The Growth Dynamics of Human Interaction*

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Abstract

Interaction is essential for the human being and it can be very useful for learning and accumulating knowledge. The effects of human (and social) interactions in the economic literature is still a quite recent topic and very much of it is empirical in nature. To study interactions between economic agents, this literature has imported many notions from sociology and anthropology, one of them being social capital. Social capital is a very broad concept that emphasizes the importance of individual relations and also social networkings. In this work, we explore the positive connection between human interactions (social capital) and human capital accumulation in an endogenous growth model. While the introduction of human capital interaction in this endogenous growth setup leads to an unstable steady-state in a simpler version of the model, where human capital is not taken as an input for the production of physical goods, we achieve saddle-path stability under a general model that contemplates a more comprehensive production technology.

Keywords: Human Capital, Social Interaction, Social Capital, Endogenous Growth, Dynamic Analysis, Stability.

JEL Classification: C61, C62, O41, Z13.

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1 Introduction

People enjoy the company of other people. It is human nature after all, we like to interact. And interaction can bring us benefits. For instance, when students talk with their colleagues about school and school courses they are in fact learning and accumulating human capital, even when they are not in the classroom. This interaction brings a positive externality, since they accumulate knowledge without class attendance. This positive externality could be smaller or larger depending on the community we are at and the family we are in. This interaction is better in an Ivy League college than in other less prestigious universities, for example, since students that attend the former are usually brighter, better connected, and engage in interesting conversations and deep discussions, in comparison with students that attend the latter. Parents more engaged in their children's school activities can also help increase their children's human capital.

To study the connection between human interaction and the accumulation of human capital, economists have introduced a new concept in economic science brought in from sociology. In sociology this human interaction is referred to as social capital.[1] Social capital is a very broad concept that emphasizes the importance of individual relations and also social networkings.

In this work we explore the positive connection between human interactions (social capital) and human capital accumulation in an endogenous growth model. We support the assumption that the connection is positive, based on the empirical literature that we refer below and also on a 2001 report from the OECD, in which the organization stresses the complementarities of both types of capital for each other accumulation and also for economic growth (OECD, 2001). We use the concept of human capital as developed by Schultz (1961) and Becker (1962). Becker (1974) also incorporated concepts of sociology and anthropology in defining his theory of social interactions. We can define our use of the concept of social capital in the words of Coleman (1988, p. S100): "Social capital, however, comes about through changes in the relations among persons that facilitate action".

Most of the research on the connection between social and human capital is empirical. The works of Coleman (1988) and Teachman et al. (1997) made the bridge between sociology and economics. The authors study the connections between social capital and school drop out in the USA and found a positive relation between social capital and academic achievement. After these two seminal studies, all other empirical studies that we encounter in the literature also found a positive relationship between human and social capital, either in studies for an individual country or for a group of countries.

For the USA, Rumberger and Lim (2008) make a very detailed survey of reasons for school drop out in highschool. The authors refer the importance of "factors associated with individual characteristics of students, and factors associated with the institutional characteristics of their families, schools, and communities" (Rumberger and Lim, 2008).

[1] For a survey of research in social capital applied mostly to economics see Durlauf and Fafchamps (2005).

Smith et al. (1992) and Israel et al. (2004) found a significant role for social capital in avoiding school drop out in the USA, although the first study addresses only the south of the country. Babcock (2008) found that having better connections with family and school community, while a student is at middle and secondary school, increases years of schooling and also the probability of attending college. His study also addresses racial heterogeneity. Dincer (2011) found, as well, that social capital increases average school years, but the author focused on existing differences between USA states. Yamamura (2011, 2012) discovered a negative relation between truancy and trust, a commonly used empirical proxy for social capital and a positive relationship between trust and language and mathematics achievement tests, respectively, for Japan. Krenz (2010) finds that social (specially family) background influences positively the number of years of education of an individual in Germany, using the German Socio-Economic Panel for 2001 and 2005 (SOEP). Also for Germany, Schmitt and Kleine (2010) found that children with good social interactions with parents, friends, and teachers have a higher probability of achieving academic success. Studies that address a set of countries include Parts (2003) that studies the relationship between human and social capital and its implications for economic development for transition countries (in particular Eastern European countries). The author finds that social capital enables a better performance of human and financial capital in acting towards an increase in better education and hence higher economic development. Robért (2003) relies on the OECD Programme for International Student Assessment (PISA) 2000 data to find a positive causality between social capital, in the form of family and teacher support, and student achievement, although the author found differences between post-socialist and non-European countries. Bjϕrnskov (2009) analyzed a group of 52 countries between 1960 and 2000 and found that trust worked as a enabler of schooling growth. A result also supported by Papagapitos and Riley (2009) that analyzed a group of 41 countries included in the World Values Survey in the 1980's and 1990's, using secondary school enrollment. Dearmon and Grier (2011) using a set of 50 countries for the period between 1976 and 2005 found a positive effect of social capital in the accumulation of both physical and human capital. Özcan and Bjϕrnskov (2011) empirically established a positive connection between trust and the United Nations (UN) Human Development Index (HDI) for the period between 1980 and 2005, being education one of the dimensions of this index.

