THRESHOLD FOR REDISTRIBUTION TO PROMOTE GROWTH

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In a neoclassical dynamic general equilibrium model we identify conditions to characterize when redistributive policies may succeed or fail in promoting economic growth. We prove that a straightforward comparison of data on exogenous diversity among the agents and an endogenous threshold that summarizes key economic fundamentals suffices the task. If the diversity is smaller than the threshold then redistribution hurts growth both in the long run and in transitions. Otherwise, it promotes growth eventually. The calibrated US economy without the education finance scheme like Benabou (2002) yields a sufficiently large threshold to imply a net negative impact of redistribution on growth. The threshold decreases with progressive education subsidy, quality of education, skill-biased technology and "neighborhood externality." With appropriate investment subsidies, a capital market enhances growth promoting potential of redistribution, because it rewards work effort with complementary wage premiums which partially mitigate the negative effect of redistribution on labor supply.

KEYWORDS: Population Diversity, Critical Minimum Variance (CMV), Redistributive Policy Package, Progressive Education Finance, Growth-Promoting Policy.

1. INTRODUCTION

The 1992 George Seltzer lecture in Minnesota by Robert Solow outlines a hypothesis that more "equity" could promote more growth through investment in human capital. Subsequently, wide varieties of dynamic general equilibrium models have emerged to establish similar hypotheses. Whether a redistributive policy could theoretically promote growth is no longer a controversial issue. Sometimes empirical conditions call for such redistribution. Benabou (2002) claims that a growth maximizing strategy for the US economy requires a significantly high degree of redistribution of resources from high income people to low income people. However, recently some others such as Erosa and Koreskova (2006) contradict that claim. They argue that empirical conditions for their benchmark US economy are such that redistribution would hurt economic growth. These apparently conflicting policy recommendations call for a resolution. We identify a general condition for a large class of economies considered in the relevant literature to show explicitly when redistribution hurts growth and when it does not. In particular, we design a

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simple algorithm to identify when economic conditions would be suitable for the government to undertake a redistributive policy to promote economic growth. We apply this new technique to resolve the apparently conflicting policy recommendations mentioned above. We leave aside welfare and equity considerations which, in general, strengthen the case for redistribution. We also note in our analysis that our algorithm could be generalized without ambiguity to address those considerations.

Subsequent to Solow's lecture on the potentially growth promoting role of a redistributive policy and the pioneering work of Galor and Zeira (1993) that links income distribution and economic growth, a new wave of PhD theses emerged from Minnesota and Chicago that examined the macroeconomic effects of human capital distribution on economic growth. Contrary to the findings of Benabou (2002), most of those works report a net negative effect of redistributive policies on economic growth (e.g., Conesa and Krueger, 2006, Caucutt, İmrohoroğlu and Kumar, 2006 and Erosa and Koreshkova, 2006).

To rationalize the apparently contradictory policy recommendations of Benabou and the above group we combine complementary features of their economies to build a generalized model. In particular, we take the pioneering work of Benabou (2002) as the main foundation and then add to it relevant features from Erosa and Koreshkova (2006), which we take as the representative of the group that contradicts Benabou's findings. Erosa and Koreskova (or EK from here on) use a specialized environment which bears marks of important differences with Benabou. EK's model treat capital goods and human capital as complementary inputs to production unlike Benabou's model which effectively treats the two types of capital as perfect substitutes. EK's model includes a social security benefit that typically retards growth and rules out any education subsidy to compensate for the distortionary effects of income taxation on saving, unlike Benabou. They also rely on a competitive market to equalize rates of return on investment in physical capital across agents. The presence of such market reduces allocative inefficiency due to trade frictions which causes unequal marginal product of capital across agents, which, in presence of diminishing returns to capital, offers potential gains from redistribution.

We first focus on an analysis without a competitive market for physical capital as in Benabou and then examine how the results change by introducing a competitive market for trading capital goods. Our general model assumes complementarity between the two forms of capital, omits social security consideration and allows for various types of subsidies that offset negative effects of redistribution on economic growth. We abstract from a social security scheme, since it obviously raises the cost and lowers the potential benefit of redistribution in a way such that one could analyze its impact on growth separately. We maintain a common assumption of Benabou and EK that heterogeneity arises only from an idiosyncratic shock which follows a lognormal distribution. The variance of this distribution plays a crucial role in our model analysis. Note that this variance may very well differ across countries depending upon ethnic composition, language varieties, religion, migration history and other country-specific characteristics of the population. Ill developed financial market and the absence of an efficient mechanism for delivering education can aggravate income inequality from one generation to another and that, in presence of diminishing returns to capital of either type, retards economic growth. Redistribution via a scheme of progressive income taxation or via a scheme of progressive education subsidy can be used to mitigate these inefficiencies and thereby to foster economic growth.

In the above generalized model we provide explicit formulas to compare redistribution led gains and losses of output of various kinds that typically arise in a large class of economies characterized by diminishing returns and trade frictions. A static gain arises in these economies from a reallocation of resources from a less productive unit to a more productive unit. A dynamic gain arises from an increased intergenerational mobility that facilitates growth of investment and efficiency. The loss of output arises from the typical adverse effect of redistribution on the supply of inputs. We provide a simple and explicit condition for a large class of economies to determine if in an arbitrarily specified economy those gains would exceed the loss or not. As a part of that condition we design a new statistic that maps economic fundamentals into the minimum threshold for the variance of idiosyncratic shocks and call it the critical minimum variance (CMV) for the economy. We show that the net gains of long-run output from a redistributive policy would be positive, if and only if the exogenous heterogeneity or diversity in the population is large enough and, in particular, exceeds the economy's CMV. However, in any economy where agents value leisure, the output must drop immediately after introduction of a redistributive policy, since the gains from redistribution appear after a lag of one period. Interestingly, we also find that once the gains exceed the loss, it continues to do so and hence from that period onwards the per capita output monotonically increases to its long-run value. We prove that if population diversity is too small relative to the economy's CMV such that the variance of idiosyncratic shock is less than the CMV, then the gains would not offset the loss sufficiently either to stop per capita output from falling or to push it up above the level prior to the introduction of the policy. Consequently, under such condition, redistributive policy would unequivocally hurt economic growth.

The CMV not only differs among economies, it also differs across different types of redistributive policy packages. Based on these differences in CMVs, we argue that apparently conflicting findings of both Benabou (2002) and Erosa and Koreshkova (2006) could be rationalized in our general model by considering alternative policy packages for redistribution. For example, if, following EK, we disallow any types of investment subsidy then the CMV for Benabou's economy would exceed his chosen value for population diversity. Consequently, by our algorithm, like EK, Benabou's model would find redistribution to be harmful for growth. Even with an education subsidy as in Benabou (2002), we show that with physical capital in the model the growth-maximizing progressive income tax rate would be zero. Unless we could allow a bequest subsidy or consider a progressive income subsidy, redistribution would hurt growth in augmented Benabou economy. Similarly, we notice that the EK economy does not satisfy a necessary condition for yielding a long-run growth benefit from redistribution unlike Benabou. When we modify the EK economy by introducing an education subsidy following Benabou's lead, we find a significant gain from some degree of redistribution compared to no redistribution, which their scheme of proportional taxation implies. The growth maximizing average marginal income tax rate in the EK economy with education subsidy turns out to be about 4.8%. If we replace the income tax scheme by a scheme of progressive education subsidy considered by Benabou then the growth-maximizing degree of progressivity in the EK economy increases significantly. Even without any education subsidy, we report a large range of parameter values for the human capital technology that EK use for which the model's CMV or the threshold for population diversity would be sufficiently low to ensure the growth maximizing progressive income tax rate to be significantly positive.

In general, we find that the presence of a market for capital goods, and its presence not as substitutes but as complementary inputs to human capital in the output generating process, not considered by Benabou, decreases those gains and increase the value of CMV making it less likely for the economy's optimal redistributive tax rate to be positive. At the same time, a redistributive policy package that allows for various types of subsidy to offset negative effects of redistribution, not considered in EK, lowers CMV and that increases the growth-promoting potential of redistribution. Thus analysis of how the CMV varies across policies and across economies would contribute significantly to rationalizing the difference between Benabou and EK's policy recommendation for the US economy as well as for appreciating why merits of redistribution may rationally vary across countries. We discover that the CMV of a country would be relatively large if it operates a unskilled labor-intensive technology, or if its education system suffers from poor quality, or if its communities are not segregated such that family connections play little role in determining one's stock of human capital, or if its agents can freely access to a competitive capital market. Consequently, by our key proposition, even with a moderate degree of heterogeneity or diversity in the agent population, countries with a combination of those characteristics find redistribution to be harmful for growth.

These findings have two important implications for policy recommendations. First, we need reliable estimates of the economic fundamentals and identify a feasible package of redistributive policies to determine whether or not the economy's CMV exceeds the statistical estimates of the degree of heterogeneity in the population that arises from the variations in ethnicity, language, migration status and other socio-political characteristics of agents. Second, the idea of globalization involving a universal policy rule for all countries would not work in this context, since country-specific differences in population, technology, preference and institutions as well as country-specific differences in population heterogeneity do matter in determining the growth promoting potential for a redistributive policy package. In particular, as a growth maximizing strategy, a less developed economy that primarily relies on unskilled labor and physical capital calls for a low degree of redistribution while a developed economy that primarily relies on skilled labor calls for a high degree of redistribution. In short, when it comes to optimal degree of redistribution, one size does not fit all.

Section 2 describes the model with a redistributive policy package involving progressive tax and transfer. Section 3 analyzes the equilibrium dynamics followed by a discussion in Section 4 of the model's steady state. Section 5 repeats the exercise under a different policy package involving progressive education finance. Section 6 includes the key proposition of the paper including the definition of CMV and some examples to illustrate its properties in different cases. Section 7 considers alternative benchmarks for the US economy and provides numerical analyses of the CMV and their implications for the growth maximizing rate of redistribution and the transitional dynamics. Section 8 summarizes our contributions along with the concluding remarks followed by the appendix containing proofs of various analytical propositions, detailed algebra to characterize competitive equilibrium and the complete list of references.

2. THE MODEL

2.1. *Preference, Technology and Endowments*

The model consists of a continuum of infinitely lived dynasties $i \in [0, 1]$. Each dynasty is made of a sequence of families consisting of individuals who live for two periods, first as a child and then as an adult. At each time period t, the dynasty is represented by a family of an adult and a child. The adult, at time period t represents the dynasty from that period onward and makes all decisions for that period subject to the constraint that she cannot pass on her debt to her child. We call this adult of the dynasty i at period t the dynastical agent i or simply agent i. The child becomes an adult in the following period and represents her dynasty.

Following Benabou (2002), the preference of the dynastical agent i at period t is given by:

$$(1) \quad \ln U_t^i = E_t \left[\sum_{n=0}^{\infty} \rho^n \left(\ln c_{t+n}^i - \left(l_{t+n}^i \right)^\eta \right) \right], \, \eta \geq 1,$$

where $c_t^i \geq 0$ and $l_t^i \in [0, 1]$ denote, respectively, consumption and labor supply by the adult of the dynasty i in period t; $\rho \in (0,1)$ is the discount factor.[3]

2.1.1. Backyard Technology

In the spirit of Benabou (2002) we assume that facing trade frictions in the capital market everyone uses a backyard technology for production[4] that follows Barro, Mankiw and Sala-i-Martin (1995) such that the output of the agent i as a function of her physical and human capital k_t^i, h_t^i and labor l_t^i satisfies

$$(2) \quad y_t^i = \left(k_t^i\right)^\lambda \left(h_t^i\right)^\mu \left(l_t^i\right)^\varepsilon, \text{ where, } \varepsilon = 1 - \lambda - \mu.$$

In Benabou's technology $\lambda = 0$ but we can interpret h to be the sum of two types of non-tradable capital (physical and human) that are perfect substitutes. In EK's technology $\varepsilon = 0$ and the two types of capital are complements or essential inputs to production. Also in EK's technology human capital of different individuals are perfect substitutes and only the average human capital affects the output of the representative firm. The use of average washes away any potential gains in national output from changes in the distribution of human capital.

2.1.2. Competitive Environment

Alternatively, along the line of EK, we may allow a competitive environment for trading capital goods and labor where the representative firm profit maximizes profit by operating a technology that closely corresponds to the backyard technology considered above. For example, by denoting $\tilde{h}^i \equiv \left(h^i\right)^{\frac{\mu}{1-\lambda}}$, $\tilde{l}^i \equiv \left(l^i\right)^{\frac{\varepsilon}{1-\lambda}}$ we may rewrite (2) as

$$(3) \quad y_{BT}^i = \left(k^i\right)^\lambda \left(\tilde{h}^i \tilde{l}^i\right)^{1-\lambda}$$

and assume that in the competitive environment (CE), the representative firm would be able to pull resources to operate the following technology:

$$(4) \quad y_{CE} = k^\lambda H^{1-\lambda}, \text{ where, } k = \int_0^1 k^i di, \, H = \int_0^1 \tilde{h}^i \tilde{l}^i di.$$

[3]Note that the intertemporal elasticity of substitution of labor, $\epsilon = \frac{1}{\eta - 1}$. We assume $\epsilon > 0$ or, equivalently, $\eta > 1$. We also consider other types of preference such as the one used in EK where $\eta = 0$, or an extension of EK where $\epsilon = 1$, or equivalently, $\eta = 2$ and discuss their implications in Section 7.

[4]Key results remain unchanged when we allow for a competitive labor market and assume that the efficiency units of labor of an agent is given by a Cobb-Douglas function of her stocks of the two forms of non-tradable capital.

Given a rental price of r per unit of capital goods and a wage rate w per unit of effective labor $\tilde{h}^i\tilde{l}^i$ the firm solves the following exercise:

$$(5) \quad \max_{k,H} \left(y_{CE} - rk - wH\right)$$

and the first order conditions of the above exercise satisfy $r = \lambda y/k$ and $w = (1-\lambda)\, y/H$. Consequently, the earning of the agent i in that competitive environment would be given by,

$$(6) \quad y^i_{CE} = rk^i + w\tilde{h}^i\tilde{l}^i \equiv y_{CE}\chi^i, \text{ where } \chi^i \equiv \left(\lambda\left(\frac{k^i}{k}\right) + (1-\lambda)\left(\frac{\tilde{h}^i\tilde{l}^i}{H}\right)\right).$$

In other words, the agent i's earning $y^i = y^i_{BT}$ for the case without a capital market and $y^i = y^i_{CE}$ for the case with a competitive capital market. In our detailed analysis we treat these two extreme cases separately in the end but allow the common notation as far as possible to get the main points across efficiently.

