and the payoff of following the strategy and produce in the second period is higher if $\lambda\geq\lambda^*(\theta)$. $\lambda^*(\theta)$ is such that $\lambda^*(1)=1$ and $\lambda^{*\prime}(\theta)<0$ for the relevant range.[13] Given that $\lambda^*(1)=1$ it is important to

[13] Notice that we can write the difference between the payoffs as

$$
\frac{1}{144}\frac{\lambda-1}{(\lambda+1)(2\lambda+1)^2}\Delta(\lambda)
$$

where

$$
\Delta(\lambda,\theta)=36\theta\lambda(\theta+\lambda-1)+\lambda\left(\lambda^2-34\lambda-1\right)-2
$$

Thus, the payoff of following the strategy and produce in the second period is higher if $\Delta(\lambda,\theta)>0$. The goal is to show that there exists a $\lambda^*(\theta)$ such that for all $\lambda\geq\lambda^*(\theta)$ we have $\Delta(\lambda,\theta)>0$.

First, define $\lambda_o(\theta)\equiv\inf\left\{\lambda\in[0,1]:\frac{\partial\Delta}{\partial\lambda}\geq 0\right\}$. Since $\frac{\partial\Delta}{\partial\lambda}$ is continuous and

$$
\frac{\partial\Delta}{\partial\lambda}(1,\theta)=36\theta(\theta+1)-66>0
$$

have some idea of the magnitude of the decrease of $\lambda^* (\theta)$ when θ increases, that is given by the following table

θ	$\lambda^* (\theta)$
1.01	0.88866
1.05	0.58164
1.1	0.38153
1.25	0.16534
5	0.00278

Now to wrap up the proof we only need to verify that given the strategies the beliefs proposed can be, whenever possible, updated by Bayes Rule. Particularly

$$\begin{aligned} \mu_i^1 \left(\bar{t}_j = 1, \vartheta_j = \frac{\theta - \frac{1-\lambda}{2}}{2\lambda + 1} \right) &= P(\tau_i = H) = \lambda \\ \mu_i^1 \left(\bar{t}_j = 2, \vartheta_j = \frac{1 - q_{1i}}{2} \right) &= P(\tau_i = L) = 1 - \lambda \\ \mu_i^2 \left(\vartheta_j = \frac{1}{3} | \bar{t}_j = 2 \right) &= \frac{P\left(\vartheta_j = \frac{1}{3} \bar{t}_j = 2\right)}{P(\bar{t}_j = 2)} = 1 \end{aligned}$$

Proof Proposition 2:

---

then $\lambda_o (\theta)$ is non-empty. Moreover, since

$$\frac{\partial^2 \Delta}{\partial \lambda^2} (\lambda, \theta) = 72\theta + 6\lambda - 68 > 0$$

then $\Delta (\lambda, \theta)$ is strictly convex in λ, which implies that $\frac{\partial \Delta}{\partial \lambda} > 0$ for $\lambda > \lambda_o (\theta)$.

Now define $\lambda^* (\theta) \equiv \inf \{\lambda \in [0, 1] : \Delta (\lambda, \theta) \geq 0\}$. Notice that

$$\Delta (1, \theta) = 36\theta^2 - 36 > 0$$

and so $\lambda^* (\theta)$ is always non-empty.

Furthermore, since $\frac{\partial \Delta}{\partial \lambda} > \Delta (\lambda, \theta)$ for $\lambda \in [0, 1]$, then $\lambda^* (\theta) > \lambda_o (\theta)$ for $\theta \geq 1$. Therefore, we know that for all $\lambda \geq \lambda^* (\theta)$ we have $\Delta (\lambda) > 0$.

Analitically is almost impossible to find this value. However, we can derive the relevant properties. With this goal define

$$F (\theta, \lambda^* (\theta)) = 36\theta\lambda (\theta + \lambda - 1) + \lambda \left(\lambda^2 - 34\lambda - 1\right) - 2 = 0$$

So,

$$\frac{\frac{\partial F}{\partial \theta}}{\frac{\partial F}{\partial \lambda}} = \frac{\partial \lambda}{\partial \theta} = -\frac{36\lambda (2\theta + \lambda - 1)}{36\theta (\theta + 2\lambda - 1) + 3\lambda^2 - 68\lambda - 1} < 0$$

for $\theta > 1$ and $\lambda \in [\lambda_o (\theta), 1]$. Thus, $\lambda^{*\prime} (\theta) < 0$ for the relevant range.

In order to prove proposition 2 we enumerate all possible strategy profiles that could be considered and explain why there cannot exist equilibria with such profiles.