In this work we theoretically model the empirically verified positive relation between social and human capital in an endogenous model of economic growth and analyze the stability and dynamics properties of the model. In the model, social capital directly contributes for the accumulation of human capital. Section 2 presents our benchmark model and in section 3 we present the dynamics of the benchmark model and also the dynamics for a more complete model, in which human capital is a production factor in final goods production. In this section we also present numerical examples in order to obtain results concerning the stability of the model. Finally, section 4 concludes.

2 The Model

We consider a two-sector representative agent endogenous growth model, where the accumulation of different forms of capital, physical and human, can be translated on a pair of differential equations.

2.1 Production Factors and Final Goods

Physical capital (K_P) will evolve according to a standard rule where net investment is produced with income (Y) that is not spent in consumption (C). Assuming a physical capital depreciation rate $\delta_P \in (0, 1)$, the equation that reflects the accumulation of physical capital will be:

$$\dot{K}_P = Y - C - \delta_P K_P, \quad K_{P0} \text{ given} \tag{1}$$

Income is generated through a simple constant returns AK production function:

$$Y = AK_P \tag{2}$$

with $A > 0$ an index of technology available at the current period.

Human capital is defined by variable K_H and it can be separated into two types of skills that serve different purposes. Specifically, we take $K_H = H_H + H_S$, with H_H the skills allocated to school attendance (in order to directly generate additional human capital) and H_S hours of human capital oriented to build and improve social capital skills. This type of allocation of human capital concerns the social relations that may benefit agents both in terms of their utility and in the way human capital could be increased. Social capital will have an impact on the availability of human capital because it can help in the accumulation of human capital.

We define an AK type production function also for human capital, $Y_H = \xi H_H$, where $\xi > 0$ is productivity of school attendance. The dynamics of human capital accumulation is represented as follows:

$$\dot{K}_H = Y_H - \delta_H K_H + \sigma \frac{H_S}{K_H} H_H, \quad K_{H0} \text{ given} \tag{3}$$

where $\delta_H \in (0, 1)$ is the depreciation or obsolescence rate for human capital, and $\sigma \in (0, 1)$ is a parameter that intends to reflect how human interaction helps in the accumulation of human capital. Human capital is produced through school attendance.

2.2 Consumers

The characterization of the growth setup is completed with the presentation of the objective function. The representative agent intends to maximize an intertemporal utility function that contains two arguments: consumption and hours spent in building and improving social capital skills; the agent withdraws utility from the consumption of physical goods and also from the interaction (socializing) with others (family, friends, and other elements of their surrounding community). Let $\rho > 0$ be the discount rate; the objective function is:

$$V_0 = \int_0^\infty U(C, H_S) \exp(-\rho t) dt \tag{4}$$

The control variables of this problem are precisely the arguments in the utility function - the agent can choose the trajectories of consumption and social capital in order to optimize her behavior. If we consider a logarithmic utility function, with both arguments exhibiting decreasing marginal utility, i.e., $U(C, H_S) = \ln(CH_S^\psi)$, where $\psi > 0$ reflects the degree of preference for social capital, we end up addressing two separate problems: the one involving the trade-off between consumption and physical capital accumulation and the one that respects the allocation of human capital to each of the two purposes that we have identified. Calculations for obtaining differential equations for the control variables are explored in the next section.