The government has a scheme of progressive income taxation and transfer. It also provides an education subsidy and a bequest subsidy to offset the negative effects of income tax on output and finance those subsidies by taxing consumption at a rate θ per unit.[5] As in Benabou (2002), we assume that agents cannot inherit debt from the previous generations. Nor can they pass on debt to their children. Consequently, at each date, the disposable income $\hat{y}^i_t$ of the agent i must equal the total expenditure on consumption c^i_t, consumption tax θc^i_t, private education expenditure e^i_t and bequest b^i_t. In other words,

$$(7) \quad \hat{y}^i_t = (1+\theta)\, c^i_t + e^i_t + b^i_t.$$

The agent receives education subsidy at a rate d per unit of her expenditure e^i_t on the child's education such that in the following period her grown up child's human capital h^i_{t+1} as a function of her innate ability ξ^i_{t+1}, external effects arising from neighborhood or family as proxied by parental human capital h^i_t, and the sum of private and public investment on her education $(1+d)\, e^i_t$, is given by,

$$(8) \quad h^i_{t+1} = \kappa\xi^i_{t+1}\left(h^i_t\right)^{\alpha}\left((1+d)\, e^i_t\right)^{\beta}.$$

The idiosyncratic shocks ξ^i_t that arise from discrepancies in innate ability or in efficiency of human capital usage are *i.i.d.* with $\ln\, \xi^i_t \sim N\left(\varphi, \sigma^2\right)$, where φ and σ^2 are constants.[6] The variance of this discrepancy plays a crucial role in this paper. The parameter $\alpha > 0$ measures the elasticity of "neighborhood externality," a phrase explored originally in Benabou (1996) in the context of human capital inequality and the parameter $\beta > 0$ measures the elasticity or effectiveness of education expenditure, which is primarily determined by the quality of the education system, on the child's human capital. Given this interpretation we refer to α and β, respectively, as parameters for the degree of "neighborhood externality" and the quality of the

[5] Bequest is usually taxed and not subsidized. We introduce bequest subsidy as a hypothetical tool for offsetting the distortionary effect of redistribution on physical capital accumulation. In principle, it could be zero or negative.

[6] Benabou assumes $\varphi = -\sigma^2/2$; but our results do not depend on that specification.

education system.

Capital goods are complementary to human capital and become obsolete at the end of each generation. A tool loses value when its user dies. Parents buy new tools for their children at a subsidized rate set by the government and leave them as bequest. To capture this feature we assume that they depreciate completely in the production process. Consequently, in the generation $t+1$, the agent i's physical capital k^i_{t+1} consists only of her parent's bequest b^i_t and a bequest subsidy from the government at the rate of v per unit of the bequest such that

$$(9) \quad k^i_{t+1} = (1+v)\, b^i_t.$$

Initial levels of k^i_0 and h^i_0 are jointly, lognormally distributed.

2.2. *Redistribution with Progressive Income Tax*

We first consider a redistributive policy package involving progressive income tax and transfer. Following Benabou (2002), the government cannot detect individual innate ability ξ^i_t and neighborhood or family effects h^i_t, but does observe individual incomes y^i_t and their expenditure on education e^i_t. Consequently, the disposable income of a typical agent at a date t is

$$(10) \quad \hat{y}^i_t \equiv \left(y^i_t\right)^{1-\tau} \left(\tilde{y}_t\right)^{\tau},$$

such that those with income higher than $\tilde{y}_t$ pay net tax while those with income below $\tilde{y}_t$ receive net transfers and the balanced-budget constraint is

$$(11) \quad \int_0^1 \left(y^i_t\right)^{1-\tau} \left(\tilde{y}_t\right)^{\tau} di = y_t,$$

where $y_t \equiv \int_0^1 y^i_t di$ denotes per-capita income, $\tilde{y}_t$ represents the break-even level of income and $\tau \leq 1$ measures the average marginal tax rate and is identified as the degree of redistribution or *progressivity* in fiscal policy. Note that on a logarithmic scale τ denotes the proportional tax rate on the log of personal income.[7]

Benabou (2002) emphasizes that elected governments do use a wide range of instruments rather than mere income taxation to redistribute resources among people and across time. Typically governments attempt to offset some of the distortionary effects of income taxes with a package of redistributive policies. In that tradition, we consider a redistributive policy-package which also includes investment subsidies financed by consumption tax such that

$$(12) \quad \theta \int_0^1 c^i_t di = d \int_0^1 e^i_t di + \nu \int_0^1 b^i_t di.$$

2.3. *Individual Optimization*

Our main objective is to characterize a test for determining 'growth-promoting potential' of a redistributive policy package. To get our main point across easily we only focus on the stationary

[7] EK use a different formula for taxation. The estimate for growth maximizing tax rate does vary when we use different tax formulas. However, the necessary and sufficient conditions that identify when redistribution hurts or promotes growth do not depend on the choice of the tax formulas.

policy sequence, $T = (\tau, d, v, \theta)$.[8] At each date t, let m_{ht}, m_{kt} denote the means and Δ^2_{ht}, Δ^2_{kt} denote the variances of $\ln h^i_t$ and $\ln k^i_t$, respectively, and let cov_t denote the covariance between $\ln h^i_t$ and $\ln k^i_t$. Suppose $M_t \equiv \left(m_{ht}, m_{kt}, \Delta^2_{ht}, \Delta^2_{kt}, cov_t\right)$. Then for the agent's dynamic optimization problem, the state variables are $(h^i_t, k^i_t, M_t; T)$, the control variables are $\left(c^i_t,\ l^i_t,\ e^i_t,\ b^i_t\right)$ and the Bellman equation satisfies

$$(13)\quad \ln U\left(h^i_t, k^i_t, M_t; T\right) = \max_{c^i_t, l^i_t, e^i_t, b^i_t} \left\{(1-\rho)\left[\ln c^i_t - \left(l^i_t\right)^{\eta}\right] + \rho E_t\left[\ln U(h^i_{t+1}, k^i_{t+1}, M_{t+1}; T)\right]\right\},$$

subject to (2), (7), (8) and (9).

Lemma 1: *The value function under fiscal redistribution is* $\ln U\left(k^i_t, h^i_t, M_t; T\right) = Z_1\left(\ln h^i_t - m_{ht}\right) + Z_2\left(\ln k^i_t - m_{kt}\right) + W_t$,

$$(14)\quad Z_1 = \frac{(1-\rho)\mu(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)},$$

$$(15)\quad Z_2 = \frac{(1-\rho\alpha)(1-\rho)\lambda(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)},$$

where aggregate welfare is $W_t = \int_0^1 \ln U\left(k^i_t, h^i_t, M_t; T\right) di$.
Proof. See Appendix.

The values of physical and human capital as expressed by their utility elasticities are respectively given by Z_1 and Z_2. Note the tax rate τ can alter these values individually but does not alter the relative value of human to physical capital, which increases with output elasticity of human capital μ, neighborhood effect α and patience ρ but remains unaffected by the quality of education β.

2.4. *Labor Supply and Savings Decisions*

The first order conditions associated with the Bellman equation given by (13) yield complete solutions to the agent's problem. We first discuss labor supply followed by investments in education and bequest.

Lemma 2: *The optimal labor supply* l_{BT} *with a backyard technology in every period is determined by constant values of parameters and tax rate* τ *:*

$$(16)\quad l_{BT} = \left(\frac{((1-\lambda-\mu)/\eta)(1-\rho\alpha)(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)}\right)^{1/\eta}.$$

If we set $\lambda = 0$, this labor supply function will be exactly the same as the one in Benabou (2002).
Proof. Appendix.

Lemma 3: *The optimal labor supply* l_{CE} *with a competitive market for capital in every period is given by*

$$(17)\quad l_{CE} = \left(\frac{(\varepsilon/\eta)(1-\rho\alpha)(1-\lambda)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)}\right)^{1/\eta}.$$

[8] A straightforward generalization of the analysis presented here could be done based on the arguments presented in page 488 of Benabou (2002).

Proof. Appendix.

Lemma 4: *If the degree of redistribution is sufficiently low then the availability of a competitive market for capital increases labour supply and the negative marginal effect of redistribution on labor supply is larger when such market is not available.*

Proof. The ratio R of labor supply under BT to that under CE satisfies

$$(18)\quad R = \left(\frac{1-\tau}{1-\lambda}\right)^{1/\eta}.$$

Clearly the ratio $R \geq 1$ if and only if $\tau \leq \lambda$. Also, as τ increases R decreases implying that in response to an increase in the average marginal tax rate τ labor supply under BT decreases more than that under CE. □

Next we consider investment propensities for the two forms of capital. We denote by s^i_{jt}, $j = 1, 2$, respectively the fraction of disposable income that agent i invests in her children's education and for her bequest and call them respectively the education investment rate and the bequest rate such that $s^i_{1t} \equiv e^i_t/\hat{y}^i_t$, $s^i_{2t} \equiv b^i_t/\hat{y}^i_t$.

Lemma 5: *The investment rate s^i_{1t} in education and the bequest rate s^i_{2t} are time invariant and decreases with the degree of progressivity τ:*

$$(19)\quad s_1 = \frac{\rho\beta\mu\,(1-\tau)}{1-\rho\alpha} \equiv (1-\tau)\,\bar{s}_1,$$

$$(20)\quad s_2 = \rho\lambda\,(1-\tau) \equiv (1-\tau)\,\bar{s}_2,$$

where $\bar{s}_1$ and $\bar{s}_2$ are the laissez-faire saving rates.
Proof. Appendix.

From (19) and (20) we note that the relative propensity of investment between human and physical capital increases with the quality of education β as well as all other factors that raises the relative utility valuation of human to physical capital. Lemmas 2, 3 and 5 spell out explicitly the negative effect of redistribution on the incentives to supply labor and capital inputs.

2.5. *Consumption Taxes, Education and Bequest Subsidies*

By the government budget constraint (12) and Lemma 5, we get

$$(21)\quad \frac{\theta\,(1-s_1-s_2)}{1+\theta} = ds_1 + vs_2.$$

By (19)-(21), it follows, therefore, the consumption tax rate θ must be set such that

$$(22)\quad \theta = \frac{\bar{s}_1 + \bar{s}_2 - (s_1+s_2)}{1-\bar{s}_1-\bar{s}_2}.$$

We can switch on the intertemporal distortions simply by setting either d or v or both equal to zero by adjusting θ accordingly. We allow for a bequest subsidy only to detect the upper limit for the optimal tax rate after eliminating the distortionary effect of income taxes on physical capital accumulation.

We now describe the equilibrium dynamics before going to the section on steady state where the key proposition of this paper lies.

3. THE EQUILIBRIUM DYNAMICS

The individual optimization problem (13) yields a set of decision rules as follows:

$$\ln c_t^i = \ln \left(1 - s_{1t}^i - s_{2t}^i\right) - \ln (1+\theta) + (1-\tau) \ln y_t^i + \tau \ln \tilde{y}_t \tag{23}$$

$$\ln e_t^i = \ln s_1 + (1-\tau) \ln y_t^i + \tau \ln \tilde{y}_t, \tag{24}$$

$$\ln b_t^i = \ln s_2 + (1-\tau) \ln y_t^i + \tau \ln \tilde{y}_t. \tag{25}$$

Together with the government's budget constraints (11) and (12) the above decision rules imply a unique sequence of aggregate state variables $\{M_t\}$ that coincides with what the agent i takes as given in (13) such that at each date $t = 0, 1, 2, ..$, the following aggregate consistency condition holds:

$$\int_0^1 y_t^i di = \int_0^1 c_t^i di + \int_0^1 e_t^i di + d \int_0^1 e_t^i di + \int_0^1 b_t^i di + v \int_0^1 b_t^i di. \tag{26}$$

We now focus on the derivation of explicit formulas for gains and losses from a redistributive policy package following the lead of Benabou (2002) with backyard technology (BT), a straightforward extension of Benabou's environment. We place at the end of the Appendix necessary characterization of the same for the other case with a market for capital goods in a competitive environment (CE) for interested readers and summarize results from the CE case that are relevant to this paper in Sections 6 and 7.

3.1. *Dynamic Path of Physical Capital, Human Capital and Income*

The decision rules derived above imply the following dynamics for individual stocks of physical and human capital and individual income for the case when agents do not have access to a capital market and use their backyard technology.

The logarithm of (9), combining with (16) and (20) yields the dynamics of physical capital for the dynasty i,

$$\ln k_{t+1}^i = \ln \bar{s}_2 + (1-\lambda-\mu)(1-\tau) \ln l + \lambda (1-\tau) \ln k_t^i + \mu (1-\tau) \ln h_t^i + \tau \ln \tilde{y}_t. \tag{27}$$

The logarithm of (8), combining with (16) and (19) yields,

$$\begin{aligned} \ln h_{t+1}^i = {} & \ln \kappa + \beta \ln \bar{s}_1 + \beta (1-\lambda-\mu)(1-\tau) \ln l + \ln \xi_{t+1}^i + \beta\lambda (1-\tau) \ln k_t^i \\ & + (\alpha + \beta\mu (1-\tau)) \ln h_t^i + \beta\tau \ln \tilde{y}_t. \end{aligned} \tag{28}$$

Substituting (27) and (28) into (2) yields the equilibrium path of income for agent i:

$$\ln y_{t+1}^{i}=\psi+(1-\alpha)(1-\lambda-\mu)\ln l+\mu\ln\xi_{t+1}^{i}+(\alpha+(\lambda+\beta\mu)(1-\tau))\ln y_{t}^{i} -\alpha\lambda(1-\tau)\ln y_{t-1}^{i}+(\lambda+\beta\mu)\tau\ln\tilde{y}_{t}-\alpha\lambda\tau\ln\tilde{y}_{t-1}, \tag{29}$$

where $\psi=\mu(\ln\kappa+\beta\ln\bar{s}_1)+\lambda(1-\alpha)\ln\bar{s}_2$ is a constant.

Note that the intergenerational persistence of human capital $p^h(\tau)\equiv\alpha+\beta\mu(1-\tau)$ and physical capital $p^k(\tau)\equiv\lambda(1-\tau)$ together imply the intergenerational persistence of income $p^y(\tau)\equiv\alpha+(\lambda+\beta\mu)(1-\tau)$ between parents and children. It is always positive but decreases with the rise of income tax rate. Consequently, intergenerational social mobility increases with redistribution. Note also that the income of the parents does not sufficiently determine children's income unlike Benabou (2002). The grandparent's income has a negative effect on grandchildren's income with the value of $\alpha\lambda(1-\tau)$. This feature resembles Becker and Tomes (1986)'s finding. Next, we characterize the dynamic path of the aggregate state variables.

3.2. *Dynamics of the Economy-wide State Variables*

Given the initial lognormal distribution, by (27) and (28) physical and human capital and income remain lognormally distributed over time such that at each date t, M_t satisfies

$$m_{kt+1}=\ln\bar{s}_2+(1-\lambda-\mu)\ln l+\lambda m_{kt}+\mu m_{ht}+\tau(2-\tau)\left(\lambda^2\Delta_{kt}^2+\mu^2\Delta_{ht}^2+2\lambda\mu cov_t\right)/2, \tag{30}$$

$$\Delta_{kt+1}^2=(1-\tau)^2\left(\lambda^2\Delta_{kt}^2+\mu^2\Delta_{ht}^2+2\lambda\mu cov_t\right), \tag{31}$$

$$\begin{aligned} m_{ht+1} &= \ln\kappa+\varphi+\beta\ln\bar{s}_1+\beta(1-\lambda-\mu)\ln l+\beta\lambda m_{kt}+(\alpha+\beta\mu)m_{ht} \\ &\quad +\beta\tau(2-\tau)\left(\lambda^2\Delta_{kt}^2+\mu^2\Delta_{ht}^2+2\lambda\mu cov_t\right)/2, \end{aligned} \tag{32}$$

$$\begin{aligned} \Delta_{ht+1}^2 &= \sigma^2+\beta^2\lambda^2(1-\tau)^2\Delta_{kt}^2+(\alpha+\beta\mu(1-\tau))^2\Delta_{ht}^2 \\ &\quad +2\beta\lambda(1-\tau)(\alpha+\beta\mu(1-\tau))cov_t, \end{aligned} \tag{33}$$

$$\begin{aligned} cov_{t+1} &= \beta\lambda^2(1-\tau)^2\Delta_{kt}^2+\mu(1-\tau)(\alpha+\beta\mu(1-\tau))\Delta_{ht}^2 \\ &\quad +\lambda(1-\tau)(\alpha+2\beta\mu(1-\tau))cov_t. \end{aligned} \tag{34}$$

Lemma 6: *The evolution of personal income is governed by a lognormal distribution such that at time t it is $\ln y_t^i\sim N(\lambda m_{kt}+\mu m_{ht}+(1-\lambda-\mu)\ln l,\,2\Lambda_t)$, and following Benabou (2002), $\Lambda_t\equiv\left(\lambda^2\Delta_{kt}^2+\mu^2\Delta_{ht}^2+2\lambda\mu cov_t\right)/2$ denotes income inequality measured by the ratio of mean to median income.*
Proof. Appendix.

