Inequality and Policy Changes: the case of a decline on inflation in open economies

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Abstract

That inequality should be taken into account to assess the effects of policy changes is a well known result, when the way different agents are affected is relevant. It is less well reported in the literarture that inequality should also be taken seriously for the analysis of policy affects on the aggregates. We show that this is indeed the case when extensive margins are the ones that dominate households decisions. In this case the efficiency effect of the decline of inflation can be much larger than the figure usually reported. Also, when countries are not closed economies, the effect of policy will depend on the inequality of the trading partner.

1 Introduction

Across developed countries the prolonged decline in the average inflation rate is perhaps the most widespread, large and sustained economic policy regime

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change. When comparing the average inflation rate in the 80's and in the last decade for most of those economies we find an average decline of around 10 percent points. This decline is described as a very positive monetary policy change, the main reason being the gain in efficiency which is associated with the regime change. However most studies consider that the aggregate effects of the that regime change does not depend on the heterogeneity in the economy. This heterogeneity is very important when the main issue is to understand the distributional effects of the inflation rate reduction. It is relevant to understand whether the effect on equity do reinforce the positive effect on efficiency, or whether the change in policy leads to a trade-off between efficiency and equity.

In this paper we want to understand whether the aggregate equilibrium can be significantly dependent on the existing degree of exogenous inequality in the economy, and whether the decline in inflation can decrease the degree of inequality associated with the initial regime with relatively higher inflation.

While on the first question there is no evidence, there is some empirical evidence that there is a strong positive correlation between average inflation and measures of income inequality in the post-war period (see Albanesi (2007)). Easterly and Fisher (2001) present indirect evidence on the distributional effects of inflation. Using household pooling data on 38 countries they find that the poor are more likely to be concerned with inflation than the rich. It seems that low income households perceive inflation as more costly.

It is important however to understand what can cause such a relation, how this can affect the aggregate equilibrium and how it is connected with more empirical cross-section evidence on portfolio holdings and payments patterns. This is the objective of this article.

Heterogeneity of households is reflected on different consumption and hours of work choices, but for the present question the main fact is that it is also reflected in wealth composition and transaction patterns. Erosa and Ventura (2002) survey some facts for households in the U.S.. First, high income individuals use cash and cash plus checks for a smaller fraction of their transactions than low income individuals. Second, the fraction of household wealth held in liquid assets decreases with income and wealth. And third, a nontrivial fraction of households does not own a checking account and/or do not use credit cards to perform transactions.

The evaluation of the welfare costs of policy changes with heterogeneous agents is in general a quite difficult task. In this article we will use as a first approach to that answer the method developed in Correia (1999). This method can be applied for a very stylized economy where it is still feasible to compute the equilibrium prices of the economy as if agents are identical, and allows for determining the qualitative effect on equity of the decline of inflation, for any given distribution of households. The use of the method implies that, although the economy is populated by heterogeneous agents, preferences and markets are such that equilibrium prices do not depend on the specific distribution of agents' characteristics. Here therefore the answer to first question is trivial and to the second is qualitatively robust to any degree of heterogeneity.

Unfortunately when we try to apply this method for the monetary model those constraints are not consistent with the cross section evidence just described above. Thus we begin by developing a model economy where the method can be applied but, although agents display the observed cross section characteristics on consumption and labor, is not able to replicate the cross section evidence on wealth composition and transaction patterns. The gain is to have a tractable fist step to get some intuition on the mechanisms through which inflation affects different agents differently. Then we will extend the model to accomplish replicating the cross section facts on payments patterns. The above method can no longer be applied here. Therefore we have to solve an heterogeneous model numerically to get the answer. How important is heterogeneity to assess the effects of a lower inflation rate? We will compare the decline in the interest rate for an economy with homogeneous agents with the one in an economy with cross section heterogeneity. In addition, since during the relevant time period most countries were open to trade and had more and more coordinated monetary policy we analyze afterwards a monetary union composed of two countries. When countries are identical including inequality results are similar to the closed economy case. The relevant question is whether the union with a country with higher (or lower) inequality alters the impact on efficiency and equity of the permanent decline in inflation.

Agents' heterogeneity has its roots[1] on differences in labor productivity and in the initial wealth held by every agent. The effect of the policy change on equity will depend on some well known and robust cross country facts of the joint distribution of these characteristics. In particular wealth is more concentrated than earnings and that these two characteristics are positively

[1] Not the fundamental ones that is not the aim of thsi paper to explain, but the near ones that allows for the characterization of the economy.

correlated in the population. In the numerical examples we will concentrate on heterogeneity coming from a given initial wealth distribution, since as show elsewhere[2] this is the dominant characteristic when analyzing the effects on the decisions of different agents.

This article focus on a stationary economy meaning that, given constant policies, prices and allocations are constant over time. Therefore I abstract from capital, and labor is the only input in production. Given stationarity, the real interest rate is constant across policies and there is a one to one relation between changes in inflation and changes in the nominal interest rate. Monetary policy is therefore characterized either by the nominal interest rate or the inflation rate, and these two prices will be used with the same meaning.

