Financial Development, Employment Heterogeneity, and Sectoral Dynamics

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Abstract

We study the implications of financial development for sectoral, employment, and aggregate dynamics in emerging economies by advancing a theory of small open economy business cycles with search frictions, firm heterogeneity, and sectoral collateral constraints. The model is broadly consistent with the differences in the employment and financing structure between large and small firms in emerging economies. Our analysis suggests that financial development reflected improved credit access for small firms and a smaller credit gap with large firms, and not higher aggregate credit shares per se, may play an important role in explaining the negative link between financial development and labor market volatility in the data.

JEL Classifications: E32, E62, J64
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1 Introduction

Small firms in developing and emerging economies face severe limitations in obtaining formal credit and consider access to finance as one of their most important obstacles (Beck and Demirgüç-Kunt, 2006; Chavis, Klapper, and Love, 2011; Beck, Demirgüç-Kunt, and Maksimovic, 2008; Ayyagari, Demirgüç-Kunt, and Maksimovic, 2012; GFDR, 2014).\(^1\) The share of total bank credit to small and medium enterprises (SMEs) is close to 20 percent in Latin America and Asia and about 30 percent in Sub-Saharan Africa. This stands in contrast to advanced economies, where the share of total credit to SMEs easily surpasses 40 percent (Table 1; IMF, 2014).\(^2\) Recent cross-country evidence also suggests that the majority of lending from formal financial institutions requires collateral (Nguyen and Qian, 2012), and the availability of pledgable collateral represents an important obstacle to access to formal financing. In turn, barriers to formal financing for micro and small firms in emerging economies are associated with reduced scale, delayed entry, and, importantly, increased search for input suppliers and greater reliance on interfirm (informal) input credit (IDB, 2005b).

Higher aggregate credit shares are considered to be an important feature of financial development (GFDR, 2014).\(^3\) Evidence from recent years suggests that firms in developing and emerging economies are experiencing consistent improvements in access to formal credit as reflected by higher shares of bank credit to the private sector (as a percent of GDP). Importantly, this process has not been restricted to larger firms alone. Indeed, commercial banks are showing a strong impetus to steadily increase the share of loan accounts for SMEs in their portfolios, where short-term capital loans dominate the services offered to SMEs.\(^4\) However, onerous collateral requirements still represent important barriers for many

\(^{1}\)This is true not only formal firms but also informal firms, that is, firms that often operate outside the regulatory and institutional framework and hire the majority of their workers without contributing to social security programs. Informal firms often lack a credit history, which places additional barriers to accessing formal financing.

\(^{2}\)Broadly similar patterns, with advanced economies having higher SME loan shares relative to developing economies, hold when loans to SMEs as a percent of GDP are considered, in which case Sub-Saharan Africa ranks at the bottom of the list. See CGAP (2010, 2013) and OECD (2014) for additional evidence.

\(^{3}\)This is the definition of financial development we adhere to in the rest of the paper. We use the terms financial development, financial or credit deepening, and improved access to credit interchangeably. Lower interest rate spreads—defined as the spread between lending and deposit rates—coupled with higher aggregate credit shares are also associated with financial development. We address the interaction between these two features of financial deepening before concluding.

\(^{4}\)For example, a series of surveys in more than 20 Latin American economies show that 96 percent of
of these firms (IFC; 2010, de la Torre et al., 2010, de la Torre et al., 2012, and GFDR, 2014; and OECD, 2014). From a labor market perspective, understanding the impact of changes in these credit constraints across firm categories is exceedingly important for three reasons. First, relative to advanced economies, developing and emerging economies have larger shares of total employment concentrated in small salaried firms. Second, in many cases, small firm employment—very often associated with informal employment in emerging economies, which is often countercyclical—can exhibit different cyclical dynamics relative to large firm employment (Bosch and Maloney, 2008; Loayza and Rigolini, 2011; Fernández and Meza, 2014). Improving credit access for small firms can then have important effects on both the composition of employment by fostering the expansion of small firms, and on the cyclical behavior of the labor market, especially as the cyclicity of small and large firm employment may change with financial deepening. Second, regardless of age, small and medium enterprises (SMEs) in emerging economies are responsible for a non-negligible fraction of gross job creation (Ayyagari, Demirguc-Kunt, and Maksimovic, 2014; GFDR 2014). By changing the behavior of job creation and the cyclicity of sectoral employment, improving access to credit for small firms could have potentially important consequences for cyclical employment movements and business cycle dynamics.

In this paper, we study the implications of financial development for employment, sectoral, and aggregate dynamics in emerging economies. We develop a small-open-economy two-sector business cycle model with search and credit frictions that features: (1) the endogenous creation of small firms and the prevalence of employment and informal financing among small salaried firms, where the latter is reflected in these firms’ reliance on (informal) input credit relationships with larger firms; and (2) the existence of imperfect and heterogeneous access to formal financing across firm categories, characterized by the presence of sectoral collateral constraints for small and large salaried firms. The inclusion of frictional banks considered SMEs as part of their overall strategy in 2013, compared to 84 percent in 2004. Similarly, more than 90 percent of the banks surveyed had an active policy to finance SMEs in 2013, compared to only 69 percent in 2004 (IDB, 2013). While the recent global financial crisis led to a temporary setback in banks’ perceptions of SMEs’ financial position, a majority of banks not only expect improvements in SMEs’ performance, but also plan for their SME portfolios to grow in the future.

Love, Martínez Pería, and Singh (2013) document that a number of countries have improved collateral registries, which facilitates the use of movable assets for collateral.

Our notion of small firms—which we explicitly outline in the model section—is associated with the way
labor markets coupled with firm heterogeneity and sectoral collateral constraints allows us to explicitly study sectoral and aggregate employment dynamics in the context of financial deepening, a theme that has received little attention in the literature.

Our main findings are twofold. First, we show that our benchmark model, calibrated to Mexico, can not only capture the cyclicality of sectoral and aggregate labor market dynamics in the data surprisingly well, but also generates high consumption and wage volatility, where the latter are two are important features of emerging economies (Boz, Durdu, and Li, 2015). Importantly, we show that the inclusion of financial shocks alongside standard productivity shocks plays a relevant role not only in generating high wage volatility, but more importantly in matching the negative link between financial development and wage volatility in the data.\(^7\)

Relative to alternative modeling approaches that abstract from the endogenous creation of small firms, our benchmark model provides several improvements in matching the cyclical dynamics of employment in emerging economies, and in capturing the empirical negative relationship between financial deepening and unemployment and wage volatility (see the next section).

Turning to the consequences of financial development, first we show that improving firms’ average access to formal financing— attained by improving firms’ borrowing capacity— is unambiguously associated with higher average wages, capital, and output levels, and to lower investment volatility, regardless of whether improved access to credit markets affects mainly large or small firms. But, from a quantitative standpoint, financial deepening leads to limited changes in the average composition of employment across firms. More importantly, whether large or small firms are disproportionately affected by improvements in credit access is critical for characterizing the changes in unemployment and wage volatility observed in the data. Specifically, we find that, for the same change in the economy’s aggregate credit as a share of GDP, improving credit access for large firms widens the credit gap between small and large firms and lowers wage volatility, but generates higher volatility in unemployment. The latter is inconsistent with the negative link between unemployment volatility and financial

\(7\)This result is related to the findings in Boz, Durdu, and Li (2015), who find that foreign interest rate shocks play a key role in capturing the high volatility of wages in emerging economies. We discuss the role of foreign interest rate shocks in our framework when we present the main results.
development in the data. Instead, when we consider an improvement in credit access for small firms, which narrows the credit gap between large and small firms and increases the share of small-firm credit in total credit, financial development results in smaller unemployment and wage fluctuations. We point to cross-country evidence on the link between financial market development and the share of total credit allocated to small firms on the one hand, and financial development and unemployment and wage volatility on the other hand, that gives general validity to the latter set of results: developed economies—which exhibit higher aggregate credit-output ratios, and lower unemployment and wage volatility—also tend to have higher shares of total credit allocated to small firms (Table 1; OECD, 2014; IMF, 2014; Figure 1).

From a quantitative standpoint, when financial development is reflected in improved credit access for small firms, the model is able to capture, respectively, 50 and 60 percent of the reduction in unemployment and wage volatility associated with a 30 percentage-point increase in the aggregate credit share in the data. This change in aggregate credit effectively takes Mexico’s share up to the average credit share in a large sample of developed and developing economies. Our findings highlight the importance of firm heterogeneity for understanding the consequences of financial deepening for labor market dynamics.

To understand the intuition behind improvements in financial development reflected in a smaller credit gap between large and small firms and the associated reduction in unemployment and wage volatility, note that small-firm employment represents an outside option to large-firm workers. Improved credit access for small firms alters the cyclicity of small-firm employment by making it less countercyclical (where the countercyclicality of small firm employment is a feature of the data). In turn, such a change alters large-firm workers’ outside options (by reducing them in recessions relative to the benchmark economy) and makes large firms’ decisions to hire and invest less sensitive to shocks. This occurs despite the fact that the value of employment to small firms has increased when these firms enjoy higher borrowing capacity and this has lead to tighter sectoral labor markets (implying, on average, higher wages across sectors and hence a lower average value of having a worker to large firms). All

\[8\] Within these references, see specifically Table 1.4 in OECD (2014), Figure 4.5 in CGAP (2010), and Figure 18 in CGAP (2013), and the third and fourth sub-figures in Figure A3.1 in IMF (2014).
in all, the volatility of vacancy postings (and therefore employment) and investment falls across sectors, and financial development ultimately results in lower unemployment volatility. When improved credit access is tilted towards large firms (resulting in a larger credit gap), a similar mechanism is at play. However, the countercyclicality of small-firm employment is reduced by less (relative to the benchmark economy, and relative to the economy with better credit conditions for small firms), such that the reduction in the sensitivity of vacancy creation among large firms is little changed. The combination of a fall in the countercyclicality of small firm employment and the small change in the variability of large firm vacancies and employment ultimately results in sharper unemployment fluctuations. Even though the same mechanism is operational, the reasons behind these contrasting results lie in the higher sensitivity of small firms to changes in their own borrowing capacity—which traces back to the average lower values of having a worker—relative to the smaller sensitivity of large firms to changes in their borrowing capacity, who, already having better access to credit compared to small firms, benefit relatively less from financial development.

Surprisingly, in the presence of both productivity and financial shocks, the link between financial development and wage volatility does not depend on whether financial deepening improves large firms’ or small firms’ borrowing capacity. However, the presence of financial shocks—shocks to borrowing capacity—does play a critical role. Intuitively, financial development fosters capital accumulation, which, for a given set of financial shocks, makes firms more resilient to shocks and reinforce the benefits of improved access to credit via higher average employment surpluses for firms. All in all, this implies that the hiring decisions of firms become less sensitive to exogenous disturbances in the economy, which, by reducing the volatility of workers’ outside options, results in lower wage volatility. Importantly, this reduction in wage volatility cannot be obtained in a version of the model that introduces foreign interest rate shocks but abstracts from financial shocks. Hence, our results suggest an important role for financial shocks in explaining the link between financial development and wage dynamics.

More broadly, our results make a subtle but important point: it is better credit access for small firms—reflected in a reduction in the credit gap with larger firms, and also, incidentally, accompanied by a higher aggregate credit share—and not higher aggregate credit
shares _per se_ that may contribute to explaining the negative empirical link between financial development and labor market volatility. Importantly, sectoral heterogeneity in the model allows us to cast our analysis through the lens of the credit gap between firm categories—as opposed to solely using the aggregate credit share across economies, as most studies have done—and provides relevant new insights into the consequences of financial deepening for employment and sectoral dynamics in emerging economies.

