Incorporating monetary policy committee judgment with DSGE models

work in progress...

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Abstract

Policymakers tend to express forecast judgment in terms of paths for macroeconomic variables rather than the structure of DSGE models. We show how existing techniques can be used to incorporate judgment and produce conditional while maintaining forecasts most consistent with the DSGE model. In addition, we suggest measures for the extent of judgment imposed on the forecasts. Our particular application uses an open economy DSGE model of the New Zealand economy. We condition point-forecasts on interest rate projections from yield curve information, the published history of RBNZ forecasts, and interest rate projections from a BVAR. We also impose judgment on the density forecasts for inflation. We argue that these techniques can help understand monetary policy committee judgment and enhance communication between policymakers and modellers.

Keywords: DSGE models; monetary policy; conditional forecasts
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1 Introduction

DSGE models deliberately abstract from many things to present a stylized, but theoretically coherent, view of the economy. Recent DSGE models have developed features (such as habit formation on the part of consumers, and price

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indexation on the part of firms) that imply DSGE models can broadly match the data and produce forecasts competitive with other benchmark models (see Smets and Wouters (2003) and Adolfson et al. (2005)). This has sparked interest of central banks which have designed DSGE models with the goal of working directly in the forecast and policy environment (see for example, Binette et al. (2004), Brubakk et al. (2005), the DSGE model in Adolfson et al. (2005), Harrison et al. (2005) and Medina and Soto (2006) amongst others).

However, policymakers bring to DSGE models a wealth of experience and accumulated knowledge that is typically not directly interpretable in terms of the structure of the DSGE model. If DSGE models are to operate effectively in the policy environment, modellers need to consider how to optimally incorporate monetary policy committee judgment.

In this paper, we use existing techniques to find the set of future structural shocks with the minimal variance that returns the MPC judgment. This unique set of shocks incorporates MPC judgment while ensuring the paths are most consistent with the DSGE model. Thus the conditional forecasts will represent the most likely outcomes, given the MPC judgment.

Furthermore, the average standard error of the introduced structural shocks forms the basis of a metric we argue is useful to inform monetary policy committees considering adopting off-model judgment. The metric could also help identify periods when judgment is playing a larger than normal role.

In particular, we use both the ‘hard’ and ‘soft’ tune techniques of Waggoner and Zha (1999) to condition point and density forecasts from the DSGE model respectively. We apply the techniques to the forecast environment of the Reserve Bank of New Zealand (RBNZ). The Reserve Bank of New Zealand has the longest history of explicit inflation targeting and a long history of publishing endogenous interest rate forecasts from the Forecast and Policy System (FPS), the RBNZ’s core policy model, which we use for the first of our exercises. Our second application conditions the DSGE projections on a market-based interest rate track and seeks to interpret the economic conditions (shocks and/or parameters) required to return the market-based interest rate track. Finally, we explore conditioning the DSGE projections on a simple BVAR model used in the policy process at the RBNZ.

Typically, to condition a set of forecasts on specific judgment for the path of a given variable (for example, a flat track for the interest rate) a unique combination of univariate shocks is added. More generally when the number

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1 We use the nomenclature in the literature and use the term ‘hard tune’ to refer to a forecast conditioned on a single forecast path. A ‘soft tune’ refers to judgment applied to a density forecast. For example, a belief that annual inflation will fall between 1 and 3 percent 90 percent probability is an example of a soft tune.
of tuned variables is equal to the number of shock types we can choose from, the judgment or the combination of shocks required is unique and the problem is a trivial one. However, when the number of shock types we can choose from exceeds the number of variables to be tuned there exists an infinite number of potential shock combinations consistent with the judgment. The problem then becomes, how to choose the most efficient combination of shocks.

We use an algorithm based on Waggoner and Zha (1999) to obtain the set of shocks with minimal variance that satisfy the conditional forecast. We extend the Waggoner and Zha (1999) algorithm to the case of rational-expectations models where future shocks and more importantly the future paths of variables are anticipated by economic agents. Technically, we expand the standard reduced-form solution of a rational-expectations model forward to take into account today’s effect of future expected events (shocks) and adjust Waggoner and Zha (1999) for this expansion.

Many central banks publish fancharts and density forecasts for key macroeconomic variables (the Bank of England and the Riksbank, for example). While these particular fancharts are not presently constructed from DSGE models, estimated DSGE models seem to offer the promise of density forecasts for macroeconomic variables. We show how the “soft tune” techniques of Waggoner and Zha (1999), that involves resampling from both the DSGE parameters and realisation of the DSGE shocks, can be used to learn about the conditioning information applied to the density forecasts. In particular we work from the Reserve Bank’s explicit inflation target to provide possible conditional judgment.

The remainder of the paper is organised in the following sections. Section 2 discusses methods of adding judgment and the central bank forecasting and policy environment. Section 3 discusses three applications using hard tunes and section 4 discusses using soft tunes. Section 5 concludes.

2 A framework for thinking about judgment

It is typical to express the DSGE model in the following manner:

\[ A_0 y_t = A_1 E_t y_{t+1} + A_2 y_{t-1} + B \varepsilon_t + C \]  

where \( y_t \) is a vector of state variables, \( \varepsilon_t \) is a vector that contains a set of model shocks, \( C \) contains a vector of constants while the matrices \( A_0, A_1, A_2 \) and \( B \) determine the dynamics of the DSGE. This general representation may contain identities and lagged economic variables which implies the vector of
model shocks may contain zeros. Also, we restrict the structural shocks to Gaussian processes where the off-diagonal elements of the $B$ matrix are zeros.

