Scale Effects, Relative Supply of Skills, Industrial Structure and Endogenous Growth

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This paper proposes an analytical mechanism to explain the data on the distribution of firms and production across high- and low-tech manufacturing sectors in a number of developed countries. It builds a model of endogenous directed technical change that considers simultaneously vertical and horizontal R&D, and explores the complementarity between them under distinct regimes of scale effects at the industry level. The model generates a broad set of balanced-growth-path results with respect to the relationship between the relative supply of skills, industrial structure, concentration and economic growth. Under "medium" positive scale effects, the model is able to account for all the empirical regularities.

Keywords: industrial structure, high-tech, low-tech, directed technical change, scale effects, supply of skills

JEL Classification: O41, D43, L11, L16

1. Introduction

The OECD classification of high- and low-tech sectors has been considered in a number of recent studies, namely in order to analyse empirically the impact of technology on industrial performance and globalisation (e.g., Pilat, Cimper, Olsen, and Webb, 2006), and on wage inequality (e.g., Cozzi and Impullitti, 2008). What our paper does is to look at the industrial structure per se, defined as the distribution of firms and production across high- and low-tech sectors. Empirical data (illustrated in Section 2) suggests a

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significant variability of industrial structures across a number of developed countries, considering the number of firms, total production and average firm size in high- vis-à-vis low-tech manufacturing sectors. However, a few regularities can be pointed out. Firstly, the number of firms and total production are systematically smaller in high- than in low-tech sectors, while average firm size is larger in the high-tech sectors. Secondly, the *relative* number of firms (i.e., the ratio of the number of firms in the high- to the low-tech sector) tends to be positively correlated to relative production (idem) across countries, but negatively correlated to relative average firm size (ibidem). Thirdly, by linking the latter evidence to the supply of skills, we find that the relative supply of skills (i.e., the ratio of more skilled workers – e.g., college graduates – to less skilled workers) tends to be positively correlated to the relative number of firms and relative production across countries, but negatively correlated to relative average firm size. Finally, the economic growth rate and a measure of concentration of economic activity tend to be, respectively, positively and negatively correlated to the relative supply of skills.

This paper takes this evidence as a point of departure. The centerpiece of our argument is that because of cross-country differences in the relative supply of skills, the distribution of firms and output across high- and low-tech sectors will also differ. To uncover the analytical mechanism through which the observed patterns in the industrial structure and its relationship to economic growth can be explained by the relative supply of skills, we propose a general equilibrium model of endogenous growth with several key ingredients. We describe them in the paragraphs that follow.

Our model incorporates endogenous directed technical change (e.g, Acemoglu, 1998; Kiley, 1999; Acemoglu and Zilibotti, 2001), such that final-goods production uses either low- or high-skilled labour with labour-specific intermediate goods, while R&D can be directed to either the low- or the high-skilled labour complementary technology; hence, "sector" herein represents a group of firms producing the same type of labour-complementary intermediate goods. As in, e.g., Cozzi and Impullitti (2008), we consider OECD high- and low-tech sectors as the empirical counterpart of the high- and low-skilled labour-complementary intermediate-good sectors.

Following a recent strand of the literature (e.g., Dinopoulos and Thompson, 1998; Peretto, 1998, 1999; Howitt, 1999; Segerstrom, 2000; Peretto and Connolly, 2007), the model considers simultaneously vertical and horizontal R&D. In particular, a quality-ladders mechanism provides the accumulation of non-physical capital (technological knowledge), whilst an expanding-variety mechanism offers the flow of new firms (new product lines). Average firm size is measured by production (sales) per firm, in turn commanded by technological-knowledge stock per firm. This framework makes economic growth and firm dynamics closely related: vertical R&D is the ultimate growth engine, while horizontal R&D builds an explicit link between aggregate and industry-level variables (the number of firms and firm size).

We consider a full lab-equipment R&D specification, whereas the literature predominantly assumes that R&D is knowledge-driven (e.g., Dinopoulos and Thompson, 1998; Peretto, 1998, 1999; Howitt, 1999; Segerstrom, 2000).[1] In the last case, the choice be-

[1] Using Rivera-Batiz and Romer (1991)'s terminology, the assumption that homogeneous final good is

tween vertical and horizontal innovation ultimately implies a division of labour between the two types of R&D. Since the total labour level is determined exogenously, in the end, the rate of extensive growth is exogenous, i.e., the balanced-growth-path (BGP) flow of new goods is a function of population growth. The alternative assumption that R&D is of the lab-equipment type implies the choice between vertical and horizontal innovation is related to the splitting of R&D expenditures, which are fully endogenous. Therefore, we endogenise the rate of extensive growth, and thereby the number of firms in each sector.

Finally, this paper allows for non-null (i.e., strictly positive or strictly negative) scale effects at the industry (sector) level, in which case the relationship between the number of firms and average firm size across sectors becomes endogenous. In lab-equipment models, the scale effects are connected with the size of profits that accrue to the R&D successful firm; a larger market expands profits and, thus, the incentives to allocate resources to R&D, thereby increasing the aggregate growth rate.[2] However, an increase in market scale also dilutes the impact of R&D outlays on innovation probability (e.g., Barro and Sala-i-Martin, 2004), due to a number of costs (e.g., Dinopoulos and Thompson, 1999) and rental protection actions by incumbents (e.g., Sener, 2008) (positively) related to market size. Depending on the effectiveness of the referred costs and actions, these may partially, totally or over counterbalance the benefits of scale to innovative activity.

Accordingly, we consider a priori four distinct regimes of (net) scale effects: "large" positive, "medium" positive, "small" positive, and negative scale effects. By focusing on the BGP, we find that, depending on the regime of scale effects, the relationship between industrial structure and the relative supply of skills may differ significantly. Indeed, our results make clear the contrast between industrial structure under "small"/"medium" positive scale effects and under negative/"large" positive scale effects. For instance, in the former case, the number of firms is greater in the sector with smaller average firm size, implying a right-skewed average-size distribution; in contrast, in the latter case, a left-skewed distribution arises, i.e., there are more firms in the sector with larger average firm size.

The model also provides a theoretical instrument to study the association between the relative supply of skills and the concentration of economic activity in each regime of scale effects. We consider a two-dimension aggregate concentration index by combining the share of the number of firms with the share of the output (sales) of each sector (high- and low-tech), i.e., the index compares sectors at the economy-wide level instead of individual firms within a sector. While the latter is the typical approach in the Industrial Organisation (IO) literature, the aggregate character of our index accommodates the view that traditional industry boundaries become decreasingly useful for economic analysis as new products and processes compete *across* industries (e.g., Kamien and Schwatrz, 1975),

the R&D input means that one adopts the "lab-equipment" version of R&D, instead of the "knowledge-driven" specification, in which labour is ultimately the only input.

[2]The basic theory of industrial structure in the Industrial Organisation literature suggests that concentration is determined by economies of scale relative to the market size. In our model, the role of market size is quite different.

which is a salient feature of our model.[3] Under "small" positive scale effects, the index displays an inverted U-shaped behaviour with respect to the relative supply of skills, with the lower boundary corresponding to *total concentration of firms in one sector and of production in the other* and the upper boundary to a *uniform distribution of firms and production across sectors*. Under either negative or "large"/"medium" positive scale effects, the index displays an U-shaped behaviour, with the lower boundary corresponding to the *uniform-distribution* case and the upper boundary to *total concentration of firms and production in a single sector*.

However, ultimately, only under the "medium" positive regime are the theoretical results able to account simultaneously for all the empirical regularities on industrial structure. While we have no knowledge of any empirical work that explicitly provides a parametrised estimate of the degree of scale effects, well-known studies like, e.g., Backus, Kehoe, and Kehoe (1992), clearly indicate the existence of *positive* scale effects at the industry (manufacturing) level for a large sample of countries.

In recognising the endogenous character of the determinants of industrial structure, the literature has frequently emphasised the interrelation between structure, and technology and innovative activity (e.g., Dasgupta and Stiglitz, 1980; Sutton, 1998). A recent strand of the literature studies the interplay between structure, innovative activity and long-term aggregate growth. Whereas some papers focus on the strategic interaction of firms in an oligopolistic framework (e.g., van de Klundert and Smulders, 1997; Peretto, 1999; Aghion, Bloom, Blundell, Griffith, and Howitt, 2005), others emphasise the role of a specific factor within a monopolistic-competition setting – e.g., the number of firms (Peretto and Smulders, 2002), average firm size (e.g., Peretto, 1998) or firm-size distribution (e.g., Thompson, 2001; Klette and Kortum, 2004). In particular, this paper relates closely to the last set of papers, as it studies the interplay between growth, innovative activity and industrial structure in the context of monopolistic competition, using average firm size as the pivotal variable.

On the other hand, there are several studies analysing the link between the distribution of economic activity across industries and growth, either empirically (e.g., Fagerberg, 2000) or theoretically (e.g., Ngai and Pissarides, 2007; Bonatti and Felice, 2008; Acemoglu and Guerrieri, 2008). They derive the implications of different sectoral Total Factor Productivity (TFP) growth rates, suggesting that countries specialised in "technologically progressive" industries (high TFP growth) enjoy higher growth rates. By analysing the same particular dimension of industrial structure, our paper is also related with this literature. However, by building on a mechanism of endogenous directed technical change, it is substantially different, namely by predicting constant TFP growth rates across sectors along the BGP (see Acemoglu and Zilibotti, 2001). Thus, concerning the link between growth and industrial structure, our results are set in *quantitative* terms, i.e., how many firms and how much production are allocated to each sector vis-à-vis the others, and not in *qualitative* terms, i.e., in which specific sector is economic activity

[3] Moreover, our purpose is to analyse concentration per se, and not to link it to the degree of market power of individual firms, as in the typical IO applications (notice that in our model firms within each sector are homogeneous monopolistic competitors).

concentrated.

The remainder of the paper has the following structure. The next section presents the evidence on the industrial structure (high- vis-à-vis low-tech manufacturing sectors) and the supply of skills for a number of developed countries. In Section 3, we present the model of directed technological change with vertical and horizontal R&D, derive the general equilibrium and analyse the BGP properties. In Section 4, we detail the comparative statics results, deriving predictions with respect to the relationship between BGP industrial structure, relative supply of skills and the regime of scale effects. Section 5 gives some concluding remarks.

2. Empirical evidence: industrial structure and growth

In this section, we present cross-country data with respect to the number of firms, production, and average firm size (production/number of firms) in manufacturing, by considering the OECD classification of high- and low-tech sectors (see Hatzichronoglou, 1997). We also collect data on the relative supply of skills, measured as the ratio of college to non-college graduates among persons employed in manufacturing. "College graduates" refers to those who have completed tertiary education (corresponding to the International Standard Classification of Education [ISCED] levels 5 and 6), while "non-college graduates" here refers to those who have only completed primary or lower-secondary education (ISCED levels 0-2).

The data concerns the 1995-2005 average and covers 23, 16 and 32 European countries regarding, respectively, the number of firms, production and the supply of skills (education attainment). The source is the Eurostat on-line database on Science, Technology and Innovation – tables "Economic statistics on high-tech industries and knowledge-intensive services at the national level" and "Annual data on employment in technology and knowledge-intensive sectors at the national level, by level of education" (available at `http://epp.eurostat.ec.europa.eu`).[4] At the aggregate level, we gather data on per capita GDP growth rates for the same period, also from the Eurostat on-line database.

Using the data on the number of firms and production, we compute an aggregate measure of concentration of economic activity, which combines the share of the number of firms with the share of production of each sector (high- and low-tech). Let $m \in \{L, H\}$ denote a specific sector: either the low- (L) or the high-tech (H) sector ; if u_m represents the share of production in sector m, and o_m represents the share of the number of firms in sector m, then the concentration index is $\mathfrak{I} \equiv u_L \cdot o_L + u_H \cdot o_H$, with $0 \leq \mathfrak{I} \leq 1$.[5]The properties of the proposed concentration index will be explained in Subsection 4.2, in

[4]Data with respect to output (Gross Value Added) is also available from the on-line OECD STAN Indicators Database (link at `http://stats.oecd.org`). According to the latter, the patterns depicted by Figure 1 are also verified for the non-European countries belonging to the OECD, namely the US, Canada, Mexico, Australia, Korea and Japan.