Case 1: *A Low Belief firm produces at period 1 and an High Belief firm produces at period 2:*

Suppose such equilibrium exists. In that case each firm produces according to the following rule

1. If firm is a Low Belief firm then :

(a) It produces at date 1;

(b) It produces $q_L = \frac{2-\theta\lambda}{6-4\lambda}$*;*

(c) If it did not produced at date 1 and the other firm produced $\bar{q}$ *at date 1, it would produce at date 2 according*

to $q_L = \frac{1-\bar{q}}{2}$, *at date 2;*

(d) If both firms did not produced at date 1, then it would produce $q_L = \frac{3-\theta}{6}$ *at date 2;*

2. If firm is a High Belief firm

(a) It produces at date 2;

(b) If it were to produce at date 1, it would produce $q_H = \frac{6\theta-2+2\lambda-6\theta\lambda+\theta\lambda^2}{4\lambda^2-14\lambda+12} = \frac{\theta\left(6-6\lambda+\lambda^2\right)+2(\lambda-1)}{(2-\lambda)(6-4\lambda)}$*;*

(c) If the other firm produces $\bar{q}$ *at date 1, it will produce at date 2 according to* $q_H = \frac{\theta-\bar{q}}{2}$, *at date 2;*

(d) If both firms do not produced at date 1, then it will produce $q_H = \frac{\theta}{3}$ *at date 2;*

The expected profit that the High Belief firm obtains from following this strategy profile is

$$\lambda\left(\frac{\theta}{3}\right)^2 + \frac{(1-\lambda)}{4}\left(\frac{6\theta-3\lambda\theta-2}{6-4\lambda}\right)^2$$

Now, suppose the High Belief firm deviates and produces in the first period. In that case its expected profit is $\frac{\left(6\theta+2\lambda-6\theta\lambda+\theta\lambda^2-2\right)^2}{8(2-\lambda)(2\lambda-3)^2}$. Notice that

$$E\left(\pi_H^1\right) = \frac{\left(6\theta+2\lambda-6\theta\lambda+\theta\lambda^2-2\right)^2}{8\left(2-\lambda\right)\left(2\lambda-3\right)^2} > \lambda\left(\frac{\theta}{3}\right)^2 + \frac{(1-\lambda)}{4}\left(\frac{6\theta-3\lambda\theta-2}{6-4\lambda}\right)^2 = E\left(\pi_H^2\right)$$

for all $\lambda \in [0,1]$ and $\theta > 1$ and so the High Beliefs firms always have incentives to deviate[14]

[14] Notice that we can write the difference between the profits as

$$E\left(\pi_H^1\right) - E\left(\pi_H^2\right) = \frac{1}{144}\frac{\lambda\Delta\left(\lambda\right)}{\left(2-\lambda\right)\left(2\lambda-3\right)^2}$$

where

$$\Delta\left(\lambda,\theta\right) = \theta^2\left(\lambda^3+31\lambda^2-66\lambda+36\right) + 36\left(1-\lambda\right)\left(\lambda\theta-1\right)$$

Case 2: *Both players produce at period 1, regardless of their beliefs:*

In this case an High Belief firm produces $\frac{3\theta+\lambda(1-\theta)-1}{6}$ while a Low Belief firm produces $\frac{2+\lambda(1-\theta)}{6}$. So, the expected profit of following the strategy for a Low Belief firm is $\frac{1}{36}(\lambda-\theta\lambda+2)^2$. Now suppose a Low Belief firm deviates and produces in the second period according to the following rule $\frac{1-q}{2}$, where q is the quantity produced by the other firm. So, the expected profits are

$$\begin{aligned} E(\pi) &= \lambda\left(\frac{1-\frac{3\theta+\lambda(1-\theta)-1}{6}}{2}\right)\left(1-\frac{1-\frac{3\theta+\lambda(1-\theta)-1}{6}}{2}-\frac{3\theta+\lambda(1-\theta)-1}{6}\right) \\ &\quad +(1-\lambda)\left(\frac{1-\frac{2+\lambda(1-\theta)}{6}}{2}\right)\left(1-\frac{1-\frac{2+\lambda(1-\theta)}{6}}{2}-\frac{2+\lambda(1-\theta)}{6}\right) \\ &= \frac{\lambda}{4}\left(\frac{7-3\theta-\lambda(1-\theta)}{6}\right)^2+\frac{1-\lambda}{4}\left(\frac{4-\lambda(1-\theta)}{6}\right)^2 \end{aligned}$$

Notice

$$\frac{\lambda}{4}\left(\frac{7-3\theta-\lambda(1-\theta)}{6}\right)^2+\frac{1-\lambda}{4}\left(\frac{4-\lambda(1-\theta)}{6}\right)^2>\frac{1}{36}(\lambda-\theta\lambda+2)^2$$

and so a Low Belief firm always have incentives to deviate and so the strategy proposed cannot be an equilibrium

Case 3: *Both players produce at period 2, regardless of their beliefs:*

This cannot be an equilibrium because if both players wait regardless of their type, then they have no information gain by waiting. If they deviate by committing to a quantity at date 1 they have a first-mover advantage gain.