When the model is expressed in terms of equation (1), the algorithms of Klein (2000) (based on the Schur decomposition) can be applied to solve for the reduced form of the model:

$$ y_t = F y_{t-1} + D + G \epsilon_t. \quad (2) $$

where $D$ is the vector of constants in equation (1) post-multiplied by the inverse of $A_0$. Using the reduced form representation, at time $t$, we can construct the $h$-step ahead forecast of the deviation of the vector of state variables $y_t$ from the vector of constants:

$$
\begin{align*}
y_{t+1|t} &= F(F y_{t-1} + D + G \epsilon_t) + D + G \epsilon_{t+1} \\
y_{t+2|t} &= F[F(F y_{t-1} + D + G \epsilon_t) + D + G \epsilon_{t+1}] + D + G \epsilon_{t+2} \\
&\vdots \\
y_{t+h|t} &= F^{h+1} y_{t-1} + (I + F^{h-1}) D + \sum_{i=0}^{h} F^h G \epsilon_{t+(h-i)}
\end{align*}
$$

(3)

Note that equation (3) decomposes forecasts of the state vector into three components: (i) the initial value of the state vector $y_{t-1}$, (ii) the vector of constants and (iii) the subsequent shock realisations $\sum_{i=0}^{h} F^h G \epsilon_{t+(h-i)}$. Clearly judgment can be added to the DSGE model via any combination of the three arguments that form the forecast variables.

Here, we focus on off-model judgment where the policymaker possesses a belief about the future path of the state vector $y_{t+h|t}$ that is exogenous to the model. Such beliefs might reasonably come from financial markets, business information visits, the acquired wisdom and experience of policymakers but are not directly related to: (i) specific beliefs about the structural parameterisation (captured in the matrices $A_0, A_1, A_2, B$); or (ii) the reduced form dynamics (captured by the matrices $F, G$, and $D$); and (iii) the vector of constants, irrespective of whether this is the structural steady-state parameters $C$ or the reduced form constants $D$.

In addition to thinking about point forecasts, we can think about adding judgment to density forecasts. Forecast distributions that account for shock uncertainty can be constructed via simulation. This is achieved by drawing from the distribution of shocks at each point $t, t+1...t+h$ and applying the matrix $F$ to each realisation of the forecasting process $y_{t+h}$.

Furthermore, if we possess distributions around the parameters in our model (and hence distributions around the elements of the forecasting matrix, $F$), by
drawing from both the parameter distributions and shocks distributions, we can produce forecasts that account for both shock and parameter uncertainty.

To work with distributions of forecasts we need to introduce some notation. We use the superscript \( j \) to denote the \( j \)-th draw from the shock distribution, where \( j \) runs from 1 to \( J \), the number of shock draws. For a given draw \( j \), the \( h \) period ahead forecast is given by

\[
y_{t+h|t}^{j} = \left( F^{j} \right)^{h+1} y_{t-1} + \left( I + \left( F^{j} \right)^{h-1} \right) D + \sum_{i=0}^{h} \left( F^{j} \right)^{h} G \epsilon_{t+(h-i)}^{j} \quad (4)
\]

This implies that \( y_{t+h}^{1:J} \) describes the entire forecast distribution of the vector of variables \( y_t \) at forecast horizon \( h \).

3 Applications

Central banks frequently operate a central monetary policy model (for example, the United Kingdom uses the Bank of England Quarterly Model (BEQM), New Zealand uses the Forecast and Policy System (FPS), and for some time Canada used the Quarterly Projection Model (QPM)). However, central banks’ rhetoric often refers to the use of a suite of models, primarily as a means of ensuring alternative beliefs and information are incorporated formally in the monetary policy committee process. This schizophrenia develops from the conflict between the desire to bring all viewpoints to bear on the monetary policy decision and a desire for a single organising framework for discussing alternative outcomes.

In this section we show how we can help resolve this conflict by using a DSGE model as a means of interpreting the forecasts from alternative models and judgment that we think is typical of the policy environment of many central banks. We focus our hard tune exercises on the policy rate track and illustrate how the DSGE model can be used to interpret the type of structural shocks most likely to generate the alternative interest rate paths. But the techniques are general enough to consider conditioning on forecasts for other key macroeconomic variables, such as output and inflation (singly or jointly).

The DSGE model we use to interpret the alternative interest rate paths is a calibrated version of a multi-sector DSGE model currently under development at the Reserve Bank of New Zealand. The open economy model consists of explicit production functions for export and non-tradable goods. Inflation processes for non-tradable goods, tradable goods and wages are characterized by quadratic adjustment costs that generate costs from monetary policy that
seeks to stabilise inflation. Description of the model is mainly relegated to a technical appendix that details the model including the optimisation problems faced by both households and firms. While the model contains some features specifically designed to address the nature of the New Zealand economy, the model contains many features common to the latest generation of DSGE models in use at several central banks. We also detail the forecast performance of the unconditional DSGE forecasts and the forecasts conditioned on the alternative monetary policy judgment.

Monetary Policy Committee (MPC) judgment can take many forms. It may be influenced by projections from satellite or indicator models, be driven by information from markets, or the policy makers intuition in general. We use three concrete examples for illustration, conditioning on the market’s implied interest rate track, the RBNZ’s historical published interest rate tracks, and the projections from a BVAR, to illustrate how interest rate tracks from different sources can be incorporated into a DSGE model using the hard tunes technique of Waggoner and Zha (1999). The model will replicate each alternative interest rate track, but because the shocks required to return each track differ, the forecasts of key macroeconomic variables will differ for each set of conditioning information. By fitting the set of model shocks with the smallest variance we uncover the conditional DSGE forecasts with highest probability.

Conditioning on market interest rates

Our particular applications include conditioning the DSGE model forecasts to a market interest rate track constructed from observations on the yield curve. This replicates the behaviour of the Bank of England monetary policy process who refrain from producing endogenous policy forecasts since the committee feels they cannot agree on the appropriate policy rule.

We use the commonly implemented Nelson and Siegel (1987) method, that fits a single function with four parameters, to explain the entire domain of the yield curve. 2 Eleven observations on zero coupon bank bill yields (evenly spaced at maturities from 30 to 180 days) and seven swap rates (maturities 1,2,3,4,5,7 and 10 years). Once the function for the yield curve has been estimated, spot rates at any point in the yield curve can be generated by integrating the estimated forward rate curve. We limit ourselves to conditioning on a market track up to eight periods into the future and thus obtain spot rates up to eight quarters in advance.

