Fiscal Rules and the Intergenerational Welfare Effects of Public Investment

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Abstract

This paper studies the intergenerational welfare effects of a public investment scale up under alternative fiscal rules. In the European Union, most member states face serious needs for public infrastructure investment. The recently-signed Fiscal Compact, however, imposes a balanced budget requirement on any type of structural spending. Using a dynamic overlapping generations model of a small open economy, I argue that such a balanced budget constraint causes public infrastructure investment to decrease the welfare of existing generations even when public capital is initially scarce. Current generations are thus likely to oppose fiscal policies aiming at higher public infrastructure spending. A golden rule that allows net investment to be debt-financed, on the other hand, may improve the welfare of both current and future generations.

JEL codes: E62, F41, H54

Keywords: Infrastructure capital, public investment, distortionary taxation, intergenerational welfare, Yaari-Blanchard overlapping generations

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1 Introduction

Three decades of declining public investment-to-GDP ratios have left most advanced economies with serious infrastructure needs. The OECD (2006, 2012) and the World Economic Forum (WEF, 2014), among other policy institutions, speak of large infrastructure gaps. According to some estimates, public capital-to-GDP ratios have decreased by about five percentage points since the 1970s (Abiad et al., 2014). A public infrastructure scale up may be necessary to sustain long-run growth. But, how to pay for it? Strong fiscal imbalances in the aftermath of the crisis have, in fact, led most advanced countries to adopt fiscal rules or tighten existing ones (Schaechter et al., 2012). European Union member states, for instance, have recently signed a Fiscal Compact, which limits structural deficits to 0.5% of GDP. In practice, this pact imposes a balanced budget requirement on structural increases to public investment spending. Can countries scale up public infrastructure when constrained by balanced budget rules? Should such rules exempt productive infrastructure spending?

This paper argues that a balanced-budget requirement on total spending—such as the Fiscal Compact—makes public infrastructure investment hard to scale up, even when the stock of public capital is initially scarce. In this fiscal scenario, a permanent increase in public investment improves the welfare of future generations but at the cost of decreasing the welfare of current generations, especially the older ones. Current generations are therefore likely to politically oppose higher levels of public infrastructure spending. Fiscal rules allowing public investment to be debt-financed, on the other hand, potentially improve the welfare of current generations as well.

I study the intergenerational welfare effects of public investment using a dynamic general equilibrium macroeconomic model of a small open economy. On the household side, the model features Blanchard-Yaari overlapping and disconnected generations of finitely-lived individuals facing a consumption-leisure choice. On the firm side, the model considers production spillovers from government-supplied public infrastructure to private firms, and capital adjustment costs. On the government side, the model considers a single tax instrument, namely, a (distortionary) labor income tax. I consider three fiscal scenarios: (i) a balanced-budget
rule, (ii) a tax-smoothing rule, and (iii) a golden rule for net investment. I derive the analytical impulse response functions and intergenerational welfare effects from the log-linearized model. I then calibrate the model for a typical economy in the euro area and simulate the macroeconomic and intergenerational welfare effects numerically.

I find two main results. First, the balanced-budget rule implies a sharp increase in the tax rate on impact, thereby severely contracting employment, output, and the market value of the private capital stock, even when the public capital stock is initially scarce. The mechanism generating this strong short-run contraction operates as follows: (1) to finance the public investment impulse, the tax rate must increase on impact; (2) the tax rate increase is multiplied by the distortionary effect on the tax base, which gives rise to a tax rate-tax base distortionary/contractionary loop; (3) the sudden drop in employment implies lower marginal returns to installed capital; (4) given capital adjustment costs, the value of installed capital (i.e., Tobin’s $q$) falls. This mechanism gives rise to channel through which the welfare of current generations, who own the existing stock of capital, is negatively affected. On the other hand, current generations gain from higher future net wages. A plausible calibration of the model shows, however, that the net welfare effect is likely to negative. Future generations, on the other hand, are only affected by the positive effect; their welfare improves.

Second, when public capital is initially scarce, public infrastructure investment can increase the welfare of both existing and future generations if the fiscal rule exempts public infrastructure spending from the balanced-budget requirement. Both the golden rule for net investment and the tax smoothing rule (which can be interpreted as a golden rule for gross investment) protect existing generations from strong tax rate increases, which tends to soften—despite not eliminating—the initial contraction. A plausible calibration of the model reveals that the negative welfare effect can be sufficiently reduced that current generations enjoy welfare gains in net terms. Of course, the higher long-run tax rates implied by public debt financing imply lower welfare gains for future generations. The results in this paper therefore make a case for exempting public infrastructure spending from balanced budget fiscal rules.

This paper relates to several strands of literature. Baxter and King (1993) and Turnovsky

\[ \text{2} \]
and Fisher (1995) study the dynamic macroeconomic effects of public investment the infinitely-lived representative agent (Ramsey) framework. A second group of papers—including Heijdra and Meijdam (2002), Basseto and Sargent (2006), and Bom and Ligthart (2014b)—study the intergenerational implications of public investment in overlapping generations models. The present paper relates closely to these studies. The model developed here is similar to Bom and Ligthart’s (2014), although the focus on intergenerational welfare is closer to Heijdra and Meijdram (2002). The latter authors, however, do not consider endogenous labor supply and distortionary taxation, which are key elements in this model. Although stressing a completely different mechanism, the policy implications derived here complement those in Basseto and Sargent (2006), whose results also call for debt-financing of public capital spending.

The paper is structured as follows. Section 2 describes the structure and elements of the model. Section 3 discusses the policy experiments conducted here, as well as the fiscal rules considered. Section 4 explains the solution method and the calibration strategy. The simulation results are discussed in Section 5, which considers in turns the macroeconomic (allocation) effects and the intergenerational welfare effects. Finally, Section 6 concludes.

2 The Model

This section presents the dynamic general equilibrium macroeconomic model of a small open economy used to investigate the intergenerational welfare effects of public investment. The model features Blanchard-Yaari overlapping generations of households, endogenous labor supply, and production spillovers from public infrastructure capital. Time is continuous. Sections 2.1–2.5 describe the elements of the model.

2.1 Individual Households

An individual household born at time \( v \leq t \) derives instantaneous utility, \( U(v, t) \), from private consumption, \( C(v, t) \), and leisure, \( L(v, t) \). I assume the constant elasticity of substitution
(CES) utility specification:

\[ U(v,t) \equiv \varepsilon_C C(v,t)^{\sigma_C^{-1}} + (1 - \varepsilon_C) \left[ 1 - L(v,t) \right]^{\sigma_C^{-1}} \sigma_C - 1, \]

where \( \varepsilon_C \) is the consumption weight in utility and \( \sigma_C \) denotes the elasticity of substitution between consumption and leisure. The individual household chooses labor supply and consumption so as to maximize lifetime utility

\[ \Lambda(v,t) = \int_t^\infty \ln U(v, \tau) e^{(\alpha + \beta)(t-\tau)} d\tau, \]

subject to the flow budget constraint

\[ \dot{A}(v,t) = (r + \beta)A(v,t) + \bar{w}(t) - X(v,t), \]

where \( \alpha \) denotes the pure rate of time preference, \( \beta \) is both the rate of birth and the instantaneous probability of death, \( r \) is the exogenously-given world rate of interest, \( A(v,t) \) is the individual stock of financial assets, and \( \bar{w}(t) \equiv w(t)[1 - L(t)] \) is the after-tax wage \( (w(t) \) and \( t_L(t) \) being the gross wage and tax rate on labor income, respectively). In this paper, I use a dot to denote a derivative with respect to time \( (e.g., \dot{A}(v,t) \equiv dA(v,t)/dt). \) In (3), \( X(v,t) \) denotes ‘full’ consumption, which is defined as

\[ X(v,t) \equiv P(t)U(v,t) \equiv \bar{w}(t)[1 - L(v,t)] + C(v,t), \]

where \( P(t) \) is a price index (derived below).