3 Social Capital Dynamics

3.1 Simple Model

Like we have stated in the previous section, we have to address two separate problems: the one involving the trade-off between consumption and physical capital accumulation and the one that respects the allocation of human capital to each of the two purposes that we have identified. The first is a trivial Ramsey-type growth setup that is straightforward to analyze. Let λ_P be a co-state variable or the shadow price associated with K_P; relatively to this problem, one computes the first-order conditions:

$$1/C = \lambda_P \tag{5}$$

$$\dot{\lambda}_P = (\rho + \delta_P - A)\lambda_P \tag{6}$$

The transversality condition is $\lim_{t\to\infty} \lambda_P \exp(-\rho t) K_P = 0$. From (5) and (6), one derives a constant growth rate for consumption:

$$\frac{\dot{C}}{C} = A - (\rho + \delta_P) \tag{7}$$

Consumption grows at a positive rate, according to (7) as long as $A > \rho + \delta_P$.

The dynamics of the trade-off between the two types of human capital allocations (H_H and H_S) are relatively more complex although, as we shall see, it can be reduced to the analysis of a one-dimensional differential equation.

Let λ_H be the shadow-price of K_H; the first-order optimality conditions are:

$$\psi = \lambda_H H_S \left(\xi - \sigma + 2\sigma \frac{H_S}{K_H}\right) \tag{8}$$

$$\dot{\lambda}_H = \left[\rho - \xi + \delta_H - \sigma\left(\frac{H_S}{K_H}\right)^2\right]\lambda_H \tag{9}$$

The transversality condition is, in this case, $\lim_{t\to\infty}\lambda_H \exp(-\rho t)K_H = 0$. This transversality condition converts into:

$$(\sigma - \xi)\frac{H_S}{K_H} - 2\sigma\left(\frac{H_S}{K_H}\right)^2 < 0 \tag{10}$$

This implies both $\frac{H_S}{K_H} \neq 0$ and $\frac{H_S}{K_H} > \frac{\sigma-\xi}{2\sigma}$.

The differentiation of equation (8) with respect to time yields:

$$\left(\xi - \sigma + 4\sigma\frac{H_S}{K_H}\right)\frac{\dot{H}_S}{H_S} = 2\sigma\frac{H_S}{K_H}\frac{\dot{K}_H}{K_H} - \left(\xi - \sigma + 2\sigma\frac{H_S}{K_H}\right)\frac{\dot{\lambda}_H}{\lambda_H} \tag{11}$$

This expression is valid under the condition $\frac{H_S}{K_H} \neq \frac{\sigma-\xi}{4\sigma}$, which is always true under the transversality condition (10), as $\frac{H_S}{K_H} > \frac{\sigma-\xi}{2\sigma} > \frac{\sigma-\xi}{4\sigma}$.

Replacing in (11) the growth rates of K_H and λ_H, that we have found in equations (3) and (9) respectively, that expression may be rearranged in order to present the growth rate of H_S as follows:

$$\begin{aligned}\frac{\dot{H}_S}{H_S} = \; & \left\{2\sigma\frac{H_S}{K_H}\left[\left(1-\frac{H_S}{K_H}\right)\left(\xi+\sigma\frac{H_S}{K_H}\right)-\delta_H\right]\right. \\ & \left. -\left[\rho-\xi+\delta_H-\sigma\left(\frac{H_S}{K_H}\right)^2\right]\left(\xi-\sigma+2\sigma\frac{H_S}{K_H}\right)\right\} \\ & \Big/ \left(\xi-\sigma+4\sigma\frac{H_S}{K_H}\right)\end{aligned} \tag{12}$$

The dynamics of human capital accumulation are addressable once we define the ratio $u = \frac{H_S}{K_H}$, i.e., the percentage of human capital that is allocated to the fostering of human interaction. Noticing that $\frac{\dot{u}}{u} = \frac{\dot{H}_S}{H_S} - \frac{\dot{K}_H}{K_H}$, we resort to equations (12) and (3) in order to compute the equation for the motion of the specified ratio. The outcome is the following:

$$\frac{\dot{u}}{u} = \frac{(\xi - \sigma + 2\sigma u)\left[\xi u - \rho - \sigma u(1-2u)\right]}{\xi - \sigma + 4\sigma u} \tag{13}$$