Proof of Proposition 3:

Since

$$\frac{\partial}{\partial\theta}\Delta(\lambda,\theta)=2\theta\left(\lambda^3+31\lambda^2-66\lambda+36\right)+36(1-\lambda)\lambda>0$$

for all $\lambda\in[0,1]$, the minimum value of $\Delta(\lambda,\theta)$ is achieved at $\theta=1$. Furthermore

$$\frac{\partial}{\partial\lambda}\Delta(\lambda,1)=3\lambda^2-10\lambda+6$$

and so $\Delta(\lambda,1)$ is increasing until $\lambda\simeq 0.78$ and so it decreases after $\lambda\simeq 0.78$. Therefore, for $\lambda\in[0,1]$ and $\theta>1$, the minimum value of $\Delta(\lambda,\theta)$ is achived at $(0,1)$ or $(1,1)$. Since

$$\begin{aligned} \Delta(0,1) &= 0 \\ \Delta(1,1) &= 2 \end{aligned}$$

then $\Delta(\lambda)>0$ for $\lambda\in[0,1]$ and $\theta>1$. This implies that $E\left(\pi_H^1\right)-E\left(\pi_H^2\right)>0$

The first step of this proof is to show that strategies are sequential rational given beliefs. We start by setting the subjective beliefs of a OE firm equal to $\bar{\pi}$. Thus, the beliefs for a rational firm coincide with the objective beliefs while the beliefs of a OE firm are such that it gives probability 1 to state $\theta^* = \theta$.

We find the optimal level of production for each type of firm in each contingency, assuming that each firm take as given the strategy of the other firm. In a second stage we will show that the beliefs $\bar{\pi}$ are really the optimal beliefs.

1) Consider the problem of a OE firm

i) If it produces in the first period: it may be that the other firm is also a OE firm and will produce at first period or else it will produce at the second period, if it is a rational firm. In this case the problem of OE firm is

$$\max_{q_O} \lambda \left(\theta - q_{OE} - \frac{\theta - \bar{\theta}_R (1-\lambda) \frac{1}{2}}{2\lambda + 1} \right) q_{OE} + (1-\lambda) \left(\theta - q_{OE} - \frac{\bar{\theta}_R - q_{OE}}{2} \right) q_{OE}$$

The solution to the problem is

$$q_{OE} = \frac{\theta - \bar{\theta}_R (1-\lambda) \frac{1}{2}}{2\lambda + 1}$$

where $\bar{\theta}_R$ is the expected value of θ^* given the obective beliefs about θ^*, that is,

$$\bar{\theta}_R = \rho(1) + \rho(\theta)\theta$$

ii) If it produces in the second period and the other firm produced a quantity q in the first period, then it must choose the quantity that solves the problem

$$\max_{q_{OE}} (\theta - q_{OE} - q) q_{OE}$$

Therefore the choice of the OE firm following this strategy is

$$q_{OE} = \frac{\theta - q}{2}$$

iii) If it produces at period 2, knowing that the other firm has not produced yet: then it infers that the other firm is rational and that it will produce $\bar{\theta}_R/3$; thus the OE firm must produce a quantity that solves the following problem:

$$\max_{q_{OE}} \left(\theta - q_{OE} - \frac{\bar{\theta}_R}{3}\right) q_{OE},$$

which leads to production of:

$$q_O = \frac{3\theta - \bar{\theta}_R}{6}.$$

2. Consider the problem of R firm

i) If it produces at the first period: it may be that the other firm is OE and will produce at first period or else it will produce at the second period, if it is a R firm. In this case the problem is

$$\max_{q_R} \lambda \left(\bar{\theta}_R - q_R - \frac{\theta - \bar{\theta}_R (1-\lambda)\frac{1}{2}}{2\lambda + 1}\right) q_R + (1-\lambda)\left(\bar{\theta}_R - q_R - \frac{\bar{\theta}_R - q_R}{2}\right) q_R.$$

The solution to this problem is:

$$q_R = \frac{\bar{\theta}_R \left(4\lambda + \lambda^2 + 1\right) - 2\theta\lambda}{6\lambda + 4\lambda^2 + 2}$$

(ii) If it produces at date 2, knowing that the other firm has produced the quantity q at period 1: then it must produce the quantity that solves the following problem:

$$\max_{q_R} \left(\bar{\theta}_R - q - q_R\right) q_R,$$

which leads to production of:

$$q_R = \frac{\bar{\theta}_R - q}{2}.$$

(iii) If it produces at period 2, knowing that the other firm has not produced at date 1: then it infers that the other firm is also rational and that he will produce $\bar{\theta}_R/3$ at date 2; thus must produce a quantity that solves the following problem:

$$\max_{q_R} \left(\bar{\theta}_R - q_R - \frac{\bar{\theta}_R}{3}\right) q_R,$$

which leads to production of:

$$q_R = \frac{\bar{\theta}_R}{3}.$$

The optimal moment for production is determined by looking at the associated expected profits:

1. Consider the OE firm's problem

a) If a OE firm produces in the first period its expected profit will be:

$$\begin{aligned} E\left(\Pi_{OE}^{1}\right) &= \lambda\left(\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\left(\theta-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}\right) \\ &+(1-\lambda)\left(\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)\left(\theta-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}-\frac{\bar{\theta}_R-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}}{2}\right) \\ &= \frac{\lambda+1}{2}\left(\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)^2 \end{aligned}$$

b) If a OE firm produces in the second period its expected profit will be

$$\begin{aligned} E\left(\Pi_{OE}^{2}\right) &= \lambda\left(\frac{\theta-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}}{2}\right)\left(\theta-\frac{\theta-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}}{2}-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}\right) \\ &+(1-\lambda)\left(\frac{3\theta-\bar{\theta}_R}{6}\right)\left(\theta-\frac{3\theta-\bar{\theta}_R}{6}-\frac{\bar{\theta}_R}{3}\right) \\ &= \lambda\left(\frac{\theta-\frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}}{2}\right)^2+(1-\lambda)\left(\frac{3\theta-\bar{\theta}_R}{6}\right)^2 \end{aligned}$$