Solving the problem by two-stage budgeting gives, in the first stage, the household’s Euler equation

\[ \frac{\dot{X}(v,t)}{X(v,t)} = \frac{\dot{U}v(t)}{U(v,t)} + \frac{\dot{P}(t)}{P(t)} = r - \alpha > 0. \]

Integrating (3) while using (5) and imposing a Non-Ponzi Game (NPG) condition for the

\(^1\)Assuming infinitely many households within each cohort, the law of large numbers implies that probabilities and frequencies coincide. The population size is thus constant and can be normalized to one.
household, \( \lim_{r \to \infty} A(v, r)e^{(r+\beta)(t-r)} = 0 \), gives full consumption as a constant fraction of financial and human wealth:

\[
X(v, t) = (\alpha + \beta) [A(v, t) + H(t)],
\]

(6)

where human wealth is defined as

\[
H(t) \equiv \int_{t}^{\infty} \tilde{w}(\tau)e^{(r+\beta)(t-\tau)}d\tau.
\]

(7)

In the second stage—i.e., given the chosen level of full consumption—the household decides on consumption and leisure so as to maximize (1). The resulting first-order conditions read

\[
C(v, t) = [1 - \omega_N(t)]X(v, t),
\]

(8)

\[
\tilde{w}(t)[1 - L(v, t)] = \omega_N(t)X(v, t),
\]

(9)

where \( \omega_N(t) \equiv (1 - \varepsilon_C)^{\sigma_C} \left( \frac{\tilde{w}(t)}{P(t)} \right)^{1-\sigma_C} \in (0, 1) \). Finally, using these two expressions in the utility specification (1) gives the expression of the price level:

\[
P(t) \equiv \begin{cases} \left[ \varepsilon_C^{\sigma_C} + (1 - \varepsilon_C)^{\sigma_C} \tilde{w}(t)^{1-\sigma_C} \right]^{\frac{1}{1-\varepsilon_C}} & \text{for } \sigma_C \neq 1 \\ \left( \frac{1}{\varepsilon_C} \right)^{\varepsilon_C} \left( \frac{\tilde{w}(t)}{1-\varepsilon_C} \right)^{1-\varepsilon_C} & \text{for } \sigma_C = 1 \end{cases}.
\]

(10)

2.2 Aggregate Households

Taking into account that each living cohort represents a fraction \( \beta e^{\beta(v-t)} \) of the total population, individual financial wealth and full consumption can be aggregated using

\[
x(t) = \int_{-\infty}^{t} x(v, t)\beta e^{\beta(v-t)}dv, \quad x = \{A, X\}.
\]

(11)

The resulting expression for the accumulation of aggregate financial wealth is

\[
\dot{A}(t) = (r + \beta)A(t) + \tilde{w}(t) - X(t).
\]

(12)
Similarly, aggregating full consumption yields

\[
\frac{\dot{X}(t)}{X(t)} = r - \alpha - \frac{\beta(\alpha + \beta)A(t)}{X(t)},
\]

which is the familiar Keynes-Ramsey rule augmented with a ‘generational turnover’ effect (i.e., the last term).

### 2.3 Firms

The goods market is perfectly competitive. Firms produce output using a Cobb-Douglas technology featuring the stock of public infrastructure capital, \(K_G(t)\), as an additional input to private capital, \(K(t)\), and labor, \(L(t)\):

\[
Y(t) = K(t)^{\varepsilon_Y} L(t)^{1-\varepsilon_Y} K_G(t)^\eta,
\]

where \(\varepsilon_Y \in (0, 1)\) is the output elasticity of private capital and \(\eta \geq 0\) is the output elasticity of public capital. Given the exogenously-given rate of interest, I postulate adjustment costs to private capital:

\[
\dot{K}(t) = \left[ \Phi \left( \frac{I(t)}{K(t)} \right) - \delta \right] K(t),
\]

where \(\Phi(\cdot)\) is a concave accumulation function (i.e., \(\Phi'(\cdot) > 0\) and \(\Phi''(\cdot) < 0\)) featuring no marginal adjustment costs at zero gross investment (i.e., \(\Phi(0) = 0\) and \(\Phi'(0) = 1\)), and \(\delta \in (0, 1)\) denotes the depreciation rate of private capital.

The representative firm maximizes the present value of its cash flow

\[
V(t) \equiv \int_t^\infty [Y(\tau) - w(\tau)L(\tau) - I(\tau)] e^{r(t-\tau)} d\tau.
\]

subject to the production technology (14) and the capital accumulation constraint (15).
first-order conditions of the firm’s optimization problem are

\[
\begin{align*}
  w(t) &= (1 - \varepsilon_Y) \frac{Y(t)}{L(t)}, \\
  1 &= q(t)\Phi'\left(\frac{I(t)}{K(t)}\right), \\
  \dot{q}(t) &= -\varepsilon_Y \frac{Y(t)}{K(t)} - q(t) \left[ \Phi\left(\frac{I(t)}{K(t)}\right) - \frac{I(t)}{K(t)} \Phi'\left(\frac{I(t)}{K(t)}\right) - (r + \delta) \right].
\end{align*}
\]

(16) (17) (18)

### 2.4 Government

The government spends on public infrastructure investment, \(I_G(t)\), and public consumption goods, \(C_G(t)\), which represents pure waste. To finance its expenditures, the government levies a proportional tax on labor income. Depending on the fiscal rule restraining its actions (see Section 3), the government may also resort to public debt financing. The government thus faces the budget constraint

\[
\dot{B}(t) = rB(t) + I_G(t) + C_G(t) - tL(t)w(t)L(t),
\]

(19)

where \(B(t)\) denotes the stock of government debt outstanding at time \(t\). Together with a government NPG condition, (19) implies the intertemporal government constraint

\[
B(t) = \int_t^{\infty} [tL(\tau)w(\tau)L(\tau) - I_G(\tau) - C_G(\tau)] e^{r(t-\tau)} d\tau,
\]

(20)

which states that the present discounted value of the stream of primary balances from time \(t\) onwards must match any positive stock of government debt outstanding at time \(t\).

Public infrastructure capital is, like private capital, subject to adjustment costs. Public capital accumulates according to

\[
K_G(t) = \left[ \Phi_G\left(\frac{I_G(t)}{K_G(t)}\right) - \delta_G \right] K_G(t),
\]

(21)

where \(\Phi_G(\cdot)\) exhibits the same properties as \(\Phi(\cdot)\) (see (15) and the subsequent description).
2.5 Market Equilibrium

I abstract from any rigidities in the labor or goods market. Equilibrium in the goods market amounts to

\[ Y(t) = C(t) + I(t) + C_G(t) + I_G(t) + Z(t), \]  

where \( Z(t) \) denotes net exports. Net foreign assets, \( F(t) \), thus evolves according to

\[ \dot{F}(t) = rF(t) + Z(t). \]  

Equilibrium in the goods market implies equilibrium in financial markets:

\[ A(t) = V(t) + B(t) + F(t), \]  

where \( V(t) = q(t)K(t) \) denotes the stock market value of private capital.

3 Policy Experiments

I study the macroeconomic and intergenerational welfare effects of a permanent and unanticipated public investment shock at time \( t = 0 \), which scales up public investment spending to a higher long-run level. Panel (a) of Figure 1 illustrates this public investment increase. As shown in Panel (b), the public investment shock gradually increases the stock of public capital toward its new long-run level.

The fiscal policy rule determines how public infrastructure investment is financed. I consider three such rules. First, I assume the government is subject to a balanced budget requirement on total spending (scenario ‘BB’, for short). The government flow budget constraint then requires the tax rate to vary so as to generate enough tax revenues for a given tax base. Imposing \( \dot{B}(t) = 0 \) in (19) and solving for the tax rate gives

\[ t_L(t) = \frac{rB(t) + I_G(t) + C_G(t)}{w(t)L(t)}. \]  

(BB scenario)
Second, I assume a tax smoothing rule (‘TS’, for short), which allows the government to issue debt so as to minimize tax distortions. Specifically, the government ensures long-run solvency by adjusting the labor tax rate once-and-for-all at the time of the shock and then uses public debt to accommodate transitional changes in the tax base. The intertemporal budget constraint (20) therefore requires the tax rate to be

\[
t_L(t) = \frac{B(0) + \int_0^\infty [I_G(\tau) + C_G(\tau)] e^{r(t-\tau)} d\tau}{\int_0^\infty [w(\tau)L(\tau)] e^{r(t-\tau)} d\tau}, \quad \text{for all } t \geq 0, \quad \text{(TS scenario)} \quad (26)
\]

which jumps at \( t = 0 \) and remains constant thereafter.

Third, I consider a golden rule (‘GR’, for short) for net public investment. In this scenario, net additions to the capital stock may be debt-financed but current expenditures—including interest payments, depreciation costs, and capital adjustment costs—must be paid out of tax revenues. Hence, changes in the stock of public debt reflect changes in the stock of public capital. By noting that \( \dot{B}(t) = \dot{K}(t) \), we can combine (19) and (21) to arrive at the implied (time-varying) tax rate

\[
t_L(t) = \frac{rB(t) + I_G(t) - \Phi_G \left( \frac{I_G(t)}{K_G(t)} \right) K_G(t) + \delta_G \dot{K}_G(t) + C_G(t)}{w(t)L(t)}. \quad \text{(GR scenario)} \quad (27)
\]

To build intuition, Figure 2 shows stylized tax rate responses to the public investment shock under the assumption of exogenous labor supply. Panel (a) illustrates the time profile of the labor tax rate under the balanced budget fiscal rule. The tax rate increases on impact to finance a higher level of public investment; as the stock of public capital gradually builds up, however, gross wages rise, the tax base expands, allowing the tax rate to gradually fall toward its new long-run level. Panel (b) shows the tax smoothing case, where the tax rate is permanently adjusted at \( t = 0 \) to its long-run level. Thus, tax smoothing requires lower impact tax adjustments but higher long-run tax rates. Finally, Panel (c) shows the golden rule case. The tax rate increases by a small amount on impact to cover the adjustment costs of public capital (otherwise it would not be affected), and then raises gradually in tandem
with the accumulation of public capital, to cover increasing interest payments and capital depreciation.

