Is informal risk-sharing less effective for the poor?
Risk externalities and moral hazard in mutual insurance

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Abstract

Poor farm-households are less keen to adopt high risk/high return technologies than rich households. Yet, the poor are more vulnerable to income shocks. We develop a model of endogenous risk-taking to explain these facts. In autarky, poor households adopt less risky production plans and obtain lower expected returns, but face higher relative risk than the rich. The introduction of risk-sharing generates negative risk externalities between agents. At the first best, the social planner imposes a homogeneous level of risk-taking in the group. At the second best, risk-taking is not enforceable and increases with insurance, generating moral hazard. Interestingly, the poor’s risk-taking behavior is more sensitive to insurance. The social planner thus mitigates risk-taking by applying a lower insurance coverage in poor groups. The introduction of risk-sharing therefore reinforces the gap between rich and poor in terms of expected income and absolute risk, while the effect on relative risk is ambiguous.

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1 Introduction

In developing countries, the ability of farm households to deal with risk is a key determinant of their daily livelihood as well as their long term economic outcome. In rural regions, formal credit and insurance markets are generally missing. In this context, households try to mitigate the effects of various types of income shocks through ex-post coping strategies, individually through buffer stocks such as cattle on the one hand, and collectively through informal insurance transfers on the other hand. However, these ex post strategies generally prove insufficient, forcing households to also adopt ex ante precautionary measures. In particular, households may adopt various production plans, crop choices as well as agricultural (traditional versus modern) techniques. These choices are characterized by a trade-off between expected returns and output risks. This trade-off is a potential source of poverty traps for the poorest, who tend to adopt low risk, low return production plans, whereas the rich tend to adopt higher risk, higher return plans. Still, interestingly, the empirical literature has shown that the poor tend to be more affected by idiosyncratic shocks than the rich.\(^1\)

In this paper, we provide a theory of endogenous risk-taking which reproduces the aforementioned stylized facts. While these results are easily obtained when agents behave in autarky, they are robust and even reinforced by the introduction of informal risk-sharing schemes. Our model takes into account two fundamental specificities of informal risk-sharing arrangements compared to classical insurance models. First, unlike standard insurance markets where shocks can be diluted over a very large number of agents, informal insurance groups are of limited size. Second, informal insurance groups cannot rely on credit to cover a deficit in the event of a bad year, whereas in developed economies, capital markets or markets for reinsurance are available. As a result, informal insurance transfers must adapt to every combination of shocks, in the sense that the insurance group’s budget must always be balanced ex post, for all states of the world.\(^2\) A direct consequence of these two features is that agents’ risk-taking behavior, which affects the distribution of shocks, generates a negative externality on their partners. More precisely, if a household takes important risks, it is likely to face large negative shocks, and these shocks need to be covered by its insurance partners. Ex ante, the post-transfer income of these partners is therefore more random. At equilibrium, risk externalities entail moral hazard in the sense that, for a given degree of risk-sharing, risk-taking is higher than the socially optimal level. This moral hazard problem affects the way agents are ready to share risks, and is a potential explanation for the fact that the poor are less well protected against shocks although they take lower risks than the rich. Interestingly, moral hazard is more severe for poor agents, because they have larger returns to risk-taking in terms of expected marginal utility. However, poor agents also are more risk averse and have a stronger willingness to share risks. We show that the moral hazard effect dominates the need for insurance among poor groups. As a result, the levels of risk-sharing and risk-taking in poor groups are lower than in richer groups.

We first solve the first best problem of the model, in which a social planner designs the risk-sharing scheme and sets agents’ levels of risk-taking under the above mentioned constraints imposed by informal insurance. We then study the case of autarky, in which agents choose their level of risk-taking, and we finally compare the first best allocation to the second best risk-sharing in which agents choose non-cooperatively their risk-taking after observing the insurance scheme.

\(^1\)References for these claims are provided in the section "Related literature".

\(^2\)Budget balance means here that the sum of interpersonal transfers is equal to zero, whereas in a formal insurance market, this constraint must only hold in expectation.
This paper is organized as follows. In Section 2, we provide a survey of the literature. In Section 3 we present the general setting and solve the social planner’s first best problem and the case of autarky. In section 4, we study the planner’s second best problem in which agents choose their risk-taking levels non cooperatively after observing the insurance scheme. In Section 5, we discuss our results and potential extensions. Section 6 concludes.

2 Related literature

The survey is articulated around three fields of the literature which interact in this paper. In the first subsection, we review the literature which treats risk-coping and risk-taking mechanisms, which our models studies simultaneously. In the second subsection, we provide a survey of moral hazard in mutual insurance and explain how endogenous risk-taking relates to this concept. Finally, we refer to the literature linking risk-taking to wealth / risk aversion and risk-sharing.

2.1 Risk-coping and risk-taking

As previously mentioned in the introduction, the results of our paper are compatible with the combination of two fundamental stylized facts about risk and poverty in developing countries: (1) poor households tend to be more affected by idiosyncratic risk (Jalan and Ravallion (1999)); (2) poor households take less risks in general (Dercon (1996), Dercon (1998)) and are in particular less keen to adopt high risk / high return technologies (Dercon and Christiaensen (2011)).

Jalan and Ravallion (1999) estimate the fraction of idiosyncratic income shocks that translates into household consumption and find that this fraction tends to be higher for the poor. The inability of the poor to protect themselves against income shocks is due to a limited access to (or use of) risk-coping strategies. These strategies are divided into three important categories.

First, households facing shocks can rely on buffer stocks such as cattle (McPeak (2004), Verpoorten (2009)) or grain (Kazianga and Udry (2006)). However, cattle is also a productive asset and selling it may threaten the household’s future livelihood. In that sense, the optimal dynamics of asset accumulation may be incompatible with consumption smoothing when mechanisms of poverty traps are at play. This has been recently attested by Carter and Lybbert (2012) in Burkina Faso. According to them, poor households may engage in asset smoothing rather than consumption smoothing when they feel that their stock of assets is close to a critical threshold, while richer households can afford to use their assets as a buffer.

Second, adjustments in terms of household composition and activities can be made, such as the use of child fostering (Akresh (2009)) and child labor (Jacoby and Skoufias (1997), Beegle et al. (2006), Gubert and Robilliard (2008), Björkman-Nyqvist (2013)).

Third, and central to this article, rural households share risk informally. As already mentioned, this strategy is however known to be imperfect as rural households do not exhaust the gains from sharing their risks, i.e. risk-sharing is incomplete (Townsend (1994); Jalan and Ravallion (1999); Hoogeveen (2002); Murgai et al. (2002)). Development economists have tried to rationalize this phenomenon. Yet, to the best of our knowledge, the classical arguments that are generally invoked in the context of formal insurance markets (i.e. adverse selection and moral hazard) have not been explicitly transposed to the case of informal groups.

Some contributions highlight the lack of contract enforceability (i.e. limited commitment) as a source
of incomplete risk-sharing (Kimball (1988), Coate and Ravallion (1993), Kocherlakota (1996), Ligon et al. (2002)).

Limited commitment implies that an agent experiencing a favorable outcome relative to his insurance partners and should be in a position to help them has an incentive to renege on this promise. However, the role of wealth has been overlooked in this literature, with the exception of Coate and Ravallion (1993). They show that informal risk-sharing is potentially more limited in scope for the poor, which is consistent with the first stylized fact motivating this paper. Indeed, the incentive compatibility condition, which restricts the rate of risk-sharing, is binding at lower levels of interpersonal transfers for the poor since their marginal utility of current income is higher. The poor are therefore more reluctant to make transfers when other agents are facing adverse shocks, which explains why they are less protected against income shocks ex post. Our analysis comes to the same conclusion under perfect commitment and endogenous risk-taking, a modeling strategy which provides a potential explanation to the two above mentioned stylized facts in a single setting.

Beside limited commitment, the existing theoretical literature has also studied private information as a another source of incomplete risk-sharing. Some papers develop models allowing them to confront predictions on risk-sharing under various types of information imperfections. For instance, Ligon (1998) provides a very general model in which agents are able to hide income and/or actions, and compares the intertemporal pattern of the second best risk-sharing arrangement to the permanent income hypothesis. Karaivanov and Townsend (2014) and Kinnan (2014) consider various regimes including hidden income, moral hazard and limited commitment hypotheses and confront them empirically. However, these papers put very little structure on the moral hazard problem and do not intend to study its consequences on risk-taking, neither do they explore the role of wealth. Both Ligon (1998) and Kinnan (2014) refer to Rogerson (1985), which studies the role of moral hazard in a dynamic Principal-Agent relationship, as a theoretical basis for the analysis of risk-sharing with moral hazard. Moral hazard in the specific context of informal risk-sharing is however quite distinct from a Principal-Agent relationship, given the interactions between multiple agents and the absence of an explicit authority managing risk-taking in the community. In contrast, we impose more structure on the type of moral hazard and study in detail the consequences of group composition on risk-sharing and risk-taking.

Third, since risk-coping strategies, including buffer stocks and risk-sharing, are imperfect especially for the poor, households also mitigate risks ex ante via risk management strategies (Dercon (2002)). These strategies, which affect the distribution and the magnitude of income shocks, take various forms. First, household may diversify their income sources, both in terms of economic activities (agricultural and non-agricultural sectors) and geographical locations (urban and rural environments) (Morduch (1995), Sarpong and Asuming-Brempong (2004)). Second, households may make use of various crop, production and technological choices. Used efficiently, i.e. on the production frontier, these strategies lead to a trade-off between expected returns and risk. As already mentioned, poor households tend to opt for low risk, low return strategies (Dercon (1996), Dercon (1998), Kurosaki and Fafchamps (2002), Dercon and Christiaensen (2011)).

The following section aims at defining the concept of moral hazard that will be used throughout the paper. It has indeed to be distinguished from the standard formulation of moral hazard in insurance problems. We argue that the pure risk-taking dimension, as opposed to the standard version of moral hazard, is particularly relevant in the context of informal risk-sharing.

\(^3\) Notice that limited commitment might also be labelled as ex post moral hazard.
2.2 Moral hazard and risk-taking

As previously mentioned, risk management strategies affect the distribution of future income. In classical insurance problems, one generally represents the way in which agents affect the distribution of outcomes in a specific form, which involves an investment in costly actions, or effort. This effort reduces the probability of facing an adverse shock (Arnott and Stiglitz (1988), Arnott and Stiglitz (1991)), and the outcome distribution under high effort is generally considered to first order stochastically dominate a low effort distribution.\footnote{Those efforts therefore result in an increase in the outcome mean. However, the impact on the outcome variance may be indeterminate in this setting. Suppose that an agent’s income \( Y \) is equal to \( y \) with probability \( (1 - p) \) and \( y - s \), with probability \( p \), where \((y, s) \in \mathbb{R}^2_+ \) and \( p \in [0, 1] \). Assume that effort \( e \) reduces the probability of facing the shock, \( p'(e) < 0 \). It is easy to see that \( \partial E[Y]/\partial e > 0 \) and that \( \partial \text{Var}(Y)/\partial e < 0 \iff p < 1/2 \).} While utility is concave to account for risk aversion, effort costs are generally a separable argument in the utility function.

Instead, we model risk management strategies as a trade-off between the expected output and the variance of income shocks. Agents allocate their resources on the production frontier, so that if they opt for high return strategies, they will face higher risks.\footnote{Risk management strategies also involve direct costs, as with technology adoption for instance. These costs are considered as implicitly deducted in the final income.}

In the context of formal insurance markets, the classical approach (effort to reduce the likelihood of adverse shock) only leads to a moral hazard problem for the insurer if insufficient effort leads to a lower outcome mean. Indeed, if riskier strategies didn’t reduce the outcome mean, but were only increasing its variance, insurer profits would remain unchanged on average, while the increase in risk would be handled thanks to the large size of developed economies’ markets, the existence of markets for reinsurance and the ability to smooth profits over time via capital markets.

In contrast, in informal insurance groups, moral hazard occurs even if the mean of shocks is unaffected by agents’ risk-taking behavior. If the group is of finite size and markets are incomplete, the risk-sharing group’s budget constraint has to be satisfied with equality.\footnote{Yet, one can imagine that the group can store resources even when capital markets are absent in order to smooth aggregate income over time. This however depends on the storage technology that is available. If this possibility remains limited, then the budget constraint may bind with a strictly positive probability, which would not affect our main results.} As previously mentioned, this implies that individual risk-taking affects the (post-transfer) income distributions of all group members. In other words, we show that the context of informal risk-sharing leads to the existence of externalities which are purely related to risk.\footnote{Note that in our model, different risk-taking strategies imply differences in shock variance as well as differences in the income mean. These differences in mean affect the lump sum component of interpersonal transfers, but the unique source of inefficiency in risk-sharing stems from pure risk externalities.}

In this sense, moral hazard may occur under the weaker concept of second order stochastic dominance (Rothschild and Stiglitz (1970)).

