Is it Safe to Follow a Speed Limit Policy?

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Abstract

This study investigates the theoretical implications of following a speed limit policy, using a backward-looking model of the economy. It compares a speed limit policy with other forms of monetary policy, such as inflation targeting and average inflation targeting. The speed limit policy is found to be inferior to both these policies. Additionally, a major limitation of the model, its failure to take account of data uncertainty, is identified.

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1. Introduction

Over the past two decades, monetary policy in OECD countries has been spectacularly successful in reducing inflation from high and variable rates to relatively stable levels of around two to four per cent. The results have been similar whether central banks have followed formal inflation targets or a more general goal of low and stable inflation[1]. In both approaches, policymakers focus on measures of both inflation and the output gap (that is the *level* of output (GDP) relative to its potential *level*) when deciding on the appropriate level of the policy instrument, the short-term interest rate. There is general agreement that the aim of monetary policy involves "stabilizing inflation around an inflation target, and stabilizing the real economy, represented by the output gap" (Svensson, cited in Walsh, 2003a, p.265). The use of the output gap reflects the belief that expected future inflation depends on the difference between actual output and the level of output that is capable of being produced, rather than on the level of output itself (Clarida et al., 1999). As a result, both the output gap and inflation appear in policymakers' target rules.

Walsh (2003a) argues that it is not clear whether the output gap has been or should be a target of monetary policy. Citing press releases from the Federal Open Market Committee (FOMC), he suggests that the Federal Reserve Board had been pursuing a speed limit policy. That is, they had been focusing on the *change* in the output gap (or *growth* in output relative to the *growth* in potential), rather than the output gap itself. Using a New Keynesian forward-looking model of the economy, Walsh examines whether targeting inflation and the change in the output gap (that is following a speed limit policy (SLP)) is superior to other forms of monetary policy.

Woodford (1999) shows that within the forward-looking New Keynesian framework, optimal discretionary policy leads to inefficient stabilization following cost-push shocks. Under commitment (that is, if the central bank could commit to a policy rule, once and for all), optimal policy imparts inertia when expectations are forward-looking. This is because the central bank's current actions directly affect the public's

[1] Formal inflation targeting was pioneered by New Zealand and subsequently adopted by other countries, such as Canada, the United Kingdom, Sweden, Finland, Israel, Spain, and Australia. In contrast, the United States has not adopted a formal target for inflation (Bernanke et al., 1999).

expectations of future inflation, a mechanism absent under discretion. However, because commitment to a rule is not a practical option, discretion is the only way that policy can be conducted in practice. Therefore, a central bank operating under discretion fails to introduce inertia.

However, Walsh (2003a) argues that if the central bank follows a speed limit policy, under which it seeks to stabilize the *change* in the output gap, the lagged output gap becomes an endogenous state variable. This then introduces inertia into policy, even under discretion. Thus the inefficiency (noted by Woodford (1999)) is reduced if a central bank follows a speed limit policy. Walsh therefore concludes that a SLP dominates inflation targeting in the standard forward-looking framework.

Nessén and Vestin (2005) demonstrate that average inflation targeting (AIT) also introduces history dependence within a forward-looking framework under discretion because of the lagged inflation in the calculation of average inflation. It causes inflation expectations to change, thereby producing superior outcomes, compared with one-period inflation targeting. Additionally, Söderströms (2005) shows that average inflation targeting is dominated by a speed limit policy in a forward-looking model.

While most discussions of optimal policy use a forward-looking model as the basis of analysis, this paper discusses the conduct of optimal policy using a variant of the backward-looking model of Ball (1997, 1999). In particular, it considers a speed limit policy and average inflation targeting, two policies not considered by Ball.

We first describe the model and outline optimal policy (Section 2). Then we evaluate whether a speed limit policy and average inflation targeting are consistent with optimal policy (Sections 3 and 4). Other policies, including strict inflation targeting, the Taylor Rule, and nominal income growth targeting, are also considered. A discussion of the theoretical findings forms Section 5, and Section 6 concludes.

2. The Backward-Looking Model.

In this backward-looking model, the economy is described by two equations:

IS relationship $$y_t = -\beta r_{t-1} + \lambda y_{t-1} + \varepsilon_t \quad (1)$$

$$\beta > 0\,,\ 0 \le \lambda \le 1$$

Phillips Curve $$\pi_t = \gamma \pi_{t-1} + \alpha y_{t-1} + \eta_t \quad (2)$$

$$\alpha > 0\,,\ 0 \le \gamma \le 1$$

where y is the output gap (the difference between real output and its potential), r is the real rate of interest, π is the difference between inflation and its target level. Alternatively, we can make a simplifying assumption, an inflation target level of zero, in which case π is the inflation rate[2]. Both ε and η are white noise shocks, while α, β, γ, and λ are all constants. This model is the same as that used by Ball (1997) except that we have the coefficient γ on π_{t-1} in the Phillips Curve (PC)[3].