By (11), it follows, therefore that the break-even income at which an agent's net tax obligation is zero satisfies:

$$\begin{aligned} \ln\tilde{y}_t &= \ln y_t+(1-\tau)\Lambda_t \\ &= \lambda m_{kt}+\mu m_{ht}+(1-\lambda-\mu)\ln l+(2-\tau)\Lambda_t. \end{aligned} \tag{35}$$

We now characterize the equilibrium dynamics with the following Lemma.

Lemma 7: *At equilibrium, the time series of the per capita income satisfies*

$$(36)\quad \ln y_{t+1} - \ln y_t = \psi + \mu\varphi + (1-\alpha)(1-\lambda-\mu)\ln l - (1-\alpha-\lambda-\beta\mu)\ln y_t - \alpha\lambda \ln y_{t-1} + \Lambda_{t+1} - \left(\alpha + (\lambda+\beta\mu)(1-\tau)^2\right)\Lambda_t + \alpha\lambda(1-\tau)^2\Lambda_{t-1},$$

where $\Lambda_{t+1} = \left(\begin{array}{c} \mu^2\sigma^2 + (\lambda+\mu\beta)^2(1-\tau)^2\lambda^2\Delta_{kt}^2 + (\alpha+(\lambda+\beta\mu)(1-\tau))^2\mu^2\Delta_{ht}^2 \\ +2\lambda\mu(1-\tau)(\alpha(\lambda+\beta\mu)+\beta\mu(\beta\mu+2\lambda)(1-\tau))\,cov_t \end{array}\right)/2.$

By imposing Benabou's assumption $\varphi = -\sigma^2/2$ and setting $\lambda = 0$, the above equation corresponds exactly to its counterpart in Benabou (2002).

In the steady state $M_t = M$. By Lemma 5, it follows that the growth rate equals zero in the steady state to which we now turn.

4. STEADY-STATE INCOME, INEQUALITY AND REDISTRIBUTION

In the steady states, $m_{kt} = m_k$, $\Delta_{kt}^2 = \Delta_k^2$, $m_{ht} = m_h$, $\Delta_{ht}^2 = \Delta_h^2$, and $cov_t = cov$. The following Proposition gives a sufficient condition for the existence of a unique steady state to which the equilibrium sequence of M_t converges.

PROPOSITION 1: *If* $(1-\alpha)(1-\lambda)-\beta\mu > 0$ *then the equilibrium sequence of* M_t *monotonically converges to a unique steady state.*
Proof. Appendix.

By (30) to (34), in the steady state, $\Lambda_t = \Lambda(\tau)$, where

$$(37)\quad \Lambda(\tau) \equiv \left(\frac{\mu(\mu+2\lambda D(\tau))}{A(\tau)-B(\tau)}\right)\left(\frac{1-\lambda(1-\tau)(\alpha+2\beta\mu(1-\tau))}{1-\lambda^2(1-\tau)^2}\right)\frac{\sigma^2}{2},$$

where $A(\tau) \equiv \left(1-(\alpha+\beta\mu(1-\tau))^2\right)(1-\lambda(1-\tau)(\alpha+2\beta\mu(1-\tau))) - 2\beta\lambda\mu(1-\tau)^2(\alpha+\beta\mu(1-\tau))^2$,

$$B(\tau) \equiv \frac{\beta^2\lambda^2\mu^2(1-\tau)^4(1+\alpha\lambda(1-\tau))^2}{1-\lambda^2(1-\tau)^2-\alpha\lambda(1-\tau)+\alpha\lambda^3(1-\tau)^3-2\beta\lambda\mu(1-\tau)^2},$$

$$D(\tau) \equiv \frac{\left(\beta\lambda^2\mu^2(1-\tau)^4+\mu(1-\tau)(\alpha+\beta\mu(1-\tau))\left(1-\lambda^2(1-\tau)^2\right)\right)}{(1-\lambda(1-\tau)(\alpha+2\beta\mu(1-\tau)))\left(1-\lambda^2(1-\tau)^2\right)-2\beta\lambda^3\mu(1-\tau)^4}.$$

By (16) the labor supply does not change with time but is a function of τ and wedenote it as $l(\tau)$; The term ψ would be a constant if both subsidies are applied. In general, it is a function of τ. In the steady state, by (36), per capita income converges to

$$(38)\quad \ln y(\tau) = \frac{\psi + \mu\varphi + (1-\alpha)(1-\lambda-\mu)\ln l(\tau) + \left(1-\alpha-((1-\alpha)\lambda+\beta\mu)(1-\tau)^2\right)\Lambda(\tau)}{(1-\alpha)(1-\lambda)-\beta\mu},$$

where ψ and $\Lambda(\tau)$ are from equations (29) and (37).

Rewriting (38) by plugging the formula for intergenerational persistence $p^y(\tau)$ from Section 3.1, we get

$$(39)\quad \ln y(\tau) = \frac{\psi + \mu\varphi + (1-\alpha)(1-\lambda-\mu)\ln l(\tau) + \left[1-\alpha-\frac{((1-\alpha)\lambda+\beta\mu)(p^y(\tau)-\alpha)^2}{(\lambda+\beta\mu)^2}\right]\Lambda(\tau)}{(1-\alpha)(1-\lambda)-\mu\beta}.$$

From (39), we note that as τ increases $p^y(\tau)$ decreases and, therefore, the coefficient of $\Lambda(\tau)$ increases reflecting an increase in the intergenerational mobility, a dynamic gain from redistribution, and that helps to increase per capita income. Alternatively, to discern the static gains from the reduction in inequality through redistribution, we can rearrange the above equation according to Benabou's format to get:

$$(40)\quad \ln y(\tau) = \frac{\ln\bar{\kappa} + (1-\alpha)(1-\lambda-\mu)\ln l(\tau) - \Omega(\tau)\Lambda(\tau)}{(1-\alpha)(1-\lambda)-\mu\beta},$$

where $\ln\bar{\kappa} \equiv \mu(\ln\kappa + \beta\ln\bar{s}_1) + (1-\alpha)\lambda\ln\bar{s}_2 + \mu\varphi + (\lambda+\mu)^2\sigma^2/2$ is a constant,

$$(41)\quad \Omega(\tau) \equiv \alpha + ((1-\alpha)\lambda+\beta\mu)(1-\tau)^2 - F(\tau) > 0$$

and $F(\tau) \equiv 1 - \dfrac{(\lambda+\mu)^2}{\left(\frac{\mu(\mu+2\lambda D(\tau))}{A(\tau)-B(\tau)}\right)\left(\frac{1-\lambda(1-\tau)(\alpha+2\beta\mu(1-\tau))}{(1-\lambda^2(1-\tau)^2)}\right)}.$

The term $\Omega(\tau)$ measures the negative effect of inequality on per capita income and corresponds to the exact same expression of Benabou (2002) if we set $\lambda = 0$. Benabou refers this adverse effect of inequality on income as "*investment reallocation effect*," which reduces to zero in a representative economy.

The equation (39) or (40) provides an explicit analytical expression for the cost and benefit of redistribution in terms of steady state income per capita and as a function of the degree τ of redistribution. A higher degree of redistribution would lower labor supply and, in the absence of education and bequest subsidies, would also lower investment in education and accumulation of physical capital. At the same time the combination of reduced income inequality and increased mobility implied by a higher degree of redistribution raises steady-state income per capita. The relative magnitude of the marginal cost and the marginal benefit from redistribution for a laissez-faire economy can be found by plugging $\tau = 0$ in the first derivative of the cost and benefit with respect to τ. Consequently, if the marginal benefit exceeds marginal cost when $\tau = 0$ then the growth maximizing degree of redistribution would be strictly greater than zero. We expand on the above intuition to spell out a condition under which redistribution promotes growth later.

Prior to presenting that result following the next section we discuss an alternative policy of redistribution considered by Benabou (2002) involving a progressive education subsidy instead of a progressive income tax rate. If one wishes to go straight to the key proposition of the paper, the next section could be skipped.

5. REDISTRIBUTION WITH PROGRESSIVE EDUCATION SUBSIDY

5.1. *The Model*

In this section, we repeat our analysis but under an alternative policy package for redistribution considered by Benabou (2002). In particular, we replace the income tax scheme by a progressive education subsidy such that with zero income tax $\hat{y}_t^i = y_t^i$ but the education subsidy scheme follows a progressivity rate τ, such that the educational investment satisfies

$$(42)\quad \hat{e}_t^i \equiv (1+d)\left(\tilde{y}_t / y_t^i\right)^{\tau} e_t^i.$$

It means that for the poor family whose income y_t^i is less than $\tilde{y}_t$ will receive net benefit and then could provide more investment in education.

Bequest b_t^i for next period's physical capital accumulation continues to be a fraction of income plus government subsidy at rate v:

$$(43)\quad k_{t+1}^i = (1+v)\, s_{2t}^i \left(k_t^i\right)^{\lambda} \left(h_t^i\right)^{\mu} \left(l_t^i\right)^{(1-\lambda-\mu)}.$$

The Bellman equation is now

$$(44)\quad \ln U\left(k_t^i, h_t^i, M_t; T\right) = \max_{s_{1t}^i, s_{2t}^i, l_t^i} \left\{ \begin{array}{c} (1-\rho)\left[\ln\left(\left(1 - s_{1t}^i - s_{2t}^i\right)/(1+\theta)\right)\right. \\ \left. +\lambda \ln k_t^i + \mu \ln h_t^i + (1-\lambda-\mu)\ln l_t^i - \left(l_t^i\right)^{\eta}\right] \\ +\rho E_t\left[\ln\left(U\left(k_{t+1}^i, h_{t+1}^i, M_{t+1}; T\right)\right)\right] \end{array} \right\},$$

with h_{t+1}^i and k_{t+1}^i are still given by (28) and (43).

Lemma 8: *The investment rate s_1, the bequest rate s_2, and the labor supply l are time invariant and decrease with the education equalization rate τ* such that

$$(45)\quad s_1 = \frac{\rho\beta\mu}{1-\rho\alpha+\rho\beta\mu\tau} \equiv \frac{\bar{s}_1}{1+\tau\bar{s}_1},$$

$$(46)\quad s_2 = \frac{\rho\lambda(1-\rho\alpha)}{1-\rho\alpha+\rho\beta\mu\tau} \equiv \frac{\bar{s}_2(1-\rho\alpha)}{1-\rho\alpha+\rho\beta\mu\tau},$$

$$(47)\quad l = \left(\frac{(1-\rho\alpha)(1-\lambda-\mu)}{\eta\left((1-\rho\alpha)(1-\rho\lambda)-\rho\beta\mu(1-\tau)\right)}\right)^{1/\eta}.$$

Proof. Appendix.

5.2. *Laws of Motion*

It follows, therefore, by (43) and (46) the decision rule for physical capital accumulation satisfies,

$$(48)\quad \ln k_{t+1}^i = \ln \bar{s}_2 + \lambda \ln k_t^i + \mu \ln h_t^i + (1-\lambda-\mu)\ln l,$$

and the decision rule for human capital accumulation is the same as (28). Consequently, by (2) the personal income of agent i evolves as follows:

$$(49)\quad \ln y^i_{t+1} = \psi + (1-\alpha)(1-\lambda-\mu)\ln l + \mu \ln \xi^i_{t+1} + (\alpha+\lambda+\beta\mu(1-\tau))\ln y^i_t - \alpha\lambda \ln y^i_{t-1} + \beta\mu\tau \ln \tilde{y}_t - \alpha\lambda\tau \ln \tilde{y}_{t-1},$$

where ψ is the same as that in (29).

The general expressions for the mean and variance of $\ln h^i_{t+1}$ are the same as (32) and (33). The mean and variance of $\ln k^i_{t+1}$ and covariance between $\ln h^i_{t+1}$ and $\ln k^i_{t+1}$ are

$$(50)\quad m_{kt+1} = \ln \bar{s}_2 + (1-\lambda-\mu)\ln l + \lambda m_{kt} + \mu m_{ht},$$

$$(51)\quad \Delta^2_{kt+1} = \lambda^2\Delta^2_{kt} + \mu^2\Delta^2_{ht} + 2\lambda\mu cov_t,$$

$$(52)\quad cov_{t+1} = \beta\lambda^2(1-\tau)\Delta^2_{kt} + \mu(\alpha+\beta\mu(1-\tau))\Delta^2_{ht} + \lambda(\alpha+2\beta\mu(1-\tau))cov_t.$$

The expression for the break-even level of income is the same as (35). Denote by Λ^E_t the same concept of inequality as before but under progressive education finance. Equations (32), (33), (50), (51) and (52) yield the following result.

Lemma 9: *The growth rate of per capita income is given by*

$$(53)\quad \ln y_{t+1} - \ln y_t = \psi + \mu\varphi + (1-\alpha)(1-\lambda-\mu)\ln l - (1-\alpha-\lambda-\beta\mu)\ln y_t - \alpha\lambda \ln y_{t-1} + \Lambda^E_{t+1} - \left(\alpha+\lambda+\beta\mu(1-\tau)^2\right)\Lambda^E_t + \alpha\lambda\Lambda^E_{t-1}.$$

This equation is different from (36) because unlike the case of progressive income tax rate, under the regime of progressive education subsidy, the disposable income is equal to the pre-tax income.

5.3. *Steady-State Income, Inequality and Redistribution*

As before, m_{ht}, m_{kt}, Δ^2_{ht}, Δ^2_{kt}, cov_t and Λ^E_t converge to their respective steady states. The steady state income inequality $\Lambda^E(\tau)$ satisfies

$$(54)\quad \Lambda^E(\tau) \equiv \frac{\mu^2(D_E(\tau)+2\lambda B_E(\tau))\sigma^2}{2\left(A_E(\tau)D_E(\tau) - 2\beta\lambda\mu(1-\tau)B^2_E(\tau)\right)},$$

where $A_E(\tau) \equiv 1 - (\alpha+\beta\mu(1-\tau))^2 - \lambda^2\left(1-\alpha^2-2\alpha\beta\mu(1-\tau)\right)$,

$B_E(\tau) \equiv \alpha\left(1-\lambda^2\right) + \beta\mu(1-\tau)$,

$D_E(\tau) \equiv (1-\alpha\lambda)\left(1-\lambda^2\right) - 2\beta\lambda\mu(1-\tau)$,

and by (53), per capita income is

$$(55)\quad \ln y(\tau) = \frac{\psi + \mu\varphi + (1-\alpha)(1-\lambda-\mu)\ln l(\tau) + \left((1-\alpha)(1-\lambda) - \beta\mu(1-\tau)^2\right)\Lambda^E(\tau)}{(1-\alpha)(1-\lambda) - \beta\mu}.$$

where $l(\tau)$ is from (47) and ψ follows the same discussion in (38). The difference between the above equation and (38) is in the last term of the numerator. This is shown in the last term of (55) that $1-\alpha-\lambda(1-\alpha)-\beta\mu(1-\tau)^2$ is less than the last term in (38) $1-\alpha-(\lambda-\alpha\lambda+\beta\mu)(1-\tau)^2$. The other terms are the same. Therefore, the per capita income in the steady state will be higher in the education finance scheme than that in the redistributive income taxation scheme.