Figure 2 clearly shows that different fiscal rules imply different timings of taxes even in a very simple setting of exogenous labor supply. Hence, both the gradual expansion of public capital and the timing of taxes contribute for public investment to cause differentiated intergenerational welfare effects. As discussed in Section 5, however, endogenous labor supply implies more complex tax rate dynamics (except for the tax smoothing scenario, of course, where the effect is qualitatively similar).

4 Solution and Calibration

I log-linearize the model around a steady state with zero net financial assets (i.e., \( F = 0 \)) and fully-backed public debt by public capital (i.e., \( B = K_G \)), which I solve analytically for the impulse response functions and intergenerational welfare effects of a public investment impulse. The Appendix provides additional details on the log-linearization procedure and analytical solution. Table A.1 in the Appendix summarizes the log-linearized model. I then calibrate the model by taking parameter values from the literature and by choosing steady-state shares to match the data of a typical small open economy in the euro area.

I calibrate the model as follows (Table 1 reports the values of the most important parameters). First, I set the shares of the main GDP components. The private consumption-to-GDP ratio \( \omega_C \equiv C/Y \) and the government expenditures-to-GDP ratio \( \omega_G \equiv G/Y \) are assumed to be 56% and 21%, in line with the averages in the eurozone countries.\(^2\) Out of total government expenditures, I take the public investment-to-GDP ratio to be 4%. Public investment ratios have typically been between 3% and 4% over the last few decades in most euro area countries, with a tendency to decrease; for a scenario of public capital scarcity, 4% is thus a conservative estimate. Assuming net financial assets (and thus net exports) to be zero in the initial steady state, the implied public consumption and private investment shares GDP are 17% and 23%.

Next, following Bom and Ligthart (2014b), I set the parameter of the private capital

\(^2\)Data are from the Eurostat.
accumulation function at $\bar{z} = 0.532$ (see Section A.1 of the Appendix), which implies capital adjustment costs of 0.2% of GDP in steady state. The analogous parameter in the public capital accumulation function, $\bar{z}_G$, is set at the same value. The depreciation rate of private capital ($\delta$) is assumed to be 10%, whereas the world interest rate ($r$) is fixed at 4%. It follows from (15) in steady state that the private investment-to-capital ratio is 11%, so that the private capital-to-GDP ratio is 2.091. Equation (17) then implies $q = 1.207$, which we can use in (18) in steady state to find a capital share of $\varepsilon_Y = 0.331$.

I assume that the stock of public debt is initially equal to the stock of public capital. Although it needs not be the case, this assumption is approximately valid in practice and obviates the need to solve explicitly for the dynamics of public debt in the golden rule case, since it goes in tandem with the stock of public capital in this case. Thus, in view of an average public debt-to-GDP ratio of 58% at the beginning of 2009 and an average public capital ratio of about 57% for advanced economies in 2011 (see Abiad et al., 2013), I set $K_G/Y = B/Y = 0.580$. Using $\bar{z}_G = 0.532$, it follows from the public capital accumulation function (21) in steady state that $\delta_G = 0.065$.

I take the benchmark value of one for the leisure-labor ratio (i.e., $\omega_{LL} = 1$), which implies households initially work half of their disposable time. In this model, however, $\omega_{LL}$ also measures the Frisch-elasticity of labor supply, which is a more controversial parameter to calibrate. Although an elasticity of one is frequently used in the literature, empirical studies with microeconomic data tend to find lower estimates, whereas the macroeconomic literature typically requires higher elasticities to explain employment fluctuations. Therefore, I also consider the alternative values of $\omega_{LL} = 0$ (inelastic labor supply) and $\omega_{LL} = 2$ (elastic labor supply). Given the assumed values of public investment, public consumption, and public debt, as well as the implied wage rate, the leisure-labor ratio pins down the initial labor tax rate at $t_L = 0.349$ via the government budget constraint (19) in steady state. From the (aggregated) first-order conditions (8) and (9), it follows that $\omega_N = 0.438$ and $\epsilon_C = 0.562$; also, from (10), $P = 1.869$.

The average household planning horizon is assumed to be 55 years, which amounts to choosing $\beta = 0.018$. The aggregated version of (9) then implies $X = 0.996$. Because, assuming
\( F = 0, A = 3.106 \) from (24), the modified Keynes-Ramsey rule (13) implies \( \alpha = 0.037 \). We can then compute the shares 
\[ \omega_K \equiv r q K / Y = 0.101, \quad \omega_X \equiv X / Y = 0.996, \quad \omega_A \equiv r A / Y = 0.124, \]
and \( \omega_{\bar{w}} \equiv \bar{w} / Y = 0.872; \) and the capital installation function elasticities \( \rho_A = 0.171 \) and \( \chi_G = 0.061 \).

Finally, the output elasticity of public capital, which measures the size of the public capital spillover, also needs to be specified. Because there is substantial disagreement over the magnitude of this parameter in the literature, I rely on the meta-analysis by Bom and Ligthart (2014a), which arrives at a value of 0.080 for core public capital provided at the general government level.

### 5 Simulation Results

#### 5.1 Macroeconomic Responses

Figure 3 shows the dynamic macroeconomic responses to a 10% permanent increase in public investment in the benchmark case of \( \omega_{LL} = 1 \) for the three fiscal rules considered. Each period of time denotes one year. Let us first consider the balanced budget scenario (denoted by solid lines). Because public capital is pre-determined, labor productivity and wages—and thus the tax base—are initially unaffected, which requires a tax rate jump to foot the bill on public investment. This increase in the tax rate distorts the labor market and depresses labor supply, which in turn increases wages. The labor contraction dominates the wage rise, however, which requires an even higher tax rate. This tax rate-tax base loop gives rise to a substantial tax rate increase on impact. Moreover, because private capital is also pre-determined, the fall in employment decreases its market value (i.e., Tobin’s \( q \)), which discourages investment and shrinks the capital stock in the subsequent periods. As a result, output falls initially and remains depressed in the short/medium run.

As public capital builds up, labor productivity improves and gross wages continue rising. The expanding tax base then allows the labor tax rate to fall, which stimulates labor supply and reinforces the tax base expansion. Again, this tax base-tax rate loop works as to dramatically decrease the tax rate, which eventually dips below its initial value. The employment
recovery has a positive effect on Tobin’s $q$, which stimulates private investment and expands the private capital stock. Output thus recovers and expands for almost four decades. The generational disconnection leads to a non-monotonic transition, however. Eventually, labor supply falls to its initial level (its long-run effect is exactly zero), increasing the tax rate and depressing private capital accumulation and output.

In the tax smoothing case (denoted by dotted lines), the tax rate jumps on impact to its long-run level, which is lower than in the balanced budget case in the short run. The impact contraction on employment and output is thus much less severe. Hence, Tobin’s $q$ barely falls on impact, having a small negative effect on private capital accumulation in the very short run. The overall recovery is much quicker than in the balanced budget case, although the medium/long run profile of labor, private capital, and output is worse. This is because interest payments on accumulated public require a higher labor tax rate after 15 or so years.

The golden rule scenario allows for an even lower tax rate in the short run by postponing a larger share of the tax burden to future generations. Thus, the tax rate jumps on impact by less than half of the increase in the tax smoothing case, causing an even smaller short-run contraction to employment and output, and no contraction at all to Tobin’s $q$ and private capital formation. Of course, because of higher interest rate payments, the tax rate adjustment in the golden rule eventually exceeds that in the other two scenarios, causing inferior transitional time profiles for employment, private capital, and output at longer horizons.

Figure 4 shows the impulse response functions in the simpler case of inelastic labor supply (i.e., $\omega_{LL} = 0$). The tax rate responses exhibit the pattern depicted in Figure 2. Because labor supply is exogenous, however, the responses of Tobin’s $q$, private capital formation, and output depend on the accumulation of public capital but are independent of the timing of taxes. Because public capital builds up monotonically, so do private capital and output.

