Price Competition when Not All Customers Know of All Firms*

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Abstract

We examine a duopoly pricing game where some customers know only of firm 1, others know only of firm 2, and some know of both firms. Firms have constant and identical marginal costs, sell homogenous goods and choose prices simultaneously. Customers observe the prices of the firms that are known to them, and they have a unitary demand. We show that there is no equilibrium in pure price strategies for this game. We find the unique mixed strategy equilibrium, and show that it has intuitively appealing comparative static properties. We then examine the two stage game in which firms advertise in stage 1 to create their customer bases, and in stage 2 play the pricing game described above. The equilibrium to the two stage game is asymmetric, and far from the Bertrand equilibrium.

Keywords: Bertrand paradox, continuous mixed strategies, imperfect information, duopoly, advertising

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*Quality assured

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1 Introduction

If one is to choose to buy some good from a firm, or choose to not buy it from the firm because there is another firm with a lower price, one must know of the firm – that is, one must be aware that the firm exits. Indeed, one of the more important functions of a firm's advertising effort would seem to be to make its potential customers aware of its existence. If we suppose that it is costly for firms to make potential customers aware of their existence and/or that it is costly for potential customers to become aware of the existence of firms, it would seem likely that not all potential customers are aware of all firms and therefore that firms have some captive consumers giving rise to market power. This, as far as we can discern, has not been directly addressed in the literature of information economics.

We tackle the problem in a very simple model in Section 2. There are two firms, they have identical constant marginal costs, and they compete in prices. Potential customers' demand for the good in question is characterized by a reservation price that is the same for all potential customers – if the good is available from at least one firm that the customer knows of, and if the lowest price is less than or equal to the reservation price, the customer buys one unit of the good from the firm with the lowest price. We show that when some potential customers are aware of both firms and some are aware of just one firm, no pure strategy price equilibrium exists. We find the unique mixed strategy equilibrium where the upper bound in the price distribution is the consumers' reservation price and the lower bound is a function of the reservation price and the extent of the overlap in the consumer bases. The equilibrium is therefore characterised by price dispersion. We examine both symmetric and asymmetric models, where there is asymmetry when one firm has more captive consumers than the other. We show that the equilibrium has some very attractive, intuitive properties. In the symmetric case, the equilibrium ranges from monopoly to a competitive equilibrium when the overlap is increased from zero to perfect. In the asymmetric case, even with perfect overlap, the larger firm has some captive consumers, which moderates the competition and gives rise to market power for both firms.

In Section 3 we explore a particular model that gives rise to partial overlap of customer bases consistent with our assumption in Section 2. Firms advertise to make potential customers aware of their existence. Consumers are perfectly unaware of the existence of either firm and cannot gain any knowledge through search, and so a firm must advertise to make its presence known to a consumer. Each firm advertises to a set of consumers drawn from a common population where the cost of adding a new customer to the firm's customer base is convex. The firms are aware that this advertising effort will lead to some consumers being on both firms' lists and others being on only one firm's list. The greater the number of draws the firms make, the more likely is each new addition to be on the other firm's list as well. This overlap of customer bases determines the extent of competition in the stage two price-setting game. We find that the equilibrium overlap in customer bases is imperfect with any marginal cost of adding a consumer to the mailing list that is positive but less than the consumers' reservation price. The higher is the marginal cost, the fewer consumers are added to the list and therefore the smaller is the overlap and higher are the equilibrium expected prices and profits enjoyed by the two firms. Furthermore, the equilibrium in the customer base game is asymmetric. The larger firm will have a higher expected price and enjoy larger equilibrium profits than the smaller firm, but both firms will enjoy positive profits.

Our model offers a new simple solution to the Bertrand Paradox. The Bertrand Paradox refers to the fact that the equilibrium in a price-setting game in a market with two firms selling homogenous products is marginal cost pricing.

This is considered a paradox because it does not seem to conform with how firms behave in markets with a small number of firms. We show that the expected price of each firm is above zero as long as there are some consumers who only know of one of the firms in the market, the expected price rises to monopoly price as the overlap of the firms customer bases goes to zero. Others have found solutions to the Paradox by assuming that firms are capacity-constrained (Edgeworth, 1897; Kreps and Scheinkman, 1983); that firms engage in tacit collusion in a repeated game, an idea introduced by Chamberlin (1929) (see for example Tirole, 1988, for a formal model); and that firms produce differentiated products (Hotelling, 1929).

While our set-up is simple and intuitively appealing, there are few papers in the literature that are similar to ours. Varian (1980) has a similar set-up where there are perfectly informed consumers as well as consumers who visit the stores at random. This gives rise to the firms using a common mixed strategy where positive probability is placed on prices from the average cost to the reservation price. Free entry guarantees zero profits. Varian does not examine a duopoly game where it is clear that the minimum price in the support of the mixed strategy is greater than cost whenever there is only imperfect overlap in the customer bases of the firms in the market. We argue that the only equilibrium in the advertising game is asymmetric, whereas Varian uses a search model where the equilibrium is symmetric.

Papers in the search literature have taken a different approach from us. There is usually a large number of firms setting prices and consumers have rational expectations on the price distribution but not the location of firms in the distribution. Consumers engage in either non-sequential search (Stigler, 1961, Burdett and Judd, 1983) or sequential search (McCall, 1965 and 1970, Nelson 1970, Burdett and Judd, 1983) to locate a low price. The search costs of consumers determines the intensity of the search as well as the price equilibrium.

Diamond (1970) showed that if consumers face the same non-zero cost for all searches beyond the first search, the only equilibrium in the market is one where all firms charge the monopoly price and where consumers do not search. Salop and Stiglitz (1977) showed that when consumers differ in their cost of search and when a search makes consumers fully informed, it is possible to have an equilibrium where the high-cost consumers do not search and there are some firms who have higher than competitive prices and serve the uninformed consumers only. Stiglitz (1989) reviews and anlyses other search technologies that result in equilbirium price dispersion. It is also possible to have equilibrium price dispersion if consumers and firms optimally choose to remain somewhat ignorant of their economic environment (Rothschild and Yaari, in Rothschild 1973). Burdett and Judd (1983) show that we can get equilibrium price dispersion in a rational expectations model even without *ex ante* consumer heterogeneity if the model exhibits *ex post* heterogeneity in consumer information due to stochasticity in the acquisition of information. The commonality of all these papers is the assumption of rational expectations and therefore that consumers know the true distribution of prices in the market. This set-up takes as given that consumers are aware of the existence of all firms and all prices charged by the firms, but not which firm charges what. This is in contrast to our model where we assume that some consumers are only aware of the existence and price of one of the firms while others know the existence and price of both firms.