The first step in the study of the dynamics of equation (13) consists in determining steady-state points, i.e., points for which $\dot{u} = 0$. Four equilibrium points are obtained from applying the previous condition: $u^* = 0$, the solution of $\xi - \sigma + 2\sigma u^* = 0$ and the two solutions of $\xi u^* - \rho - \sigma u^*(1-2u^*) = 0$. The corner solution $u^* = 0$ and $u^* = \frac{\sigma-\xi}{2\sigma}$ can be ruled out by the transversality condition for human capital, as can be verified by observing (10).[2] The remaining solutions are, respectively,

$$u_1^* = \frac{\sigma - \xi - \sqrt{(\xi-\sigma)^2 + 8\sigma\rho}}{4\sigma} \tag{14}$$

$$u_2^* = \frac{\sigma - \xi + \sqrt{(\xi-\sigma)^2 + 8\sigma\rho}}{4\sigma} \tag{15}$$

Not both of the above equilibrium values are feasible under constraint $u^* \in (0,1)$. Relatively to point u_1^* this will be a value below zero and, therefore, it can be neglected and also excluded by the transversality condition, since it is useless for the analysis. Additionally, u_2^* is a positive value, and it remains below 1 for $\sqrt{[\xi-\sigma]^2 + 8\sigma\rho} < \xi + 3\sigma$.

Let us analyze the local stability of the steady-state value for u_2^*.

Proposition 1 *If* $u_2^* \in (0,1)$ *exists, then this is an unstable steady-state value.*

Proof. We differentiate the right hand side of equation (13); then, we evaluate in the vicinity of the steady-state u_2^*. We find

$$\left.\frac{d\dot{u}}{du}\right|_{u_2^*} = \frac{1}{2}\left[\xi - \sigma + \sqrt{(\xi-\sigma)^2 + 8\sigma\rho}\right]$$

To the above expression it must correspond a positive value; therefore, steady-state point u_2^* must be unstable. Any initial point close to u_2^* will not converge to it. ■

Notice also that

$$1 - u_2^* = \frac{\xi + 3\sigma - \sqrt{(\xi-\sigma)^2 + 8\sigma\rho}}{4\sigma} > 0 \text{ if } \sqrt{(\xi-\sigma)^2 + 8\sigma\rho} < \xi + 3\sigma \tag{16}$$

The above value is the percentage of human capital that is directly allocated to the generation of additional human capital. Looking at this value and to the equilibrium percentage of human skills dedicated to social interaction, as presented in (15), we infer the following:

- A higher productivity in school attendance, i.e., a higher value of ξ, shifts human capital resources from social interaction towards schooling $\left(\frac{\partial u_2^*}{\partial \xi} = -\frac{1 + \frac{(\sigma-\xi)}{\sqrt{(\xi-\sigma)^2+8\sigma\rho}}}{4\sigma} < 0\right)$;
- There is an equivocal qualitative impact of the change in parameter σ on the steady-state amount of social capital. An increase in σ will increase the amount of social capital if and only if $\sigma(\xi - 8\rho) > \xi(1+\xi)$

[2] As $\left.\frac{d\dot{u}}{du}\right|_{u_{0,2}^*} < 0$, we can define $u_{0,2}^*$ as a point to which the economy converges in limit. However, as it is not an optimal point and the function $\dot{u}$ is undefined in that point, we do not analyse this possibility.

$$\left(\frac{\partial u_2^*}{\partial \sigma} = \frac{\xi(\sigma-\xi)-8\sigma\rho}{4\sigma^2\sqrt{(\xi-\sigma)^2+8\sigma\rho}} + \frac{\xi}{4\sigma^2}\right).$$

In what follows, we illustrate the above results with one numerical example. Consider the array of parameters $(\xi;\sigma;\rho) = (0.04;0.3;0.02)$. For this example, we can confirm that the only steady-state value is feasible; observe that

$$\sqrt{(\xi-\sigma)^2 + 8\sigma\rho} < \xi + 3\sigma \Leftrightarrow 0.34 < 0.94$$

Having checked that u_2^* is an admissible steady-state value, we may compute it for the assumed example: $u_2^* = 0.5$. This value for the share of human capital allocated to foster human capital interaction is quite high, even though we can wonder that several nontypical activities in the concept of time allocated to human capital interaction, namely some tasks performed as working hours, could also be considered as pertaining to that type of activity.[3] The extension presented below will show how it is possible to decrease the value of this share. Proposition 1 indicates that the steady-state is an unstable equilibrium value. Transversality condition (10) rules out $u = 0$ and $u = 0.43(3)$ as optimal points; thus the only optimal allocation is $u = 0.4708$ but any point other than this would not converge to it as it is an unstable allocation. Figure 1 presents the phase diagram respecting the characterized situation.