Aggregate demand is summarized by the behaviour of income (Hall & Mankiw, 1994). Equation (1) is an IS relationship, where output depends negatively on the lagged real interest rate, positively on lagged output, and a demand shock. The coefficient λ measures the persistence the output gap from one period to another. β represents the impact that policy has on real output, with a one-period lag.

The supply side of the economy is represented by equation (2), a backward-looking Phillips curve. Current inflation depends positively on lagged inflation, positively on lagged output and a supply (or cost-push) shock. α represents the impact of the output gap on inflation, while γ measures the persistence of inflation. In describing a backward-looking Phillips curve, Taylor (1994) says that it summarizes price adjustments in the economy. When real GDP rises above (falls below) potential GDP, inflation increases (decreases), with a lag because of the stickiness of prices.

This model makes the simplifying assumption that the monetary authority has complete control over the real interest rate, r. The backward-looking structure of the model leads to lags in impact of policy. The IS relationship shows that there is a one

[2] Ball (1997) defines π as the difference between inflation and its average level.
[3] When $\gamma = 1$, our model is equivalent to Ball's.

period lag from a change in the interest rate to a change in output, while the PC indicates a further one-period lag from changes in output to affect inflation. Thus, there is a one-period lag from a change in policy to a change in output, and a two-period lag to a change in inflation, as represented by:

$$r_t \rightarrow y_{t+1} \rightarrow \pi_{t+2}$$

This lag structure of the model provides the model with a dose of realism, supported by empirical evidence. Walsh (2003b, p.503-504) notes that shocks to the federal funds rate had effects on output that were felt over several periods, while the response of inflation was "much more delayed". Svennson (1997) highlights the difficulty in conducting monetary policy because of these lags.

The strengths of the model are its simplicity and realism. However, it ignores the role that the expectation of future inflation has on current inflation. Additionally, it ignores the Lucas critique[4]. Ball (1997) also assumes that the IS and PC equations remain unchanged across different policy regimes.

2.1 Determining Optimal Policy

In this section, we briefly outline the procedure for solving the model. Using this adapted version of Ball's (1997) model, we derive a specification for efficient (optimal) policy.

The policymaker is concerned about the variability of output and inflation. His/her objective is to minimize a loss function consisting of a weighted average of the variance of output and the variance of inflation:

$$E\left[L_t\right] = V(y_t) + \mu V(\pi_t) \qquad (3)$$

where μ is the policymaker's aversion to the variance of inflation relative to the variance of output. Equation (3) may be thought of as society's loss function, which the policymaker is attempting to minimize.

[4] The Lucas critique expresses doubt on the ability to use parameters of econometric models for evaluating alternative public policies, because selection of a policy itself alters the actions of economic agents.

The policymaker who follows a policy of one-period inflation targeting (henceforth referred to as inflation targeting) chooses a policy rule that is a weighted average of next period's expected output gap and inflation[5].

$$\theta E_t y_{t+1} + E_t \pi_{t+1} = 0 \tag{4}$$

The target value for the output gap and the rate of inflation (or rather, its deviation from its target value) is zero (Froyen & Guender, 2007). The parameter θ represents the weight that the policymaker's places on the output gap relative to the rate of inflation when setting policy. Optimal policy involves choosing the value of θ that minimizes the loss function (3).

Combining this policy rule with the IS and PC equations yields the policymaker's reaction function:

$$r_t = \frac{\gamma}{\beta\theta}\pi_t + \left(\frac{\alpha + \lambda\theta}{\beta\theta}\right) y_t \tag{5}$$

The reaction function, specifies how the policymaker adjusts the rate of interest to changes in the output gap and the rate of inflation. In the event of a one-percentage point rise (fall) in the rate of inflation, the policymaker raises (lowers) interest rates by $\gamma/\beta\theta$ percentage points. Similarly, following a one percentage rise (fall) in the output gap, the policymaker responds by raising (lowering) the interest rate by $(\alpha+\lambda\theta)/\beta\theta$ percentage points.

Backdating equation (5) by one period to reflect r_{t-1} and inserting this into the IS equation (1), we obtain the reduced form of the IS equation, given policy:

$$y_t = -\frac{\gamma}{\theta}\pi_{t-1} - \frac{\alpha}{\theta} y_{t-1} + \varepsilon_t \tag{6}$$

The variances of the output gap and of inflation can now be computed[6].

$$V(y_t) = \frac{\left(-2\alpha\gamma\theta - \theta^2 + \gamma^2\theta^2\right)\sigma_\varepsilon^2 - \gamma^2\sigma_\eta^2}{\alpha^2 - 2\alpha\gamma\theta - \theta^2 + \gamma^2\theta^2} \tag{7}$$

$$V(\pi_t) = \frac{-\alpha^2\theta^2\sigma_\varepsilon^2 + \left(\alpha^2 + 2\alpha\gamma\theta - \theta^2\right)\sigma_\eta^2}{\alpha^2 - 2\alpha\gamma\theta - \theta^2 + \gamma^2\theta^2} \tag{8}$$

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[5] The form of this policy rule differs from, but is algebraically equivalent to, that of Ball (1997). Both obtain the same expression for the optimal parameter θ^*.