Papers in the advertising literature are closer related to our approach. Butters (1977) examines a situation where consumers are aware of the existence of the firms only through advertising and, if informed, purchase at most one unit of the good from the firm advertising the lowest price, a set-up very similar to ours. The analysis of Butters looks at the cumulative distribution of advertised prices and sale prices in the market, the latter taking into account that each consumer chooses the lowest price advetised to her. Therefore, Butter's focus is on the market and not on the behaviour of each individual firm. Furthermore, Butters' analysis is collapsed to a single stage game where firms send ads with a distribution of prices. As we examine a two-stage game where firms choose their customer bases in stage one and price distributions in stage two, the results of our price-setting game have wider applicability than just our advertising model. The main differences of our results are that our equilibrium is asymmetric whereas the Butters equilibrium is symmetric. Also, the minimum price in the equilibrium price distribution in Butters is likely to be smaller than ours.[1] However, our comparative static results are similar. We both find that the number of consumers who buy the good approaches the population when the marginal cost of an ad approaches zero. We also both find that the cheaper are the ads (in our terminology, the cheaper it is to add a consumer to the customer base), the lower is the mean of the sales price distribution (in our terminology, the expected transaction price). Butters also examines optimal non-sequential consumer search in conjunction with advertising, and Robert and Stahl (1993) extent the analysis to include optimal sequential search. Bester and Petrakis (1995) examine a model where consumers are spacially disperesed and know of their local store, but may also be informed of the price of the other firm through advertising. The equilibrium may involve random advertising where firms advertise and offer low prices at positive probability and do not advertise and have high prices at remaining probability. In the equilibrium, consumers are always aware of both firms' prices either because they receive an ad or because they do not receive one and deduce that the firm is charging a high price. As a result, our set-up with partial overlap of consumer bases cannot be derived from Bester and Petrakis model.

[1]For example, if we apply our assumptions of two firms and zero production cost and look at the limiting case where the marginal cost of an ad (in our terminology, the marginal cost of adding a consumer to a firm's customer base) goes to zero, we find that the minimum price goes to half of the consumers reservation price while Butters finds it goes to zero.

The rest of the paper is organized as follows. Section 2 discusses the equilibrium in the pricing game or the stage two equilibrium in the game where customer bases are chosen in stage one and prices are set in stage two. Subsection 2.1 establishes that there is no pure strategy equilibrium, Subsection 2.2 examines the mixed strategy equilibrium, first when customer bases are symmetric and second when customer bases are asymmetric. Section 3 examines the stage one problem where the firms advertise their presence to consumers. Section 4 concludes.

2 The Pricing Problem

Imagine a situation with two firms, 1 and 2, competing to sell some homogeneous good. There are many potential customers, and each of them has the same reservation price for the good, $R > 0$. If offered the good at a price $p \leq R$, they will buy 1 unit of the good, and if offered the good at higher price they will not buy the good. For simplicity, we imagine that the marginal costs of both firms are 0.

Some customers know only of firm 1, some know only of firm 2, and some know of both. We denote the total number of customers who know of firm i by N_i, and the number who know of both firms by M. We assume that $N_1 \geq N_2 > 0$ and $N_2 \geq M > 0$. In this section we look at the pricing problem of these firms. The problem is novel in that each firm has some potential customers who are ignorant of the existence of the other firm, and some who are not.

2.1 No Equilibrium in Pure Price Strategies

We can restrict attention to non-negative prices, less than or equal to the reservation price. So, we suppose that the strategy space of firm i is

$$S_i = \{p_i | 0 \leq p_i \leq R\}, i = 1, 2 \qquad (1)$$

The (expected) payoff function of firm i is then

$$\begin{aligned} \pi_i(p_i, p_j) &= p_i N_i \text{ if } p_i < p_j \\ &= p_i(N_i - \frac{M}{2}) \text{ if } p_i = p_j \\ &= p_i(N_i - M) \text{ if } p_i > p_j \end{aligned} \qquad (2)$$

Here we are assuming that when prices are identical, the M customers who know of both firms choose either firm with probability $\frac{1}{2}$.

The best response function of firm i is

$$\begin{aligned} B_i(p_j) &= p_j - \varepsilon \text{ if } \lambda_i R < p_j \leq R \\ &= R \text{ if } p_j \leq \lambda_i R, \end{aligned} \qquad (3)$$

where $\lambda_i \equiv \frac{N_i - M}{N_i}$. When $p_j = R$, firm i's best response is $B_i(p_j) = p_j - \varepsilon$, where ϵ is a small positive number that is arbitrarily close to zero, since firm i attracts just $N_i - \frac{M}{2}$ customers if she matches j's price and N_i if she undercuts j's price[2]. When $p_j < R$, firm i's best response is either $p_i = R$, which generates profit equal $R(N_i - M)$, or $p_i = p_j - \varepsilon$, which generates profit approximately equal to $p_j N_i$. When $p_j = \lambda_i R$,these options are equally attractive. So, if $p_j > \lambda_i R$, i's best response is to undercut j's price, $B_i(p_j) = p_j - \varepsilon$, and if $p_j \leq \lambda_i R$ i's best response is to fully exploit its captive market, $B_i(p_j) = R$.

[2]Strictly speaking firm i's best response is not well defined in this case since there is no largest price smaller than p_j, but for current purposes it is helpful to define the best response as $p_j - \epsilon$.

The best responses are illustrated in Figure 1, and it is apparent that there is no equilibrium in pure strategies. What drives the nonexistence result is that not all customers know of all firms. This would seem to be an awkward result since there is nothing bizarre about this pricing problem. In particular, it would seem to be not at all uncommon for some customers to be ignorant of the existence of some firms. Given that information is costly, the improbable occurrence would seem to be the case in which all customers know of all firms. Below we show that there is a mixed strategy equilibrium to this pricing problem, and that it has some very nice properties.

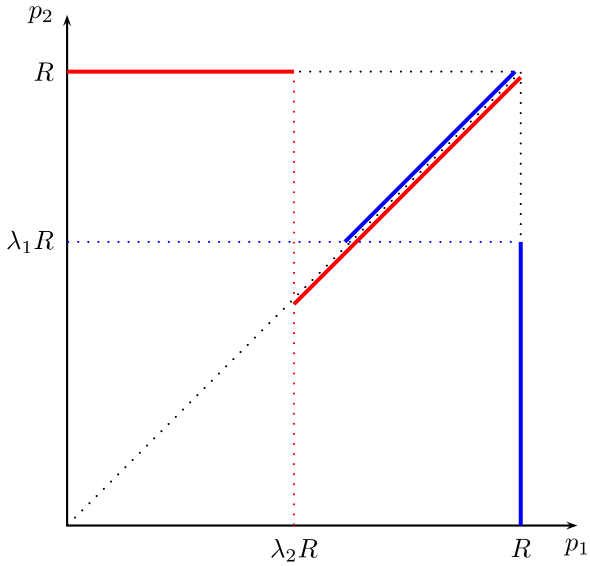

Figure 1: Best response functions for firms 1 and 2 for pure strategies: BR_1 is in blue and BR_2 is in red

2.2 A Mixed Strategy Equilibrium

For clarity we begin by reviewing what we are trying to achieve in this section. We are looking for two probability distributions of prices, or two price density functions (DFs), one for each firm. It is convenient to work with cumulative density functions (CDFs), $F_1(p_1)$ for firm 1 and $F_2(p_2)$ for firm 2. So the two CDFs, $F_1(p_1)$ and $F_2(p_2)$, are the endogenous variables in this problem.

Let $\Pi_i(p_i|F_j)$ denote firm i's expected profit, given its own price, p_i, and the other firm's CDF, F_j, and let Π_i^* denote the expected profit of firm i in the mixed strategy equilibrium. A mixed strategy equilibrium is characterized by the following properties.