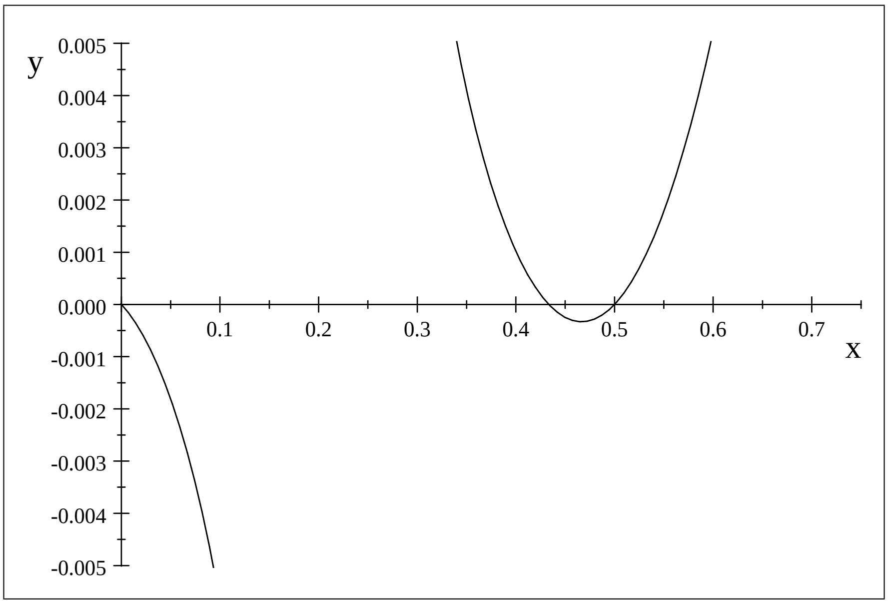

Figure 1

Any other combination of parameters would yield a similar situation.

[3]If we sum-up hours tipically considered as socializing such as caring and helping others, civic, organizational and religious activities, socializing and communicating and telephone calls, mail, and e-mail messages, we reach 19% of the total non-leisure time according to the USA time use survey, 2010 (http://www.bls.gov/news.release/atus.t12.htm). We consider non-leisure time the following: working and working-related activities, caring for others, eating and drinking, telephone calls, mail and e-mail and organizational, civic, and religious activities.

We have studied how the decision of allocation of human capital between schooling and promotion of social interaction is determined by the functioning of the economic system and by the preferences of the representative agent. Schooling creates knowledge; social interaction allows to increase knowledge and it also generates utility. With these simple assumptions in mind, we have arrived to a dynamic setting that is insightful in terms of the possible outcomes it can achieve. In fact, the consideration of human capital interaction in this endogenous growth setup implied the emergence of a unique unstable optimal equilibrium. The following extension of the model aims to better characterize the outcomes of the model and eventually to point out the conditions on which stable equilibria can arise.

3.2 Model with Human Capital as a Production Input

One of the main drawbacks of the former analysis is that human capital could be solely allocated to production of human capital and to human capital interaction, and it did not contribute to the production of the final good. This extension considers that human capital also contributes to the generation of final goods, allocating some resources to its production, and aims to show that multiple equilibria may exist, and one of the equilibrium points may be saddle-path stable.

$$Y = AK_P^{\beta}H_Y^{1-\beta},\ 0<\beta<1 \tag{17}$$

with $H_H = K_H - H_Y - H_S$. All the first order conditions (5)-(9) are maintained, except (6), which is now: $\dot{\lambda}_P = (\rho + \delta_P - \frac{\beta Y}{K_P})\lambda_P$ and there is an additional one for the control variable H_Y which is $\lambda_H(\xi + \sigma u_S) = \lambda_P(1-\beta)Y/H_Y$, where $u_S = \frac{H_S}{K_H}$ now stands for the former u. Using equations (1), (3), (17), and (11) together with this new optimality condition for H_Y, we reach the following dynamic equations for $\upsilon = Y/K$, $\chi = C/K$ and u_S, together with a static equation that relates u_S and u_Y, evaluated at the steady-state.