Since inflation is a source of revenue for central banks (and eventually to governments) comparing policy regimes associated with different inflation rates while maintaining other taxes is not a complete exercise. Different revenues from the inflation tax should be compensated by an increase of alternative tax rates. This article develops a revenue neutral exercise where the decline of the inflation tax is accompanied with an increase of consumption (VAT) taxation. Since this is the more similar tax to the inflation tax, we can in this way stress the effects which come from the specific channels of inflation.

How a given distribution of households faced with a given inflation tax decide and it compares with the decisions of a different distribution of households and the difference the aggregation of those decisions impose on general equilibrium is the first objective. If the aggregate equilibrium is affected by the distribution how can we measure the effects of the change on the inflation rate both on efficiency and on equity. The results of this paper are the following: contrary to most of the literature, the model economy able to replicate cross section data, we found that the aggregate effect for a given policy is highly dependent of exogenous heterogeneity. This dependence is very clear for significative inflation rates and disappears for very low levels of inflation. therefore the effect of the decline on efficiency is highly dependent on the existing heterogeneity. For the very simple numerical example it can be very significative for an economy with a high degree of inequality. The effects on equity always reinforce the effects on efficiency. There is no trade-off.

[2] See for example Correia (2010).

1.1 The closed economy model

The monetary economy is populated by agents that decide over consumption and leisure as well as over the means of payment. Households hold money because it is an alternative means of payment to costly credit. Credit services are produced by a transaction technology that uses labor as input. As discussed below, whether this technology has constant returns to scale or not is determinant for the ability of the model to replicate cross section facts, and will have an important role on the evaluation of the effects of different inflation regimes.

The economy is well described by a monetary general equilibrium model where the credit technology is $s = l(m, C)$, where s is time used on transactions paid with credit, and m and C represent, respectively, real balances and consumption. There is no physical capital and the production technology of the consumption good is linear in labor with a unitary coefficient. The government must finance a constant exogenous government expenditures, and collects revenues from the inflation tax and from a tax on consumption expenditures. There is a set of households indexed by i which can be differentiated by their labor productivity and their initial financial wealth in real terms, represented respectively by E_i and A_{i0}[3].

Stationarity allows us to concentrate in momentary preferences. We choose an utility function linear in consumption[4]

$$v_i = C_i - \chi N_i^{\varphi},\ \chi > 0, \varphi > 1. \tag{1}$$

The stationary budget constraint is given by the following expression

$$(1+\tau_c)C_i + wl\left(m_{i,}C_i\right) + Rm_i = wE_iN_i + (1-\beta)\,A_{i0}, \tag{2}$$

where τ_c represents the tax on consumption expenditures, w the wage paid by firms, R the nominal interest rate and N_i total hours of work.

The choice of real money demand is such that the cost of one additional unity of money, R, should equalize the benefit in terms of reduction of transaction costs, measured by net wage times the decline in hours necessary for transactions with credit, which is given by $wl_m\left(m_{i,}C_i\right)$.

[3]For the numerical analyis we will impose $E_i = 1$.

[4]See Correia (1999) to justify this type of preferences.

1.2 First stage - constant returns to scale in transactions

When the technology of transactions is constant returns to scale, CRS, we can say that for a given ratio of money to consumption, $\frac{m_i}{C_i}$, the marginal and average labor productivity on transactions do not depend on the level of consumption. As an example let us suppose that $l\left(m_{i,}C_i\right) = (k\left(1-\frac{m_i}{C_i}\right)^2)C_i$. In this case the optimal choice of money, $R = wl_m\left(m_{i,}C_i\right)$, is given by

$$\frac{m_i}{C_i} = (1 - \frac{R}{2kw}) \leq 1, (= 1 \text{ for } R = 0) \tag{3}$$

This expression has the basic money demand characteristics, namely that money demand increases with the amount of transactions and declines with the opportunity cost of money, the nominal interest rate[5]. We can state that:

Result 1: When the transaction technology is CRS, $\frac{m_i}{c_i}$ is identical across households. The quantity of money rich and poor agents hold is a fixed fraction of their transactions.

In this case we can rewrite the budget constraint as

$$P_cC_i = wE_iN_i + (1-\beta)\,A_{i0}, \tag{4}$$

where $P_c \equiv (1+\tau_c) + wk\left(1-\frac{m_i}{C_i}\right)^2 + R\frac{m_i}{C_i}$, and $\frac{m_i}{C_i}$ is given by equation (3).

