# Optimal Capital Taxation with Entrepreneurial Risk

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**Abstract**

We examine the optimal taxation of capital in a Ramsey setting of a general-equilibrium heterogeneous-agent economy with uninsurable idiosyncratic investment or capital-income risk. We find that the optimal capital-income tax is positive. This result is due both to the ex ante insurance or ex post redistribution aspect of the tax, and to a novel general-equilibrium insurance role of the tax that allows for the possibility of increased capital accumulation when the capital tax increases.

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# 1 Introduction

We study optimal capital taxation in an environment where agents face uninsurable idiosyncratic investment or capital-income risk. Such risk is empirically important for entrepreneurs and business owners, who hold most of an economy's wealth, despite being only a small fraction of its population. In this context, capital taxation raises an interesting tradeoff: On the one hand, it comes at the usual cost, as it distorts agents' saving decisions. On the other hand, it has benefits, as it provides agents with partial insurance against idiosyncratic investment risk. This suggests that a positive tax on capital income could be welfare-improving.

However, we show that this ex-ante insurance or ex-post redistribution aspect of the tax is not, on its own, sufficient to generate the possibility of a positive optimal capital tax. In particular, in the small open economy version of our model, where the interest rate is exogenously fixed, the optimal capital tax is in fact zero, despite the fact that the tax does reduce the effective variance of risk agents face. By contrast, when the economy is closed and the interest rate adjusts to clear the bond market, the optimal tax on capital is positive. This means that the positive-optimal-tax result hinges on general-equilibrium effects, operating through the adjustment of the interest rate. In other words, the rationale for positive capital taxation in our framework is motivated by an indirect or general-equilibrium insurance aspect of the tax. Here, an increase in the capital tax reduces the effective variance of risk (and allows for some direct redistribution). This reduction in risk reduces the demand for precautionary saving, and increases the interest rate in equilibrium. In turn, the increase in the interest rate might increase wealth and, with decreasing absolute risk aversion, wealthier agents may then be willing to undertake more risk, leading to increased capital accumulation. Therefore, the additional general-equilibrium effect of the tax on the aggregates and on insurance provision through the interest rate generates the result that the optimal capital-income tax is positive.

The result that a positive tax on capital income might actually stimulate capital accumulation was first shown in Panousi (2011), in a framework similar to the one we employe here. That paper did not solve for the optimal taxation problem; rather, it performed comparative-static exercises with respect to the capital tax, suggesting that the optimal tax might be positive. Our present paper verifies and formalizes this intuition, by fully solving the social planner problem of maximizing ex ante welfare, in a Ramsey framework of optimal taxation, though also allowing for the possibility of lump-sum transfers.

Our modelling framework builds on Angeletos (2007), who develops a variant of the neoclassical growth model that allows for idiosyncratic investment risk, and studies the effects of such risk on macroeconomic aggregates. Agents own privately-held businesses that operate under constant returns to scale. These private businesses are subject to idiosyncratic risk that the agents cannot diversify away. However, agents are not exposed to labor-income risk, and they can also freely borrow and lend in a riskless bond. Abstracting from borrowing constraints, labor-income risk, and other market frictions, isolates the impact of the idiosyncratic investment risk, and preserves tractability of the model. Finally, there is a government, imposing proportional taxes on capital- and labor- income, along with a non-contingent lump-sum tax or transfer.

## 2 Related Literature

This paper relates to the macroeconomic literature on optimal taxation. Most of this literature has focused either on complete markets or on labor-income risk. Chamley (1986), Judd (1985), and Atkeson, Chari and Kehoe (1999) have established the result of zero optimal capital taxation when markets are complete. Gourio (2008) quantitatively examines the effects of taxation in a neoclassical growth model with heterogeneous agents. He finds that a majority would lose from elimination of the capital-income tax, since this is accompanied by increases in the labor tax, which affect most of the population negatively. Correia (1996) shows that, in the neoclassical model, capital income should not be taxed in the steady state, if every factor of production can be taxed at the optimal rate. By contrast, when there are restrictions on the taxation of production factors, the tax rate on capital income in the steady state is different from zero.

Aiyagari (1995) extended the complete-markets framework to include uninsurable labor-income risk and borrowing constraints, and found that the optimal capital tax is positive in the long run. Chamley (2003) argued that the it might be best to think of the reasoning behind Aiyagari's positive optimal capital tax as related to the ex-ante insurance or ex-post redistribution aspect of the tax: Namely, the planner taxes agents with high income realizations, and subsidizes agents with low income realizations, thereby equalizing consumption across different types of agents. The capital tax in our framework performs a similar function (direct insurance). However, we show that in our framework this direct insurance aspect of the tax, when not accompanied by indirect or general-equilibrium insurance provision, in not sufficient to generate a positive optimal tax.