So, the difference between the payoffs is

$$E\left(\Pi_{OE}^{1}\right)-E\left(\Pi_{OE}^{2}\right)=\frac{1}{144}\frac{1-\lambda}{(2\lambda+1)^2}\left(14\bar{\theta}_R^2-25\bar{\theta}_R^2\lambda^2-25\bar{\theta}_R^2\lambda+24\bar{\theta}_R\theta\lambda^2+24\bar{\theta}_R\theta\lambda-48\bar{\theta}_R\theta+36\theta^2\right)$$

and so the payoff of follow the strategy and produce in period 1 is higher for all values of λ and θ.

2. Consider the R firm's problem

a) If a R firm produces in the first period its expected profit will be:

$$
\begin{aligned}
E\left(\Pi_R^1\right) &= \lambda\left(\bar{\theta}_R - \frac{\theta-\bar{\theta}_R\left(1-\lambda\right)\frac{1}{2}}{2\lambda+1} - \frac{\bar{\theta}_R\left(4\lambda+\lambda^2+1\right)-2\theta\lambda}{6\lambda+4\lambda^2+2}\right)\left(\frac{\bar{\theta}_R\left(4\lambda+\lambda^2+1\right)-2\theta\lambda}{6\lambda+4\lambda^2+2}\right) \\
&\quad +\left(1-\lambda\right)\left(\bar{\theta}_R - \frac{\bar{\theta}_R\left(4\lambda+\lambda^2+1\right)-2\theta\lambda}{6\lambda+4\lambda^2+2} - \frac{\bar{\theta}_R - \frac{\bar{\theta}_R\left(4\lambda+\lambda^2+1\right)-2\theta\lambda}{6\lambda+4\lambda^2+2}}{2}\right)\left(\frac{\bar{\theta}_R\left(4\lambda+\lambda^2+1\right)-2\theta\lambda}{6\lambda+4\lambda^2+2}\right) \\
&= \frac{1}{8\left(\lambda+1\right)\left(2\lambda+1\right)^2}\left(\theta_R - 2\theta\lambda + 4\lambda\theta_R + \lambda^2\theta_R\right)^2
\end{aligned}
$$

b)If it produces in the second period its expected profit will be

$$
\begin{aligned}
E\left(\Pi_R^2\right) &= \lambda\left(\bar{\theta}_R - \frac{\theta-\bar{\theta}_R\left(1-\lambda\right)\frac{1}{2}}{2\lambda+1} - \frac{\bar{\theta}_R - \frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}}{2}\right)\left(\frac{\bar{\theta}_R - \frac{\theta-\bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}}{2}\right) \\
&\quad +\left(1-\lambda\right)\left(\bar{\theta}_R - \frac{\bar{\theta}_R}{3} - \frac{\bar{\theta}_R}{3}\right)\frac{\bar{\theta}_R}{3} \\
&= \frac{\lambda}{4}\left(\bar{\theta}_R - \frac{\theta-\bar{\theta}_R\left(1-\lambda\right)\frac{1}{2}}{2\lambda+1}\right)^2 + \left(1-\lambda\right)\left(\frac{\bar{\theta}_R}{3}\right)^2
\end{aligned}
$$

So, the difference between the payoffs

$$
E\left(\Pi_R^1\right) - E\left(\Pi_R^2\right) = \frac{1}{144}\frac{\lambda-1}{\left(\lambda+1\right)\left(2\lambda+1\right)^2}\Delta\left(\lambda,\theta\right)
$$

$$
\Delta\left(\lambda,\theta\right) = 36\theta\lambda\left(\theta+\lambda\theta_R-\theta_R\right) + \lambda\theta_R^2\left(\lambda^2-34\lambda-1\right) - 2\theta_R^2
$$

Let $\lambda^*\left(\theta\right) \equiv \inf\left\{\lambda \in [0,1] : \Delta\left(\lambda,\theta\right) \geq 0\right\}$. We can show that $\lambda^*\left(\theta\right)$ is non-empty and the payoff of following the strategy and produce in the second period is higher if $\lambda \geq \lambda^*\left(\theta\right)$. $\lambda^*\left(\theta\right)$ is such that $\lambda^*\left(1\right) = 1$ and $\lambda^{*\prime}\left(\theta\right) < 0$ for the relevant range.[15]

[15] Notice that we can write the difference between the payoffs as

$$
\frac{1}{144}\frac{\lambda-1}{\left(\lambda+1\right)\left(2\lambda+1\right)^2}\Delta\left(\lambda\right)
$$

where

$$
\Delta\left(\lambda,\theta\right) = 36\theta\lambda\left(\theta+\lambda\theta_R-\theta_R\right) + \lambda\theta_R^2\left(\lambda^2-34\lambda-1\right) - 2\theta_R^2
$$