$$\begin{aligned}
g_\chi &= (\beta-1)\upsilon + \chi - \rho && (18)\\
g_\upsilon &= (\beta-1)(\upsilon-\chi-\delta_P) + (1-\beta)\left[(\xi+\sigma u_S)(1-u_S-u_Y)-\delta_H\right] && (19)\\
g_{u_S} &= \frac{(\xi-\sigma+2\sigma u_S)(2\sigma u_S^2 + (\xi+\sigma u_S)u_Y + (\xi-\sigma)u_S - \rho)}{\xi-\sigma+4\sigma u_S} && (20)\\
u_Y &= \frac{(1-\beta)u_S(\xi-\sigma+2\sigma u_S)\upsilon}{\psi\chi(\xi+\sigma u_S)} && (21)
\end{aligned}$$

Solving for the steady-state with $g_\chi = g_\upsilon = g_{u_S} = 0$, we obtain multiple equilibria, from which we state the variable u_S.[4]

[4] Since the other expressions are rather cumbersome, they are *not displayed; they are* available upon request.

$$u_S = \frac{\sigma - \xi}{2\sigma} \tag{22}$$

$$u_S = \frac{(1-u_Y)\sigma - \xi - \sqrt{8(\rho - u_Y\xi)\sigma + (\xi - (1-u_Y)\sigma)^2}}{4\sigma} \tag{23}$$

$$u_S = \frac{(1-u_Y)\sigma - \xi + \sqrt{8(\rho - u_Y\xi)\sigma + (\xi - (1-u_Y)\sigma)^2}}{4\sigma} \tag{24}$$

which, except for the presence of u_Y,are very similar to the ones previously obtained, namely (14) and (15). The equilibrium (22) is not admissible as $u_S = \frac{\sigma-\xi}{2\sigma}$ would imply that by (8) would yield $\psi = 0$ (a contradiction), meaning that the welfare benefit of allocating resources to human interaction would not be equal to the benefit it will bring to human capital accumulation. Similarly to what happened in the simpler model, (24) would provide positive values for u_S and this will be the only feasible equilibrium. It remains to be studied if this equilibrium is stable or not. The Jacobian that results from the system (18)-(21) is rather complex and it has been impossible to draw sufficient conditions for stability. In face of that, we recur to numerical examples to obtain results concerning the stability of the system.

3.2.1 Numerical Examples

In this section we present some numerical exercises and discuss them. First, we discuss sets of reasonable values for the parameters in the model. Then, we calculate the steady-state values for the main variables in the economy, specifically the allocation of resources to human capital interaction (H_S), for increasing values of the parameter that governs interaction between human capital and time allocated to social capital (σ). We also calculate the eigenvalues of the system formed by (18)-(21) and state the negative roots which indicate that the system is stable. Furthermore we also check the transversality condition, stating if the exercise satisfies the condition on human capital $\lim_{t\to\infty} \lambda_H \exp(-\rho t) K_H = 0$. With this, we want to discover if the instability result that we have highlighted on the simpler model, can be overcome in the more complete model.

For the values of parameters, we consider $\beta = 0.4$; $\xi = 0.04$; $\delta_H = \delta_P = 0.05$; $\rho = 0.02$; $\psi = 0.5$ (results in Table 1) and alternatively $\xi = 0.09$ and $\rho = 0.04$ (results in Table 2). The share of physical capital income β, depreciation rates (δ_H and δ_P) and the discount rate (ρ) were set according to previous literature confirmed also by empirical evidence, when available. For the parameter that weights welfare driven by social capital, there is not empirical guidance. Thus, we follow Roseta-Palma *et al.* (2010) in setting $\psi = 0.5$. We have concluded that oscillations on this value would not introduce much differences to our results although it has influence on the precise value of the allocation of human resources to human capital interaction. The considered values for parameters ξ and ρ are quite usual in the literature (see e.g. Caballero and Jaffe, 1993 for ρ and Funke and Strulik, 2000 and Iacopetta, 2011 for ξ). The empirical assessment of σ is not direct, as there are not estimated regressions between human capital and time or human capital dedicated to interaction or social capital. However there are empirical estimations between human capital and other measures of social capital, such as trust. Generally it is wise to assume that human capital interaction builds trust. For example, (robust) values for the effect of trust on test performance scores oscillate from 0.2 to 0.92 in Yamamura (2011) for Japan. Also, (robust) values for the effect of trust on schooling, for a 52-countries sample, oscillate between 0.212 to 0.657 in Bjϕrnskov (2009), an interval that gets smaller when we analyze regressions on Dearmon and Grier (2011), with coefficients that oscillate from 0.245 to 0.272. In Dincer (2011), for the USA, the same estimate is around 0.8. Thus, given those estimations but also the fact that the cited measures are at most proxies for σ, we consider intervals that begin near 0 and end on 0.3.