Note that in this case the effective price of consumption, P_c, includes the direct tax on consumption and the indirect cost due to payments. This one depends on the opportunity cost of holding cash, R, on the cost of labor used in credit, w, as well as on the transactions technology. Given that this effective price of consumption is identical across households the budget constraint, (4) can be used, and there is still a representative agent that describes the aggregate economy.[6]

Given the optimal decisions of every agent, its indirect utility can be written as

$$v_i = \left[\frac{\left[wE_i/P_c\right]^{\frac{\varphi}{\varphi-1}}}{(\chi\varphi)^{\frac{1}{\varphi-1}}}(1-\frac{1}{\varphi}) + (1-\beta)\,A_{i0}/P_c\right] \tag{5}$$

[5] Note that for a cash in advance economy, where $\frac{m_i}{C_i} \equiv 1$,for any R, the inflation tax and the consumption tax are equivalent.

[6] Or the economy is amenable to Gorman aggregation.

The representative agent of this economy, $i = r$, is characterized by $E_r = 1$ and $A_{r0} = 0$. When welfare is computed as the utility of the representative agent, or corresponds to the efficiency level of the economy, it is well known, see Correia and Teles (1996), that:

Result 2: (Friedman Rule) In a second best environment, to maximize the utility of the representative agent governments should abstain from taxing money, i. e. the government should follow the Friedman rule and set the nominal interest rate to zero. The gain will not depend on the specific joint distribution of wealth and labor efficiency that characterizes the economy.

Government expenditures should be financed with consumptions taxes and/or labor income taxes. Using (5), as well as the characteristics of the representative agent, we can write the utility of the representative agent as $v_r = \frac{[w/P_c]^{\frac{\varphi}{\varphi-1}}}{(\chi\varphi)^{\frac{1}{\varphi-1}}}(1-\frac{1}{\varphi})$. Because to decline the inflation rate increases the utility of the representative agent, it is immediate to conclude that the decline of inflation, compensated an increase in the tax on consumption, leads to an increase of the net real effective wage, w/P_c.

As was said before, in this first stage of the analysis, given that CRS transactions technologies allow for Gorman aggregation, the aggregate equilibrium is invariant to the specific joint distribution of wealth and labor efficiency. In addition in this case we can use a simple method to rank policies by their effects on inequality.[7] The simplicity of the methodology[8] allows for the development of economic intuition on the channels through which policy changes affect equity.

It also implies some restrictions on the multivariate distribution of characteristics across agents, but we will not take this as a cost since the class of characteristics for which the methodology is valid covers the most relevant cases of heterogeneity used in general equilibrium aggregate models, namely the heterogeneity in private wealth or the heterogeneity in labor efficiency. The question of the effect on equity of different inflation rates is equivalent to the comparison of the distribution across households of the burden from

[7] This methodology was developed in Correia (1999).

[8] The conditions to use that methos are rather strong for the case under study. As just stated in result 1, they are obtained at the cost of imposing a degenerate distribution in the money to consumption ratio across households. Only policy measures that yield an equilibrium in which all agents face the same prices can be discussed.

inflation versus the distribution of the burden of a tax on consumption.

How to compare those distributions is therefore the question.

A short description of equity evaluation The assumption of Gorman aggregation is equivalent to assuming indirect utility functions affine on characteristics, which can be represented by $v_i = \alpha(p)F(E_i) + \gamma(p)A_i$, where p is the vector of equilibrium prices faced by every household.

We have shown in (5) that for the specific case under analysis in this section, $\alpha(p)F(E_i) = \frac{[wE_i/P_c]^{\frac{\varphi}{\varphi-1}}}{(\chi\varphi)^{\frac{1}{\varphi-1}}}(1-\frac{1}{\varphi})$ and that $\gamma(p) = (1-\beta)/P_c$. To rank welfare distributions by equity we use the so called relative differential concept. When agents are ordered by decreasing welfare, meaning that $i > j$ implies $v_i < v_j$, we say that policy 2 dominates policy 1 in equity terms iff

$$\frac{v_i^2}{v_j^2} > \frac{v_i^1}{v_j^1}, \text{ for } i > j \tag{6}$$

The intuition for this condition is quite simple: suppose we compare any two households in the economy, agent i and agent j, where the first is poor (meaning that has a lower welfare, or income[9]). Then $\frac{v_i}{v_j} < 1$. When by the policy change this ratio increases, it means that the poor household is less distant from the richer one, that is, their economic situation is more equal than before. When this is true for every two agents then we say that the policy change leads to a more equal society, or that inequality declined.

Therefore the question is to understand how policy changes alter equilibrium prices, and then whether that change of prices in the economy increases $\frac{v_i}{v_j}$.