Thus, the payoff of following the strategy and produce in the second period is higher if $\Delta\left(\lambda,\theta\right) > 0$. The goal is to

The next step of this proof is to show that the beliefs $\bar{\pi}$ are really the optimal beliefs. The well being of OE firm if it chooses $\hat{\pi} = \bar{\pi}$ is

$$\begin{aligned} W(\bar{\pi}) &= \frac{\lambda+1}{4}\left(\frac{\theta - \bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)^2 - \\ &- \frac{\left(12\theta^2\lambda + 4\theta^2 + 4\theta\lambda^2\theta_R - 20\theta\lambda\theta_R - 8\theta\theta_R - \lambda^3\theta_R^2 - 7\lambda^2\theta_R^2 + 5\lambda\theta_R^2 + 3\theta_R^2\right)}{16(2\lambda+1)^2} \\ &= \frac{1}{8(2\lambda+1)^2}\left(2\theta\theta_R - 4\theta^2\lambda + 10\theta\lambda\theta_R + \lambda^3\theta_R^2 + 3\lambda^2\theta_R^2 - 3\lambda\theta_R^2 - \theta_R^2\right) \end{aligned}$$

On the other hand if it chooses $\hat{\pi} = \pi$ then the well being is

$$W(\pi) = \frac{\lambda}{4}\left(\bar{\theta}_R - \frac{\theta - \bar{\theta}_R(1-\lambda)\frac{1}{2}}{2\lambda+1}\right)^2 + (1-\lambda)\left(\frac{\bar{\theta}_R}{3}\right)^2$$

So, $W(\bar{\pi}) > W(\pi)$ if

$$\theta < \frac{1}{18\lambda}\theta_R\left(24\lambda + 3 + 9\lambda^2 + (1+2\lambda)\sqrt{3}\sqrt{2\lambda + 7\lambda^2 + 3}\right) = \bar{\theta}$$

Therefore, for $\theta < \bar{\theta}$ we have that $\hat{\pi} = \bar{\pi}$ are the "optimal" beliefs

Now to finish the proof we need to verify that given the strategies the beliefs proposed can be,

show that there exists a $\lambda^*(\theta)$ such that for all $\lambda \geq \lambda^*(\theta)$ we have $\Delta(\lambda, \theta) > 0$.

First, define $\lambda_o(\theta) \equiv \inf\left\{\lambda \in [0,1] : \frac{\partial \Delta}{\partial \lambda} \geq 0\right\}$. Since $\frac{\partial \Delta}{\partial \lambda}$ is continuous and

$$\frac{\partial \Delta}{\partial \lambda}(1, \theta) = 36\theta\left(\theta + \theta_R^2\right) - 66 > 0$$

then $\lambda_o(\theta)$ is non-empty. Moreover, since

$$\frac{\partial^2 \Delta}{\partial \lambda^2}(\lambda, \theta) = 2\theta_R(36\theta - 34\theta_R + 3\lambda\theta_R) > 0$$

then $\Delta(\lambda, \theta)$ is strictly convex in λ, which implies that $\frac{\partial \Delta}{\partial \lambda} > 0$ for $\lambda > \lambda_o(\theta)$.

Now define $\lambda^*(\theta) \equiv \inf\{\lambda \in [0,1] : \Delta(\lambda, \theta) \geq 0\}$. Notice that

$$\Delta(1, \theta) = 36\left(\theta^2 - \theta_R^2\right) > 0$$

and so $\lambda^*(\theta)$ is always non-empty.

Furthermore, since $\frac{\partial \Delta}{\partial \lambda} > \Delta(\lambda, \theta)$ for $\lambda \in [0,1]$, then $\lambda^*(\theta) > \lambda_o(\theta)$ for $\theta \geq 1$. Therefore, we know that for all $\lambda \geq \lambda^*(\theta)$ we have $\Delta(\lambda) > 0$.

Analitically is almost impossible to find this value. However, we can derive the relevant properties. With this goal define

$$F(\theta, \lambda^*(\theta)) = 36\theta\lambda(\theta + \lambda\theta_R - \theta_R) + \lambda\theta_R^2\left(\lambda^2 - 34\lambda - 1\right) - 2\theta_R^2 = 0$$

So,

$$\frac{\frac{\partial F}{\partial \theta}}{\frac{\partial F}{\partial \lambda}} = \frac{\partial \lambda}{\partial \theta} = -\frac{36\lambda(2\theta - \theta_R + \lambda\theta_R)}{36\theta(\theta + 2\lambda\theta_R - \theta_R) + \theta_R^2\left(3\lambda^2 - 68\lambda - 1\right)} < 0$$

for $\theta > 1$ and $\lambda \in [\lambda_o(\theta), 1]$. Thus, $\lambda^{*\prime}(\theta) < 0$ for the relevant range.

whenever possible, updated by Bayes Rule. Particularly

$$\begin{aligned}
\mu_i^1\left(\bar{t}_j = 1, \vartheta_j = \frac{\theta - \bar{\theta}_R\,(1-\lambda)\,\frac{1}{2}}{2\lambda + 1}\right) &= \lambda \\
\mu_i^1\left(\bar{t}_j = 2, \vartheta_j = \frac{\bar{\theta}_R - q_{1i}}{2}\right) &= 1 - \lambda \\
\mu_i^2\left(\vartheta_j = \frac{\bar{\theta}_R}{3} | \bar{t}_j = 2\right) &= 1
\end{aligned}$$

Proof of Proposition 4:

In order to find the Subgame Perfect Nash Equilibrium we solve the game by backward induction.