[6] According to a procedure outlined in Hendry (1995, pp.111-112).

Minimizing the loss function (3) with respect to θ yields the optimal value of the policy parameter, the weight on the output gap in the policy rule[7]:

$$\theta^* = \frac{1-\gamma^2+\alpha^2\mu+\sqrt{4\alpha^2\gamma^2\mu+\left(-1+\gamma^2-\alpha^2\mu\right)^2}}{2\alpha\gamma\mu} \tag{9}$$

Notice that θ^* includes two parameters from the Phillips curve: α (the responsiveness of inflation to the lagged output gap) and γ (the weight on lagged inflation), as well as μ (the policymaker's (or society's) aversion to the variability of inflation). The higher the policymaker's aversion to variations in inflation, the lower is θ^* (the optimal policy parameter)[8].

By inserting parameter values into the variance equations, we are able to show graphically the trade-off between the variance of inflation and the variance of the output gap as an optimal policy frontier[9].

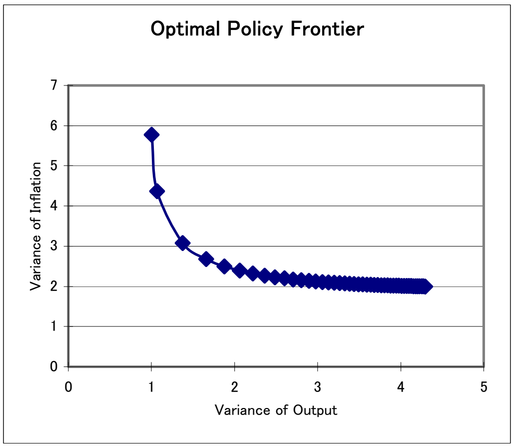

Figure 1. Optimal Policy Frontier.

Different points on the frontier represent different values of μ (aversion to inflation variability relative to output variability). Moving from left to right along the frontier represents increasing aversion to inflation variability (higher values of μ). Any point above the frontier is a combination of variances representing a policy that is less

[7] Setting $\gamma = 1$ simplifies (9) to the result obtained by Ball (1997):

$$\theta^* = \frac{\alpha\mu+\sqrt{4\mu+\alpha^2\mu^2}}{2\mu}.$$

[8] As $\mu \to \infty, \theta \to \alpha/\gamma$ and as $\mu \to 0, \theta \to \infty$.

[9] Parameter values, $\alpha = 0.4, \sigma_\varepsilon^2 = 1$ and $\sigma_\eta^2 = 1$ are taken from Ball (1997). We here choose $\gamma = 0.9$ as a value less than unity.

efficient than optimal policy. Any point below the frontier would be more efficient, but, given policy rule (4), not achievable.

Ball (1997) also shows that both strict inflation targeting (SIT) and gradual inflation targeting (GIT) are forms of efficient policy. GIT results in the same trade-off between inflation and output variability represented by the optimal policy frontier, while SIT is represented by a single point at the extreme lower-right of that frontier.

3. Speed Limit Policy

As stated in the introduction, the speed limit is shorthand for the change in the output gap, and is defined as $y_t - y_{t-1}$. Incorporating this in the policy rule results in:

$$\theta^{SL}\left[E_t y_{t+1} - y_t\right] + E_t \pi_{t+1} = 0 \qquad (10)$$

Combining the IS and PC equations with this policy rule yields the following reaction function:

$$r_t = \frac{\gamma}{\beta\theta^{SL}}\pi_t + \frac{\alpha - \theta^{SL}\left(1-\lambda\right)}{\beta\theta^{SL}} y_t \qquad (11)$$

Comparing this reaction function with that of optimal policy (5), notice that the coefficient on π_t has the same form as that for equation (5), while the coefficient on y_t is smaller and not unambiguously positive. Its sign depends on the size of the structural parameters α and λ and the policy parameter θ^{SL}.

Backdating equation (11) by one period and inserting it into the IS equation (1) reveals the response of output to policy.

$$y_t = -\frac{\gamma}{\theta^{SL}}\pi_{t-1} + \frac{\theta^{SL} - \alpha}{\theta^{SL}} y_{t-1} + \varepsilon_t \qquad (12)$$

Combining this equation with the Phillips curve (2), we are able to calculate the variances of the output gap (y_t) and inflation (π_t):

$$V\left(y_t\right) = \frac{\left(2\alpha\gamma\theta^{SL} + (\theta^{SL})^2 - \gamma(\theta^{SL})^2 - \gamma^2(\theta^{SL})^2 + \gamma^3(\theta^{SL})^2\right)\sigma_\varepsilon^2 + \left(\gamma^2 + \gamma^3\right)\sigma_\eta^2}{-\alpha^2 + \alpha^2\gamma + 2\alpha\theta^{SL} - 2\alpha\gamma^2\theta^{SL}} \qquad (13)$$

$$V\left(\pi_t\right) = \frac{-\left(\alpha(\theta^{SL})^2 + \alpha\gamma(\theta^{SL})^2\right)\sigma_\varepsilon^2 + \left(\alpha - \alpha\gamma - 2\theta^{SL}\right)\sigma_\eta^2}{\alpha - \alpha\gamma - 2\theta^{SL} + 2\gamma^2\theta^{SL}} \qquad (14)$$