Using 5 let us begin by analyzing the easier case where agents are identical in labor productivity $E_i = E_j$. In this case we can write the relative welfare between agent i and j:

$$\frac{v_i}{v_j} = \frac{\gamma(p) + A_{i0}}{\gamma(p) + A_{j0}}$$

[9]Since utility is given by $\frac{[wE_i/P_c]^{\frac{\varphi}{\varphi-1}}}{(\chi\varphi)^{\frac{1}{\varphi-1}}}(1-\frac{1}{\varphi}) + \beta A_{i0}/P_c$, it can be read as a measure of income.

where

$$\gamma(p) = \frac{P_c \left[w/P_c\right]^{\frac{\varphi}{\varphi-1}}}{\beta\left(\chi\varphi\right)^{\frac{1}{\varphi-1}}}(1-\frac{1}{\varphi}) = \frac{P_c^{\frac{-1}{\varphi-1}}\left(w\right)^{\frac{\varphi}{\varphi-1}}}{\left(\chi\varphi\right)^{\frac{1}{\varphi-1}}}(1-\frac{1}{\varphi}) \tag{7}$$

As $A_{i0} < A_{j0}$, $\frac{v_i}{v_j} < 1$ and the change in policy increase relative welfare if $\gamma(p)$ increases.

Compensating inflation with a VAT tax The decline of inflation (a lower R) is compensated by an increase of the VAT tax (or the tax on consumption) to maintain tax revenues. The net real wage, w, is not affect by the change of policy. Then using result 2, which states that the net real effective wage, w/P_c, increases with the decline of inflation. The reason why in this case the lower inflation tax is efficient is that it declines the effective consumption price. As $\frac{m_i}{C_i} \leq 1$, and $\frac{m_r}{C_r} < 1$, the base of the consumption tax is higher then the base of the inflation tax. This means that although the tax on consumption increases it increases by less than the decline of the nominal interest rate. Other way to understand this result is to see that in the limit, when the nominal interest rate is zero and credit is not used as payment, the inflation tax is equivalent to the consumption tax. But when households decide to use credit for a share of payments, it is because at the existing interest rate, the cost of transactions is lower. Again the decline in R implies a positive income effect, that to be compensated implies an smaller increase in τ_c. We can write $\gamma(p)$ as the second expression in 7, and since $\varphi > 1$, a decline on P_c increases $\gamma(p)$.

We can summarize this in:

Result 3: A decline in inflation compensated by an increase in the consumption tax rate improves welfare distribution, when $E_i = E_j$[10].

Note that the robustness of this result, that the decrease of inflation compensated with a consumption tax reduces inequality, is not obvious even in this very simple set-up without the knowledge of the specific distribution of wealth. Since we know that the main effect of the change of policy is

[10] This is not an important assumption because it can be shown that when $E_i < E_j$ the same result is obtained, as long as wealth is more concentrated than earnings, as described epirirically.

the decline of the effective price of consumption what we can immediately guarantee is that richer agents with positive levels of initial wealth would gain by two reasons: first because the value of initial wealth in terms of consumption is higher, $(1-\beta)\, A_{j0}/P_c$; and second because the net real effective wage increases. Therefore households with non-negative wealth, as the representative agent, increase welfare given the proposed change of policy. The same cannot be said in the extreme case for agents that have negative initial wealth. As with richer agents they benefit from the the higher net effective wage, but since they are debtors, and the effective value of debt increases with the decline of the effective price, it would not be clear, just by analyzing the expression for their utility, why the first effects would always dominate this last one, and the poor is better off given the policy change. Note however this is always true, without any other channel in addition to the ones described, because:

Result 4: A decline in inflation compensated by an increase in the consumption tax leads to a Pareto movement.

Using result 3, the decline on inflation leads to an increase in $\frac{v_i}{v_r}$, for every i. Given result 2 the welfare of the representative agent, v_r, increases and therefore v_i also increases for every $i < r$. As we described for $i > r$ welfare increases. Therefore the proposed policy increases welfare for every household in this economy and therefore leads to a Pareto movement, but the poor, even when a debtor, increases welfare by more than the richer, the creditor. This result is important to develop the intuition for the results in

this paper.

1.2.1 Economies of scale

After analyzing the case where transactions technology is constant returns to scale, let us correct for the cross section evidence on payment patters. As before the stationary budget constraint can be written as:

$$(1+\tau_c)C_i + wl\,(m_{i,}C_i) + Rm_i = wE_iN_i + (1-\beta)\,A_{i0},$$

Let us assume now that the transactions technology $l\,(m_{i,}C_i)$ is no more homogeneous of degree one, and that it can be given, for example, by:

$$l\left(m_{i,}C_i\right) = k(1 - \left(\frac{m_i}{C_i}\right))^2 C_i + (1 - \left(\frac{m_i}{C_i}\right))\overline{N}, \tag{8}$$

where the main difference from the one used in the last sub-section is the inclusion of a cost that does not depend on the total amount of transactions but uniquely on the share of transactions paid with credit. It is a fixed cost for a given share. When this technology is used to compute the household optimal decision on money holdings we obtain that:

$$\left(\frac{m_i}{C_i}\right) = 1 - \frac{R}{2wk} + \frac{\overline{N}}{2kC_i} \tag{9}$$

It is immediate to conclude that, for $\overline{N} > 0$, the larger is C_i the smaller is the share of transactions realized with cash. That is:

Result 5: When transaction technologies are increasing returns to scale, $\frac{m_i}{C_i}$ is no more constant across households. Rich agents hold a lower share of cash to transactions than poor agents.