We can rearrange the equation (55) following Benabou's format to get

$$(56)\quad \ln y(\tau) = \frac{\ln\bar{\kappa} + (1-\alpha)(1-\lambda-\mu)\ln l(\tau) - \Omega(\tau)\Lambda^E(\tau)}{(1-\alpha)(1-\lambda)-\beta\mu},$$

where $\ln\bar{\kappa}$ is described in (40), $\Omega(\tau) \equiv \lambda + \alpha(1-\lambda) + \beta\mu(1-\tau)^2 - F_E(\tau) > 0$ and

$$F_E(\tau) \equiv 1 - \frac{(\lambda+\mu)^2\left(A_E(\tau)D_E(\tau) - 2\beta\lambda\mu(1-\tau)B_E^2(\tau)\right)}{\mu^2\left(D_E(\tau)+2\lambda B_E(\tau)\right)}.$$

Redistribution with a progressive education subsidy also incur positive and negative effect on income in a way that is similar to what we discussed earlier with income tax. In the following section we develop a simple algorithm to compare the relative magnitude of these effects to determine when a positive degree of redistribution would be necessary to maximize economic growth.

6. THE CRITICAL MINIMUM VARIANCE (CMV) OF INNATE ABILITY

6.1. *Definition*

To spell out this condition in practical terms for the policy makers, we design a statistic which we call the critical minimum variance (CMV). We show that if and only if the variance of innate ability in an economy exceeds a minimum threshold then a redistributive policy in that economy promotes growth. In other words, sufficiently low degrees of inequality do not call for a redistributive policy to promote growth. In particular, we have the following proposition:

PROPOSITION 2: *There is a critical minimal value of the variance (CMV) of innate ability σ^* such that the long-run output maximizing degree of redistribution $\tau > 0$ if and only if $\sigma^2 > \sigma^*$.*

Proof: Note the function of income inequality (37) or (54) could be written like

$$(57)\quad \Lambda(\tau) \equiv \Phi(\tau)\sigma^2.$$

The steady state income per capita given by (38) and (55) could be represented by a general formula

$$(58)\quad \ln y = \frac{\Psi(\tau) + \varkappa(\tau)\Lambda(\tau)}{c_1},$$

where $\Psi(\tau) \equiv \mu(\ln\kappa + \beta\ln(1+d)s_1(\tau)) + \lambda(1-\alpha)\ln(1+v)s_2(\tau) + (1-\alpha)(1-\lambda-\mu)\ln l(\tau)$ and by assumption $c_1 \equiv (1-\alpha)(1-\lambda)-\beta\mu > 0$ and $\varkappa(\tau) \equiv 1-\alpha-((1-\alpha)\lambda+\beta\mu)(1-\tau)^2$ under the income taxation scheme and $\varkappa(\tau) \equiv (1-\alpha)(1-\lambda)-\beta\mu(1-\tau)^2$ under education finance. Differentiating both sides of (58) with respect to τ yields

$$(59)\quad c_1\frac{\partial\ln y(\tau)}{\partial\tau} = \Psi'(\tau) + \varkappa(\tau)\Phi'(\tau)\sigma^2 + \varkappa'(\tau)\Phi(\tau)\sigma^2.$$

Note $\Phi'(\tau) = \Phi(\tau) * \mu(\tau)$ such that $\mu(0) = -1$ and $\mu'(0) = 0$.It follows, therefore, $\varkappa(0)\Phi'(0) + \varkappa'(0)\Phi(0) > 0$ and that $\frac{\partial^2 \ln y(\tau)}{\partial \tau^2} |_{\tau=0}$exists and that makes $\frac{\partial \ln y(\tau)}{\partial \tau}$ continuous at $\tau = 0$. Let τ^* denote the growth maximizing degree of redistribution. Clearly, continuity of $\frac{\partial \ln y(\tau)}{\partial \tau}$ at $\tau = 0$ implies that $\tau^* > 0$ if and only if

$$\frac{\partial \ln y(\tau)}{\partial \tau} |_{\tau=0} > 0.$$

$$\Longleftrightarrow \varkappa(0)\Phi'(0)\sigma^2 + \varkappa'(0)\Phi(0)\sigma^2 > -\Psi'(0).$$

$$\Longleftrightarrow \sigma^2 > \frac{-\Psi'(0)}{\varkappa(0)\Phi'(0) + \varkappa'(0)\Phi(0)}.$$

We define the RHS of the above inequality, which does not depend on σ, as the Critical Minimum Variance (CMV) and denote it by σ^* such that

$$(60) \quad \sigma^* \equiv \frac{-\Psi'(0)}{\varkappa(0)\Phi'(0) + \varkappa'(0)\Phi(0)} = f(\alpha, \beta, \lambda, \mu, \rho, \eta).$$

It follows, therefore, that $\tau^* > 0$ if and only if $\sigma^2 \geq \sigma^*$. □

The denominator of (60) is strictly greater than zero, since $c_1 > 0$. Consequently, $\sigma^* \geq 0$ if and only if $\Psi'(0) < 0$. One corollary of the above result is that in a homogeneous economy with no idiosyncratic uncertainty (i.e., $\sigma^2 = 0$) which can be represented by a single agent, if $\Psi'(0) < 0$, then $\tau^* = 0$. The above proposition signifies the importance of heterogeneity behind the argument for a growth promoting policy of redistribution.

We call this new statistic defined by (60), the **Critical Minimum Variance** or CMV, because it represents the **minimum** threshold for the **variance** of idiosyncratic uncertainty and, because according to Proposition 2, its value turns out to be **critical** in determining whether or not a growth maximizing policy calls for redistribution.

PROPOSITION 3: *If* $\sigma^2 < \sigma^*$ *then for any given* $\tau > 0$ and for all $t > 0$, $\ln y_t(\tau) < \ln y(0)$.
Proof. See Appendix.

The above proposition answers the question we ask in the title of our paper. It states that if the exogenous diversity in the population of decision making adults measured by σ^2 is too small and, in particular, smaller than the endogenously determined threshold for diversity, the CMV or σ^*, allowed in the economy then any redistributive policy of the types we discussed in the paper would lower output per capita permanently. In other words, if the population is homogeneous relative to the insignificant variations permitted in the economy, redistribution would hurt economic growth.

The value of CMV does vary with the design of the redistributive policy package and they can be ordered. Let us denote CMV_{NS} as CMV with no subsidy applied, CMV_{ES} as CMV with only education subsidy applied and CMV_{BS} as CMV with both education and bequest subsidies applied. Proposition 4 summarizes the relative magnitudes of these CMVs.

PROPOSITION 4: *The value of* CMV_{NS} *would be equal to or greater than the* CMV_{ES} *which is equal to or greater than* CMV_{BS}*.*

Proof: See Appendix.

By (60), CMV is a function of parameters $(\alpha, \beta, \lambda, \mu, \rho, \eta)$ and always non-negative and may not have an upper bound. If the CMV of an economy is infinite then the optimal income tax rate would be always zero in that economy. If the CMV equals to zero then the optimal income tax rate will be always positive. If it is strictly greater than zero but has a finite value then the optimal income tax rate could be positive or zero depending on the value of the variance of the innate ability. We now discuss a few special cases to illustrate how a simple diagnostic test involving the CMV of an economy enables us to determine when redistribution may hurt or promote growth.

6.2. *Examples*

Example 1: $\lambda = \beta = 0$. In this case, the CMV is infinity. Therefore, by Proposition 2 all types of redistribution hurt growth irrespective of the other characteristics of the economy related to preference, technology and institution. Consequently $\tau^* = 0$ in those economies. This example is similar to those considered by Benabou except for the quality of the education system which is too poor to allow the government to offset the negative effect of redistribution on growth. We, therefore, argue that poor agents' access to high quality school is a prerequisite for a redistributive policy to be growth promoting in Benabou's model. The redistribution does provide a static gain in per capita income but only temporarily, since it does not change the distribution of human capital which determines the distribution of income in the next period. In other words, redistribution has no impact on long run distribution of income. Consequently, the benefit from redistribution is only temporary but cost of redistribution is permanent and large.

Example 2: $\mu = 0$. In this case also the CMV is infinity. Consequently, as in the previous example, any redistributive policy would hurt economic growth. The diminishing returns to physical capital would imply convergence of wealth in the long run. Although everyone leaves the same fraction of their disposable income as bequest for their children, the income growth rate would be relatively high for a person with relatively low stock of capital due to diminishing returns. The following equation describes that catch-up effect:

$$(61)\quad g_t^i \equiv \frac{y_{t+1}^i - y_t^i}{y_t^i} = \left(\bar{s}_2 l^{(1-\lambda)(1-\tau)} \left(\tilde{y}_t\right)^{\tau}\right)^{\lambda} \left(\frac{1}{k_t^i}\right)^{\lambda(1-\lambda(1-\tau))} - 1.$$

In this case, the long-run inequality Λ would be given by the steady state variance of physical capital, which equals zero by (61). Consequently, redistribution is not relevant in steady state.

Example 3: $\beta = 0$. We note from Example 1 that in Benabou (2002), the elasticity of children's human capital to education β is required to be positive to justify a growth promoting role of redistribution. In our general model that result can be extended only if we allow a perfectively competitive market for capital goods since the CMV would be infinity. However, if we rule out a competitive market for capital it is no longer the case. In particular, if $\beta = 0$ then under the income tax scheme:

$$(62)\quad \sigma^* = \frac{(1-\lambda-\mu)(1+\lambda)\left(1-\lambda^2\right)(1+\alpha)(1-\alpha\lambda)^2}{\eta\lambda\mu^2(1-\rho\lambda)\left(1+\alpha\lambda^2\right)}.$$

If technology requires both human and physical capital (i.e., λ, $\mu > 0$) then, by (62), the CMV would have a finite value. Consequently, by Proposition 2, a redistributive policy could be growth promoting even when $\beta = 0$ in a sufficiently diverse economy in which the variance of idiosyncratic shock is large enough to exceed the value of above CMV. Note also that if, in addition, the technology does not require any unskilled labor (i.e., $\lambda + \mu = 1$) then the CMV equals zero and hence redistribution would always be growth promoting irrespective of the extent of diversity among the heterogenous population in the economy. We conclude, therefore, that the quality of the educational system is not a prerequisite for a government to consider a redistributive policy in promoting economic growth. Fellman (2001) and Schroyen (2003) demonstrate the merits of a positive redistributive income tax without considering the role of education. Moreover, in modern economies that use a technology with little role for unskilled labor the growth maximizing policy would require a significantly positive degree of redistribution even when the education system may be marred with the poor quality public schools.

Under the education finance scheme, the CMV turns out to be infinity implying such policy to be harmful for growth under all circumstances. Clearly, poor quality education system does not serve as a useful vehicle for a redistributive policy to deliver growth. It turns out that this represents the only special case when CMV under education finance exceeds that under income tax such that redistribution is more likely to hurt growth under education finance than under income tax in a country where quality of education does not meet a minimum standard.

Example 4: $\lambda = 0$. This case is exactly what Benabou (2002) considers. We get two explicit formulas of the CMV for his model under the two different schemes of redistribution:

Under the income tax scheme:

$$(63)\quad \sigma^* = \frac{(1-\rho\alpha)(1-\alpha)(1-\mu)(1-\alpha-\beta\mu)(1+\alpha+\beta\mu)^2}{\eta\beta\mu^3(1-\rho(\alpha+\beta\mu))}.$$

Note that the CMV decreases with a lower intertemporal elasticity of labor ($\epsilon = \frac{1}{\eta-1}$), with a greater share of human capital μ in the production technology, with a better quality of education system β and as the CMV decreases, by Proposition 2, the likelihood of growth promoting potential of a redistributive policy increases. These findings are consistent with what Benabou (2002) reports.

Under the education finance scheme:

$$(64)\quad \sigma^* = \frac{\rho(1-\alpha)(1-\mu)(1-\alpha-\beta\mu)(1+\alpha+\beta\mu)^2}{\eta\mu^2(1-\rho(\alpha+\beta\mu))}.$$

Dividing (64) by (63), we get $\frac{\rho\beta\mu}{1-\rho\alpha}$ which is less than one. From that, we can infer that the likelihood of a redistributive policy to promote growth would be higher if it involves a progressive education subsidy than if it involves a progressive income tax rate. We extend this result in the following section also for the general case when all parameters have strictly positive values. One clear implication for this result is that the models such as the one used by EK are biased against finding growth potential of a redistributive policy than those such as Benabou which incorporates a progressive education subsidy as a part of redistribution.

These four examples illustrate that our understanding of how the economic fundamentals influence the value of CMV is quite important for policy designs. We now report findings from numerical simulations regarding the properties of the CMV and the growth maximizing rate of redistribution for the general case.

7. QUANTITATIVE ANALYSIS

7.1. *Parameter Values*

To simulate the value of CMV in the general case we need to assign values to parameters representing economic fundamentals. They are the output elasticity of unskilled labor ε and that of physical and human capital λ and μ, respectively, the 'neighborhood externality' parameter α, measured by the elasticity of children's human capital with respect to parent's human capital, the quality of education system β, measured by the elasticity of children's human capital with respect to education expenditure, agent's attitude towards patience ρ and intertemporal elasticity of labor supply ϵ which identifies the preference parameter η. In addition, to determine the growth maximizing degree of redistribution τ^*, one would also need to estimate the variance of the idiosyncratic shock σ^2 that reveals the extent of exogenous heterogeneity or in-born diversity among the people within the country.

Barro, Mankiw and Sala-i-Martin (1995), BMX from here on, provide estimates of the output share of labor ε and the output shares of physical and human capital, λ and μ, respectively. We use these values from BMX and values of other parameters from Benabou (2002) to re-examine growth-maximizing values of τ^* in Benabou's model. In the same vein, to re-examine the findings of Erosa and Koreshkova (2006) we derive the parameter values from various numbers reported by EK for their calibrated economy. For example, we take the value of capital share λ from EK and $1 - \lambda$ will be the share of human capital μ because in Erosa and Koreshkova (2006), they did not consider unskilled labor in the production technology. We also consider an extension of the EK model (EK*) with distortionary effect on the labor supply. In that economy, we choose the share of human capital μ to match the value of the laissez-faire investment rate s_1 to the share (3.8%) of private expenditure on education in GNP and then the share of labor ε equals $1 - \lambda - \mu$. EK estimate the time discount around 0.93 a year. Therefore, we set $\rho = 0.16$ for a generation (25 years) to correspond to EK's case. The estimates of the parameters for EK's human capital technology imply corresponding values for the elasticity of children's human capital with respect to parental human capital α and the elasticity of children's human capital with respect to expenditure on education β. The human capital technology in our model includes input of parental human capital but no input of parental time. In EK the time input adversely affects the beneficial role of redistribution. So our specification biases the EK model in favor of generating a positive effect of redistribution on growth.

EK use log utility without any allowance for leisure which can be thought of a special case of Benabou with $\eta = 0$. Alternatively, to interpret EK's utility in our general model we can set the value of $\eta = 2$ such that the intertemporal elasticity of substitution ($\epsilon = \frac{1}{\eta-1}$) equals its corresponding value in EK, which is unity. Another option that we consider is to set the value of the laissez-faire labor supply in our model to one third, which is the value used in EK. Given the calibrated values of other parameters, the above condition implies a value of $\eta = 1.56$. We consider all of these three cases.

The idiosyncratic shock follows a common lognormal distribution assumed by both Benabou and EK but the specific values of the mean and variance differ between Benabou and EK.

The following table provides a comparison of other parameter values.