Figure 5 reports the results for the case of elastic labor supply (i.e., $\omega_{LL} = 2$). As shown by Bom and Ligthart (2014b), elastic labor supply gives rise to cyclical dynamic responses to public investment in the case of a balanced budget rule. The dramatic jump of the labor tax rate on impact greatly exacerbates the short-run contraction of employment, private capital, and output, but also the ensuing recovery and expansion. The long-run effects are identical.
to the benchmark case of $\omega_{LL} = 1$, but the transition is cyclical. The cyclical dynamics are preserved in the golden rule case, where the tax rate is also time-varying, although the responses are much less pronounced. The size of the labor supply elasticity is less relevant in the tax smoothing scenario; the impulse responses are virtually equal to the case $\omega_{LL} = 1$.

5.2 Intergenerational Welfare Effects

Figure 6 shows the intergenerational welfare effects of public investment for the three fiscal rules. It shows the birth date in the horizontal axis (negative for existing generations and positive for future generations) and the change in lifetime utility in the vertical axis. As shown in the Appendix, the welfare of existing generations is measured at the time of the shock, whereas the welfare of future generations is measured at birth.

Panel (a) of Figure 6 shows the results for the benchmark case of $\omega_{LL} = 1$. In the balanced budget case (solid line), current and future generations are affected very unequally: whereas the latter enjoy substantial welfare gains—particularly those born 30 to 40 years after the public investment scale up—the former generally suffer welfare losses. The intuition is straightforward: although all generations benefit from higher after-tax wages in the long run, older generations experience a sharp devaluation of the capital stock of they have accumulated over the course of their lives (see Tobin’s $q$ response in Figure 3). Very old generations suffer the most because they own larger shares of the capital stock. Future generations always benefit because they don’t inherit capital at birth.

How to mitigate (or outright eliminate) the negative welfare effects on current generations? By relaxing the tax burden that falls on existing generations. In fact, the tax smoothing (dotted line) and golden rule (dashed line) scenarios allow for lower tax rate adjustments on impact, causing less severe employment and output contractions in the short run, which reduce (in the tax smoothing case) or eliminate (in the golden rule case) the negative initial effect on market value of installed capital (i.e., Tobin’s $q$). Hence, existing generations generally benefit (except perhaps extremely old ones in the tax smoothing case) from higher public investment if they are partially exempted from higher tax rates. Of course, these positive welfare effects come at the cost of lower welfare gains for future generations, who then face
higher tax rates.

Panel (b) shows the intergenerational welfare effects in the simpler case of inelastic supply of labor (i.e., \( \omega_{LL} = 0 \)). As in Heijdra and Meijdam (2002), as long as public capital is initially scarce, both current and future generations gain from an increase in public investment. Because the labor market is not distorted in this case, Tobin’s \( q \) does not fall on impact, irrespective of the fiscal rule in place. Hence, public investment improves the welfare of current generations even under a balanced budget fiscal rule. Comparing Panels (a) and (b) shows that a balanced budget fiscal rule is critical for the welfare of existing generations if labor is endogenous—even for mild elasticities of labor supply. Public debt financing, on the other hand, prevents current generations from losing, thereby presumably increasing public acceptability for a higher level of public investment.

Panel (c) illustrates the intergenerational welfare effects for the case of elastic labor supply (i.e., \( \omega_{LL} = 2 \)). The results are qualitatively similar to those in Panel (b). Quantitatively, however, current generations experience much higher welfare losses, whereas (some) future generations enjoy even higher welfare gains. In short, a higher labor supply elasticity exacerbates the uneven distribution of welfare effects across generations.

5.3 Quantitative Effects

Table 2 reports the macroeconomic long-run and impact multipliers as well as the welfare effects of a permanent and unanticipated impulse to public investment. It shows the results for the benchmark case of unitary labor supply elasticity and for the alternative values \( \omega_{LL} = 0 \) and \( \omega_{LL} = 2 \); for each of these cases, the table considers the balanced budget (BB), tax smoothing (TS), and golden rule (GR) fiscal scenarios.

Public investment generates sizable long-run effects on the capital stock, output, private investment and private consumption. The output multiplier reaches a value of virtually three, fueled by a private capital multiplier above six, which in turn requires a large investment multiplier of about 0.7. Note that these values are independent of both the labor supply elasticity and the fiscal rule—because labor is not affected in the long run for \( \sigma_C = 1 \), private capital is pinned down by the exogenously-given interest rate. The multiplier of private
consumption exceeds one in the balanced-budget case, but decreases below unity in the tax smoothing and golden rule scenarios, because of interest payments on public debt accumulated during transition.

The large positive long-run multipliers come at the cost of large negative impact multipliers, however. For a unitary elasticity of labor supply (i.e., $\omega_{LL} = 1$), the balanced-budget rule implies a strong tax rate increase and a contraction in employment, giving rise to an output multiplier of -1.58 and a private investment multiplier of -0.58. The private consumption multiplier is also negative, but close to zero. Fiscal scenarios involving public debt—i.e., tax smoothing and golden rule—decrease, in absolute value, the negative impact multipliers of output and investment, and move the consumption response into positive terrain.

If labor is exogenous (i.e., $\omega_{LL} = 0$), the impact multiplier of output is nil and the investment multiplier is highly positive and invariant to the fiscal rule. Intuitively, because the labor market is not distorted, Tobin’s $q$ increases with the prospect of higher returns to private capital. The consumption is then positive even in the balanced-budget case, although it is again higher in the tax smoothing and golden rule scenarios. Conversely, a more elastic labor supply (i.e., $\omega_{LL} = 2$) exacerbates the negative impact multipliers of output and private investment, whereas the consumption multiplier becomes positive even under a balanced-budget rule (note, however, that this aggregate effect hides substantial variation across cohorts, as discussed in the previous section).

The welfare effects are always positive in the cases considered in Table 2. In the balanced-budget case, instantaneous utility rises on impact mainly because of the strong increase in leisure brought about by the tax rate jump. In the tax smoothing and golden rule cases, the impact increase utility is less dominated by leisure and more by consumption. In the long run, instantaneous utility is totally dominated by the increase in consumption, because leisure unaffected. Hence, long-run utility gains are larger in the balanced budget case, followed by the tax smoothing and golden rule. Exogenous labor supply implies lower aggregate utility gains on impact but higher long-run aggregate utility gains. Elastic labor supply has the opposite effects.

Finally, the last row of Table 2 displays the lifetime utility change of an infinitely-lived
representative individual—i.e., the change in (2) for \( \beta = 0 \). By computing the lifetime welfare changes of an infinitely-lived household, this welfare measure integrates the utility changes of current generations over their lifetime (discounted at the rate of time preference \( \alpha \)) and the utility changes of future generations (also discounted at the rate \( \alpha \)). It can thus be seen as an intergenerational welfare measure, where future generations are discounted at the rate of time preference. This measure is always positive for the parameters considered, reflecting that a permanent public investment impulse is welfare-improving from an intergenerational viewpoint. Because of the interest payments in public debt, the welfare gains are higher for the balanced budget rule than for tax smoothing and golden rule. The welfare gains are also higher for lower elasticities of labor supply, due to the lower labor market distortions.

6 Conclusions

This paper studied the intergenerational welfare effects of a permanent public investment impulse. Motivated by the recently-adopted Fiscal Compact between European Union member states, I focused on the balanced-budget as the benchmark fiscal rule. The paper provided a mechanism through which a balanced-budget permanent increase in public investment may decrease the welfare of living generations, even when public capital is initially scarce. In short, the balanced-budget requirement implies strong tax rate adjustments in the short run, which tend to depress employment and decrease the stock market value of installed capital. I showed that, for plausible parameter values, this mechanism gives rise to welfare losses for existing generations—who own the capital stock—that outweigh the welfare gains from higher future net wages. Future generations, on the contrary, always benefit from more public investment, as long as public capital is initially scarce. I showed, for plausible parameter values, that fiscal rules exempting public investment from the balanced budget requirement can make public investment improve the welfare of both existing and future generations.
Figure 1: Public Investment Shock and Public Capital Accumulation

(a) Public Investment

(b) Public Capital

Notes: The permanent public investment shock is assumed to be unanticipated.