The remainder of this paper is organized as follows. Section 2 presents a series of stylized facts that guide our modeling assumptions, and provides an overview of related literature and our main contributions relative to this literature. Section 3 develops the model. Section 4 describes the model’s calibration, and Section 5 presents the main results. Finally, Section 6 concludes.

## 2 Stylized Facts, Modeling Implications, and Related Literature

### 2.1 Facts and Modeling Implications

The following facts, summarized in Figure 1, motivate the structure of formal credit markets in the model. First, economies with a higher share of domestic credit to the private sector (as a percent of GDP) have higher shares of small and large firms with a bank loan or a line of credit and higher shares of firms that use banks to meet their working capital. Second, the value of collateral required to obtain a loan is decreasing in the share of aggregate private domestic credit. Third, hiring costs—proxied by the ratio of the minimum wage to value added per worker—are also decreasing in the aggregate credit share. Fourth, economies with higher domestic credit shares tend to have lower unemployment and wage volatility.

Turning to the financing structure of firms, a majority of micro and small firms in several emerging economies face onerous limitations in accessing formal credit and are therefore

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9See the Appendix for more details about the data. Given data limitations, the facts pertaining to access to bank loans, collateral value (or collateral requirements), and working capital financing through banks are only applicable to formal firms (see the World Bank Enterprise Surveys for more). However, the data is available by firm size.
heavily reliant on interfirm input credit relationships in order to successfully enter markets and start production (IDB, 2005b).\textsuperscript{10} While comparable cross-country micro data on informal financing is extremely limited and almost nonexistent, a number of studies highlight the importance of input suppliers and informal, input-based credit from other firms for micro and small firms in developing economies (McMillan and Woodruff, 1999; Demirgüç-Kunt and Maksimovic, 2001; Beck et al., 2008; GFDR, 2014).\textsuperscript{11} Finally, as previously pointed out, more developed economies (which also have higher aggregate credit shares) have a higher proportion of total credit allocated to SMEs (IMF, 2014; OECD, 2014).

The model we develop is consistent with the above facts. Our modeling framework is a modification of Epstein and Finkelstein Shapiro (2015) (henceforth EFS, 2015)—who develop a business cycle search model with employment and firm heterogeneity, endogenous small firm entry, and input credit relationships among small firms consistent with emerging economies—by introducing formal financial frictions via sectoral collateral constraints for small and large salaried firms in an open economy setting. Financial development is then characterized by an exogenous relaxation of the collateral constraints for either large or small firms.

\subsection*{2.2 Related Literature}

The international business cycle literature has emphasized the role of interest rate shocks in explaining business cycles, consumption, and current-account dynamics in emerging economies (see Neumeyer and Perri, 2005; Li, 2011; Chang and Fernández, 2013, among many). More recently, business cycle models with more microfounded financial frictions (Bernanke, Gertler, and Gilchrist, 1999; Kiyotaki and Moore, 1997; Calstrom and Fuerst, 1997; Jermann and Quadrini, 2012; Liu, Wang, and Zha, 2013; Iacoviello, 2014) have not only been incorporated into open-economy settings (see Mendoza, 2010, and others), but have also

\footnote{In a detailed study on microenterprise and small-firm financing that exploits Mexican census data (including both formal and informal firms), Pavón (2010) documents that input suppliers (which act as informal financing providers) are the most important source of external financing, accounting for almost 70 percent of total external financing (commercial banks are the second most important source of financing). This share is decreasing in firm size (but remains important), with commercial banks becoming increasingly important as firms grow.}

\footnote{For more on the prevalence of input-based as opposed to cash-based informal credit in developing economies, see Burkart and Ellingsen (2004).}
been modified to include frictional labor markets. Such is the case for both closed-economy models for advanced economies (Liu, Miao, and Zha, 2013; Monacelli, Quadrini, and Trigari, 2012; Petrosky-Nadeau, 2013; Garín, 2014), as well as models of emerging economies (Lama and Urrutia, 2010; Boz, Durdu, and Li, 2015; Fernández and Herreño, 2012; Altug and Kabaca, 2014; Buera, Fattal-Jaef, and Shi, 2014; Finkelstein Shaprio and González Gómez, 2014). A growing literature has also placed attention on the implications of costly financial intermediation in a business cycle context.\textsuperscript{12} Also, with increasing evidence on financial intermediation and financial constraints in developing countries (Beck, 2007), several empirical studies have explored the connection between financial development, growth, and aggregate volatility (Beck, Demirgüç-Kunt, Laeven, and Levine, 2004; Beck and Demirgüç-Kunt, 2006; Beck, Lundberg, and Majnoni, 2006; Pinheiro, Rivadeneyra, and Teignier, 2013; Manganelli and Popov, 2012; Wang and Wen, 2013; Bhattacharya and Patnaiky, 2015; Dabla-Norris and Srivival, 2013; Sahay et al., 2015).

However, very few studies on financial development and aggregate fluctuations have explored the importance of sectoral heterogeneity, and of those studies, none have studied the role of the labor market structure and the cyclical dynamics of employment within a context of financial deepening.\textsuperscript{13} Accordingly, this paper makes two main contributions. First, we show how sectoral firm heterogeneity—partly reflected in differences in access to external financing across firm categories—plays a role in matching empirical facts about employment dynamics in emerging economies, which is particularly relevant given the recent emphasis on improving formal credit access for SMEs in these economies. Second, by building on a labor market and firm-financing structure consistent with emerging economies, we are able to study the consequences of financial development for sectoral labor market dynamics and for the cyclical behavior of wages, and how the latter influence aggregate fluctuations. More


\textsuperscript{13}See, for example, Beck, Demirgüç-Kunt, Laeven, and Levine (2004), and Manganelli and Popov (2012), and more recently, Dabla-Norris, Ji, Townsend, and Unsal (2015) who study the link between financial inclusion, GDP, and inequality. However, none of these recent studies address the impact of financial development on labor markets. Our results provide a theoretical rationale behind Manganelli and Popov’s (2012) findings that sectoral reallocation plays a relevant role in explaining the link between financial development and aggregate volatility. Our findings are also broadly consistent with Beck et al.’s (2004) conclusion that financial development has both important sectoral and aggregate implications.
broadly, our work provides a plausible rationale—rooted in the differential access to credit by firms with distinct employment and financing characteristics, and the relative contribution of these firms to aggregate employment fluctuations—that can reconcile the existing empirical evidence on the link between financial development and labor market dynamics.

3 The Model

Relative to the framework in EFS (2015), we: (1) abstract from self-employment; (2) assume a small open economy with patient and impatient households; (3) assume small firms use both internally-accumulated capital and input credit from large firms; and (4) introduce sectoral collateral constraints and financial intermediaries.

There are two types of households, patient and impatient, and two categories (classes) of firms, small and large salaried firms. Productivity differs across firm categories but is the same for all firms in a given category.\(^{14}\) The output from each firm category is used as an input by a representative, perfectly-competitive final goods firm that in turn produces a final consumption good sold in a perfectly competitive market. There is no labor force participation margin.

Following EFS (2015), large firms accumulate capital, post vacancies to hire workers, and choose the share of capital they want to use within the firm. The remaining, unused capital is supplied to households via frictional capital markets. There is neither entry nor exit of large firms. Consistent with the prevalence of input credit relationships among small firms highlighted in the previous section, households use matched capital (input credit relationships) from large firms for the creation and operation of small firms.\(^{15}\) We label the capital supplied by large firms (who act as capital suppliers) as input credit to the small firm sector. In the absence of idiosyncratic productivity, we can frame our discussion of large

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\(^{14}\)That is, there are no idiosyncratic productivity differences between firms—only sectoral productivity differences across firm categories.

\(^{15}\)EFS (2015) also include the endogenous creation of self-employed firms. Given our emphasis on financial deepening among salaried firms, we abstract from self-employment to have a more transparent environment. Our main results carry through if we include self-employment. We note that our environment is related to the sequence of events in Nicoletti and Pierrard (2006), where small firms must first establish a capital match before hiring a worker. In contrast to their setup, our small-firm sector accommodates multi-worker small firms, and as we show below, small firms accumulate capital that is used for collateral and production.
firms through a representative firm perspective.

Patient households provide deposits to perfectly competitive financial intermediaries (owned by these households) and consume the returns from these deposits. In contrast, impatient households receive income from salaried workers in small and large firms. They also own small salaried firms and, among other things, make all the decisions regarding input demand for these firms. Specifically, impatient households spend resources to search for capital from large firms—required for the creation and operation of small firms—accumulate capital internally, and post vacancies to hire salaried workers.\(^{16}\) Capital obtained from input credit relationships with large firms and internally-accumulated capital are complements in production. Each small salaried firm uses one and only one unit of matched capital, along with salaried labor and internally accumulated capital, to produce.\(^{17}\) As we discuss below, this last feature implies straightforward aggregation in the small firm sector. Coupled with the absence of idiosyncratic productivity within firm categories, this allows us to frame our modeling of the decisions made by small salaried firms in terms of decisions taken by households (who own these firms) at the sectoral level (see EFS, 2015).

Both large and small firms pledge their internally-accumulated capital as collateral to borrow funds from financial intermediaries, where these funds are partially used to finance the firms’ wage bills, and therefore face standard, sector-specific collateral constraints (Kiyotaki and Moore, 1997; Iacoviello, 2014; and others).\(^{18}\)

Small firms differ from large firms in three respects: (1) are endogenously created; (2) are less capital intensive; and (3) continue to rely on input credit relationships with large firms to obtain part of the production capital they need to operate, in addition to their internally accumulated capital. The assumption that new small firms can have access to working capital funds from financial intermediaries without facing additional search frictions is in line with

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\(^{16}\)We need two types of households to allow small firms—owned by impatient households, who make all decisions for small firms from an input demand perspective—to borrow from financial intermediaries. We note that the costs of becoming a small salaried firm by also posting vacancies to hire salaried workers (as outlined below) can easily accommodate (part of) the costs of having access formal financing.

\(^{17}\)This implies that the measure of matched capital in the small firm sector is also the measure of small firm owners (see EFS, 2015, for more on this modeling assumption).

\(^{18}\)Initially, we assume that the capital obtained from input credit relationships cannot be used as collateral. We relax this assumption as part of our robustness checks and show that our main results still hold (see the Appendix for more details).
evidence suggesting that financial intermediaries use large firms who act as input suppliers to smaller firms in order to locate potential small firms that may be willing and able to use formal credit (de la Torre et al., 2010). Thus, as long as small firms maintain input credit relationships with large firms, financial intermediaries can locate small firms without facing additional frictions and can provide working-capital financing (and small firms do not have to incur additional costs to find a financial intermediary). One key distinction between large and small firms—besides a lower capital intensity among small firms—is that, despite being able to accumulate capital internally, small firms continue to rely on input credit relationships with large firms and external financing from financial intermediaries in order to operate. This assumption is in line with the relative intensity of input credit by small firms, and the more prevalent joint use of input credit and formal credit by these firms relative to larger firms (World Bank Enterprise Surveys).

3.1 Final Goods Production

Total output aggregates large firm output and small firm output using the production function \( y_t = y(y_{l,t}, y_{s,t}) \). Thus, the problem of the final goods producer is

\[
\max_{y_{l,t}, y_{s,t}} \left[ y_t - p_{l,t} y_{l,t} - p_{s,t} y_{s,t} \right].
\]

The solution to this problem yields standard expressions for the relative prices \( p_{l,t} \) and \( p_{s,t} \).