[5]Index $\mathfrak{I}$ can be seen as an adaptation of the well-known Herfindahl index, with the latter applied to a two-sector context, that is, the Herfindahl index calculated by considering the share of output of each sector $m \in \{L, H\}$, such that $\mathfrak{I}_{\mathcal{H}} \equiv u_L^2 + u_H^2$, where $1/2 \leq \mathfrak{I}_H \leq 1$. For other uses of the Herfindahl index at the aggregate level, see, e.g., Thompson (2001) and Laincz (2009).

light of the developed theoretical model.

While empirical data, as illustrated by Figures 1 and 2, suggests a significant variability of industrial structures across countries by considering the number of firms, total production and average firm size in high- vis-à-vis low-tech sectors, interesting regularities stand out: (i) the number of firms and total production are smaller in high- than in low-tech sectors (i.e., the *relative* number of firms and relative production are below unity) in all countries; (ii) average firm size is larger in the high-tech sectors (i.e., *relative* average firm size is above unity) in all countries; (iii) the relative number of firms and relative production tend to be positively correlated to the relative supply of skills; (iv) the relative average firm size tends to be negatively correlated to the relative supply of skills. The evidence in (iii) and (iv) is reported in logs (see Figure 2), since, as it will be shown in Section 3, our theoretical model predicts a log-log linear relationship between the relative number of firms, relative production, relative firm size and the relative supply of skills in BGP. As regards concentration, our index always takes values between 0.5 and 1, which means, as shown in Subsection 4.2, that firms and production tend to concentrate in the same sector within each country in the sample. Furthermore, the concentration index tends to be negatively correlated to the relative supply of skills, while the latter is positively correlated to the economic growth rate.

[Figure 1 goes about here]

[Figure 2 goes about here]

Observe that, in order to get a clearer picture of the behaviour of the number of firms, total production and firm size across sectors and countries, we only focus on the high- and low-tech sectors, which are the extreme OECD categories for manufacturing. That is, we leave out of our empirical illustration the medium-high and the medium-low categories also considered in the OECD classification. Likewise, concerning the supply of skills, we leave out of our analysis the upper secondary/post secondary education levels (ISCED levels 3-4). This way, we avoid the arbitrariness that involves the computation of the ratios when all categories are considered, since the theory offers no guidance on whether a specific intermediate category should incorporate the denominator or the numerator.[6]

To sum up, although the data above is limited and essentially of qualitative nature, and although there are many factors that may explain the observed patterns of industrial structure, empirical evidence suggests that we should be thinking of explanations featuring differences in the relative supply of skills. In the next section, the analytical mechanism behind the described patterns of industrial structure is analysed by building on an analytically tractable theoretical model that dichotomises between “high-tech” and ”low-tech” sectors, and between “high-skilled” and ”low-skilled” labour.

[6] As soon as all categories are considered, a number of alternative ratios are a priori admissible. However, the cross-country patterns exhibited by these ratios may differ considerably. The issue of the arbitrary choice involved in breaking the population into two skill groups has been addressed by, e.g., Acemoglu and Zilibotti (2001).

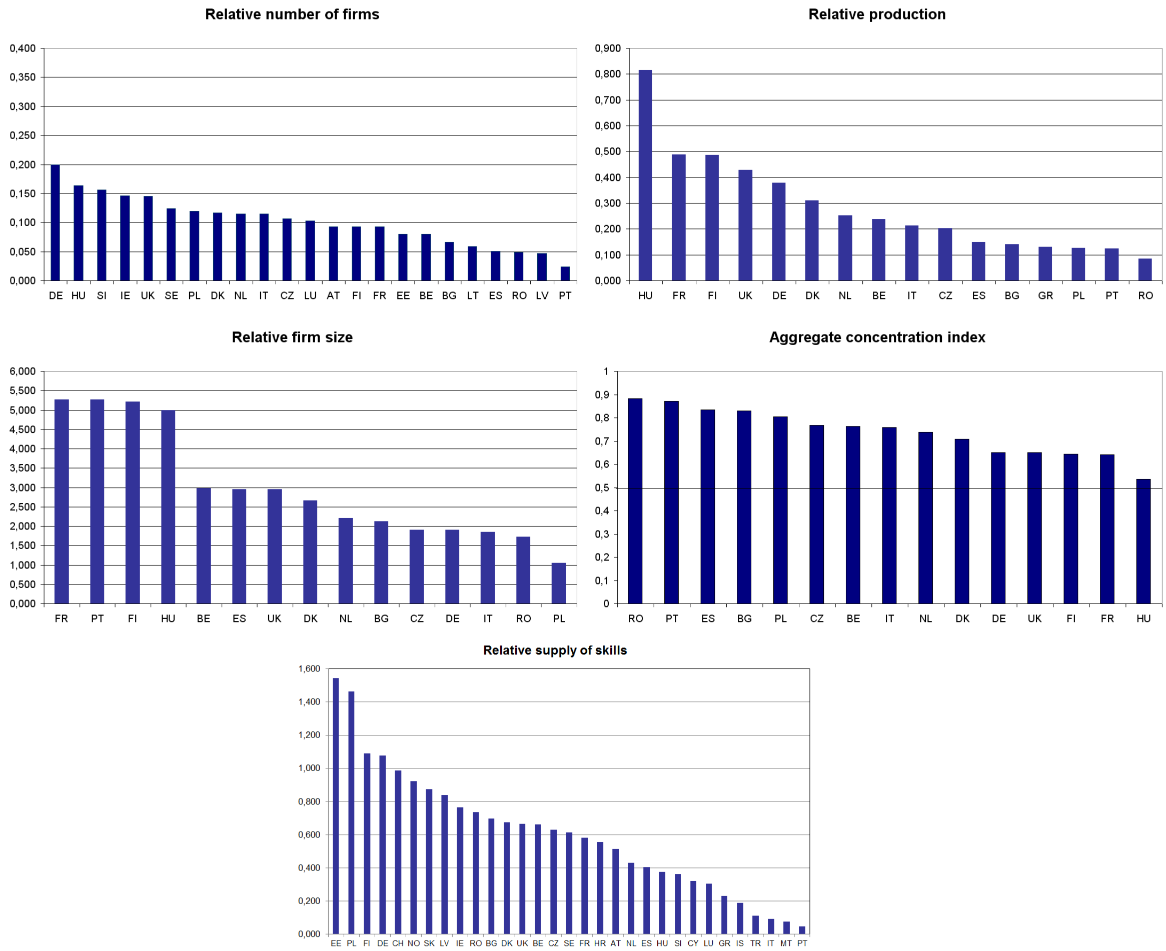

Figure 1: The relative number of firms, relative production, relative average firm size, aggregate concentration index, and relative supply of skills in a sample of European countries, 1995-2005 average.

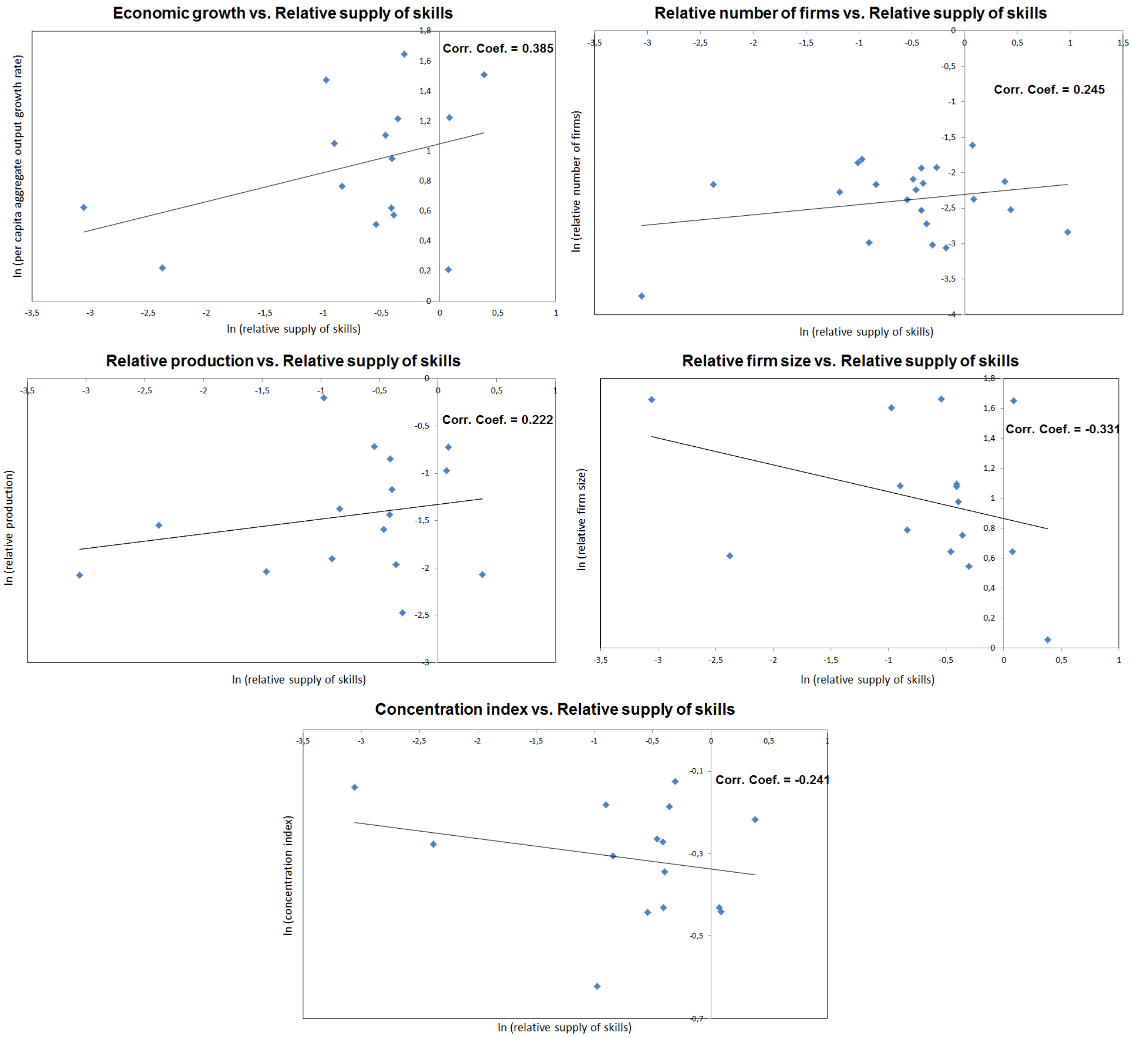

Figure 2: The economic growth rate, relative number of firms, relative production, relative average firm size and concentration index vis-à-vis the relative supply of skills – cross-country correlations in a sample of European countries, 1995-2005 average.

3. The model

The model used herein is well-known from Acemoglu and Zilibotti (2001), augmented with vertical R&D, as introduced in Afonso (2006), and developed under varying regimes of scale effects. Thus, we study a directed technological change model with vertical and horizontal R&D, built into a closed-economy dynamic general equilibrium setup where a single competitively-produced final good can be used in consumption, production of intermediate goods and R&D. The economy is populated by a fixed number of infinitely-lived households who inelastically supply one of two types of labour to final-good firms: low-skilled, L, and high-skilled labour, H. The final good is produced by a continuum of firms, indexed by $n \in [0,1]$, to which two substitute technologies are available: the "Low" (respectively, "High") technology uses a combination of L (H) and a continuum of L-(H-)specific intermediate goods indexed by $\omega_L \in [0, N_L]$ ($\omega_H \in [0, N_H]$).