2.3 Wealth, risk aversion and risk-taking

When possibilities to share risk are absent or limited, it is natural to expect that risk-taking will be positively related to risk tolerance, or wealth. The classical theory of entrepreneurship builds on this relationship. Kihlstrom and Laffont (1979) produce a general equilibrium theory of occupational choices. Under missing insurance markets and without any informal possibility to share risk, they show that the identity of entrepreneurs as well as the size of their firms is directly explained by wealth when preferences are characterized
by decreasing absolute risk aversion. More recently, Newman (2007) falsified this prediction by adding the possibility of risk-sharing. In this paper, he shows that a setting based on wealth heterogeneity, endogenous risk-taking and risk-sharing with moral hazard may lead to implausible predictions, namely that the poor become the entrepreneurs. Newman (2007)’s model differs from our approach in several ways. First, it adopts the standard approach to moral hazard described in the previous subsection (first order stochastic dominance induced by a costly effort). Second, Newman (2007)’s setting considers risk-sharing with a continuum of agents, which prevents the problem of the imperfect diversification of risks that is inherent to informal risk-sharing groups motivated in our paper. These differences lead us to divergent conclusions. Indeed, he finds that moral hazard is more severe for the rich in the sense that, to produce the incentive compatible level of effort, they need to bear more risk. In other words, moral hazard in the classical approach leads the rich to receive a lower insurance coverage. In contrast, we find that moral hazard mainly prevents the poor from sharing risk efficiently. As developed below, the reason is that the poor are more sensitive to a marginal increase in their insurance coverage, so that their response in terms of risk-taking generates more negative externalities. The reason for this opposition in predictions is therefore due to the absence of risk externalities in Newman’s setting.

Fischer (2013), which examines risky investments and risk-sharing within microfinance groups, also shares similarities with our paper. He compares the performance of alternative contractual forms, such as individual liability, joint liability, and equity contracts, and shows that joint liability, which fosters peer monitoring, may hamper risky investments, thereby reducing the profitability of the economic activities financed by micro-loans. As in our model, he finds that agents who are more risk tolerant / richer may engage more in risk-sharing. The mechanisms behind this similarity are however different. In Fischer (2013), risk tolerant agents tend to invest more in the risky assets and therefore have more risk to share than risk averse agents. In our model, the presence of risk externalities plays again a crucial role to explain this result. Moral hazard in risk-taking leads to lower insurance coverage. Since, as previously mentioned, moral hazard is more prevalent among poor, these agents end up being less insured than the rich.

3 The general model

3.1 Technology and preferences

Let us consider a set $H = \{1, ..., n\}$ of $n$ households which may engage in risk-sharing. Income $y_h$ is random and its distribution is affected by the household’s risk management choices embodied by the decision variable $\sigma_h \in \mathbb{R}_+$, which captures the level of risk taken by household $h$. We assume that the first and second moments of $y_h$ are affected by $\sigma_h$ in the following way:

\[
E(y_h; \sigma_h) = \mu(\sigma_h),
\]

\[
Var(y_h; \sigma_h) = \sigma_h^2,
\]

where $\mu(0) \geq 0$, $\mu'(\sigma) > 0$ and $\mu''(\sigma) < 0$. The fact that $\sigma_h$ increases both the mean and the variance of $y_h$ implies a trade-off between risk and expected return. One can interpret this representation as the production
frontier of the set of technologies available to households.\(^8\) Income can therefore be written as:

\[ y_h = \mu (s_h) + \sigma_h s_h, \]  

(1)

where \(s_h \in \mathbb{R}\) is a random shock of mean \(E(s_h; \sigma_h) = 0\) and variance \(Var(s_h; \sigma_h) = 1\). Shocks are independent between households: \(s_i \perp s_j\), for all \(i \neq j\) in \(H\).\(^9\) A state of the world \(S\) is a specific realization of all households’ income shocks: \(S = (\sigma_1 s_1, \ldots, \sigma_n s_n) \in \mathbb{R}^n\). Households can protect themselves against these shocks via an informal risk-sharing arrangement within the group. In this arrangement, they commit to make reciprocal income-contingent transfers.\(^10\) As argued in the introduction, informal insurance groups cannot rely on credit to cover potential deficits. A direct consequence of these characteristics is that the transfer scheme must be budget-balanced for all possible states of the world. This implies that, contrary to classical insurance problems, each household’s transfer is a function of all the shocks faced by all households: \(t_h = t_h (S)\), and budget balance imposes that for all \(S\),

\[ \sum_{h \in H} t_h (S) = 0. \]

The vector of income transfers received by each household is denoted by \(T = (t_1, \ldots, t_n)' \in \mathbb{R}^n\). A risk-sharing arrangement maps a vector of shocks \(S\) into a vector of transfers \(T\) in the following way:

\[ T = L + \Gamma' S, \]  

(2)

where \(L = (l_1, \ldots, l_n)' \in \mathbb{R}^n\) is a vector of lump sum transfers, i.e. transfers that are independent of \(S\), and where the \((n \times n)\) matrix \(\Gamma\) determines how the realization of shocks in the group affect all members’ transfers:

\[
\Gamma = \begin{pmatrix}
-\alpha_1 & \gamma_{12} & \cdots & \cdots & \gamma_{1n} \\
\gamma_{21} & -\alpha_2 & \cdots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & & \ddots & -\alpha_{n-1} & \gamma_{n-1,n} \\
\gamma_{n1} & \cdots & \cdots & \gamma_{n,n-1} & -\alpha_n
\end{pmatrix}
\]

The diagonal element \(\Gamma_{h,h} = -\alpha_h\) represents the share of household \(h\)’s shock which is insured by the group, while the off-diagonal element \(\gamma_{jh}\) represents the share of household \(j\)’s shock that household \(h\) commits to cover. The budget constraint reduces the scheme’s total number of parameters by imposing the following structure.

**Lemma 1** The risk-sharing arrangement’s budget constraint imposes that

\(^8\) Any technological choice below this frontier would be inefficient as, from this point, it would be possible to strictly increase the expected income without increasing risk.

\(^9\) Notice that a covariate shock could easily be added. It would simply imply that some fraction of the variance cannot be reduced by risk-sharing at the group level. However, since one of our objectives is to describe pure risk externalities, ignoring covariate shocks reinforces our point. Indeed, we show that individual risk-taking generates externalities even if shocks are independent.

\(^10\) In order to describe the mechanism behind risk externalities in the clearest way, we assume that these transfers are enforceable. In this way, the implications of moral hazard in risk-taking are clearly distinct from the limited commitment argument.
1. \[ \sum_{h \in H} t_h = 0, \]
2. for all \( h \in H \), \( \alpha_h = \sum_{i \in H \setminus \{h\}} \gamma_{hi}. \)

**Proof.** Provided in Appendix 1.

For the budget constraint to be satisfied, two conditions need to be met. On the one hand, the sum of lump sum transfers should be zero, otherwise the group would generate a surplus or a deficit on average. In particular, it can be easily seen that in the state of the world where \( S = (0, \ldots, 0)' \), the budget constraint would be violated. On the other hand, the insured fraction of household \( h \)’s shock, \( \alpha_h \), is equal to the sum of the fractions of \( h \)’s shock that the other members commit to cover, \( \sum_{i \in H \setminus \{h\}} \gamma_{hi}. \) It is worth noting that the informal insurance’s budget constraint makes full insurance impossible since under full insurance, the risk-sharing arrangement would be such that \( \Gamma = -I_n \), where \( I_n \) is a \((n \times n)\) identity matrix, which is incompatible with Lemma 1.

Making use of (2), we can write the transfer received by household \( h \) as

\[
t_h (S; L, \Gamma) = l_h - \alpha_h \sigma_h s_h + \sum_{i \in H \setminus \{h\}} \gamma_{ih} \sigma_i s_i.
\]

The consumption level of household \( h \) after transfer is obtained by combining (1) and (3):

\[
c_h (S; L, \Gamma) = w_h + y_h + t_h = w_h + \mu (\sigma_h) + l_h + (1 - \alpha_h) \sigma_h s_h + \sum_{i \in H \setminus \{h\}} \gamma_{ih} \sigma_i s_i,
\]

where \( w_h \) is household \( h \)’s wealth.\(^{11}\) The mean and variance of consumption are

\[
E(c_h; \sigma_h, L) = w_h + \mu (\sigma_h) + l_h,
\]

\[
Var(c_h; \Gamma, \Sigma) = (1 - \alpha_h)^2 \sigma_h^2 + \sum_{i \in H \setminus \{h\}} \gamma_{ih}^2 \sigma_i^2,
\]

by independence between \( s_i \) and \( s_j \), for all \( i \neq j \) in \( H \). The consumption equation (4) shows that informal risk-sharing may allow a household \( h \) to reduce its exposure to its own income shock by \( \alpha_h \). This fraction of the shock is supported by the other group members since by Lemma 1, \( \alpha_h = \sum_{i \in H \setminus \{h\}} \gamma_{hi}. \) The fact that shocks are passed to other members affects these members’ utilities through their consumption variance (see 6). This implies that risk-sharing is a source of risk externalities, since households no longer fully internalize the adverse effects of their risk-taking behavior, which now affects other members. This will be the case as soon as off-diagonal elements of \( \Gamma \) are different from zero (i.e. \( \gamma_{ij} \neq 0 \)), which is imposed by any insurance scheme’s budget constraint.\(^{12}\)

Informal risk-sharing allows household \( h \) to reduce its own shock variance to \((1 - \alpha_h)^2 \sigma_h^2\) (see equation 6), while other members, who support part of this risk, increase their variance by \( \gamma_{ih}^2 \sigma_i^2\). The interest of sharing risk is that the aggregate impact of \( \gamma_{ih}^2 \left[ \left(1 - \sum_{i \in H \setminus \{h\}} \gamma_{hi}\right)^2 + \sum_{i \in H \setminus \{h\}} \gamma_{ih}^2 \right] \sigma_h^2 \), is always smaller than the risk in autarky, \( \sigma_h^2. \)

\(^{11}\) Alternatively, one can interpret \( w \) as the non-random component of income. This constant income flow is positively determined by household’s assets. Comparative statics with respect to \( w \) or with respect to household’s assets are therefore equivalent.

\(^{12}\) One could argue that risks are also shared with agents outside the community, such as migrants, allowing the group to survive to structural losses. However, as soon as the risk cannot be fully diversified, a residual risk remains at the group level, which is the mechanism on which we concentrate.
To illustrate this, let us consider the homogeneous case in which all agents have identical wealth. In this case, we know by Lemma 1 that $\alpha_i = \alpha$ and $\gamma_{hi} = \alpha / (n - 1)$ for all $i, h$. The aggregate impact in the group of $\sigma_h^2$ then boils down to $1 - \alpha (2 - \alpha n / (n - 1))$, which is equal to 0 (i.e. risks are fully diversified) when $n$ tends to infinity and $\alpha$ tends to 1. In other words, when the group is of infinite size, full risk-sharing ($\alpha \to 1$) allows perfect diversification and completely suppresses risk externalities. Summing up, when capital and insurance markets are missing and informal risk-sharing groups are of finite size, individual risk-taking generates risk externalities. These externalities will generate moral hazard problems when actions are not contractible.

Agents are risk averse and derive utility from consumption. Agents have different wealth endowments but identical preferences represented by the utility function $u(c)$, with $u' > 0$ and $u'' < 0$. Let us denote absolute risk aversion of household $h$ by $a_h = -u''(c_h)/u'(c_h)$ and relative risk aversion by $r_h = -c_h u''(c_h) / u'(c_h)$.

In this paper, we focus on the case where informal risk-sharing takes place under moral hazard, namely when risk-taking is unenforceable. Before turning to that case, we briefly examine two polar cases of this model: (1) the first best allocation, where risk-taking is assumed enforceable and (2) autarky, where households are prevented from sharing risk.

### 3.2 The first best allocation

We first consider the first best problem of a social planner who designs the risk-sharing scheme and is able to enforce households’ risk-taking $\sigma_h$. This planner seeks to maximize a social welfare function $W$ aggregating the expected utilities of all households in the group, with respect to the vector of lump sum transfers $L$, the risk-sharing arrangement $\Gamma$, and the risk-taking profile $\Sigma$:

$$\max_{L, \Gamma, \Sigma} W = \sum_{h \in H} \lambda_h E u(c_h),$$

where $\lambda_h$ is the Pareto weight attributed to household $h$.

The first order condition with respect to any lumpsum transfer $l_{ij}$ imposes that

$$\frac{\partial W}{\partial l_{ij}} = 0 \iff \lambda_i E[u'(c_i)] = \lambda_j E[u'(c_j)], \forall \{i, j\} \subset H. \quad (8)$$

As expected, lump sum transfers aim at redistributing income, and this redistribution is a function of the vector of Pareto weights.

Recalling that the consumption level is given by equation (4), the first order condition with respect to any $\gamma_{ij}$ can be written as

$$\frac{\partial W}{\partial \gamma_{ij}} = 0 \iff \lambda_i E[u'(c_i) \sigma_i s_i] = \lambda_j E[u'(c_j) \sigma_j s_i], \forall \{i, j\} \subset H. \quad (9)$$

The term $E[u'(c_i) \sigma_i s_i]$ is the covariance between household $i$’s income shock and its marginal utility of consumption. Because marginal utility is decreasing, this covariance is negative. When $\gamma_{ij}$ increases at the margin, i.e. when household $i$ transfers more of its shock to $j$, household $i$’s shock induces a lower disutility for $i$. Conversely, $E[u'(c_j) \sigma_j s_i]$ is the (negative) covariance between $i$’s income shock and $j$’s marginal utility.

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13 The solutions of the first best, the autarky and the second best will be presented both in the general and homogeneous cases.