3.2 Households and Small Firms

3.2.1 Patient Households and Optimization

Following related literature, patient households do not supply labor. They own financial intermediaries, taking their profits as given, and supply deposits to financial intermediaries. Patient households choose consumption of the final good \( c_{p,t} \) (whose price is normalized to one) and deposits to financial intermediaries \( d_t \) to maximize \( \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t_p U(c_{p,t}) \), where: \( \mathbb{E} \) is the expectation operator; \( \beta_p \in (0, 1) \) is the patient household’s subjective discount factor; and the properties of flow utility \( U \) are: \( U'(c) > 0 \); and \( U''(c) < 0 \). The patient household’s
budget constraint is
\[ c_{p,t} + d_t = R_{d,t-1}d_{t-1} + \Pi_{f,t}, \]
where \( R_{d,t-1} \) is the gross return on deposits. \( \Pi_{f,t} \) are profits from financial intermediaries. The first-order conditions yield a standard Euler equation for deposits: \( U'(c_{p,t}) = R_{d,t}\beta_p E_t U'(c_{p,t+1}) \). In what follows, we define \( \Xi^p_{t+1|t} \equiv \beta_p U'(c_{p,t+1})/U'(c_{p,t}) \). We assume that patient households have a higher subjective discount factor than impatient households. This guarantees that the borrowing constraint for small firms is binding in the neighborhood of the steady state (Iacoviello, 2014). We make a similar assumption regarding a lower subjective discount factor for large firms.

### 3.2.2 Small Firm Sector

Following EFS (2015), and to simplify sectoral aggregation and given the absence of idiosyncratic productivity, we focus on a symmetric Nash equilibrium in which every operating small firm is identical and hence frame our discussion in terms of the small firm sector. This implies that total profits from the small firm sector, \( \Pi_{s,t} \), can be written as:

\[
\Pi_{s,t} = \left[ p_{s,t}z_{s,t}F(n_{s,t}, k_{s,t}, o_{s,t}) - w_{s,t}n_{s,t} \right] - r_{s,t}o_{s,t} - \kappa(v_{s,t})o_{s,t} - i_{s,t} + l_{s,t} - R_{s,t-1}l_{s,t-1},
\]

Above: \( p_{s,t} \) is the price of sectoral output relative to that of the final good; \( z_{s,t} \) is exogenous sectoral productivity; the production function \( F(\cdot, \cdot, \cdot) \) is increasing and concave in each of its arguments and exhibits constant returns to scale; \( n_{s,t} \) is the measure of salaried workers in the small firm sector; \( o_{s,t} \) is the measure of (all) small firm owners (recall that, given our assumption of one unit of matched capital per small firm, \( o_{s,t} \) is also the measure of matched capital owned by large firms that is used for production in the small firm sector); \( k_{s,t} \) is the measure of internally-accumulated capital used for production and pledged as collateral in the small firm sector; \( w_{s,t} \) is the Nash real wage paid by small firms; and \( r_{s,t} \) is the Nash capital rental rate for matched capital in each small firm. In addition, each small firm posts vacancies \( v_{s,t} \) in order to recruit additional labor, \( \kappa(v_{s,t}) \) is a convex function of \( v_{s,t} \) that captures the cost of posting vacancies, and \( i_{s,t} \) denotes total small firm investment.
in capital $k_{s,t}$.\textsuperscript{19} Finally, the small firm sector can borrow a total of funds $l_{s,t}$ from financial intermediaries, for which they pay a gross interest rate $R_{s,t-1}$ on funds borrowed in the previous period.\textsuperscript{20} In what follows, we denote $y_{s,t} = z_{s,t} F(n_{s,t}, k_{s,t}, o_{s,t})$ as total small firm output.

The small firm sector is subject to: a law of motion for internally-accumulated capital,

$$k_{s,t+1} = (1 - \delta)k_{s,t} + i_{s,t} - \Psi \left( \frac{k_{s,t+1}}{k_{s,t}} \right) k_{s,t},$$

(2)

where $\delta$ is the capital depreciation rate and $\Psi (k_{s,t+1}/k_{s,t})$ is a standard convex capital adjustment cost; and also to a borrowing constraint that aggregates to:

$$R_{s,t} l_{s,t} \leq \eta_{s,t} k_{s,t+1} - \phi w_{s,t} n_{s,t}.$$  

(3)

Above, $R_{s,t} l_{s,t}$ denote total liabilities from borrowing, and $\eta_{k_{s,t}}$ determines the firms’ borrowing capacity, which has mean $\overline{\eta}_s$ and follows a stochastic process. We label shocks to $\eta_{s,t}$ as financial shocks. Finally, $\phi$ determines the fraction of the wage bill that must be paid in advance (Neumeyer and Perri, 2005; Iacoviello, 2014).

### 3.2.3 Employment-State Evolution

All sectoral job finding and filling probabilities are endogenous (discussed further below), and all job separation probabilities are exogenous. The probabilities of finding a job and losing a job in a large firm are $f_{l,t}$ and $\rho', \rho'$, respectively. Thus, from a labor supply perspective the impatient household’s perceived law of motion for salaried employment in large firms, $n_{l,t}$, is:

$$n_{l,t+1} = (1 - \rho') (n_{l,t} + f_{l,t} u_t),$$

(4)

\textsuperscript{19}See Merz and Yashiv (2007) for more on convex vacancy posting costs.

\textsuperscript{20}The above expression can be rewritten as

$$\Pi_{s,t} = \left[ p_{s,t} z_{s,t} F \left( \frac{n_{s,t}}{o_{s,t}}, k_{s,t}, 1 \right) \right] o_{s,t} - \left[ w_{s,t} \frac{n_{s,t}}{o_{s,t}} \right] o_{s,t} - r_{s,t} o_{s,t} - \kappa (v_{s,t}) o_{s,t} - i_{s,t} + l_{s,t} - R_{s,t-1} l_{s,t-1},$$

where, given our assumption regarding $F(\cdot, \cdot, \cdot)$, the two terms in brackets denote revenue and the wage bill per, respectively, per small firm.
where: $u_t$ denotes unemployment.

Similar to Finkelstein Shapiro (2014) and EFS (2015), the creation of firms arises from a matching process between household resources spent on capital search and the supply of unused capital by large firms. We denote total household capital search expenditures by $s_t$. These resources are matched with capital supplied by large firms with probability $f_{k,t}$. When a successful capital match occurs, an unemployed individual becomes a small firm owner (equivalently, a new small firm is created). Capital matches are destroyed with exogenous probability $\rho^o$. Salaried workers are separated from small firms with probability $\rho^s$.

It follows that the perceived evolution of small firm owners is given by:

$$o_{s,t+1} = (1 - \rho^o) [o_{s,t} + s_t f_{k,t}].$$ (5)

From the small firm sector’s labor demand perspective, the evolution of small-firm employment is:

$$n_{s,t+1} = (1 - \rho^o)(1 - \rho^s) [n_{s,t} + v_{s,t} q_{s,t} o_{s,t}],$$ (6)

where $q_{s,t}$ is the probability with which a small firm fills a vacancy and the probability of finding salaried employment in a small firm is $f_{s,t}$. We can define aggregate unemployment as:

$$u_t \equiv 1 - n_{l,t} - n_{s,t} - o_{s,t}.$$

### 3.2.4 Impatient Households

The impatient household owns small salaried firms. Following EFS (2015), in order to keep the solution to the household’s labor-supply and -demand problems explicitly decentralized, we assume that from the point of view of small firms the household maximizes utility as a demander of labor. As such, the household chooses consumption of the final good $c_{i,t}$, foreign debt $b_t^*$, desired demand for salaried workers by small firms $n_{s,t+1}$, the desired measure of small firm owners $o_{s,t+1}$, the desired measure of total capital search expenditures $s_t$, and

---

21 This can be interpreted as a startup cost. It is necessary but not sufficient to successfully create a small firm.
internally-accumulated capital by small firms $k_{s,t+1}$, vacancies posted per small firm $v_{s,t}$, and borrowed funds among small firms $l_{s,t}$, to maximize $E_0 \sum_{t=0}^{\infty} \beta_i^t U(c_{i,t})$, where $\beta_i \in (0, 1)$ is the household’s subjective discount factor (with $\beta_i < \beta_p$) and the utility function satisfies $U' > 0$ and $U'' < 0$, subject to: the budget constraint

$$c_{i,t} + \kappa(s_t) + T_t + R_{t-1}^* b_{t-1}^* = \Pi_{s,t} + w_{s,t} n_{s,t}^s + w_{l,t} l_{l,t} + b_t^*,$$

and equations (2), (3), (5), and (6), where $n_{s,t}^s$ is small-firm employment from the perspective of the household (equal to $n_{s,t}$ in equilibrium). In the household’s budget constraint, expenditures are comprised of consumption $c_{i,t}$, total capital search expenditures $\kappa(s_t)$, where $\kappa'(s_t) > 0$ and $\kappa''(s_t) > 0$, and lump-sum taxes $T_t$. Additionally, households receive income from large-firm salaried workers $w_{l,t} n_{l,t}$, where $w_{l,t}$ is the large-firm wage, and from small firm salaried workers $w_{s,t} n_{s,t}^s$. Note that given the absence of labor force participation decisions, the impatient household takes its perceived evolution (that is, from a supply perspective) of salaried employment in each firm type as given. $R_{t-1}^*$ is the gross foreign interest rate on foreign debt. Following Schmitt-Grohé and Uribe (2003), we assume the following functional form: $R_t^* = 1 + r^* + \eta_b [\exp(b_t^* - b^*) - 1]$, where the adjustment cost function $\eta_b [\exp(b_t^* - b^*) - 1]$, $\eta_b > 0$, induces stationarity in foreign debt holdings, $r^*$ is the steady-state net foreign interest rate, and $b^*$ is steady state foreign debt.

### 3.2.5 Impatient Household Optimality Conditions

First, we obtain a standard Euler equation for foreign debt

$$U'(c_{p,t}) = R_t^* \beta_p E_t U'(c_{p,t+1}).$$

The impatient household’s optimal decision over capital search expenditures $s_t$ is given by

$$\frac{\kappa'(s_t)}{f_{k,t}} = (1 - \rho_o) E_t \Xi_{t+1|t} \left\{ p_{s,t+1} z_{s,t+1} F_{a,t+1} - r_{a,t+1} - \kappa(v_{s,t+1}) + \frac{\kappa'(s_{t+1})}{f_{k,t+1}} \right\},$$

where: we define $\Xi_{t+1|t} = \beta_i U'(c_{i,t+1})/U'(c_{i,t})$ as the impatient household’s stochastic discount factor. This condition shows that the expected marginal cost of spending resources on
capital search is equal to the expected marginal benefit of having an additional household member operating a firm in the small firm sector. The latter is given by the net return on a unit of matched capital, $p_{s,t+1}z_{s,t+1}F_{o_s,t+1} - r_{o,t+1}$, the cost of continuing to open vacancies as a small firm owner $\kappa(v_{s,t+1})$, and the continuation value of the capital relationship.

Let $\lambda_{s,t}$ be the multiplier on the small firm sector’s borrowing constraint (normalized by the marginal utility of large-firm consumption). The optimal choice of vacancies posted by existing small firms is

$$
\frac{\kappa'(v_{s,t})}{q_{s,t}} \frac{1}{(1 - \rho^s)(1 - \rho^o)} = \mathbb{E}_t \Xi_{t+1}^i \left\{ p_{s,t+1}z_{s,t+1}F_{n_s,t+1} - w_{s,t+1} \left[ 1 + \lambda_{s,t+1}\phi \right] \right\}.
$$

This equation is a standard job creation condition, adjusted for the fact that small firms borrow resources to finance their wage bill. The interpretation of this expression is straightforward: the left-hand side is the expected marginal cost of posting a vacancy and the right-hand side is the expected marginal benefit. The right-hand side is comprised of the marginal product of labor net of the wage plus the continuation value of the relationship. Note that the fact that small firms borrow resources increases both the per-worker wage bill since $\lambda_s > 0$.