Potential entrants can devote resources to either horizontal or vertical R&D, and directed to either the high- or the low-skilled labour-complementary technology. Horizontal R&D increases the number of industries, N_m, $m \in \{L, H\}$, in the m-complementary intermediate-good sector,[7] while vertical R&D increases the quality level of the good of an existing industry, indexed by $j_m(\omega_m)$. Then, the quality level $j_m(\omega_m)$ translates into productivity of the final producer from using the good produced by industry ω_m, $\lambda^{j_m(\omega_m)}$, where $\lambda > 1$ measures the size of each quality upgrade. By improving on the current best quality j_m, a successful R&D firm will introduce the leading-edge quality $j_m(\omega_m)+1$ and thus render inefficient the existing input. Hence, the monopoly in ω_m is temporary.

3.1. Production and price decisions

This section briefly describes the familiar components of Acemoglu and Zilibotti's (2001) model, augmented with vertical R&D. Aggregate output at time t is defined as $Y_{tot}(t) = \int_0^1 P(n,t)Y(n,t)dn$. Each final-good firm n has a constant-returns-to-scale technology using low- and high-skilled labour and a continuum of labour-specific intermediate goods with measure $N_m(t)$, $m \in \{L, H\}$, so that $N_{tot}(t) = N_L(t) + N_H(t)$ and

$$\begin{array}{rcl} Y(n,t) & = & A\left[\int_0^{N_L(t)} \left(\lambda^{j_L(\omega_L,t)} \cdot X_L(n,\omega_L,t)\right)^{1-\alpha} d\omega_L\right] [(1-n)\cdot L(n)]^{\alpha} + \\ & & +A\left[\int_0^{N_H(t)} \left(\lambda^{j_H(\omega_H,t)} \cdot X_H(n,\omega_H,t)\right)^{1-\alpha} d\omega_H\right] [n \cdot H(n)]^{\alpha} \end{array}, \quad 0<\alpha<1, \tag{1}$$

where $A > 0$ is the total factor productivity, $L(n)$ and $H(n)$ are the labour inputs used by n and α is the labour share in production, and $\lambda^{j_m(\omega_m,t)} \cdot X_m(n, \omega_m, t)$ is the input of m-complementary intermediate good ω_m measured in efficiency units at time t.[8] A relative productivity advantage of each labour type is determined by terms n and $(1-n)$, implying that H is relatively more productive for larger n, and vice-versa. As explained below, at each t, there is a competitive equilibrium threshold $\bar{n}$, which is endogenously determined, where the switch from one technology to the other becomes advantageous,

[7] Henceforth, we will refer to the "m-complementary intermediate-good sector" as "sector m".

[8] In equilibrium, only the top quality of each ω_m is produced and used; thus, $X_m(j, \omega_m, t) = X_m(\omega_m, t)$.

so that each firm n produces exclusively with one technology, either L- or H-technology.

Final producers take the price of their final good, $P(n,t)$, wages, $w_m(t)$, and input prices $p_m(\omega_m,t)$ as given. From the profit maximisation conditions, the demand of intermediate good ω_m by firm n is

$$\begin{aligned} X_L(n,\omega_L,t) &= (1-n)\cdot L(n)\cdot\left[\tfrac{A\cdot P(n,t)\cdot(1-\alpha)}{p_L(\omega_L,t)}\right]^{\frac{1}{\alpha}}\lambda^{j_L(\omega_L,t)\left(\frac{1-\alpha}{\alpha}\right)} \\ X_H(n,\omega_H,t) &= n\cdot H(n)\cdot\left[\tfrac{A\cdot P(n,t)\cdot(1-\alpha)}{p_H(\omega_H,t)}\right]^{\frac{1}{\alpha}}\lambda^{j_H(\omega_H,t)\left(\frac{1-\alpha}{\alpha}\right)} \end{aligned}. \tag{2}$$

Intermediate-good sector m consists of a continuum $N_m(t)$ of industries. There is monopolistic competition if we consider the whole sector: the monopolist in industry $\omega_m\in[0,N_m(t)]$ fixes the price $p_m(\omega_m,t)$ but faces an isoelastic demand curve, $X_L(\omega_L,t)=\int_0^{\bar{n}}X_L(n,\omega_L,t)dn$ or $X_H(\omega_H,t)=\int_{\bar{n}}^1 X_H(n,\omega_H,t)dn$ (see (2)). Intermediate goods are non-durable and entail a unit marginal cost of production, in terms of the final good, whose price is taken as given. Profit in ω_m is thus $\pi_m(\omega_m,t)=(p_m(\omega_m,t)-1)\cdot X_m(\omega_m,t)$, and the profit maximising price is a constant markup over marginal cost,

$$p_m(\omega_m,t)\equiv p=\frac{1}{1-\alpha}>1,\ m\in\{L,H\}. \tag{3}$$

Given $\bar{n}$ and (3), the final-good output can be rewritten as

$$Y(n,t)=\begin{cases} A^{\frac{1}{\alpha}}P(n,t)^{\frac{1-\alpha}{\alpha}}\cdot(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}}\cdot(1-n)\cdot L(n)\cdot Q_L(t) & ,\,0\le n\le\bar{n} \\ A^{\frac{1}{\alpha}}P(n,t)^{\frac{1-\alpha}{\alpha}}\cdot(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}}\cdot n\cdot H(n)\cdot Q_H(t) & ,\,\bar{n}\le n\le 1 \end{cases}, \tag{4}$$

where the aggregate quality index

$$Q_m(t)=\int_0^{N_m(t)}q_m(\omega_m,t)d\omega_m,\ q_m(\omega_m,t)\equiv\lambda^{j_m(\omega_m,t)\left(\frac{1-\alpha}{\alpha}\right)},\ m\in\{L,H\}, \tag{5}$$

measures the technological-knowledge level in each sector m. With competitive final-good producers, economic viability of either L- or H-technology relies on the relative productivity and price of labour, as well as on the relative productivity and prices of intermediate goods, due to complementarity in production. The endogenous threshold $\bar{n}$ then follows from equilibrium in the inputs markets, and relies on the determinants of economic viability of the two technologies, such that $\bar{n}(t)=1/\left[1+(H/L\cdot Q_H(t)/Q_L(t))^{1/2}\right]$. $\bar{n}$ implies that L- (H-)complementary technology is exclusively used by final-good firms indexed by $n\in[0,\bar{n}]$ ($n\in[\bar{n},1]$), and it can be related to the ratio of price indices of final goods produced with L- and H-technologies,

$$\frac{P_H(t)}{P_L(t)}=\left(\frac{\bar{n}(t)}{1-\bar{n}(t)}\right)^{\alpha},\ \text{where}\ \begin{cases} P_L(t)=P(n,t)\cdot(1-n)^{\alpha}=e^{-\alpha}\bar{n}(t)^{-\alpha} \\ P_H(t)=P(n,t)\cdot n^{\alpha}=e^{-\alpha}(1-\bar{n}(t))^{-\alpha} \end{cases}. \tag{6}$$

To get (6), one first defines the price indices, P_L and P_H, by recognising that, in equilibrium, the marginal value product, $\frac{\partial}{\partial m(n)}\left(P(n,t)Y(n,t)\right)$, must be constant over n, implying that $P(n,t)^{\frac{1}{\alpha}} \cdot (1-n)$ and $P(n,t)^{\frac{1}{\alpha}} \cdot n$ must be constant over $n \in [0,\bar{n}]$ and $n \in [\bar{n},1]$, respectively. Then, by considering that at $\bar{n}$ the L- and the H- technology firms must break even, P_L and P_H are related with $\bar{n}$.

From (2), (3) and (6), the optimal intermediate-good production , $X_m(\omega_m)$, is derived and, thus, the optimal profit accrued by the monopolist in ω_m is

$$\pi_m(\omega_m,t) = \pi_0 \cdot m \cdot P_m(t)^{\frac{1}{\alpha}} \cdot q_m(\omega_m,t) \ \ , \ m \in \{L,H\}\,, \tag{7}$$

where $\pi_0 \equiv A^{\frac{1}{\alpha}}\,(1-\alpha)^{\frac{2}{\alpha}}\,\alpha/(1-\alpha)$ is a positive constant. Total intermediate-good optimal production, $X_{tot} \equiv X_L + X_H \equiv \int_0^{N_L} X_L(\omega_L)d\omega_L + \int_0^{N_H} X_H(\omega_H)d\omega_H$, and total final-good optimal production, $Y_{tot} \equiv Y_L + Y_H \equiv \int_0^{\bar{n}} P(n)Y(n)dn + \int_{\bar{n}}^1 P(n)Y(n)dn$, are, respectively,

$$X_{tot}(t) = A^{\frac{1}{\alpha}} \cdot (1-\alpha)^{\frac{2}{\alpha}} \cdot \left(P_L(t)^{\frac{1}{\alpha}} \cdot L \cdot Q_L(t) + P_H(t)^{\frac{1}{\alpha}} \cdot H \cdot Q_H(t)\right) \tag{8}$$

and

$$Y_{tot}(t) = A^{\frac{1}{\alpha}} \cdot (1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot \left(P_L(t)^{\frac{1}{\alpha}} \cdot L \cdot Q_L(t) + P_H(t)^{\frac{1}{\alpha}} \cdot H \cdot Q_H(t)\right). \tag{9}$$

3.2. R&D

We consider two R&D sectors, one targeting vertical innovation and the other targeting horizontal innovation. We assume that the pools of innovators performing the two types of R&D are different. We also take the simplifying assumptions that both vertical and horizontal R&D are performed by (potential) entrants, and that successful R&D leads to the set-up of a new firm in either an existing or in a new industry (e.g., Howitt, 1999; Segerstrom, 2000; Cozzi and Spinesi, 2006; Strulik, 2007; Gil, Brito, and Afonso, 2012). There is perfect competition among entrants and free entry in R&D business.

3.2.1. Vertical R&D

As is common in the literature (e.g., Aghion and Howitt, 1992; Barro and Sala-i-Martin, 2004, ch. 7),, each new design is granted a patent and thus a successful innovator retains exclusive rights over the use of his/her good. By improving on the current top quality level $j_m(\omega_m,t)$, a successful R&D firm earns monopoly profits from selling the leading-edge input of $j_m(\omega_m,t)+1$ quality to final-good firms. A successful innovation will instantaneously increase the quality index in ω_m from $q_m(\omega_m,t) = q_m(j_m)$ to $q_m^+(\omega_m,t) = q_m(j_m+1) = \lambda^{(1-\alpha)/\alpha} q_m(\omega_m,t)$. In equilibrium, lower qualities of ω_m are priced out of business.