2 Several countries (including Belgium, Finland, France, Italy and Spain) estimated their zero coupon yield curve using the Nelson-Siegel method and reported their results to the BIS in the late 1990s (see Bank for International Settlements (2005). Other countries used the extended Nelson-Siegel method popularised by Svensson (1994) and spline methods, see Fisher et al. (1995)
Conditioning on RBNZ published forecasts

We also condition on the long history of the published endogenous forecast track from the Reserve Bank’s FPS model. Implicitly, this tests whether there is information in an alternative model that may improve the forecast performance of the DSGE. In addition, we can use the techniques outlined in section 2 to recover the types of structural shocks likely to be most consistent with the FPS interest rate track.

The Reserve Bank of New Zealand is unique in publishing a long history of endogenous interest tracks, mostly determined by the Forecast and Policy System (FPS), the RBNZ’s core model. The FPS model shares many features with the Bank of Canada’s Quarterly Projection Model (QPM) and has been described as a second generation macroeconomic model. By uncovering the DSGE shocks that are consistent with the interest rate track produced by the FPS model we can gain an understanding to the differences across the two models at a chosen forecast horizon.

Conditioning on forecasts from a Bayesian VAR

Finally, we condition on the interest rate forecasts from a Bayesian VAR model currently in use in the policy environment at the Reserve Bank of New Zealand. We show how the BVAR forecasts can be viewed in relation to the DSGE model to generate a structural interpretation, often absent from discussion of statistical model forecasts which tend to be predicated on time series properties of data series.

We choose a Bayesian VAR in particular because BVARs have been shown to produce good forecasting performance (see Litterman (1986) and Lees et al. (2007) for the case of New Zealand). Conditioning directly on aspects of the BVAR forecasts may be considered an alternative to applying the full blown DSGE-VAR methodology of Del Negro and Schorfheide (2004) in the policy process.

3.1 Forecast performance

Before turning to assess various judgment exercises, we test the forecast properties of both the unrestricted DSGE forecasts, and the DSGE forecasts conditioned on alternative information. We want to show that the DSGE model,

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3 The published interest tracks produced by FPS also include policy makers judgment in the form of shocks to the model.
and the DSGE model conditioned on alternative interest rate information are reasonable models to take forecasting.

Table 1 details the forecast performance of the alternative models. For each model, we construct forecasts of each variable, up to eight quarters ahead, at each quarter from 2000Q1 to 2004Q4. The forecast errors are cumulated from quarters 1 to $h$, except for interest rates which are forecast errors in levels.

Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>RMSEs from the DSGE conditioned on alternative interest rate information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h-step ahead</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>-------------------</td>
<td>----</td>
</tr>
<tr>
<td>Unconditional DSGE Model Forecasts</td>
<td></td>
</tr>
<tr>
<td>90-day rate</td>
<td>0.353</td>
</tr>
<tr>
<td>consumption growth</td>
<td>0.610</td>
</tr>
<tr>
<td>annual inflation</td>
<td>0.188</td>
</tr>
<tr>
<td>Conditioned on Market Interest Rate Track</td>
<td></td>
</tr>
<tr>
<td>90-day rate</td>
<td>0.551</td>
</tr>
<tr>
<td>consumption growth</td>
<td>0.596</td>
</tr>
<tr>
<td>annual inflation</td>
<td>0.245</td>
</tr>
<tr>
<td>Conditioned on BVAR interest rate forecasts</td>
<td></td>
</tr>
<tr>
<td>90-day rate</td>
<td>0.430</td>
</tr>
<tr>
<td>consumption growth</td>
<td>0.605</td>
</tr>
<tr>
<td>annual inflation</td>
<td>0.228</td>
</tr>
<tr>
<td>Conditioned on Published FPS interest rate forecasts</td>
<td></td>
</tr>
<tr>
<td>90-day rate</td>
<td>0.409</td>
</tr>
<tr>
<td>consumption growth</td>
<td>0.586</td>
</tr>
<tr>
<td>annual inflation</td>
<td>0.192</td>
</tr>
</tbody>
</table>

Consumption growth is expressed in percentage deviations in the level from actuals, effectively cumulating growth rate errors.
Since the DSGE model is calibrated based on data from 1992Q1 to 2006Q4. We cannot make strong statements about relative forecasting performance since the DSGE model incorporates some information not available to the other forecasting models. A full assessment of the forecast performance of the DSGE model would need to treat the issue of real-time data more rigorously.

However, Matheson (2006) shows that for New Zealand, AR(1) forecasts of annual CPI inflation return Root Mean Square Errors (RMSEs) for annual inflation of 0.56, 1.05 and 1.58 at horizons of 2, 4 and 8 quarters respectively. The DSGE model improves on this benchmark across the unconditional forecasts and all the conditioned forecasts. Other benchmarks such as the RMSEs behind the comparisons in Lees et al. (2007) suggest reasonable forecast performance from the DSGE model and the conditioned policy tracks. In terms of forecasting at least, the conditioning exercises we consider appear reasonable directions in which to push the model. Furthermore, conditioning on alternative interest paths does not lead to a stark deterioration in forecasting performance.

3.2 Hard tunes for point forecasts

In order to gain the most from our series of applications, we apply our techniques to an episode within the recent history of the RBNZ and focus on the policy situation at the end of 2003. Furthermore, we compare forecasts conditioned using the multi shock approach to the standard limited case, where interest rate tracks are conditioned on a set of univariate monetary policy shocks only.

In the December 2003 statement, the Reserve Bank left the Official Cash Rate on hold at 5.0%. Inflation in imported goods had fallen over recent quarters due to an appreciation in the exchange rate but “underlying domestic inflation pressure” was expected to lift over the next year or so. In the Monetary Policy Statement that accompanied the decision, Governor Alan Bollard commented: “...small increases in the OCR may be required over the year ahead to ensure that inflation remains comfortably within the target range over the medium term.” This was reflected in the track for the ninety day interest rate, published in the Monetary Policy Statement, that was consistent with some increase in rates over 2004 and 2005.

At that point, market rates were more sanguine about the inflation outlook and growth outlook, and suggested a much smaller tightening over the forthcoming quarters. Figure 1 shows the FPS track implied by an additional two policy tightenings by the end of 2005 relatively to financial markets. In addition, the figure shows that forecasts from a BVAR would have suggested a relatively
benign interest rate track.