- if $F_i(p_i) > 0$, then $\Pi_i(p_i|F_j) = \Pi_i^*$
- if $F_i(p_i) = 0$, then $\Pi_i(p_i|F_j) \leq \Pi_i^*$

If firm 1 charges $p_1 = R$ and firm 2 charges $p_2 < R$, firm 1 will only be able to sell to the $(N_1 - M)$ consumers who know of firm 1 alone, and its profit is $\pi_1(R, p_2) = R(N_1 - M)$. Also, we know that $\pi_1(R, R) = R(N_1 - \frac{M}{2})$ and therefore $\pi_1(R, p_2) \geq R(N_1 - M)$. Therefore, firm 1's expected profit in a mixed strategy equilibrium can be no smaller than $R(N_1 - M)$: that is, $\Pi_1^* \geq R(N_1 - M)$. This in turn implies that in a mixed strategy equilibrium, prices $p_1 < \lambda_1 R$ must get a zero probability weight, since if $p_1 < \lambda_1 R$, $\pi_1(p_1, p_2) \leq p_1 N_1 < \lambda_1 R N_1 = R(N_1 - M)$. So, in a mixed strategy equilibrium, $F_1(p_1) = 0$ for all $p_1 < \lambda_1 R$.

Analogously, in a mixed strategy equilibrium, $F_2(p_2) = 0$ for all $p_2 < \lambda_2 R$. But we can extend the range of firm 2's prices that must get a 0 probability weight in a mixed strategy equilibrium. First notice that since $N_2 \leq N_1$, $\lambda_2 R \leq \lambda_1 R$. Then consider any price $p_2 < \lambda_1 R$, say $p_2 = \overline{p}$. We showed above that in a mixed strategy equilibrium, $F_1(\overline{p}) = 0$, so when $p_2 = \overline{p} < \lambda_1 R$, firm 2 is necessarily the firm with the lower price. Consequently $\Pi_2(p_2|F_1) = p_2 N_2$, an increasing function of p_2 so long as $p_2 < \lambda_1 R$. But, as the following argument by contradiction shows, this implies that $F_2(p_2)$ must be 0 for all $p_2 < \lambda_1 R$. Suppose instead that there is some $p_2 < \lambda_1 R$, say $\widehat{p}_2$, such that $F_2(\widehat{p}_2) > 0$. Since $\Pi_2(p_2|F_1)$ is an increasing function of p_2 for $p_2 < \lambda_1 R$, there exists a $\widetilde{p}_2$ in interval $(\widehat{p}_2, \lambda_1 R\ )$ such that $\Pi_2(\widehat{p}_2|F_1) < \Pi_2(\widetilde{p}_2|F_1) \leq \Pi_2^*$. But this contradicts the assumption that $F_2(\widehat{p}_2) > 0$, since in a mixed strategy equilibrium all prices that get a positive probability weight must generate Π_2^*.

These considerations establish the following result:

Result 1. *In any mixed strategy equilibrium,* $F_1(p_1) = 0$ *for all* $p_1 < \lambda_1 R$*, and* $F_2(p_2) = 0$ *for all* $p_2 < \lambda_1 R$*.*

This suggests that we look for a mixed strategy equilibrium in which the support of the two price distributions is $[\lambda_1 R, R]$.

2.2.1 The Symmetric Case: $N_1 = N_2 = N$

Since it is the easiest case, we begin by finding the mixed strategy equilibrium for the symmetric case in which $N_1 = N_2 = N$ and $\lambda_1 = \lambda_2 = \lambda \equiv \frac{N-M}{N}$. Consistent with Result 1, assume that the $F_2(p_2) = 0$ for all $p_2 < \lambda R$. Assume too that there are no mass points in $F_2(p)$. Then if $p_1 > p_2$, firm 1's profit is $p_1(N - M)$ since the M customers who know of both firms choose to buy from firm 2. Since there are no mass points in F_2, the probability that $p_1 > p_2$ is just $F_2(p_1)$. On the other hand, if $p_1 < p_2$, firm 1's profit is $p_1 N$ since the M customers who know of both firms now choose to buy from firm 1. The probability that $p_1 < p_2$ is just $1 - F_2(p_1)$. Hence, firm 1's expected profit as a function of p_1 and F_2 is just

$$\Pi_1(p_1|F_2) = p_1(N - M)F_2(p_1) + p_1 N(1 - F_2(p_1)). \tag{4}$$

Above we argued that $\Pi_1^* \geq R(N-M)$. This suggests that we ought to look for an $F_2(p)$ such that $\Pi_1(p_1|F_2)$ is equal to $R(N-M)$ for all $p_1 \in [\lambda R, R]$. That is, we ought to look for $F_2(p)$ such that

$$p_1(N-M)F_2(p_1) + p_1 N(1 - F_2(p_1)) = R(N-M). \tag{5}$$

Solving (5) for F_2 we get

$$F_2(p_1) = \frac{p_1 N - R(N-M)}{p_1 M} \tag{6}$$

Notice the following: $F_2(\lambda R) = 0$; $F_2(R) = 1$; $F_2' > 0$; F_2 has no mass points. So, this functions has all the properties of CDF, and it has no mass points.

So, define $F(p)$ as follows:

$$\begin{aligned} F(p) &= \frac{pN - R(N-M)}{pM} \text{ if } \lambda R \leq p \leq R \\ &= 0 \text{ otherwise.} \end{aligned} \tag{7}$$

Then, differentiating with respect to p, we get the corresponding price DF, $f(p)$,

$$\begin{aligned} f(p) &= \frac{R(N-M)}{Mp^2} \text{ if } \lambda R \leq p \leq R \\ &= 0 \text{ otherwise.} \end{aligned} \tag{8}$$

As we showed above, if $F_2(p) = F(p)$, then $\Pi_1(p_1|F_2) = R(N-M) > 0$ for all $p_1 \in [\lambda R, R]$. Further, if $p_1 < \lambda R$, $\Pi_1(p_1|F_2) = p_1 N < R(N-M)$. Given the symmetry of the problem, if $F_1(p) = F(p)$, then $\Pi_2(p_2|F_1) = R(N-M) > 0$ for all $p_2 \in [\lambda R, R]$, and if $p_2 < \lambda R$, $\Pi_2(p_2|F_1) = p_2 N < R(N-M)$. So, we have found the mixed strategy equilibrium for the symmetric case.

Result 2. *If $N_1 = N_2 = N$, then $F_1(p) = F(p)$ and $F_2(p) = F(p)$ constitute a mixed strategy equilibrium in which $\Pi_1^* = \Pi_2^* = R(N-M)$.*

In the symmetric case the overlap in the set of potential customers, M, is bounded below by 0 and above by N. Figure 2 illustrates the density functions for different values of M, given N. When $M = 0$, we have two monopolists using the monopoly price R as their equilibirum pure strategy and enjoying monopoly profits RN. At the other extreme, when $M \to N$, the equilbrium approaches the pure strategy Bertrand model, where the firms are mixing between prices $[0, R]$ with probability that approaches zero applied on prices above 0. Consequently the firms make profits that approach zero. So, as M increases from 0 to N, profit of both firms declines smoothly from the monopoly level, RN, to the Bertrand level, 0. This is an appealing property.