Let $I_m^i(j_m)$ denote the Poisson arrival rate of vertical innovations (vertical-innovation rate) by potential entrant i in industry ω_m, at a cost of $\Phi_m(j_m)$ units of the final good, when the highest quality is j_m. The rate $I_m^i(j_m)$ is independently distributed across firms, across industries and over time, and depends on the flow of resources $R_{v,m}^i(j_m)$ committed by entrants at time t. As in, e.g., Barro and Sala-i-Martin (2004, ch. 7),$I_m^i(j_m)$ features constant returns in R&D expenditures, $I_m^i(j_m) = R_{v,m}^i(j_m)/\Phi_m(j_m)$. The cost $\Phi_m(j_m)$ is assumed to be symmetric within sector m, such that

$$\Phi_m(j_m) = \zeta \cdot m^\epsilon \cdot q_m(j_m + 1), \; m \in \{L, H\}, \tag{10}$$

where $\zeta \equiv \zeta_L = \zeta_H > 0$ is a constant fixed (flow) cost, and $\epsilon \geq 0$. Equation (10) incorporates a complexity effect (e.g., Barro and Sala-i-Martin, 2004, ch. 7; Etro, 2008), implying vertical-R&D dynamic decreasing returns to scale (i.e., decreasing returns to cumulated R&D). That is, the larger the level of quality, q_m, the costlier it is to introduce a further jump in quality.[9] Moreover, (10) also implies that an increase in market scale, m, dilutes the effect of R&D outlays on innovation probability. This captures the idea that the difficulty of introducing new qualities and replacing old ones is positively related to the market size measured by employed labour (e.g., Barro and Sala-i-Martin, 2004), due to coordination, organisational and transportation costs (e.g., Dinopoulos and Thompson, 1999), as well as rental protection actions by incumbents (e.g., Sener, 2008);[10] however, depending on the effectiveness of that costs and actions, they may partially ($0 < \epsilon < 1$), totally ($\epsilon = 1$) or over ($\epsilon > 1$) counterbalance the scale benefits on profits (see (7)), which accrue to the R&D successful firm each t. Thus, as shown later, there may be, respectively, positive, null or negative net scale effects on economic growth, as measured by $1 - \epsilon$. Aggregating across i in ω_m, we get $R_{v,m}(j_m) = \sum_i R_{v,m}^i(j_m)$ and $I_m(j_m) = \sum_i I_m^i(j_m)$, and thus

$$I_m(j_m) = R_{v,m}(j_m) \cdot \frac{1}{\zeta \cdot m^\epsilon \cdot q_m(j_m + 1)}, \; m \in \{L, H\}. \tag{11}$$

As the terminal date of each monopoly arrives as a Poisson process with frequency $I_m(j_m)$ per (infinitesimal) increment of time, the present value of a monopolist's profits is a random variable. Let $V_m(j_m)$ denote the expected value of an incumbent with current quality level $j_m(\omega_m, t)$,[11]

[9] As usual in the literature, the fact that Φ_m depends linearly on q_m implies that the increasing difficulty of creating new product generations over t exactly offsets the increased rewards from marketing higher quality products; see (10) and (7). This allows for constant vertical-innovation rate over t and across ω_m in BGP (on *asymmetric* equilibrium in quality-ladders models and its growth consequences, see Cozzi, 2007).

[10] Sener (2008) contrasts the effects of rental protection actions with the expanding variety and the dynamic decreasing returns to R&D as scale-removal mechanisms within a quality-ladders model with knowledge-driven R&D specification. Observe, however, that the dynamic decreasing returns to R&D, as first introduced by Segerstrom (1998), and represented by the term $1/q_m(j_m + 1)$ in (10), are neither necessary nor sufficient for the purpose of scale removal in a model with lab-equipment specification (though it plays a crucial role in guaranteeing a Poisson rate constant over ω_m and hence the existence of a symmetric equilibrium; see fn. 9). The same applies to the expanding variety mechanism, as it is clear if we let $\epsilon = 0$ in our results below.

[11] We assume that entrants are risk-neutral and, thus, only care about the expected value of the firm.

$$V_m(j_m) = \pi_0 \cdot m \cdot q_m(j_m) \int_t^{\infty} P_m(s)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s (r(v)+I_m(j_m(v)))dv} ds,\ m \in \{L, H\}, \quad (12)$$

where r is the equilibrium market real interest rate, and $\pi_0 m q_m(j_m) = \pi_m(j_m) P_m^{-\frac{1}{\alpha}}$, given by (7) and (6), is constant in-between innovations. Free-entry prevails in vertical R&D such that the condition $I_m(j_m) \cdot V_m(j_m+1) = R_{v,m}(j_m)$ holds, which implies that

$$V_m(j_m+1) = \Phi_m(j_m) = \zeta \cdot m^{\epsilon} \cdot q_m(j_m+1),\ m \in \{L, H\}. \quad (13)$$

Next, we determine $V_m(j_m+1)$ analogously to (12), then consider (13) and time-differentiate the resulting expression. Therefore, if we also consider (7), we get the arbitrage condition facing a vertical innovator

$$\zeta = \frac{\pi_0 \cdot L^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}}}{r(t) + I_L(t)} = \frac{\pi_0 \cdot H^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}}}{r(t) + I_H(t)}, \quad (14)$$

It has two implications:[12] the present value of "basic" profits $\pi_0 \cdot m^{1-\epsilon} \cdot P_m(t)^{\frac{1}{\alpha}}$, using the effective rate of interest $r + I_m$ as a discount factor, should be equal to the fixed cost of entry; and the rates of entry are symmetric across industries $I_m(\omega_m, t) = I_m(t)$.

After solving equation (11) for $R_{v,m}(\omega_m, t) = R_{v,m}(j_m)$ and aggregating across industries ω_m, we determine total resources devoted to vertical R&D, $R_{v,m}(t)$, $R_{v,m}(t) = \int_0^{N_m(t)} R_{v,m}(\omega_m, t)\, d\omega_m = \int_0^{N_m(t)} \zeta \cdot m^{\epsilon} \cdot q_m^{+}(\omega_m, t) \cdot I_m(\omega_m, t)\, d\omega_m$. As the innovation rate is industry independent, then

$$R_{v,m}(t) = \zeta \cdot m^{\epsilon} \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_m(t) \cdot Q_m(t),\ m \in \{L, H\}. \quad (15)$$

3.2.2. Horizontal R&D

Variety expansion arises from R&D aimed at creating a new intermediate good. Again, innovation is performed by a potential entrant and there is free entry; thus, the new good is produced by new firms. Under perfect competition and constant returns to scale at the firm level, instantaneous entry is obtained as $\dot{N}_m^e(t) = R_{h,m}^e(t)/\eta_m(t)$, where $\dot{N}_m^e$ is the contribution to the instantaneous flow of new m-complementary intermediate goods by R&D firm e at a cost of η_m units of the final good and $R_{h,m}^e$ is the flow of resources devoted to horizontal R&D by innovator e at time t. The cost η_m is assumed

[12] Observe that, from (7) and (11), we have $\frac{\dot{\pi}_m(\omega_m,t)}{\pi_m(\omega_m,t)} - \frac{1}{\alpha}\frac{\dot{P}_m(t)}{P_m(t)} = I_m(\omega_m,t) \cdot \left[\dot{j}_m(\omega_m,t) \cdot \left(\frac{\alpha}{1-\alpha}\right) \cdot \ln\lambda\right]$ and $\frac{\dot{R}_{vm}(\omega_m,t)}{R_{vm}(\omega_m,t)} - \frac{\dot{I}_m(\omega_m,t)}{I_m(\omega_m,t)} = I_m(\omega_m,t) \cdot \left[\dot{j}_m(\omega_m,t) \cdot \left(\frac{\alpha}{1-\alpha}\right) \cdot \ln\lambda\right]$. Thus, if we time-differentiate (13) by considering (12) and the equations above, we get $r(t) = \frac{\pi_m(j_m+1) \cdot I_m(j_m)}{R_{vm}(j_m)} - I_m(j_m+1)$, which can then be re-written as (14).

to be symmetric within sector m. Then, $R_{h,m}(t) = \sum_e R^e_{h,m}(t)$ and $\dot{N}_m(t) = \sum_e \dot{N}^e_m(t)$, implying

$$R_{h,m}(t) = \eta_m(t) \cdot \dot{N}_m(t),\ m \in \{L, H\}\,. \tag{16}$$

We assume that the cost of setting up a new variety (cost of horizontal entry) is increasing in the number of existing varieties, N_m,

$$\eta_m(t) = \phi \cdot m^{\delta} \cdot N_m(t)^{\sigma},\ m \in \{L, H\}\,, \tag{17}$$

where $\phi \equiv \phi_L = \phi_H > 0$ is a constant fixed (flow) cost, and $\sigma > 0$ and $\delta \geq 0$. Through the dependence of η_m on N_m, equation (17) incorporates a complexity effect (e.g., Evans, Honkapohja, and Romer, 1998; Barro and Sala-i-Martin, 2004, ch. 6), implying horizontal-R&D dynamic decreasing returns to scale. That is, the larger the number of existing varieties, the costlier it is to introduce new varieties. Moreover, (17) also implies that an increase in market scale, m, dilutes the effect of R&D outlays on the innovation rate. Again, this may happen due to coordination, organisational and transportation costs related to market size (e.g., Dinopoulos and Thompson, 1999), which may partially ($0 < \delta < 1$), totally ($\delta = 1$) or over ($\delta > 1$) counterbalance the scale benefits on profits.

Each horizontal innovation results in a new intermediate good whose quality level is drawn randomly from the distribution of existing varieties (e.g., Dinopoulos and Thompson, 1998; Howitt, 1999). Thus, the expected quality level of the horizontal innovator is

$$\bar{q}_m(t) = \int_0^{N_m(t)} \frac{q_m(\omega_m, t)}{N_m(t)} d\omega_m = \frac{Q_m(t)}{N_m(t)},\ m \in \{L, H\}\,, \tag{18}$$

As his/her monopoly power will be also terminated by the arrival of a successful vertical innovator in the future, the benefits from entry are

$$V_m(\bar{q}_m) = \pi_0 \cdot m \cdot \bar{q}_m(t) \int_t^{\infty} P_m(s)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s [r(\nu) + I_m(\bar{q}_m(v))]d\nu} ds,\ m \in \{L, H\}\,, \tag{19}$$

where $\pi_0 m \bar{q}_m = \bar{\pi}_m P_m^{-\frac{1}{\alpha}}$. The free-entry condition, $\dot{N}_m \cdot V(\bar{q}_m) = R_{hm}$, by (16), simplifies to

$$V_m(\bar{q}_m) = \eta_m(t),\ m \in \{L, H\}\,. \tag{20}$$

Substituting (19) into (20) and time-differentiating the resulting expression, yields the arbitrage equation facing a horizontal innovator

$$r\,(t) + I_m(t) = \frac{\bar{\pi}_m(t)}{\eta_m\,(t)},\ m \in \{L, H\}\,. \tag{21}$$

3.2.3. No-arbitrage arbitrage conditions

No-arbitrage in the capital market requires that the two types of investment – vertical and horizontal R&D – yield equal rates of return; otherwise, one type of investment dominates the other and a corner solution obtains. Thus, if we equate the effective rate of return $r + I_m$ for both types of entry, from (14) and (21), we get the *intra-sector* no-arbitrage conditions

$$\bar{q}_m(t) = \frac{Q_m(t)}{N_m(t)} = \frac{\eta_m(t)}{\zeta \cdot m^{\epsilon}}, \; m \in \{L, H\} \tag{22}$$

which are a key ingredient of the model. They equate the cost of the horizontal, η_m, to the average cost of vertical R&D, $\bar{q}_m \zeta m^{\epsilon}$.

Following a similar reasoning, if we equate the effective rate of return for both sectors, e.g., by considering (14), an *inter-sector* no-arbitrage condition obtains

$$I_H(t) - I_L(t) = \frac{\pi_0}{\zeta} \left(H^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}} - L^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}} \right). \tag{23}$$

3.3. Households

The economy is populated by a fixed number of infinitely-lived households who consume and collect income from investments in financial assets (equity) and from labour. Households inelastically supply low-skilled, L, or high-skilled labour, H. Thus, total labour supply, $L + H$, is exogenous and constant. We assume consumers have perfect foresight concerning the technological change over time and choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximise discounted lifetime utility

$$U = \int_0^{\infty} \left(\frac{C(t)^{1-\theta} - 1}{1 - \theta} \right) e^{-\rho t} dt, \tag{24}$$

where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, subject to the flow budget constraint

$$\dot{a}(t) = r(t) \cdot a(t) + w_L(t) \cdot L + w_H(t) \cdot H - C(t), \tag{25}$$

where a denotes households' real financial assets holdings. The initial level of wealth $a(0)$ is given and the non-Ponzi games condition $lim_{t\rightarrow\infty} e^{-\int_0^t r(s)ds} a(t) \geq 0$ is imposed. The optimal consumption path Euler equation and the transversality condition are standard,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} \cdot (r(t) - \rho) \tag{26}$$

$$\underset{t\rightarrow\infty}{lim}\, e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0. \tag{27}$$

3.4. The balanced-growth path

The aggregate financial wealth held by all households is $a(t) = a_L(t)+a_H(t) = \sum_{m=L,H} \int_0^{N_m(t)} V_m(\omega_m, t)d\omega_m$ which, from the arbitrage condition between vertical and horizontal entry, yields $a(t) = \eta_L(t) \cdot N_L(t) + \eta_H(t) \cdot N_H(t)$. Taking time derivatives and comparing with (25), we get an expression for the aggregate flow budget constraint which is equivalent to the product market equilibrium condition

$$Y_{tot}(t) = X_{tot}(t) + C(t) + R_h(t) + R_v(t) \tag{28}$$

where $R_h = R_{h,L} + R_{h,H}$ and $R_v = R_{v,L} + R_{v,H}$.