Consider the game generated by choosing $\delta = 0$. In this case, we have a game with an informed firm and with an uniformed firm. Using the results in Normann (2002) we know that we have three pure strategies equilibria in that subgame: both firms play Cournot in the first period, Stackelberg equilibrium with uniformed firm as leader and Stackelberg equilibrium with informed firm as leader.

Now, consider the game generated by choosing $\delta = 1$. This is the standard game proposed by Hamilton and Slutsky (1990) with three pure strategies equilibria: one Cournot equilibrium in the first period and two Stackelberg equilibria.

By Lemma 2 we know that any strategy profile where firm with low mechanism chooses $\delta = 0$ and to produce in period 2 is not a Subgame Perfect Nash Equilibrium. Thus, we can exclude these strategies from the possible equilibria candidates

Suppose we have a Cournot outcome when $\delta = 0$. Therefore, low mechanism firm's profits are higher by choosing $\delta = 0$ if firms compete *à la* Cournot or low mechanism firm is Stackelberg follower when $\delta = 1$. In a similar way, if low mechanism firm is Stackelberg leader when $\delta = 0$, its profits are higher by choosing $\delta = 0$ if firms compete *à la* Cournot or low mechanism firm is Stackelberg follower when $\delta = 1$.

Suppose low mechanism firm is Stackelberg leader when $\delta = 1$, its profits are always higher by choosing $\delta = 1$. On the other hand, if firms compete *à la* Cournot or low mechanism firm is Stackelberg follower when $\delta = 1$, low mechanism firm's profits are higher by choosing $\delta = 1$ if low mechanism firm is Stackelberg follower when $\delta = 1$

DESCRIPTION OF STRATEGY PROFILES THAT GENERATE THE OUTCOMES OF PROPOSITION 4

1. $\Delta = 0, \chi_{HM}(\theta^*, 0) = 1, \chi_{HM}(\theta^*, 1) = 1, \chi_{LM}(\theta^*, 0) = 1, \chi_{LM}(\theta^*, 1) = 1, \vartheta_{HM}(1, 1, \theta^*, 0) = \frac{2\theta^* - p\theta - (1-p)}{3}, \vartheta_{LM}(1, 1, \theta^*, 0) = \frac{2[p\theta + (1-p)] - 2\theta^*}{3}, \vartheta_{HM}(1, 1, \theta^*, 1) = \vartheta_{LM}(1, 1, \theta^*, 1) = \frac{\theta^*}{3}$

2. $\Delta = 0, \chi_{HM}(\theta^*, 0) = 1, \chi_{HM}(\theta^*, 1) = 1, \chi_{LM}(\theta^*, 0) = 1, \chi_{LM}(\theta^*, 1) = 2, \vartheta_{HM}(1, 1, \theta^*, 0) = \frac{2\theta^* - p\theta - (1-p)}{3}, \vartheta_{LM}(1, 1, \theta^*, 0) = \frac{2[p\theta + (1-p)] - 2\theta^*}{3}, \vartheta_{HM}(2, 1, \theta^*, 1) = \frac{\theta^*}{2}, \vartheta_{LM}(2, 1, \theta^*, 1) = \frac{\theta^*}{2} - \frac{q_{HM}}{2}$

3. $\Delta = 0, \chi_{HM}(\theta^*, 0) = 2, \chi_{HM}(\theta^*, 1) = 1, \chi_{LM}(\theta^*, 0) = 1, \chi_{LM}(\theta^*, 1) = 1, \vartheta_{HM}(1, 2, \theta^*, 0) = \frac{\theta^*}{2} - \frac{q_{LM}}{2}, \vartheta_{LM}(1, 2, \theta^*, 0) = \frac{p\theta + (1-p)}{2}, \vartheta_{HM}(1, 1, \theta^*, 1) = \vartheta_{LM}(1, 1, \theta^*, 1) = \frac{\theta^*}{3}$

4. $\Delta = 0, \chi_{HM}(\theta^*, 0) = 2, \chi_{HM}(\theta^*, 1) = 1, \chi_{LM}(\theta^*, 0) = 1, \chi_{LM}(\theta^*, 1) = 2, \vartheta_{HM}(1, 2, \theta^*, 0) = \frac{\theta^*}{2} - \frac{q_{LM}}{2}, \vartheta_{LM}(1, 2, \theta^*, 0) = \frac{p\theta + (1-p)}{2}, \vartheta_{HM}(2, 1, \theta^*, 1) = \frac{\theta^*}{2}, \vartheta_{LM}(2, 1, \theta^*, 1) = \frac{\theta^*}{2} - \frac{q_{HM}}{2}$