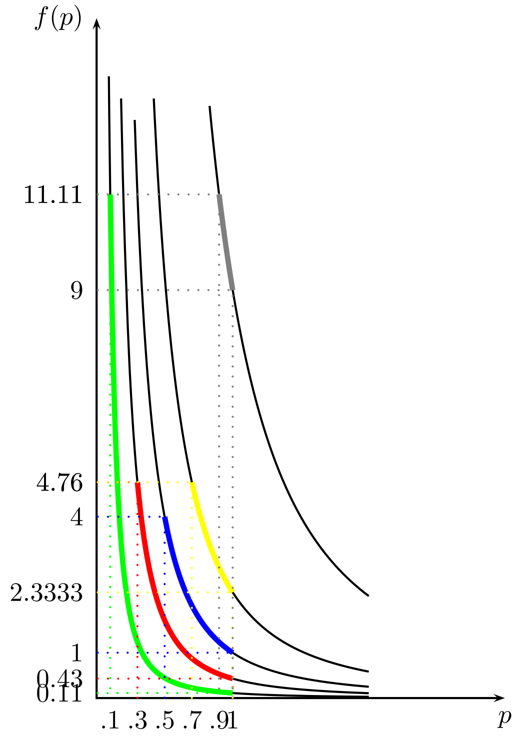

Figure 2: Equilibrium density functions and price support for $N = 10$ and $R = 1$ when $M = 1$ (in gray) $M = 3$ (in yellow) $M = 5$ (in blue), $M = 7$ (in red), and $M = 9$ (in green)

The expected price charged by a firm in the symmetric mixed strategy equilibrium is given by

$$E(p) = \int_{p=\lambda R}^{R} pf(p)dp = \int_{p=\lambda R}^{R} \frac{R(N-M)}{Mp} dp = \frac{R(N-M)}{M} \ln\left(\frac{N}{N-M}\right). \tag{9}$$

While (9) is the price the $2(N-M)$ consumers who know of one firm only expect to pay, the M consumers who know of both firms' prices will only pay the minimum of the two. As the probability that a price p charged by one of the firms is the lower of the two prices is $1-F(p)$, and because there are two firms, the expected minimum price is just

$$\begin{aligned} E(\min(p)) &= 2\int_{p=\lambda R}^{R} pf(p)\left(1-F(p)\right)dp \qquad (10)\\ &= \frac{2R(N-M)}{M^2}\left(M-(N-M)\ln\left(\frac{N}{N-M}\right)\right) \end{aligned}$$

Therefore, the expected transaction price is

$$\begin{aligned} ET(p) &= \frac{2(N-M)}{2N-M}E(p) + \frac{M}{2N-M}E(\min(p)) \qquad (11)\\ &= \frac{2R(N-M)}{2N-M}. \end{aligned}$$

The expected transaction price in (11) goes to R as $M \to 0$ and to 0 as $M \to N$ and therefore it equals the expected price in (9) at the extreme values of M. However, for interior values of M, $ET(p) < E(p)$.

2.2.2 The Asymmetric Case: $N_2 < N_1$

Now let's consider the asymmetric case where $N_2 < N_1$. Assume that $M > 0$[3]. We adopt a different expositional strategy – we first write out the equilibrium DFs and CDFs and then show that they are indeed equilibrium distributions.

The equilibrium CDF for firm 2 is

$$\begin{aligned} F_2(p) &= \frac{pN_1 - R(N_1-M)}{pM} = \frac{N_1(p-\lambda_1 R)}{pM} \text{ if } \lambda_1 R \leq p \leq R \qquad (12)\\ &= 0 \text{ otherwise} \end{aligned}$$

[3]The case where $M = 0$ is discussed separately.

The corresponding DF is

$$
\begin{aligned}
f_2(p) &= \frac{R(N_1 - M)}{Mp^2} \text{ if } \lambda_1 R \le p < R \\
&= 0 \text{ if } p < \lambda_1 R \text{ or if } p = R
\end{aligned}
\tag{13}
$$

Notice that in the asymmetric case, $f_2(R) = 0$ – that is, the probability density for $p_2 = R$ is 0, not $\frac{N_1 - M}{MR}$ as one might expect. The fact that $f_2(R) = 0$ instead of $\frac{N_1 - M}{MR}$ has no effect on the CDF because the definite integrals of $f_2(p)$ over the sets $[\lambda_1 R, R]$ and $[\lambda_1 R, R)$ are identical.

Notice that $F_2(\lambda_1 R) = 0, F_2(R) = 1$, and $F_2'(p) > 0$ for $\lambda_1 R \le p < R$, so $F_2(p)$ has the attributes of a CDF. In addition it has no mass points.

Firm 1's expected profit as a function of p_1 and F_2 is then

$$
\Pi_1(p_1|F_2) = p_1(N_1 - M)F_2(p_1) + p_1 N_1(1 - F_2(p_1)) \tag{14}
$$

It is then straight forward to show the following:

$$
\Pi_1(p_1|F_2) = R(N_1 - M) = \lambda_1 R N_1 \text{ for all } p_1 \in [\lambda_1 R, R] \tag{15}
$$

$$
\Pi_1(p_1|F_2) = p_1 N_1 < \lambda_1 R N_1 \text{ for all } p_1 < \lambda_1 R \tag{16}
$$

The equilibrium price DF for firm 1 is

$$
\begin{aligned}
f_1(p) &= \frac{N_2}{N_1} f_2(p) = \frac{R\lambda_1 N_2}{Mp^2} \text{ if } \lambda_1 R \le p < R \\
&= 0 \text{ if } p < \lambda_1 R \\
m_1(p) &= 1 - \frac{N_2}{N_1} \text{ if } p = R
\end{aligned}
\tag{17}
$$

Notice the mass point in this DF at $p = R$. Notice also that

$$
\int_{\lambda_1 R}^{R} \frac{R\lambda_1 N_2}{Mp^2} dp = \frac{N_2}{N_1}
$$

So a fraction $\frac{N_2}{N_1}$ of the probability mass for firm 1's prices is distributed on the half open interval $[\lambda_1 R, R)$, where density at any point is equal to $\frac{R\lambda_1 N_2}{Mp^2}$. The remainder of the probability mass is all piled on R. The corresponding CDF is

$$
\begin{aligned}
F_1(p) &= \frac{N_2}{N_1}F_2(p) = \frac{N_2(p-\lambda_1 R)}{pM} \text{ if } \lambda_1 R \le p < R \qquad (18)\\
&= 0 \text{ if } p < \lambda_1 R\\
F_1(R) &= 1
\end{aligned}
$$

Notice that $F_1(\lambda_1 R) = 0, F_1(R) = 1$, and $F_1'(p) > 0$ for $\lambda_1 R \le p < R$, so $F_1(p)$ has the attributes of a CDF.