The dynamic general equilibrium is defined by the allocation $\{X_m(\omega_m, t), \omega_m \in [0, N_m(t)], t \geq 0, m \in \{L, H\}\}$, by the prices $\{p_m(\omega_m, t), \omega_m \in [0, N_m(t)], t \geq 0, m \in \{L, H\}\}$ and by the aggregate paths $\{C(t), N_m(t), \; Q_m(t), I_m(t), r(t), t \geq 0, m \in \{L, H\}\}$, such that: (i) consumers, final-good firms and intermediate-good firms solve their problems; (ii) free-entry and no-arbitrage conditions are met; and (iii) markets clear.

We now derive and characterise the BGP. Let $g_y \equiv \dot{y}/y$ represent the growth rate of $y(t)$. Along the BGP, the aggregate resource constraint (28) is satisfied with Y_{tot}, X_{tot}, C, R_v and R_n growing at the same constant rate. By considering (9) and by time-differentiating (22), the following necessary conditions for the existence of a BGP are derived: (i) the asymptotic growth rates of consumption and of the quality indices are constant and equal to the economic growth rate, $g_C = g_{Q_L} = g_{Q_H} = g$; (ii) the asymptotic growth rates of the number of varieties are constant and equal, $g_{N_L} = g_{N_H}$; (iii) the vertical-innovation rates and the final-good price indices are asymptotically trendless, $g_{I_L} = g_{I_H} = g_{P_L} = g_{P_H} = 0$; and (iv) the asymptotic growth rates of the quality indices and the number of varieties are monotonously related, $g_{Q_L}/g_{N_L} = g_{Q_H}/g_{N_H} = (\sigma + 1)$, $g_{N_m} \neq 0$, $m \in \{L, H\}$.

It results from (i) and (ii) that $I_L = I_H$ along the BGP. Using this in (23) to derive P_H/P_L, and then the latter in (6) to solve in order to $Q \equiv Q_H/Q_L$, we get

$$\tilde{Q} = \left(\frac{H}{L}\right)^{1-2\epsilon}. \tag{29}$$

On the other hand, if we assume that the number of industries, N, is large enough to treat Q as time-differentiable and non-stochastic, then we can time-differentiate (18) to get $\dot{Q}_m(t) = \int_0^{N_m(t)} \dot{q}(\omega, t)d\omega + q(N, t)\dot{N}(t)$. After some algebraic manipulation of the latter, we can write, for $I_m > 0$,

$$g_{Q_m} = \Xi \cdot I_m + g_{N_m} \; m \in \{L, H\}. \tag{30}$$

where $\Xi \equiv \left(\lambda^{\frac{1-\alpha}{\alpha}} - 1\right)$ denotes the quality shift. Next, solve (26) with respect to r and note that, along the BGP, $g_C = g_{Q_L} = g_{Q_H} = g$, to get $r = \rho + \theta g$. The latter, combined with $g = (\sigma+1) \cdot g_{N_L} = (\sigma+1) \cdot g_{N_H}$, (30) and (14), and under the condition $\tilde{r}_{0m} - \rho > 0$, yields the positive endogenous growth rates

$$\tilde{g}_{Q_L} = \tilde{g}_{Q_H} = \tilde{g} = \frac{\frac{\Xi}{\theta}\left(\tilde{r}_{0L} - \rho\right)(\sigma + 1)}{\Xi(\sigma + 1) + \frac{1}{\theta}\sigma} > 0, \tag{31}$$

$$\tilde{g}_{N_L} = \tilde{g}_{N_H} = \frac{\frac{\Xi}{\theta}\left(\tilde{r}_{0L} - \rho\right)}{\Xi(\sigma + 1) + \frac{1}{\theta}\sigma} > 0, \tag{32}$$

$$\tilde{I}_L = \tilde{I}_H = \frac{\sigma}{\Xi}\tilde{g}_{N_L} = \frac{\sigma}{\Xi}\tilde{g}_{N_H} > 0, \tag{33}$$

where $\sim$ denotes a steady-state value, $\tilde{r}_{0L} \equiv \pi_0 L^{1-\epsilon}\tilde{P}_L^{\frac{1}{\alpha}}/\zeta = \tilde{r}_{0H} \equiv \pi_0 H^{1-\epsilon}\tilde{P}_H^{\frac{1}{\alpha}}/\zeta$, $\tilde{P}_L = e^{-\alpha}\left[1 + (H/L)^{1-\epsilon}\right]^{\alpha}$ and $\tilde{P}_H = e^{-\alpha}\left[1 + (H/L)^{\epsilon-1}\right]^{\alpha}$. Thus, our model predicts, under a sufficiently productive technology, a BGP with constant positive growth rates, g and g_{N_m}, where the former exceeds the latter by the growth of intermediate-good quality; to verify this, just use (18) to get to get $\dot{Q}_m/Q_m - \dot{N}_m/N_m = I_m \cdot \Xi$, which is positive if $I_m > 0$. This implies that the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the well-known view that industrial growth proceeds both along an intensive and an extensive margin. Variety expansion is sustained by endogenous technological-knowledge accumulation (independently of population growth), as the expected growth of intermediate-good quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an entry cost, in spite of its increase with N_m. Moreover, it is clear from (31) that there may be positive, null or negative net scale effects on economic growth, as measured by $1 - \epsilon$.

From the expressions for X_L and X_H (see (8)) and for N_L and N_H (see (22)), combined with (29), we derive the expressions for relative intermediate-good output and the relative number of firms (i.e., sector H vis-à-vis sector L),

$$\tilde{X} \equiv \left(\frac{\tilde{X_H}}{X_L}\right) = \left(\frac{H}{L}\right)^{1-\epsilon}, \tag{34}$$

$$\tilde{N} \equiv \left(\frac{\tilde{N_H}}{N_L}\right) = \left(\frac{H}{L}\right)^{\frac{1-\epsilon-\delta}{\sigma+1}}. \tag{35}$$

4. Relative supply of skills, industrial structure, and growth

Since we wish to confront our theoretical results with the data on production for a number of countries and data is presented in a quality-adjusted base by the national statistics offices (see, e.g., Eurostat, 2001), we find it convenient to compute production also in quality-adjusted terms. Reiterating the steps as in Subsection 3.1, we find total intermediate-good quality-adjusted production to be (e.g., with $m = L$) $\mathfrak{X}_L =$

$\int_0^{N_L}\int_0^{\bar{n}} \lambda^{j_L(\omega_L)} \cdot X_L(n,\omega_L) dn d\omega_L = A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{2}{\alpha}} P_L^{\frac{1}{\alpha}} L\breve{Q}_L$, where $\breve{Q}_L = \int_0^{N_L} \lambda^{j_L(\omega_L)\frac{1}{\alpha}} d\omega_L$, and $\mathfrak{X}_{tot} = \mathfrak{X}_L + \mathfrak{X}_H$. We cannot find an explicit algebraic expression for the BGP value of $\breve{Q}_m$. However, as shown in Appendix A, we can build an adequate proxy for $\breve{Q}_m$ as a function of Q_m and N_m, which is $\widehat{\breve{Q}_m} = Q_m^{\frac{1}{1-\alpha}} \cdot N_m^{-\left(\frac{\alpha}{1-\alpha}\right)}$. Accordingly, we define the proxy $\widehat{\mathfrak{X}_m} = X_m \cdot (Q_m/N_m)^{\frac{\alpha}{1-\alpha}}$ for $\mathfrak{X}_m$. Thus, bearing in mind (29), (34) and (35), and by measuring firm size as production (or sales) per firm, we consider the following quality-adjusted measures of relative production and relative firm size along the BGP,

$$\left(\tilde{\widehat{\mathfrak{X}}}\right) = \tilde{X} \cdot \left(\frac{\tilde{Q}}{N}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{H}{L}\right)^{\mathcal{D}_1(\epsilon,\delta)}, \tag{36}$$

$$\frac{\left(\tilde{\widehat{\mathfrak{X}}}\right)}{\tilde{N}} = \left(\frac{\tilde{X}}{N}\right) \cdot \left(\frac{\tilde{Q}}{N}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{H}{L}\right)^{\mathcal{D}_2(\epsilon,\delta)}, \tag{37}$$

where $\mathcal{D}_1(\epsilon,\delta) \equiv \{\alpha\delta + 1 - \alpha + \sigma - \epsilon\left[1 + (1+\alpha)\sigma\right]\} / \left[(\sigma+1)(1-\alpha)\right]$ and $\mathcal{D}_2(\epsilon,\delta) \equiv \{\delta + \sigma - \epsilon\left[\alpha + (1+\alpha)\sigma\right]\} / \left[(\sigma+1)(1-\alpha)\right]$. As one can see, the elasticity of m in vertical R&D cost, ϵ, has an important role in the determination of the sign of the relationship between the relative supply of skills and both relative production and relative firm size. Thus, we define the critical values for ϵ, $\bar{\epsilon} \in \{\bar{\epsilon}_1, \bar{\epsilon}_2\}$, such that $\mathcal{D}_1(\bar{\epsilon}_1) = 0$ and $\mathcal{D}_2(\bar{\epsilon}_2) = 0$,

$$\bar{\epsilon}_1 = \frac{1 - \alpha + \sigma + \alpha\delta}{1 + (1+\alpha)\sigma}, \tag{38}$$

$$\bar{\epsilon}_2 = \frac{\sigma + \delta}{\alpha + (1+\alpha)\sigma}. \tag{39}$$

Given $0 < \alpha < 1$, and $0 \leq \delta < \bar{\delta}(\alpha)$, where $\bar{\delta}(\alpha) < 1$, then $0 < 1 - \alpha \leq \bar{\epsilon}_1 < 1/(1+\alpha) < 1$, $0 \leq \bar{\epsilon}_2 < 1/(1+\alpha) < 1$ for $\sigma > 0$, and $\bar{\epsilon}_2 < \bar{\epsilon}_1$ for σ finite.

We wish to discuss the comparative statics of the BGP concerning the impact of changes in the relative supply of skills, H/L, on the relative number of firms, N, relative production (sales), $\widehat{\mathfrak{X}}$, and relative average firm size, $\widehat{\mathfrak{X}}/N$.[13] This exercise is carried out by considering four regimes of net scale effects on economic growth: (i) "large" positive (corresponding to $0 \leq \epsilon < \bar{\epsilon}_2$); (ii) "medium" positive ($\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$); (iii) "small" positive ($\bar{\epsilon}_1 < \epsilon < 1$); and (iv) negative scale effects ($\epsilon > 1$). Notice that $\epsilon = 0$ implies full positive net scale effects (e.g., Acemoglu and Zilibotti, 2001), $\epsilon = 1$ implies null net scale effects (e.g., Barro and Sala-i-Martin, 2004) – in this case, H/L has no impact on N –, while $\epsilon = \bar{\epsilon}_2$ (respectively, $\epsilon = \bar{\epsilon}_1$) implies parcial positive net scale effects – in this case, H/L has no impact on $\widehat{\mathfrak{X}}/N$ (on $\widehat{\mathfrak{X}}$). We find that, as the considered regime of scale effects changes, the relationship between relative supply of skills and industrial structure may differ significantly.