This money demand replicates the facts that we quote in the beginning of this article. Agents differ on m/C depending on the total volume of transactions. There is a group of households for which $C_s < \frac{w\overline{N}}{R}$ that do not use credit for transactions. They use just cash for payments and therefore

1) $m_i = C_i$, $i < s$.

The other subset of the population for which $C_j > \frac{w\overline{N}}{R}$, decide to use both cash and credit for payments. However they decide to use more credit the higher is the transactions level, that is the richer they are, and therefore the higher the wealth, the lower its cash to wealth ratio. For this group money demand is given by

2) $m_j = \left(1 - \frac{R}{2wk}\right) C_j + \frac{\overline{N}}{2k}$, $j > s$.

Then we can write the budget constraint for every household as

$$P_{ci}C_i + w(1 - \left(\frac{m_i}{C_i}\right))\overline{N} = wE_iN_i + (1-\beta)\,A_{i0}, \tag{10}$$

The effective price of consumption is now specific to each household, includes the cost of holding money and the average cost of using credit, and given by

$$P_{ci} = (1 + \tau_c) + R\left(\frac{m_i}{C_i}\right) + wk(1 - \left(\frac{m_i}{C_i}\right))^2 \qquad (11)$$

In addition we can observe that the heterogeneity of this price comes uniquely due to the share of payments done with cash[11], which as stated in result 5, is now different across agents.

In addition it is straightforward to compute that

$$\partial \frac{P_j}{\left(\frac{m_j}{C_j}\right)} = R - 2wk(1 - \left(\frac{m_j}{C_j}\right)) = \frac{w\overline{N}}{C_j} > 0$$

which implies the following result:

Result 6: With economies of scale in the transactions technology there is a non degenerate distribution of $\frac{m}{C}$ across households. Poor agents consume less, have a higher share of money and a higher effective price of consumption. The existence of inflation with this type of technology is an endogenous source of inequality.

Since the main objective of this article is to understand the connection between different equilibria associated with a changing inflation rate and inequality this result is quite important. It explains that, when the monetary model economy is able to replicate payments facts, the mere existence of inflation implies that different agents react to inflation deciding to participate or not in the credit market. The existence of fixed costs in the use of credit implies that the effective price of consumption is higher for those agents that do not have an advantage in using credit. And for those that use it, the richer they are the lower is the effective price of consumption. This extensive margin on the decision will be crucial to understand how the aggregates will depend on exogenous heterogeneity. Also the effect on equity is amplified since, as we just described, inflation amplifies the exogenous inequality.

Before entering on the model solution we can see that the relative price of consumption across agents depends on the inflation level. Inflation, directly and through m/C, affects the relative effective price of consumption:

$$\frac{d\frac{P_{ci}}{P_{cj}}}{dR} > 0$$

[11] This is the advantage of the proposed transactions technology.

We had described before that the price for the poorer household is higher than for the richer, i.e., the relative price $\frac{P_{ci}}{P_{cj}}$, for $i < j$, is greater than one. Now we see that when inflation increases both households face a higher price but because the richer households have higher advantage for substituting cash by credit, the price faced by the richer households increases less than that faced by poorer households. Then we can say that:

Result 7: With economies of scale in the transactions technology, inflation acts like a regressive tax on consumption. With a higher inflation the effective tax rate on consumption increases more for poorer than for richer households.

Inflation is thus not just an additional source of inequality but the increase of inflation is regressive. As we are analyzing the effects of the decline of the inflation tax we can say that, if the rest of the analysis would be maintained, the decline of inflation would work as a progressive policy. Through this additional channel now discussed, the decline in inflation would reduce inequality.

As mentioned in the beginning, with this transactions technology which display increasing returns, in this monetary economy it is no longer possible to aggregate households decisions and to compute equilibrium prices that do not depend explicitly on the underlying distribution. A necessary condition for aggregation is that prices faced by different agents should be identical, and we just showed that this is not the case with increasing returns, since the effective price of consumption is household' specific.

To pursue this work the construction, calibration and numerical computation of the equilibrium in this non-aggregable heterogeneous agent model is necessary, even for the very simple comparisons of stationary equilibria. The results would then be always conditional on the specific calibration, either of the parameters that command the aggregate behavior, or of the proposed joint distribution of characteristics across households.

1.2.2 The numerical solution

Although being an heterogeneous non aggregable model we constructed a quite simple algorithm to solve for the general equilibrium, for a given policy. This algorithm is described in the appendix[12]. Here we present the results

[12] Also the algoritm that for each change of policy, the decline in the interest rate, computes the associated increase of the consumption tax able to finance the same government

obtained for a given calibration of the economy. Namely we show how the heterogeneity across agents deliver significantly different aggregate outcomes, and to what extent relative individual outcomes are also affected by initial relative characteristics.