Fig. 1. Interest rate forecast path: December 2003

To interpret the differences in the forecasts, we tune the DSGE model to each of the three alternative interest rate paths in turn, finding the minimal set of shocks that returns the forecast paths. Figure 2 shows the conditional forecasts of other key macroeconomic variables when the DSGE model is conditioned on the three alternative interest rate tracks, allowing all fourteen shocks to vary. As a comparison, figure 3 shows the conditional forecasts when we tune the DSGE model to each of the three alternative interest rate paths using only univariate monetary policy shocks.  

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4 In this case, the judgment required to return each track is unique, only one set of monetary policy shocks is consistent with each outcome.
Fig. 2. Forecast paths conditioned on interest rate tracks: All shocks

![Graphs showing forecast paths for QoQ Inflation Annualised, Tradable Inflation, Non-tradable Inflation, and Consumption Growth conditioned on interest rate tracks.]

Fig. 3. Forecast paths conditioned on interest rate tracks: Monetary policy shocks

![Graphs showing forecast paths for QoQ Inflation Annualised, Tradable Inflation, Non-tradable Inflation, and Consumption Growth conditioned on monetary policy shocks.]

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In the multivariate shock case, inflation and consumption growth are broadly similar across the three tuned models. However compared to the no judgment paths, inflation is a little lower, while consumption growth is a little stronger. Across all cases, the DSGE model is tuned to interest rate tracks that call for higher interest rates initially, relative to the model’s baseline, no judgment (endogenous) track. Because we allow the algorithm to choose from all fourteen shocks in the model, the forecasts show the most likely outcomes for the key macroeconomic variables, given the respective interest rate tracks.

The monetary policy rule in the model is both a function of inflation outcomes and monetary policy shocks (deviations from the rule). As a consequence the interest rate tracks in the tuned cases are lower due to both decreased inflation pressure and to the monetary authority setting interest rates at a level lower than their rule suggests. Lower interest rates allow slightly higher consumption growth.

When we tune the alternative interest rate tracks using only monetary policy shocks, inflation and consumption growth are higher than in the no judgment case. By only adding monetary policy shocks, we imply the lower interest rate tracks are solely due the monetary authority deviating from their rule. By setting interest rates at a level lower than their rule dictates, inflation and consumption growth are allowed to increase. Given the interest rate rule is a fair representation of how policy is set, and in the absence of further information, allowing the algorithm to choose from all the shocks in the model when adding judgment is going to return the highest probability forecasts for the other key macroeconomic variables.

To gain further intuition and understanding about the implications of particular monetary policy judgment, we can examine where the algorithm has chosen to put the shocks in the model. Figure 4 depicts the shocks for the multivariate case while figure 5 displays the case for the univariate monetary policy shocks. Because many of the shocks are trivial in the multivariate case, figure 4 displays the six largest shocks.
Fig. 4. Implied DSGE shocks: December 2003

Fig. 5. Implied univariate interest rate shocks: December 2003
Clearly, the DSGE model interprets the initial higher interest paths as consistent with positive monetary policy shocks in the initial period. The multivariate algorithm also attributes positive shocks to both tradable and non-tradable prices that policy responds to, since the least squares metric penalises attributing the shock to a single part of the model. In the following periods the monetary policy and inflation shocks are negative reflecting the tuned interest rate tracks being lower than in the no judgment scenario. This is consistent with the algorithm choosing shocks so that monetary policy is partially an endogenous response to inflation outcomes and partially a response to the monetary authority deviating from their rule.

Figure 5 above shows that larger monetary policy shocks are required to match the alternative interest rate tracks in the univariate shock case than in the multivariate shock case. Leeper and Zha (2003) suggest that interpreting these shocks as monetary policy shocks may be problematic since persistent deviations from the policy rule will cause agents to revisit their expectations about the how policy will be implemented in the future. Allowing all the shocks in the model to adjust in the face of judgment limits the extent to which large monetary policy shocks are required to return a specific policy path.

Usefully, the average standard error of the requisite shocks for returning the particular policymaker judgment can be used as a measure of the extent to which judgement is contrary to the model output. Policymakers or monetary policy committees might reconsider imposing particular off-model judgment that appears less likely relative to alternative paths or less likely relative to the degree of judgment added historically. Furthermore, one could calculate the mean of the judgment shocks as a measure of the overall direction of the judgment that is added to the model.

We define the JASE (Judgment Average Standard Error) as the average of the model standard errors required to return policymaker judgment. For the examples we consider, returning the interest rate track given by the FPS track gives the highest JASE number of 0.078. The tracks implied by the yield curve and the BVAR return JASE numbers of 0.075 and 0.069 respectively. This suggests that more judgment is required on average to match the DSGEs policy response to the published FPS track, than is required to match the market track or the BVAR track.

3.3 Soft tunes for density forecasts

Mervyn King has emphasized the need for monetary policy committees to consider the full distribution of alternative policies. The DSGE model we operate is log-linear and relative to second-generation models such as FPS, it
is computationally cheap to generate forecasts drawn from the distribution of parameters and possible shocks.

Monetary policy committees will naturally want to place judgment on the density forecasts from the model, in the same manner that monetary policy committees will want to place judgment on point forecasts from a specific model. We use the soft tunes techniques to impose judgment on the density forecast.

Both Cogley et al. (2005) and Dowd (2007) have noted that the fancharts produced by the Bank of England have tended to overstate inflation risks such that the predictive density is wider than the observed distribution of inflation. The Bank of England’s forecasts are not derived from a specific model and this judgment is the agreed monetary policy committee have applied.

For our exercise we stand the Bank of England approach on its head by taking the distribution from the DSGE model and assume that the monetary policy committee desires that the forecast densities for inflation are reasonable given the historical range for inflation. We believe that it is perfectly reasonable to assume that inflation will remain between 0 and 5% for the eight quarters beginning in 2004Q1. We then remove the draws from the distribution that lie outside this interval. Figure 6 below shows the density forecast for inflation with the distributions removed coloured green.

Fig. 6. Soft tune for inflation: December 2003
Cogley et al. (2005) suggest using entropy to adjust the fan chart such that the density forecast matches the monetary policy judgment with regard to a particular moment, most likely, the mean or standard deviation. The Kullback-Leibler Information Criteria is used as the metric for finding the minimal shift in the forecast density to return the required moments (see Robertson et al. (2005) for example). However, this method necessarily treats the forecast density for each variable in turn, such that adjusting the forecast density for inflation has no implication for other forecast densities in the model.