[13] Henceforth, the ~ is omitted for the sake of simplicity. The comparative statics concerning the remaining structural parameters and endogenous variables can be found in Gil, Brito, and Afonso (2010).

4.1. Number of firms, production and firm size

4.1.1. "Cross-country" perspective

Below we summarise the main results with respect to the impact of changes in the relative supply of skills on industrial structure, as characterised by the number of firms, output and average firm size in sector H vis-à-vis sector L. This exercise can be interpreted as a *cross-section* comparison of industrial structures between countries with different levels of the relative supply of skills.

Proposition 1. Let a country have a *larger* relative supply of skills. Then it will have:

(i) A *larger* relative number of firms, production and firm size, if there are "large" positive scale effects ($0 \leq \epsilon < \bar{\epsilon}_2$);

(ii) A *larger* relative number of firms and production but a *smaller* relative firm size, if there are "medium" positive scale effects ($\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$);

(iii) A *larger* relative number of firms but a *smaller* relative production and firm size, if there are "small" positive scale effects ($\bar{\epsilon}_1 < \epsilon < 1$);

(iv) A *smaller* relative number of firms, production and firm size, if there are negative scale effects ($\epsilon > 1$).

To better understand the link between the relative supply of skills, H/L, and the industrial structure, observe that the results above stem from the different response of $\widehat{\mathfrak{X}}$ and N to H/L (see (29) and (35)). This mirrors the dominant impact of scale effects on $\widehat{\mathfrak{X}}$ through the vertical-innovation channel, imposing only through the latter on the horizontal-innovation channel, and thereby affecting N.[14] Furthermore, the effect on N is dampened by the friction in horizontal entry dynamics introduced by the entry-cost function (17). The higher sensitivity of $\widehat{\mathfrak{X}}$ to scale effects is clear: $\widehat{\mathfrak{X}}$ is constant with respect to H/L when $\epsilon = \bar{\epsilon}_1 < 1$ (i.e., incomplete removal of scale effects), while N is constant with respect to H/L when $\epsilon = 1$ (i.e., exact removal of scale effects).

This also explains the effect of the elasticity of the horizontal entry cost function,σ, on the threshold $\bar{\epsilon} \in \{\bar{\epsilon}_1, \bar{\epsilon}_2\}$: the lower σ, the lower $\bar{\epsilon}$, i.e., the higher the degree of scale effects below which there is falling relative production and firm size with H/L. These results make clear the contrast between industrial structure under "small"/"medium" positive scale effects and under either negative or "large" positive scale effects. Figure 3 depicts the theoretical relationship between the relative supply of skills and the relative number of firms, production and firm size under the different regimes of (net) scale effects.

[Figure 3 goes about here]

[14] Given the postulated horizontal entry technology and our lab-equipment specification, the vertical-innovation mechanism ultimately commands the horizontal entry dynamics along the BGP, meaning that a BGP with increasingly costly entry only occurs because entrants expect incumbency value to grow propelled by quality-enhancing R&D.

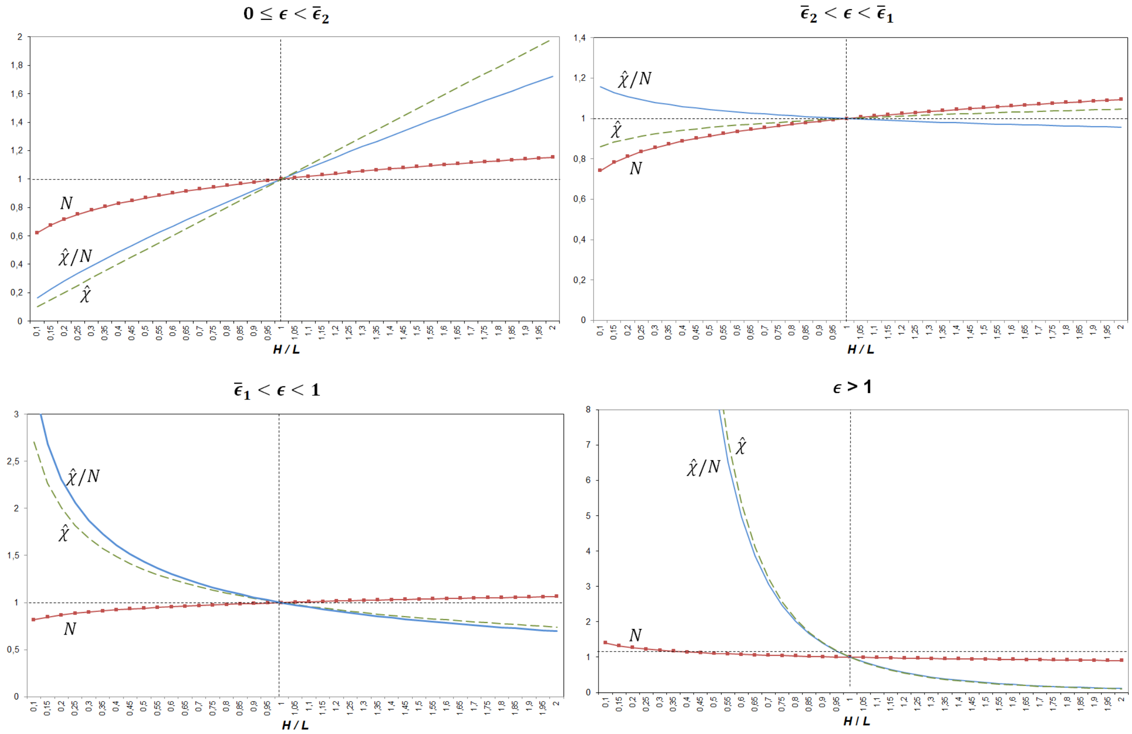

Figure 3: Relative number of firms, production and average firm size under the different regimes of scale effects: "large" positive ($0 \leq \epsilon < \bar{\epsilon}_2$) (upper-left panel); "medium" positive ($\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$) (upper-right panel); "small" positive ($\bar{\epsilon}_1 < \epsilon < 1$) (lower-left panel); and negative ($\epsilon > 1$) (lower-right panel). Considered parameter values: $\sigma = 2$, $\alpha = 0.6$, and, respectively, $\epsilon = 0.3$, $\epsilon = 0.56$, $\epsilon = 0.7$ and $\epsilon = 1.5$.

Cross-country evidence in Figure 2 suggests that the (log of the) relative supply of skills is (a) positively correlated with the (log of the) relative number of firms and relative production, but (b) negatively correlated with the (log of the) relative average firm size. Then, it is clear from Proposition 1 that both (a) and (b) are compatible with the predictions of our model in the case of "medium" positive scale effects ($\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$).

4.1.2. "Intra-country" perspective

Now, we look at the industrial structure in an *intra-country* perspective, by focusing on the comparison between the relative number of firms, relative output and relative average firm size, for a given level of relative supply of skills.

Proposition 2. A given relative supply of skills *below* unity corresponds to (vice versa for a ratio *above* unity):

(i) A relative number of firms, production and firm size *below* unity, if there are "large" positive scale effects ($0 \leq \epsilon < \bar{\epsilon}_2$);

(ii) A relative number of firms and production *below* and a relative firm size *above* unity, if there are "medium" positive scale effects ($\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$);

(iii) A relative number of firms *below* and a relative production and firm size *above* unity, if there are "small" positive scale effects ($\bar{\epsilon}_1 < \epsilon < 1$);

(iv) A relative number of firms, production and firm size *above* unity, if there are negative scale effects ($\epsilon > 1$).

From Proposition 2, we conclude that, provided the relative supply of skills is different from unity, $H/L \neq 1$, and the degree of scale effects is different from zero, $\epsilon \neq 1$, the industrial structure is characterised by a skewed BGP distribution of average firm size between sectors L and H as follows. Under "small" or "medium" positive scale effects, the number of firms is greater in the sector with smaller average firm size (the relative number of firms above unity and the relative firm size below unity, or vice versa), implying a right-skewed average-size distribution. In contrast, negative or "large" positive scale effects imply a left-skewed average-size distribution, i.e., there are more large-average firms than small-average firms (both the relative number of firms and firm size above unity, or vice versa).[15] Thus, only in the case of positive but "small"/"medium" scale effects does average-size distribution obtain a skewness in line with the IO stylised facts on the size distribution of *individual* firms (see, e.g., Sutton, 1997; Cabral and Mata, 2003). The mechanism behind this result is the same as the one described in Proposition 1 (see again Figure 3).

According to the data in Figure 1, the relative number of firms and relative production display values systematically below unity. This is in line with our results with respect to

[15] If $\epsilon = \bar{\epsilon}_2$ and $H/L \neq 1$, the two sectors have a different number of firms, but their average size is the same. If $\epsilon = 1$ and $H/L \neq 1$, the opposite is true: the two sectors exhibit the same number of firms, but with different average size. If $H/L = 1$, the average-firm-size distribution is uniform (same number of firms and same average firm size), whatever $\epsilon \geq 0$.

the relative number of firms and production given a relative supply of skills: below unity, if scale effects are "large" or "medium" positive ($0 \leq \epsilon < \bar{\epsilon}_1$); above unity, if scale effects are negative ($\epsilon > 1$). On the other hand, the data shows relative firm size systematically above unity. This is compatible, in theory, with relative production and the relative number of firms below unity – implying a right-skewed firm size distribution – if we have a relative supply of skills below unity combined with "medium" positive scale effects ($\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$).

4.2. Concentration index

4.2.1. Relative supply of skills and concentration

We are interested in the relationship between the level of relative supply of skills and a measure of concentration of economic activity. Concentration measures widely used in the IO literature are, e.g., the k-firm concentration ratio and the Herfindahl index.

However, to choose the concentration measure, one must take into account two particular features of our model. Firstly, the relevant "market" to measure the degree of concentration is established at the economy-wide level, since the economy comprises a continuum of monopolistic industries producing imperfectly substitutable goods. Secondly, the equilibrium values of the relevant variables (namely, output and the number of firms) are non-stationary, since they are defined along a BGP characterised by positive growth rates. Due to the first feature, the k-firm concentration ratio is meaningless and, owing to the second feature, the typical Herfindahl index is not appropriate (it tends to zero as N_m grows at the rate $g_N > 0$ along the BGP).

Since a similar reasoning applies to other commonly used concentration indices and we are solely interested in measuring the concentration of economic activity across sectors (with no forceful link to the concept of market power), we propose an own measure. We consider a two-dimension aggregate concentration index, by combining the "market share" of the number of firms, N_m/N_{tot}, with the "market share" of the quality-adjusted output (sales), $\widehat{\mathfrak{X}}_m/\widehat{\mathfrak{X}}_{tot}$, of each sector $m \in \{L, H\}$, as follows

$$\mathfrak{I} \equiv \frac{\widehat{\mathfrak{X}}_L}{\widehat{\mathfrak{X}}_{tot}} \cdot \frac{N_L}{N_{tot}} + \frac{\widehat{\mathfrak{X}}_H}{\widehat{\mathfrak{X}}_{tot}} \cdot \frac{N_H}{N_{tot}} = \frac{\widehat{\mathfrak{X}} \cdot N + 1}{\widehat{\mathfrak{X}} \cdot N + \widehat{\mathfrak{X}} + N + 1} \tag{40}$$

The proposed concentration index compares sectors at the economy-wide level instead of individual firms within a sector and is constant along the BGP (see (35) and (36)). The aggregate character accommodates the view, noted by, e.g., Kamien and Schwatrz (1975), that traditional industry boundaries seem to become decreasingly useful for the purpose of economic analysis as new products and processes compete *across* industry lines, which is a salient feature of our model.