Figure 2: Stylized Tax Rate Adjustment Under the Various Financing Scenarios

(a) Balanced Budget

(b) Tax Smoothing

(c) Golden Rule

Notes: The figures assume exogenous labor supply.
Figure 3: Macroeconomic Responses to a Public Investment Impulse: $\omega_{LL} = 1$

Tax rate  
Labor  
Gross Wages  
Tobin’s q  
Private capital  
Output  

Notes: The public investment shock amounts to $I_G = 0.1$ (i.e., 10% of the initial level).
Figure 4: Macroeconomic Responses to a Public Investment Impulse: $\omega_{LL} = 0$

Notes: The public investment shock amounts to $\bar{I}_G = 0.1$ (i.e., 10% of the initial level).
Notes: The public investment shock amounts to $I_G = 0.1$ (i.e., 10% of the initial level).
Figure 6: Intergenerational Welfare Effects of a Public Investment Impulse

Panel (a): $\omega_{LL} = 1$

Panel (b): $\omega_{LL} = 0$

Panel (c): $\omega_{LL} = 2$

Notes: The public investment shock amounts to $\tilde{I}_G = 0.1$ (i.e., 10% of the initial level).
Table 1: Chosen and Implied Parameter Values in the Benchmark Model

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter/Share</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel (a): Chosen Values</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private consumption-to-GDP ratio</td>
<td>( \omega_C \equiv C/Y )</td>
<td>0.560</td>
</tr>
<tr>
<td>Government expenditures-to-GDP ratio</td>
<td>( \omega_G \equiv G/Y )</td>
<td>0.210</td>
</tr>
<tr>
<td>Public investment-to-GDP ratio</td>
<td>( \omega_I \equiv I_G/Y )</td>
<td>0.040</td>
</tr>
<tr>
<td>Parameter of private capital installation function</td>
<td>( \bar{z} )</td>
<td>0.532</td>
</tr>
<tr>
<td>Parameter of public capital installation function</td>
<td>( \bar{z}_G )</td>
<td>0.532</td>
</tr>
<tr>
<td>Depreciation rate of private capital</td>
<td>( \delta )</td>
<td>0.100</td>
</tr>
<tr>
<td>Rate of interest</td>
<td>( r )</td>
<td>0.040</td>
</tr>
<tr>
<td>Public capital-to-GDP ratio</td>
<td>( K_G/Y )</td>
<td>0.580</td>
</tr>
<tr>
<td>Public debt-to-GDP ratio</td>
<td>( B/Y )</td>
<td>0.580</td>
</tr>
<tr>
<td>Leisure-labor ratio</td>
<td>( \omega_{LL} \equiv (1 - L)/L )</td>
<td>1.000</td>
</tr>
<tr>
<td>Birth/death rate</td>
<td>( \beta )</td>
<td>0.018</td>
</tr>
<tr>
<td>Elasticity of substitution between consumption and leisure</td>
<td>( \sigma_C )</td>
<td>1.000</td>
</tr>
<tr>
<td>Output elasticity of public capital</td>
<td>( \eta )</td>
<td>0.080</td>
</tr>
<tr>
<td><strong>Panel (b): Implied Values</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public consumption-to-GDP ratio</td>
<td>( \omega_C \equiv C_G/Y )</td>
<td>0.170</td>
</tr>
<tr>
<td>Private investment-to-GDP ratio</td>
<td>( \omega_I \equiv I/Y )</td>
<td>0.230</td>
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<tr>
<td>Output elasticity of private capital</td>
<td>( \varepsilon_Y )</td>
<td>0.331</td>
</tr>
<tr>
<td>Depreciation rate of public capital</td>
<td>( \delta_G )</td>
<td>0.065</td>
</tr>
<tr>
<td>Pure rate of time preference</td>
<td>( \alpha )</td>
<td>0.037</td>
</tr>
<tr>
<td>Preference weight of consumption in utility</td>
<td>( \varepsilon_C )</td>
<td>0.562</td>
</tr>
<tr>
<td>Firm ownership income-to-GDP ratio</td>
<td>( \omega_K \equiv rqK/Y )</td>
<td>0.101</td>
</tr>
<tr>
<td>Full consumption-to-GDP ratio</td>
<td>( \omega_X \equiv X/Y )</td>
<td>0.996</td>
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<tr>
<td>Financial income-to-GDP ratio</td>
<td>( \omega_A \equiv rA/Y )</td>
<td>0.124</td>
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<tr>
<td>Net wages-to-GDP ratio</td>
<td>( \omega_{\bar{w}} \equiv \bar{w}/Y )</td>
<td>0.872</td>
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<tr>
<td>Elasticity of the marginal private capital installation function</td>
<td>( \rho_A )</td>
<td>0.171</td>
</tr>
<tr>
<td>Elasticity of the public capital installation function</td>
<td>( \chi_G )</td>
<td>0.061</td>
</tr>
</tbody>
</table>
Table 2: Macroeconomic and Welfare Effects of a Permanent Increase in Public Investment

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{LL} = 1$</th>
<th></th>
<th>$\omega_{LL} = 0$</th>
<th></th>
<th>$\omega_{LL} = 2$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BB</td>
<td>TS</td>
<td>GR</td>
<td>BB</td>
<td>TS</td>
<td>GR</td>
</tr>
<tr>
<td>Panel (a): Long-Run Multipliers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>2.9892</td>
<td>2.9892</td>
<td>2.9892</td>
<td>2.9892</td>
<td>2.9892</td>
<td>2.9892</td>
</tr>
<tr>
<td>Investment</td>
<td>0.6875</td>
<td>0.6875</td>
<td>0.6875</td>
<td>0.6875</td>
<td>0.6875</td>
<td>0.6875</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.2851</td>
<td>0.9529</td>
<td>0.5355</td>
<td>1.2851</td>
<td>0.9190</td>
<td>0.5355</td>
</tr>
</tbody>
</table>

Panel (b): Impact Multipliers

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Private capital</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Output</td>
<td>-1.5765</td>
<td>-0.7527</td>
<td>-0.3734</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Investment</td>
<td>-0.5606</td>
<td>-0.0725</td>
<td>0.0859</td>
<td>0.3031</td>
<td>0.3031</td>
<td>0.3031</td>
</tr>
<tr>
<td>Consumption</td>
<td>-0.0017</td>
<td>0.1174</td>
<td>0.1572</td>
<td>0.0260</td>
<td>0.0854</td>
<td>0.1490</td>
</tr>
</tbody>
</table>

Panel (c): Welfare Effects

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous utility (impact)</td>
<td>0.5486</td>
<td>0.3252</td>
<td>0.2143</td>
<td>0.0260</td>
<td>0.0854</td>
<td>0.1490</td>
</tr>
<tr>
<td>Instantaneous utility (long run)</td>
<td>0.6878</td>
<td>0.5100</td>
<td>0.2866</td>
<td>1.2851</td>
<td>0.9190</td>
<td>0.5355</td>
</tr>
<tr>
<td>Lifetime utility change</td>
<td>0.0242</td>
<td>0.0236</td>
<td>0.0232</td>
<td>0.0429</td>
<td>0.0425</td>
<td>0.0421</td>
</tr>
</tbody>
</table>

Notes: ‘BB’, ‘TS’, and ‘GR’ stand for balanced budget, tax smoothing, and golden rule, respectively. For a generic variable $x(\cdot)$, the impact multiplier is defined as $dx(0)/dI_G$ (i.e., immediately after the shock), whereas the long-run multiplier is given by $dx(\infty)/dI_G$, where $x(\infty)$ denotes the new steady-state value of $x$. Similarly, the impact utility effect is $dU(0)/dI_G$ and the long-run utility effect is $dU(\infty)/dI_G$, where $U(\cdot)$ aggregates individual instantaneous utility in (1) as described by (11). The effect on lifetime utility is measured by the change in discounted stream of utility of an infinitely-lived representative agent (given by (2) with $\beta = 0$.)
Appendix

This Appendix provides extra details on the structure and solution of the model. A complete derivation of the analytical results in this paper is reported in a separate Mathematical Appendix; see Bom (2015).

A.1 Capital Accumulation Functions

Following Bom and Ligthart (2014b), I assume the following specification for the private capital accumulation function:

$$
\Phi(x) \equiv a [\ln(x + \bar{z}) - \ln \bar{z}],
$$

(A.1)

where $\bar{z}$ is an exogenous constant. The function is defined in the region where $x + \bar{z} > 0$—i.e., for $x \in (-\bar{z}, \infty)$. The first and second-order derivatives are:

$$
\Phi'(x) = a(x + \bar{z})^{-1} > 0,
$$

(A.2)

$$
\Phi''(x) = -a(x + \bar{z})^{-2} < 0.
$$

(A.3)

We wish to have that adjustment costs are zero at $x = 0$, which is the case if $\Phi'(0) = 1$, implying $a = \bar{z}$. The elasticity of the $\Phi'(x)$ function (which is used in the log-linearized model) is defined as:

$$
\rho_A \equiv -\frac{x\Phi''(x)}{\Phi'(x)} = \frac{x}{x + \bar{z}},
$$

(A.4)

from which we deduce that $0 < \rho_A < 1$ for the relevant region with positive values for $x$.