The optimal choice over capital accumulation $k_{s,t+1}$ yields a capital Euler equation that takes into account the small firms’ borrowing constraint:

$$
[1 - \lambda_s \eta_{s,t}] = \mathbb{E}_t \Xi_{t+1}^i \left\{ p_{s,t+1}z_{s,t+1}F_{k_s,t+1} + (1 - \delta) \right\}.
$$

Note that, all else equal, an increase in $\eta_{k_s}$ reduces the marginal cost of accumulating capital. Finally, the optimal choice over borrowing is given by

$$
1 - R_{s,t}\lambda_{s,t} = \mathbb{E}_t \Xi_{t+1}^i R_{s,t}.
$$

\footnote{For expositional purposes, we abstract from including capital adjustment costs in this expression. However, we do include these costs in the quantitative analysis.}
The left-hand side of this last expression denotes the expected marginal benefit from borrowing funds, which is decreasing in the tightness of the collateral constraint, \( \lambda_s \). The right-hand side denotes the expected marginal cost of borrowed funds (see Iacoviello, 2014).

### 3.3 Large Firms

Large firms choose consumption \( c_{l,t} \), desired salaried employment \( n_{l,t+1} \), vacancies \( v_{l,t} \), capital \( k_{l,t+1} \), the desired amount of capital in the small firm sector \( o_{s,t+1} \) (supplied via frictional capital markets), borrowed funds \( l_{t,t} \), and the fraction of capital to be used within the firm \( \omega_t \) to maximize \( E_0 \sum_{t=0}^{\infty} \beta_t U(c_{l,t}) \). We assume that the discount factor satisfies \( \beta_l < \beta_p \), which guarantees that the borrowing constraint of large firms—shown below—is binding in the neighborhood of the steady state, and that \( U \) satisfies \( U''(c_l) > 0 \) and \( U''(c_l) < 0 \) (Iacoviello, 2014). The large firm is subject to the budget constraint

\[
c_{l,t} = p_{l,t} z_{l,t} F(n_{l,t}, \omega_t k_{l,t}) - w_{l,t} n_{l,t} - \kappa(v_{l,t}) - i_{l,t} \\
+ r_{s,t} o_{s,t} + l_{t,t} - R_{l,t-1} l_{t-1} + (\rho^o - \delta) o_{s,t} - (1 - \rho^o) q_k t (1 - \omega_t) k_{l,t},
\]

where: \( p_{l,t} \) is the price of sectoral output relative to that of the final good; \( z_{l,t} \) is exogenous sectoral productivity; \( F(\cdot, \cdot) \) is a constant-returns-to-scale production function that is increasing and concave in each of its arguments; large-firm vacancies \( v_{l,t} \) are posted at a cost \( \kappa(v_{l,t}) \), where \( \kappa' > 0 \) and \( \kappa'' > 0 \); \( i_{l,t} \) is large-firm investment; rental income \( r_{s,t} o_{s,t} \) is received from supplying capital to small firms via frictional capital markets; and \( R_{l,t-1} \) is the gross interest rate paid on borrowed funds. Finally, the separated capital from small firms (net of depreciation) represents an additional source of revenue, whereas matched capital, \( (1 - \rho^o) q_k t (1 - \omega_t) k_{l,t} \), must be subtracted from the firm’s revenue. In the preceding, \( q_k t \) is the firm’s probability of forming a new capital match. The large firm’s maximization problem is also subject to the perceived evolution of salaried employment

\[
n_{l,t+1} = (1 - \rho^l)(n_{l,t} + v_{l,t} q_k t),
\]
where \( q_t \) is the large firm’s job filling probability, the perceived evolution of capital in the small firm sector

\[
o_{s,t+1} = (1 - \rho) [o_{s,t} + (1 - \omega)k_{l,t}q_{k,t}],
\]

and the evolution of the firm’s total capital stock

\[
k_{l,t+1} = (1 - \delta)k_{l,t} + i_{l,t} - \Psi \left( \frac{k_{l,t+1}}{k_{l,t}} \right) k_{l,t},
\]

where firms face standard convex capital adjustment costs \( \Psi (k_{l,t+1}/k_{l,t}) \). In addition, the firm faces a borrowing constraint:

\[
R_{l,t}l_{t} \leq \eta_{l,t}k_{l,t+1} - \phi w_{l,t}n_{l,t}, \quad (7)
\]

where: \( \eta_{l,t} \) determines the firms’ borrowing capacity, which has mean \( \bar{\eta}_l \) and follows a stochastic process. Similar to small firms, we label shocks to \( \eta_{l,t} \) as financial shocks. Parameter \( \phi \) denotes the fraction of the wage bill financed with external funds.

### 3.3.1 Large-Firm Optimality Conditions

Let \( \lambda_{l,t} \) be the multiplier on the large firm’s borrowing constraint (normalized by the marginal utility of large-firm consumption). The optimal choice over total capital yields a capital Euler equation (again, abstracting from capital adjustment costs for expositional simplicity):

\[
[1 - \lambda_{l,t} \eta_{l,t}] = \mathbb{E}_{t} \Xi_{t+1|t} \left\{ p_{t,t+1} z_{l,t+1} \omega_{k_{l,t+1}} + (1 - \delta) \right\},
\]

where: \( \Xi_{t+1|t} \equiv \beta tU''(c_{l,t+1})/U'(c_{l,t}) \). Similar to the case of small firms, the multiplier on the borrowing constraint affects the firm’s optimal choice over capital. We obtain a standard job creation condition, adjusted for the fact that firms must borrow in advance to pay for the wage bill:

\[
\frac{k'(v_{l,t})}{q_{l,t}} = (1 - \rho')\mathbb{E}_{t} \Xi_{t+1|t} \left\{ p_{t,t+1} z_{l,t+1} n_{l,t+1} - w_{l,t+1} [1 + \lambda_{l,t+1} \phi] + \frac{k'(v_{l,t+1})}{q_{l,t+1}} \right\}.
\]
The left-hand side of this expression captures the expected marginal cost of posting a vacancy. The right-hand-side of this expression includes the marginal product of salaried labor net of the wage plus the continuation value of the employment relationship.

Similar to Finkelstein Shapiro (2014), the firm’s optimal supply of capital is

\[
\frac{p_{l,t}z_{l,t}F_{\omega k_{l,t}} + (1 - \rho^o)q_{k,t}}{q_{k,t}}
= (1 - \rho^o)\frac{r_{s,t} + (\rho^o - \delta)}{\frac{p_{l,t+1}z_{l,t+1}F_{\omega k_{l,t+1}} + (1 - \rho^o)q_{k,t+1}}{q_{k,t+1}}},
\]

Intuitively, this expression equates the expected marginal cost of devoting a unit of capital to matching (adjusting for the fact that a matched unit this period remains idle before becoming productive next period) to the expected marginal benefit. The latter is given by the rental rate from an additional capital relationship, \(r_s\), the value of a separated unit of capital (net of depreciation), \((\rho^o - \delta)\), and the continuation value.

Finally, the optimal choice over borrowing is given by:

\[
1 - R_{l,t}\lambda_{l,t} = \mathbb{E}_t \Xi_{t+1|t} R_{l,t}.
\]

Once again, the left-hand side of this expression captures the marginal benefit from borrowed funds, and the right-hand side captures the marginal cost of those funds.

### 3.4 Matching Processes

The functions \(m_{l,t} = m_l(v_{l,t}, u_t)\) and \(m_{s,t} = m_s(v_{s,t}o_{s,t}, u_t)\) are standard constant-returns-to-scale matching functions that are increasing and concave in each of their arguments. The job-finding probabilities are given by: \(f_{s,t} = m_{s,t}/u_t\) and \(f_{l,t} = m_{l,t}/u_t\). In turn, the job-filling probabilities are given by: \(q_{s,t} = m_{s,t}/(v_{s,t}o_{s,t})\) and \(q_{l,t} = m_{l,t}/v_{l,t}\). The capital matching function is \(m_{k,t} = m_k((1 - \omega_t)k_{l,t}, s_t)\). The household’s probability of finding a capital supplier per unit of expenditure on capital search and the large firm’s probability of matching its unused capital are \(f_{k,t} = m_{k,t}/s_t\) and \(q_{k,t} = m_{k,t}/((1 - \omega_t)k_{l,t})\), respectively. Labor market tightness for small and large firms are similarly defined as \(\theta_{s,t} \equiv (v_{s,t}o_{s,t})/u_t\) and \(\theta_{l,t} \equiv v_{l,t}/u_t\), respectively.
respectively. Finally, capital market tightness is defined as \( \theta_{k,t} \equiv s_t/(1 - \omega_t) k_{l,t} \). The matching function properties are such that all salaried job-finding probabilities are increasing in market tightness while \( f_{k,t} \) is decreasing in capital market tightness.

### 3.5 Nash Bargaining

Wages and the capital rental rate in the economy are determined via Nash bargaining. For expositional briefness, the following value functions are presented in the Appendix. The value of having an additional salaried worker from the large firm’s perspective is \( J_{l,t} \). In turn, \( J_{s,t} \) is the value to a large firm of having a capital relationship with a small firm owner. The value to the small firm sector of having an additional worker and an additional capital relationship with large firms are \( J_{o,t} \) and \( W_{o,t} \), respectively. For the household, the values of having a household member in employment in a large firm, employment in a small firm, and unemployment are \( W_{l,t} \), \( W_{s,t} \), and \( W_{u,t} \), respectively.

The exogenous and constant bargaining power of workers who negotiate wages with large and small firms are \( \chi_l \in (0,1) \) and \( \chi_s \in (0,1) \), respectively. Similarly, the exogenous and constant rental-rate bargaining power of small firm owners is \( \chi_o \in (0,1) \).

The implicit expressions for the Nash wages are given by

\[
\frac{\chi_l}{(1 - \chi_l) [1 + \lambda_{l,t} \phi]} J_{l,t} = (W_{l,t} - W_{u,t}) ,
\]

for \( w_{l,t} \), and

\[
\frac{\chi_s}{(1 - \chi_s) [1 + \lambda_{s,t} \phi]} J_{o,t} = (W_{s,t} - W_{u,t}) ,
\]

for \( w_{s,t} \). The implicit expression for the Nash rental rate \( r_{s,t} \) is given by

\[
\frac{\chi_o}{(1 - \chi_o)} (J_{s,t} - (1 - \delta)) = (W_{o,t} - W_{u,t}) .
\]

Note that the outside option of large firms is the value of a unit of capital net of depreciation \( (1 - \delta) \). Given the presence of differences in the stochastic discount factors between impatient households and large firms, no closed-form solution for the wages and the rental rate can be obtained. However, it is worth pointing out that each sectoral wage is increasing in
market tightness in its own sector and in other sectors. Thus, the hiring decisions of firms in a given sector—which are in part affected by access to formal financing and hence the level of financial development—will have spillover effects over the decisions of firms in other sectors via changes in sectoral wages. A similar claim holds for the Nash rental rate paid by small firm owners, which depends in part on the outside options in large-firm and small-firm salaried employment (and hence sectoral labor market tightness).\textsuperscript{23}

3.6 Financial Intermediaries

Financial intermediaries are perfectly competitive. Financial intermediaries demand deposits $d_t$ from patient households, obtain revenue $R_{l,t-1}l_{l,t-1}$ and $R_{s,t-1}l_{s,t-1}$ from loans to large and small firms, and make loans $l_{l,t}$ and $l_{s,t}$. They must also pay back $R_{d,t-1}d_{t-1}$ to patient households. The problem of financial intermediaries is to choose loans to large firms $l_{l,t}$ and small firms $l_{s,t}$ to maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \Xi_{t} \Pi_{f,t}$ subject to

$$\Pi_{f,t} = d_t + R_{l,t-1}l_{l,t-1} + R_{s,t-1}l_{s,t-1} - l_{l,t} - l_{s,t} - R_{d,t-1}d_{t-1},$$

and the budget constraint

$$l_{l,t} + l_{s,t} = d_t,$$

where all deposits are devoted to loans. The first-order conditions imply that, in equilibrium, $R_{l,t} = R_{s,t} = R_{d,t}$.\textsuperscript{24}

3.7 Closing the Model

The government uses lump-sum taxes to finance expenditures, and its budget constraint is $T_t = g_t$, where $g_t$ is exogenous government spending. The resource constraint of the economy

\textsuperscript{23}One can see this explicitly in a simpler version of the model where there are no collateral constraints, and households own all firms. See Finkelstein Shapiro (2014) or EFS (2015) for a more detailed discussion.