The soft tune implies discarding model parameters and sequences of shocks that do not match the monetary policy committee judgment. This has implications not only for the forecast density for inflation but for forecast densities of all the other variables in the model. Thus the monetary policy committee is confronted by the impact of their judgment on other macroeconomic variables. Figure 7 depicts the impact of our judgment exercise on consumption and inflation.

In addition, this analysis suggests the percentage of draws that are inconsistent with the judgment applied to the forecast density is a natural metric for the degree of judgment added. For this particular example, 9.4% of the draw are removed. The requirement that annual inflation lies between 0% and 5% is not particularly onerous for the DSGE model to replicate.
4 Conclusion

Monetary policy committee judgment is most often expressed in terms of observable paths for key macroeconomic variables rather than the deep parameters and shocks that make up DSGE models. However, several easily implemented techniques allow the addition of judgment to both point and density forecasts produced by macroeconomic models. Using a multiple shock approach allows judgment to enter forecasts with the least amount of disruption to the consistency of the model. Certainly, there appears little to suggest formal modeling of the economy makes it difficult to incorporate policymakers’ off-model judgment. The structure that DSGE models impose on forecasts implies that they can assist in the interpretation of other forecasts in the policy environment.

5 Appendix

Within this section we list the equations and optimisation processes behind the DSGE model. Details of the model calibration and implied model dynamics are given in Beneš et al. (2006).

Non-tradable sector. We set up a continuum of firms indexed on \([0, 1]\) producing a differentiated variety of non-tradable goods.

The production function of firm \(j \in [0, 1]\) is

\[
Y^n_{jt} = (h_{jt} - h_0)^\gamma_n \cdot a^n_t,
\]

where \(Y^n_{jt}\) denotes the level of output, \(h_{jt}\) is the amount of labour (hours worked) demanded by the firm, \(h_0\) is the overhead (or subsistence) level of labour, \(a^n_t\) is the level of technology in the non-tradable sector, and \(\gamma_n \in (0, 1]\) is labour’s share of non-tradable production. A certain amount of labour is required for the firm to open their doors. This implies a short-run elasticity of the labour demand schedule with procyclical labour productivity (see Rotemberg and Woodford (1999) for example).

As a result of optimisation on the behalf of all other agents in the model the firm faces a downward-sloping demand curve for its own output

\[
Y^n_{jt} = c^n_{jt} + g^n_{jt} + j^n_{jt} = (c^n_t + g_t + j^n_t) \cdot (p^n_{jt}/P^n_t)^{-1/(\mu^n - 1)},
\]

\(^5\) Then, a diminishing marginal product of labour (ensured by \(\gamma_n < 1\)) entail the dependence of real marginal costs on the scale of production which is an empirical regularity observed in most industries.
where \( p^n_jt \) is the final price of non-tradable good \( j \), \( P^n_t \) is a CES aggregate price index defined over the whole non-tradable variety, and \( c^n_{jt} \), \( g^n_{jt} \) and \( j^n_{jt} \) are the quantities of good \( j \) demanded by the household, the government, and the landlord, respectively. In addition, \( c^n_t \), \( g^n_t \) and \( j^n_t \) are the corresponding CES aggregate non-tradable demand indices with the elasticity of substitution \( \mu^n_t / (\mu^n_t - 1) \).

Labour is paid in advance and non-tradable firms have to borrow from the financial intermediary to pay the wage bill in advance. This creates demand for one-period bank loans available at a lending rate \( i^\ell_t \),

\[
\ell_{jt} = W_t h_{jt},
\]

where \( W_t \) is the nominal wage rate.

Each non-tradable firm chooses \( Y^n_{jt}, p^n_{jt}, h_{jt}, \) and \( \ell^n_{jt} \) to maximise the expected sum of its present and future instantaneous earnings,

\[
\Phi^n_{jt} = p^n_{jt} Y^n_{jt} - \ell^n_{jt-1} (1 + i^\ell_{t-1}),
\]

discounted by the household’s shadow value of wealth, subject to (5), (6), (7), and a sticky-price constraint.

**Tradeable sector.** There is also a continuum of importers, or retail distributors, indexed on \([0, 1]\). Each importer purchases homogeneous import goods abroad, differentiates them costlessly, and sells them to consumers in the home country under its own brand.

The production function of importer \( j \in [0, 1] \) is simply

\[
Y^n_{jt} = m^n_{jt} \cdot a^n_t,
\]

where \( Y^n_{jt} \) denotes the supply of tradable consumption good \( j \), \( m^n_{jt} \) is the quantity of the import goods needed for its production, and \( a^n_t \) is a technology process introduced to allow for permanent changes in the real exchange rate.

The technology term, \( a^n_t \), is a unit-root process that can account for permanent shifts in the relative price of imports and domestically sold tradables without altering the steady-state nominal expenditure shares of imports or tradables.

The importer faces a downward-sloping demand curve for their own output derived from the household’s and the exporter’s expenditure minimisation,

\[
Y^n_{jt} = c^n_{jt} + j^n_{jt} = (c^n_t + j^n_t) \cdot \left( \frac{p^n_{jt}}{P^n_t} \right)^{-1 / (\mu^n_t - 1)},
\]

where \( p^n_{jt} \) is the final price of tradable good \( j \), \( P^n_t \) is a CES aggregate price index defined over the whole tradable variety, \( c^n_{jt} \) and \( j^n_{jt} \) is the quantity of the good.
demanded by the household (consumption) and the exporter (investment), respectively. Also $c_t^r$ and $j_t^r$ are the corresponding CES aggregate tradable demand indices with the elasticity of substitution $\mu_t^r/(\mu_t^r - 1)$.

Each importer chooses $c_{jt}^r$, $p_{jt}^r$, $m_{jt}^r$ to maximise the expected sum of its present and future instantaneous earnings,

$$\Phi_{jt}^r = p_{jt}^r Y_{jt}^r - P_t^m m_{jt},$$

where $P_t^m = P_t^m^*/S_t$, discounted by the household’s shadow value of wealth, subject to (8), (9), and a sticky-price constraint similar to that of non-tradable firms, with $P_t^m^*$ being the foreign-currency price of imports, and $S_t$ the nominal exchange rate.

