Propositions versus events

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Existing epistemic models fall in two distinct classes: there are epistemic logics which add knowledge axioms to the *calculus of propositional logic*; and there are belief systems which define rules for the generation of knowledge within some *event space* framework. Since real-life agents think (reason, know, believe) in terms of *propositions* rather than in terms of mathematical objects such as measurable subsets of some state space (i.e., *events*), the description of reasoning processes within the framework of propositional logic appears to be first choice. To consider events rather than propositions is therefore only justifiable under the assumption that reasoning processes can be equivalently described within the event space approach. More specifically, the justification of the event space approach as an equally appropriate framework for epistemic models is based on the two presumptions that

1. Every proposition a of propositional logic can be equivalently interpreted as some event A.

2. An agent's measure of confidence in the truth of proposition a is then given as his subjective probability attached to event A.

While the first assumption seems to be innocuous, its meaning is rather murky at best. After all, events are defined as subsets of some state space whereas propositions of propositional logic are defined very differently, namely, by an iterative procedure that generates *well-formulated formulas*. Moreover, while the primitives of propositional logic are so-called *atomic propositions*, the primitives of the event space approach are *states of the world*.

In this paper we construct for a given propositional language an event space such that we associate with every proposition of the language a unique event which we call the

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canonical interpretation of this proposition. Thus, our construction proposes a specific way by which every proposition a of propositional logic can be interpreted as an event A of an appropriate state space. As our main formal contribution we prove a determination theorem stating that a proposition is a theorem of the calculus of propositional logic if and only if its canonical interpretation is the universal event, i.e., the whole state space. While every proposition has a unique canonical interpretation, our determination theorem implies that the converse is not true. That is, when we consider events instead of propositions we reduce the complexity of the propositional language to a much simpler space.

With respect to the second assumption we argue that it is only justifiable for perfectly rational agents. Put differently, whenever agents are boundedly rational the reduction from propositional logic to the event space approach results in an inappropriate description of the agent's reaoning processes. By our construction of a canonical interpretation there exists a homeomorphism between the set of states of the world and the set of possible *truth conditions*, (i.e., all possible truth-value combinations of atomic propositions). Thus, if A is the canonical interpretation of the proposition a, then the agent's subjective probability attached to the event A can be equivalently interpreted as the agent's subjective probability that some truth condition holds at which a is true. That is, the event space approach would be an appropriate framework for epistemic models under the condition that the agent's measure of confidence in the truth of propositions is given as a probability measure for a state space that consists of all possible truth conditions.

This last condition, however, is very strong. Since a theorem is true at all truth conditions, the agent's measure of confidence in the truth of a theorem must equal one, i.e., certainty. The same conclusion follows from our determination theorem which shows that every theorem must be associated with the universal event, i.e., the event that occurs by definition with probability one. The second assumption is therefore only plausible if we suppose *logical omniscience* on part of the agents. That is, the agent has to know all logical theorems. But such a logical omniscience is impossible for boundedly rational agents who are prone to computational limitations or/and errors.

While there exist several approaches within the propositional logic that avoid the unrealistic assumption of logical omniscience, our investigation has demonstrated that this is impossible for the event space approach. Epistemic models based on the event space approach are forced by the very structure of this approach to assume logical omniscience. We therefore conclude that while the results of the event space may apply to perfectly rational agents, their relevance for boundedly rational agents remains unclear.