\textsuperscript{24}The Appendix discusses the results from introducing costly financial intermediation, which generates a spread between lending and deposit rates (see, for example, Cúrdia and Woodford, 2010). Our results remain the same if we characterize financial development by a joint improvement in borrowing capacity and a reduction in intermediation costs and interest rate spreads.
is given by

\[ y_t = c_t + g_t + i_{t,t} + i_{s,t} + \kappa(s_t) + \kappa(v_{s,t}) + \kappa(v_{s,t})o_{s,t} + R_{t-1}^*b_{t-1}^* - b_t^* , \]

where: vacancy postings and capital search are resource costs; aggregate consumption is \( c_t = c_{p,t} + c_{s,t} + c_{l,t} \); and household consumption is \( c_{h,t} = c_{p,t} + c_{i,t} \). Moreover, the trade balance is defined as \( tb_t \equiv R_{t-1}^*b_{t-1}^* - b_t^* \).

## 4 Calibration

### 4.1 Functional Forms and Shocks

Total output is given by \( y_t = \left[ \gamma_a y_t^\phi_a + (1 - \gamma_a) y_t^\phi_a \right]^{\frac{1}{\phi_a}} \) where: \( \gamma_a \in (0,1) \); and \( \phi_a \leq 1 \). The production function for large firms is \( F(n_{t,t}, \omega_t k_{t,t}) = (n_{t,t})^{1-\alpha_l} (\omega_t k_{t,t})^{\alpha_l} \) with \( \alpha_l \in (0,1) \). The production function for small firms is given by \( F(n_{s,t}, k_{s,t}, o_{s,t}) = (n_{s,t})^{1-\alpha_s} [(o_{s,t})^{\gamma_s} (k_{s,t})^{1-\gamma_s}]^{\alpha_s} \) where: \( \alpha_s, \gamma_k \in (0,1) \). Since large firms tend to be more capital intensive, we assume that \( \alpha_l > \alpha_s \). Also, \( \gamma_k \) determines the relative importance of capital obtained via input credit in the production function of small firms. All matching functions are Cobb-Douglas. For large firms, \( m_{l,t} = M_l (u_t)^\xi (v_{l,t})^{1-\xi} \). For existing small firms, \( m_{s,t} = M_s (u_t)^\xi (v_{s,t} o_{s,t})^{1-\xi} \), and \( m_{k,t} = M_k (s_t)^\xi ((1 - \omega_t) k_{l,t})^{1-\xi} \). In the preceding functions, \( M_j \) is the matching efficiency parameter and \( \xi \) is the matching elasticity parameter for \( j \in \{l,s,k\} \).

The remaining functional forms are as follows. We assume quadratic capital adjustment costs given by \( \Psi(k_{j,t+1}/k_{j,t}) = (\varphi_{k_j}/2) (k_{j,t+1}/k_{j,t} - 1)^2 \) for \( j = l,s \). The cost of searching for capital and the costs of posting vacancies for each firm type are: \( \kappa(s_t) = \psi_k(s_t)^{\eta_k} \), with \( \psi_k > 0 \) and \( \eta_k > 1 \); and \( \kappa(v_{i,t}) = \psi_v(v_{i,t})^{\eta_v} \), with \( \psi_v > 0 \), \( \eta_v > 1 \) for \( j \in \{l,s\} \) respectively (Merz and Yashiv, 2007). The utility function for all agents is of the constant relative risk aversion type: \( u(c) = c^{1-\sigma}/(1 - \sigma) \).

The exogenous shock processes are as follows. Sectoral productivity follows an AR(1) process with a common aggregate shock \( \ln z_{i,t} = (1 - \varrho_z) \ln(\tilde{z}_i) + \varrho_z \ln z_{i,t-1} + \epsilon_i^z \) where: \( i \in \{l,s\} \), \( \tilde{z}_i \) is a constant, \( \varrho_z < 1 \), and \( \epsilon_i^z \sim N(0,\sigma_z^2) \) denotes the aggregate productivity. \(^{25}\)Note that the elasticity of substitution is \( 1/(1 - \phi_a) \).
shock. Note that steady-state firm productivity can be different across sectors, but all firms face the same (aggregate) shock. Similarly, sectoral borrowing capacity also follows an AR(1) process with a common shock: 

\[ \ln \eta_{k,i,t} = \left(1 - \varrho_\eta\right) \ln(\eta_{k,i}) + \varrho_\eta \ln \eta_{k,i,t-1} + \varepsilon_\eta^t \]

where: \( i \in \{l,s\} \), \( \eta_{k,i} \) is a parameter, \( \varrho_\eta < 1 \), and \( \varepsilon_\eta^t \sim N(0, \sigma^2_\eta) \).

4.2 Parameters from Literature

We choose Mexican data to calibrate the model, as Mexico is widely used in the literature on emerging-market business cycles. We assume a time period of one quarter. The capital shares for large firms, \( \alpha_l \), and small firms, \( \alpha_s \), are set to 0.32 and 0.27, respectively. The subjective discount factor for patient households, \( \beta_p \), is set to 0.985, a common value in the literature (see Boz, Durdu, and Li, 2015), and the subjective discount factors for impatient households, \( \beta_i \), and large firms, \( \beta_l \), are set to 0.885.\(^{26}\) The capital depreciation rate \( \delta \) is 0.025. The working capital parameter for the wage bill, \( \phi \), is set to 1 (Iacoviello, 2014). The curvature of the capital search and vacancy posting costs, \( \eta_j \), for \( j \in \{k,l,s\} \) is set to 2.\(^{27}\)

Following the search literature, we set the bargaining power of all types of workers and small firm owners, \( \chi_j \), for \( j \in \{l,s,o\} \) equal to the common matching elasticity, \( \xi = 0.50 \). The main conclusions hold if the Hosios condition does not hold or if we impose sectoral matching elasticities.

As a benchmark, the elasticity parameter for the total output aggregator, \( \phi_a \), is set to 0.7, implying imperfect substitutability. We set \( \rho^l = 0.03 \) and \( \rho^s = 0.03 \) such that the separation rates are consistent with the evidence in Bosch and Maloney (2008).\(^{28}\) We set the persistence of the productivity processes to \( \varrho_z = 0.94 \) (Aguiar and Gopinath, 2007). Following the international business cycle literature, the constant-relative-risk-aversion in the utility

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\(^{26}\)This guarantees that the collateral constraint multiplier is always positive in both the benchmark calibration and in the financial development experiments we perform. A lower value relative to existing studies is needed given the search structure of the model, which generates more volatile collateral multipliers. However, the main conclusions of the paper remain unchanged with higher values for the discount factors.

\(^{27}\)The total resource cost from vacancy posting in the benchmark calibration is around 1 percent of output, in line with the literature (see, for example, Boz, Durdu, and Li, 2015).

\(^{28}\)Note that the actual separation rate for small-firm employment depends on \( \rho^o \), which we calibrate below using a specific target, so that the actual separation rate of workers in small firms is higher than the separation rate for workers in large firms.
function, $\sigma$, is set equal to 2 for all agents. Finally, similar to EFS (2015), we use the evidence for Mexico in Busso et al. (2012) to capture the productivity differentials between small and large firms, which yields $\tau_s=1$ and $\tau_l=2.2189$ (see the Appendix for more details).

### 4.3 Calibrated Parameters

Using evidence on the share of credit to the private sector as a percent of GDP from the World Bank’s World Development Indicators, we choose the borrowing capacity parameter for large firms to match an average credit-to-GDP ratio for Mexico of 0.20. This yields $\overline{\eta}_k=0.308$. In turn, based on OECD (2012), we choose the borrowing capacity of small firms so that the share of credit to small firms accounts for 10 percent of total credit. This yields $\overline{\eta}_s=0.840$. The importance of matched capital in total small firm capital, $\gamma_k$, is set to 0.65, which matches the allocation of capital among small firms documented in Busso, Fazio, and Levy (2012).

Exogenous government spending $g$ and steady-state foreign debt $b^*$ represent 10.2 percent of output and 30 percent of (annual) output, respectively (Aguiar and Gopinath, 2007), so that $g=0.057$ and $b^*=0.674$.

We calibrate the matching efficiency parameters for each matching function to capture the shares of employment in each firm category, where $n_l$ is 0.40, $n_s$ is 0.48, and $o_s$ is 0.07. The respective matching efficiency parameters are $M_l=0.075$, $M_s=0.495$, and $M_k=0.145$. We choose the parameters in the vacancy cost function, $\psi_j$ for $j \in \{l,s,e\}$ so that the cost of a vacancy represents 3.5 percent of sectoral wages (Levy, 2007). This yields $\psi_l=0.013$ and $\psi_s=0.013$.

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29 As a benchmark, we abstract from introducing asymmetries in risk aversion across agents. Assuming that $\sigma = 2$ for households, following the international business cycle literature, and $\sigma = 1$ for large firms and financial intermediaries, as in Iacoviello (2014), does not change the main conclusions of the paper.

30 Note that while this value is higher than the borrowing capacity parameter for large firms, the capital stock of large firms is considerably larger than the one for small firms, which in turn allows large firms to access more funds from financial intermediaries. Thus, these values are not inconsistent with large firms’ better access to formal financing.

31 As we show below, as part of our experiments we vary this share with the degree of financial development to account for the decrease in reliance on input (informal) credit as access to formal financing increases.