A related but different normative exercise to that in Aiyagari (1995) is conducted by Davila, Hung, Krusell and Rios-Rull (2005), in the spirit of Geanakoplos-Polemarchakis. Albanesi (2006) considers optimal taxation in a two-period model of entrepreneurial activity, in a Mirrlees constrained-efficiency setting. The benefit of her approach is that the source of incomplete risk-sharing is endogenously specified, and that there are no ad hoc restrictions on the tax instruments. However, her model does not allow for dynamics, or for general-equilibrium effects like those we study. In general, the extensive theoretical work on taxation originating from the Mirrlees tradition focuses on labor-income risk. This literature shows that, if insurance is limited due to the presence of asymmetric information, then it may be best to restrict free access to savings. This result has in turn been interpreted as a justification for capital taxation. Some additional examples include Diamond and Mirrlees (1978), Golosov, Kocherlakota, and Tsyvinski (2003), Kocherlakota, (2005), Albanesi and Sleet (2006), Golosov, Troshkin, Tsyvinsky and Weinzierl (2010), and Werning (2011). Farhi and Werning (2008) study optimal nonlinear taxation of labor and capital in a political economy model with heterogeneous agents, where policies are chosen sequentially over time, without commitment, as the outcome of democratic elections. They find that credible policies show a concern for future inequality and that capital taxation emerges as an efficient redistributive tool for this purpose.

The overlapping-generations literature has often found support for positive optimal capital taxation. Conesa, Kitao and Krueger (2009) quantitatively characterize the optimal capital and labor income tax in an overlapping generations model with idiosyncratic uninsurable income shocks and permanent productivity differences across households. They find that the optimal capital-income tax rate is significantly positive at 36 percent, mainly driven by the life-cycle structure of the model. Domeij and Heathcote (2004) perform a similar exercise. Uhlig and Yanagawa (1996) show that higher capital income taxes lead to faster growth in an overlapping generations economy with endogeneous growth. Erosa, A., and M. Gervais (2002), Garriga (2003), and Peterman (2011) examine optimal taxation in life-cycle economies with human capital.

A strand of the public finance literature has examined the effects of capital taxation on risk taking, mostly in a partial equilibrium framework. Some examples include Domar and Musgrave (1944), Stiglitz (1969), Ahsan (1974), Sandmo (1977), and Kanbur (1981). Our paper is most closely related to Varian (1980), who assumes that differences in observed income are due to exogenous differences in luck. In a two-period model of endogenous saving, he finds that the optimal capital-income tax is positive due to the trade-off it involves between the distortion in the saving decision and the provision of social insurance through redistribution. Our paper extends Varian's result to infinite horizon, and to the inclusion of production functions and sources of safe income. Furthermore, our paper corrects Varian's rationale for a positive optimal capital tax. We show that, in the open-economy version of our model, where the interest rate is exogenously fixed, the optimal capital tax is in fact zero, despite the redistribution it entails. By contrast, when the economy is closed and the interest rate adjusts to clear the bond market, the optimal capital tax is positive. This shows that the ex ante insurance or ex post redistribution aspect of the tax alone is not sufficient to outweigh the distortion of the tax on savings. Therefore, the positive capital tax is due to a general-equilibrium insurance aspect of the tax, operating through the interest rate.

We focus on entrepreneurial risk, because such risk is in fact empirically relevant, even in a financially developed country like the United States. For example, Moskowitz and Vissing-Jorgensen (2002), among others, find that about 80 percent of all private equity is owned by agents who are actively involved in the management of their own firm, and for whom such investment constitutes at least half of their total net worth. It seems plausible, then, that entrepreneurial risk must be even more prevalent in less developed economies. Furthermore, Panousi and Papanikolaou (2011) show that the negative relationship between idiosyncratic risk and the investment of *publicly-traded* firms in the United States is stronger in firms where the mangers hold a larger fraction of the firm's shares. Combined, these findings strengthen the empirical applicability of our model setup, because they demonstrate that a large fraction of total investment in the United States is sensitive to idiosyncratic risk, through the risk aversion of the agents making the investment decisions.

The paper is organized as follows. All proofs are delegated to the appendix.

# 3 Benchmark model

Time is continuous and indexed by $t \in [0, \infty)$. There is a continuum of agents distributed uniformly over $[0, 1]$.