5. $\Delta = 1, \chi_{HM}(\theta^*, 0) = 2, \chi_{HM}(\theta^*, 1) = 2, \chi_{LM}(\theta^*, 0) = 1, \chi_{LM}(\theta^*, 1) = 1, \vartheta_{HM}(1, 2, \theta^*, 0) = \frac{\theta^*}{2} - \frac{q_{LM}}{2}, \vartheta_{LM}(1, 2, \theta^*, 0) = \frac{p\theta + (1-p)}{2}, \vartheta_{HM}(1, 2, \theta^*, 1) = \frac{\theta^*}{2} - \frac{q_{LM}}{2}, \vartheta_{LM}(1, 2, \theta^*, 1) = \frac{\theta^*}{2}$

6. $\Delta = 1, \chi_{HM}(\theta^*, 0) = 1, \chi_{HM}(\theta^*, 1) = 2, \chi_{LM}(\theta^*, 0) = 1, \chi_{LM}(\theta^*, 1) = 1, \vartheta_{HM}(1, 1, \theta^*, 0) = \frac{2\theta^* - p\theta - (1-p)}{3}, \vartheta_{LM}(1, 1, \theta^*, 0) = \frac{2[p\theta + (1-p)] - 2\theta^*}{3}, \vartheta_{HM}(1, 2, \theta^*, 1) = \frac{\theta^*}{2} - \frac{q_{LM}}{2}, \vartheta_{LM}(1, 2, \theta^*, 1) = \frac{\theta^*}{2}$

7. $\Delta = 1, \chi_{HM}(\theta^*, 0) = 1, \chi_{HM}(\theta^*, 1) = 2, \chi_{LM}(\theta^*, 0) = 2, \chi_{LM}(\theta^*, 1) = 1, \vartheta_{HM}(2, 1, \theta^*, 0) = \frac{\theta^*}{2}, \vartheta_{LM}(2, 1, \theta^*, 0) = \frac{\theta^*}{2} - \frac{q_{HM}}{2}, \vartheta_{HM}(1, 2, \theta^*, 1) = \frac{\theta^*}{2} - \frac{q_{LM}}{2}, \vartheta_{LM}(1, 2, \theta^*, 1) = \frac{\theta^*}{2}$

8. $\Delta = 1, \chi_{HM}(\theta^*, 0) = 1, \chi_{HM}(\theta^*, 1) = 1, \chi_{LM}(\theta^*, 0) = 2, \chi_{LM}(\theta^*, 1) = 1, \vartheta_{HM}(2, 1, \theta^*, 0) = \frac{\theta^*}{2}, \vartheta_{LM}(2, 1, \theta^*, 0) = \frac{\theta^*}{2} - \frac{q_{HM}}{2}, \vartheta_{HM}(1, 1, \theta^*, 1) = \vartheta_{LM}(1, 1, \theta^*, 1) = \frac{\theta^*}{3}$

9. $\Delta = 1, \chi_{HM}(\theta^*, 0) = 1, \chi_{HM}(\theta^*, 1) = 1, \chi_{LM}(\theta^*, 0) = 2, \chi_{LM}(\theta^*, 1) = 2, \vartheta_{HM}(2, 1, \theta^*, 0) = \frac{\theta^*}{2}, \vartheta_{LM}(2, 1, \theta^*, 0) = \frac{\theta^*}{2} - \frac{q_{HM}}{2}, \vartheta_{HM}(2, 1, \theta^*, 1) = \frac{\theta^*}{2}, \vartheta_{LM}(2, 1, \theta^*, 1) = \frac{\theta^*}{2} - \frac{q_{HM}}{2}$
$\left(\frac{2[p\theta + (1-p)] - 2\theta^*}{3}\right)^2 - \frac{\theta^*}{16}$

Proof of Proposition 5:

Fix outside option $\delta = 0$. Notice that $\pi^F_{LM}(\delta = 1) < \pi^C_{LM}(\delta = 0) < \pi^L_{LM}(\delta = 0)$, that is, if firm with low mechanism chooses $\delta = 1$ and $t = 2$, then its profit is always lower than if it had chosen $\delta = 0$. Therefore, if firm with low mechanism firm chooses $\delta = 1$, then firm with high mechanism antecipates that low mechanism firm will not choose to be a follower. So by forward induction we can rule out the SPNE's

$$\{\delta = 0,\ \ (L, F)\ \ \textit{if}\ \ \delta = 0\ \ \textit{and}\ \ (F, L)\ \ \textit{if}\ \ \delta = 1\}$$

$$\{\delta = 0,\ \ (C, C)\ \ \textit{if}\ \ \delta = 0\ \ \textit{and}\ \ (F, L)\ \ \textit{if}\ \ \delta = 1\}$$

Fix outside option $\delta = 1$. If firm with low mechanism chooses $\delta = 0$, then it never chooses $t = 2$, because for the remain SPNE's the equilibrium profits of choose $\delta = 0$ and $t = 2$ are always lower than the equilibrium profits associated with choose $\delta = 1$. So, by Forward Induction the following SPNE can also be ruled out

$$\{\delta = 1,\ \ (F, L)\ \ \textit{if}\ \ \delta = 0\ \ \textit{and}\ \ (L, F)\ \ \textit{if}\ \ \delta = 1\}$$

$$\{\delta = 1,\ \ (F, L)\ \ \textit{if}\ \ \delta = 0\ \ \textit{and}\ \ (C, C)\ \ \textit{if}\ \ \delta = 1\}$$

$$\{\delta = 1,\ \ (F, L)\ \ \textit{if}\ \ \delta = 0\ \ \textit{and}\ \ (F, L)\ \ \textit{if}\ \ \delta = 1\}$$