Using the optimal choice of money given by equation (9) and substituting it in the definition of idiosyncratic consumption price given by (11), we can write the intratemporal decision of households as

$$N_i = \{\frac{1}{\epsilon}\left[\frac{wE_i}{P_{ci} + g'(C_i)C_i - w\overline{N}f'(C_i)}\right]\}^{\frac{1}{\chi}} \tag{12}$$

where $g(C_i) \equiv P_{ci}$ and $f(C_i) \equiv \frac{m_i}{C_i}$.

Using the budget constraint (10) we can write it as

$$g(C_i)C_i + w(1 - f(C_i))\overline{N} = wE_iN_i(C_i) + (1 - \beta)\,A_{i0} \tag{13}$$

If markets are competitive, firms with a linear technology equate the wage, w to that constant productivity, and therefore $w = z$.

For every interest rate and tax on consumption we can use the set of equations given by (13), one for each agent, to compute the consumption in equilibrium for every agent.

The equilibrium would therefore be characterized by the set $\{C_i, N_i,\ P_{ci},\ \frac{m_i}{C_i}, R, \tau_c, w = z\}$, such that equations (9), (11), (12) are (13) are satisfied.

1.2.3 Effects of heterogeneity on efficiency and equity

Let us suppose that the interest rate is constant and study the effects on the aggregate and the individual equilibrium of a change of the distribution of characteristics in the economy. That is we will maintain government expenditures, and the interest rate and change the economy from an economy where agents are identical because they have the same stock of initial wealth, to an economy where, although the aggregate wealth is maintained, its distribution is more and more unequal. That is we want to analyze the effects on equilibrium of a pure redistribution of wealth across agents in this economy. If the equilibrium reacts to the redistribution, as we will describe, the tax on consumption that satisfies the budget constraint for the same government expenditures and for a constant interest rate would change.

The results are shown in Figure 1, and can be summarized as,

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expenditure, is described in the appendix.

Result 8: The relation between inequality and efficiency is not monotonous. When we compare a representative economy with an economy with a substantial degree of heterogeneity efficiency declines with inequality. A pure redistribution increases efficiency. The consumption tax rate necessary to finance government expenditures increases always with inequality.

1.2.4 The change of the inflation tax

When agents are not identical, Figure 2 shows the effects of the decline of inflation associated with a decline of the nominal interest rate from a tax of around 10% to a constant level of 3%. They are summarized as result 9.

Result 9: The decline in inflation has a significative effect on efficiency and a positive effect on equity. Even without a pure redistribution a more equal economy can be achieved simultaneously with a more efficient one. The magnitude of these effects does not depend on the heterogeneity of the economy.

2 The inflation tax in open economies

How important for the questions in the last sections is the hypothesis that the country was well represented by a closed economy? Here we consider a standard international macroeconomic model. Monetary policy is such that the exchange rate is fixed or, alternatively, countries belong to a monetary union. Let us assume a world of two countries with identical tastes, technologies and population. We denote the home country with H and the foreign country with F. Firms use technologies that are linear in labor, and productivity is identical across goods and across countries.

Government expenditures are identical across goods and countries. The monetary authority of the monetary union issues the common currency, that is distributed endogenously across countries in order to satisfy demand. Monetary policy sets the interest rate, which is the instrument of monetary policy. Seigniorage is divided equally across countries.

There are union-wide markets for the goods but the market for labor is segmented across countries. Labor is homogeneous and mobile inside each country but immobile across countries. There is a non-state contingent nominal asset traded across countries. If there was perfect mobility of labor the terms of trade channel of the monetary transmission mechanism would be closed.

Households Preferences are identical across households, and given by (1). Aggregate consumption of household i that lives in country H, C_i, is defined as:

$$C_i = C_{i,h}^{\alpha} C_{i,f}^{1-\alpha}, \text{ with } 0 < \alpha < 1, \text{ for all } t,, \tag{14}$$

where $C_{i,h}$ corresponds to consumption of household i that lives in country H of the good produced at home, and $C_{i,f}$ corresponds to consumption of household i that lives in country H of the good produced in the foreign country.

By standard procedures it can be shown that price index in each country is given by $P = \Phi P_h^{\alpha} P_f^{1-\alpha}$, with $\Phi = \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}$, where P_h is the price of the good produced in country H and P_f is the price of the good produced in country F. The assumption that every consumer good is tradable and the assumption of identical preferences implies an identical consumer price index across countries. One of the first-order conditions for all households is that the terms of trade or relative price, $p \equiv \frac{P_f}{P_h}$, must be equal to the relative consumption, $\frac{C_{i,h}}{C_{i,f}}$, times the relative preference, $\Theta = \frac{1-\alpha}{\alpha}$. A similar condition holds for the households of country F and for the expenditures of each government, as we assume that the governments choose the consumption of each good that minimizes the aggregate level of expenditures. The aggregate consumption of household i in country F, C_i^*, and the aggregate government consumption of country H, G, and the aggregate government consumption of country F, G^*, are given by