The next step is to determine the optimal value of θ^{SL}. Minimizing the loss function

$$E\left[L_t\right] = V(y_t) + \mu V(\pi_t) \tag{3}$$

with respect to θ^{SL}, we obtain the optimal value of the policy parameter, θ^{SL}.

$$\theta^{SL} = \frac{\alpha}{2(1+\gamma)} + \frac{\sqrt{\left(1-2\gamma^2+\gamma^4+2\alpha^2\mu+2\alpha^2\gamma^2\mu+\alpha^4\mu^2\right)\sigma_\varepsilon^2\left(\alpha^2\sigma_\varepsilon^2+4\gamma^2\sigma_\eta^2\right)}}{2(1+\gamma)\left(1-2\gamma+\gamma^2+\alpha^2\mu\right)\sigma_\varepsilon^2} \tag{15}$$

The reason we use the same loss function (3) as that used when considering inflation targeting is that we assume that society cares about the output gap and inflation. The policymaker therefore judges the performance of its policy (whatever policy rule it is using) in terms of the variance of output gap and the variance of the inflation.

The distinctive feature of θ^{SL} is that it depends on the variances of demand and cost-push shocks (σ_ε^2 and σ_η^2), while the optimal parameters for policies considered in Sections 2 and 4 do not[10]. The policymaker is therefore faced with a signal extraction problem, which requires him/her to determine the origin of disturbance that causes the changes in the observed inflation or output gap, before setting policy.

Inserting θ^{SL} into the variance equations (13) and (14), and using parameter values as in Section 2, we obtain the policy frontier, illustrated in Figure 2.

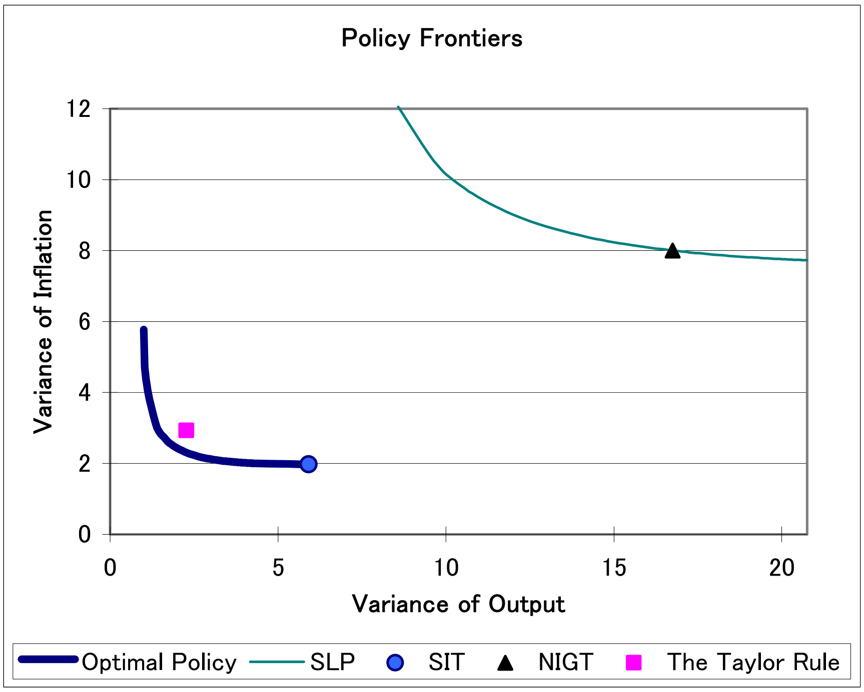

Figure 2. Policy Frontiers: Comparing Different Policies.

[10] See Appendix 2 for a comparison of optimal policy parameters.

Also shown is the optimal policy frontier we obtained from Section 2. It is immediately obvious that the speed limit policy as specified produces variances of output and inflation far greater than the optimal policy. It is therefore much less efficient. We therefore conclude that the speed limit policy (which targets the change in the output gap) is vastly inferior to the optimal policy, which targets the output gap itself[11].