TABLE I

BENCHMARK PARAMETERS

	Benabou with physical capital	EK	EK*(EK with unskilled labor)
λ	0.30	0.35	0.35
μ	0.50	0.65	0.50
ε	0.20	0.00	0.15
α	0.35	0.30	0.30
β	0.40	0.45	0.45
φ	-0.50	0.00	0.00
σ^2	1.00	0.94	0.94
η	6.00	0.00	2.00 or 1.56
ρ	0.40	0.16	0.16
κ	3.81	0.47	0.47

We note a small difference in technology between Benabou and the EK economy. Relative to other inputs the technology in EK values human capital more intensively than in Benabou. In EK* economy, we consider changing the balance changes in the other direction. Regarding human capital accumulation, Benabou's economy allows a larger extent of "neighborhood externality" α than EK and that hinders intergenerational mobility in Benabou relative to EK. Also, Benabou considers an education system of poorer quality β than EK and that reduces the beneficial impact of redistribution on intergenerational mobility in Benabou relative to EK which, however, do not allow for an education subsidy unlike Benabou. Consequently, a better quality of education in EK has little effect in raising the potency of a redistributive policy in fostering economic growth. Agents in Benabou show a greater degree of patience ρ and have a greater intertemporal elasticity of substitution of labor than in EK and that tend to raise the distortionary effect of redistribution on labor and the two forms of investments in Benabou relative to EK. In the EK* economy we, however, introduce a greater degree of distortionary effect of redistribution on labor supply than in Benabou.

We thus learn how the benchmark economies differ and could guess how these differences create biases in one way or the other. However, these comparisons do not help us to make a comparison of the overall bias of the EK relative to Benabou in determining the growth potency of a redistributive policy. Consequently, we argue that the comparison of the CMVs of these two economies would economize our time, since we would only need to compare one statistic which, by Proposition 2, sufficiently determines the relative growth potency of a redistributive policy for a given degree of heterogeneity in the population. With that insight, we now proceed to compare the growth maximizing redistributive policies in the above two economies and in other economies in the neighborhood of these two economies.

7.2. *Benabou's Economy in the Generalized Model*

Benabou (2002) "maximizes out" physical capital from the three factor production function that we consider in this paper to estimate parameters of his two factor model. Presumably a generalization of his model that works out the full implications of the underlying three factor production function would make little difference in terms of policy recommendations.

TABLE II

CMV AND OPTIMAL REDISTRIBUTION RATE τ

AUGMENTED BENABOU ECONOMY WITH BT($\sigma^2 = 1$)				
	σ_Y^*	τ_Y^*	σ_E^*	τ_E^*
No Subsidy	3.25	0.00	0.56	18.3%
Education Subsidy Only	1.71	0.00	0.30	33.1%
Both Education and Bequest Subsidies	0.21	26.6%	0.04	51.4%

From Table II, we can see that total income is maximized at about $\tau_Y^* = 26.6\%$ with income tax and transfer and at about $\tau_E^* = 51.4\%$ under education finance. The corresponding output gains with respect to laissez-faire are 4.5% and 5.8% respectively. These values are comparable to what Benabou (2002) reports.

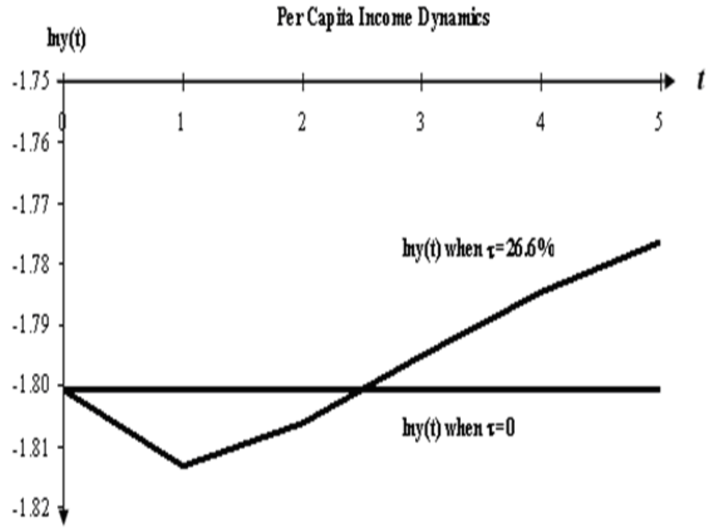

Figure 1: In the Benabou economy, redistribution with both subsidies accompanied by an optimal tax rate of 26.6% (see, Table II.) increases lny(t) also in the transition path.

However, Figure 1 illustrates a stark fact that, irrespective of the redistribution mechanism, with valued leisure the per capita output must fall in the first period and remain below the pre-existing level for more than two periods. Moreover, if the policy package includes only education subsidy then $CMV > \sigma^2$ and hence by Proposition 2 $\tau_Y^* = 0$. Consequently, by Proposition 3, redistribution would lower per capita income permanently. In other words, if bequest subsidy is not available then even in Benabou's economy, a positive redistribution from the rich to the poor would be harmful to growth. Moreover, if we consider a weight on leisure $\eta = 1.2$, which lies in the middle of various estimates used in the RBC literature, then even when both subsidies are applied, $CMV > \sigma^2$ such that redistribution would hurt growth both in the short run as well as in the long run. We also note that when redistributive policies do not accommodate any subsidy as in EK, $CMV > \sigma^2$. Consequently, Benabou's policy recommendation concurs with EK and that partially resolves their apparent conflict.

Redistribution with a progressive education subsidy promotes growth in Benabou's benchmark US economy whether or not there are subsidies. However, it may not be true in other countries. For example, if the technology of a country puts relatively less emphasis on skilled labor and more emphasis on unskilled labor (e.g., $\lambda = 0.30$, $\mu = 0.17$ and $\varepsilon = 0.53$) then even with both education and bequest subsidy and even under education finance the CMV exceeds unity making it greater than Benabou's assumed value of the variance of innate ability. Consequently, in such an economy redistribution would be harmful to economic growth.

Next we turn to understanding the policy implications for the EK economy.

7.3. ***EK Economy in the Generalized Model***

By using EK and EK*s parameter values, we can get positive optimal tax rates which are shown in Table III.

TABLE III

CMV AND OPTIMAL OPTIMAL REDISTRIBUTION RATE τ

EK and EK* ECONOMY WITH CE ($\sigma^2 = 0.94$)	EK		EK*	
	σ_Y^*	τ_Y^*	σ_Y^*	τ_Y^*
No Subsidy	1.39	0.0%	3.34	0.0%
Education Subsidy Only	0.64	4.8%	1.77	0.0%
Both Education and Bequest Subsidies	0.00	51.5%	0.04	39.6%

The optimal redistributive income tax rate is zero when, following EK, we do not allow any subsidy. This result provides a rationalization of EK's finding. Interestingly, however, if we modify EK's policy package to include education subsidy like Benabou then the CMV drops significantly to make the growth maximizing progressive tax rate (i.e., the average marginal tax rate) to be 4.8% and if we include both education and bequest subsidies then the optimal tax rate increases to 51.5% which is much higher than what we get in the extended Benabou model.

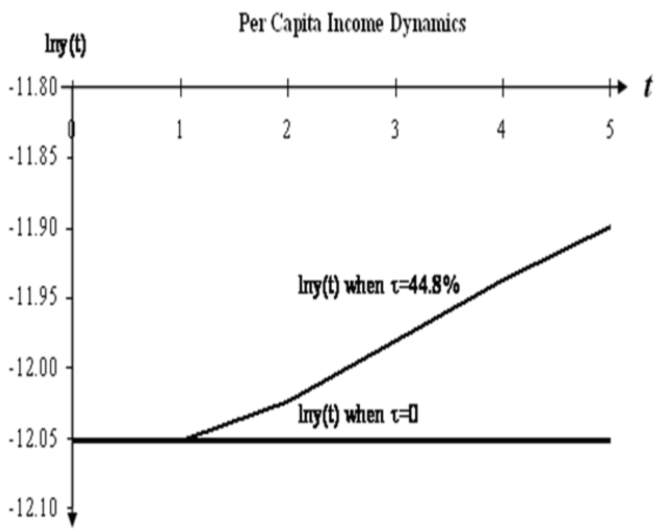

Figure 2: In the EK economy, redistribution with both subsidies accompanied by an optimal tax rate of 44.8% (see, Table III.) increases lny(t) also in the transition path.

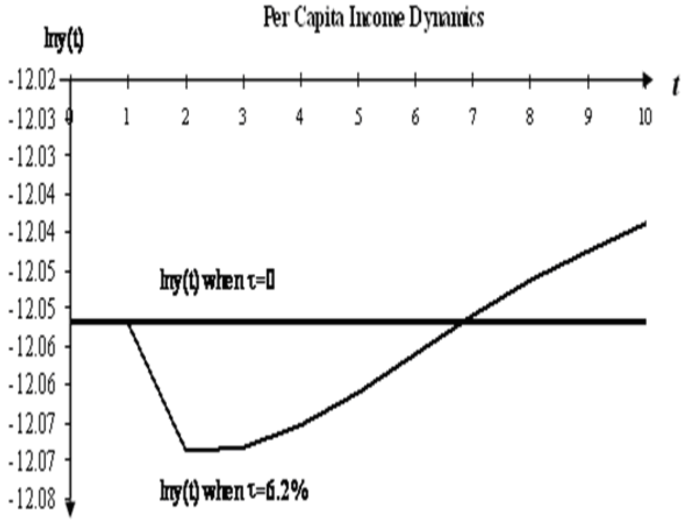

Figure 3: In the EK economy, redistribution with education subsidy alone with an optimal tax rate of 6.2% (see, Table III.) hurts growth in the first period but promotes it afterwards.

Moreover, we note from Figures 2 that along the transition path per capita income increases in all periods following the introduction of redistributive policy with two subsidies. When the policy package includes income tax and education subsidy only (see Figure 3) then per capita income drops immediately after the introduction of the policy but it increases and exceeds the level that it would be at with no redistribution after about six periods. In EK* economy, the optimal rate of redistribution could be as high as 39.6% when both subsidies are applied. In other words, the apparent conflict in the policy recommendations by Benabou and EK would disappear if the design of the optimal policy ignores short-run considerations as both of them do.

Next we examine what modification of the EK economy we can make simply by changing α or β such that redistribution would always help to promote growth in their model. We find that either when α is greater than 0.39 or β is greater than 0.55, σ_Y^* is less than σ^2.

TABLE IV

CMV AROUND EK* US ECONOMY ($\sigma^2 = 0.94$)

	$\sigma_Y^* \begin{pmatrix} \alpha = 0.55 \\ \mu = 0.52 \end{pmatrix}$		$\sigma_Y^* \begin{pmatrix} \beta = 0.65 \\ \mu = 0.55 \end{pmatrix}$	
No Subsidy	0.90	(+)	0.77	(+)
Education Subsidy Only	0.40	(+)	0.30	(+)
Both Education and Bequest Subsidies	0.01	(+)	0.01	(+)

Similarly, in the EK* economy with $\eta = 2$, Table IV shows that when $\alpha = 0.55$ and $\mu = 0.52$ we find that $\sigma_Y^* < \sigma^2$ whether or not we allow any subsidy.

Alternatively, a higher value of β implies a lower value of CMV. Table IV reports that when $\beta = 0.65$ and $\mu = 0.55$, $\sigma_Y^* < \sigma^2$ irrespective of the nature of the redistributive policy.

We conclude from the data presented in Table IV that an introduction of a strong neighborhood effect or a good quality public education system in an otherwise EK economy results in a clear call for redistribution as a growth promoting strategy. Thus the generalized model identifies a zone of agreement between Benabou and EK regarding policy recommendations.

7.4. Other Economies Around the Benchmark USA

We now consider minor variations of the parameter values used by EK and Benabou to represent other empirically reasonable economies in the vicinity of the benchmark US economy considered by them. By comparing the CMV of each of those economies and the variance of innate ability σ^2 in the economy's population, we could easily tell whether redistribution will hurt or promote economic growth. Policymakers may economize their time simply by focusing on a comparison of CMV and the σ^2 as a diagnostic test to check if a redistributive policy would be at all worthy of a consideration or not.

In this subsection, we examine the properties of CMV to determine how the likelihood of the growth-promoting potential of a redistributive policy varies across countries with different economic fundamentals. To characterize cross-country differences in CMV, we summarize our observations that appear to be robust under a large number of numerical simulations in the form of a few Propositions illustrated by Figures 4-8. The controlled values of the parameters that we report in each of these Figures 4-8 are generally around the estimates reported in Table I with minor exceptions which we make only to ensure that the range of CMVs in our illustrations nests the range of the variance of innate ability considered by EK and Benabou. However, the graphical profiles of the CMVs that characterize cross-country variations of a redistributive policy's effectiveness in promoting growth typically do not change their shapes as we consider other parameter values. In each figure we describe the variations of CMV both under income tax and under education finance but with education subsidy only (i.e., without the unrealistic bequest subsidy) which can be compared with various real world scenarios. One robust observation that we note from Figures 4, 6, 7 and 8 that CMV under income tax always exceed CMV under education finance. Consequently, a redistributive policy with a progressive education subsidy is more likely to be growth promoting than the same with a progressive income tax and transfer as considered in EK.

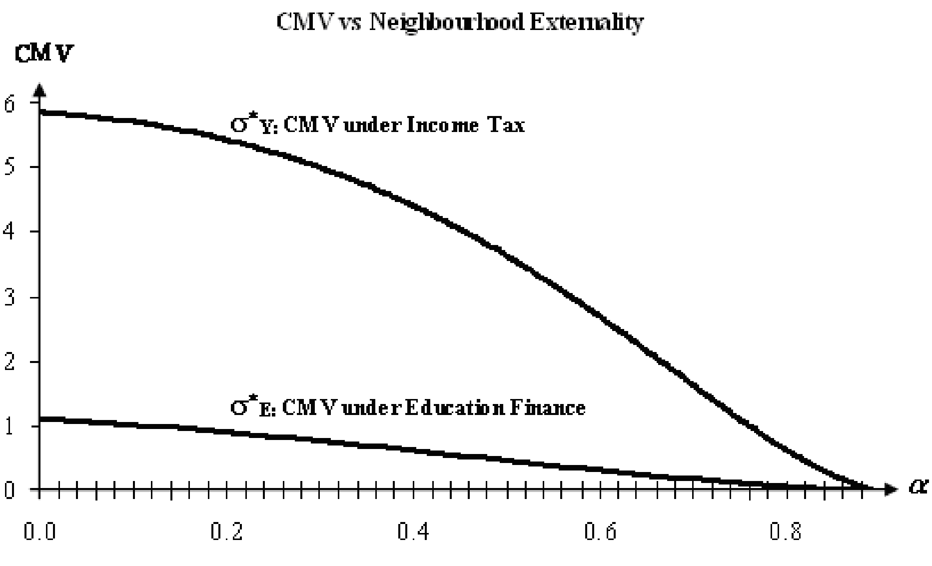

Figure 4: $\lambda = 0.3$, $\mu = 0.5$, $\varepsilon = 0.2$, $\beta = 0.15$, $\rho = 0.5$, $\eta = 1.5$.

We find (see Figure 4) that the CMV decreases with the parameter α that measures "neighborhood externality" of the type Benabou (2002) highlights. We interpret this parameter also to represent the degree of segregation in a society that prohibits interactions among families based on cast, creed, race, language and on other non-tradable characteristics that hinder spillover of knowledge across families. Because such segregation helps to confine knowledge within families and creates knowledge-gaps across communities, that could be exploited by suitable policies of redistribution to generate growth benefits for everyone.