$$C_i^* = C_{i,h}^{*\alpha} C_{i,f}^{*1-\alpha}, \tag{15}$$

$$G = G_h^{\alpha} G_f^{1-\alpha}, \tag{16}$$

and

$$G^* = G_h^{*\alpha} G_f^{*1-\alpha}, \tag{17}$$

respectively, where C_h^* denotes the foreign consumption of the good produced in country H, C_f^* denotes the foreign consumption of the good produced in country F, G_h and G_f are the government expenditures of country H in the good produced in H and in the good produced in F, respectively. And G_h^* and G_f^* are the government expenditures of country F in the good produced in H and in the good produced in F, respectively. We adopt the convention of indexing with a star the variables concerning country F. As referred, the relation between relative consumptions and the terms of trade is

$$
\begin{aligned}
p_t &= \Theta \frac{C_{i,h}}{C_{i,f}} = \Theta \frac{C_{i,h}^*}{C_{i,f}^*} && (18) \\
&= \Theta \frac{G_h}{G_f} = \Theta \frac{G_h^*}{G_f^*}, \text{ for all } i. && (19)
\end{aligned}
$$

As before the stationary budget constraint can be written as:

$$
(1+\tau_c) C_i + wl\left(m_{i,} C_i\right) + Rm_i = wE_i N_i + (1-\beta)\, A_{i0}, \tag{20}
$$

where now w, m_i and A_{i0} are expressed in units of the composite good.

Again, for the transactions technology given by (8), the optimal choice for the ratio between consumption and money, $\frac{m_i}{C_i}$, is given by equation (9). After replacing for the optimal decision of $\frac{m_i}{C_i}$ and for the transactions technology in (20) we obtain the budget constraint (21)

$$
g(C_i) C_i + w(1 - f(C_i))\overline{N} = wE_i N_i(C_i) + (1-\beta)\, A_{i0}, \tag{21}
$$

where $g(C_i) \equiv P_{ci}$, being P_{ci} given by (11) and $f(C_i) \equiv \frac{m_i}{C_i}$. There are similar budget constraints for the households of country F

$$
g(C_i^*) C_i^* + w^*(1 - f(C_i^*))\overline{N} = wE_i^* N_i^*(C_i) + (1-\beta)\, A_{i0}^*, \tag{22}
$$

The intratemporal decision of the households of country H implies

$$
N_i = \left\{ \frac{wE_i}{\epsilon} \frac{1}{g(C_i) + g'(C_i) C_i - w\overline{N} f'(C_i)} \right\}^{\frac{1}{\chi}}. \tag{23}
$$

There are similar conditions for the households of country F,

$$
N_i^* = \left\{ \frac{w^* E_i^*}{\epsilon} \frac{1}{g(C_i^*) + g'(C_i^*) C_i^* - w\overline{N} f'(C_i^*)} \right\}^{\frac{1}{\chi}}. \tag{24}
$$

Since markets are competitive and firms have a linear technology in labor, the nominal wage, W is equal to the nominal constant productivity,

$$
W = zP_h.
$$

The real wage that shows in the budget constraints, w, is a function of the terms of trade,

$$w = \frac{W}{P} = \frac{zP_h}{P} = \frac{z}{\Phi}\left(\frac{1}{p}\right)^{1-\alpha}. \tag{25}$$

There is a similar condition for the nominal wage paid by foreign firms, W^*,

$$W^* = zP_f.$$

Thus, the real wage that shows in the budget constraints of the foreign household is

$$w^* = \frac{zP_f}{P} = \frac{z}{\Phi}p^{\alpha}. \tag{26}$$

We assume, as is standard in this literature, that aggregate public expenditures are exogenous and that the consumption tax rates adjust to satisfy the government budget constraints of country H and F,

$$\tau_c \sum_i \left(C_{i,h} + pC_{i,f}\right) + R\sum_i m_i = G_h + pG_f + (1-\beta)\, B_0, \tag{27}$$

and

$$\tau_c^* \sum_i \left(C_{i,h}^* + pC_{i,f}^*\right) + R\sum_i m_i^* = G_h^* + pG_f^* + (1-\beta)\, B_0^*, \tag{28}$$

where B_0 and B_0^* are the initial public debt of country H and F, respectively. Since we want to emphasize the case of similar countries we assume that $B_0 + \sum_i A_{i0} = B_0^* + \sum_i A_{i0}^* = 0$.