The proposition below characterises the behaviour of the concentration index with respect to the relative supply of skills, H/L.

Proposition 3. The concentration index $\mathfrak{I}$ displays:

(i) A U-shaped behaviour with respect to the relative supply of skills, such that $1/2 \leq \mathfrak{I} \leq 1$, with the minimum $\mathfrak{I} = 1/2$ at $H/L = 1$, if there are either "large"/"medium" positive ($0 \leq \epsilon < \bar{\epsilon}_1$) or negative scale effects ($\epsilon > 1$);

(ii) An inverted U-shaped behaviour with respect to the relative supply of skills, such that $0 \leq \mathfrak{I} \leq 1/2$, with the maximum $\mathfrak{I} = 1/2$ at $H/L = 1$, if there are "small" positive scale effects ($\bar{\epsilon}_1 < \epsilon < 1$).

The boundaries to the concentration index are to be interpreted as follows: $\mathfrak{I} = 1$ refers to total concentration of firms and production in a single sector and $\mathfrak{I} = 0$ refers to total concentration of firms in one sector and of production in the other. We identify the extreme case $\mathfrak{I} = 1$ as industry concentration of type I and $\mathfrak{I} = 0$ as industry concentration of type II. In contrast, $\mathfrak{I} = 1/2$ implies a uniform distribution of firms and production across sectors.[16] ,[17]

By interpreting this exercise again as a cross-country comparison of industrial structures, we conclude that, depending on the regime of scale effects, the industrial structure of a given country is characterised by different concentration patterns. In the case of either negative or "medium"/"large" positive scale effects, given a relative supply of skills *below* unity, a country with a higher relative supply of skills is expected to have a *lower* concentration index, with $\mathfrak{I} = 1/2$ as the lower boundary, i.e., *a uniform distribution*; given a relative supply of skills *above* unity, a country with a higher relative supply of skills is expected to have a *higher* concentration index, with $\mathfrak{I} = 1$ as the upper boundary, i.e., *concentration of type I*. In the case of "small" positive scale effects, given a relative supply of skills *below* unity, a country with a higher relative supply of skills is expected to have a *higher* concentration index, with $\mathfrak{I} = 1/2$ as the upper boundary; given a relative supply of skills *above* unity, a country with a higher relative supply of skills is expected to have a *lower* concentration index, with $\mathfrak{I} = 0$ as the lower boundary, i.e., *concentration of type II*.

Figure 4 illustrates the non-monotonic relationship between the concentration index and the relative supply of skills, H/L. Observe that the concentration index exhibits a symmetric behaviour with respect to H/L in geometric terms, i.e., if we define the function $\mathfrak{I} \equiv \mathfrak{I}(H/L)$, for a given ϵ, then it can be shown that $\mathfrak{I}(H/L) = \mathfrak{I}\left((H/L)^{-1}\right)$.[18]

[16] To be rigourous, when $\epsilon = \bar{\epsilon}_1$ or $\epsilon = 1$, then $\mathfrak{I} \to 1/2$ for $H/L > 1 \wedge H/L \to \infty$ or $H/L < 1 \wedge H/L \to 0$. We interpret these as "degenerate" cases associated with specific levels of scale effects, i.e., those implied by $\epsilon \in \{\bar{\epsilon}_1, 1\}$.

[17] Instead of using (40), we could have calculated a version of the Herfindahl index by considering solely the share of the quality-adjusted output of each sector $m \in \{L, H\}$, $\mathfrak{I}_{\mathcal{H}} \equiv \left(\frac{\hat{\mathfrak{x}}_L}{\hat{\mathfrak{x}}_{tot}}\right)^2 + \left(\frac{\hat{\mathfrak{x}}_H}{\hat{\mathfrak{x}}_{tot}}\right)^2 = \frac{(\hat{\mathfrak{x}})^2+1}{(\hat{\mathfrak{x}})^2+2\hat{\mathfrak{x}}+1}$, which is is also constant along the BGP. However, since the number of firms, N_m and N_{tot}, is not considered in $\mathfrak{I}_{\mathcal{H}}$, this index does not allow one to distinguish concentration of type I from concentration of type II. In fact, see that $1/2 \leq \mathfrak{I}_{\mathcal{H}} \leq 1$.

[18] Firstly note, from (36) and (35), that $\left((H/L)^{-1}\right)^{\mathcal{D}_1} = 1/\hat{\mathfrak{X}}$ and $\left((H/L)^{-1}\right)^{\frac{1-\epsilon}{\sigma+\gamma+1}} = 1/N$. Secondly, let $\mathfrak{I}_{1/\hat{\mathfrak{x}}} = \frac{\frac{1}{\hat{\mathfrak{x}} \cdot N}+1}{\frac{1}{\hat{\mathfrak{x}} \cdot N}+\frac{1}{\hat{\mathfrak{x}}}+\frac{1}{N}+1}$. Finally, multiply both numerator and denominator by $\hat{\mathfrak{X}} \cdot N$, to get $\mathfrak{I}_{1/\hat{\mathfrak{x}}} = \frac{1+\hat{\mathfrak{X}} \cdot N}{1+\hat{\mathfrak{x}} \cdot N+\hat{\mathfrak{x}}+N} = \mathfrak{I}$ (see (40)).

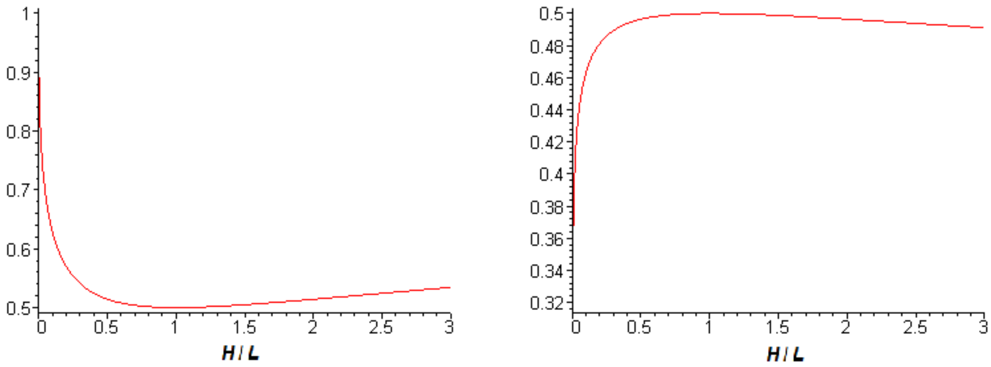

Figure 4: Concentration index in the case of: "medium"/"large" positive scale effects ($0 \leq \epsilon < \bar{\epsilon}_1$) or negative scale effects ($\epsilon > 1$) (left panel); "small" positive scale effects ($\bar{\epsilon}_1 < \epsilon < 1$) (right panel). Considered parameter values: $\sigma = 2$ and $\alpha = 0.6$.

[Figure 4 goes about here]

Computation of the concentration index by using the data in Figure 1 shows that the index and the relative supply of skills display a negative correlation across countries. This is in line with the theoretical results if there are negative or "large"/"medium" positive scale effects combined with a relative supply of skills below unity (in the case of concentration of type I), or if there are "small" positive scale effects with the relative supply of skills above unity (concentration of type II).

On the other hand, the concentration index displays values systematically above 1/2 in the data, hence implying industry concentration of type I. This is compatible with the theoretical results in the case of negative or "large"/"medium" positive scale effects.

4.2.2. Concentration and economic growth

Now, we explore the association between the concentration index, $\mathfrak{I}$, and the economic growth rate, g, by considering the impact of changes in the relative supply of skills, H/L, on both variables. The following proposition summarises the results concerning the relationship between the relative supply of skills and the economic growth rate.

Proposition 4. Assume that an increase in the relative supply of skills occurs with either $\Delta H, \Delta L > 0$ or $\Delta H > |\Delta L|$, if $\Delta H > 0$ and $\Delta L < 0$. Then, the economic growth rate is:

(i) Increasing in the relative supply of skills, if there are positive scale effects ($0 \leq \epsilon < 1$);

(ii) Decreasing in the relative supply of skills, if there are negative scale effects ($\epsilon > 1$).

The proof for Proposition 4 is immediate if one considers (31) along with the expressions for $\tilde{r}_{0m}$ and $\tilde{P}_m$.

Cross-country evidence in Figure 2 suggests that the relative supply of skills is positively correlated with the per capita growth rate, which is compatible with the predictions of our model in the case of positive scale effects ($0 \leq \epsilon < 1$).[19] These results are compatible with those regarding the concentration index (see Section 4.2.1), if we have "large"/"medium" positive scale effects combined with a relative supply of skills below unity. One should then expect a negative relationship between concentration (of type I) and the economic growth rate both in the data and in the model.

It is clear from the above how the general equilibrium nature of our model allows for the simultaneous determination of the economic growth rate and the industrial structure through the effect of the relative supply of skills on those two variables. Finally, it should also be clear that our results with respect to the association between growth and industrial structure concern the *quantitative* dimension of the latter (i.e., how many firms and how much production are allocated to each sector vis-à-vis the others) and not the *qualitative* dimension (i.e., the concentration of economic activity in a specific type of sector), as pursued by the literature of structural change (e.g., Fagerberg, 2000; Bonatti and Felice, 2008).

4.3. Regime of scale effects implied by the data

We find useful to make a final assessment of the model by focusing on the regime of scale effects that is implied by the data. As shown throughout Section 4, the model generates a number of results, which depend on the posited regime of scale effects and on the level of relative supply of skills (above or below unity). The model is able to account simultaneously for all the empirical regularities concerning the cross-country correlations of the variables of interest (Figure 2) under the condition $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$, which corresponds to the regime of "medium" positive scale effects. This regime combined with the condition that the level of the relative supply of skills is below unity, $H/L < 1$, also allows the model to successfully address the empirical regularities concerning the levels (systematically above or below unity) of the variables of interest (Figure 1).

But how plausible are these conditions from an empirical point of view? Regarding the level of (net) scale effects, the size and position of the interval $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$ varies with the elasticities of the horizontal entry cost function, σ and γ (see (38) and (39)), such that $\lim_{\sigma\to\infty} \bar{\epsilon}_1 = \lim_{\sigma\to\infty} \bar{\epsilon}_2 = 1/(1+\alpha)$, and $\bar{\epsilon}_1|_{\sigma=0} = 1-\alpha$ and $\bar{\epsilon}_2|_{\sigma=0} = 0$. For the sake of concreteness, let $\alpha = 0.6$, which is the standard value for the labour share in production in the endogenous-growth literature; then, under $\sigma = 0$, we must impose the condition $0 < \epsilon < 0.4$, while, under $\sigma \to \infty$, the condition is $\epsilon = 0.625$.[20] We have no knowledge

[19]It must also be noted that the cross-section data that we are using from the Eurostat supports the assumption that an increase in H/L occurs with either $\Delta H, \Delta L > 0$ or $\Delta H > |\Delta L|$, if $\Delta H > 0$ and $\Delta L < 0$, stated in Proposition 4.

[20]The described dependence of $\bar{\epsilon} \in \{\bar{\epsilon}_1, \bar{\epsilon}_2\}$ on σ and γ makes clear the role of the horizontal-entry mechanism in explaining the data on industrial structure: without entry (corresponding to $\sigma, \gamma \to \infty$; note that $\lim_{\sigma\to\infty} \tilde{g} = \lim_{\gamma\to\infty} \tilde{g} = \tilde{g}_{no-entry}$), the selected interval for ϵ collapses into the single point

of any empirical work that explicitly provides a parametrised estimate of the degree of scale effects, but well-known studies like, e.g., Backus, Kehoe, and Kehoe (1992), clearly indicate the existence of *positive* scale effects at the industry (manufacturing) level for a large sample of countries. On the other hand, since in our model the scale effects at the sector level show up at the aggregate level (see the impact of H and L in g, in (31)), it may be argued that our results conflict with the endogenous-growth debate over the counterfactual character of positive scale effects when assessed by the relationship between aggregate scale variables and the economic growth rate. However, two points can be made in this regard: firstly, our theoretical results may still be considered compatible with the empirical results at the aggregate level since the latter only unequivocally reject the existence of *large* positive scale effects (e.g., Backus, Kehoe, and Kehoe, 1992; Jones, 1995).