Similarly, I assume the public capital accumulation, $\Phi_G(x)$, to follow (A.1) with parameters $a_G = \bar{z}_G$. Because the first and second derivatives of $\Phi_G(x)$ are analogous to (A.2) and (A.3), it follows that the elasticity of the public capital accumulation function (which is used in the log-linearized model) is

$$
\chi_G \equiv x\Phi'_G(x) = x \left(\frac{\bar{z}_G}{x + \bar{z}_G}\right) > 0.
$$

(A.5)
A.2 Log-Linearization

I log-linearize the model around a steady state with \( F(0) = 0 \). The results are reported in Table A.1. Notational conventions are:

\[
\tilde{x}(t) \equiv \frac{dx(t)}{x}, \quad \dot{x}(t) \equiv \frac{d\tilde{x}(t)}{x} = \frac{\dot{x}(t)}{x}, \tag{A.6}
\]

where \( x \) is the steady-state value of \( x(t) \). For a number of variables I use a slightly different notation; asset-like variables (e.g., \( H, F, B \) and \( A \)) are defined as

\[
\tilde{x}(t) \equiv \frac{rdx(t)}{Y}, \quad \dot{x}(t) \equiv \frac{rd\tilde{x}(t)}{Y}, \tag{A.7}
\]

and for the labor-income tax rate I use

\[
\tilde{t}_L(t) \equiv \frac{dt_L(t)}{1 - t_L}. \tag{A.8}
\]

We will make use of the Laplace transform technique, which allows us to analyze time-varying fiscal shocks. The Laplace transformation of \( x(t) \) evaluated at \( s \) is given by

\[
\mathcal{L}\{x, s\} \equiv \int_0^\infty x(t)e^{-st}dt. \tag{A.9}
\]

Intuitively, \( \mathcal{L}\{x, s\} \) represents the present value of \( x(t) \) using \( s \) as the discount rate.

A.3 Public Investment Shock and Financing Scenarios

I consider a permanent and unanticipated public investment shock:

\[
\tilde{I}_G(t) = \tilde{I}_G, \quad \text{for } t \geq 0, \tag{A.10}
\]

which affects the stock of public capital according to

\[
\tilde{K}_G(t) = (1 - e^{-\chi t}) \tilde{I}_G, \quad \text{for } t \geq 0. \tag{A.11}
\]
where $\chi_G$ is defined in (A.5).

Total public spending is financed by proportional labor taxes and/or public debt. The log-linearized tax rates for the three financing scenarios are:

(i) Balanced budget:

$$\tilde{t}_L(t) = \frac{\omega_I \bar{I}_G}{(1 - \varepsilon_Y)(1 - t_L)} - \bar{\theta}_L \left[ \bar{L}(t) + \bar{w}(t) \right], \quad (A.12)$$

where $\bar{\theta}_L \equiv t_L/(1 - t_L)$.

(ii) Tax smoothing:

$$\tilde{t}_L(t) = \frac{\omega_I \bar{I}_G}{(1 - \varepsilon_Y)(1 - t_L)} - r \bar{\theta}_L \left[ \mathcal{L}\{\bar{L}, r\} + \mathcal{L}\{\bar{w}, r\} \right] \equiv \tilde{t}_L \quad (A.13)$$

(iii) Golden rule:

$$\tilde{t}_L(t) = \left( \frac{\omega_I - \chi_G \bar{y}_G}{(1 - \varepsilon_Y)(1 - t_L)} - \bar{\theta}_L \left[ \bar{L}(t) + \bar{w}(t) \right] \right) \tilde{r}_L + \bar{y}_G \left[ r + \chi_G \right] \tilde{K}_G(t). \quad (A.14)$$

The following specification nests the tax rate in the various financing scenarios:

$$\tilde{t}_L(t) = d_D \left\{ \left( \frac{\omega_I - d_G \chi_G \bar{y}_G}{(1 - \varepsilon_Y)(1 - t_L)} - \bar{\theta}_L \left[ \bar{L}(t) + \bar{w}(t) \right] \right) \tilde{r}_L + \bar{y}_G \left[ r + \chi_G \right] \tilde{K}_G(t) \right\} + d_S \tilde{t}_L. \quad (A.15)$$

where:

$$d_D = \begin{cases} 
1, & \text{if variable labor tax rate (balanced budget or golden rule)}, \\
0, & \text{if tax smoothing}.
\end{cases} \quad (A.16)$$

and:

$$d_G = \begin{cases} 
1, & \text{if golden rule} \\
0, & \text{otherwise}.
\end{cases} \quad (A.17)$$

Of course, $d_S = 1 - d_D$. Hence, the balanced budget scenario is obtained by setting $d_D = 1$
and $d_G = 0$, the golden rule is obtained by setting $d_D = 1$ and $d_G = 1$, and the labor tax-smoothing case is obtained by setting $d_D = 0$.

**A.4 Model Solution**

This section describes the solution of the log-linearized model. The model can be split into a static and a dynamic system. I discuss each system in turn. The solution of the dynamic system generates impulse response functions for the (log-linearized) capital stock, Tobin’s $q$, financial assets, and full consumption. The detailed derivations are described in the Mathematical Appendix. Here, I report the main results.

**A.4.1 The Static System**

By using (TA.1.13)-(TA.1.15) in (TA.1.10) we can write the labor supply equation as:

$$\tilde{L}(t) = \tilde{\omega}_{LL}[(\tilde{w}(t) - \tilde{t}_L(t))] - \omega_{LL}\tilde{X}(t),$$

(A.18)

where $\tilde{\omega}_{LL} \equiv \omega_{LL} + \bar{\sigma}_L = \omega_{LL}[1 + (\sigma_C - 1)(1 - \omega_N)]$.

Equations (A.15), (A.18), and (TA.1.13)-(TA.1.15) can be written in matrix notation as:

$$\begin{bmatrix}
1 & -1 & -1 & 
1 & 1 & -1 & \varepsilon_Y & 0 & 
0 & 1 & -\tilde{\omega}_{LL}d_D\tilde{b}_L & -\tilde{\omega}_{LL}(1 + d_D\tilde{b}_L)
\end{bmatrix}
\begin{bmatrix}
\tilde{Y}(t) \\
\tilde{L}(t) \\
\tilde{w}(t)
\end{bmatrix}
= 
\begin{bmatrix}
0 & 
\tilde{K}^*(t) & 
Z^*(t)
\end{bmatrix},$$

(A.19)

where

$$Z^*(t) \equiv -d_D \frac{\tilde{\omega}_{LL}}{(1 - \varepsilon_Y)(1 - \tilde{t}_L)} \left[ (\omega_G - d_G\chi_G\tilde{y}_G)\tilde{I}_G + d_G\tilde{y}_G(r + \chi_G)(1 - e^{-\chi_Gt})\tilde{I}_G \right]$$

$$-d_S\tilde{\omega}_{LL}\tilde{I}_L - \omega_{LL}\tilde{X}(t),$$

(A.20)

and $\tilde{K}^*(t) = \varepsilon_Y\tilde{K}(t) + \eta\tilde{K}_G(t)$. Using (A.11), we find

$$\tilde{K}^*(t) = \varepsilon_Y\tilde{K}(t) + \eta(1 - e^{-\chi_Gt})\tilde{I}_G.$$  

(A.21)
The solution of the static system is

\[
\begin{bmatrix}
\tilde{Y}(t) \\
\tilde{L}(t) \\
\tilde{w}(t)
\end{bmatrix}
= \begin{bmatrix}
1 & -1 & -1 \\
1 & -1 + \varepsilon_Y & 0 \\
0 & 1 - \tilde{\omega}_{LL}d_D\tilde{t}_L & -\tilde{\omega}_{LL}(1 + d_D\tilde{t}_L)
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
\tilde{K}^*(t) \\
Z^*(t)
\end{bmatrix}
\]

where the last equality follows from inverting the square matrix on the right-hand side of the first line while using (A.20) and (A.21). Following these steps, the implied \(\xi_{ij}\)'s concerning output are:

\[
\xi_{yk} = \frac{\varepsilon_Y(1 + \tilde{\omega}_{LL})}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}, \quad \xi_{gx} = \frac{(1 - \varepsilon_Y)\tilde{\omega}_{LL}}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}
\]

\[
\xi_{yy} = \frac{\eta(1 + \tilde{\omega}_{LL}) - d_d\tilde{d}_LL(1 + \theta_L)\tilde{y}_G(r + \chi_G)}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}, \quad \xi_{sy} = \frac{\tilde{\omega}_{LL}(1 + \theta_L)(\tilde{\omega}_{LL}^2 - d_G\chi_G\tilde{y}_G)}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}
\]

For employment, the coefficients are given by:

\[
\xi_{lx} = \frac{\tilde{\omega}_{LL}(1 + \theta_L)}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}, \quad \xi_{id} = \frac{1}{1 - \varepsilon_Y} \frac{\tilde{\omega}_{LL}(1 + \theta_L)(\tilde{\omega}_{LL}^2 - d_G\chi_G\tilde{y}_G)}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}
\]