**Export sector.** The perfectly competitive export sector is represented by a single firm behaving competitively in both input and output markets. The production function of the export firm is

$$X_t = (k_{t-1}^x)^{1-\gamma_x} \cdot (a_t^x)^{\gamma_x},$$

(10)

where $X_t$ denotes the production of export goods, $k_{t-1}^x$ is the time $t-1$ quantity of capital available for export production at time $t$, $a_t^x$ is the level of export technology, and $\gamma_x$ is capital’s share of export production. Note that capital has to be in place one period in advance for production to take place.\footnote{This ensures that exporting firms cannot simply boost production in response to a booming world export price by utilising labour from the non-tradable sector.}

Capital, $k_t$, and investment, $j_t^x$, are related in a non-linear Cobb-Douglas way,

$$k_t^x = (k_{t-1}^x)^{(1-\delta)} (j_t^x / \delta)^{\delta}$$

(11)

where $j_t^x$ is gross investment, and exp $\alpha_x$ is the steady-state gross rate of growth in the export sector. Gross investment $j_t^x$ is a CES bundle of individual types of tradable goods sold, however, at perfectly flexible prices. We assume the same elasticity of substitution $1/(\mu_t^r - 1)$ as for the household.

The firm takes export prices, $P_t^x = P_t^x^*/S_t$, as given, and chooses $X_t$ and $k_t^x$ to maximise the expected sum of its present and future earnings,

$$\Phi_t^x = P_t^x X_t - P_t^x j_t^x,$$

discounted by the household’s shadow value of wealth, subject to (10) and (11).

**Housing sector.**
In the present version of the model the housing sector is meant to be a placeholder that will be developed in the future but facilitates the parameterisation of the steady-state nominal expenditure ratios. We assume that there is a representative landlord who uses non-tradable resources as residential investments, \( I^r_t \), demanding them in a fixed proportion to private consumption,

\[
P^r_t I^r_t = \nu_r \cdot P^n_t c^n_t,
\]

where \( \nu_r > 0 \), and that residential investments do not affect the household’s marginal utility functions.

There is a single representative household consisting of a continuum of members indexed on \([0, 1]\). Each member of the household is endowed with differentiated labour skills. The household’s lifetime preferences are given by

\[
\sum_{k=0}^{\infty} \beta^k \exp \epsilon^{cd}_t \left( \log \Gamma_{t+k} - \int_0^1 H_{jt+k} \mathrm{d}j \right),
\]

where \( \Gamma_t \) is a CES real consumption index defined over tradable and non-tradable consumption, \( H_{jt} \) are the hours worked by member \( j \), \( \beta \in (0, 1) \) is the discount factor, and \( \epsilon_t^{cd} \) is an intertemporal preference, or consumption demand, shifter.\(^7\)

We assume a unit elasticity of intratemporal substitution between tradable and non-tradable consumption in the long-run. However, we achieve lower short-run elasticities by following Ravn et al. (2006) and introducing deep habit into consumption behaviour

\[
\Gamma_t = \left( \frac{c^t_{\tau t} - \chi b^t_{\tau t}}{c^n_{\tau t} - \chi b^n_{\tau t}} \omega \right)^{1-\omega},
\]

where \( c^t_{\tau t} \) and \( c^n_{\tau t} \) are themselves CES indices introduced below and \( b^t_{\tau t} \) and \( b^n_{\tau t} \) are the habit reference levels in tradable and non-tradable consumption respectively. The parameter \( \omega \in (0, 1) \) determines the tradable expenditure share of total consumption in the long-run and \( 0 < \chi < 1 \) is the weight of habit.\(^8\)

Furthermore, the tradable and non-tradable consumption indices, \( c^t_t \) and \( c^n_t \), are defined over the respective varieties of consumption goods and have the

\(^7\) Utility is of the log form in consumption and linear in the disutility of labour such that the intertemporal elasticity of substitution in consumption is unitary, whereas the Frisch elasticity of labour substitution is infinity.

\(^8\) Deep habit not only reduces the short-run intratemporal elasticity but also the intertemporal elasticity. The magnitude of these short-run elasticities depend, in general, on the relative distance between today’s consumption and the habit reference levels. The reference levels are, in turn, determined by the previous period’s consumption. However, we exclude these effects from the household’s optimization.
standard continuous Dixit and Stiglitz (1977) CES form, with possibly time-varying elasticities of substitution equal to $1/(\mu_\tau^\tau - 1) > 1$ and $1/(\mu_n^n - 1) > 1$.

This gives rise to a system of demand curves for individual goods $\forall j \in [0, 1]$:

$$c^\tau_{jt} = c^\tau_t \cdot (p^\tau_{jt}/p^\tau_t)^{-\mu_\tau^\tau/(\mu_\tau^\tau - 1)},$$ (14a)

$$c^n_{jt} = c^n_t \cdot (p^n_{jt}/p^n_t)^{-\mu_n^n/(\mu_n^n - 1)},$$ (14b)

where we use the underlying expenditure-minimizing price indices, $P^\tau_t$ and $P^n_t$.

The household’s optimisation problem proceeds as follows. The household decides on the intertemporal consumption and savings plan using deposit accounts, $d_t$, as the only available asset (yielding a deposit rate $i^d_t$). The household takes labour income, $\int_0^1 w_{jt} H_{jt} d j$, as given, maximizing (12) subject to a budget constraint,

$$d_t + \int_0^1 (p^n_{jt} c^n_{jt} + p^\tau_{jt} c^\tau_{jt}) d j = d_{t-1} (1 + i^d_{t-1})$$

$$+ \int_0^1 w_{jt} H_{jt} d j + \Phi_t + \Psi_t - T_t,$$ (15)

in which $\int_0^1 p^n_{jt} c^n_{jt} d j$ and $\int_0^1 p^\tau_{jt} c^\tau_{jt} d j$ are total spending on non-tradable and tradable consumption, respectively,

$$\Phi_t = \int_0^1 \Phi^n_{jt} d j + \int_0^1 \Phi^\tau_{jt} d j + \Phi^\tau_t + \Phi^\tau^f,$$

is the sum of domestic firms’ instantaneous earnings transferred to the household in each period, $\Psi_t$ are the factor payments received from the financial intermediary (see subsection 9) and $T_t$ is a lump-sum government tax.