Firm 2's expected profit as a function of p_2 and F_1 is then

$$
\begin{aligned}
\Pi_2(p_2|F_1) &= p_2(N_2 - M)F_1(p_2) + p_2 N_2(1 - F_1(p_2)) \text{ for all } p_2 \in [0, R) \qquad (19)\\
&= R\left((1-\frac{N_2}{N_1})(N_2 - \frac{M}{2}) + \frac{N_2}{N_1}(N_2 - M)\right) \text{ if } p_2 = R
\end{aligned}
$$

Notice that $\Pi_2(\lambda_1 R|F_1) = \lambda_1 R N_2$, since when $p_2 = \lambda_1 R$, firm 2 is the low price firm with probability 1. Although the calculations are slightly tedious, it straightforward to show that

$$\Pi_2(p_2|F_1) = \lambda_1 R N_2 \text{ for all } p_2 \in [\lambda_1 R, R) \qquad (20)$$

$$\Pi_2(p_2|F_1) = p_2 N_2 < \lambda_1 R N_2 \text{ for all } p_2 < \lambda_1 R \qquad (21)$$

$$\Pi_2(p_2|F_1) = \lambda_1 R N_2 - R\frac{N_2 M}{2N_1} < \lambda_1 R N_2 \text{ if } p_2 = R \qquad (22)$$

In particular, it might be useful to show that despite the mass point firm 1 has in its CDF at $p_1 = R$, firm 2 does not have an incentive to use a pure strategy of $p_2 = R - \varepsilon$ because it does not gain in profit by doing so:

$$\Pi_2(R - \varepsilon|F_1) = \lambda_1 R N_2. \qquad (23)$$

We have shown that given the CDFs in (12) and (18) and the corresponding DFs in (13) and (17), firm i enjoys the same profit for all prices for which it plays with positive probability and lower profit for prices it plays with zero probability for $i = 1, 2$. Therefore, we have found a mixed strategy equilibrium and have the following result:

Result 3. *If $N_1 > N_2$, then $f_1(p)$ defined in equation (17) and $f_2(p)$ defined in equation (13) constitute a mixed strategy equilibrium in which $\Pi_1^* = \lambda_1 R N_1$ and $\Pi_2^* = \lambda_1 R N_2$.*

Now, if $M = 0$, the two firms become monopolies charging pure strategy prices R[4], as in the symmetric case. However, the equilibrium when $M \to N_2$ is quite different from the symmetric case ($M \to N$) where both firms effectively charged zero prices. In the asymmetric case, as $M \to N_2 < N_1$, firm 1 mixes between prices in $[\frac{N_1 - N_2}{N_1} R, R]$, and firm two mixes between prices in $[\frac{N_1 - N_2}{N_1} R, R)$ with both players attaching positive probability to all prices in their equilibrium price supports such that firm 1 still has a mass point at $p = R$. Consequently, expected profits remain above zero. That is to say, even if all of firm 2's customers know of firm 1 as well (but not vice versa due to the asymmetry), the profit-margins are maintained above zero due to firm 1's opportunity to sell to its $(N_1 - M)$ captive customers that limits the incentive of firm 1 to undercut firm 2's price.

The expected price charged by a firm 2 in the asymmetric mixed strategy equilibrium is given by

$$E(p_2) = \int_{p=\lambda_1 R}^{R} p f(p_2) dp = \int_{p=\lambda_2 R}^{R} \frac{R(N_1 - M)}{Mp} dp = \frac{R(N_1 - M)}{M} \ln\left(\frac{N_1}{N_1 - M}\right). \tag{24}$$

Firm 2's expected price in (24) goes to R as $M \to 0$ and to $\frac{R(N_1 - N_2)}{N_2} \ln\left(\frac{N_1}{N_1 - N_2}\right)$ as $M \to N_2$. The expected price charged by a firm 1 in the asymmetric mixed strategy equilibrium is

$$E(p_1) = \frac{N_2}{N_1} \frac{R(N_1 - M)}{M} \ln\left(\frac{N_1}{N_1 - M}\right) + \left(1 - \frac{N_2}{N_1}\right) R. \tag{25}$$

[4] Notice that we assumed that $M > 0$ for the above discussion. However, when $M = 0$, it is easy to show that $f_1(R) = 1$ and $f_2(R) = 1$.

Firm 1's expected price in (25) goes to R as $M \to 0$ and to $\frac{R(N_1-N_2)}{N_1} \ln\left(\frac{N_1}{N_1-N_2}\right) + \left(1-\frac{N_2}{N_1}\right) R$ as $M \to N_2$. The expected minimum price paid by consumers is

$$\begin{aligned} E\left(\min(p)\right) &= \int_{p=\lambda_1 R}^{R} \left(pf_1(p)\left(1-F_2(p)\right) + pf_2(p)\left(1-F_1(p)\right)\right) dp \qquad (26) \\ &= \int_{p=\lambda_1 R}^{R} \left(p\frac{N_2}{N_1} f_2(p)\left(1-F_2(p)\right) + pf_2(p)\left(1-\frac{N_2}{N_1}F_2(p)\right)\right) dp \\ &= \frac{R(N_1-M)}{N_1 M^2}\left(M\left(N_1+N_2\right) - 2N_1N_2\right)\ln\left(\frac{N_1}{N_1-M}\right) + \frac{2N_2(N_1-M)R}{N_1 M}. \end{aligned}$$

Therefore, the expected transaction price is

$$\begin{aligned} ET(p) &= \frac{N_1-M}{N_1+N_2-M}E(p_1) + \frac{N_2-M}{N_1+N_2-M}E(p_2) + \frac{M}{N_1+N_2-M}E\left(\min(p)\right) \\ &= \frac{N_1+N_2}{N_1}\frac{(N_1-M)R}{N_1+N_2-M}. \qquad (27) \end{aligned}$$

The expected transaction price in (11) goes to R as $M \to 0$. As $M \to N_2$, $ET(p) \to \frac{(N_1-N_2)^2}{(N_1)^2} R$.

3 Choosing the Customer Bases

In this section we briefly extend the game to analyze firm choices with respect to the size of their customer bases. We have in mind an advertising effort in which firms manage their customer bases by varying their expenditure on advertising. We imagine a population of potential customers of size H. We assume that initially all of these potential customers are ignorant of the firms and that they become aware of them only through the advertising efforts of the firms.

We denote the cost of making one's firm known to $N \leq H$ of the potential customers by the function $A(N)$. We assume that the function A is increasing and convex in its argument: $A' > 0$ and $A'' > 0$.

Given N_1 and N_2, what should we assume about M, the overlap in the two set of customers? Clearly, if both N_1 and N_2 are small relative to H it seems likely that there will be very little overlap, whereas if both are large relative to

H there will be substantial overlap. It also seems sensible to suppose that the overlap ought to be an increasing function of both N_1 and N_2. Beyond this, there is not a lot more that can be said with conviction.

For concreteness we make a somewhat stronger assumptions. In particular, we assume that any particular person in the population is just as likely to be included in a firm's set of potential customers as is any other person. So, the probability that any person in firm i's set of potential customers is also included in firm j's set of potential customers is simply $\frac{N_j}{H}$, and therefore the expected overlap of the two set of potential customers is

$$M = \frac{N_1 N_2}{H} \tag{28}$$

Given this assumption, if we suppose that there is a two stage game in which firms simultaneously choose their customer bases in stage 1 and simultaneously choose prices in stage 2, the firms' objective functions in stage 1 are

$$\begin{aligned} V_1(N_1, N_2) &= R[N_1 - \frac{N_1 N_2}{H}] - A(N_1) \\ &= N_1 R[1 - \frac{N_2}{H}] - A(N_1) \text{ if } N_1 \geq N_2 \end{aligned} \tag{29}$$

$$\begin{aligned} V_1(N_1, N_2) &= R[N_1 - \frac{N_1}{N_2}\frac{N_1 N_2}{H}] - A(N_1) \\ &= N_1 R[1 - \frac{N_1}{H}] - A(N_1) \text{ if } N_1 < N_2. \end{aligned} \tag{30}$$

Notice that (30) is not a function of N_2. Then, differentiating, we get

$$\begin{aligned} \frac{\partial V_1(N_1, N_2)}{\partial N_1} &= R[1 - \frac{N_2}{H}] - A'(N_1) \text{ if } N_1 \geq N_2 \\ &= R[1 - \frac{2N_1}{H}] - A'(N_1) \text{ if } N_1 < N_2. \end{aligned} \tag{31}$$

So there is a discontinuity in this derivative at $N_1 = N_2$. When N_1 and N_2 are not too different, the marginal value of an added potential customer is bigger for the firm with the larger customer base.