4. Average Inflation Targeting

Another option is for the policymaker to target average inflation[12]. For simplicity average inflation is defined over two periods:

$$\overline{\pi_t} = \tfrac{1}{2}\left(\pi_t + \pi_{t-1}\right) \qquad (16)$$

The resulting policy rule, targeting the output gap and average inflation in two periods hence is:

$$\theta^{AIT} E_t y_{t+1} + E_t \overline{\pi_{t+2}} = 0 \qquad (17)$$

Combining this with the IS and PC equations yields the reaction function:

$$r_t = \frac{\gamma(1+\gamma)}{(\alpha + 2\theta^{AIT})\beta}\pi_t + \frac{\alpha(1+\gamma) + (\alpha + 2\theta^{AIT})\lambda}{(\alpha + 2\theta^{AIT})\beta} y_t \qquad (18)$$

Solving the model and minimizing the loss function (3) with respect to θ^{AIT} results in a trade-off between inflation and the output variability that is exactly the same as that of optimal policy, as represented by the optimal policy frontier (Figure 1)[13].

The intuition behind AIT's achieving the same result as optimal policy is that the policy rules for both contain the same variables (expected output and expected inflation). In the case of AIT, it is expected inflation in two periods time that is

[11] These results are based on a closed economy model. An open economy model, with the real exchange rate in the IS equation and the change in the real exchange rate in the Phillips curve, was also considered. Because it was not possible to obtain a closed form analytical solution for the optimal policy parameter, we were not able to determine the variances of y_t and π_t. However, since the coefficients on π_t and y_t in the reaction functions for IT and SLP have exactly the same form as those for the closed economy, we can argue that because the speed limit policy is shown to be inefficient in comparison to optimal policy in the closed economy framework, it will also be inefficient in the open economy model.

[12] The Reserve Bank of Australia targets inflation over the business cycle. Likewise, the Reserve Bank of New Zealand targets inflation over the medium term. Therefore, in a sense, both central banks practice average inflation target.

[13] The equation for θ^{AIT} is given in Appendix 2.

targeted. But since $E_t\pi_{t+2}$ depends on $E_t\pi_{t+1}$, the policy rule can also be expressed in terms of E_ty_{t+1} and $E_t\pi_{t+1}$. Similarly, a policy targeting expected inflation two periods hence, with a policy rule in the form of $\theta^{**}E_t y_{t+1} + E_t\pi_{t+2} = 0$, also results in the same optimal policy frontier. Rules (such as SLP and NIGT) which include additional variables produce different results.

5. Discussion

In the above analysis, we have seen that the speed limit policy is inferior to optimal policy within the backward-looking framework. Average inflation targeting and some alternative policy rules (SIT, GIT, and a modified policy rule targeting expected inflation two periods hence) have been identified as forms of optimal policy, and are therefore all superior to the speed limit policy. Additionally, nominal income growth targeting (NIGT) can be shown to be a special form of the speed limit policy. This is because the form of the policy rule used for NIGT (shown in Appendix 1) is equivalent to the policy rule for the SLP (10), when $\theta^{SL} = 1$.

The outcomes of these different policy options are displayed in Figure 2. SIT is represented by the point on the optimal policy frontier with the lowest possible inflation variance. The Taylor Rule is shown to be inefficient, but not highly so, being close to the optimal policy frontier. Finally, NIGT is also inefficient – represented by a point on the SLP frontier[14].

In comparing different policies, it is informative to consider the simple interest rate rules. How do the coefficients in the reaction function for different policies differ? And how do they change as the policymaker's aversion to inflation variability changes? By inserting values for the structural parameters of the model into these response functions, we are able to see how the coefficients change, that is, how the policymaker would respond to changes in inflation and the output gap.

[14] Ball (1997) finds NIGT not merely inefficient, but disastrous, as output and inflation follow non-stationary processes – that is, output and inflation tend to have infinite variances. This is the case when $\gamma = 1$.

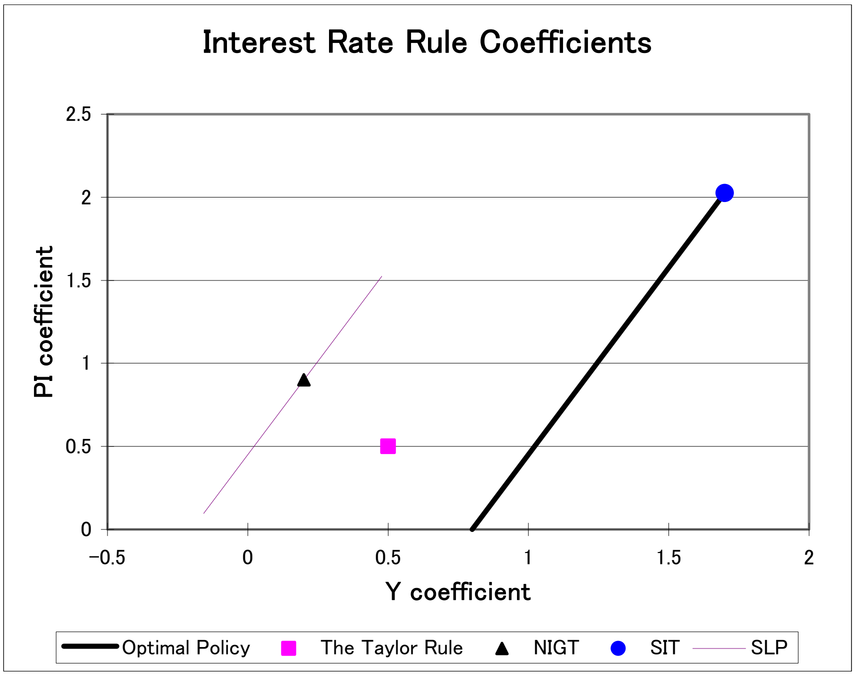

Figure 3. Coefficients in the Simple Interest Rate Rules: Comparing Policies.