Property 1: *A country with a relatively segregated society has a lower CMV than another that allows greater interactions across communities.*

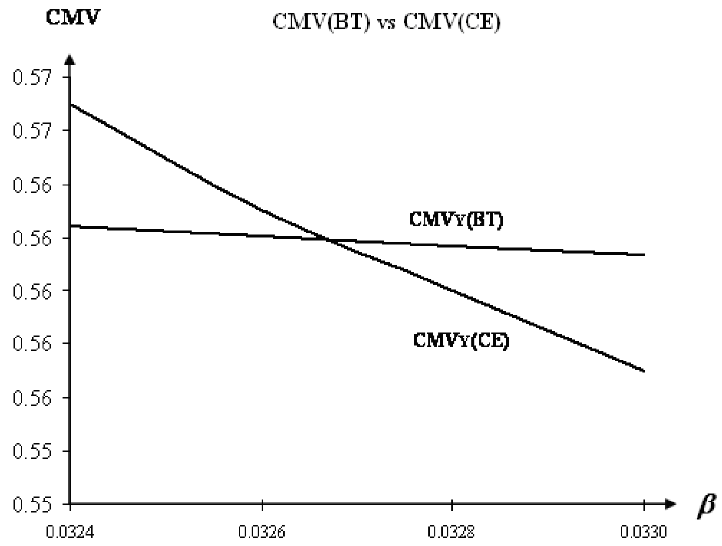

Figure 5: $CMV_Y\ (BT)$. and $CMV_Y\ (CE)$ with BMX parameters.

We also find (see Figure 5) that the CMVs under income tax regime either with a backyard technology or with a market for capital in a competitive environment decreases with β. Clearly, the higher quality of the education system the better is the potential for a growth promoting redistributive policy to be effective. We argue, therefore, if the education system in a country is marred by poor quality public schools then the CMV of the economy is likely to be large and, consequently, the likelihood of a redistributive policy to hurt growth would be high as well. Moreover, we detect a critical value for the quality of education such that if the quality lies above that standard then the growth-promoting potential of the income tax and transfer scheme would be significantly greater in a competitive environment than in a model without a market for capital, contrary to most other cases we discussed in this paper.

Property 2: *A country with a better quality education system has a lower CMV.*

By Properties 1 and 2, a redistributive policy would be more effective in promoting growth in a relatively segregated society and in a country with a better quality of education system.

Assuming the degree of heterogeneity as measured by σ^2 falls within the range [0.90, 1.00] that accommodates Benabou and EK, by Proposition 2 and Figures 4-5, it follows that in countries with $\alpha < 0.17$ and $\beta < 0.23$ redistribution either based on education finance or income tax would likely to hurt growth while in countries with $\alpha > 0.76$ and $\beta > 0.83$ redistribution of either type would likely to promote growth.

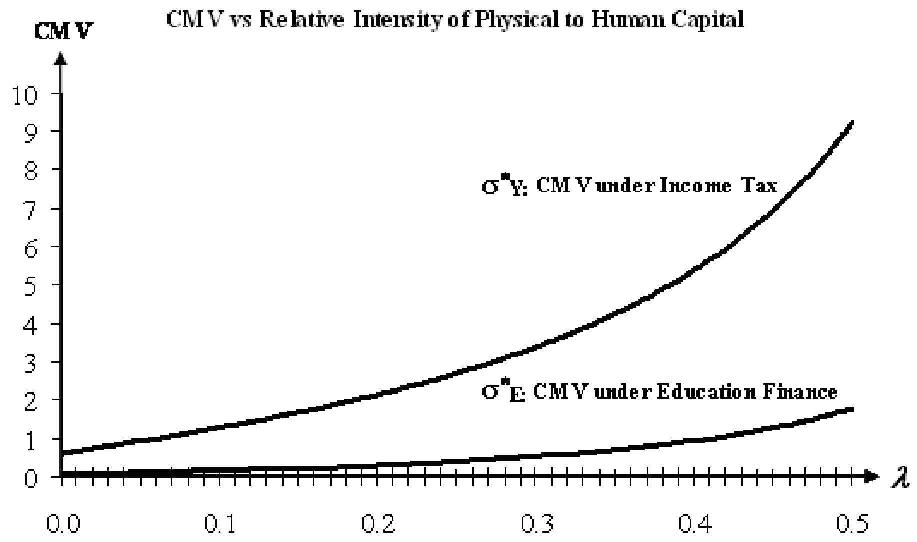

Figure 6: $\varepsilon = 0.2$, $\mu = 1 - \lambda - \varepsilon$, $\alpha = 0.25$, $\beta = 0.3$, $\rho = 0.4$, $\eta = 2$.

Figure 6 provides an illustration of a robust finding that given a fixed share of labor ε, the CMV increases as share of capital λ increases and the share of human capital μ decreases. This finding implies that if the technology of a country relies more on physical capital and less on human capital the likelihood of redistribution to hurt growth increases. Harmful effects of inequality arise primarily from the difference in the marginal product of human capital due to the complementarity between the child's innate ability shock and parental human capital. As the relative importance of human capital diminishes the growth augmenting "investment reallocation effect" from reduced inequality also diminishes. Consequently, a redistributive policy becomes less effective in countries which operate a machine oriented technology rather than a skill-intensive one.

Property 3: *A country that operates a relatively capital intensive technology rather than a skill-intensive technology tends to have a high value of CMV.*

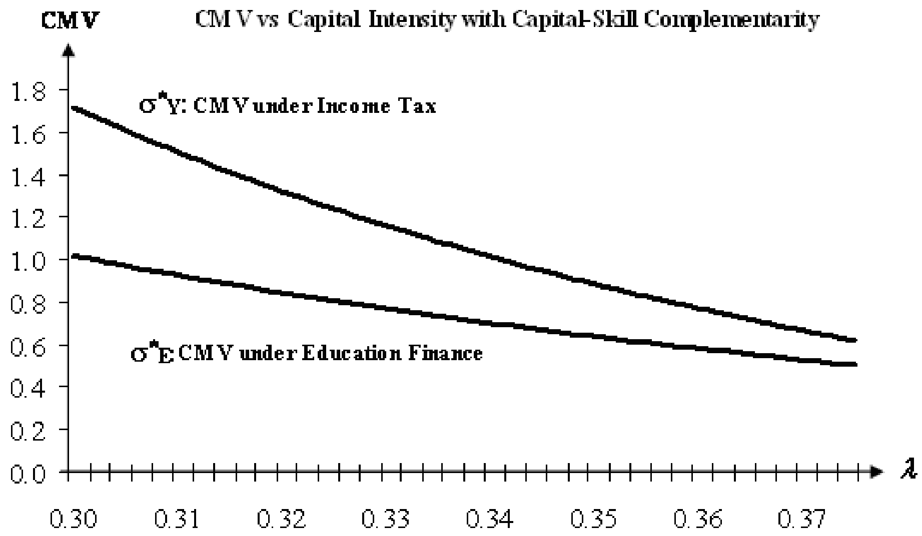

Figure 7: $\mu = 5/3{*}\lambda$, $\varepsilon = 1 - \lambda - \mu$. $\alpha = 0.35$, $\beta = 0.4$, $\rho = 0.4$, $\eta = 6$ for σ_Y^*; $\alpha = 0.1$, $\beta = 0.1$, $\rho = 0.5$, $\eta = 1.5$, for σ_E^*.

Next, we consider cross-country differences that arise from choice of technology and stages of development. Countries in their earlier stages of development typically rely on technology with a large intensity of unskilled labor. In the later stages of development it adopts technology that relies more intensely on machines and tools and requires skilled labor to operate those machines due to a hypothesize "capital-skill complementarity". In the process unskilled labor becomes less important and gets replaced by modern machines. To capture this hypothesized path of development, in our simulations, we decrease ε and increase λ and μ proportionately by keeping the ratio of the two capital shares same as implied by the BMX estimates. We then examine how CMVs differ across countries which are in different stages along this development path. We report a robust observation in Figure 7 that the CMV decreases as the technology replaces unskilled labor by machines accompanied by their skilled operators. We conclude, therefore, growth promoting potential of a redistributive policy increases as a country modernizes its technology along its hypothesized development path mentioned above.

Property 4: *In the presence of capital-skill complementarity, the less labor intensive a technology is the smaller would be the CMV of an economy.*

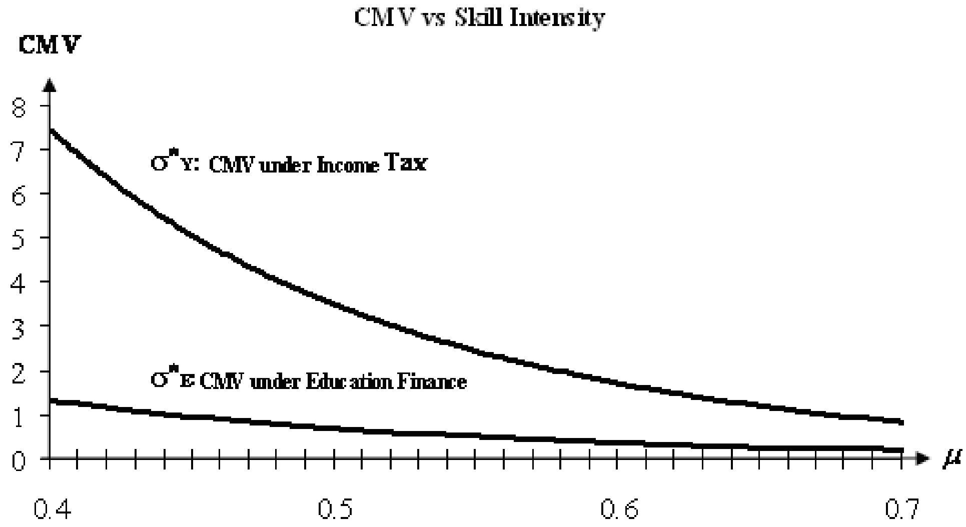

Figure 8: $\lambda = 0.3$, $\varepsilon = 1 - \lambda - \mu$, $\alpha = 0.3$, $\beta = 0.3$, $\rho = 0.5$, $\eta = 1.5$.

Figure 8 shows that given a class of technology with a constant output elasticity of physical capital, the CMV would be lower in countries with relatively high income share of human capital μ and with relatively low income share of unskilled labor ε. This feature creates an important difference in policy recommendations in two countries that place equal importance in the use of machines and tools and other capital goods but value skilled and unskilled labor differently. The one with a relatively skilled labor or human capital intensive technology would have a relatively low CMV and would find redistributive policies to be more likely to promote growth.

Property 5: *A country with relatively high skill intensity tends to have a low value of CMV.*

7.5. *CMV with A Competitive Market for Capital Goods*

The simulations of the benchmark economies presented in the Table V below, show that in a country where agents have a free access to a capital market it is more likely that $CMV_{CE}^{NS} \geq CMV_{BT}^{NS}$ than $CMV_{CE}^{ES} \geq CMV_{BT}^{ES}$. Consequently, the potential gains from reduction in inequality with redistribution are relatively small when agents can access a competitive market for capital and their investments are not subsidized. Clearly, the gains from redistribution decrease as trade friction decreases. Interestingly, however, simulations demonstrate without exception that the above inequality changes in the opposite direction, if both bequest and education subsidies are available such that $CMV_{CE}^{BS} < CMV_{BT}^{BS}$. We provide an explanation for this surprising result based on Lemma 4 as follows: the equilibrium interest rate in the CE case turns out to be higher than the same in the BT case. This interest rate premium that accompanies a competitive market for capital partially offsets the negative effect of income tax on labor supply. This phenomenon explains the results described in Lemma 4 as well as the fact that $CMV_{CE}^{BS} < CMV_{BT}^{BS}$. Consequently, when both subsidies are available, the presence of a competitive market for capital in the model increases the likelihood for a growth maximizing degree of progressivity to be strictly positive contrary to the case without subsidy that EK considered. Also, note that in the EK* economy $CMV_{CE}^{ES} < CMV_{BT}^{ES}$. The reason is similar. Moreover, if the technology does not value unskilled labor as in EK then $CMV_{CE}^{BS} = CMV_{BT}^{BS} = 0$. Consequently, redistribution would always promote growth in those environments. However, if the quality of the education system is very poor such that the value of β approaches zero then CMV_{CE}^{BS} jumps to infinity while CMV_{BT}^{BS} remain finite and is given by (62). Consequently, if a country has a sufficiently poor quality education system then redistribution would always hurt growth in a competitive equilibrium while it may not do so if the country also lacks a capital market and has a sufficiently diverse population.

TABLE V

Compare CMV_{BT} and CMV_{CE}

		CMV_{BT}	CMV_{CE}
BMX	No Subsidy	3.25	3.86
	Education Subsidy Only	1.71	1.91
	Both Education and Bequest Subsidies	0.21	0.06
EK*	No Subsidy	3.11	3.34
	Education Subsidy Only	1.79	1.77
	Both Education and Bequest Subsidies	0.34	0.04
EK	No Subsidy	1.22	1.39
	Education Subsidy Only	0.56	0.64
	Both Education and Bequest Subsidies	0.00	0.00

8. CONCLUSION

The paper makes three separate contributions.

First, it combines Benabou (2002) and Erosa and Koreshkova (2006), which emphasize complementary features of an economy, to develop a generalized framework for analyzing growth-promoting potentials of redistributive policies. Within that generalized framework it characterizes explicitly the condition when a government cannot successfully pursue a policy of redistribution to promote economic growth, either in the short run or in the long run. It also characterizes the condition when a redistributive policy package can promote economic growth at least in the long-run. Most importantly, it designs a new statistic that economizes policy analysis by summarizing the essential elements of a large number of multidimensional conditions into a single dimensional analysis. It offers an algorithm for the policy makers, interested in maximizing growth, to determine exactly when to redistribute and when not to redistribute.

Second, it reconciles the apparently conflicting policy recommendations and conflicting conclusions regarding the growth promoting potentials of redistributive policies between Benabou (2002) and Erosa and Koreshkova (2006). We provide comparisons of numerical simulations of the time series of per capita income with and without redistribution in different economies corresponding to those considered by Benabou (2002) and Erosa and Koreshkova (2006) approximately and with alternative policy packages. We demonstrate how two alternative specifications of the human capital technology can produce agreement between the policy recommendations by Benabou and EK.

Third, it demonstrates through numerical simulations how the growth-promoting potentials of a redistributive policy package vary across countries with different characteristics. Based on these simulation results we argue that governments of different countries, which pragmatically aim to maximize economic growth, could rationally disagree regarding the choice of the policy package for redistribution. We provide a handful of illustrations based on numerical simulations that highlight the importance of country specific differences in the design of growth enhancing policies. A large and complementary role of physical and human capital relative to unskilled labor in technology such as what we observe in a modern economy raises the growth-promoting potential of a redistributive policy and calls for redistribution. A greater "neighborhood-externality" and a better quality education system too call for a greater degree of redistribution.

In general, the optimal package for redistribution varies widely among economies with different fundamentals and with different degree of variability of the idiosyncratic shocks. A software tool based on our algorithm can help policymakers in different countries to identify conditions when Solow's vision of "growth with equity" would actually work.