Finally, for the wage rate the coefficients are:

\[
\xi_{lw} = \frac{\varepsilon_Y(1 - \tilde{\omega}_{LL}d_D\tilde{t}_L)}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}, \quad \xi_{wx} = \frac{\tilde{\omega}_{LL}\varepsilon_Y}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}
\]

\[
\xi_{al} = \frac{(1 - \varepsilon_Y)\tilde{\omega}_{LL}}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}, \quad \xi_{ld} = \frac{\tilde{\omega}_{LL}(1 + \theta_L)(\tilde{\omega}_{LL}^2 - d_G\chi_G\tilde{y}_G)}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}
\]

\[
\xi_{aw} = \frac{\varepsilon_Y(1 - \tilde{\omega}_{LL}d_D\tilde{t}_L)}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}, \quad \xi_{ws} = \frac{\tilde{\omega}_{LL}\varepsilon_Y}{1 + \tilde{\omega}_{LL}[\varepsilon_Y(1 + d_D\theta_L) - d_D\theta_L]}
\]

Note that the system (A.22) is conditional on the state variables, which must be determined
We want to derive the path for after-tax wages. This is written as
\[ \tilde{w}(t) = \tilde{w}(t) - \tilde{t}_L(t) \]

where we have made use of (A.15) and the quasi-reduced form for \( \tilde{w}(t) \) in the last row of (A.22). The coefficients are given by:

\[ \begin{align*}
\xi_{\tilde{w}k} &\equiv \xi_{wk} + d_D \bar{\theta}_L (\xi_{lk} + \xi_{wk}) \\
\xi_{\tilde{w}x} &\equiv \xi_{wx} + d_D \bar{\theta}_L (\xi_{lx} + \xi_{wx}) \\
\xi_{\tilde{w}g} &\equiv \xi_{wg} + d_D \bar{\theta}_L (\xi_{lg} + \xi_{wg}) - \frac{d_D d_G (1+\bar{\theta}_L) \bar{y}_G (r+\chi_G)}{1-\epsilon_Y} \\
\xi_{\tilde{w}d} &\equiv \xi_{wd} + d_D \bar{\theta}_L (\xi_{ld} + \xi_{wd}) - \frac{(1+\bar{\theta}_L)(\omega_I^* - d_G \chi_G \bar{y}_G)}{1-\epsilon_Y} \\
\xi_{\tilde{w}s} &\equiv \xi_{ws} + d_D \bar{\theta}_L (\xi_{ls} + \xi_{ws}) - 1.
\end{align*} \]

A.4.2 The Dynamic System

The dynamic system can be written in terms of one matrix equation of the form:

\[ \begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{q}}(t) \\ \dot{\tilde{X}}(t) \\ \dot{\tilde{A}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{\bar{\omega}_L}{\rho_A \omega_K} & 0 & 0 \\ \frac{\bar{\omega}_X}{\omega_K} (1 - \xi_{bk}) & r & -\frac{\bar{\omega}_X}{\omega_K} \xi_{qx} & 0 \\ 0 & 0 & r - \alpha & -\frac{\bar{\omega}_A}{\omega_A} \\ r \omega_{\bar{w}} \xi_{\tilde{w}k} & 0 & r(\omega_{\bar{w}} \xi_{\tilde{w}x} - \omega_X) & r \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{q}(t) \\ \tilde{X}(t) \\ \tilde{A}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma_q(t) \\ \gamma_A(t) \end{bmatrix} \]

where the shock terms are given by:

\[ \begin{align*}
\gamma_q(t) &\equiv -\frac{\bar{\omega}_Y}{\omega_K} \left[ \xi_{yg}(1 - e^{-\chi_G t}) \tilde{I}_G + \xi_{yd} d_D \tilde{I}_G + \xi_{ys} d_S \tilde{t}_L \right], \\
\gamma_A(t) &\equiv r \omega_{\bar{w}} \left[ \xi_{\tilde{w}g}(1 - e^{-\chi_G t}) \tilde{I}_G + \xi_{\tilde{w}d} d_D \tilde{I}_G + \xi_{\tilde{w}s} d_S \tilde{t}_L \right].
\end{align*} \]

For future reference, we write the shock terms in the following compact form:

\[ \gamma_i(t) = \pi_{iP} + \pi_{iD} e^{-\chi_G t}, \quad \text{for } i = q, A, \]

30
with:

\[ \pi_{qp} \equiv \frac{r\varepsilon Y}{\omega K} \left( (\xi_{yg} + \xi_{yd}d)\tilde{I}_G + \xi_{ys}d\tilde{S}_L \right), \]
\[ \pi_{qt} \equiv -\frac{r\varepsilon Y\xi_{yg}}{\omega K} \tilde{I}_G, \]
\[ \pi_{Ap} \equiv -r \left[ (\omega\bar{w}\xi_{\bar{w}g} + \omega\bar{w}\xi_{\bar{w}d}d)\tilde{I}_G + \omega\bar{w}\xi_{\bar{w}s}d\tilde{S}_L \right], \]
\[ \pi_{At} \equiv r\omega\bar{w}\xi_{\bar{w}g}\tilde{I}_G. \]

The dynamic system (A.24) embeds two important special cases. First, exogenous labor supply (i.e., \( \omega_{LL} = 0 \)), yields \( \xi_{gx} = 0 \), implying that the \( [\tilde{q}(t), \tilde{K}(t)] \) system can be solved independent of the \( [\tilde{X}(t), \tilde{A}(t)] \) system. Second, infinitely lived households (i.e., \( r = \alpha \)) imply that the third row of (A.24) consists of zeros only. The knife-edge condition \( r = \alpha \) yields a hysteretic steady state. In general, for plausible parameter values, the dynamic system (A.24) is saddle path-stable—i.e., it possesses two negative (stable) roots and two positive (unstable) roots. Moreover, for large labor supply elasticities (i.e., larger values of \( \omega_{LL} \)), the roots are potentially complex (see Bom and Ligthart, 2014b).

### A.4.3 Impact Jumps

Let \( \delta_{ij} \) denote the typical element in the \( i \)-th row and \( j \)-th column of the \( 4 \times 4 \) Jacobian matrix of the dynamic system (A.24). Also, define

\[ \phi(x) \equiv (x - \delta_{33})(x - \delta_{42}) - \delta_{34}\delta_{43}. \quad (A.28) \]

Then, one can show that the impact effects (immediately upon the shock, at \( t = 0 \)) on the jumping variables—i.e., Tobin’s \( q \) and full consumption—are given by

\[
\begin{bmatrix}
\tilde{q}(0) \\
\tilde{X}(0)
\end{bmatrix} =
\begin{bmatrix}
\phi(r_1^*) + \delta_{23}\delta_{34}\omega K & \delta_{23}(r_1^* - \delta_{22}) \\
\phi(r_2^*) + \delta_{23}\delta_{34}\omega K & \delta_{23}(r_2^* - \delta_{22})
\end{bmatrix}^{-1}
\begin{bmatrix}
\phi(r_1^*)\mathcal{L}\{\gamma_q, r_1^*\} + \delta_{23}\delta_{34}\mathcal{L}\{\gamma_A, r_1^*\} \\
\phi(r_2^*)\mathcal{L}\{\gamma_q, r_2^*\} + \delta_{23}\delta_{34}\mathcal{L}\{\gamma_A, r_2^*\}
\end{bmatrix},
\]

(A.29)

where \( r_1^* \) and \( r_2^* \) denote the positive (unstable) characteristic roots of the Jacobian matrix.
A.4.4 Impulse Responses

Solving the dynamic system (A.24) means finding the reduced-form impulse response functions for the variables in that system during transition to the new steady state. Define the following temporary transition and permanent adjustment terms:

\[
\begin{align*}
T_1(h_1^*, h_2^*, t) &\equiv \frac{e^{h_1^*t} - e^{h_2^*t}}{h_1^* - h_2^*}, \\
T_2(h_1^*, h_2^*, t) &\equiv \frac{h_1^*e^{h_1^*t} - h_2^*e^{h_2^*t}}{h_1^* - h_2^*} = \frac{dT_1(h_1^*, h_2^*, t)}{dt}, \\
T_3(h_1^*, h_2^*, \chi G, t) &\equiv \frac{1}{h_1^* - h_2^*} \left( \frac{e^{h_1^*t} - e^{-\chi G t}}{h_1^* + \chi G} - \frac{e^{h_2^*t} - e^{-\chi G t}}{h_2^* + \chi G} \right), \\
A(h_1^*, h_2^*, t) &\equiv \frac{1}{h_1^* - h_2^*} \left( \frac{e^{h_1^*t} - 1}{h_1^*} - \frac{e^{h_2^*t} - 1}{h_2^*} \right) = T_3(h_1^*, h_2^*, 0, t).
\end{align*}
\]

where \( h_1^* \) and \( h_2^* \) denote the negative (stable) roots. One can then show that, in the case of simple roots,\(^3\) the impulse response function for the capital is given by

\[
\bar{K}(t) = \delta_{12} \bar{q}_0 T_1(h^*, \theta h, t) - \delta_{12} \frac{\pi_q \phi(-\chi G) + \pi_A \delta_{23}\delta_{34}}{(r_1^* + \chi G)(r_2^* + \chi G)} T_3(h^*, \theta h, \chi G, t)
+ \delta_{12} \frac{\pi_q \delta_{34}\delta_{43} - \delta_{22}\delta_{33} - \pi_A \delta_{23}\delta_{34}}{r_1^* r_2^*} A(h^*, \theta h, t). \tag{A.34}
\]