## 3.1 Households and preferences

All agents are endowed with one unit of time. Preferences are logarithmic over consumption, $c$, i.e. $U_t = E_t \int_t^{\infty} e^{-\beta s} U(c_s)\, ds$, where $\beta > 0$ is the discount rate and $U(c) = ln(c)$.

## 3.2 Firms and idiosyncratic risk

An entrepreneur owns and runs a firm operating a production function $F(k) = Ak$, where $k$ is capital input. The entrepreneur can invest only in the capital of his own firm. This capital investment in his firm is subject to idiosyncratic uninsurable risk. The idiosyncratic shocks are i.i.d., and hence there is no aggregate uncertainty. An entrepreneur can also save in a riskless bond, $b$. Agents do not receive wage or transfer income.

The financial wealth of an entrepreneur $i$, denoted by $x_t^i$, is the sum of his holdings in private capital, $k_t^i$, and in the riskless bond, $b_t^i$, i.e. $x_t^i = k_t^i + b_t^i$. The evolution of $x_t^i$ is:

$$dx_t^i = (1 - \tau_t)\, d\pi_t^i + [\, (1 - \tau_t)\, R_t\, b_t^i - c_t^i \,]\, dt\,, \tag{1}$$

where $d\pi_t^i$ are the firm's profits (capital income), $R_t$ is the interest rate on the riskless bond, $\tau_t$ is the proportional capital-income tax, and $c_t^i$ is consumption. A no-Ponzi-game condition is imposed, ensuring the non-negativity of consumption.

Firm profits are given by:

$$d\pi_t^i = [\, A\, k_t^i \, - \delta\, k_t^i \,]\, dt \, + \, \sigma\, k_t^i\, dz_t^i\,, \tag{2}$$

where $F(k) = Ak$ is the production function, $A$ is the marginal product of capital, and $\delta$ is the mean depreciation rate in the aggregate economy. Idiosyncratic risk is introduced through $dz_t^i$, a standard Wiener process that is i.i.d. across agents and across time. Literally taken, $dz_t^i$ represents a stochastic depreciation shock. However, these shocks can also be modelled or interpreted as stochastic productivity shocks. The scalar $\sigma$ measures the amount of undiversified idiosyncratic risk, and it is an index of market incompleteness, with higher $\sigma$ corresponding to a lower degree of risk-sharing, and with $\sigma = 0$ corresponding to complete markets.

### 3.3 Government

At each point in time the government taxes capital income and bond income at the rate $\tau_t$. Part of the tax revenue is used by the government for own consumption at the deterministic rate $G_t$. This government spending does not affect the utility from private consumption or the production technology. The government budget constraint is:

$$dB_t^g = [\, \tau_t \,( A - \delta\, ) \int_i k_t^i + \tau_t \, R_t \, B_t^g - G_t \,] \, dt \,, \tag{3}$$

where $B_t^g$ denotes the level of government assets (i.e. minus the level of government debt). A no-Ponzi game condition is imposed to rule out explosive debt accumulation.

## 4 Equilibrium

This section characterizes individual behavior and the general equilibrium in the economy. The initial position of the economy is given by the distribution of $(k_0^i, b_0^i)$ across households. An equilibrium is a deterministic sequence of prices $\{R_t\}_{t\in[0,\infty)}$, a deterministic sequence of policies $\{\tau_t, G_t\}_{t\in[0,\infty)}$, a deterministic macroeconomic path $\{C_t, K_t, Y_t, X_t\}_{t\in[0,\infty)}$, and a collection of individual contingent plans $(\{c_t^i, k_t^i, b_t^i, x_t^i\}_{t\in[0,\infty)})$ for $i \in [0, 1]$, such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the bond market clears, $\int_t b_t^i = 0$, in all $t$; (iii) the government budget constraint (3) is satisfied in all $t$; and (iv) the aggregates are consistent with individual behavior, $C_t = \int_i c_t^i$, $K_t = \int_i k_t^i$, $Y_t = \int_i F(k_t^i) = A \int_i k_t^i$, $X_t = \int_i x_t^i$, in all $t$.

### 4.1 Individual behavior

Agents' production function, $F$, exhibits constant returns to scale, and hence optimal profits are linear in own capital:

$$d\pi_t^i = r \, k_t^i \, dt + \sigma \, k_t^i \, dz_t^i \,, \tag{4}$$

where $r \equiv A - \delta$. Here, $r$ is an entrepreneur's expectation of the net return to his capital prior to the realization of his idiosyncratic shock, as well as the mean of the realized returns in the cross-section of firms, since there is no aggregate uncertainty. As in Angeletos (2007), the key result here is that entrepreneurs face linear, albeit risky, returns to their investment.