32 According to Mexico’s National Survey on Urban Employment and National Survey on Occupation and Employment, the share of self-employment in Mexico is close to 23 percent (a lower bound). Around 30 percent of those self-employed work in firms with more than one worker (that is, in salaried firms). Hence the share of small firm owners. In the model, the self-employed are accounted for in the share of workers in small firms. Explicitly including endogenous self-employment, as in EFS (2015), does not change the main results and only makes the model more complex.
\[ \psi_s = 0.011. \] The parameter that determines the cost of capital search represents 3 months of small firm wages (McKenzie and Woodruff, 2008), so that \( \psi_k = 0.317. \) The destruction rate for small salaried firms is chosen to have a job-separation rate for small-firm salaried workers of 8 percent (Bosch and Maloney, 2008), so that \( \rho^o = 0.022. \) We use the targets \( p_l y_l / y = 0.55 \) and \( p_s y_s / y = 0.45 \) to calibrate the output shares in the CES aggregates (Busso et al., 2012, and Enterprise Surveys). This yields \( \gamma_a = 0.340. \)

We assume that the investment adjustment cost parameters \( \varphi_{k_l} \) and \( \varphi_{k_s} \) are the same and choose their single value to obtain a relative volatility of investment for Mexico of 2.78, as in the data, so \( \varphi_k = 16.45. \) We set the debt elasticity parameter to a small enough number that guarantees stationary debt holdings and does not affect aggregate dynamics, \( \eta_b = 0.012. \) Finally, the standard deviation of aggregate productivity is chosen to match a volatility of output of 2.39 percent over the same time period, so \( \sigma_z = 0.0170, \) the persistence of borrowing capacity shocks is chosen to match the persistence of output, so that \( \varphi_{\eta} = 0.90, \) and the volatility of borrowing capacity shocks is set to match the cyclicality of wages, which yields \( \sigma_{\eta} = 0.0302. \)

## 5 Quantitative Results

In this section we first show results for the benchmark model as well as alternative versions of the model. These results validate both our overall modeling framework and benchmark calibration. Then, we focus on the steady-state and business cycle implications of financial development. In particular, we present results showing steady-state variables under different financial-development equilibria obtained through changes in credit access for either large or small firms, and then we show the economy’s response to temporary aggregate shocks under these different scenarios. The Appendix provides a brief discussion of several robustness checks.

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\(^{33}\)Smaller values for these cost parameters do not change the main results of the paper.

\(^{34}\)The share of value added in large (small) firms in Mexico are documented in Busso et al. (2012) (Table 5, Formality/Legality Status). These targets are a combination of the rows with information by formality/legality status as well as information on the share of firms with access to formal financing from the World Bank Enterprise Surveys for Mexico. We use this information as opposed to information solely based on firm size since formality and access to formal financing are strongly related.
5.1 Business Cycle Moments for Benchmark Economy

Table 2 shows key empirical business cycle statistics for Mexico and compares them to: (1) the benchmark model (with productivity and financial shocks); (2) a one-sector, small open economy business cycle search model with collateral constraints for firms; (3) a two-sector version of (2); (4) the benchmark model with no capital search frictions; (5) the benchmark model with productivity and foreign interest rate shocks (no financial shocks); and (6) the benchmark model with only productivity shocks. The top four rows of the table show the second moments we target in the benchmark calibration.

Comparison of Table 2’s second and third columns (the data and the benchmark model) shows that the benchmark specification captures qualitatively and generally quantitatively as well, the following properties of the data: a relative volatility of consumption and wages larger than one; the high procyclicality of large firm employment and the countercyclicality of small firm employment; the procyclicality of the large-firm and small-firm job finding rates (including the fact that the former is higher than the latter); the countercyclicality of unemployment; and the persistence of unemployment. As in most of the search and

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35We log-linearize the model around the non-stochastic steady state and use a first-order approximation to the equilibrium conditions. The model is simulated for 2100 periods. The first 100 periods are discarded, and a Hodrick-Prescott filter with smoothing parameter 1600 is applied to the remaining series. All second moments are computed using the filtered series, as we would do in the data.

36The empirical moments for large-firm and small-firm employment, as well as their respective job finding rates, are from Bosch and Maloney (2008). We discuss the mapping between the model and the data below. The relative volatility of consumption and wages, the cyclicality of unemployment, and the persistence of output and unemployment in the data are from Boz, Durdu, and Li (2015).

37A caveat regarding the moments for small-firm salaried employment: due to data limitations and in order to compare sectoral employment dynamics in the model to the data, we assume that the majority of formal salaried workers are in large firms and the majority of informal salaried workers are in small firms, as in EFS (2015). This explains why, given our mapping between the data and the model, small firm employment is countercyclical: informal salaried employment (and self-employment) tend to be countercyclical in the data (see Bosch and Maloney, 2008; Fernández and Meza, 2015; Loayza and Rigolini, 2011). More importantly, our mapping is broadly consistent with the evidence on informality across firm size classes in Busso, Fazio, and Levy (2012). Having said that, within the context of our model with formal financing, this mapping is only an approximation as many small firms (which are generally informal) may not have access to formal financing. We do not think the mapping we use is problematic since a (small) share of small firms do have access to formal financing (see the World Bank Enterprise Surveys). Moreover, recent evidence suggests that banks are starting to lend to the informal sector (see Ruiz, 2011). One alternative to explicitly address potential issues with our mapping to the data would be to modify the model to separate small firms between formal (facing collateral constraints) and informal (not being able to borrow), but this would introduce additional complexity without providing further insights or changing the main conclusions of the paper. More importantly, it is worth highlighting that the model is consistent with specific second moments of the labor market that we do not target, which adds validity to our framework despite being unable to have a perfect comparison between employment in small firms in the model and in the data.
matching literature, the model is unable to generate enough unemployment volatility, despite
the presence of financial frictions \( \sigma_u / \sigma_y \) is lower than one). The other shortcoming of the
model is the acyclicality of the trade balance-to-output ratio relative to the data.

To highlight the relative strengths of our benchmark model, note that simpler models (ei-
ther a more standard one-sector model or a two-sector model without endogenous small-firm
creation) perform worse in matching some of the non-targeted moments in the data (compare
columns numbered (1), (2), and (3) in Table 2).\(^{38}\) Comparing columns numbered (1) and (4)
show that, while abstracting from capital search frictions (but maintaining the endogenous
creation of small firms) yields similar results to those from the benchmark economy, the
presence of capital search frictions does generate, among other things, higher unemployment
and wage volatility, and hence a slightly better fit with the data.

Finally, comparing columns numbered (1), (5), and (6) shows that the inclusion of both
productivity and financial shocks (the benchmark model) provides a better fit with the
data relative to a version of the benchmark model with either both productivity and foreign
interest rate shocks, or with productivity shocks alone, even though all three versions match
the data relatively well, especially compared to simpler models (such as those in columns (2)
and (3)).\(^{39}\) As we show below, the presence of financial shocks plays a key role in matching
the stylized facts regarding financial development and labor market dynamics.

5.2 Financial Development

We mimic the process of financial development by changing the average borrowing capacity
for firms. We then compare the benchmark economy to the following two cases, where
borrowing capacity changes so that the change in the aggregate credit share is identical
across cases.\(^{40}\) First, we consider an equilibrium with higher borrowing capacity for large
firms \( \bar{\eta}_l \). We refer to this first case as a situation with improved credit access for large firms.

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\(^{38}\)While the one-sector model is able to match certain moments reasonably well, we note that performing
the financial development experiments described further below yield counterfactual results relative to the
facts presented in Section 2).

\(^{39}\)As pointed out above, the benchmark model with productivity and financial shocks cannot generate a
higher countercyclicity of the trade balance-output ratio, which the model with productivity and foreign
interest rate shocks can (by construction, and only if these two shocks are negatively correlated, as in Boz,
Durdu, and Li, 2015).

\(^{40}\)See Dabla-Norris et al. (2015) for a related approach.
Second, we consider an equilibrium with higher borrowing capacity for small firms, $\eta_s$. We refer to this second case as a situation with improved credit access for small firms. For both cases, we generate a 30 percentage point increase in the average share of credit in the economy, from 20 percent of output to 50 percent.41

5.2.1 Steady-State Implications

Table 3 shows steady state results for the benchmark model and the two equilibria discussed above. Inspection of the table reveals that, compared to the benchmark economy (second column), financial development is associated with higher output, capital, and average wages, regardless of whether large or small firms are the ones to experience an improvement in credit access (the third column, and the fourth and fifth columns, respectively). Moreover, despite the large change in the share of total credit, the steady-state compositional changes in the labor market are quantitatively very small.

Table 3 also shows how the gap between the average value of having an additional worker to a large firm and the value of having an additional worker to a small firm, $(J_l - J_o)$ changes with financial development. With improved credit access for large firms (a change in $\eta_l$), this gap widens whereas the opposite occurs when credit access improves for small firms (a change in $\eta_s$). Something similar occurs when we consider a measure of the credit gap, given by the absolute value of the difference between the amount of credit for large firms and the amount of credit for small firms, $|l_l - l_s|$. When $\eta_l$ increases, the credit gap widens. Conversely, when $\eta_s$ increases, the credit gap shrinks. These two results help explaining the differences in cyclical dynamics for the labor market for an identical change in the average credit share—one of the key results of the paper—which we discuss next.

5.2.2 Cyclical Dynamics

First, consider how the economy’s volatility changes with financial development. This is summarized in Table 4 for some labor market and macroeconomic variables. Looking at the benchmark specification (with productivity and financial shocks), it is clear that, qual-

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41 This implies increasing the average credit share in Mexico to the average credit share in the sample of 182 countries for which we have data on credit to the private sector.
itatively, financial development results in lower output, investment, and total consumption volatility, with changes in $\eta_s$ having a stronger effect and the reduction in investment volatility being quantitatively stronger as well. Turning to the volatility of unemployment and wages, we see that, while an increase in $\eta_s$ reduces both volatilities, an increase in $\eta_l$ reduces the volatility of wages but increases the volatility of unemployment. Recalling the link between unemployment volatility and financial development in Figure 1, this last result is inconsistent with the stylized facts.

Table 2 showed that, while the benchmark specification matches the data reasonably well, it fails to capture the countercyclicality of the trade balance-output ratio, whereas a version of the benchmark model with productivity and foreign interest rate shocks improves the model fit in this respect. Such a version of the model, however, cannot replicate the negative relationship between financial development and both unemployment and wage volatility. As shown in Table 4, raising either $\eta_l$ or $\eta_s$ results in lower unemployment volatility, but in higher wage volatility. A similar conclusion holds when we consider a model with only productivity shocks: financial development reflected in a higher $\eta_l$ is associated with higher volatility in unemployment and wages; in turn, financial development reflected in a higher $\eta_s$ reduces unemployment volatility, but raises wage volatility. In particular, regardless of whether we consider a model with foreign interest rate shocks or financial shocks, an increase in $\eta_s$ is associated with a reduction in unemployment volatility. Thus, the presence of either of these two shocks is not essential for capturing the change in unemployment dynamics, but proves crucial to generating factual changes in wage dynamics.

To understand the intuition behind the results from the benchmark specification, it proves useful to consider the impulse responses of the economy to productivity shocks. Figures 2, 3 and 4 show the response of the economy to an unexpected temporary negative (1 standard deviation) aggregate productivity shock for three cases: (1) the benchmark economy (second column of Table 3); (2) the economy with a higher credit-to-output ratio via improved credit access for large firms (third column of Table 3); and (3) the economy with a higher credit-to-output ratio obtained through improved credit access for small firms (fourth column of Table 3). Similar to Table 3, all financial development experiments generate the same change in the average (or steady-state) aggregate credit-to-output ratio.
The first thing to note is that, for the change in financial development considered, improved credit access has a very limited quantitative impact on the dynamics of total output and consumption (even though consumption falls by slightly less when $\eta_s$ increases). However, the same cannot be said of investment and the labor market variables in general. Under both financial deepening scenarios, investment contracts by less, with the contraction being smaller when small firms benefit from improved credit access. This occurs because large-firm investment accounts for the bulk of total investment, even when $\eta_s$ increases (see the capital stocks in Table 3). As we discuss below, an increase in $\eta_s$ has a more stabilizing effect on large-firm employment and investment relative to a change in $\eta_l$, such that the smaller volatility in large-firm investment drives total investment dynamics. Second, the most important differences between changes in $\eta_l$ and $\eta_s$ are observed for unemployment and the wage. This represents one of the key results of the paper. Indeed, when financial development is reflected in improved credit access for small firms, both unemployment and wages become less volatile. The opposite takes place when financial development materializes via improved credit access for large firms.