14 The description of the first and second best allocation could be easily extended to the case of preference heterogeneity.
of consumption. Taken in absolute value, \( E[u'(c_j)\sigma_i s_i] \) represents the marginal increase in household \( j \)'s disutility caused by its exposure to household \( i \)'s income risk through the risk-sharing arrangement. Loosely speaking, condition (9) states that at the first best, the parameter of risk-sharing \( \gamma_{ij} \) must be such that the utility loss generated by \( i \)'s income risk is equitably borne by all households within the group (up the Pareto weights). This allows to achieve efficient risk-sharing.

Finally, the first order condition with respect to a household’s individual level of risk-taking is given by

\[
\frac{\partial W}{\partial \sigma_i} = \lambda_i E[u'(c_i)(\mu'(\sigma_i) + (1 - \alpha_i)s_i)] + \sum_{h \in H \setminus \{i\}} \lambda_h E[u'(c_h)\gamma_{ih}s_i] = 0, \forall i \in H.
\]

Making use of the two preceding conditions, we obtain

\[
\frac{\partial W}{\partial \sigma_i} = 0 \iff \mu'(\sigma_i) E[u'(c_i)] + E[u'(c_i)s_i] = 0, \forall i \in H, \quad (10)
\]

where use has been made of the fact that \( \alpha_i = \sum_{h \in H \setminus \{i\}} \gamma_{ih} \), by Lemma 1. Condition (10) states that the benefit of risk-taking must equal its marginal cost when shocks are efficiently shared within the group. \( \mu'(\sigma_i) \) is indeed the marginal increase in expected income allowed by higher risk-taking. Further combining those optimality conditions leads to the following proposition, where \( \tau_h = 1/a_h (E(c_h)) \) denotes the risk tolerance of household \( h \) evaluated at its expected consumption level \( E(c_h) \).

**Proposition 1** At the first best,

1. the risk-sharing arrangement \( \Gamma^{FB} \) can be approximated by \( \tilde{\Gamma}^{FB} \), which is such that for all pair \( \{i,j\} \subset H \),

\[
\tilde{\gamma}_{ij}^{FB} = \frac{\tau_j}{\sum_{h \in H} \tau_h}, \quad (11)
\]

\[
\tilde{\alpha}_i^{FB} = 1 - \frac{\tau_i}{\sum_{h \in H} \tau_h}; \quad (12)
\]

2. risk-taking \( \Sigma^{FB} \) is homogeneous and can be approximated by \( \tilde{\Sigma}^{FB} = (\tilde{\alpha}^{FB}, \ldots, \tilde{\alpha}^{FB}) \), where \( \tilde{\alpha}^{FB} \) is such that

\[
\frac{\mu'(\tilde{\alpha}^{FB})}{\tilde{\sigma}^{FB}} = \frac{1}{\sum_{h \in H} \tau_h}. \quad (13)
\]

**Proof.** Provided in Appendix 2. ■

Let us discuss the intuitions behind the first best levels of risk-sharing on the one hand, and risk-taking on the other hand. The first best risk-sharing arrangement \( \Gamma^{FB} \), (11) and (13), imposes that individual income shocks \( s_h \) are shared across households according to their relative level of risk tolerance: \( \tilde{\gamma}_{ij}^{FB} = \tau_j/ \sum_{h \in H} \tau_h \). The higher \( j \)'s risk tolerance \( \tau_j \), the higher the share of \( i \)'s income shock that is transmitted to \( j \). More risk-tolerant households also bear more of their own shock, since \( 1 - \tilde{\alpha}_i^{FB} = \tau_i/ \sum_{h \in H} \tau_h \). Therefore, the relative level of risk tolerance determines the way in which the total sum of income shocks within the group, \( \sum_{i \in H} s_i \), is shared between households. Indeed, at the first best, post-transfer consumption boils down to:

\[
c_h(S, \tilde{\Gamma}^{FB}, \tilde{\Sigma}^{FB}) = w_h + \tilde{\tau}_h^{FB} + \mu(\tilde{\sigma}_h^{FB}) + \sum_{i \in H} \frac{\tau_i}{\tilde{\tau}_i} \tilde{\sigma}_i^{FB} \sum_{i \in H} s_i. \quad (14)
\]
It is important to point out that, at the first best allocation, \( z_{ij}^{FB} = 1 - \hat{\alpha}_{ij}^{FB}, \forall h \in H \setminus \{j\}, \forall j \). This means that a household \( j \) bears as much of its own risk as of the risk of any other household in the group.

As regards risk-taking \( \Sigma^{FB} \), two things are worth mentioning. First, despite potential differences in wealth and therefore in the degree of absolute risk aversion, all agents take the same level of risk at the first best. This is due to the fact that the planner can reallocate consumption among households and at the same time prevent them from adjusting their risk-taking behavior. This does not hold anymore at the second best, where \( \Sigma \) is not enforceable.

Second, the socially optimal risk-taking level is determined by the willingness to bear risk at the group level, namely the aggregate risk tolerance \( \sum_{h \in H} \tau_h \). One can indeed see from (13) that the first best level of risk-taking \( \hat{\sigma}^{FB} \) is increasing in aggregate risk tolerance \( \sum_{h \in H} \tau_h \). This implies that the socially optimal level of risk-taking increases mechanically with the size of the insurance group, which is due to a better diversification of risks.

Finally, Corollary 1 describes the first best allocation when the group is homogeneous in wealth.

**Corollary 1** If the group is homogeneous in wealth, the first best risk-sharing arrangement \( \Gamma^{FB} \) can be approximated by \( \hat{\Gamma}^{FB} \), which is such that \( z_{ij}^{FB} = 1/n, \forall \{i, j\} \subset H \) and \( \hat{\alpha}_{ih}^{FB} = (n-1)/n, \forall h \in H \).

**Proof.** Provided in Appendix 2. ■

### 3.3 Autarky

In the absence of risk-sharing, \( L^A = 0_{(n \times 1)} \) and \( \Gamma^A = 0_{(n \times n)} \) the consumption equation (4) reduces to

\[
c_h^A = w_h + \mu(\sigma_h) + \sigma_h s_h,
\]

where the superscript \( A \) denotes autarky.\(^{15}\) The next propositions provide some results on the relationship between risk-taking and wealth in autarky. This relationship will be further analyzed in the presence of risk-sharing arrangements in the following section.

**Proposition 2** In autarky, equilibrium risk-taking \( \hat{\sigma}_h^A \) increases with wealth if and only if preferences exhibit decreasing absolute risk aversion (DARA):

\[
\frac{\partial \hat{\sigma}_h^A}{\partial w_h} \geq 0 \iff \alpha' \leq 0.
\]

**Proof.** Provided in Appendix 3. ■

Proposition 2 is an extension of a well-known result of the classical portfolio theory (Arrow (1965)). The canonical model of portfolio choice is a particular case of our model for which the function \( \mu(\sigma) \) is linear.\(^{16}\) Arrow (1965) shows that the investment in the risky asset \( \sigma \) increases with wealth \( w \) if and only if preferences exhibit DARA in this linear case, while we show it for a more general production technology.

\(^{15}\)We present the case of autarky here because it introduces individual risk-taking, which is naturally followed by non-cooperative risk-taking in the second best analysis.

\(^{16}\)Indeed, assume for instance that \( \mu(\sigma) = \mu \sigma \). In this case, \( w \) can be interpreted as initial wealth. Part of it \( (w - \sigma) \) is invested in a safe asset, whose return is normalized to unity, while \( \sigma \) is invested in the risky asset whose return amounts to \( (1 + \mu) \). \( \mu \) is then the random excess return over the safe return. In this setting, final wealth is indeed \( w + \mu \sigma \).
Corollary 2 Under decreasing absolute risk aversion,

1. The poor have a lower expected income:

\[ a' \leq 0 \implies \frac{\partial E\left(y^A_h\right)}{\partial w_h} = 1 + \mu' \left(\sigma^A_h\right) \frac{\partial \sigma^A_h}{\partial w_h} > 0. \]

2. The poor bear lower absolute risk:

\[ a' \leq 0 \iff \frac{\partial \text{Var}\left(y^A_h\right)}{\partial w_h} = \frac{\partial (\sigma^A_h)^2}{\partial w_h} \geq 0. \]

Beyond expected income and absolute risk, it is also interesting to look at the impact of wealth on relative risk. Relative risk is defined here by the coefficient of variation of income \( \nu^A_y \), which is the following ratio:\(^1\)

\[ \nu^A_y = \sqrt{\text{Var}\left(y^A_h\right)/E\left(y^A_h\right)} = \sigma^A\left(w\right)/\left(w + \sigma^A\left(w\right)\right). \] (15)

**Proposition 3** The poor bear higher relative risk if preferences exhibit increasing relative risk aversion (IRRA):

\[ r' \geq 0 \implies \frac{d\nu^A_y}{dw_h} < 0. \]

**Proof.** Provided in Appendix 3.

This result is a second extension of the portfolio choice theory. Arrow (1965) shows that the proportion of the risky asset in an agent’s portfolio is non-increasing in initial wealth under IRRA, a well admitted property of utility functions in risk theory.\(^2\) With our notations, Arrow (1965) states that \( \nu^A_y \) is non increasing in \( w \) under IRRA. We find the same result in autarky while allowing \( \mu (\sigma) \) to be potentially non-linear.

Summing up, DARA is a necessary and sufficient condition for the poor to take lower absolute risk in autarky, while IRRA implies that in relative terms, this risk is higher than the rich. We will show in the next section how the introduction of the second-best risk-sharing arrangement affects these statements. In what follows, we maintain the assumption that preferences exhibit DARA (\( a' \leq 0 \)) and IRRA (\( r' \geq 0 \)).

### 4 Second best analysis: risk-sharing with moral hazard

Let us now study the case in which the social planner is not able to enforce households’ risk-taking behavior. In this case, the planner sets the transfer scheme \( (L, \Gamma) \) in the first stage, while households decides on their risk-taking level \( \sigma_h \) simultaneously and non-cooperatively in the second stage. The transfer scheme \( (L, \Gamma) \) still maps shocks \( S \) into transfers \( T \), but as we will see, the insurance scheme now affects individuals’ risk-taking \( \Sigma \). In this second best analysis, the planner’s problem is therefore to maximize the social welfare function by setting the insurance scheme \( (L, \Gamma) \), anticipating the scheme’s impact on the Nash equilibrium risk-taking profile \( \Sigma^N \).

\(^1\) Expected consumption and its variance are given by equations (5) and (6), where the risk-sharing parameters are evaluated at their autarkic values, that is for \( L^A = 0_{(n \times 1)} \) and \( \Gamma^A = 0_{(n \times n)} \).

\(^2\) Equivalently, the wealth elasticity of demand for cash is at least equal to 1 if and only if preferences exhibit IRRA.
4.1 Moral hazard

We start by solving the household’s individual risk-taking problem, considering the risk-sharing arrangement as given. The following Lemma characterizes the equilibrium non-cooperative risk-taking profile $\Sigma^N$.

**Lemma 2 Non-cooperative risk-taking:**

1. The equilibrium non-cooperative risk-taking profile is given by $\Sigma^N (L, \Gamma) = (\sigma^N_1 (L, \Gamma), \ldots, \sigma^N_n (L, \Gamma))$, where $\sigma^N_h$ is such that

   $$
   \mu' (\sigma^N_h) E [u' (c_h)] + (1 - \alpha_h) E [u' (c_h) s_h] = 0.
   $$

2. The equilibrium non-cooperative risk-taking profile can be approximated by $\Sigma^N (L, \Gamma) = (\tilde{\sigma}^N_1 (L, \Gamma), \ldots, \tilde{\sigma}^N_n (L, \Gamma))$, where $\tilde{\sigma}^N_h$ is such that

   $$
   \mu' (\tilde{\sigma}^N_h) = \frac{\partial \tilde{\pi}_h}{\partial \sigma^N_h} = a_h (1 - \alpha_h)^2 \sigma^N_h,
   $$

   where Pratt’s approximation of the risk premium $\tilde{\pi}_h$ is

   $$
   \tilde{\pi}_h \simeq \frac{a_h}{2} \left[ (1 - \alpha_h)^2 \sigma^2_h + \sum_{i \in H \setminus \{h\}} \gamma^2_{ih} \sigma^2_i \right].
   $$

**Proof.** Taking the transfer scheme as given, households maximize expected utility $E_S [u (c_h (S; L, \Gamma))]$, where $c_h (S; L, \Gamma)$ is given by equation (4). $\sigma^N_h$ is obtained by taking the first order condition of the household’s utility maximization problem with respect to $\sigma_h$. We approximate this condition by a Taylor expansion described in Appendix 2. This approximation reveals that the marginal cost of risk-taking is equal to $\partial \tilde{\pi}_h / \partial \sigma^N_h$, where $\tilde{\pi}_h$ is Pratt’s approximation of the risk premium.

The risk-taking level of a household $h$ at the Nash equilibrium, as given by equation (17) can be interpreted in the following way: the marginal benefit of risk-taking must equal its private marginal cost.\(^{19}\) The marginal benefit is $\mu' (\tilde{\sigma}^N_h)$, which measures the marginal increase in household $h$’s expected income as $\sigma_h$ increases. The private marginal cost of risk-taking is represented by the marginal increase in the risk premium.