As regards the condition imposed on the level of the relative supply of skills, while it is in accordance with the empirical evidence concerning the majority of the countries in our sample, Germany, Finland, Poland and Estonia display a relative supply of skills above unity (see Figure 1).

However, the apparently strict conditions imposed on ϵ and H/L on the theoretical side can be relaxed to some extent if we slightly modify the model in order to allow for heterogeneity in: (i) the flow fixed costs to horizontal R&D, i.e., $\phi_L \neq \phi_H$ (see (17)); (ii) the flow fixed costs to vertical R&D, i.e., $\zeta_L \neq \zeta_H$ (see (10)); and/or (ii) labour efficiency, by replacing H and L with hH and lL in (1), where $h > l \geq 1$ (thus capturing an *absolute*-productivity advantage of H over L), while the scale-effect removal in (10) continues to refer to market size measured by *physical* labour, H and L. Assumptions (i)-(iii), applied together or alone, move the interval for ϵ (such that $\epsilon < \bar{\epsilon}_2$ or $\epsilon > \bar{\epsilon}_1$ may occur) under which, given a relative supply of skills *below* unity, the model predicts a level of the variables of interest (below or above unity) that is in accordance with the empirical patterns. On the other hand, assumption (i) is also able to relax the constraint on the relative supply of skills, by generating the empirically correct level of the variables of interest for a given relative supply of skills *above* unity, if properly combined with $\epsilon > \bar{\epsilon}_1$.

Thus, let us re-write (35)-(37) by considering $\phi_L \neq \phi_H$ in (17). Reiterating the same steps as in Section 3, we get

$$\tilde{N} = \left(\frac{H}{L}\right)^{\frac{1-\epsilon}{\sigma+1}} \left(\frac{\phi_H}{\phi_L}\right)^{\frac{-1}{\sigma+1}},$$

$$\left(\tilde{\hat{\mathfrak{X}}}\right) = \left(\frac{H}{L}\right)^{\mathcal{D}_1(\epsilon,\delta)} \left(\frac{\phi_H}{\phi_L}\right)^{\frac{\alpha}{(\sigma+1)(1-\alpha)}},$$

$\epsilon = 1/(1+\alpha)$; hence only three regimes of scale effects exist: the "large" positive, the "small" positive and the negative scale-effects regime.

$$\frac{\left(\tilde{\hat{\mathfrak{X}}}\right)}{\tilde{N}} = \left(\frac{H}{L}\right)^{\mathcal{D}_2(\epsilon,\delta)} \left(\frac{\phi_H}{\phi_L}\right)^{\frac{1}{(\sigma+1)(1-\alpha)}}.$$

Similar expressions can be derived by considering $\zeta_L \neq \zeta_H$ or $h \neq l$ in Section 3. It is then straightforward to show that a number of new theoretical scenarios are possible.

Proposition 5.

(i) For a given relative supply of skills *below* unity: **(a)** a ratio $\phi_H/\phi_L > 1$ (or $\zeta_H/\zeta_L < 1$ or $h/l > 1$) exists such that an interval $\bar{\epsilon}_2^{'} < \epsilon < \bar{\epsilon}_1^{'}$, where $\bar{\epsilon}_2^{'} < \bar{\epsilon}_2$ and $\bar{\epsilon}_1^{'} < \bar{\epsilon}_1$, also exists that is compatible with the empirically observed levels (above or below unity) of the variables of interest; **(b)** a ratio $\phi_H/\phi_L < 1$ (or $\zeta_H/\zeta_L > 1$ or $h/l < 1$) exists such that an interval $\bar{\epsilon}_2^{'} < \epsilon < \bar{\epsilon}_1^{'}$, where $\bar{\epsilon}_2^{'} > \bar{\epsilon}_2$ and $\bar{\epsilon}_1^{'} > \bar{\epsilon}_1$, also exists that is compatible with the empirically observed levels of the variables of interest.

(ii) For a given relative supply of skills *above* unity, a ratio $\phi_H/\phi_L > 1$ exists such that an interval $\epsilon > \bar{\epsilon}_1^{'}$, where $\bar{\epsilon}_1^{'} > \bar{\epsilon}_1$, also exists that is compatible with the empirically observed levels of the variables of interest.

While we must a priori (by construction) exclude the case of $h/l < 1$, in the end the pertinence of having the ratios ϕ_H/ϕ_L and ζ_H/ζ_L above or below unity is an empirical issue. Furthermore, it follows from the analysis above that if we consider the hypothesis of ϕ_H/ϕ_L, ζ_H/ζ_L or h/l varying across countries, then this will allow the model to accommodate, at least to some extent, the observation that the degree of scale effects at the sectoral level may also vary across countries.

5. Concluding remarks

This paper studies a specific dimension of the industrial structure, that of the distribution of firms and production across high- and low-tech sectors, by building a general-equilibrium endogenous-growth model of directed technical change with simultaneous vertical and horizontal R&D.

The model seeks to devise an analytical mechanism to explain the data on the number of firms, production and average firm size in high- vis-à-vis low-tech manufacturing sectors in a number of developed countries. It also provides an instrument to study the association between the concentration of economic activity, measured at the aggregate level, and long-run aggregate growth. The model allows us to accommodate the view that the relationship between industrial structure, innovative activity and aggregate growth is not causal: they are simultaneously (endogenously) determined.

While there are many factors that may explain the observed patterns of industrial structure, qualitative empirical evidence suggests that we should be thinking of explanations featuring differences in the relative supply of skills. Our results hinge on the assumption that scale effects connected to the size of profits that, in each period, accrue to the R&D successful firm in each sector may be negative, positive or null. By focusing on the BGP, we then find that, as the regime of scale effects changes, the relation between industrial structure and the relative supply of skills may differ significantly. If anything, the confrontation with the data suggests that the empirical relevance of our results depends on the existence of "medium" positive scale effects. We thus underline the practical importance of distinguishing between "small", "medium" and "large" positive scale effects, since the predictions of the model suffer a change of sign due to the "regime switch".

However, in the end, whether the relative supply of skills can account for a substantial fraction of the patterns of industrial structure is an empirical question. Thus, on the empirical side, further research should be devoted to fill the data gap on the magnitude of scale effects, in order to assess the quantitative relevance of our mechanism. Some effort might also be devoted to the collection of data on the flow fixed costs to vertical and horizontal R&D. Moreover, a larger set of countries, in particular including the US and Japan, is desirable in order to guarantee robustness of empirical results.

Finally, the study of the transitional dynamics should be an objective for future work. The qualitative characterisation of the local dynamics properties might allow one to find to what extent variations in a country's initial conditions (namely the inherited number of firms and stock of technological knowledge) lead to non-monotonic time paths of industrial structure towards the BGP. On the other hand, given the role played by factor endowment and scale effects, it should be only natural to extend the model to an open-economy framework, in particular focusing on intermediate-good international trade and its impact on the cross-country industrial structure.

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Appendix

A. Proxy for quality-adjusted production

Given that j follows a Poisson distribution with parameter $I \cdot t$, that is, we have $j \sim Po(I \cdot t)$ over $[0,t]$, then

$$\begin{aligned} E\left(\lambda^{j\frac{1}{\alpha}}\right) &= E\left(\left(\lambda^{\frac{1}{\alpha}}\right)^{j}\right) = \sum_{j=0}^{\infty}\left(\lambda^{\frac{1}{\alpha}}\right)^{j}\frac{e^{-It}\left(It\right)^{j}}{j!} = \\ &= e^{It\lambda^{\frac{1}{\alpha}}}e^{-It}\sum_{j=0}^{\infty}\frac{e^{-It\lambda^{\frac{1}{\alpha}}}\left(It\lambda^{\frac{1}{\alpha}}\right)^{j}}{j!} = \\ &= e^{It\lambda^{\frac{1}{\alpha}}}e^{-It} = e^{-It(1-\lambda^{\frac{1}{\alpha}})} \end{aligned}$$

$$\begin{aligned} E\left(\lambda^{j\frac{1-\alpha}{\alpha}}\right) &= E\left(\left(\lambda^{\frac{1-\alpha}{\alpha}}\right)^{j}\right) = \sum_{j=0}^{\infty}\left(\lambda^{\frac{1-\alpha}{\alpha}}\right)^{j}\frac{e^{-It}\left(It\right)^{j}}{j!} = \\ &= e^{It\lambda^{\frac{1-\alpha}{\alpha}}}e^{-It}\sum_{j=0}^{\infty}\frac{e^{-It\lambda^{\frac{1-\alpha}{\alpha}}}\left(It\lambda^{\frac{1-\alpha}{\alpha}}\right)^{j}}{j!} = \\ &= e^{It\lambda^{\frac{1-\alpha}{\alpha}}}e^{-It} = e^{-It(1-\lambda^{\frac{1-\alpha}{\alpha}})} \end{aligned}$$

Next, consider the random variables $\mathcal{Z} \equiv \lambda^{j\frac{1-\alpha}{\alpha}}$ and $\mathcal{K} \equiv \lambda^{j\frac{1}{\alpha}}$, as well as the sum of the random variables $\mathcal{Z}_i$, i.i.d. of $\mathcal{Z}$, in $Q_m = \sum_i^{N_m} \mathcal{Z}_{mi}$, and $\mathcal{K}_i$, i.i.d. of $\mathcal{K}$, in $\breve{Q}_m = \sum_i^{N_m} \mathcal{K}_{mi}$, $m \in \{L, H\}$. Then, for a given N_m, we get

$$E(Q_m) = N_m e^{-I_m t(1-\lambda^{\frac{1-\alpha}{\alpha}})} \tag{41}$$

$$E(\breve{Q}_m) = N_m e^{-I_m t(1-\lambda^{\frac{1}{\alpha}})} \tag{42}$$

Using $\ln(v+1) \approx v$ for v small enough, (41) and (42) can be rewritten as follows

$$E(Q_m) = N_m e^{I_m t\left(\frac{1-\alpha}{\alpha}\right)\ln\lambda} = N_m \lambda^{I_m t\left(\frac{1-\alpha}{\alpha}\right)} \tag{43}$$

$$E(\breve{Q}_m) = N_m e^{I_m t\left(\frac{1}{\alpha}\right)\ln\lambda} = N_m \lambda^{I_m t\left(\frac{1}{\alpha}\right)}. \tag{44}$$

Thus, $E(\breve{Q}_m)/E(Q_m) = \lambda^{I_m t\left(\frac{1}{\alpha}-\frac{1-\alpha}{\alpha}\right)} = \lambda^{I_m t}$, which goes to ∞ as $t \to \infty$. However, given (43) and (44), we also have

$$\{E(Q_m)\}^{\left(\frac{1}{1-\alpha}\right)} N_m^{-\left(\frac{\alpha}{1-\alpha}\right)} = N_m \lambda^{I_m t\left(\frac{1}{\alpha}\right)} = E(\breve{Q}_m) \tag{45}$$

Since, in our model, Q_m is treated as a continuous deterministic variable, we consider the following proxy, $\widehat{\breve{Q}}_m$, as a deterministic version of (45)

$$\widehat{\breve{Q}}_m = Q_m^{\frac{1}{1-\alpha}} \cdot N_m^{-\left(\frac{\alpha}{1-\alpha}\right)}$$

It can then be shown that $\breve{Q}_m/\widehat{\breve{Q}}_m = \text{constant}$.