Combinations of the coefficients on inflation and the output gap are shown graphically in Figure 3 (and summarized in Appendix 3) [15]. The heavy line on the right of the graph traces the combination of coefficients associated with optimal policy. Moving from left to right along that line represents increasing values of μ (the policymaker's aversion to inflation variability relative to output variability). Notice that as μ increases, the coefficient on inflation increases – but so does the coefficient on the output gap. This is because, through the PC, the output gap influences inflation in the next period. Thus a policy that acts strongly against inflation must also lean heavily against the output gap in the setting of the interest rate.

The point at the far right end of that line represents SIT, which leans hard against both inflation and the output gap. Notice that the point for the Taylor Rule, with coefficients of 0.5 for both inflation and the output gap, lies to the left of the optimal policy line. This confirms Ball's (1997) finding that it does not lean heavily enough against the output gap.

[15] Here, for illustration, we use $\alpha = 0.4$, $\beta = 1.0$, $\gamma = 0.9$, $\lambda = 0.8$, $\sigma_\varepsilon^2 = 1$ and $\sigma_\eta^2 = 1$, which (except for γ) are the parameters suggested by Ball (1997).

Finally, the combination of coefficients associated with the speed limit policy lies further to the left of the optimal policy line and for low values of μ the coefficient on the output gap is negative[16]. This line passes through the point representing NIGT (again illustrating that the latter policy is in practice, a special case of SLP).

For any value of the coefficient on inflation, the coefficient on the output gap in the SLP reaction function is smaller by 1.0 than the associated coefficient for optimal policy. In considering why this should be, it is instructive to compare the reaction functions of optimal policy and SLP, with their terms rearranged:

IT (Optimal Policy)
$$r_t = \frac{\gamma}{\beta\theta^*}\pi_t + \left(\frac{\alpha}{\beta\theta^*} + \frac{\lambda}{\beta}\right)y_t \qquad (19)$$

SLP
$$r_t = \frac{\gamma}{\beta\theta^{SL}}\pi_t + \left(\frac{\alpha}{\beta\theta^{SL}} - \frac{1}{\beta} + \frac{\lambda}{\beta}\right)y_t \qquad (20)$$

When the policy rules for optimal policy and SLP have the same coefficient on inflation, that is when the two policy parameters are equal ($\theta^* = \theta^{SL}$), we can see that the coefficient on y_t in equation (20) is $1/\beta$ smaller than that for equation (19). Using a parameter value of $\beta = 1$ (as in the above illustrations), the coefficient on the output gap is smaller by one in the case of SLP. Appendix 3 provides a numerical illustration of this point. For example, for a coefficient on inflation of 0.5, the output gap coefficient under SLP is only 0.022, while that for optimal policy is 1.022. However, it is important to note that a given coefficient on inflation under SLP represents a much larger aversion to inflation variability (μ) than does the same coefficient under optimal policy.

The above clearly shows that, when following a speed limit policy, the policymaker under-responds to changes in the output gap (in comparison to optimal policy). In fact, for any given level of aversion to inflation variability (μ), the policymaker, following a SLP, responds less forcefully not only to the output gap but also to inflation when setting policy. This is illustrated in Appendix 3: for μ = 2.0, the coefficients on inflation and the output gap would be 0.786 and 1.149 respectively under inflation targeting, while they would be only 0.506 and 0.247 under SLP, with the latter policy response leading to higher inflation and output variability.

[16] This confirms the observation on page 7 that the coefficient on the output gap in the reaction function is not unambiguously positive.

Intuitively, the reason for the poor performance of a speed limit policy is that a speed limit does not carry the same information as the output gap. Some important information (whether real output is above or below potential) has been lost. It is this information that is essential for providing an effective response to future inflation, because inflation in the next period depends on the current output gap. This is illustrated graphically in Figure 4. At points A and B, the output gap is negative and positive respectively, while at both points the speed limit is positive. By taking account of the speed limit only, the policy maker would respond in a similar manner at both points (tightening monetary policy), even though at point A the economy is below potential. This is the source of increased variability.