Let us comment on the positive and normative aspects of $\sigma^N_h$. First, in the light of equation (17), one can see that risk-taking increases with household wealth under DARA, $\partial \tilde{\sigma}^N_h / \partial \omega_h > 0$. Therefore, as soon as group members have different levels of wealth, risk-taking is not homogeneous as in the first best. Also, household $h$’s risk-taking increases with its own rate of insurance coverage: $\partial \tilde{\sigma}^N_h / \partial \alpha_h > 0$: the larger the household’s risk coverage through risk-sharing, the higher the level of risk it takes. Let us define $\epsilon_h$ as the elasticity of non-cooperative risk-taking to the rate of coverage $\alpha_h$, a concept that will be used later on to highlight the impact of moral hazard on risk-taking:\(^{20}\)

$$
\epsilon_h = \frac{\partial \tilde{\sigma}^N_h}{\partial \alpha_h} \frac{\alpha_h}{\tilde{\sigma}^N_h} = - \frac{2 \alpha_h (1 - \alpha_h) a_h}{\mu'' (\tilde{\sigma}^N_h) (1 - \alpha_h)^2 a_h - \sigma^N_h \mu' (\tilde{\sigma}^N_h) (1 - \alpha_h)^2 a_h} > 0.
$$

Second, as regards the normative aspects of $\sigma^N_h$, the private marginal benefit of non-cooperative risk-taking, $\mu' (\sigma^N_h) E [u' (c_h)]$, is equal to its private marginal cost $(1 - \alpha_h) E [u' (c_h) s_h]$, which by definition

\(^{19}\)Externalities are highlighted in the next proposition.

\(^{20}\)This formula is obtained by applying the implicit function theorem to equation (17). The denominator is negative if and only if the second order condition (SOC) of the household maximization problem is satisfied. We show in Appendix 4 that IRRA is a sufficient condition for the SOC to be satisfied.
does not incorporate the risk externalities generated by \( \sigma^N_h \) on other members (i.e. the shares of \( h \)'s risk that are borne by its insurance partners). As a result, \( \sigma^N_h \) is too high as compared to the social optimum, leading to moral hazard as stated in the next proposition.

**Proposition 4 Moral hazard:** For all insurance arrangement \( \Gamma \), non-cooperative risk-taking is always larger than the socially optimal level:

\[
\sigma^N_h (\Gamma) > \sigma^{FB}_h (\Gamma),
\]

where \( \sigma^{FB}_h (\Gamma) \) is the social planner’s first best level of risk-taking for any given \( \Gamma \).

**Proof.** This result is obtained by a simple comparison of the planner’s first order condition with respect to household \( h \)'s risk-taking and household \( h \)'s first order condition at the non-cooperative solution for a given insurance scheme \( \Gamma \). These conditions are respectively given by

\[
\frac{\partial W}{\partial \sigma_h} = 0 \iff \mu' (\sigma^N_h) E [u' (c_h)] + (1 - \alpha_h) E [u' (c_h) s_h] + \frac{1}{\lambda_h} \sum_{j \in H \setminus \{h\}} \lambda_j E [u' (c_j) \gamma_{hj} s_h] = 0,
\]

\[
\frac{\partial E u(c_h)}{\partial \sigma_h} = 0 \iff \mu' (\sigma^N_h) E [u' (c_h)] + (1 - \alpha_h) E [u' (c_h) s_h] = 0.
\]

Since \( u'' < 0 \), \( E [u' (c_j) \gamma_{hj} s_h] < 0 \), \( \forall j \in H \setminus \{h\} \), we can conclude that \( \sigma^{FB}_h (\Gamma) < \sigma^N_h (\Gamma) \) for all \( \Gamma \).

This proposition states that non-cooperative risk-taking is always higher than the risk-taking level that would have been chosen by the social planner, had this risk-taking been enforceable. The reason thereof is that, while household \( h \) only considers its private cost and benefit of risk-taking, the social planner also takes into account the externalities generated by risk-taking on the other group members, as highlighted by the following term:

\[
\sum_{j \in H \setminus \{h\}} \lambda_j \frac{\partial E u_j}{\partial \sigma_h} = \sum_{j \in H \setminus \{h\}} \lambda_j E [u' (c_j) \gamma_{hj} s_h] < 0.
\]

This result may appear at odds with empirical observations of risk-taking behaviors in developing countries, which highlight that risk-taking is limited (see, for instance Kurosaki and Fafchamps (2002)). However, since risk-taking is generally excessive, risk-sharing arrangements are adjusted to mitigate the moral hazard problem, which leads to limited insurance, as will be shown below. As a result, under the second best risk-sharing arrangement, it may well be that \( \sigma^N_h (\Gamma^{SB}) < \sigma^{FB} \), as shown in the next proposition. This proposition describes the design of the transfer scheme \((L^{SB}, \Gamma^{SB})\) set by the planner in the first stage of the game (i.e. before individual risk-taking is chosen), anticipating that households will adopt the non-cooperative level of risk-taking \( \Sigma^N (\Gamma) \).

### 4.2 The second best allocation

Let us turn to the description of the second best risk-sharing arrangement. At the second best, the planner maximizes the welfare function \( W = \sum_{h \in H} \lambda_h E u(c_h) \) with respect to the transfer scheme \((L, \Gamma)\), subject to the incentive compatibility condition (16).

The first order condition with respect to the lump sum transfer writes

\[
\frac{\partial W}{\partial l_{ij}} = -\lambda_i E [u' (c_i)] + \lambda_j E [u' (c_j)] + \sum_{h \in H \setminus \{i\}} \lambda_h \frac{\partial E u_h}{\partial \sigma_i} + \sum_{h \in H \setminus \{j\}} \lambda_h \frac{\partial E u_h}{\partial \sigma_j} = 0. \tag{20}
\]
The lump sum transfer still aims at redistributing income according to the Pareto weights $\lambda$. However, the lump sum transfer now also allows to manage risks within the group since it affects risk-taking behavior: by DARA, the household sending a lump sum transfer takes fewer risks ($\partial \sigma^N_i / \partial x_{ij} < 0$), while the household receiving it becomes richer and takes more risks ($\partial \sigma^N_j / \partial x_{ij} > 0$). This phenomenon affects the utility of all other agents in the group ($\partial E_u/\partial \sigma_j = \gamma_{jh} E[u'(c_h) s_j] < 0$, $\forall h \in H \setminus \{j\}$.\footnote{Notice that $\partial E_u/\partial \sigma_i = \partial E_u/\partial \sigma_j = 0$, by the envelope theorem.} As a result, the fact for a household of sending a positive lump sum transfer generates positive externalities for the group, while receiving one generates negative externalities.\footnote{It should be noted that these effects cancel each other out in homogeneous groups (and with equal Pareto weights), in which case, the condition for second best lump sum transfer boils down to its first best counterpart.}

Regarding the risk-sharing coefficient $\gamma_{ij}$, the first order condition is as follows:

$$\frac{\partial W}{\partial \gamma_{ij}} = -\lambda_i E[u'(c_i) \sigma_i s_i] + \lambda_j E[u'(c_j) \sigma_i s_i] + \frac{\partial \sigma^N_i}{\partial \gamma_{ij}} \sum_{h \in H \setminus \{i\}} \lambda_h \frac{\partial E_u}{\partial \sigma_i} + \frac{\partial \sigma^N_j}{\partial \gamma_{ij}} \sum_{h \in H \setminus \{j\}} \lambda_h \frac{\partial E_u}{\partial \sigma_j} = 0. \quad (21)$$

The parameter $\gamma_{ij}$ is set so as to share risk efficiently. But again, at the second best, changes in the risk-sharing arrangement affect risk-taking behavior. The effect is twofold. On the one hand, since $\alpha_i = \sum_{j \in H \setminus \{i\}} \gamma_{ij}$, a marginal increase in $\gamma_{ij}$ implies ceteris paribus a marginal increase in $\alpha_i$, the rate of coverage of household $i$. This increase in risk coverage induces household $i$ to take more risk $\partial \sigma^N_i / \partial \alpha_i > 0$ as shown above. On the other hand, an increase in $\gamma_{ij}$ increases the level of background risk faced by household $j$. A substantial literature has studied conditions required on utility functions to ensure that the impact of background risk on individual risk-taking is negative.\footnote{Eeckhoudt and Kimball (1992) and Kimball (1993) obtained a sufficient condition called “standard risk aversion”, which states that both absolute risk aversion and absolute prudence be both decreasing. Gollier and Scarmure (1994) have shown an alternative condition such that risk aversion must not only be decreasing, but also convex. Gollier and Pratt (1996) present an alternative condition called “risk vulnerability”.}

In our case, the use of Taylor approximations on the first order conditions eliminates this effect. Indeed, it can be seen from (17) that $\gamma_{ih}$ does not enter into the implicit formulation of $\sigma^N_h$. Combining the first order conditions, we obtain the following result.

**Proposition 5** At the second best, the risk-sharing arrangement $\Gamma^{SB}$ can be approximated by $\tilde{\Gamma}^{SB}$, which is such that for all pair $\{i, j\} \subset H$,

$$\tilde{\gamma}_{ij}^{SB} = \frac{\tau_j}{\lambda_j u_j} \left( \sum_{h \in H} \frac{\tau_h}{\lambda_h u_h} + \frac{\tau_i}{\lambda_i u_i} \epsilon_i \right)^{-1},$$

$$\tilde{\alpha}_i^{SB} = 1 - \frac{\tau_i}{\lambda_i u_i} (1 + \epsilon_i) \left( \sum_{h \in H} \frac{\tau_h}{\lambda_h u_h} + \frac{\tau_i}{\lambda_i u_i} \epsilon_i \right)^{-1}, \quad (22)$$

**Proof.** Provided in Appendix 5. \blacksquare
apparent in equation (22), which shows that a household level of informal insurance coverage decreases with its responsiveness in terms of risk-taking $\epsilon_i$.\textsuperscript{24} This means that households more prone to taking risks when they are insured receive less insurance under the second best, other things equal. Interestingly, we will show below that $\epsilon_i$ is higher for the poor. As regards the presence of marginal utilities of all the agents ($\lambda_h u_h'$) in (22), they would have been equal and would have canceled out as in the first best if lumpsum transfers did not have the additional effect of affecting risk-behaviors.\textsuperscript{25}

Proposition 5 also reveals that, for any pair of identical households $\{i, j\} \subset H$, we have $\gamma_{ij}^{SB} = \gamma_{ji}^{SB} < 1 - \alpha_i^{SB} = 1 - \alpha_j^{SB}$. This tells us that, at the second best, households bear more of their own shock as of the others’ shocks. This property of the second best arrangement allows to temper moral hazard, but at the cost of an incomplete and hence imperfect risk-sharing.

### 4.3 Risk-sharing, risk-taking and wealth in homogeneous groups

As pointed out in numerous empirical works (Ahlin (2010), Gine et al. (2010), Attanasio et al. (2012)), risk-sharing groups tend to be composed of individuals with similar wealth levels.\textsuperscript{26} We study in this section how groups which are homogeneous in terms of wealth levels behave in terms of risk-sharing and risk-taking. In this context, we describe the mechanisms explaining why poor households tend to share and take less risks than rich households.

**Proposition 6** At the second best in a homogeneous group, the risk-sharing arrangement $\Gamma^{SB}$ can be approximated by $\tilde{\Gamma}^{SB}$, which is such that $\forall \{i, j\} \subset H$,

\[
\tilde{\gamma}_{ij}^{SB} = \frac{1}{n + \epsilon}, \quad \tilde{\alpha}_i^{SB} = \frac{n - 1}{n + \epsilon}.
\]

**Proof.** Provided in Appendix 5. □

As shown in the proof, the system of first order conditions in the homogeneous case is satisfied for a symmetric transfer scheme, i.e. $L = 0_{(n \times 1)}$ and $\Gamma = \Gamma^H$, where $\Gamma^H$ is such that $\alpha = \alpha_h$, $\forall h \in H$ and $\gamma_{ij} = \alpha / (n - 1)$. A direct consequence of this is that the solution relies on a single parameter, $\alpha$. Showing the first order condition with respect to this unique parameter provides interesting insights:

\[
\frac{dW}{d\alpha} = \frac{\partial W}{\partial \alpha} + \frac{\partial W}{\partial \sigma} \frac{\partial \sigma}{\partial \alpha},
\]

where

\[
\frac{\partial W}{\partial \alpha} \geq 0 \iff \alpha \leq \tilde{\alpha}^{FB},
\]

\[
\frac{\partial W}{\partial \sigma} = \alpha \frac{n}{n - 1} \sum_{i \in H \backslash \{h\}} E[u'(c_h') s_i] < 0.
\]

The case of homogeneous groups offers a clear illustration of the trade-off faced by the planner. Indeed, in the light of equation (23), it can be seen that the effect of a marginal increase in $\alpha$ is twofold. On the one hand, $\frac{\partial W}{\partial \alpha} \geq 0 \iff \alpha \leq \tilde{\alpha}^{FB}$,

\[
\frac{\partial W}{\partial \sigma} = \alpha \frac{n}{n - 1} \sum_{i \in H \backslash \{h\}} E[u'(c_h') s_i] < 0.
\]

\textsuperscript{24}Indeed, $\frac{\partial \tilde{\alpha}_i^{SB}}{\partial \gamma_{ij}^{SB}} < 0 \iff \frac{\gamma_{ij}^{SB}}{n^2 \tilde{\gamma}_{ij}^{SB}} \sum_{h \in H \backslash \{i\}} \frac{\gamma_{ih}^{SB}}{\lambda_h u_h'} > 0$, which is always true.