For the remain SPNE's we have that the profits when the equilibrium outcomes is such that $\delta = 1$ are higher than the profits when the equilibrium outcomes is such that $\delta = 0$. So by forward induction we can rule out the SPNE's

$$\{\delta = 0,\ \ (C, C)\ \ \textit{if}\ \ \delta = 0\ \ \textit{and}\ \ (C, C)\ \ \textit{if}\ \ \delta = 1\}$$

$$\{\delta = 0,\ \ (L, F)\ \ \textit{if}\ \ \delta = 0\ \ \textit{and}\ \ (C, C)\ \ \textit{if}\ \ \delta = 1\}$$

Therefore, the unique SPNE'S outcomes that survive the elimination of strategies that not satisfy the forward induction refinement are the outcomes

$$\{\delta = 1, (L, F)\ \text{ if } \delta = 0 \text{ and } \ (L, F)\ \text{ if } \delta = 1\}$$

and

$$\{\delta = 1, (C, C) \text{ if } \delta = 0 \text{ and } (L, F) \text{ if } \delta = 1\}$$

Perfect Bayesian Equilibrium for a generic demand specification

Suppose that the demand is given by $P(Q) = P^\tau(Q)$ where $P^\tau(Q) \in \left\{P^L(Q), P^H(Q)\right\}$ and $P^L(Q) < P^H(Q)$. Let $q_*^\tau(q)$ be defined implicitly by

$$P^\tau\left(q + q_*^\tau\right) + q_*^\tau \frac{\partial}{\partial q_*^\tau} P^\tau\left(q + q_*^\tau\right) = 0$$

Furthermore, define

$$q_H^1 = \arg\max_q \lambda P^H\left(q + q_H^1\right) q + (1-\lambda) P^H\left(q + q_*^L(q)\right) q$$

$$q_H^2 = \arg\max_q P^H(\bar{q} + q) q = q_*^H(\bar{q})$$

$$q_H^{2C} = q_*^H\left(q_P^{2C}\right)$$

$$q_L^1 = \arg\max_q \lambda P^L\left(q + q_H^1\right) q + (1-\lambda) P^L\left(q + q_*^L(q)\right) q$$

$$q_L^2 = q_*^L(\bar{q})$$

$$q_L^{2C} = q_*^L\left(q_L^{2C}\right)$$

Consider the following assumptions

Assuption 1 (A1):

$$\begin{aligned}
\lambda P^H\left(2q_H^1\right) q_H^1 + (1-\lambda) P^H\left(q_H^1 + q_*^L\left(q_H^1\right)\right) q_H^1 &> \lambda P^H\left(q_H^2 + q_H^1\right) q_H^2 + (1-\lambda) P^H\left(q_H^{2C} + q_*^L\left(q_L^{2C}\right)\right) q_H^{2C} \\
&= \lambda P^H\left(q_*^H\left(q_H^1\right) + q_H^1\right) q_*^H\left(q_H^1\right) + \\
&\quad + (1-\lambda) P^H\left(q_*^H\left(q_L^{2C}\right) + q_*^L\left(q_L^{2C}\right)\right) q_*^H\left(q_L^{2C}\right)
\end{aligned}$$

Assuption 2 (A2):

$$\begin{aligned}\lambda P^L\left(q_L^1+q_H^1\right)q+(1-\lambda)P^L\left(q_L^1+q_*^L\left(q_L^1\right)\right)q_L^1 &< \lambda P^L\left(q_L^2+q_H^1\right)q_L^2+(1-\lambda)P^L\left(q_L^{2C}+q_*^L\left(q_L^{2C}\right)\right)q_L^{2C} \\ &= \lambda P^L\left(q_*^L\left(q_H^1\right)+q_H^1\right)q_*^L\left(q_H^1\right)+(1-\lambda)P^L\left(2q_L^{2C}\right)q_L^{2C}\end{aligned}$$

Proposition: *If A1 and A2 are satisfied, there is a Weak Perfect Bayesian with the following strategy profiles:*

1. *If firm is High Belief then* :
   (a) *It produces at date 1;*
   (b) *It produces* q_H^1*;*
   (c) *If it did not produced at date 1 and the other firm produced* $\bar{q}$ *at date 1, it would produce at date 2 according to* $q_*^H(\bar{q})$, *at date 2;*
   (d) *If both firms did not produced at date 1, then it would produce* q_H^{2C} *at date 2;*
2. *If firm is Low Belief*
   (a) *It produces at date 2;*
   (b) *If it were to produce at date 1, it would produce* $q_L = q_L^1$*;*
   (c) *If the other firm produces* $\bar{q}$ *at date 1, it will produce at date 2 according to* $q_*^L(\bar{q})$, *at date 2;*
   (d) *If both firms do not produced at date 1, then it will produce* q_L^{2C} *at date 2;*