Table 1 - Steady-State values and Stability (calibration set 1)

σ	χ	υ	u_S	u_Y	Stable Roots	Transv. Cond.?	Other Feasible SS?
$\beta = 0.4$; $\xi = 0.04$; $\delta_H = \delta_P = 0.05; \rho = 0.02; \psi = 0.5$							
0.001	0.08	0.10	0.20	0.30	-0.038	Yes	No
0.01	0.08	0.10	0.23	0.28	-0.038	Yes	No
0.05	0.08	0.10	0.33	0.21	-0.044	Yes	No
0.1	0.10	0.14	0.40	0.16	-0.053	Yes	No
0.2	0.14	0.20	0.44	0.10	-0.075	Yes	No
0.3	0.18	0.26	0.46	0.07	-0.098	Yes	No

Results in Table 1 indicate that for the set of parameter values and given a reasonable interval for σ, the feasible steady-state is always saddle-path stable, overcoming the instability problem that was present in the simpler model. In this exercise the allocation of human capital to social capital and human interaction oscillates between 20% to 46%, a value that probably is above the reality as discussed above. However, the values for the macroeconomic variables deviate from the empirical reasonable values. In particular, the consumption to capital ratio is about 10% and values for the output to capital ratio are below the empirically reasonable values that are around 20%. Because of that we present the same exercise but for alternative values of ξ and ρ which we discovered that have crucial influence on the values of the consumption-capital and output to capital ratios.

Table 2 - Steady-State values and Stability (calibration set 2)

σ	χ	υ	u_S	u_Y	Stable Roots	Transv. Cond.?	Other Feasible SS?
$\beta = 0.4$; $\xi = 0.09$; $\delta_H = \delta_P = 0.05; \rho = 0.04; \psi = 0.5$							
0.001	0.18	0.27	0.17	0.27	-0.085	Yes	No
0.01	0.18	0.23	0.19	0.26	-0.086	Yes	No
0.05	0.18	0.23	0.24	0.24	-0.088	Yes	No
0.1	0.19	0.25	0.30	0.20	-0.095	Yes	No
0.2	0.21	0.30	0.38	0.28	-0.113	Yes	No
0.3	0.25	0.35	0.41	0.12	-0.134	Yes	No

Table 2 presents values for $\chi \approx 0.2$ and $\upsilon = 0.25$, in line with the empirical evidence. The value for u_S oscillate now between 17% and 41%, values that are yet somewhat above the ones attributed by the time use survey for the typical activities linked with socializing. According to the division of time reported by the time use survey for USA in 2010, from the average 6.47 non-leisure hours, 54% were spent on working or working-related activities and 19% in activities linked to socialization or interaction. The remaining is attributed in our model to learning, which can be at school or not. Thus, we have tried to find a combination of parameters that fits the most important empirical validated values for the variables in the model, which is the set $\beta = 0.3$; $\xi = 0.06$; $\delta_H = \delta_P = 0.05$; $\rho = 0.04$; $\psi = 0.005$ and $\sigma = 0.095$. This combination would predict the following: $\chi = 0.1881$, $\upsilon = 0.211572$, $u_S = 0.19116$ and $u_Y = 0.50854$. Furthermore this would constitute a saddle-path steady-state.

4 Conclusion

We built an endogenous growth model in which we considered human capital interaction, meaning that agents allocate time to interact with each other. This allocation of resources for socializing activities is seen to have a positive effect on the accumulation of human capital. Besides that, given human nature, agents like to interact such that these resources allocated to interaction have also a positive effect in utility. We investigate the consequences of introducing such feature in an endogenous growth model. We first exemplify how a simple model can highlight these effects and concluded that the only optimal solution is an unstable equilibrium. However, when considering a more general setup where human capital interaction is present in the same manner as previously modelled in the literature, saddle-path stability is achieved under quite reasonable conditions. We also show that the model is capable of replicating the main macroeconomic statistics for a stable steady-state.

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