\textsuperscript{25}In the homogeneous case, these indirect effects cancel out, simplifying the expression, as see in the next section.

\textsuperscript{26}The question of the endogenous composition of insurance groups is discussed in Section 5.
hand, it has a positive direct effect on welfare as long as risk-sharing is incomplete, namely lower that its first best level.\textsuperscript{27} On the other hand, a higher rate of risk-sharing stimulates risk-taking $\partial \sigma^N / \partial \sigma > 0$, which is detrimental to welfare. Indeed, we know by Proposition 4 that risk-taking is excessive at equilibrium. Therefore, $\partial W / \partial \sigma < 0$. This is apparent in expression (24): $\sum_{i \in H \setminus \{k\}} E [u'(c_h) s_i]$ is indeed the sum of the negative correlations between household $h$’s marginal utility and the other households’ income shocks. This term captures the increase in risk externalities supported by $h$ following a general increase in $\sigma$, the level of risk adopted by the other households.

Naturally, the negative effect is strengthened if households’ risk-taking decision is more sensitive to their rate of coverage $\alpha$, so that $\alpha^{SB}$ is a decreasing function of $\epsilon$. Proposition 6 states that the second best level of risk-sharing in homogeneous groups $\alpha^{SB}$ is approximately equal to $(n - 1) / (n + \epsilon)$, implying that it is indeed lower under high responsiveness of risk-taking to risk-sharing $\epsilon$.

In the next proposition, we address the following question: how do groups composed of poor households share their risks compared to rich groups? In order to answer this question, a first step is therefore to see how the wealth level $w$ affects this elasticity $\epsilon$.

**Lemma 3** The elasticity of risk-taking to risk-sharing is decreasing in the wealth level: $\partial \epsilon / \partial w < 0$.

**Proof.** Taking the derivative of $\epsilon$, as given by equation (19), with respect to $w$, we obtain

$$\frac{\partial \epsilon}{\partial w} < 0 \iff \mu''(\sigma) a' + \sigma_4 \mu'(\sigma) (1 - \alpha)^2 [aa'' - (a')^2] > 0.$$ 

Since $\mu''(\sigma) a' \geq 0$, a sufficient condition is that $aa'' - (a')^2 > 0$. This condition is satisfied for a wide set of utility functions such as the hyperbolic absolute risk aversion (HARA).\textsuperscript{28} \hfill \blacksquare

This lemma states that poor households’ risk-taking behavior is more sensitive to insurance as compared to the rich’s. The reason for this is twofold. On the one hand, under decreasing absolute risk aversion, their autarkic level of risk-taking is lower (by Proposition 2). On the other hand, marginal returns to risk-taking are decreasing ($\mu''(\sigma) < 0$). Therefore, for any given reduction of their cost of risk-taking (determined by their risk premium), the poor can seize higher marginal returns $\mu'(\sigma)$ than the rich. Moreover, since the risk premium is proportional to $\alpha$, an increase in insurance coverage $\alpha$ decreases the poor’s risk premium more than the rich’s.\textsuperscript{29} Hence, following a marginal increase in $\alpha$, the poor’s optimal increase in risk-taking $\sigma$ is proportionally higher than the rich’s.

For these reasons, the moral hazard problem is more severe among poor households. Note that this result is general and independent of the group composition since it is only based on household optimization for any given insurance arrangement (see Lemma 2). This result has important consequences on risk-sharing, as stated in the next proposition.

**Proposition 7** The impact of wealth on risk-sharing

At the second best, poor groups, which are more subject to moral hazard, share less risk than rich groups:

$$\frac{\partial \alpha^{SB}}{\partial w} > 0.$$ 

\textsuperscript{27}Indeed, holding $\sigma$ constant, the value of $\alpha$ such that $\partial W / \partial \alpha = 0$ is by definition $\tilde{\alpha}^{FB}$.

\textsuperscript{28}The class of HARA utility functions, which encompasses the CRRA, exponential and quadratic utility functions, is such that $u(c) = \epsilon \left( \frac{c}{\epsilon + \nu} \right)^{1-\epsilon} \nu$, which leads to $a(c) a''(c) - (a'(c))^2 = \frac{\epsilon^2 (\epsilon - 1) \nu^2}{(c + \nu)^2} > 0$.

\textsuperscript{29}Indeed, in the light of expression (18), it is easy to see that $\partial^2 \tilde{\alpha} / \partial \alpha \alpha_h < 0$, $\forall \alpha_h \in [0, 1]$, so that the reduction of $\tilde{\alpha}_h$ is stronger for the poor for whom $\alpha$ is higher.

As explained above, a high responsiveness of risk-taking to risk-sharing strengthens the adverse effects of risk-sharing in terms of moral hazard. It follows that poor groups, where this effect is more pronounced, should adopt a lower rate of informal risk-sharing.

Corollary 3 The poor support a higher fraction of their income shock than the rich:

\[ \frac{\partial^2 c_h}{\partial w \partial \sigma_h s_h} = -\frac{\partial \alpha^{SB}}{\partial w} < 0. \]

Proof. Referring to the consumption equation (4) and recalling that an income shock is equal to \( \sigma_h s_h \), we can see that indeed \( \partial c_h / \partial \sigma_h s_h = 1 - \alpha_h \) and hence \( \partial^2 c_h / \partial w \partial \sigma_h s_h = -\partial \alpha^{SB} / \partial w < 0. \) ■

Let us now analyze whether poor groups also tend to take less risks than rich ones.

Proposition 8 Under second best risk-sharing, risk-taking increases with wealth:

\[ \frac{d \sigma^N}{dw} = \frac{\partial \sigma^N}{\partial w} + \frac{\partial \sigma^N}{\partial \alpha} \frac{\partial \alpha^{SB}}{\partial w} > 0. \]

Proof. Applying the implicit function theorem to the equilibrium non-cooperative risk-taking level (17), we have that \( \partial \sigma^N / \partial w > 0 \) and \( \partial \sigma^N / \partial \alpha > 0 \). Combining with \( \partial \alpha^{SB} / \partial w > 0 \) (by Proposition 7), one obtains the result. ■

Corollary 4 Under second best risk-sharing,

1. The poor have a lower expected income:

\[ \frac{dE(y)}{dw} = 1 + \mu' \left( \sigma^{SB} \right) \frac{d \sigma^{SB}}{dw} > 0. \]

2. The poor have lower absolute income risk:

\[ \frac{d \text{Var}(y)}{\partial w} = \frac{d \left( \sigma^{SB} \right)^2}{dw} > 0. \]

Two effects differentiate risk-taking between poor and rich groups. First, under decreasing absolute risk aversion, the rich are more risk-tolerant, which induces them to take more risks, ceteris paribus. Second, by Proposition 7, we know that rich groups share more risks. Because receiving more insurance induces agents to take more risks, this second effect reinforces the first. Taken together, Corollary 3 and Proposition 8 are therefore in line with the two stylized facts according to which poor households tend to be more affected by idiosyncratic shocks and are less keen to adopt high risk / high return technologies.

As a final exercise, we analyze the relationship between wealth and the relative risk borne by households, as defined by the coefficient of variation of consumption \( \nu_c = \sqrt{\text{Var}(c)} / E(c) \), at equilibrium. In autarky, the coefficient of variation of consumption and the coefficient of variation of income are by definition equivalent. Risk-sharing breaks this equivalence as it reduces the coefficient of variation of consumption \( \nu_c \) as compared to the coefficient of variation of income \( \nu_y \). At the symmetric equilibrium, we have

\[ \nu_c = \chi(\alpha) \nu_y, \] (25)
where $\nu_y = \sigma / (w + \mu(\sigma))$, and\(^{30}\)

$$\chi(\alpha) = \sqrt{(1 - \alpha)^2 + \frac{\alpha^2}{m - 1}} \in [0, 1] \forall \alpha \in [0, 1],$$

$$\chi'(\alpha) < 0, \forall \alpha < \tilde{\alpha}^{FB}.$$  \(26\)

**Proposition 9** Under second best risk-sharing, the poor have a higher coefficient of variation of consumption if and only if

$$\frac{d\nu_c}{dw} = \chi'(\alpha^{SB}) \frac{\partial \alpha^{SB}}{\partial w} \nu_y + \chi(\alpha^{SB}) \frac{d\nu_y}{dw} < 0,$$

where

$$\frac{d\nu_y}{dw} = \frac{\partial \nu_y}{\partial w} + \frac{\partial \nu_y}{\partial \sigma} \left[ \frac{\partial \sigma^{SB}}{\partial w} + \frac{\partial \sigma^{SB}}{\partial \alpha} \frac{\partial \alpha^{SB}}{\partial w} \right].$$

**Proof.** This result obtains by taking the total derivative of $\nu_c$ (25) with respect to wealth. \(\blacksquare\)

Under second best risk-sharing, a variation of wealth affects the coefficient of variation of consumption $\nu_c$ in two ways.

On the one hand, an increase in wealth increases the rate of risk-sharing $\partial \tilde{\alpha}^{SB} / \partial w > 0$, by Proposition 7. Since $\chi'(\tilde{\alpha}^{SB}) < 0$, this effect of $w$ through $\tilde{\alpha}^{SB}$ on $\nu_c$ is negative.

On the other hand, $\nu_c$ is affected by $\nu_y$, which is affected in various ways by wealth. The first direct effect of wealth on $\nu_y$ is negative ($\partial \nu_y / \partial w < 0$). Indeed, a higher level of wealth mechanically increases the denominator of $\nu_y$. Second, wealth has an indirect, positive impact on $\nu_y$ through risk-taking. Indeed, we know that wealth has a positive impact on risk-taking, both directly ($\partial \sigma^{SB} / \partial w > 0$) and through risk-sharing ($\partial \sigma^{SB} / \partial \alpha (\partial \alpha^{SB} / \partial w) > 0$). This increase in risk-taking has a positive impact on $\nu_y$ ($\partial \nu_y / \partial \sigma > 0$).\(^{31}\)

**Proposition 10** Under second best risk-sharing, the poor have a higher coefficient of variation of income if and only if

$$\frac{d\nu_y}{dw} < 0 \iff \epsilon_{\sigma, w} < \frac{w}{w + \mu(SB) - \sigma^{SB} \mu'(\sigma^{SB})},$$

where

$$\epsilon_{\sigma, w} = \left[ \frac{\partial \sigma^{SB}}{\partial w} + \frac{\partial \sigma^{SB}}{\partial \alpha} \frac{\partial \alpha^{SB}}{\partial w} \right] \frac{w}{\sigma^{SB}}.$$  

**Proof.** Provided in Appendix 7. \(\blacksquare\)

Proposition 10 highlights the condition under which wealth and the coefficient of variation of income are negatively associated. It is actually similar to its counterpart in the case of autarky.\(^{32}\) The only difference pertains to the fact that the elasticity of risk-taking to wealth is higher under second best risk-sharing than under autarky since risk-taking is also enhanced by a higher rate of risk-sharing: $\partial \tilde{\alpha}^{SB} / \partial \alpha > 0$ and $\partial \tilde{\alpha}^{SB} / \partial w > 0$.

In order to provide more insights into the relationship between wealth and $\nu_c$, let us decompose $d\nu_c / dw$ so as to disentangle the role played by second best risk-sharing from the direct effects of wealth on $\nu_c$. This decomposition is as follows

$$\frac{d\nu_c}{dw} = \chi(\alpha) \left[ \frac{\partial \nu_y}{\partial w} + \frac{\partial \nu_y}{\partial \sigma} \frac{\partial \sigma^{SB}}{\partial w} \right] + \frac{\partial \alpha^{SB}}{\partial w} \left[ \chi'(\alpha^{SB}) \nu_y + \chi(\alpha^{SB}) \frac{\partial \nu_y}{\partial \sigma} \frac{\partial \sigma^{SB}}{\partial \alpha} \right].$$

\(^{30}\)Notice that $\chi(\alpha)$ is minimized for $\alpha = \tilde{\alpha}^{FB}$.

\(^{31}\)See Appendix 3.

\(^{32}\)See Appendix 3.
The first term of this expression measures the total effect of wealth on $\nu_c$ for a given level of risk-sharing $\alpha$. This total effect is composed of a negative mechanical effect and a positive indirect effect through an increase in risk-taking. We show in appendix 7 that this total effect is unambiguously negative under IRRA, for any given value of $\alpha$. This is actually an extension of the result presented in Proposition 3 where we show that the total effect of wealth on $\nu_c$ is negative in autarky, namely for $\alpha = 0$.

As shown by the second term, wealth also has an impact on the planner’s strategy through an increase in risk-sharing ($\partial \alpha^S \partial w > 0$). This increase in risk-sharing has two opposite effects on $\nu_c$. First, a decrease in $\nu_c$ occurs mechanically via $\chi' (\alpha^S)$. Second, better insured households take more risks, thereby increasing relative risk ($\partial \nu_y / \partial \sigma > 0$). As a result, the net effect on $\nu_c$ of an increase in risk-sharing $\alpha$ is indeterminate.

### 5 Discussion

The results of this paper can be extended in various ways, for instance by using alternative solution concepts for the definition of the risk-sharing scheme and by endogenizing the composition of insurance groups. Also, alternative explanations of the stylized facts might be considered. We discuss these three points here.