For Tobin’s \( q \), the impulse response function is

\[
\bar{q}(t) = \left[ (r_1^* + r_2^* - \delta_{22} - \delta_{33}) \bar{q}(0) + \delta_{23} \bar{X}(0) - (\pi_q + \pi_q t) \right] T_1(h^*, \theta h, t)
+ \bar{q}(0) T_2(h^*, \theta h, t) + \frac{\chi G \left[ \pi_q \phi(-\chi G) + \delta_{23}\delta_{34}\pi_A \right]}{(r_1^* + \chi G)(r_2^* + \chi G)} T_3(h^*, \theta h, \chi G, t). \tag{A.35}
\]

For full consumption, one finds

\[
\bar{X}(t) = \left[ \delta_{34} \omega_K \bar{q}(0) + (r_1^* + r_2^* - 2\delta_{22}) \bar{X}(0) \right] T_1(h^*, \theta h, t) + \bar{X}(0) T_2(h^*, \theta h, t)
- \frac{\delta_{34} \delta_{12}\delta_{41} \pi_q t + \psi(-\chi G) \pi_A t}{(r_1^* + \chi G)(r_2^* + \chi G)} T_3(h^*, \theta h, \chi G, t) + \frac{\delta_{12}\delta_{34}\delta_{21} \pi_A - \delta_{41} \pi_q t}{r_1^* r_2^*} A(h^*, \theta h, t), \tag{A.36}
\]

\(^3\)If roots are complex, the temporary transition and adjustment terms have to be adjusted; see Bom (2015) for details.
where $\psi(x) \equiv x(x - \delta_{22}) - \delta_{12}\delta_{21}$. Finally, the time-profile of the stock of financial assets is

$$
\tilde{A}(t) = [\omega_K(r_1^* + r_2^* - \delta_{22} - \delta_{33})\tilde{q}(0) + \delta_{33}\tilde{X}(0) - (\pi_{Ap} + \pi_{At})]T_1(h^*, \theta_h, t) \\
+ \omega_K\tilde{q}(0)T_2(h^*, \theta_h, t) + \frac{(\chi G + \delta_{33})[\delta_{12}\delta_{41}\pi_A + \psi(-\chi G)\pi_A]}{(r_1^* + \chi G)(r_2^* + \chi G)}T_3(h^*, \theta_h, \chi G, t) \\
+ \frac{\delta_{12}\delta_{33}(\delta_{41}\pi_A' - \delta_{21}\pi_A)}{r_1^*r_2^*}A(h^*, \theta_h, t).
$$

(A.37)

Finding the impulse responses of the remaining variables is straightforward.

### A.5 Intergenerational Welfare Effects

Lifetime utility at time $t$ of a household born at $v \leq t$ is given by

$$
\Lambda(v, t) \equiv \int_{t}^{\infty} \ln U(v, \tau)e^{(\alpha + \beta)(t-\tau)}d\tau \\
= \int_{t}^{\infty} [\ln X(v, \tau) - \ln P(\tau)]e^{(\alpha + \beta)(t-\tau)}d\tau.
$$

(A.38)

By using that $X(v, \tau) = X(v, t)e^{(r-\alpha)(\tau-t)}$, for $\tau \geq t$, working out the integral in (A.38), and differentiating, one arrives at an expression for the change in lifetime welfare:

$$
(\alpha + \beta)d\Lambda(v, t) = \tilde{X}(v, t) - (\alpha + \beta)\int_{t}^{\infty} \tilde{P}(\tau)e^{(\alpha + \beta)(t-\tau)}d\tau.
$$

(A.39)

At this stage, we have to distinguish between existing generations at the time of the shock (i.e., those for which $v \leq 0$) and future generations (i.e., those for which $v > 0$).

#### A.5.1 Existing Generations ($v \leq 0$)

We know from (6) that

$$
X(v, 0) = (\alpha + \beta)[A(v, 0) + H(0)].
$$

(A.40)

Using that $X(v, 0) = X(v, v)e^{-(r-\alpha)v}$ and differentiating, one finds in a couple of steps that

$$
\tilde{X}(v, 0) = \frac{1 - e^{-(r-\alpha)v}}{\omega_A}A(0) + \frac{e^{-(r-\alpha)v}}{\omega_H}H(0),
$$

(A.41)
where $\omega_A \equiv rA/Y$ and $\omega_H \equiv rH/Y$. Equation (A.41) can be plugged into (A.39) to evaluate the welfare change of cohort $v$ at the time of the shock ($t = 0$):

$$d\Lambda(v, 0) = 1 - e^{(r-\alpha)v}A(0) + \frac{e^{(r-\alpha)v}}{\omega_H(\alpha + \beta)}H(0) - L\{\tilde{P}, \alpha + \beta\}. \quad (A.42)$$

A.5.2 Future Generations ($v > 0$)

Welfare of future generations is evaluated at birth—i.e., at $t = v > 0$. In the absence of bequests, future generations are born without financial assets, so that $X(v, v) = (\alpha + \beta)H(v)$. Log-linearizing and substituting in (A.39) for $t = v$ gives the welfare change of future generations:

$$d\Lambda(v, v) = \frac{\tilde{H}(v)}{\omega_H(\alpha + \beta)} - \int_{v}^{\infty} \tilde{P}(\tau)e^{(\alpha+\beta)(v-\tau)}d\tau. \quad (A.43)$$
Table A.1: Summary of the Log-Linearized Model

(a) Dynamic Equations:

\[ \dot{\tilde{K}}(t) = \frac{r\omega_I}{\omega_K} \left[ \tilde{I}(t) - \tilde{K}(t) \right] \]  
\[ \dot{\tilde{q}}(t) = r\tilde{q}(t) - \frac{r\varepsilon_Y}{\omega_K} \left[ \tilde{Y}(t) - \tilde{K}(t) \right] \]  
\[ \dot{\tilde{X}}(t) = (r - \alpha) \left[ \tilde{X}(t) - \frac{\tilde{A}(t)}{\omega_A} \right] \]  
\[ \dot{\tilde{A}}(t) = r \left[ \tilde{A}(t) + \omega_\bar{w}\tilde{\bar{w}}(t) - \omega_X\tilde{X}(t) \right] \]  
\[ \dot{\tilde{B}}(t) = r \left[ \tilde{B}(t) + \omega_G^I\tilde{I}_G + \omega_G^C\tilde{C}_G - \theta_L(1 - t_L)\tilde{I}_L(t) - \theta_Lt_L(\tilde{L}(t) + \tilde{\bar{w}}(t)) \right] \]  
\[ \dot{\tilde{K}}_G(t) = \chi_G \left[ \tilde{I}_G - \tilde{K}_G(t) \right] \]  

(b) Static Equations:

\[ \tilde{q}(t) = \rho_A \left[ \tilde{I}(t) - \tilde{K}(t) \right] \]  
\[ \tilde{\bar{w}}(t) = \tilde{Y}(t) - \tilde{L}(t) \]  
\[ \tilde{Y}(t) = \varepsilon_Y\tilde{K}(t) + (1 - \varepsilon_Y)\tilde{L}(t) + \eta\tilde{K}_G(t) \]  
\[ \tilde{L}(t) = \omega_{LL} \left[ \tilde{\bar{w}}(t) - \tilde{\bar{w}}_N(t) - \tilde{X}(t) \right] \]  
\[ \tilde{C}(t) = -\frac{\omega_N}{1 - \omega_N} \tilde{\bar{w}}_N(t) + \tilde{X}(t) \]  
\[ \tilde{F}(t) = \tilde{A}(t) - \omega_K \left[ \tilde{q}(t) + \tilde{K}(t) \right] - \tilde{B}(t) \]  

(c) Definitions:

\[ \tilde{P}(t) = \omega_N\tilde{\bar{w}}(t) \]  
\[ \tilde{\bar{w}}_N(t) = (1 - \sigma_C) \left[ \tilde{\bar{w}}(t) - \tilde{P}(t) \right] \]  
\[ \tilde{\bar{w}}(t) = \tilde{\bar{w}}(t) - \tilde{I}_L(t) \]
References