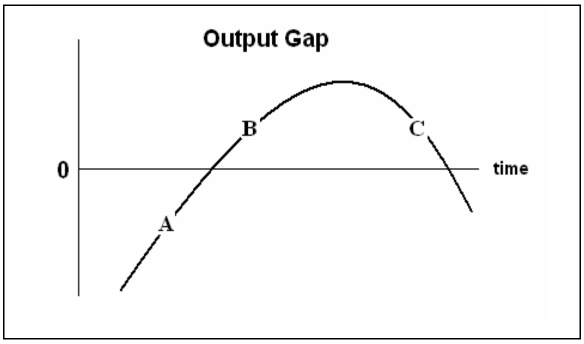

Figure 4. The Output Gap

For fine-tuning of policy, it may be informative for a policymaker to know, given the current level of the output gap, whether the *change* in the output gap is positive or negative, for example, whether a positive output gap is represented by a point like B or C in Figure 4. He/she may decide to respond less forcefully at point C than at point B. This reasoning may have been behind the FOMC's interest in the speed limit, as observed by Walsh (2003a).

However, whether the output gap itself is positive or negative is of greater importance than the sign of the speed limit. The policy maker may well want to know the size and sign of the speed limit, but only after he/she knows the size and sign of the output gap. The FOMC's interest in the speed limit does not imply that the output gap itself was no longer of interest, as Walsh (2003a) appears to have assumed.

Our result, that the speed limit policy is inferior to both inflation targeting and average inflation targeting, is model-specific. In contrast, the studies by Walsh

(2003a) and Nessén and Vestin (2005), using a forward-looking framework, show SLP and AIT to be superior to inflation targeting. The reason that Walsh (2003a) finds that a speed limit policy dominates inflation targeting, is due to the structure of the forward-looking model, in which policy, unrealistically, has contemporaneous effects on output and inflation.

Which framework is better for policy evaluation is open to debate. In most contemporary academic literature, the forward-looking model is favoured because of its microeconomic foundations and inclusion of inflation expectations. However, because the backward-looking model realistically represents the lags in the response to changes in the interest rate, it remains of interest to central bankers.

In assessing these frameworks it is important to consider one major limitation that both share – namely, a failure to take account of the availability of timely data and data uncertainty. Both frameworks assume that the actual current levels of inflation and the output gap are known by the policymaker when he/she sets interest rates. However, the levels of these variables are not known until a later date and, in the case of the output gap, there is considerable revision (Orphanides & van Nordon, 2002). It can be shown that, in practice, data uncertainty surrounding the output gap leads the policymaker to act more cautiously in setting policy than is suggested by the above analysis (Orphanides, 2003).

6. Conclusion

In this study, we have shown that the speed limit policy is inferior to optimal policy (inflation targeting) within the framework of a backward-looking model, because when targeting the speed limit, the policymaker does not lean heavily enough against the output gap. Additionally, when considering alternative policies, we found that average inflation targeting is a form of optimal policy within the backward-looking framework.

These results are at variance with those from studies based on forward-looking models where the speed-limit policy and average inflation targeting are superior to inflation targeting. However, when assessing the benefits of one model against the other, we must be aware of a major limitation of both – their failure to consider data uncertainty.

In future research, the model could be enhanced by including the international environment (taking account of the exchange rate) as well as the term structure of interest rates. Most significantly, developing a model which takes account of data uncertainty would eliminate the major limitation of our theoretical framework. The aim would be to overcome the inconsistency between our theory, which concludes that optimal policy must lean heavily against the output gap, and practice, which, taking account of data uncertainty, responds to a lesser degree to the output gap.

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Appendix 1. Policy Rules and Reaction Functions.

Policy	Policy Rule	Reaction Function
Inflation Targeting (IT) Optimal Policy	$\theta^{*}E_t y_{t+1} + E_t\pi_{t+1} = 0$	$r_t = \frac{\gamma}{\beta\theta^{*}}\pi_t + \frac{\alpha+\lambda\theta^{*}}{\beta\theta^{*}}y_t$
A Modified Policy Rule (MPR)	$\theta^{**}E_t y_{t+1} + E_t\pi_{t+2} = 0$	$r_t = \frac{\gamma^2}{\beta(\alpha+\theta^{**})}\pi_t + \frac{\alpha\gamma+(\alpha+\theta^{**})\lambda}{\beta(\alpha+\theta^{**})}y_t$
Strict Inflation Targeting (SIT)	$E_t\pi_{t+2} = 0$	$r_t = \frac{\gamma^2}{\alpha\beta}\pi_t + \frac{\gamma+\lambda}{\beta}y_t$ *
Gradual Inflation Targeting (GIT)	$E_t\pi_{t+2} = \frac{\gamma\theta^{GIT}-\alpha}{\theta^{GIT}}E_t\pi_{t+1}$	$r_t = \frac{\gamma}{\beta\theta^{GIT}}\pi_t + \frac{\alpha+\lambda\theta^{GIT}}{\beta\theta^{GIT}}y_t$
Average Inflation Targeting (AIT)	$\theta^{AIT}E_t y_{t+1} + E_t\overline{\pi_{t+2}} = 0$	$r_t = \frac{\gamma(1+\gamma)}{(\alpha+2\theta^{AIT})\beta}\pi_t + \frac{\alpha(1+\gamma)+(\alpha+2\theta^{AIT})\lambda}{(\alpha+2\theta^{AIT})\beta}y_t$
Nominal Income Growth Targeting (NIGT)	$E_t y_{t+1} - y_t + E_t\pi_{t+1} = 0$	$r_t = \frac{\gamma}{\beta}\pi_t + \frac{\alpha-(1-\lambda)}{\beta}y_t$ *
Speed Limit Policy (SLP)	$\theta^{SL}\left[E_t y_{t+1} - y_t\right] + E_t\pi_{t+1} = 0$	$r_t = \frac{\gamma}{\beta\theta^{SL}}\pi_t + \frac{\alpha-\theta^{SL}(1-\lambda)}{\beta\theta^{SL}}y_t$
The Taylor Rule	-	$r_t = 0.5\pi_t + 0.5y_t$