First, as regards the solution concept used to derive the risk-sharing scheme, the second best allocation determined by a social planner can also be obtained via a centralized bargaining process at the group level (see Delpierre et al. (2014)). Furthermore, we have studied in Delpierre et al. (2014), the case of a decentralized bargaining process where all potential pairs of agents set a specific transfer scheme so as to maximize the pair’s joint surplus. This solution concept appears more realistic in an environment where enforcement devices are lacking, as it does not rely on the strong coordination imposed by the classical second best approach. The comparison between this setting and the second best provides interesting insights. This decentralized bargaining generates group overlaps, which leads to insufficient internalization of risk externalities within pairs since these externalities also hit all other partners. Moral hazard in risk-taking is therefore more problematic than at the second best. This lack of coordination at the group level strengthens the moral hazard mechanism, which reinforces the conclusions we draw in the present paper.

Second, we have not explicitly tackled the process of group formation in this paper. This does not necessarily imply that the group is strictly exogenous in our model, since agents receiving an (endogenously) low risk coverage might be seen as excluded from the group. While we leave the study of group formation for future research, let us briefly discuss here the relevance of the case of homogeneous groups. Recent research is somewhat inconclusive regarding the assortative pattern of risk-sharing groups. Legros and Newman (2007) and Chiappori and Reny (2015) build matching models with non-transferable utility and obtain that pairs that are formed with the aim of sharing risks are negatively assortative on risk preferences. Still, recent empirical evidence tends to show that groups are homogeneous. For instance, Attanasio et al. (2012) observe positive assortative matching (PAM) on risk attitudes in Columbian communities. In accordance with the theoretical work of Ghatak (2000), group lending mechanisms, which also entail informal risk-sharing, are also characterized by PAM as highlighted by Ahlin (2010) or Gine et al. (2010). This discrepancy between the predictions of matching models and empirical evidence on the composition of risk-sharing groups may result from the imperfections of risk-sharing arrangements. Indeed, both Legros and Newman (2007) and Chiappori and Reny (2015) assume efficient risk-sharing, which obviously corresponds to the first best in our model. While the sources of inefficiency such as limited commitment and moral hazard may be overcome.

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33 We must acknowledge, however, that part of our results have been obtained in the particular case of homogeneous groups.
within couples, they tend to be more difficult to handle in larger groups. In other words, these models may be perfectly suited to study the marriage market, but not necessary the formation of informal groups of larger sizes. In their study of agricultural contracts, Ghatak and Karaivanov (2014) show that in the presence of incentive constraints, a second best contractual arrangement may produce an opposite assortative pattern as compared to the first best. We argue that this might be the case for risk-sharing groups as well.

The following intuitions, which need to be confirmed in future work, indicate that indeed NAM under first best risk-sharing and PAM under second best risk-sharing is a plausible outcome.

Under first best risk-sharing, the joint surplus generated by a heterogeneous pair is higher than the surplus generated by a homogeneous pair since a more risk neutral agent (the rich under DARA) can offer more insurance to the more risk averse (the poor). Provided the risk neutral agent can capture a sufficiently large share of this surplus, the decreasing difference condition is satisfied and NAM follows.\textsuperscript{34}

As regards the second best, the sources of imperfection studied in this paper are risk externalities and moral hazard. Therefore, under second best risk-sharing, a poor agent does not necessarily want to get matched with a rich since the latter tends to take higher levels of risks. This mechanism might lead to PAM, since the poor, whose willingness to avoid additional risks is highest, are also those who take fewer risks.

Third, and finally, one might consider potential alternative explanations for the core finding that poor households have lower expected consumption and face higher risks. While we have tackled the problem in the context of an informal risk-sharing arrangement with imperfections, a dynamic model of credit and investment under limited liability might yield similar results. Indeed, liability constraints being more binding for the poor, they tend to be excluded from capital markets (Ghosh et al. (2001)). Therefore, they are prevented from seizing remunerative opportunities, while at the same time, they cannot rely on credit to smooth consumption over time.

### 6 Conclusion

The analysis conducted in this paper has been motivated by a twofold empirical observation: on the one hand, poor farm households are reluctant to adopt technologies and to take risks that would allow them to obtain higher returns. On the other hand, despite taking less risks, they tend to suffer from a higher exposure to idiosyncratic income shocks. We propose a theoretical framework which provides a potential explanation to these phenomena. First, as a benchmark, we show that poor households indeed take less risks and earn lower expected income than the rich, but that the relative risk faced by the poor is larger than that of the rich. Second, we show that these results are robust to the introduction of informal risk-sharing arrangements subject to limited group sizes and missing insurance and capital markets.

Within this set of imperfections, we have analyzed the interactions between households’ risk-taking behavior and risk-sharing arrangements under various regimes, i.e. the first best where the planner determines both the risk-sharing mechanism and agents’ risk-taking, and the second best in which households instead choose their level of risk-taking. Our representation of risk-taking strategies is based on a trade-off between

\textsuperscript{34} The increasing (decreasing) difference condition for PAM (NAM, respectively) states that a good type’s willingness to pay for being associated with a good type is higher than a bad type’s. This implies that PAM is a stable equilibrium, while NAM is not. This condition is only valid under transferable utility (TU). TU holds in risk-sharing problems under restrictive conditions (See Schulhofer-Wohl (2006)). Legros and Newman (2007) have however introduced a generalized version of this condition, which holds under non-transferable utility. The important point is that the intuition behind this condition is preserved.
expected return (mean income) and risk (variance of income). While a standard setting with formal markets leads to efficient outcomes under this representation, this is not the case under the specific framework of informal risk-sharing. On the one hand, due to their reduced size, informal groups achieve a limited level of risk diversification. On the other hand, missing capital and reinsurance markets imply that the group's transfers must adapt to all potential realizations of shocks. As a result, individual behavior affects all group members through the mutual insurance scheme, leading to risk externalities. These externalities naturally lead to moral hazard problems in terms of excessive risk-taking.

We show that the second best insurance arrangement limits the degree of risk-sharing in order to mitigate moral hazard. Furthermore, we investigate the role played by household wealth on risk-taking and risk-sharing. Interestingly, we show that the moral hazard problem is stronger for poor households, whose risk-taking behavior is more sensitive to insurance. In order to mitigate moral hazard, the social planner’s second best risk coverage is thus lower in poor groups than in rich groups. This leads to the result that, compared to the case of autarky, the introduction of risk-sharing schemes reinforces the gap between rich and poor households in terms of risk-taking and expected income.

References


Appendix 1: proof of Lemma 1

For the budget constraint to be satisfied with equality, the sum of transfers over the whole group, which we denote by $B(S)$, needs to be zero for all $S \in R^n$. Making use of equation (2), the budget constraint can be written as

$$B(S) = \sum_{h \in H} t_h(S) = 0 \iff \sum_{h \in H} \left[ l_h + \sum_{j \in H} \gamma_{jh} \sigma_j s_j \right] = 0, \forall S \in R^n$$

First notice that in the particular case where $S = (0, ..., 0)'$, this condition implies that $\sum_{h \in H} l_h = 0$. Therefore, we can rewrite the budget constraint as

$$B(S) = 0 \iff \sum_{h \in H} \sum_{j \in H} \gamma_{jh} \sigma_j s_j = 0, \forall S \in R^n$$

$$\iff \sigma_1 s_1 \sum_{h \in H} \gamma_{1h} + \cdots + \sigma_n s_n \sum_{h \in H} \gamma_{nh} = 0, \forall S \in R^n. \quad (28)$$

Therefore, in the particular case where $S = (\sigma_1 s_1, 0, ..., 0)'$, with $\sigma_1 s_1 \neq 0$, condition (28) is satisfied if and only if $\sum_{h \in H} \gamma_{1h} = 0$. Similarly, when $S = (0, \sigma_2 s_2, 0, ..., 0)'$, with $\sigma_2 s_2 \neq 0$, we need to have $\sum_{h \in H} \gamma_{2h} = 0$, and so forth. Applying the same reasoning to the $n$ income shocks gives the condition of Lemma 1.

Appendix 2: proof of Proposition 1 and of Corollary 1

8.1 Proof of Proposition 1

The first step of the proof of Proposition 1 consists in approximating the terms that appear in the optimality conditions: $E[u'(c_i) s_i]$, $E[u'(c_j) s_i]$ and $E u'(c_i)$. This allows us to find explicit solutions for the first best parameters of risk-sharing $\Gamma$. Recall that the consumption level under risk-sharing is given by

$$c_i(S; L, \Gamma) = w_i + \mu(\sigma_i) + l_i + (1 - \alpha_i) \sigma_i s_i + \sum_{j \in H \setminus \{i\}} \gamma_{j i} \sigma_j s_j.$$
Let us take a second order Taylor expansion of \( u'(c_i) s_i \) in the neighborhood of \( s_i = 0 \) and \( \sum_{j \neq i} \gamma_{ij} \sigma_j s_j = 0 \). For the ease of exposition, let us denote \( \Theta_{-i} = \sum_{j \neq i} \gamma_{ij} \sigma_j s_j \).

\[
u'(c_i) s_i \approx u'(c_i) s_i \bigg|_{s_i=\Theta_{-i}=0} + s_i \frac{\partial u'(c_i) s_i}{\partial s_i}\bigg|_{s_i=\Theta_{-i}=0} + \Theta_{-i} \frac{\partial u'(c_i) s_i}{\partial \Theta_{-i}}\bigg|_{s_i=\Theta_{-i}=0} + \frac{s_i^2}{2} \frac{\partial^2 u'(c_i) s_i}{\partial s_i^2}\bigg|_{s_i=\Theta_{-i}=0} + \frac{s_i \Theta_{-i}}{2} \frac{\partial^2 u'(c_i) s_i}{\partial s_i \partial \Theta_{-i}}\bigg|_{s_i=\Theta_{-i}=0},
\]

where

\[
\begin{align*}
\frac{\partial u'(c_i) s_i}{\partial s_i} &= u'(c_i) + u''(c_i) (1 - \alpha_i) \sigma_i s_i, \\
\frac{\partial u'(c_i) s_i}{\partial \Theta_{-i}} &= u''(c_i) s_i, \\
\frac{\partial^2 u'(c_i) s_i}{\partial s_i^2} &= 2u''(c_i) (1 - \alpha_i) \sigma_i + u'''(c_i) (1 - \alpha_i)^2 \sigma_i^2 s_i, \\
\frac{\partial^2 u'(c_i) s_i}{\partial \Theta_{-i}^2} &= u'''(c_i) s_i, \\
\frac{\partial^2 u'(c_i) s_i}{\partial s_i \partial \Theta_{-i}} &= u'''(c_i) (1 - \alpha_i) \sigma_i s_i + u''(c_i).
\end{align*}
\]

Evaluating those terms at \( s_i = \Theta_{-i} = 0 \), substituting and simplifying, we obtain

\[
u'(c_i) s_i \approx u' (w_i + l_i + \mu(\sigma_i)) s_i + s_i^2 (1 - \alpha_i) \sigma_i u''(c_i) + \frac{s_i \Theta_{-i}}{2} u''(c_i).
\]

Taking expectations,

\[
E \left[ u'(c_i) s_i \right] \approx (1 - \alpha_i) \sigma_i u''(w_i + l_i + \mu(\sigma_i)),
\]

(29)

where use has been made of the fact that \( E(s_i) = 0 \) and that \( E(s_i^2) = Var(s_i) = 1 \), without loss of generality. Also, \( E(s_i \Theta_{-i}) = Cov(s_i, \Theta_{-i}) = 0 \), by independence between \( s_i \) and the other households' idiosyncratic shocks. A similar procedure leads to

\[
E \left[ u'(c_i) s_i \right] \approx \gamma_{ij} \sigma_j u''(w_j + l_j + \mu(\sigma_j)),
\]

(30)

\[
E u'(c_i) \approx u'(w_i + l_i + \mu(\sigma_i)).
\]

(31)

Making use of these approximations, first order conditions with respect to \( l_{ij} \) (equation (8)) and \( \gamma_{ij} \) (equation (9)) become respectively

\[
\lambda_i u'(w_i + l_i + \mu(\sigma_i)) - \lambda_j u'(w_i + l_j + \mu(\sigma_j)) = 0,
\]

(32)

\[
\lambda_i (1 - \bar{\alpha}_i) \bar{\sigma}_i^2 u''(w_i + l_i + \mu(\sigma_i)) - \lambda_j \bar{\gamma}_{ij} \bar{\sigma}_j^2 u''(w_j + l_j + \mu(\sigma_j)) = 0.
\]

(33)

Combining both equalities, we find

\[
(1 - \bar{\alpha}_i) a_i = \bar{\gamma}_{ij} a_j \iff \bar{\gamma}_{ij} = (1 - \bar{\alpha}_i) \frac{a_i}{a_j} = (1 - \bar{\alpha}_i)^{FB} \frac{\tau_j}{\tau_i}.
\]

(34)

Recall that, by definition, \( \alpha_i = \sum_{j \in H \setminus \{i\}} \gamma_{ij} \). Summing over all \( j \in H \setminus \{i\} \),

\[
\bar{\alpha}_i^{FB} = (1 - \bar{\alpha}_i^{FB}) \frac{1}{\tau_i} \sum_{j \in N \setminus \{i\}} \tau_j.
\]
Solving for $\hat{\alpha}_i^{FB}$, we obtain

$$\hat{\alpha}_i^{FB} = 1 - \frac{\tau_i}{\sum_{h \in H} \tau_h},$$

with $\tau = 1/a$.