The evolution of financial wealth for an entrepreneur is described by:

$$dx_t^i = [\, (1 - \tau_t) \, r \, k_t^i + (1 - \tau_t) \, R_t \, b_t^i - c_t^i \,] \, dt + \sigma \, (1 - \tau_t) \, k_t^i \, dz_t^i \,. \tag{5}$$

The first term captures the expected rate of growth of wealth, and it shows that wealth grows when saving exceeds consumption expenditures. The second term captures the effect of idiosyncratic risk.

Because of the homotheticity of the preferences and the linearity of the budget constraint (5) in assets, the optimal individual policy rules will be linear in wealth, $x_t^i$. Hence, for given prices and policies, an agent's consumption-saving problem reduces to a tractable homothetic problem as in Samuelson's and Merton's classic portfolio analysis. The following Lemma then characterizes optimal individual behavior.

**Lemma 1.** *Let* $\{R_t\}_{t\in[0,\infty)}$ *and* $\{\tau_t, G_t\}_{t\in[0,\infty)}$ *be equilibrium price and policy sequences. Then, equilibrium consumption, capital and bond holdings for household* $i$ *are given by* $c_t^i = m_t x_t^i$, $k_t^i = \phi_t x_t^i$, *and* $b_t^i = (1-\phi_t)x_t^i$, *where* $m_t$ *is the marginal propensity to consume out of wealth, and* $\phi_t$ *is the fraction of wealth invested in capital. In addition,* $m_t$ *and* $\phi_t$ *are given by:*

$$(i)\ m_t = \beta \text{ for all } t \quad \text{and} \quad (ii)\ \phi_t = \frac{(1-\tau_t)r - (1-\tau_t)R_t}{\sigma^2(1-\tau_t)^2}\,. \tag{6}$$

Condition ($6i$) is essential the Euler condition: With logarithmic preferences and liner budget, the marginal propensity to consume is constant over time, and equal to the discount rate in preferences. In other words, the income and substitution effect on consumption from a change in the saving return exactly cancel out with logarithmic preferences. Condition ($6ii$) simply says that the fraction of wealth invested in the risky asset is increasing in the risk premium, $\mu_t = (1-\tau_t)r-(1-\tau_t)R_t$, and decreasing in the effective variance of risk, $\sigma^2(1-\tau_t)^2$.

Using (5) and Lemma (1), we get the following characterization for individual consumption dynamics.

**Lemma 2.** *The evolution of individual consumption, investment, and wealth is characterized by:*

$$\frac{dc_t^i}{c_t^i} = \frac{dx_t^i}{x_t^i} = (\rho_t - \beta)\,dt + \sigma\,(1-\tau_t)\,\phi_t dz_t^i\,, \tag{7}$$

*where* $\rho_t = (1-\tau_t)\,\phi_t\,r + (1-\tau_t)\,(1-\phi_t)\,R_t$ *is the mean portfolio return or mean return to saving. Solving for* $c_t^i$, *we get:*

$$c_t^i = c_0^i \cdot exp\,[\int_0^t (\hat{\rho}_t - \beta)\,ds \,+\, \int_0^t \sigma(1-\tau_s)\phi_s\,dz_s^i\,]\,, \tag{8}$$

*where* $\hat{\rho}_t = \rho_t - \frac{1}{2}\sigma^2(1-\tau_s)^2\phi_t^2 = (1-\tau_t)\,\phi_t\,r + (1-\tau_t)\,(1-\phi_t)\,R_t - \frac{1}{2}\sigma^2(1-\tau_s)^2\phi_t^2$ *is the risk-adjusted return to saving.*

The fact that investment is subject to undiversifiable idiosyncratic risk introduces a wage between the marginal product of capital and the risk-free rate, so that $R_t < \rho_t < r$. Since agents face risk in their consumption stream, stemming from the risk in their private businesses, they have a precautionary saving motive, represented by the term $\frac{1}{2}\sigma^2(1-\tau_s)^2\phi_t^2$ in the risk-adjusted return to saving, so that $\hat{\rho}_t < \rho_t$. From this and (7), it follows that, if $\hat{\rho}_t > \beta$, then individual wealth and consumption grow at a positive rate.