At the same time, each member of the household sets its own wage rate, $w_{jt}$, and labour supply measured by hours worked, $H_{jt}$. These decisions maximize the same utility function (12) subject to (15), and a sticky-wage constraint. Aggregate consumption, everyone else’s wage rate and labour effort, is taken as given.

Finally, we assume that individual labour skills are bundled into a CES labour aggregate, $h_t$, with a possibly time-varying elasticity of substitution $1/(\mu^h_t - 1) > 1$; the labour aggregate is then supplied to non-tradable firms as homogeneous labour input. As a consequence, we get the following system of problem which then becomes:

$$b^\tau_t = \pi^\tau_{t-1} \cdot \exp \alpha_x$$

$$b^n_t = \pi^n_{t-1} \cdot \exp \alpha_n$$

where $\exp \alpha_x$ and $\exp \alpha_n$ is the steady-state gross rate of growth in tradable and non-tradable consumption, respectively.
downward-sloping demand curves for individual labour skills $\forall j \in [0, 1]$: 

$$H_j(t) = h_t \cdot (w_{jt}/W_t)^{-1/(\mu_\tau - 1)}.$$ 

(16)

**Financial sector.**

There is a single representative risk-neutral financial intermediary who behaves competitively. The intermediary collects deposits, $D_t$, from the household, paying a deposit rate $i^d_t$ on them, and supplies loans to firms, $L_t$, charging a lending rate $i^\ell_t$. Financial intermediation is, however, assumed to be a costly activity and the production of deposits and loans incurs private costs.\(^{10}\)

We assume that the total amount paid by the intermediary as costs is ultimately received by the household. The net surplus or need of funds, $D_t - L_t$, is cleared with the international interbank market and with assumed risk-neutrality of agents, the following form of UIP holds instantaneously,

$$1 + i_t = (1 + i^*_t) \cdot E_t(S_t/S_{t+1}) \cdot \exp z_t,$$

where $i_t$ is the home policy rate set by the central bank, $i^*_t$ is the foreign nominal interest rate, and $z_t$ is an exogenous risk premia, for example.

The intermediary chooses $D_t$ and $L_t$ to maximise instantaneous earnings

$$\Phi^f_{t+1} = D_t(i_t - i^d_t) + L(i^\ell_t - i_t) - \Psi_t,$$

(17)

taking $i_t$, $i^d_t$, and $i^\ell_t$ as given, where $\Psi_t = f(D_t, L_t)$ is the cost of intermediation. We describe the cost function by the first-order Taylor expansion of the marginal cost of deposits and marginal cost of loans, around a balanced growth path in which $D_t = L_t$,

$$f_D(D_t, L_t) \approx \zeta_0 + \zeta_D \cdot \log D_t/L_t$$

(18)

$$f_L(D_t, L_t) \approx \zeta_0 + \zeta_L \cdot \log L_t/D_t$$

(19)

with $\zeta_0$, $\zeta_D$, $\zeta_L > 0$.

If tradable and non-tradable prices and nominal wages were flexible, we ob-

\(^{10}\) This is an industrial organisation approach to banking and we adopt the financial sector setup from Diaz-Gimenez et al. (1992) extended with cyclical fluctuations in the lending and deposit spreads as in Edwards and Végh (1997).
serve

\[ p_{jt}^{n} = \mu_{t}^{n} \cdot \frac{W_{t}(h_{jt} - h_{0})}{\gamma_{n} Y_{jt}^{n}} \cdot \frac{1 + i_{t}^{n}}{1 + i_{t}^{n}}, \]

\[ p_{jt}^{T} = \mu_{t}^{T} \cdot \frac{P_{t}^{m*}}{S_{t}}, \]

\[ w_{jt}^{\dagger} = \mu_{t}^{w} \cdot \kappa \Pi_{t} \Gamma_{t}. \]

where \( \dagger \)'s denote flexible price equilibria. However, we make prices and wages sticky by assuming quadratic price adjustment costs. As in Rotemberg (1982) we now express a firms’ lifetime profit function using its second-order Taylor log-expansion calculated around the flexible-price equilibrium.\(^{11}\) That is,

\[ -\sum_{k=0}^{\infty} \beta^{k} \left[ \xi_{n} \cdot (\log p_{jt}^{n} - \log p_{jt}^{n\dagger} - \frac{1}{\xi_{n}} \epsilon_{p_{jt}^{n}}^{m})^{2} + (\Delta \log p_{jt}^{n} - \Delta \log P_{t}^{n})^{2} \right], \quad (20) \]

for the non-tradable sector, and

\[ -\sum_{k=0}^{\infty} \beta^{k} \left[ \xi_{T} \cdot (\log p_{jt}^{T} - \log p_{jt}^{T\dagger} - \frac{1}{\xi_{T}} \epsilon_{p_{jt}^{T}}^{m})^{2} + (\Delta \log p_{jt}^{T} - \Delta \log P_{t}^{T})^{2} \right], \quad (21) \]

for the tradable sector. Analogously, we can approximate the second-order effects of wage stickiness on the household’s utility,

\[ -\sum_{k=0}^{\infty} \beta^{k} \left[ \xi_{w} \cdot (\log w_{jt} - \log w_{jt}^{\dagger} - \frac{1}{\xi_{w}} \epsilon_{w_{jt}}^{\ell_{s}})^{2} + (\Delta \log w_{jt} - \Delta \log W_{t})^{2} \right]. \quad (22) \]

The three new parameters, \( \xi_{n}, \xi_{T}, \xi_{w} > 0 \), describe the importance of the cost of deviating from the flexible price equilibrium relative to the cost of departing from the market average price or wage changes; the higher \( \xi \) the more flexible the prices or wages are. Finally, \( \epsilon_{p_{jt}^{n}}^{m}, \epsilon_{p_{jt}^{T}}^{m}, \) and \( \epsilon_{w_{jt}}^{\ell_{s}} \) are a non-tradables cost shifter, a tradables cost shifter, and a labour supply shifter, respectively.