The intuition behind these results traces back to the impact of changes in average (steady-state) access to formal financing on sectoral vacancy postings (and hence sectoral market tightness), and how these changes spill over into other sectors via wages, thereby affecting sectoral dynamics. First, consider an increase in $\eta_s$ and compare the outcome to the benchmark economy. Consider first the steady state. The rise in $\eta_s$ expands the borrowing capacity of the small firm sector, fosters small-firm capital accumulation (by reducing the marginal cost of investment for a given collateral multiplier), and raises the average (steady-state) value of having a worker in the small firm sector, $J_o$ (see Table 3). In turn, this makes vacancy postings by small firms less responsive to the same set of shocks, which, in our case, implies that vacancies and employment become less countercyclical relative to the benchmark economy (see the last two columns of Table 4).^{43}

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42 This is driven mainly by the smaller contraction in the consumption of large firms, $c_l$. Also, while not shown, the response of large-firm output does vary somewhat more than total output across scenarios. The high persistence of consumption, which takes a long time to return to trend, is partly responsible for the model’s success in capturing a high relative volatility of consumption.

43 In fact, small firm vacancies become procyclical, thereby generating an initial contraction in small firm employment. This stands in contrast with the expansion of small firm employment after a negative productivity shock in the benchmark economy, consistent with the its countercyclicality (in both the model
Recall that sectoral vacancy postings affect sectoral market tightness, which in turn influence wages across sectors via changes in workers’ outside options. Then, all else equal, the fall in the countercyclicality of small firm vacancies exerts less upward pressure on large firm wages, which, in turn, stabilizes the contraction in the value of having a worker in the large firm sector, $J_l$. This makes large firm vacancies less volatile, which ultimately leads to a smaller contraction in large firm employment that partly offsets the fall in the countercyclicality of small firm salaried employment and contributes to limiting the rise in unemployment.

In addition, there is another mechanism at play that contributes to the smaller rise in unemployment, mainly the dynamics of small firm owners. As we pointed out above, large firm vacancies contract by less, which, all else equal, reduces the contraction in large-firm market tightness and improves individuals’ outside options relative to the benchmark economy. This puts downward pressure on the Nash rental rate paid by small firm owners (not shown), which cushions the fall in the value of being a small firm owners and ultimately results in a smaller contraction in the measure of small firm owners. This further limits the rise in unemployment. All told, an improvement in borrowing capacity for small firms reduces unemployment volatility, and the change in the countercyclicality of salaried employment, by affecting the decisions of large firms via changes in large-firm workers’ outside options, plays a key role.

The behavior of equilibrium wages results from the combination of changes in large-firm and small-firm labor market tightness, the marginal product of labor, and capital market tightness. We noted above that small-firm vacancies (and hence small-firm market tightness) become less countercyclical and put downward pressure on wages, whereas large-firm vacancies (and hence large-firm market tightness) fall by less and put upward pressure on wages. Initially, the former leads to a sharper initial contraction in average wages (driven mainly by small firm wages), but both the smaller contraction in large firm employment and small firm owners ultimately limit the contraction in wages in the aftermath of the shock, resulting in less volatile wages.

Now consider an increase in $\bar{\eta}_l$ and focus on the steady state first. Similar to the change
a higher $\eta_l$ raises the incentive to accumulate capital among large firms for a given collateral multiplier, thereby raising the average marginal product of labor, and the average value to large firms of having an additional worker. In turn, this fosters the creation of more large-firm vacancies, which puts upward pressure on large-firm market tightness. All else equal, this puts upward pressure on both large-firm and small-firm wages. This leads the small firm sector to reduce capital accumulation, which reduces the value of having a worker, and vacancy postings. The higher value to a large firm of having a worker makes large-firm vacancy postings less responsive to shocks, whereas the opposite occurs for small firms.

In particular, the reduction in the average value to a small firm of having a worker makes vacancy postings by these firms less countercyclical, which all else equal contributes more to the rise in unemployment relative to the benchmark economy. Coupled with the fact that large firm employment still contracts after the shock (albeit by a smaller amount), the behavior of small firm employment ultimately translates into a larger expansion in unemployment. This also explains why equilibrium wages contract by more relative to both the benchmark economy and the case with a higher $\eta_s$: the fall in the countercyclicality of small firm employment, which reflects less attractive outside options to workers relative to the benchmark economy, acts as an endogenous mechanism that puts further downward pressure on wages. The fact that the response of small firm employment is not as strong as in the benchmark scenario further fuels the continued downward pressure on wages, which results in larger deviations from trend in the aftermath of the shock.

It is worth noting that the mechanisms through which financial development affects labor market dynamics are the same: financial development alters the cyclicality of sectoral employment, and such a change ultimately determines how responsive unemployment will be. If the mechanism is the same, what explains the differences in dynamics that arise from identical changes in the aggregate credit share? While the change in the aggregate credit share is the same (30 percentage points, from 0.20 to 0.50), the share of large-firm credit in total credit only increases 6 percentage points (from 0.90 to 0.964) when $\eta_l$ increases, whereas the share of small-firm credit in total credit increases 55 percentage points (from 0.10 to 0.65) when $\eta_s$ increases (see Table 3). Given differential average levels of capital between sectors,
this implies that small firms benefit disproportionately more from improvements in borrowing capacity for the same change in the aggregate credit share (that is, the change in $\eta_s$ is larger than the change in $\eta_l$ for a given change in the aggregate credit share).\footnote{Specifically, the benchmark economy is calibrated to have $\eta_l = 0.3079$ and $\eta_s = 0.5987$. To increase the aggregate credit share from 0.20 to 0.50, we use either $\eta_l = 0.4427$ (if borrowing capacity improves for large firms), or $\eta_s = 0.8395$ (if borrowing capacity improves for small firms). The percentage change is similar for each case (around 40 percent), but the level matters for the magnitude of the response of firms to these changes.} Coupled with the fact that the average value of employment to small firms is initially lower, the change in credit access for small firms proves much more beneficial for these firms than a similar change for large firms. The resulting reduction in the sensitivity of small firm decisions to shocks (due to better access to credit) affects large firms via changes in sectoral wages, which ultimately makes employment and investment decisions among large firms less volatile as well. While a similar mechanism is at play when $\eta_l$ changes, it is not as powerful given the relatively higher levels of access to formal credit by large firms relative to small firms.

5.2.3 The Role of Financial Shocks for Wage Dynamics

While looking at changes in the credit gap between large and small firms proves informative in assessing how unemployment volatility will change with financial development, Table 4 suggests that looking solely at the credit gap is less useful in characterizing wage dynamics. Importantly, as suggested above, the presence of financial shocks plays a key role in explaining the reduction in wage volatility when financial development is reflected in improved credit access for small firms.

While we would expect labor market variables to be more volatile as external financing becomes increasingly important (and therefore the shocks afflicting firms’ borrowing capacity can become more relevant), Table 4 shows that, under the benchmark specification, both large-firm and small-firm wages become less volatile with financial development under the benchmark specification, which includes financial shocks. Intuitively, as we argued previously, the rise in average borrowing capacity boosts the accumulation of capital, fosters vacancy postings, and increases the average value of a worker to the firms that benefit from improved credit access. Thus, for the same financial shock and relative to the benchmark economy, the economy with improved credit access is less sensitive to shocks, and given the
larger stock of capital, more resilient to financial shocks. This explains why wages become 
less volatile, regardless of whether financial development is reflected in a higher $\eta_l$ or $\eta_s$. 
This does not occur in the presence of foreign interest rate shocks since these shocks are 
reflected mainly on the (impatient) household side. Given that impatient households make 
all decisions for firms, interest rate shocks have a direct impact on investment decisions and 
on the value of having a worker to small firms. Thus, these shocks make hiring decisions 
more volatile, which translates into more volatile labor market tightness and ultimately more 
volatile small firm wages. In the case of an increase in $\eta_l$, the effect of interest rate shocks on 
small firm decisions is magnified as a higher $\eta_l$ adversely affects employment creation, and 
the value of a worker in the small firm sector.

5.2.4 Implications and Empirical Validation

The model suggests that on its own financial development need not lead to a reduction in 
the volatility of the labor market. Furthermore, our results suggest that looking at changes 
in the share of aggregate credit in the economy is not sufficient to adequately characterize 
the link between financial deepening and labor market volatility. Importantly, looking at the 
credit gap between large and small firms may be informative in determining the direction 
of change in the volatility of unemployment, and accounting for the relevance of financial 
shocks as firms’ borrowing capacity improves is key for understanding differences in wage 
dynamics.

As we alluded to in the introduction, the fact that sectoral reallocation matters for the 
aggregate implications of financial development is in line with existing empirical studies 
(Beck et al., 2004; Manganelli and Popov, 2012). An important contribution of our work 
is to provide a tractable framework to explore the implications of financial development for 
sectoral and aggregate employment dynamics, which have received little attention in existing 
literature. Moreover, the model’s ability to capture the negative link between financial 
development and unemployment and wage volatility is broadly consistent with the empirical 
evidence (mentioned in the introduction), as more developed economies not only have higher 
credit-to-output ratios, but also a higher share of total credit devoted to loans to smaller 
firms and lower unemployment and wage volatility (Table 1; CGAP, 2010, 2013; OECD,
To determine the extent to which the model can quantitatively explain the changes in labor market volatility as financial development takes place in the data, we compute the change in unemployment and wage volatility in response to a 30 percentage point increase in the average share of aggregate credit (as in Table 3), and compare this change to the data. The results are presented in Table 5. The results show that, when financial development is a result of an increase in $\eta_s$ (associated with a smaller credit gap), the model can capture 50 percent of the change in unemployment volatility, and 60 percent of the change in wage volatility, respectively, in the data. These results suggest that the model does reasonably well in capturing a non-negligible quantitative change in volatility associated with higher average credit shares across economies.

5.2.5 Robustness Checks

The Appendix briefly discusses the main results when considering alternative modeling assumptions and model variations. These include introducing costly financial intermediation, allowing for capacity utilization in the small firm sector, introducing hiring costs in the collateral constraints of firms, and assuming that sectoral productivity improves along with improved credit access, among others.

6 Conclusion

Access to finance represents one of the most important constraints for small firms in emerging economies. Motivated by recent efforts to foster financial inclusion for small firms in many emerging economies, we build a two-sector business cycle search model with collateral constraints consistent with the employment and firm-financing structure of these economies that allows us to explore the implications of financial development for sectoral, labor market, and aggregate dynamics.

The model successfully captures key features of emerging economy labor market dy-

\footnote{Within these references, see specifically Table 1.4 in OECD (2014), Figure 4.5 in CGAP (2010), Figure 18 in CGAP (2013), and the third and fourth sub-figures in Figure A3.1 in IMF (2014).}
namics; it also generates high consumption and wage volatility, in line with the prevalent characteristics of these economies. We show that financial development *per se* can have an ambiguous impact on labor market volatility. If financial development is reflected in improved credit access for large firms only, which in turn leads to a widening of the credit gap between large and small firms, financial deepening is associated with lower wage volatility, but sharper unemployment fluctuations. Conversely, an identical change in financial development reflected instead in improved credit access for small firms narrows the credit gap between large and small firms and reduces both fluctuations in wages and unemployment. We find that the presence of financial shocks plays an important role in generating the negative link between financial development and wage volatility in the data. Moreover, our model is able to capture a non-negligible share of the change in both unemployment and wage volatility as financial deepening takes place, but only if financial development is reflected in improved credit access for small firms. We provide supporting cross-country evidence showing that more financially-developed economies not only have higher aggregate shares of credit, but more importantly higher shares of total credit allocated to SMEs and lower unemployment and wage volatility. This gives additional validity to our results.