Suppose that

$$A(N) = vH[\ln(H) - \ln(H - N)]. \tag{32}$$

and assume that $v \leq R$ so that the marginal cost of getting the first customer is no greater than the customer's reservation price. Then

$$A'(N) = \frac{vH}{H - N}. \tag{33}$$

Setting (31) equal to zero tells us how firm 1 behaves optimally when it chooses N_1 above N_2 and below N_2:

$$BR_1^H(N_2) = H - \frac{vH}{R(1 - \frac{N_2}{H})} \text{ if } N_1 \geq N_2 \tag{34}$$

$$BR_1^L(N_2) = \frac{3H}{4} - \frac{\sqrt{1 + \frac{8v}{R}}H}{4} \text{ if } N_1 < N_2. \tag{35}$$

Firm 2 's optimal choises above and below N_1 are:

$$BR_2^H(N_1) = H - \frac{vH}{R(1 - \frac{N_1}{H})} \text{ if } N_2 \geq N_1 \tag{36}$$

$$BR_2^L(N_1) = \frac{3H}{4} - \frac{\sqrt{1 + \frac{8v}{R}}H}{4} \text{ if } N_2 < N_1. \tag{37}$$

Notice that the above are not strictly speaking best response functions because they do not necessarily define a unique response for firm i to the other firm's choice of N. We have illustrated these "local best response functions" in Figure 3 for $0 < v < R$, where we can observe that there are four candidates for Nash equilibria in the game, namely (N^H, N^L), (N^L, N^H), $(\widetilde{N}, \widetilde{N})$ and (N^L, N^L). The profits in the four Nash equilibrium candidates are defined as follows:

$$\begin{aligned}
V_1(N^H, N^L) &= V_2(N^L, N^H) \equiv V^{HL} \\
V_1(N^L, N^H) &= V_2(N^H, N^L) \equiv V^L \\
V_1(\widetilde{N}, \widetilde{N}) &= V_2(\widetilde{N}, \widetilde{N}) \equiv V^{HH} \\
V_1(N^L, N^L) &= V_2(N^L, N^L) \equiv V^L
\end{aligned}$$

Notice that there is a range of values for $N^L \leq N_2 \leq \widetilde{N}$ for which we have two local best-responses for firm 1. Similarly, when $N^L \leq N_1 \leq \widetilde{N}$, firm 2 has two local best responses. We need to identify the unique best response for each firm in order to know which of the Nash equilibrium candidates remain. This depends on the relative sizes of the equilibrium profits given above:

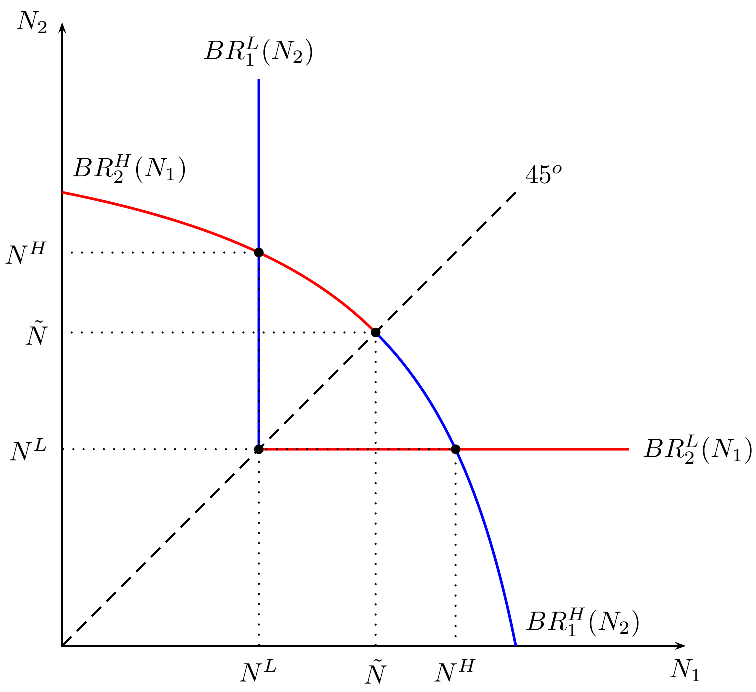

Figure 3: BRFN1N2: Best response functions and four Nash equilibrium candidates in customer base game

$V^{HL} < V^L$. If this condition holds, the best response functions are as shown in Figure 4 and we have a unique symmetric Nash equilibrium (N^L, N^L). We will show that $V^{HL} \geq V^L$, and therefore the symmetric Nash equilibrium (N^L, N^L) does not exist.

$V^{HL} > V^L > V^{HH}$. Define $V_1^*(BR_1^H(N_2), N_2)$ as the profit of firm one when

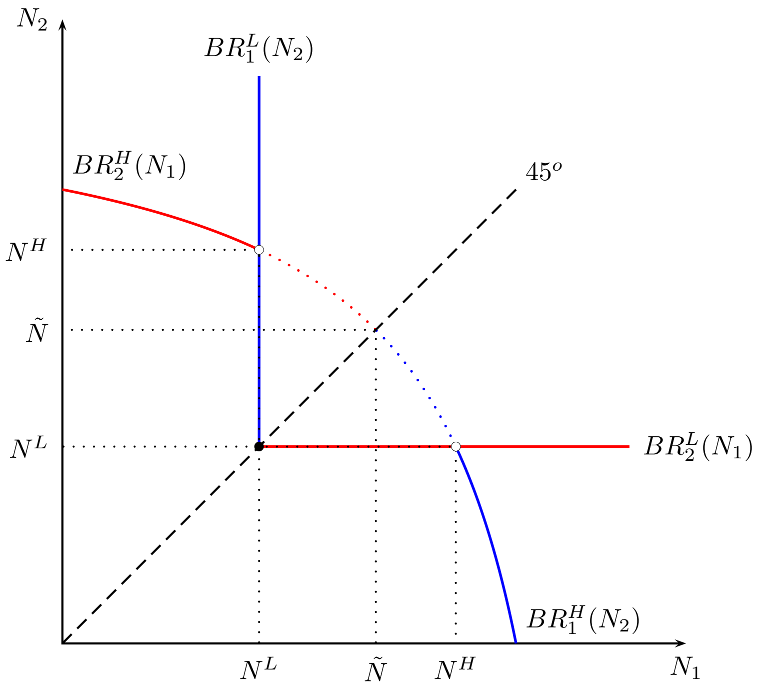

Figure 4: BRFN1N2LL: Best response functions and a unique symmetric Nash equilibrium in customer base game (N_L, N_L) when $V^{HL} < V^L$

it plays its high best response to N_2 when $N^L \leq N_2 \leq \widetilde{N}$. As $\frac{\partial V_1^*(BR_1^H(N_2),N_2)}{\partial N_2} <$ 0 and $\frac{\partial V_1^*(N^L)}{\partial N_2} = 0$, if we define $\widehat{N}$ such that $V_1^*(BR_1^H(\widehat{N}), \widehat{N}) = V_2^*(\widehat{N}, BR_2^H(\widehat{N})) \equiv$ V^L, such $\widehat{N}$ must satisfy $\widetilde{N} > \widehat{N} > N^L$ if $V^{HL} > V^L > V^{HH}$. Therefore, the unique best response function for firm 1 is

$$\begin{aligned} BR_1(N_2) &= H - \frac{vH}{R(1-\frac{N_2}{H})} \text{ if } N_2 \leq \widehat{N} \\ &= \frac{3H}{4} - \frac{\sqrt{1+\frac{8v}{R}}H}{4} \text{ if } N_2 > \widehat{N} \end{aligned} \tag{38}$$

Firm 2'a best response function is symmetric. These best response functions are illustrated in Figure 5 from where we can see that we have two asymmetric Nash equilibria (N^H, N^L) and (N^L, N^H).