**Government.**

The government purchases non-tradable goods and levies a lump-sum tax on the household to finance this spending. The government’s budget is balanced at each moment and the tax is set according to a simple rule,

\[ T_{t} = \int_{0}^{1} p_{jt}^{n} g_{jt} \, dj. \]

We assume that the government wishes to minimize real distortions incurred by public demand for privately produced goods. The government then adopts the household’s system of demand curves for individual goods so

\[ g_{jt} = g_{t} \cdot (p_{jt}^{n}/P_{t}^{n})^{-1/(\mu_{t}^{n} - 1)}, \]

\(^{11}\)Note that first-order terms vanish by construction.
where \( g_t \) is an overall CES consumption index defined over individual quantities \( g_{jt} \) with the household’s elasticity of substitution \( 1/(\mu^n_t - 1) \). We close the fiscal policy with an exogenous process for \( g_t \) such that the government’s total nominal expenditures, \( \int_0^t p^n_j g_{jt} \text{d}j \), are kept in a fixed proportion \( \nu_g \) to total nominal value added (i.e. gross domestic product).

The central bank is an inflation targeter that uses a simple policy rule to stabilise inflation. In particular, we use

\[
\dot{i}_t = \rho_i \dot{i}_{t-1} + (1 - \rho_i) \left[ \dot{i} + \kappa_n \left( P^n_t - \pi_n \right) + \kappa_T \left( \dot{P}^T_t - \pi_T \right) \right] + \epsilon^{mp}_t. \tag{23}
\]

where \( \dot{i}_t = \log(1 + i_t) \), \( \kappa_T \) and \( \kappa_n \) determine the reaction of the central bank to tradables and non-tradables inflation, respectively, with \( \pi_T \) and \( \pi_n \) being, respectively, the implicit tradables and non-tradables inflation targets adjusted appropriately for the systematic inflation differential given by the productivity growth wedge between the export and non-tradable sectors, \( \alpha_x - \alpha_n \), and \( \epsilon^{mp}_t \) is a monetary policy surprise.

The exogenous preference and technology processes evolve as follows.

Export sector technology:

\[
a^x_t = a^x_{t-1} + \alpha_x + \epsilon^{ax}_t.
\]

Non-tradable sector technology:

\[
a^n_t = a^n_{t-1} + \alpha_n + \epsilon^{an}_t.
\]

Tradeable sector technology (real exchange rate trend):

\[
a^\tau_t = a^\tau_{t-1} + \epsilon^{a\tau}_t.
\]

Wage markup:

\[
\mu^w_t = \mu^w_{t-1} + \epsilon^{wm}_t.
\]

Terms of trade:

\[
\tau_t = \tau_{t-1} + \epsilon^{tot}_t,
\]

where \( \tau_t = \log \left( \frac{P^x_t}{P^m_t} \right) \).

**Non-tradable sector.**

\[
Y^n = (H - H_0)\gamma_n \cdot a^n \tag{24}
\]

\[
P^n = \mu_n \cdot \frac{W(H + H_0)}{\gamma_n Y^n} \tag{25}
\]

The price of the non-tradable good is equal to a mark up over the marginal cost in the steady-state.

\[
Y^n = C^n + G + J^n \tag{26}
\]
The supply of non-tradable goods is equal to the demand for non-tradable goods.

**Tradeable sector.**

\[ P^\tau = \mu \cdot P^m / \alpha^\tau \]  
(27)

The price of the imported tradable good in the home country is a mark-up over the foreign price.

\[ Y^\tau = M \cdot a^\tau \]  
(28)

The amount of imported goods is equal to the aggregate demand for them adjusted for the technology term; the supply of imported goods at a given price is unlimited as Home is a small economy.

**Export sector.**

\[ X = (K^x)^{1-\gamma_x} \cdot (a^x)^{\gamma_x}, \]  
(29)

The first order conditions to determine the optimal choice of capital and investment are:

\[ \lambda' = \frac{1}{1 + i^d} \left[ (1 - \gamma_x)P^x_{+1}X_{+1}/K^x + \lambda'_{+1} \cdot (1 - \delta) \cdot K^x_{+1}/K \right], \]  
(30)

\[ \lambda' \cdot \delta \cdot K^x / J^x = \mu \cdot P^m, \]  
(31)

where \( \lambda' \) is the Lagrange multiplier associated with the capital accumulation equation (11), and can be therefore interpreted as the shadow value of capital.

**Household.**

\[ \Gamma = (C^\tau)^{\omega} \cdot (C^n)^{1-\omega} \]  
(32)

\[ H^\tau = C^\tau \]  
(33)

\[ H^n = C^n \]  
(34)

The consumption index collapses into the Cobb-Douglas aggregation of tradable and non-traded consumption, and the habit reference level follows actual consumption.

\[ \omega \cdot \Pi \Gamma = (1 - \chi) \cdot P^\tau C^\tau \]  
(35)

\[ (1 - \omega) \cdot \Pi \Gamma = (1 - \chi) \cdot P^n C^n \]  
(36)

Expenditures on tradable consumption and expenditures on non-tradable consumption are fixed proportions of total expenditures.

\[ W = \mu_w \cdot \kappa \cdot \Pi \Gamma \]  
(37)

The nominal wage rate is a mark-up over the marginal rate of substitution between consumption and labour, which is proportional to nominal consumption and independent of hours worked for our particular utility function.
Financial intermediation.

\[ i^d = i - \zeta_0 \]
\[ i^e = i + \zeta_0 \]

The costs of intermediation give rise to permanent spreads between the lending, policy and deposit rates.

\[
(1 + i^*)/(1 + i) = S_{+1}/S \tag{38}
\]

No-arbitrage condition in the international financial market (the UIP).

\[ L_t = D_t \tag{39} \]

The net supply or need of funds is zero.
References


Lees, Kirdan, Troy Matheson and Christie Smith (2007). Open economy DSGE-VAR forecasting and policy analysis - head to head with the RBNZ published forecasts.