* SIT and NIGT have no θ in the policy rule or reaction function. SIT is an extreme case of the Modified Policy Rule, where $\theta^{**} = 0$. NIGT can be considered a special case of SLP, where $\theta^{SL} = 1$.

Appendix 2. Optimal Policy Parameters

IT Optimal Policy	$\theta^{*}=\frac{1-\gamma^{2}+\alpha^{2}\mu+\sqrt{4\alpha^{2}\gamma^{2}\mu+\left(-1+\gamma^{2}-\alpha^{2}\mu\right)^{2}}}{2\alpha\gamma\mu}$
MPR	$\theta^{**}=\frac{1-\gamma^{2}+\alpha^{2}\mu+\sqrt{4\alpha^{2}\mu+\left(-1+\gamma^{2}-\alpha^{2}\mu\right)^{2}}}{2\alpha\mu}$
GIT	$\theta^{GIT}=\frac{1-\gamma^{2}+\alpha^{2}\mu+\sqrt{4\alpha^{2}\gamma^{2}\mu+\left(-1+\gamma^{2}-\alpha^{2}\mu\right)^{2}}}{2\alpha\gamma\mu}$
AIT	$\theta^{AIT}=\frac{1+\gamma-\gamma^{2}-\gamma^{3}+\alpha^{2}\mu-\alpha^{2}\gamma\mu+\sqrt{\left(1+\gamma\right)^{2}\left(1-2\gamma^{2}+\gamma^{4}+2\alpha^{2}\mu+2\alpha^{2}\gamma^{2}\mu+\alpha^{4}\mu^{2}\right)}}{4\alpha\gamma\mu}$
SLP	$\theta^{SL}=\frac{\alpha}{2\left(1+\gamma\right)}+\frac{\sqrt{\left(1-2\gamma^{2}+\gamma^{4}+2\alpha^{2}\mu+2\alpha^{2}\gamma^{2}\mu+\alpha^{4}\mu^{2}\right)\sigma_{\varepsilon}^{2}\left(\alpha^{2}\sigma_{\varepsilon}^{2}+4\gamma^{2}\sigma_{\eta}^{2}\right)}}{2\left(1+\gamma\right)\left(1-2\gamma+\gamma^{2}+\alpha^{2}\mu\right)\sigma_{\varepsilon}^{2}}$

Appendix 3. Simple Interest Rate Rules: Coefficients and Variances.

	μ	θ	Simple Interest Rate Rule		$V(\pi)$	$V(y)$
			π coeff.	**y coeff.**		
Optimal Policy (including IT GIT AIT MPR)	0	∞	**0**	**0.8**	6.10526	1.0
	0.79918	1.8	**0.5**	**1.02222**	2.79425	1.55378
	2.0	1.14501	**0.786019**	**1.14934**	2.39213	2.06184
	2.75	1.0	**0.9**	**1.2**	2.29333	2.29333
	∞	0.44444	**2.025**	**1.7**	1.97	5.91062
SIT	-	-	**2.025**	**1.7**	1.97	5.91062
Speed Limit Policy	0	9.32481	**0.0965167**	**- 0.157104**	24.1744	7.06804
	1.94677	1.8	**0.5**	**0.02222**	9.35155	11.2444
	2.0	1.7798	**0.505674**	**0.247441**	9.31451	11.3175
	9.3125	1.0	**0.9**	**0.2**	8.0	16.75
	∞	0.590502	**1.52413**	**0.477389**	7.62517	26.7202
NIGT	-	-	**0.9**	**0.2**	8.0	16.75
The Taylor Rule	-	-	**0.5**	**0.5**	2.924803	2.272317

The above values are based on parameter values: $\alpha = 0.4$, $\beta = 1.0$, $\gamma = 0.9$, $\lambda = 0.8$, $\sigma_\varepsilon^2 = 1$ and $\sigma_\eta^2 = 1$. When μ = zero or infinite, the associated values of θ, π-coeff., y-coeff., $V(\pi)$, and $V(y)$ represent limits as μ goes to zero or infinite, respectively.