### 4.2 General equilibrium

At any point in time, the aggregates do not depend on the extend of wealth inequality, because individual consumption and investment are linear in individual wealth. Aggregating the policy rules across agents and imposing market-clearing for the risk-free bond, we arrive at the following characterization of the general equilibrium.

**Lemma 3.** *In equilibrium, the aggregate dynamics satisfy the following system:*

$$dK_t = [AK_t - \delta K_t - C_t - G_t]dt\,, \tag{9}$$

$$m_t = \beta\,, \tag{10}$$

$$0 = (1-\phi_t)K_t \tag{11}$$

Condition (9) is the aggregate resource constraint of the economy. It follows from aggregating budgets and policies across all households, using the government budget constraint, bond market clearing $B_t = 0$, and $r = A - \delta$. Condition (10) is the Euler equation for logarithmic preferences. Condition (11) represents bond market clearing. It follows from aggregating bond holdings across agents and using the fact that the bond is in zero net supply. This condition holds for all values of the capital stock if and only if $\phi_t = 1$. In other words, $\phi_t = 1$ is essentially the condition that guarantees bond market clearing in the closed economy. Using ($6ii$), this implies that in the equilibrium of the closed economy:

$$R_t = r - \sigma^2(1-\tau_t) \tag{12}$$

In other words, there is a positive relationship between the interest rate and the capital-income tax. The intuition behind this positive relationship stems from the general equilibrium insurance aspect of the capital tax. In particular, when the tax increases, the effective variance of risk, $\sigma(1-\tau_t)$, falls. This reduces the demand for precautionary saving, and leads to an increase in the interest rate in general equilibrium. Put differently, since agents are faced with a lower variance of risk when the tax increases, they will require a higher interest rate for the same amount of saving. Hence, the interest rate is increasing in the tax in general equilibrium.

## 5 The planner's problem

The social planner's objective is to choose the capital-income tax that maximizes ex ante expected utility, subject to the conditions for individual optimization and general equilibrium in section (4).

The planner's objective function is the ex ante expected utility across agents, i.e. a weighted sum of agents' value functions, where the weights depend on the asset holding of each agent as a fraction of aggregate asset holdings:

$$max_{\{\tau_t\}_{t=0}^{\infty}} \int_i V(x_0^i;\tau_t)\, p(x_0^i;\tau_t)\, dx_0^i\,. \tag{13}$$

Here, $\int_i \equiv E_{-1}$ for the ex ante welfare calculation across agents. The value function for an agent with initial wealth $x_0^i$ at $t = 0$, given the tax sequence $\{\tau_t\}_{t=0}^{\infty}$, is the solution to the problem $V(x_0^i;\{\tau_t\}_{t=0}^{\infty}) = max_{c^i,\phi^i}\, E_0 \int_0^{\infty} e^{-\beta t}\, ln(c_t^i)\, dt$, subject to constraint (5). The weight used for an agent of wealth $x_0^i$ is $p(x_0^i;\tau_t)$. Without loss of generality, we assume that at $t = 0$ the wealth distribution is concentrated at one point, so that all agents hold the same amount of capital, equal to the economy-wide aggregate capital stock, and therefore receive the same weight in the planner's objective, $p(x_0^i;\tau_t) = 1$.

When maximizing his objective, the planner has to take into account the constraints from section (4). These constraints are as follows. First, equation (8) describing the evolution of individual consumption. This equation incorporates the conditions for individual optimization from Lemma (1). From (8), or equivalently from (7), the resource constraint of the economy follows by aggregation. Second, bond market clearing captured by $\phi_t = 1$ or $b_t^i = 0$ for all $i$. Third, the government budget constraint (3). Using these constraints, the planner maximizes (13) with respect to $\{\tau_t\}_{t=0}^{\infty}$.[1] The following proposition characterizes the planner's problem.

**Proposition 1.** *The social planner chooses the capital income tax so as to solve the following problem:*

$$max_{\{\tau_t\}_{t=0}^{\infty}} \int_i V(k_0^i;\tau_t)\, dk_0^i \quad s.t. \tag{14}$$

$$V(k_0^i;\{\tau_t\}_{t=0}^{\infty}) = \int_0^{\infty} e^{-\beta t}\{lnJ(\{\tau_s\}_{s=0}^{\infty}) + \int_0^{\infty} g(\tau_s,\phi_s(\tau_s),R_s(\tau_s))ds\}dt \tag{15}$$

$$G_t = \tau_t\,(A-\delta)\,K_t \tag{16}$$

$$\phi_t = 1 \tag{17}$$

*where* $J \equiv J(\{\tau_t\}_{t=0}^{\infty}) = c_0^i = \beta k_0^i$, *and* $g(\tau_s,\phi_s(\tau_s),R_s(\tau_s)) = \hat{\rho}_t - \beta$.