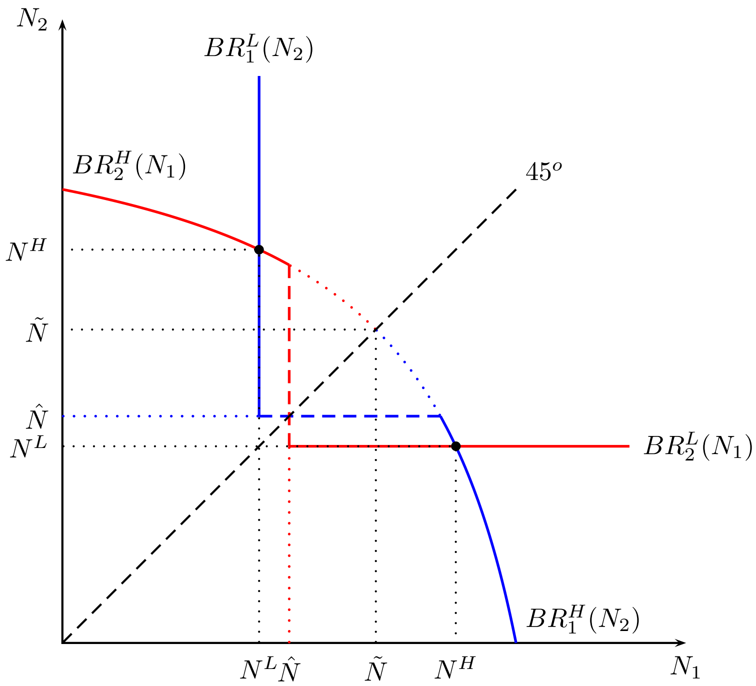

Figure 5: BRFN1N2HL: Best response functions and two Nash equilibria in customer base game (N_H, N_L) and (N_L, N_H) when $V^{HL} > V^L > V^{HH}$

$V^{HH} > V^L$. If this condition holds, the best response functions are as shown in Figure 6, and we have a symmetric Nash equilibrium $(\widetilde{N}, \widetilde{N})$ as well as two asymmetric Nash equilibria (N^H, N^L) and (N^L, N^H). The symmetric equilibrium is unstable because any movement away from it will lead to the convergence to one of the asymmetric equilibria. We will show that $V^{HH} \leq V^L$, and therefore a symmetric NE $(\widetilde{N}, \widetilde{N})$ does not exist.

N^H and N^L can be found by setting (34) equal to (37):

$$N^L = \frac{3H}{4} - \frac{\sqrt{1 + \frac{8v}{R}}H}{4} \tag{39}$$

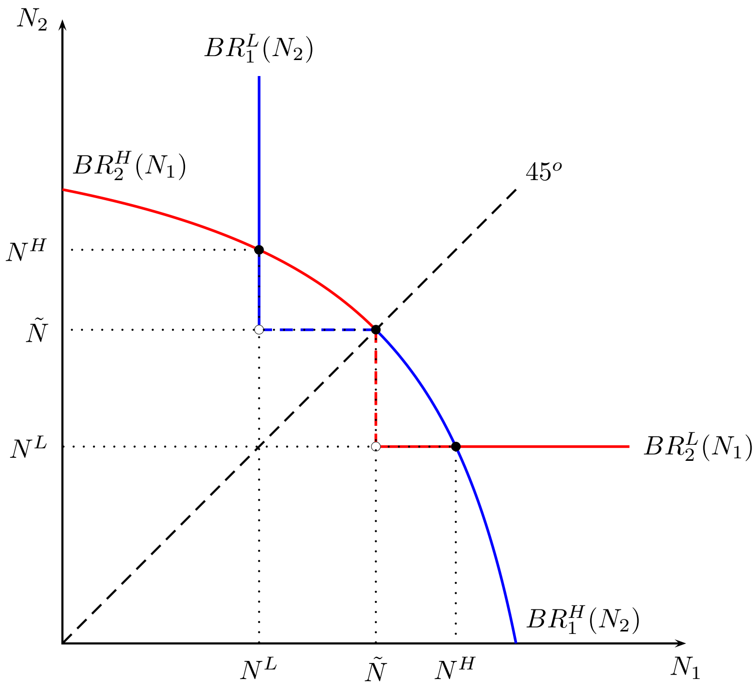

Figure 6: BRFN1N2HH: Best response functions and three Nash equilibria in customer base game (N^H, N^L), $(\tilde{N}, \tilde{N})$ and (N^L, N^H) when $V^{HH} > V^L$

and

$$N^H = 2N^L = \frac{3H}{2} - \frac{\sqrt{1 + \frac{8v}{R}}H}{2}. \tag{40}$$

Notice that $N^L \geq 0$ and $N^H \geq 0$ because $v \leq R$. Furthermore, $N^H < H$ and $N^L < \frac{H}{2}$ if $v > 0$ and $N^H = H$ and $N^L = \frac{H}{2}$ if $v = 0$. So, in an asymmetric equilibrium, the overlap of the firms is $M \leq \frac{H}{2}$.

$\widetilde{N}$ can be found by imposing symmetry on (34) :

$$\widetilde{N} = H - \sqrt{\frac{v}{R}}H. \tag{41}$$

Again, $\widetilde{N} \geq 0$ because $v \leq R$. Also, $\widetilde{N} < H$ if $v > 0$ and $\widetilde{N} = H$ if $v = 0$.

The maximized profits are

$$V^{HL} = \frac{HR}{4}\left(1+\sqrt{1+\frac{8v}{R}}\right) - vH - vH\ln\left(\frac{2}{-1+\sqrt{1+\frac{8v}{R}}}\right) \tag{42}$$

$$V^{L} = \frac{HR}{8}\left(1+\sqrt{1+\frac{8v}{R}}\right) - \frac{vH}{2} - vH\ln\left(\frac{4}{1+\sqrt{1+\frac{8v}{R}}}\right) \tag{43}$$

$$V^{HH} = vH\left(\sqrt{\frac{R}{v}} - 1 - \ln\left(\sqrt{\frac{R}{v}}\right)\right) \tag{44}$$

We can see that when $v = R$, $V^{HL} = V^{L} = V^{HH} = 0$. Also, when $v = 0$, $V^{HL} = \frac{HR}{2}$, $V^{L} = \frac{HR}{4}$, and $V^{HH} = 0$.