APPENDIX

PROOFS OF LEMMAS 1, 2, 3 AND 5:

Case 1: For the case with backyard technology, by (2), (7), (8) and (9) we rewrite (13) as follows:

(A.1)

$$\ln U\left(k_t^i, h_t^i, M_t; T\right) = \max_{s_{1t}^i, s_{2t}^i, l_t^i} \left\{ \begin{array}{l} (1-\rho)\left[\ln\left(\left(1-s_{1t}^i-s_{2t}^i\right)/(1+\theta)\right)\right. \\ +(1-\tau)\left(\lambda \ln k_t^i+\mu \ln h_t^i+(1-\lambda-\mu)\ln l_t^i\right)+\tau \ln \tilde{y}_t-\left.\left(l_t^i\right)^{\eta}\right] \\ +\rho E_t\left[\ln U\left(k_{t+1}^i, h_{t+1}^i, M_{t+1}; T\right)\right] \end{array} \right\},$$

subject to

(A.2) $h_{t+1}^i = \kappa\left((1+d)\, s_{1t}^i\right)^{\beta} \xi_{t+1}^i \left(k_t^i\right)^{\beta\lambda(1-\tau)} \left(h_t^i\right)^{\alpha+\beta\mu(1-\tau)} \left(l_t^i\right)^{\beta(1-\lambda-\mu)(1-\tau)} \left(\tilde{y}_t\right)^{\beta\tau}$, and

(A.3) $k_{t+1}^i = (1+v)\, s_{2t}^i \left(k_t^i\right)^{\lambda(1-\tau)} \left(h_t^i\right)^{\mu(1-\tau)} \left(l_t^i\right)^{(1-\lambda-\mu)(1-\tau)} \left(\tilde{y}_t\right)^{\tau}$.

The first-order conditions of (A.1) with respect to the saving rates and labor supply are

(A.4) $$\frac{1-\rho}{1-s_{1t}^i-s_{2t}^i} = \rho\left[\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial s_{1t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial s_{1t}^i}\right],$$

(A.5) $$\frac{1-\rho}{1-s_{1t}^i-s_{2t}^i} = \rho\left[\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial s_{2t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial s_{2t}^i}\right],$$

(A.6)

$$(1-\rho)\,\eta\left(l_t^i\right)^{\eta} = (1-\rho)(1-\lambda-\mu)(1-\tau) + \rho\left\{\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial \ln l_t^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial \ln l_t^i}\right\},$$

where $\partial \ln k_{t+1}^i/\partial s_{1t}^i = 0$, $\partial \ln k_{t+1}^i/\partial s_{2t}^i = 1/s_{2t}^i$, $\partial \ln h_{t+1}^i/\partial s_{1t}^i = \beta/s_{1t}^i$, $\partial \ln h_{t+1}^i/\partial s_{2t}^i = 0$, $\partial \ln k_{t+1}^i/\partial l_{1t}^i = (1-\lambda-\mu)(1-\tau)/l_{1t}^i$ and $\partial \ln h_{t+1}^i/\partial l_{1t}^i = \beta(1-\lambda-\mu)(1-\tau)/l_{1t}^i$. We guess the value function as: $\ln U\left(k_t^i, h_t^i, M_t; T\right) = Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t$. Then by substituting this value function into (A.1), we get

(A.7)

$$Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t = (1-\rho)\begin{pmatrix} \ln\left(1-s_{1t}^i - s_{2t}^i\right)/(1+\theta) \\ +(1-\lambda-\mu)(1-\tau)\ln l_t^i + \tau\ln\tilde{y}_t - \left(l_t^i\right)^\eta \end{pmatrix}$$
$$+ (1-\rho+\rho\beta Z_1+\rho Z_2)\lambda(1-\tau)\ln k_t^i$$
$$+ ((1-\rho+\rho\beta Z_1+\rho Z_2)\mu(1-\tau)+\rho\alpha Z_1)\ln h_t^i$$
$$+\rho\left\{\begin{array}{l} Z_1\begin{pmatrix} \ln\kappa + \beta\ln(1+d)s_{1t}^i + \varphi \\ +\beta(1-\lambda-\mu)(1-\tau)\ln l_t^i + \beta\tau\ln\tilde{y}_t \end{pmatrix} \\ +Z_2\left(\ln(1+v)s_{2t}^i + (1-\lambda-\mu)(1-\tau)\ln l_t^i + \tau\ln\tilde{y}_t\right) + B_{t+1} \end{array}\right\}.$$

Taking partial differentials with respect to $\ln k_t^i$ and $\ln h_t^i$ yields

(A.8) $$Z_1 = (1-\rho+\rho\beta Z_1+\rho Z_2)\mu(1-\tau)+\rho\alpha Z_1$$

(A.9) $$Z_2 = (1-\rho+\rho\beta Z_1+\rho Z_2)\lambda(1-\tau)$$

Rearranging (A.8) and (A.9), we verify the guess and confirm the existence of (A.7) and thus Lemma 1 under BT is established.

Note that the above optimization problem (A.7) is strictly concave. Consequently, (A.4)—(A.6) are sufficient for the optimization exercise and the Lemmas 2 and 5 follow immediately after we substitute (14) and (15) into (A.4)—(A.6).

Case 2: For the case with a market for capital in a competitive environment, by (7), (8) and (9), we rewrite (13) as follows:

(A.10) $$\ln U\left(k_t^i, h_t^i, M_t; T\right) = \max_{s_{1t}^i, s_{2t}^i, l_t^i}\left\{\begin{array}{l} (1-\rho)\left[\ln\left(1-s_{1t}^i-s_{2t}^i\right) - \ln(1+\theta) + \ln\hat{y}_t^i - \left(l_t^i\right)^\eta\right] \\ +\rho E_t\left[\ln U\left(k_{t+1}^i, h_{t+1}^i, M_{t+1}; T\right)\right] \end{array}\right\}$$

subject to

(A.11) $$\ln k_{t+1}^i = \ln(1+\nu)s_{2t}^i + (1-\tau)\ln\chi_t^i + \ln y_t + \tau(1-\tau)\Delta_{yt}^2/2,$$ and

(A.12) $$\ln h_{t+1}^i = \ln\kappa + \ln\xi_{t+1}^i + \alpha\ln h_t^i + \beta\begin{pmatrix} \ln(1+d)s_1 + \tau\ln\tilde{y}_t \\ +(1-\tau)\left(\ln y_t + \ln\chi_t^i\right) \end{pmatrix}$$

We guess that each agent provides the same level of labor, i.e., $\tilde{l}_t^i = \tilde{l}_t$. Consequently, the first-order condition of (A.10) with respect to labor supply yields

(A.13) $$(1-\rho)\eta\left(l_t^i\right)^{\eta-1} = (1-\rho)\frac{\partial\ln\hat{y}_t^i}{\partial\ln l_t^i} + \rho\left(\frac{\partial\ln U_{t+1}^i}{\partial\ln h_{t+1}^i}\frac{\partial\ln h_{t+1}^i}{\partial\ln l_t^i} + \frac{\partial\ln U_{t+1}^i}{\partial\ln k_{t+1}^i}\frac{\partial\ln k_{t+1}^i}{\partial\ln l_t^i}\right),$$

where $\partial\ln\hat{y}_t^i/\partial l_{1t}^i = \varepsilon/l_t^i$, $\partial\ln h_{t+1}^i/\partial\ln l_t^i = \beta\varepsilon/l_t^i$, $\partial\ln k_{t+1}^i/\partial\ln l_t^i = \varepsilon/l_t^i$. We guess the value function as: $\ln U\left(k_t^i, h_t^i, M_t; T\right) = Z_1\ln h_t^i + Z_2\ln k_t^i + B_t$. Then substituting this value function into (A.10), we get

(A.14)

$$Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t = \begin{array}{l} \rho \left(Z_1 \left(\ln \kappa + \beta \ln(1+d) + \varphi \right) + Z_2 \ln \left(1+v\right) \right) - (1-\rho) \ln \left(1+\theta\right) \\ +(1-\rho) \ln \left(1 - s_{1t}^i - s_{2t}^i\right) + \rho \left(\beta Z_1 \ln s_{1t}^i + Z_2 \ln s_{2t}^i \right) \\ + \left(1 - \rho + \rho \left(\beta Z_1 + Z_2\right)\right) \tau \left(1-\tau\right) \Delta_{yt}^2 / 2 \\ + \left(1 - \rho + \rho \left(\beta Z_1 + Z_2\right)\right) \left(\lambda \ln k_t + (1-\lambda) \ln \left(\tilde{h}_t \right) \right) \\ + \left(1 - \rho + \rho \left(\beta Z_1 + Z_2\right)\right) \varepsilon \ln l_t^i - (1-\rho) \left(l_t^i \right)^\eta \\ + \left(1 - \rho + \rho\beta Z_1 + \rho Z_2\right) (1-\tau) \ln \left(\lambda \left(\frac{k_t^i}{k_t} \right) + (1-\lambda) \left(\frac{\tilde{h}_t^i}{\tilde{h}_t} \right) \right) \\ + \rho\alpha Z_1 \ln h_t^i + \rho B_{t+1}. \end{array}$$

where $\tilde{h}_t \equiv \int_0^1 \tilde{h}_t^i di$. Taking partial differentials with respect to $\ln k_t^i$ and $\ln h_t^i$ yield

$$\text{(A.15)} \quad (1-\rho\alpha) Z_1 \left(\lambda \left(\frac{k_t^i}{k_t} \right) + (1-\lambda) \left(\frac{\tilde{h}_t^i}{\tilde{h}_t} \right) \right) = \left(1 - \rho + \rho\beta Z_1 + \rho Z_2\right) (1-\tau) \mu \left(\frac{\tilde{h}_t^i}{\tilde{h}_t} \right),$$

$$\text{(A.16)} \quad Z_2 \left(\lambda \left(\frac{k_t^i}{k_t} \right) + (1-\lambda) \left(\frac{\tilde{h}_t^i}{\tilde{h}_t} \right) \right) = \left(1 - \rho + \rho\beta Z_1 + \rho Z_2\right) (1-\tau) \lambda \left(\frac{k_t^i}{k_t} \right).$$

Taking integral of above two equations and rearranging yield the same formulas as (A.8) and (A.9). Then we verify the guess and confirm the existence of (A.14). Thus the Lemma 1 under CE is established.

The first-order conditions of (A.10) with respect to the saving rates yield the same format as (A.4) and (A.5). It follows, therefore, unlike the formula for labor, the formula for optimal saving rates don't vary between BT and CE. Note also the above optimization problem (A.14) is strictly concave. Consequently, (A.4), (A.5) and (A.13) are sufficient for the optimization exercise and Lemmas 3 and 5 follow immediately after we substitute (14) and (15) into (A.13). The guess that each agent provides the same level of labor is also verified. □

PROOFS OF LEMMAS 6 AND 7: By assumption, at the initial date $t = 0$, physical and human capitals are lognormally distributed. By (27) and (28), it follows, therefore, that k_t^i and h_t^i remain lognormally distributed over time. Taking logarithm on (2) yields

$$\text{(A.17)} \quad \ln y_t^i = \lambda \ln k_t^i + \mu \ln h_t^i + (1-\lambda-\mu) \ln l_t^i.$$

By (16), it follows that the mean of $\ln y_t^i$ is

$$\text{(A.18)} \quad E_t \left[\ln y_t^i \right] = \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu) \ln l.$$

The variance of $\ln y_t^i$ is the sum of variances of $\ln k_t^i$, $\ln h_t^i$ plus the covariance of these two variables

$$\text{(A.19)} \quad \text{var} \left[\ln y_t^i \right] = \lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t.$$

The mean of y_t^i, following Crow and Shimizu (1988)'s description about properties of moment generating function on lognormal distribution on page 9, is

$$\text{(A.20)} \quad y_t = E_t \left[y_t^i \right] = \exp \left(E_t \left[\ln y_t^i \right] + \frac{1}{2} \text{var} \left[\ln y_t^i \right] \right).$$

The median of y_t^i is

(A.21) $y_{t,median} = \exp\left(E_t\left[\ln y_t^i\right]\right).$

Therefore, the income inequality Λ_t which, following Benabou (2002), is defined as the ratio of mean to median income in logarithm is

(A.22) $\log\left(\frac{y_t}{y_{t,median}}\right) = \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right] = \left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2 \equiv \Lambda_t.$

This proves Lemma 6.

To prove Lemma 7, we proceed by taking logarithm of (A.20) to get

(A.23) $\ln y_t = \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l + \Lambda_t.$

Similarly, the mean of $\left(y_t^i\right)^{1-\tau}$ is

$$\ln E_t\left[\left(y_t^i\right)^{1-\tau}\right] = (1-\tau)E_t\left[\ln y_t^i\right] + \frac{(1-\tau)^2}{2}\operatorname{var}\left[\ln y_t^i\right],$$

$\Rightarrow$

(A.24) $\ln E_t\left[\left(y_t^i\right)^{1-\tau}\right] = (1-\tau)\left(\lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l\right) + (1-\tau)^2\Lambda_t.$

Taking the difference between before and after tax income yields

(A.25) $\ln y_t - \ln E_t\left[\left(y_t^i\right)^{1-\tau}\right] = \lambda\tau m_{kt} + \mu\tau m_{ht} + (1-\lambda-\mu)\tau\ln l + \tau(2-\tau)\Lambda_t.$

It means that $\tau\ln\tilde{y}_t = \lambda\tau m_{kt} + \mu\tau m_{ht} + (1-\lambda-\mu)\tau\ln l + \tau(2-\tau)\Lambda_t$, then we can get (35).

Expectation of next period individual income is:

(A.26)
$$\begin{aligned} E_t\left[\ln y_{t+1}^i\right] &= \psi + \mu\varphi + (\lambda+\beta\mu)\tau\ln\tilde{y}_t - \alpha\lambda\tau\ln\tilde{y}_{t-1} + (1-\alpha)(1-\lambda-\mu)\ln l \\ &\quad + (\alpha + (\lambda+\beta\mu)(1-\tau))E_t\left[\ln y_t^i\right] - \alpha\lambda(1-\tau)E_t\left[\ln y_{t-1}^i\right], \end{aligned}$$

where ψ is the same as that in (29).

From (A.20), we know

(A.27) $E_t\left[\ln y_t^i\right] = \ln E_t\left[y_t^i\right] - \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right].$

Combining (A.27) with (A.26) yields:

(A.28)

$$\ln E_t\left[y_{t+1}^i\right]-\frac{1}{2}\operatorname{var}\left[\ln y_{t+1}^i\right]=\psi+\mu\varphi+(1-\alpha)(1-\lambda-\mu)\ln l$$
$$+(\lambda+\beta\mu)\tau\ln\tilde{y}_t-\alpha\lambda\tau\ln\tilde{y}_{t-1}$$
$$+(\alpha+(\lambda+\beta\mu)(1-\tau))\left(\ln E_t\left[y_t^i\right]-\frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]\right)$$
$$-\alpha\lambda(1-\tau)\left(\ln E_t\left[y_{t-1}^i\right]-\frac{1}{2}\operatorname{var}\left[\ln y_{t-1}^i\right]\right).$$

Substituting (35) into (A.28) yields (36). This proves Lemma 7. □

PROOF OF PROPOSITION 1: Writing the system of linear equations (30) to (34) in a matrix form, we get

(A.29) $M_{t+1}=A_0+A_1 * M_t$,

where

$$M_{t+1}\equiv\begin{bmatrix} m_{kt+1}\\ m_{ht+1}\\ \Delta_{kt+1}^2\\ \Delta_{ht+1}^2\\ \text{cov}_{t+1}\end{bmatrix},\ A_0\equiv\begin{bmatrix}\ln\bar{s}_2+(1-\lambda-\mu)\ln l\\ \ln\kappa+\varphi+\beta\ln\bar{s}_1+\beta(1-\lambda-\mu)\ln l\\ 0\\ \sigma^2\\ 0\end{bmatrix},$$

$$A_1\equiv\begin{bmatrix}\lambda & \mu & \tau(2-\tau)\lambda^2/2 & \tau(2-\tau)\mu^2/2 & \tau(2-\tau)\lambda\mu\\ \beta\lambda & \alpha+\beta\mu & \beta\tau(2-\tau)\lambda^2/2 & \beta\tau(2-\tau)\mu^2/2 & \beta\tau(2-\tau)\lambda\mu\\ 0 & 0 & (1-\tau)^2\lambda^2 & (1-\tau)^2\mu^2 & 2\lambda\mu(1-\tau)^2\\ 0 & 0 & (1-\tau)^2\beta^2\lambda^2 & (\alpha+\beta\mu(1-\tau))^2 & 2\beta\lambda(1-\tau)(\alpha+\beta\mu(1-\tau))\\ 0 & 0 & (1-\tau)^2\beta\lambda^2 & \mu(1-\tau)(\alpha+\beta\mu(1-\tau)) & \lambda(1-\tau)(\alpha+2\beta\mu(1-\tau))\end{bmatrix}.$$

The sequence M_t converges to a unique steady state if and only if all eigenvalues of A_1, denoted as $S(A_1)$, are less than one such that M solves the following fixed point problem

(A.30) $M=A_0+A_1 * M$.