We will now set aside conditions (16) and (17) for a moment. From (15), the value function consists of two parts. The first part, reflected in the term $J(\cdot)$, captures the fact that time-zero consumption depends on the entire sequence of optimal tax rates in the future. The second part, reflected in the term $g(\cdot)$, shows that the value function is increasing in $g$, i.e. it is increasing in the difference between the risk-adjusted return to saving and the marginal propensity to consume. Since the planner's objective is increasing in $V(\cdot)$, it follows that the optimal tax should solve the

[1] The planner's preferences could be modified to include utility from government spending, $G_t$.

following first order condition:

$$\frac{dV}{d\tau_s} = \frac{dV}{dJ}\frac{dJ}{d\tau_s} + \frac{dV}{dg_s}\frac{dg_s}{d\tau_s}\,, \tag{18}$$

subject to the constraints in proposition 1. Here, $dV/dJ = \int_0^\infty e^{-\beta t}dt/J = 1/(\beta J) > 0$, and $dV/dg_s = \int_s^\infty e^{-\beta t}dt = \beta^{-1}e^{-\beta s} > 0$. The crucial terms in (18)are then $dJ/d\tau_s$ and $dg_s/d\tau_s$. We will now examine each one of them in turn.

In general, when there are sources of safe income in the economy, such as wage income or transfer income, total effective wealth, $w_t^i$, for an agent will be the sum of financial wealth, $x_t^i$, and safe wealth, $h_t^i$, so that $c_0^i = \beta w_0^i = \beta(x_0^i + h_0^i)$. In that case, $h_0^i$ is the present discounted value of wage and transfer income, and as such it depends on the entire dynamic path for the tax, directly as well as indirectly, through the effects of the tax on the aggregate capital stock and the interest rate. These effects are captured in the term $dJ/d\tau_s$.

In addition, when there are sources of safe income, then $\phi_t = K_t/W_t = K_t/(K_t + H_t) < 1$, and therefore the total effect of the tax on $g$, i.e. on the difference between the risk-adjusted return to saving and the marginal propensity to consume, is:

$$\frac{dg_s}{d\tau_s} = \frac{\partial g_s}{\partial \tau_s} + \frac{\partial g_s}{\partial \phi_s}\frac{\partial \phi_s}{\partial \tau_s} + \frac{\partial g_s}{\partial R_s}\frac{\partial R_s}{\partial \tau_s} = -R_s + (1-\tau_s)(1-\phi_s)\frac{dR_s}{d\tau_s} \tag{19}$$

where $\partial g_s/\partial \tau_s = -R_s\,,\ \partial g_s/\partial \phi_s = 0\,,\ \partial g_s/\partial R_s = (1-\tau_s)(1-\phi_s)$, using the formula for optimal portfolio allocation from (6). In other words, the total effect of the tax on $g$ consists of a direct part, $\partial g_s/\partial \tau_s$, and an indirect part, capturing the effects of the tax on the risk-adjusted return to saving through portfolio reallocation, $\partial g_s/\partial \phi_s \cdot \partial \phi_s/\partial \tau_s$ , and interest-rate adjustment, $\partial g_s/\partial R_s \cdot \partial R_s/\partial \tau_s$. The direct effect of the tax on $g$ is unambiguously negative and it captures the fact that an increase in the tax reduces the mean return to saving. The first term in the indirect effect of the tax is zero from the envelope condition, because the optimal choice of $\phi_t$ maximizes the risk-adjusted return to saving, $\hat{\rho}_t$. The second term in the indirect effect of the tax captures the effect of the tax on the interest rate, and through that on portfolio reallocation, saving return, and $g$. The appendix shows that $\partial R_t/\partial \tau_t > 0$, even when $\phi_t < 1$ (equation (12) demonstrates this for the case of $\phi_t = 1$): An increase in the tax reduces the effective variance of risk and hence the demand for precautionary saving, and leads to an increase in the interest rate. This increase in the interest rate makes the bond more attractive, and tends to reduce investment in the risky asset, which has a higher mean return. Intuitively, the effect of this on the return to saving will be smaller, if $\phi_t$ was small to begin with. Or, the increase in the interest rate will affect the return to saving, and therefore $g$, more, when $1-\phi_t$ is higher.