The equilibrium condition in the home good market is

$$\sum_i C_{i,h} + \sum_i C_{i,h}^* + G_h + G_h^* + z\sum_i l_i = z\left(\sum_i E_i N_i\right) \tag{29}$$

There is a similar market clearing condition for the foreign good,

$$\sum_i C_{i,f} + \sum_i C_{i,f}^* + G_f + G_f^* + z\sum_i l_i^* = z\left(\sum_i E_i^* N_i^*\right). \tag{30}$$

Using (18), (19), (29) and (30) we get

$$\frac{\alpha}{1-\alpha}p\left(\sum_i E_i N_i^* - l_i^*\right) = \left(\sum_i E_i N_i - l_i\right) \tag{31}$$

We take the interest rate interest rate, R, the initial wealth levels $\{A_{i0}, A^*_{i0}, B_0, B^*_0\}$ and the aggregate government expenditures vector, $\{G, G^*\}$, as exogenous variables. It is straitforward to verify that the number of static equilibrium equations displayed above is exactly equal to the number of the endogenous variables which are $\{C_i, C_{i,h}, C_{i,f}, L_i, N_i, m_i, w, G_h, G^*_h, \tau_c, C^*_i, C^*_{i,h}, C^*_{i,f}, L^*_i, N^*_i, m^*_i, w^*, G_f, G^*_f, \tau^*_c, p\}$.

The algorithm used to compute the equilibrium variables is pretty basic. It has two stages. In the first stage we compute all the equilibrium variables for a set of tax rates $\{\tau_c, \tau^*_c\}$. In the second stage we check if the set of tax rates considered can be part of an equilibrium. Thus, we write l_i and $\frac{m_i}{C_i}$ from (8) and (9) as a function of C_i. Similarly we determine l^*_i and $\frac{m^*_i}{C^*_i}$ as a function of C^*_i. For given $\{\tau_c, \tau^*_c, R\}$ the vector $\{C_i, N_i, w\}$ is determined from (21), (23) and (25) as a function of p, and similarly the vector $\{C^*_i, N^*_i, w^*\}$ is determined from (22), (24) and (26) as a function of p. Once we introduce the variables $\{N_i, l_i, N^*_i, l^*_i\}$, which we just determined as a function of p, in equation (31) we obtain the equilibrium value for p. Given the terms of trade, p, and the aggregates $\{C_i, C^*_i, G, G^*\}$ from (14), (15)-(17), and (18)-(19) we obtain $\left\{C_{i,h}, C_{i,f}, C^*_{i,h}, C^*_{i,f}, G_h, G^*_h, G_f, G^*_f\right\}$. Finally, we check if the vector of tax rates $\{\tau_c, \tau^*_c\}$ satisfies the government budget constraints, (27) and (28). If not we go back to stage one, in which we compute all the equilibrium variables for a new set of tax rates. The new candidate tax rate will be slightly lower if the public revenue exceeds the public expenditure and it will be slightly higher otherwise. We keep iterating until (27) and (28) are satisfied.

Results Using the equilibrium conditions in the appendix we can verify that, for a given interest rate, heterogeneity can play a large role in the equilibrium. With the interest rate equal to 10% we saw that in closed economies heterogeneity is amplified when households have different stocks of initial wealth. Here we can see that when exogenous heterogeneity, distribution of initial wealth, differs across countries the terms of trade in equilibrium is different from 1. In Figure 3 we maintain the distribution of wealth of the home country and compute the general equilibrium for different distributions of the foreign country. When those distributions are identical (difference across households inside each country equal to 0.16), the equilibrium terms of trade is one and the equilibrium of the open and closed economies coincide. When the foreign country is more unequal the terms of trade ($p \equiv \frac{p_f}{p_h}$) increases which implies a decline of efficiency in the home country. Therefore

Result 10: For a given distribution of characteristics in the home country, the more equal the foreign country the higher the efficiency of the home country. This mechanisms comes from the decline in the terms of trade associated with the increase in inequality in the foreign country.

This same result can be seen in Figure 4 where, for a representative agent in the foreign country, efficiency in the home country is 4% higher than when distribution is very unequal in the foreign country.

Result 11: The effect on inequality in home efficiency has a much lower effect on efficiency at home than the same change in inequality in the foreign country. The effect on equity depends mainly on home inequality but inequality increases with inequality abroad.

0.5% (representative agent) versus 2.8%.

Result 12: The gain on efficiency of the decline on inflation is higher the more unequal is the foreign country. The same happens with equity.

0.3% (representative agent in foreign) versus 1.7%.

3 Conclusions

When households heterogeneity is taken into account a change in equilibrium, driven for example by a policy change, implies distributional effects: different agents are affected differently by the new equilibrium. In This paper we have shown that, contrary to most heterogeneous agent economies, the aggregate equilibrium is also significantly affected by the heterogeneity. Economies identical but for the exogenous household distribution have different aggregate equilibria. While in the literature there is a strong connection between inflation and inequality, there is to our knowledge no strong results on how the connection between changes in inflation and the aggregate outcome is influenced by the underlying heterogeneity.

Therefore to understand the gains from the decline in inflation, we need to take a position on the jointly distribution that characterizes agents in the economy. In addition to the understanding of the positive effects on equity of that positive change, the results on efficiency are very large, when compared with the usually reported in the literature once we take a position on welfare distribution in line with empirical evidence.

We conjecture that this result could be extended to any framework where the extensive margin of agent's decision dominates the intensive margin, and could in this way to be a quite general result.

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