Using this expression in equation (34), allows to find the first best value of any given $\gamma_{ij}$:

$$\hat{\gamma}_{ij}^{FB} = \left(1 - \hat{\alpha}_i^{FB}\right) \frac{a_i}{a_j} \frac{\tau_j}{\sum_{h \in H} \tau_h}.$$

This completes the proof of the first point of Proposition 1.

Regarding the second point of the proposition, we need to show that risk-taking is homogeneous across households. The first order condition with respect to risk-taking is given by equation (10), which can be rewritten as

$$\frac{\partial W}{\partial \sigma_i} = 0 \iff \mu' \left(\hat{\sigma}_i^{FB}\right) = \frac{E[u'(c_i) s_i]}{E[u'(c_i)]}.$$

Replacing the terms on the right hand side by their approximations (expressions (31) and (29)) and substituting for the first best value of $\alpha_i$ (12), we obtain

$$\mu' \left(\hat{\sigma}_i^{FB}\right) = \frac{\tau_i}{\sum_{h \in H} \tau_h} \frac{\hat{\sigma}_i^{FB} a_i}{a_i}.$$

Since by definition $\tau_i = 1/a_i$, we end up with

$$\frac{\mu' \left(\hat{\sigma}_i^{FB}\right)}{\hat{\sigma}_i^{FB}} = \frac{1}{\sum_{h \in H} \tau_h}.$$

The function $\mu' (\sigma) / \sigma$ on the left hand side is given by the available technology and is identical across households, while the right hand side only depends on aggregate risk tolerance within the group $\sum_{h \in H} \tau_h$, which is also constant across households. Therefore, risk-taking is homogeneous.

### 8.2 Proof of Corollary 1

Let us turn to the proof of Corollary 1. For the sake of simplicity and coherence, assume that under homogeneous wealth, the planner sets equal Pareto weights: $\lambda_i = \lambda_j, \forall \{i, j\} \subset H$.

We have shown above that risk-taking was homogeneous at the first best. This is a fortiori true in homogeneous groups. Making use of this information, equation (32) tells us that $E(c_i) = E(c_j), \forall \{i, j\} \subset H$. Since wealth levels are equal across households, it follows that lump sum transfers are equal to zero in homogeneous groups $L^{FB} = 0_{(n \times 1)}$.

With $L^{FB} = 0_{(n \times 1)}$, equal weights, equal wealth and homogeneous risk-taking, the first order condition with respect to $\gamma_{ij}$ (33) boils down to

$$1 - \hat{\alpha}_i^{FB} = \hat{\gamma}_{ij}^{FB}.$$

Summing over $j \in H \setminus \{i\}$,

$$(n - 1) \left(1 - \hat{\alpha}_i^{FB}\right) = \sum_{j \in H \setminus \{i\}} \hat{\gamma}_{ij}^{FB}.$$

Making use of the fact that $\alpha_i = \sum_{j \in H \setminus \{i\}} \gamma_{ij}$, by Lemma 1, we end up with $\hat{\gamma}_{ij}^{FB} = 1/n, \forall \{i, j\} \subset H$ and $\hat{\alpha}_h^{FB} = (n - 1)/n, \forall h \in H$. 

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9 Appendix 3: Proof of Propositions 2 and 3

9.1 Proof of Proposition 2

In autarky, the first order condition with respect to risk-taking \( \sigma_h \) is given by

\[
\frac{\partial E[u(c_h^A)]}{\partial \sigma_h} = \mu'(\sigma_h) E[u'(c_h^A)] + E[u'(c_h^A) s_h] = 0.
\]

We take the approximations of \( E[u'(c_h^A)] \) and \( E[u'(c_h^A) s_h] \) as given by equations (31) and (29) and evaluate them at the autarkic values of the risk-sharing parameters, that is for \( L^A = 0_{(n \times 1)} \) and \( \Gamma^A = 0_{(n \times n)} \). Substituting, we obtain

\[
\mu'(\tilde{\sigma}_h^A) - \tilde{\sigma}_h^A a(w_h + \mu(\tilde{\sigma}_h^A)) = 0.
\]

By an application of the implicit function theorem on equation (35), we find that comparative statics of risk taking with respect to household wealth are given by

\[
\frac{\partial \tilde{\sigma}_h^A}{\partial w_h} \geq 0 \iff \frac{\partial^2 E[u(c_h^A)]}{\partial w_h \partial \sigma_h} \geq 0 \iff a'(w_h + \mu(\tilde{\sigma}_h^A)) \leq 0,
\]

which proves Proposition 2.

9.2 Proof of Proposition 3

Comparative statics of \( \nu_y^A \) (15) with respect to household wealth are given by

\[
\frac{d\nu_y^A}{dw_h} = \frac{\partial \nu_y^A}{\partial w_h} + \frac{\partial \nu_y^A}{\partial \sigma_h} \frac{\partial \sigma_h}{\partial w_h},
\]

where

\[
\frac{\partial \nu_y^A}{\partial w_h} = \frac{-\sigma_h^A}{[w_h + \mu(\sigma_h^A)]^2},
\]

\[
\frac{\partial \nu_y^A}{\partial \sigma_h} = \frac{w_h + \mu(\sigma_h^A) - \sigma_h^A \mu'(\sigma_h^A)}{[w_h + \mu(\sigma_h^A)]^2},
\]

\[
\frac{\partial \sigma_h^A}{\partial w_h} \approx \frac{-\mu''(\sigma_h^A) + a_h + \sigma_h^A a_h' \mu'(\sigma_h^A)}{[w_h + \mu(\sigma_h^A)]^2},
\]

by an application of the implicit function theorem on equation (35). Notice that \( \partial \nu_y^A / \partial \sigma_h \) is always positive under our assumptions on the shape of \( \mu(\sigma) \).\(^{35}\)

Substituting, one obtains after simplifications that

\[
\frac{d\nu_y}{dw} < 0 \iff \epsilon_{\sigma,w} = \frac{\partial \sigma_h}{\partial \sigma_h} \frac{w_h}{w + \mu(\sigma) - \sigma \mu'(\sigma)} < \frac{w}{w + \mu(\sigma) - \sigma \mu'(\sigma)},
\]

which leads to

\[
\frac{d\nu_y^A}{dw_h} < 0 \iff [w_h + \mu(\sigma_h^A)] a'(w_h + \mu(\sigma_h^A)) + a(w_h + \mu(\sigma_h^A)) > \mu''(\sigma_h^A),
\]

\(^{35}\) Indeed,

\[
w + \mu(\sigma) - \sigma \mu'(\sigma) > 0 \iff \frac{w + \mu(\sigma)}{\sigma} > \mu'(\sigma),
\]

and we have that

\[
w + \mu(\sigma) > \mu(\sigma) > \mu'(\sigma).
\]

The second relationship obtains for \( \mu(0) \geq 0 \) and \( \mu''(\sigma) < 0 \), which we assume. Under those assumptions, the average product is always larger than the marginal product.
where $\mu'' (\sigma^N_h) < 0$, by assumption. Notice that the left hand side of this condition corresponds to the derivative of the coefficient of relative risk aversion. Indeed

\[
r'(c) = \frac{\partial (ca(c))}{\partial c} = ca'(c) + a(c).
\]

Hence, $\Delta \nu^A / dw_h < 0 \iff r'_h \geq 0$.

10 Appendix 4: The second order condition of the risk-taking decision at the second best

Making use of equation (17), we have that

\[
\frac{\partial^2 E u(c_h)}{\partial \sigma^2_h} < 0 \iff \mu'' (\sigma^N_h) - (1 - \alpha_h)^2 a_h - \sigma^N_h \mu' (\sigma^N_h) (1 - \alpha_h)^2 a'_h < 0,
\]

where $\mu'' (\sigma^N_h) < 0$, by assumption. It follows that

\[
\frac{\partial^2 E u(c_h)}{\partial \sigma^2_h} < 0 \iff a_h + \sigma^N_h \mu' (\sigma^N_h) a'_h \geq 0 \iff r'_h \geq 0.
\]

Indeed, since $r'(c) = ca'(c) + a(c)$,

\[
a_h + \sigma^N_h \mu' (\sigma^N_h) a'_h \geq a_h + (w_h + \mu (\sigma^N_h)) a'_h \geq 0 \iff r'_h \geq 0.
\]

The former relationship is obtained by observing that

\[
a_h + \sigma^N_h \mu' (\sigma^N_h) a'_h' \geq a_h + (w_h + \mu (\sigma^N_h)) a'_h' \iff \sigma^N_h \mu' (\sigma^N_h) \leq w_h + \mu (\sigma^N_h),
\]

since $a'_h \leq 0$ under DARA. And indeed, $\sigma^N_h \mu' (\sigma^N_h) < \mu (\sigma^N_h) < w_h + \mu (\sigma^N_h)$. Recalling that $\mu (0) \geq 0$, $\mu' (\sigma) > 0$ and $\mu'' (\sigma) < 0$, one can easily see that the average product $\mu (\sigma) / \sigma$ is always larger than the marginal product $\mu' (\sigma)$, so that this relationship always holds.

11 Appendix 5: Proof of Propositions 5 and 6

11.1 Proof of Proposition 5

The first order condition with respect to the risk-sharing parameter $\gamma_{ij}$ is given by equation (21). Since $\partial E u_h / \partial \gamma_{ij} = \gamma_{ih} E [u' (c_h) s_i]$ (the same for $j$), this equation can be rewritten as

\[
\frac{\partial W}{\partial \gamma_{ij}} = -\lambda_i \sigma_i E [u' (c_i) s_i] + \lambda_j \sigma_j E [u' (c_j) s_i] + \frac{\partial \sigma^N_i}{\partial \gamma_{ij}} \sum_{h \in H \setminus \{i\}} \lambda_h \gamma_{ih} E [u' (c_h) s_i] + \frac{\partial \sigma^N_j}{\partial \gamma_{ij}} \sum_{h \in H \setminus \{j\}} \lambda_h \gamma_{jh} E [u' (c_h) s_j] = 0.
\]

In order to find explicit solutions for the second best values of $\gamma_{ij}$ and $\alpha_i$, we make use of the approximations of $E [u' (c_i) s_i]$ and $E [u' (c_j) s_i]$ (for $i \neq j$) as computed in Appendix 2 (expressions (29) and (30), respectively). Substituting those terms into (21), one obtains

\[
\frac{\partial W}{\partial \gamma_{ij}} \approx -\lambda_i (1 - \tilde{\alpha}_i) \tilde{\sigma}_i^2 u'' (E(c_i)) + \lambda_j \tilde{\gamma}_{ij} \tilde{\sigma}_j^2 u'' (E(c_j))
\]

\[
+ \frac{\partial \sigma^N_i}{\partial \gamma_{ij}} \sum_{h \in H \setminus \{i\}} \lambda_h \tilde{\gamma}_{ih} \tilde{\sigma}_i u'' (E(c_h)) + \frac{\partial \sigma^N_j}{\partial \gamma_{ij}} \sum_{h \in H \setminus \{j\}} \lambda_h \tilde{\gamma}_{jh} \tilde{\sigma}_j u'' (E(c_h)) = 0,
\]

where $\mu'' (\sigma^A_c) < 0$, by assumption. Notice that the left hand side of this condition corresponds to the derivative of the coefficient of relative risk aversion. Indeed

\[
r'(c) = \frac{\partial (ca(c))}{\partial c} = ca'(c) + a(c).
\]

Hence, $\Delta \nu^A / dw_h < 0 \iff r'_h \geq 0$.
where 
\[ E(c_h) = w_h + l_h + \mu(\bar{\sigma}_h), \forall h \in H. \]

By Lemma 1, we know that \( \partial \bar{\sigma}_i^N / \partial \gamma_{ij} = \partial \bar{\sigma}_i^N / \partial \alpha_i \). Also, in the light of equation (17), we see that \( \partial \bar{\sigma}_j^N / \partial \gamma_{ij} = 0 \). As a consequence and according to the definition of the elasticity of risk-taking to the insurance coverage \( \epsilon_i \) (equation (19)), we can write

\[ \lambda_i (1 - \tilde{\alpha}_i) u''(E(c_i)) = \lambda_j \tilde{\gamma}_{ij} u''(E(c_j)) + \epsilon_i \sum_{h \in H \setminus \{i\}} \lambda_h \tilde{\gamma}_{ih} u''(E(c_h)) = 0. \tag{36} \]

Summing over all \( j \in H \setminus \{i\} \), we find

\[ (n - 1) (1 - \tilde{\alpha}_i) \lambda_i u''(E(c_i)) = \sum_{j \in H \setminus \{i\}} \tilde{\gamma}_{ij} \lambda_j u''(E(c_j)) + \sum_{h \in H \setminus \{i\}} \tilde{\gamma}_{ih} \lambda_h u''(E(c_h)) \]

\[ \iff 1 - \tilde{\alpha}_i = \sum_{j \in H \setminus \{i\}} \tilde{\gamma}_{ij} \lambda_j u''(E(c_j)) \left( \frac{1}{n - 1} + \epsilon_i \tilde{\gamma}_{ij} \tilde{\alpha}_i \right). \]