In the present model specification, we have eliminated sources of safe income such as wages and transfers. Therefore, one immediate simplification is that, since $c_0^i = \beta x_0^i = \beta k_0^i$ and $k_0^i$ is exogenously given, $dJ/d\tau_s = 0$. Furthermore, since $b_t^i = 0$ for all $i$ and $t$, or since $\phi_t = 1$, we also have that $\partial g_s/\partial \tau_s = -R_s$. Using this discussion, proposition 1, and equation (12), the following lemma summarizes the case of the economy with no safe income.

**Lemma 4.** *When all income is risky, the optimal capital-income tax solves:*

$$\frac{dg_s}{d\tau_s} = -R_s = -[r - \sigma^2(1-\tau_t)] = 0 \qquad (20)$$

*subject to the government budget constraint* (16)*. Furthermore:*
*(i) In the small open economy version of the model, the optimal tax is zero, i.e.* $\tau^{opt} = 0$*.*
*(ii) In the closed economy version of the model, the optimal tax is* $\tau^{opt} = 1 - \frac{r}{\sigma^2}$*. The optimal tax is nonzero, it is positive when the variance of risk is sufficiently high compared to the mean return of the risky asset* ($\sigma^2 > r$)*, and it is constant over time.*

Lemma (4) shows that, when all income is risky, the optimal capital tax simply maximizes $g$, namely it maximizes the difference between the risk-adjusted return to saving, $\hat{\rho}_t$ and the marginal propensity to consume, $\beta$. Part (i) of the lemma follows immediately form (20), for $R_s = R$ for all $s$, exogenously given: there is no interior optimum and the planner's objective is negative, which means that the optimal tax is zero. Hence, in the open economy, the direct insurance provision of the tax, i.e. the fact that the tax reduces the effective variance of risk, $\sigma(1-\tau_t)$, is not strong enough to outweigh the distortion the tax creates on investment. From part (ii) of the lemma, substituting the formula for the optimal tax into the bond-market-clearing condition (12), we get that $R_t = 0$ in the closed economy equilibrium with the optimal tax. In other words, the interest rate in the closed economy adjusts in such a way, that no agent wants to hold the safe asset in equilibrium. This ensures that there is really no safe income in this economy. The optimal tax also has to satisfy (16), so that optimal government spending is given by $G_t^{opt} = (1 - \frac{r}{\sigma^2})\, r\, K_t$.

Lemma (4), as well as equation (19), demonstrate that our result about a positive optimal capital income tax hinges on general equilibrium effects of the tax on the interest rate, and subsequently on the portfolio reallocation and the return to saving, which are absent in the open economy. Put differently, the result that the optimal capital tax is positive does not hinge on our assumption that the tax system provides full loss offset (a direct insurance effect of the tax): In the open economy, the tax still provides full loss offset, but this effect is not strong enough to outdo the standard distortionary effect of the tax on savings. By contrast, in the closed economy, it is the general-equilibrium effect of the tax through the interest rate that generates additional or indirect insurance provision and therefore makes a positive tax optimal.

The result that the optimal capital-income tax is positive in our framework is reminiscent of Varian (1980). Varian examines the optimal consumption-saving problem in a two-period model with exogenously risky income (luck). He finds that the optimal marginal tax rate in a linear capital-income system is between 0 and 1. This is because, as income contains a random component, a system of redistributive taxation contributes to reducing the variance of after-tax income. This benefit from social insurance might outweigh the distortionary effect of the tax on saving, leading to a positive optimal capital-income tax. Our $Ak$ framework also essentially takes income as exogenously risky, and generalizes Varian's result into an infinite horizon setting. It is generally acknowledged that taxation results derived in a two-period model do not necessarily carry through in the infinite-horizon version of that model. However here, it turns out that Varian's two-period result does in fact hold when the horizon is infinite: The optimal capital-income tax in an economy with exogenously risky income (and no source of safe income) is positive. Nonetheless, our paper actually corrects the intuition in Varian. We show that the optimality of the positive capital tax is not due to the direct redistributional effect of the tax, as Varian concludes, but to the indirect or general equilibrium insurance effect of the tax, operating through the adjustment of the interest rate to clear the bond market.