We can express $V^{HL} - V^{L}$ the following ways:

$$V^{HL} - V^{L} = H\left[\frac{R}{8}\left(1+\sqrt{1+\frac{8v}{R}}\right) - \frac{v}{2} + v\ln\left(\frac{2\left(-1+\sqrt{1+\frac{8v}{R}}\right)}{\left(1+\sqrt{1+\frac{8v}{R}}\right)}\right)\right] \tag{45}$$

I can't seem to prove analytically that this difference is non-negative. However, the difference is linear in H, so the difference depends only on v and R. From Excel simulations, it appears to me that $V^{HL} \geq V^{L} \geq V^{HH}$ with equality holding only when $v = R$, which suggests that when $0 < v < R$, only the two asymmetric equilibria exist and the two symmetric ones do not exist. Assuming that it is in fact true that $V^{HL} > V^{L} > V^{HH}$ when $0 < v < R$, there are only two Nash equilibria: (N^{H}, N^{L}), and (N^{L}, N^{H}), where

$$N^{H} = 2N^{L} = \frac{H}{2}\left(3 - \sqrt{1+\frac{8v}{R}}\right) \tag{46}$$

and

$$N^{L} = \frac{H}{4}\left(3 - \sqrt{1+\frac{8v}{R}}\right). \tag{47}$$

The profit of the firm with the larger customer base equals

$$V^{H} = \frac{HR}{4}\left(1+\sqrt{1+\frac{8v}{R}}\right) - vH - vH\ln\left(\frac{2}{-1+\sqrt{1+\frac{8v}{R}}}\right). \tag{48}$$

and the profit of the firm with the smaller customer base equals

$$V^L = \frac{HR}{8}\left(1+\sqrt{1+\frac{8v}{R}}\right) - \frac{vH}{2} - vH\ln\left(\frac{4}{1+\sqrt{1+\frac{8v}{R}}}\right) \tag{49}$$

The overlap in the customer bases is

$$M = H\left(\frac{5}{4}+\frac{v}{R}-\frac{3}{4}\sqrt{1+\frac{8v}{R}}\right). \tag{50}$$

M has its maximum $M=\frac{H}{2}$ when $v=0$, and decreases smoothly to zero as v increases to R. The total number of customers who know of at least one firm is

$$N_1+N_2-M = H\left(1-\frac{v}{R}\right). \tag{51}$$

(51) goes to H as $v\to 0$ and to 0 as $v\to R$.

The larger firm's proportion of captive consumers in the equilibrium is

$$\lambda^H = \frac{1}{4}\left(1+\sqrt{1+\frac{8v}{R}}\right) \tag{52}$$

and thus the lower bound of the price support for the equilibrium price support is

$$\lambda^H R = \frac{\frac{R}{2}\left(1+\sqrt{1+\frac{8v}{R}}\right)-2v}{3-\sqrt{1+\frac{8v}{R}}} \tag{53}$$

Notice that when $v=0$, the lower bound of the equilibrium price support in (53) is $\frac{R}{2}$. As v increases from 0, the lower bound of the equilibrium price support increases from $\frac{R}{2}$ smoothly and at the limit when v approaches R the lower bound of the equilibrium price support approaches R as well. This happens because M has its maximum $M=\frac{H}{2}$ when $v=0$, and decreases smoothly to zero as v increases to R.

The equilibrium CDF for the firm with the smaller clientele is

$$\begin{aligned} F^L(p) &= \frac{4-\frac{R}{p}\left(1+\sqrt{1+\frac{8v}{R}}\right)}{3-\sqrt{1+\frac{8v}{R}}} \text{ if } \lambda^H R \le p \le R \\ &= 0 \text{ otherwise} \end{aligned} \tag{54}$$

and the corresponding DF is

$$
\begin{aligned}
f^L(p) &= \frac{R}{p^2}\left(\frac{1+\sqrt{1+\frac{8v}{R}}}{3-\sqrt{1+\frac{8v}{R}}}\right) \text{ if } \lambda^H R \le p < R \qquad (55)\\
&= 0 \text{ if } p < \lambda^H R \text{ or if } p = R
\end{aligned}
$$

For the larger firm, these are

$$
\begin{aligned}
F^H(p) &= \frac{2-\frac{R}{2p}\left(1+\sqrt{1+\frac{8v}{R}}\right)}{3-\sqrt{1+\frac{8v}{R}}} \text{ if } \lambda^H R \le p < R \qquad (56)\\
&= 0 \text{ if } p < \lambda^H R\\
&= 1 \text{ if } p = R
\end{aligned}
$$

and

$$
\begin{aligned}
f^H(p) &= \frac{R}{2p^2}\left(\frac{1+\sqrt{1+\frac{8v}{R}}}{3-\sqrt{1+\frac{8v}{R}}}\right) \text{ if } \lambda^H R \le p < R \qquad (57)\\
&= 0 \text{ if } p < \lambda^H R\\
m^H(p) &= \frac{1}{2} \text{ if } p = R.
\end{aligned}
$$

The expected price charged by the lower-priced firm is

$$
E(p_L) = R\left(\frac{1+\sqrt{1+\frac{8v}{R}}}{3-\sqrt{1+\frac{8v}{R}}}\right)\ln\left(\frac{4}{1+\sqrt{1+\frac{8v}{R}}}\right) \qquad (58)
$$

and it goes to $E(p_L) = \ln(2)R$ as $v \to 0$ ($M \to N_2$). The expected price charged by the higher-priced firm is

$$
E(p_H) = \frac{R}{2}\left(\frac{1+\sqrt{1+\frac{8v}{R}}}{3-\sqrt{1+\frac{8v}{R}}}\right)\ln\left(\frac{4}{1+\sqrt{1+\frac{8v}{R}}}\right) + \frac{R}{2} \qquad (59)
$$

and it goes to $E(p_1) = \frac{\ln(2)R}{2} + \frac{R}{2}$ as $v \to 0$ ($M \to N_2$). The expected transaction price is

$$
ET(p) = \frac{\frac{3R}{8}\left(1+\sqrt{1+\frac{8v}{R}}\right) - \frac{3v}{2}}{1-\frac{v}{R}}. \qquad (60)
$$

The expected transaction price in (11) goes to R as $M \to 0$. As $v \to 0$ ($M \to N_2$), $ET(p) \to \frac{3}{4}R$.

4 Conclusions

We have investigated price competition in a two-firm, simultaneous move, homogeneous good setting where only some of each firm's customers know of the other firm. We found that this game has no pure strategy equilibrium. When the firms have customer bases of the same size, we found a mixed strategy equilibrium where the firms are randomizing between prices that are strictly above marginal cost (whenever the customer base overlap is not perfect) and are therefore enjoying positive profits. When the customer bases have no overlap, we have two monopolies charging monopoly prices, and when the overlap is perfect, we have marginal cost pricing and zero profits. Therefore, we have found a simple solution to the Bertrand paradox that arises because of imperfect information, and an expected equilibrium price that varies smoothly between monopoly and marginal cost pricing when the overlap in the customer bases goes from zero to complete.

We also find the mixed strategy equilibrium when the size of the customer bases is asymmetric. The larger firm has a mass point at the monopoly price and the smaller firm puts zero weight on the monopoly price when the customer base overlap is positive. This asymmetric version of the model provides a monopoly pricing solution when the overlap in customer bases is zero but randomized pricing strictly above marginal cost for both firms when all the customers of the smaller firm know of the larger firm but when there are some customers of the larger firm who do not know of the existence of the smaller firm.

Last, we look at a two-stage game where firms advertise to manage their customer bases and then choose prices simultaneously to maximize profit. We find that the stage 1 game has two equilibria that are asymmetric in that one firm has a larger customer base than the other firm.

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