Using \( \alpha_i = \sum_{h \in N \setminus \{i\}} \gamma_{ih} \), we obtain

\[ \sum_{j \in H \setminus \{i\}} \tilde{\gamma}_{ij} \lambda_j u''(E(c_j)) \left[ \frac{1}{\lambda_j u''(E(c_j))} + \frac{1}{\lambda_i u''(E(c_i))} \left( \frac{1}{n - 1} + \epsilon_i \tilde{\gamma}_{ij} \tilde{\alpha}_i \right) \right] = 1. \tag{37} \]

Recalling that \( \tau_h = 1/A_h = u'_h / u''_h \), this leads to

\[ \tilde{\gamma}_{ij}^{SB} = \frac{1}{\lambda_j u''_j} \left( \sum_{h \in H} \frac{1}{\lambda_h u''_h} + \frac{1}{\lambda_i u''_i} \epsilon_i \right)^{-1} = \frac{\tau_j}{\lambda_j u''_j} \left( \sum_{h \in H} \frac{\tau_h}{\lambda_h u''_h} + \frac{\tau_i}{\lambda_i u''_i} \epsilon_i \right)^{-1}. \]

With this expression of \( \tilde{\gamma}_{ij} \), condition (37) is indeed satisfied. To see this, let us compute

\[ \sum_{j \in H \setminus \{i\}} \frac{\lambda_j u''_j}{\lambda_j u''_j} \left( \sum_{h \in H} \frac{1}{\lambda_h u''_h} + \frac{1}{\lambda_i u''_i} \epsilon_i \right)^{-1} \left[ \frac{1}{\lambda_j u''_j} + \frac{1}{\lambda_i u''_i} \left( \frac{1}{n - 1} + \epsilon_i \tilde{\gamma}_{ij} \tilde{\alpha}_i \right) \right] \]

\[ = \sum_{h \in H} \frac{1}{\lambda_h u''_h} \left[ \sum_{h \in H} \frac{1}{\lambda_h u''_h} + \frac{1}{\lambda_i u''_i} \epsilon_i \sum_{j \in H \setminus \{i\}} \tilde{\gamma}_{ij} \alpha_i \right] = 1. \]

The expression of \( \tilde{\alpha}_i^{SB} \) can then be found by using the definition of \( \alpha_i \) and the second best value of \( \gamma_{ij} \).

Finally, the condition for the second best level of risk-taking for household \( i \in H \) comes from the expression of the Nash equilibrium (17), where \( \alpha \) is replaced by its second best value:

\[ \mu' \left( \tilde{\alpha}_i^{SB} \right) = \frac{\tau_i}{\lambda_i u_i} (1 + \epsilon_i)^2 \left( \sum_{h \in H} \frac{\tau_h}{\lambda_h u_h} + \frac{\tau_i}{\lambda_i u_i} \epsilon_i \right)^{-2}. \tag{38} \]

### 11.2 Proof of Proposition 6

As in the analysis of the first best in homogeneous groups (see Appendix 2), we assume equal Pareto weights: \( \lambda_i = \lambda_j, \forall \{i, j\} \subset H \). The proof proceeds in two steps. The first step consists in showing that the first order condition with respect to the lump sum transfer (20) is satisfied for a symmetric transfer structure, which
we define as \( L = 0_{(n \times 1)} \) and \( \Gamma = \Gamma^H \), where \( \Gamma^H \) is such that \( \alpha = \alpha_h \), \( \forall h \in H \) and \( \gamma_{ij} = \alpha / (n - 1) \), for all pair \( \{i, j\} \subset H \).\(^{36}\)

Imposing the symmetric structure to the transfer scheme, we can see that risk-taking is homogeneous. Indeed, under \( L = 0_{(n \times 1)} \) and \( \Gamma = \Gamma^H \), the equilibrium condition non-cooperative risk-taking profile (equation (17)) is such that

\[
\mu' \left( \tilde{\sigma}^N \right) - \tilde{\sigma}^N (1 - \alpha)^2 A \left( w + \mu \left( \tilde{\sigma}^N \right) \right) = 0.
\]

Under homogeneous preferences and equal wealth levels, risk-taking is then homogeneous as nothing in this condition is specific to household \( h \). Third, let us substitute expressions (30) and (31) into equation (20). This allows us to rewrite the first order condition with respect to the lump sum transfer as

\[
\frac{\partial W}{\partial l_{ij}} \simeq 0 \iff -u' \left( E(c_i) \right) + u' \left( E(c_j) \right) + \frac{\partial \sigma^N_i}{\partial l_{ij}} \sum_{h \in H \setminus \{i\}} \gamma_{ih}^2 \sigma_i u'' \left( E(c_h) \right) + \frac{\partial \sigma^N_j}{\partial l_{ij}} \sum_{h \in H \setminus \{j\}} \gamma_{jh}^2 \sigma_j u'' \left( E(c_h) \right) = 0.
\]

We now evaluate this condition at the symmetric transfer scheme where \( L = 0_{(n \times 1)} \) and \( \Gamma = \Gamma^H \). Under equal wealth, homogeneous risk-taking and with \( L = 0_{(n \times 1)} \), we have \( E(c_i) = E(c_j) \), so that the first two terms cancel each other out. With \( \gamma_{ij} = \gamma, \forall i \neq j \), we can also see that the sum of externalities appearing in the third and fourth term are equal:

\[
\sum_{h \in H \setminus \{i\}} \gamma^2 \sigma^N_i u'' \left( E(c) \right) = \sum_{h \in H \setminus \{j\}} \gamma^2 \sigma^N_j u'' \left( E(c) \right).
\]

Finally, by an application of the implicit function theorem to equation (17), one can see that

\[
\frac{\partial \sigma^N_i}{\partial l_i} \leq 0 \iff \frac{\partial \sigma^N_i}{\partial l_i} \leq \frac{\partial \sigma^N_j}{\partial l_j} \iff \frac{\partial \sigma^N_i}{\partial l_i} \leq \frac{\partial \sigma^N_j}{\partial l_j}.
\]

At a symmetric equilibrium (equal weights, equal homogeneous risk-taking and symmetric transfer scheme), it is easy to see that \( \partial \sigma^N_i / \partial l_i = \partial \sigma^N_j / \partial l_j \). Since \( \partial \sigma^N_i / \partial l_i = -\partial \sigma^N_i / \partial l_i \) and \( \partial \sigma^N_j / \partial l_j = \partial \sigma^N_j / \partial l_j \) (\( l_h \) is the lump sum transfer received by household \( h \)), condition (39) is indeed satisfied with equality under the symmetric transfer scheme.

In a second step, we determine the value of \( \alpha \) that maximizes the planner’s objective function (7) under a symmetric transfer scheme and with equal Pareto weights: Under a symmetric transfer scheme, consumption can be written as\(^{37}\)

\[
c_h(S; L, \Gamma) = w + \mu \left( \sigma^N \right) + \left[ (1 - \alpha) s_h + \frac{\alpha}{n - 1} \sum_{i \in H \setminus \{h\}} s_i \right] \sigma^N.
\]

The first order condition with respect to \( \alpha \) is therefore determined by equations (23), (24) displayed in Section 4.3 and:

\[
\frac{\partial W}{\partial \alpha} = n \alpha \left[ \frac{1}{n - 1} \sum_{i \in H \setminus \{h\}} E \left[ u' \left( c_h \right) s_i \right] - E \left[ u' \left( c_h \right) s_h \right] \right] > 0.
\]

\(^{36}\)Notice that, as soon as one supposes that \( \gamma_{ij} = \gamma, \forall i \neq j \), the insurance budget constraint is satisfied if and only if \( \gamma = \alpha / (n - 1) \), by Lemma 1.

\(^{37}\)We show in Appendix 5 that a symmetric transfer scheme generates a homogeneous risk-taking profile at equilibrium in homogeneous groups.
Substituting and making use of the approximations provided in Appendix 2 (equations (29) and (30)), we end up with

\[ \frac{dW}{d\alpha} \simeq n (\sigma^N)^2 u'' (w + \mu (\sigma^N)) \left[ \frac{1 + \epsilon}{n - 1} \alpha^{SB} - (1 - \alpha^{SB}) \right] = 0 \]
\[ \iff \alpha^{SB} \simeq \frac{n - 1}{n + \epsilon}. \]

12 Appendix 6: Proof of Proposition 7

Let us reproduce here the first order condition that determines the second best value of $\alpha$ in homogeneous groups (see the proof of Proposition 6):

\[ \frac{\partial W}{\partial \alpha} \simeq n (\sigma^N)^2 u'' (w + \mu (\sigma^N)) \left[ \frac{1 + \epsilon}{n - 1} \alpha^{SB} - (1 - \alpha^{SB}) \right] = 0. \]

By an application of the implicit function theorem,

\[ \frac{\partial \alpha^{SB}}{\partial w} = - \frac{\partial^2 W}{\partial w \partial \alpha} \left( \frac{\partial^2 W}{\partial \alpha^2} \right)^{-1} > 0 \iff \frac{\partial^2 W}{\partial w \partial \alpha} > 0. \]

Indeed, $\frac{\partial^2 W}{\partial \alpha^2} < 0$, by the second order condition of the planner's optimization problem.

\[ \frac{\partial^2 W}{\partial w \partial \alpha} = n (\sigma^N)^2 u'' (w + \mu (\sigma^N)) \left[ \frac{1 + \epsilon}{n - 1} \alpha^{SB} - (1 - \alpha^{SB}) \right] + n (\sigma^N)^2 u'' (w + \mu (\sigma^N)) \frac{\alpha^{SB}}{n - 1} \frac{\partial \epsilon}{\partial w} \]
\[ = n (\sigma^N)^2 u'' (w + \mu (\sigma^N)) \frac{\alpha^{SB}}{n - 1} \frac{\partial \epsilon}{\partial w} > 0 \iff \frac{\partial \epsilon}{\partial w} < 0. \]

Indeed, $\frac{1 + \epsilon}{n - 1} \alpha^{SB} - (1 - \alpha^{SB}) = 0$, by the first order condition.

13 Appendix 7: Proof of Proposition 10

Under second best risk-sharing in homogeneous groups, the coefficient of variation of income writes

\[ \nu_y = \frac{\sigma^{SB} (w, \alpha^{SB} (w))}{w + \mu (\sigma^{SB} (w, \alpha^{SB} (w)))}. \]

The total derivative of $\nu_y$ is therefore given by

\[ \frac{d\nu_y}{dw} = \frac{\partial \nu_y}{\partial w} + \frac{\partial \nu_y}{\partial \sigma} \left[ \frac{\partial \sigma^{SB}}{\partial w} + \frac{\partial \sigma^{SB}}{\partial \alpha} \frac{\partial \alpha^{SB}}{\partial w} \right], \]

where

\[ \frac{\partial \nu_y}{\partial w} = - \frac{\sigma^{SB}}{[w + \mu (\sigma^{SB})]^2}, \]
\[ \frac{\partial \nu_y}{\partial \sigma} = \frac{w + \mu (\sigma^{SB}) - \sigma^{SB} \mu' (\sigma^{SB})}{[w + \mu (\sigma^{SB})]^2}. \]

Substituting and rearranging, one readily finds that

\[ \frac{d\nu_y}{dw} < 0 \iff \epsilon_{\sigma,w} < \frac{w}{w + \mu (\sigma^{SB}) - \sigma^{SB} \mu' (\sigma^{SB})}. \]
where
\[ \epsilon_{\sigma,w} = \left[ \frac{\partial \sigma_{SB}}{\partial w} + \frac{\partial \sigma_{SB}}{\partial \alpha} \frac{\partial \alpha_{SB}}{\partial w} \right] \frac{w}{\sigma_{SB}}. \]

We now need to show that
\[ r'(w + \mu(\sigma_{SB})) \geq 0 \implies \frac{\partial \nu_y}{\partial w} + \frac{\partial \nu_y}{\partial \sigma} \frac{\partial \sigma_{SB}}{\partial w} < 0. \]

Making use of equations (40) and (41), we obtain that
\[ \frac{\partial \nu_y}{\partial w} + \frac{\partial \nu_y}{\partial \sigma} \frac{\partial \sigma_{SB}}{\partial w} < 0 \iff \frac{\partial \sigma_{SB}}{\partial w} < \frac{\sigma_{SB}}{w + \mu(\sigma_{SB}) - \sigma_{SB} \mu'(\sigma_{SB})}, \]

where
\[ \frac{\partial \sigma_{SB}}{\partial w} = -\frac{-\sigma_{SB}(1 - \alpha)^2 A'}{\mu''(\sigma_{SB}) - (1 - \alpha)^2 A' - \sigma_{SB}(1 - \alpha)^2 A' \mu'(\sigma_{SB})}, \]

by an application of the implicit function theorem to equation (17). Hence, the condition becomes
\[ \frac{\partial \nu_y}{\partial w} + \frac{\partial \nu_y}{\partial \sigma} \frac{\partial \sigma_{SB}}{\partial w} < 0 \iff \left[ w + \mu(\sigma_{SB}) \right] a' + a = r' > \frac{\mu''(\sigma_{SB})}{(1 - \alpha)^2} < 0. \]

Therefore, \( r'(w + \mu(\sigma_{SB})) \geq 0 \) is a sufficient condition to ensure that \( \frac{\partial \nu_y}{\partial w} + \frac{\partial \nu_y}{\partial \sigma} \frac{\partial \sigma_{SB}}{\partial w} < 0. \)