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# 6 Appendix

## 6.1 The model with CRRA preferences

In this section, we continue to assume that production is $AK$ and that there is no safe income, but we allow preferences to be CRRA, with $\theta = 1/\gamma \neq 1$. The only difference from the benchmark model is that the marginal propensity to consume out of wealth, $m_t$, will now not be equal to the discount rate, $\beta$, but will instead be characterized by an Euler equation. In this case, (7) becomes:

$$\frac{dc_t^i}{c_t^i} = [\ (1-\tau_t)\, r - m_t\ ]\, dt + \sigma\, (1-\tau_t)\, dz_t^i\, . \tag{21}$$

and (8) becomes:

$$c_t^i = c_0^i\, exp[\int_0^t (\ (1-\tau_s) r - m_s - \frac{1}{2}\sigma^2 (1-\tau_s)^2\ )\ ds\ +\ \int_0^t \sigma(1-\tau_s)\, dz_s^i\ ]\, . \tag{22}$$

Since preferences are homothetic, individual consumption is log-normally distributed. Let $\mu_c$ and $\sigma_c$ denote the mean and variance of individual consumption, respectively. Taking logs in (22) gives:

$$ln\, c_t^i = ln\, c_0^i\ +\ \int_0^t (\ (1-\tau_s) r - m_s - \frac{1}{2}\sigma^2(1-\tau_s)^2\ )\ ds\ +\ \int_0^t \sigma(1-\tau_s)\, dz_s^i\ \sim\ N(\mu_c, \sigma_c) \tag{23}$$

where

$$\mu_c = ln\, c_0^i\ +\ \int_0^t (\ (1-\tau_s) r - m_s - \frac{1}{2}\sigma^2(1-\tau_s)^2\ )\ ds \tag{24}$$

and

$$\sigma_c^2 = \int_0^t \sigma^2 (1-\tau_s)^2\, ds \tag{25}$$

With CRRA preferences, utility will also be log-normal:

$$lnu_t^i = ln(\frac{1}{1-\gamma}) + (1-\gamma) lnc_t^i\ \sim\ N(\mu_u, \sigma_u) \tag{26}$$

where

$$\mu_u = (1-\gamma)\mu_c + ln(1-\gamma) \tag{27}$$

and

$$\sigma_u^2 = (1-\gamma)^2\, \sigma_c^2 \tag{28}$$

The planner maximizes:

$$V_0 = E_{-1} \int_0^\infty e^{-\beta t} u(c_t^i) dt = \int_0^\infty e^{-\beta t} E_{-1}[u(c_t^i)] dt = \int_0^\infty e^{-\beta t} exp\{\mu_u + \frac{1}{2}\sigma_u^2 dt\} \tag{29}$$

After some algebra, we can write the planner's objective as:

$$V_0 = \int_0^{\infty} e^{-\beta t} \frac{c_0^{i,1-\gamma}}{1-\gamma} exp\{\int_0^t [(1-\gamma) g(\tau_s)] ds\} dt \tag{30}$$

where

$$g(\tau_s) = (1-\tau_s) r - m_s(\tau_s) - \frac{\gamma}{2}\sigma^2 (1-\tau_s)^2 \tag{31}$$

Note that $V_0$ is always increasing in $g(\tau_s)$, and that $\tau_s$ only enters $V_0$ through $g(\tau_s)$. Hence, by maximizing $g(\tau_s)$ with respect to $\tau_s$, we are actually maximizing $V_0$. More formally:

$$\frac{dV_0}{d\tau_s} = \frac{dV_0}{dg(\tau_s)} \frac{dg(\tau_s)}{d\tau_s} \text{ and } \frac{dV_0}{d\tau_s} = 0 \text{ iff } \frac{dg(\tau_s)}{d\tau_s} = 0 \tag{32}$$

Taking the first order condition of $g(\tau_s)$ with respect to $\tau_s$ yields:

$$-r - m_s'(\tau_s) + \gamma \sigma^2 (1-\tau_s) = 0 \tag{33}$$

where $m_s'(\tau_s)$ is the derivative of the marginal propensity to consume with respect to the tax, to be calculated from the Euler equation. Equation (33) defines the optimal tax. Note that we can write this equation as:

$$\tau_s = 1 - \frac{r + m_s'(\tau_s)}{\gamma \sigma^2} < 1\,, \tag{34}$$

and we need $\gamma\sigma^2 > r + m_s'(\tau_s)$ for a positive optimal tax. For the optimal tax to be an equilibrium, it needs to clear the bond market, which means that the optimal tax will be an equilibrium if and only if $R_t = m_t'(\tau_t)$. If the marginal propensity to consume does not depend on the tax (as in the case of logarithmic preferences, where $m_t = \beta$), then $R_t = m_t'(\tau_t) = 0$ in equilibrium of the optimal tax, as we found in the previous section. Note also that, in general, the optimal tax will not be constant, unless the marginal propensity to consume is constant, as the case of logarithmic preferences.