To prove convergence, we solve $\det|A_1-S(A_1)I|=0$, where I is identity matrix, and get

$$S(A_1)=\begin{bmatrix}\lambda\\ \alpha+\beta\mu\\ \lambda^2(1-\tau)^2\\ (\alpha+\beta\mu(1-\tau))^2\\ \lambda(1-\tau)(\alpha+2\beta\mu(1-\tau))\end{bmatrix}.$$

By assumption $\lambda<1$ and $(1-\alpha)(1-\lambda)-\beta\mu>0$, which implies $\alpha+\beta\mu(1-\tau)<1$ and that implies $\alpha + 2\beta\mu(1-\tau) < 1 + \beta\mu(1-\tau)$. Also $\lambda + \mu < 1$ implies $\lambda(1+\beta\mu(1-\tau)) < (1-\mu)(1+\beta\mu(1-\tau)) < 1$. It follows therefore, that $\lambda(1-\tau)(\alpha+2\beta\mu(1-\tau)) < 1$. Thus, $S(A_1) < 1$, and that proves convergence of M_t. Moreover, $S(A_1) < 1$ also implies $I - A_1$ is nonsingular. Consequently, (A.30) has a unique solution. Or, equivalently, a unique steady state exists.Non-singularity of $I - A_1$ also implies A_1 is a monotone matrix and hence $\{M_t\}$ constitutes a monotone sequence. □

PROOFS OF LEMMA 8: Similarly, in the education finance scheme, we guess the value function $U\left(k_t^i, h_t^i, M_t; T\right)$ is still in the form: $\ln U\left(k_t^i, h_t^i, M_t; T\right) = Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t$. Substituting into (44) yields

(A.31)

$$\begin{aligned} Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t = \;& \rho\left(Z_1\left(\ln\kappa + \beta\ln(1+d) + \varphi\right) + B_{t+1}\right) - (1-\rho)\ln(1+\theta) \\ & + (1-\rho)\ln\left(1 - s_{1t}^i - s_{2t}^i\right) + \rho\left(\beta Z_1 \ln(1+d)\, s_{1t}^i + Z_2 \ln(1+v)\, s_{2t}^i\right) \\ & + \rho\beta Z_1 \tau \ln\tilde{y}_t + \left(1 - \rho + \rho\beta Z_1(1-\tau) + \rho Z_2\right)(1-\lambda-\mu)\ln l_t^i - (1-\rho)\left(l_t^i\right)^\eta \\ & + \left(1 - \rho + \rho\beta Z_1(1-\tau) + \rho Z_2\right)\lambda \ln k_t^i \\ & + \left(\left(1 - \rho + \rho\beta Z_1(1-\tau) + \rho Z_2\right)\mu + \rho\alpha Z_1\right)\ln h_t^i. \end{aligned}$$

Taking partial differentials of (A.31) with respect $\ln k_t^i$ and $\ln h_t^i$ yield

(A.32) $$Z_1 = \frac{(1-\rho)\mu}{(1-\rho\lambda)(1-\rho\alpha) - \rho\beta\mu(1-\tau)},$$

(A.33) $$Z_2 = \frac{(1-\rho\alpha)(1-\rho)\lambda}{(1-\rho\lambda)(1-\rho\alpha) - \rho\beta\mu(1-\tau)}.$$

The differences between (14), (15) and (A.32), (A.33) are the absence of $1-\tau$ multiplying

numerator and $\rho\lambda$ in the denominator. First-order conditions for the saving rates are unchanged and still given by equations (A.4) and (A.5). For the labor supply, it becomes

(A.34) $$(1-\rho)\eta\left(l_t^i\right)^\eta = (1-\rho)(1-\lambda-\mu) + \rho\left(\frac{\partial\ln U_{t+1}^i}{\partial\ln h_{t+1}^i}\frac{\partial\ln h_{t+1}^i}{\partial\ln l_t^i} + \frac{\partial\ln U_{t+1}^i}{\partial\ln k_{t+1}^i}\frac{\partial\ln k_{t+1}^i}{\partial\ln l_t^i}\right).$$

Strict concavity of (A.31) implies that (A.4), (A.5) and (A.34) are sufficient for optimality and thus Lemma 8 is established. □

PROOF OF LEMMA 9: By using the property of moment generating function on log-normal distribution, we could get $\ln\tilde{y}_t$ with the same format of function as (35) except that Λ_t is replaced by Λ_t^E. In the education finance scheme, human capital accumulation takes the same format as (28) in the redistributive income taxation scheme while the function of physical capital accumulation is from (48). Then the expectation of next period individual income is

(A.35)

$$\begin{aligned} E_t\left[\ln y_{t+1}^i\right] = \;& \psi + \mu\varphi + \beta\mu\tau\ln\tilde{y}_t - \alpha\lambda\tau\ln\tilde{y}_{t-1} + (1-\alpha)(1-\lambda-\mu)\ln l \\ & + \left(\alpha + \lambda + \beta\mu(1-\tau)\right)E_t\left[\ln y_t^i\right] - \alpha\lambda E_t\left[\ln y_{t-1}^i\right]. \end{aligned}$$

Substituting (35) into the above function and combining with (A.27) yield (53). □

PROOF OF PROPOSITION 3: By (32) and (30) under income tax regime or by (50) under education finance regime following the introduction of redistributive policies labor supply and hence per capita income decreases contemporaneously while the mean of human and physical capital changes, if at all, only after one period. Also, if $\sigma^2 < \sigma^*$ then, by Proposition 2, $\lim_{t\to\infty} \ln y_t(\tau) < \ln y(0)$. Consequently, following redistribution per capita output must remain below the pre-redistribution level for the first and last few periods. Finally, by Proposition 1, following the initial decrease, the per capita income changes monotonically, i.e., it either continues to decrease or increases monotonically. It follows, therefore, that in no date $t > 0$ the per capita income can exceed $y(0)$ from below and Proposition 3 is proved. □

PROOF OF PROPOSITION 4: (60) gives the general formula of CMV. By comparing the values of CMV under different redistributive policy packages we note that the numerator of (60) with both subsidy, denoted as N_{BS}, equals $\frac{1}{\eta}\frac{(1-\alpha)(1-\lambda-\mu)(1-\rho\alpha)}{(1-\rho\alpha)(1-\rho\lambda)-\rho\beta\mu}$. The same with only education subsidy, denoted by N_{ES}, equals to $N_{BS}+\lambda(1-\alpha)$ and the one with no subsidy, $N_{NS} = N_{ES} + \beta\mu$. Consequently, $N_{NS} \geq N_{ES} \geq N_{BS}$ since λ, μ, α and β are non-negative and hence $CMV_{NS} \geq CMV_{ES} \geq CMV_{BS}$. □

Competitive Equilibrium: Key Equations

The logarithm of (9), combining with (6) and (10) yields the dynamics of physical capital for the dynasty i,

(A.36) $\ln k^i_{t+1} = \ln(1+\nu)\, s^i_{2t} + (1-\tau)\ln\chi^i_t + \ln y_t + \tau(1-\tau)\,\Delta^2_{yt}/2$

The logarithm of (8), combining with (6) and (10) yields

(A.37) $\ln h^i_{t+1} = \ln\kappa + \ln\xi^i_{t+1} + \alpha\ln h^i_t + \beta(\ln(1+d)s_1 + \tau\ln\tilde{y}_t + (1-\tau)(\ln y_t + \ln\chi^i_t))$

Given the initial lognormal distribution, by (A.36) and (A.37) physical and human capital and income remain lognormally distributed over time[9] such that at each date t, state variables M_t under CE satisfies

(A.38) $m_{kt+1} = \ln(1+\nu)\, s_2 + \ln y_t - (1-\tau)^2\,\Delta^2_{yt}/2$

(A.39) $\Delta^2_{kt+1} = (1-\tau)^2\,\Delta^2_{yt}$

[9]The sum of dependent lognormal random variables have been discussed at some length in the literature, e.g., Naus (1969, 1973), Abu-Dayya and Beaulieu (1994) and Yue (2000), but no closed form is known for their distribution. Therefore, we need an assumption that the sum of k^i_t and $\tilde{h}^i_t$ is approximately lognormal. This approximation is not correct in the strict mathematical sense, it is empirically proved that individual income is distributed lognormally approximately, e.g., Roy (1950), Aitchison and Brown (1957) and Fonseca and Tayman (1989).

(A.40)
$$m_{ht+1} = \ln \kappa + \varphi + \beta \ln(1+d)s_1 + \alpha m_{ht}$$
$$+\beta \left((1-\tau) \left(\ln y_t - \frac{1}{2}\Delta_{yt}^2 \right) + \tau \ln \tilde{y}_t \right)$$

(A.41) $\Delta_{ht+1}^2 = \sigma^2 + \alpha^2 \Delta_{ht}^2 + \beta^2 (1-\tau)^2 \Delta_{yt}^2 + 2\alpha\beta (1-\tau) COV_{\ln ht, \ln \chi t}$

(A.42) $cov_{t+1} = \alpha (1-\tau) COV_{\ln ht, \ln \chi t} + \beta (1-\tau)^2 \Delta_{yt}^2,$

where $COV_{\ln ht, \ln \chi t}$ denotes the covariance between $\ln h_t^i$ and $\ln \chi_t^i$. By the property of the moment generating for the bivariate normal distribution, we have

(A.43) $COV_{\ln ht, \ln \chi t} = \ln \left(\lambda e^{cov_t} + (1-\lambda) e^{\frac{\mu}{1-\lambda}\Delta_{ht}^2} \right).$

Because of the assumption that the sum of k_t^i and $\tilde{h}_t^i$ is approximately lognormal, i.e., $\ln \chi_t^i$ distributes normally, Δ_{yt}^2 denotes the variance of $\ln \chi_t^i$. By the property of moment generating function on lognormal distribution, we have

(A.44) $$\Delta_{yt}^2 = \ln \begin{pmatrix} 1 + \left(\frac{\lambda}{e^{m_{kt}+\frac{1}{2}\Delta_{kt}^2}} \right)^2 \sigma_{kt}^2 + \left(\frac{1-\lambda}{e^{\frac{\mu}{1-\lambda}\left(m_{ht}+\frac{\mu}{1-\lambda}\Delta_{ht}^2/2\right)}} \right)^2 \sigma_{\tilde{h}t}^2 \\ + \frac{2\lambda(1-\lambda)}{e^{m_{kt}+\frac{1}{2}\Delta_{kt}^2+\frac{\mu}{1-\lambda}\left(m_{ht}+\frac{\mu}{1-\lambda}\Delta_{ht}^2/2\right)}} \sigma_{kt,\tilde{h}t} \end{pmatrix},$$

where σ_{kt}^2, $\sigma_{\tilde{h}t}^2$ and $\sigma_{kt,\tilde{h}t}$ denote the variance of k_t^i, $\tilde{h}_t^i$ and covariance between k_t^i and $\tilde{h}_t^i$. Then, by the property of moment generating function on lognormal distribution, it is shown as follows

(A.45) $\sigma_{kt}^2 = \left(e^{\Delta_{kt}^2} - 1 \right) e^{2m_{kt}+\Delta_{kt}^2},$

(A.46) $\sigma_{\tilde{h}t}^2 = \left(e^{\left(\frac{\mu}{1-\lambda}\right)^2 \Delta_{ht}^2} - 1 \right) e^{2\frac{\mu}{1-\lambda}m_{ht}+\left(\frac{\mu}{1-\lambda}\right)^2 \Delta_{ht}^2},$

(A.47)
$$\sigma_{kt,\tilde{h}t} = \exp \left(m_{kt} + \left(\frac{\mu}{1-\lambda} \right) m_{ht} + \frac{1}{2} \left(\Delta_{kt}^2 + \left(\frac{\mu}{1-\lambda} \right)^2 \Delta_{ht}^2 \right) \right) \left(\exp \left(\frac{\mu}{1-\lambda} cov_t \right) - 1 \right).$$

By using the same idea as discussed under backyard technology, therefore, the break-even income under CE is

(A.48) $\tau \ln \tilde{y}_t = \tau \ln y_t + \tau (1-\tau) \Delta_{yt}^2/2$

Note equations (A.38) - (A.48) constitutes a system of 11 equations and 11 variables (m_{kt+1}, m_{ht+1}, Δ_{kt+1}^2, Δ_{ht+1}^2, cov_{t+1}, Δ_{yt}^2, $COV_{\ln ht, \ln \chi t}$, σ_{kt}^2, $\sigma_{\tilde{h}t}^2$, $\sigma_{kt,\tilde{h}t}$, $\ln \tilde{y}_t$) which we solve using MatLab. We find that the system always converges to a unique steady state supporting Proposition 1. Also, numerical simulations show that Propositions 2 and 3 hold without an exception. In particular, by plotting the derivative of $\ln y(\tau)$ with respect to τ at $\tau = 0$ as a function of σ^2, we derive σ^* numerically as follows. We define $g(\sigma^2) \equiv \frac{\partial \ln y(\tau)}{\partial \tau} |_{\tau=0}$, such that σ^* satisfies $g(\sigma^2) = 0$.

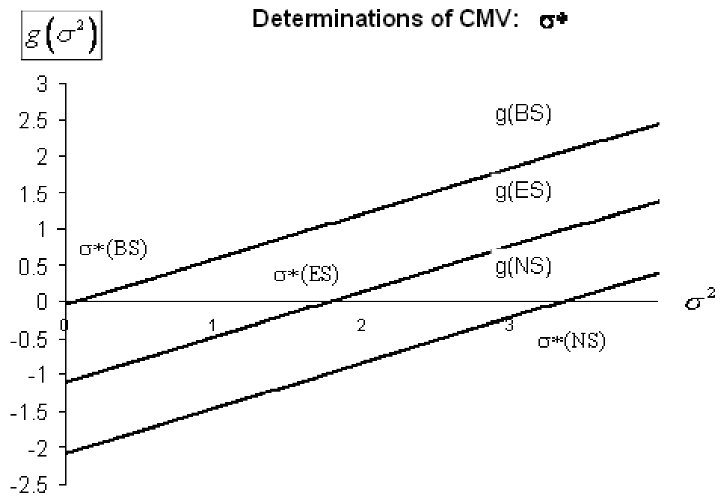

Figure 6: Determinations of CMV for EK* economy with CE.

From above Figure, for example we see for the EK* economy, $g(0) < 0$ and $g' > 0$. It follows therefore, that there exists a $\sigma^* > 0$. Moreover, for any given $\sigma^2 > 0$, $g_{BS}(\sigma^2) > g_{ES}(\sigma^2) > g_{NS}(\sigma^2)$. Consequently, $\sigma^*_{NS} > \sigma^*_{ES} > \sigma^*_{BS}$.

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