Our work makes four contributions. First, it highlights the importance of differentiating between small and large firms for characterizing the impact of financial development on sectoral and labor market dynamics. Second, it provides a first characterization of the impact of financial development on labor market dynamics, a topic that has received little attention. Third, it points to a mechanism rooted in firm heterogeneity that can potentially explain the link between labor market volatility and financial development. Finally, it suggests particular conditions under which financial deepening may be accompanied by lower labor market volatility. This last point may prove particularly relevant for policymakers as emerging economies continue to experience financial deepening.
References


Figure 1: Empirical Evidence: Financial Development, Small Firms, and Labor Market Volatility

Note: The lines in each of the subfigures represent trend lines.
### Table 1: SME Access to Finance Across Regions

<table>
<thead>
<tr>
<th>Region</th>
<th>Loans to SMEs (% Total Loans)</th>
<th>Loans to SMEs (% of GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central and Eastern Europe</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>Middle-East and North Africa</td>
<td>9</td>
<td>3.5</td>
</tr>
<tr>
<td>Latin America</td>
<td>20.5</td>
<td>6</td>
</tr>
<tr>
<td>Asia</td>
<td>20.5</td>
<td>15.5</td>
</tr>
<tr>
<td>Caucasus and Central Asia</td>
<td>21</td>
<td>5.5</td>
</tr>
<tr>
<td>Sub-Saharan Africa</td>
<td>29.5</td>
<td>5</td>
</tr>
<tr>
<td>Advanced Economies</td>
<td>45</td>
<td>26</td>
</tr>
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</table>

Source: Figure A3.1 in IMF (2014).

### Table 2: Business Cycle Statistics, Data vs. Model

<table>
<thead>
<tr>
<th>Targeted Second Moments</th>
<th>Data (1)</th>
<th>Benchmark Model (2)</th>
<th>One Sector Model (3)</th>
<th>Two Sector Model (4)</th>
<th>Benchmark No Capital Search (5)</th>
<th>Benchmark Interest Rate Shocks (6)</th>
<th>Benchmark, only TFP Shocks</th>
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</thead>
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<tr>
<td>$\sigma_{yt}$</td>
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<td>2.390</td>
<td>2.390</td>
<td>2.390</td>
<td>2.390</td>
<td>2.390</td>
<td>2.390</td>
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<tr>
<td>$\sigma_{yt}/\sigma_{yt}$</td>
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<td>2.780</td>
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<td>$\text{corr}(y_t, y_{t-1})$</td>
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<td>0.758</td>
<td>0.736</td>
<td>0.735</td>
<td>0.745</td>
<td>0.769</td>
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<tr>
<td>$\text{corr}(w_t, y_t)$</td>
<td>0.560</td>
<td>0.562</td>
<td>0.563</td>
<td>0.563</td>
<td>0.563</td>
<td>–</td>
<td>–</td>
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<tr>
<td>$\text{corr}(\theta_t/y_t, y_t)$</td>
<td>$-0.750$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>$-0.755$</td>
<td>–</td>
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</table>

<table>
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<tr>
<th>Non-Targeted Moments</th>
<th>$\sigma_{ct}/\sigma_{yt}$</th>
<th>1.260</th>
<th>1.143</th>
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<th>0.740</th>
<th>1.181</th>
<th>3.152</th>
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<td>1.361</td>
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</tr>
<tr>
<td>$\sigma_{ut}/\sigma_{yt}$</td>
<td>6.280</td>
<td>0.709</td>
<td>0.312</td>
<td>0.538</td>
<td>0.353</td>
<td>0.959</td>
<td>0.534</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(u_t, y_t)$</td>
<td>0.740</td>
<td>0.688</td>
<td>0.664</td>
<td>0.422</td>
<td>0.640</td>
<td>0.674</td>
<td>0.961</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(s_t, y_t)$</td>
<td>$-0.470$</td>
<td>$-0.440$</td>
<td>–</td>
<td>0.015</td>
<td>$-0.199$</td>
<td>$-0.100$</td>
<td>$-0.726$</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(u_t, y_t)$</td>
<td>$-0.780$</td>
<td>$-0.665$</td>
<td>$-0.446$</td>
<td>$-0.119$</td>
<td>$-0.524$</td>
<td>$-0.748$</td>
<td>$-0.730$</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(f_t, y_t)$</td>
<td>0.798</td>
<td>0.809</td>
<td>0.692</td>
<td>0.696</td>
<td>0.810</td>
<td>0.956</td>
<td>0.990</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(s_t, y_t)$</td>
<td>0.366</td>
<td>0.500</td>
<td>–</td>
<td>0.079</td>
<td>0.339</td>
<td>0.804</td>
<td>0.634</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(u_t, u_{t-1})$</td>
<td>0.840</td>
<td>0.809</td>
<td>0.835</td>
<td>0.426</td>
<td>0.862</td>
<td>0.745</td>
<td>0.793</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(\theta_t/y_t, y_t)$</td>
<td>$-0.750$</td>
<td>$-0.022$</td>
<td>0.592</td>
<td>0.656</td>
<td>$-0.058$</td>
<td>–</td>
<td>$-0.246$</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(w_t, y_t)$</td>
<td>0.560</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.872</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(y_t, y_{t-1})$</td>
<td>0.750</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.729</td>
<td></td>
</tr>
</tbody>
</table>
Table 3: Impact of Financial Development on Steady State

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark Calibration</th>
<th>Increase in $\eta_l$</th>
<th>Increase in $\eta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.562</td>
<td>0.583</td>
<td>0.571</td>
</tr>
<tr>
<td>$k_l$</td>
<td>0.829</td>
<td>1.011</td>
<td>0.827</td>
</tr>
<tr>
<td>$k_s$</td>
<td>0.274</td>
<td>0.275</td>
<td>0.414</td>
</tr>
<tr>
<td>Total Capital</td>
<td>1.102</td>
<td>1.286</td>
<td>1.241</td>
</tr>
<tr>
<td>$n_l$</td>
<td>0.400</td>
<td>0.407</td>
<td>0.394</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.480</td>
<td>0.473</td>
<td>0.485</td>
</tr>
<tr>
<td>$o_s$</td>
<td>0.070</td>
<td>0.071</td>
<td>0.072</td>
</tr>
<tr>
<td>$u$</td>
<td>0.050</td>
<td>0.0495</td>
<td>0.0492</td>
</tr>
<tr>
<td>Average wage</td>
<td>0.347</td>
<td>0.359</td>
<td>0.355</td>
</tr>
<tr>
<td>$w_l$</td>
<td>0.381</td>
<td>0.398</td>
<td>0.388</td>
</tr>
<tr>
<td>$w_s$</td>
<td>0.317</td>
<td>0.326</td>
<td>0.327</td>
</tr>
<tr>
<td>$J_l$</td>
<td>0.745</td>
<td>0.800</td>
<td>0.737</td>
</tr>
<tr>
<td>$J_o$</td>
<td>0.189</td>
<td>0.183</td>
<td>0.197</td>
</tr>
<tr>
<td>$(J_l - J_o)$</td>
<td>0.556</td>
<td>0.617</td>
<td>0.540</td>
</tr>
<tr>
<td>$l_l/l$</td>
<td>0.900</td>
<td>0.964</td>
<td>0.349</td>
</tr>
<tr>
<td>$l_s/l$</td>
<td>0.100</td>
<td>0.036</td>
<td>0.651</td>
</tr>
<tr>
<td>$l/y$</td>
<td>0.200</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>$</td>
<td>l_l - l_s</td>
<td>$</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Notes: $l$ denotes total credit and is given by the sum of credit to large and small firms: $l_l$ and $l_s$, respectively. Changes in parameters with $l$ ($s$) subscripts refer to changes in parameters associated with financial access for large firms (small firms). $(J_l - J_o)$ denotes the difference between the value of having an additional worker to a large firm, $J_l$, and the value of having an additional worker to a small firm, $J_o$. 

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Table 4: Changes in Volatility and Employment Correlations for Benchmark Model and Different Shock Modalities

<table>
<thead>
<tr>
<th>Benchmark Specification</th>
<th>$\sigma_{w_t}$</th>
<th>$\sigma_{w_{s,t}}$</th>
<th>$\sigma_{w_{l,t}}$</th>
<th>$\sigma_{c_t}$</th>
<th>$\sigma_{c_{h,t}}$</th>
<th>$\sigma_{y_t}$</th>
<th>$\rho(n_{s,t}, y_t)$</th>
<th>$\rho(n_{l,t}, y_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Economy</td>
<td>3.606</td>
<td>3.815</td>
<td><strong>3.687</strong></td>
<td>1.693</td>
<td>2.732</td>
<td>2.962</td>
<td>6.646</td>
<td>2.390</td>
</tr>
<tr>
<td>Increase in $\eta_l$</td>
<td>3.579</td>
<td>3.507</td>
<td><strong>3.500</strong></td>
<td><strong>1.885</strong></td>
<td>2.635</td>
<td>2.963</td>
<td>5.080</td>
<td>2.373</td>
</tr>
<tr>
<td>Increase in $\eta_s$</td>
<td>3.244</td>
<td>3.357</td>
<td><strong>3.299</strong></td>
<td><strong>1.318</strong></td>
<td>2.586</td>
<td>2.989</td>
<td>4.280</td>
<td>2.351</td>
</tr>
</tbody>
</table>

| TFP, Interest Rate Shocks|                |                  |                  |                |                |                |                |                |
| Benchmark Economy        | 2.380          | 4.238            | **3.252**        | **2.293**      | 7.534          | 10.74          | 6.646          | 2.390          | -0.100          | 0.674          |
| Increase in $\eta_l$     | 2.791          | 4.735            | **3.682**        | **2.244**      | 7.325          | 10.48          | 4.395          | 2.356          | 0.447           | 0.591          |
| Increase in $\eta_s$     | 2.263          | 4.484            | **3.328**        | **1.846**      | 7.335          | 10.61          | 3.810          | 2.335          | 0.734           | 0.576          |

| Only TFP Shocks          |                |                  |                  |                |                |                |                |                |
| Benchmark Economy        | 2.349          | 2.120            | **2.232**        | **1.276**      | 2.611          | 2.934          | 6.646          | 2.390          | -0.726          | 0.961          |
| Increase in $\eta_l$     | 2.755          | 2.417            | **2.581**        | **1.334**      | 2.541          | 2.935          | 4.557          | 2.373          | -0.593          | 0.840          |
| Increase in $\eta_s$     | 2.325          | 2.235            | **2.280**        | **1.039**      | 2.523          | 2.961          | 4.061          | 2.353          | -0.112          | 0.972          |

Notes: $\sigma_{c_{h,t}}$, $\sigma_{c_t}$, and $\sigma_{w_t}$ denote the volatility of household consumption, total consumption, and average wages, respectively. $\rho(n_{s,t}, y_t)$ and $\rho(n_{l,t}, y_t)$ denote the contemporaneous correlations of small-firm employment and large-firm employment with output.
Figure 2: Response to a One Percent Negative Productivity Shock (Quarters after Shock)
Figure 3: Response to a One Percent Negative Productivity Shock (Quarters after Shock)
Figure 4: Response to a One Percent Negative Productivity Shock (Quarters after Shock)
Table 5: 30 Percentage Point Change in Financial Development and Change in Unemployment and Wage Volatility – Data vs. Model

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \sigma_u / \Delta (l/y)$</th>
<th>$\Delta \sigma_w / \Delta (l/y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>-0.0255</td>
<td>-0.0218</td>
</tr>
<tr>
<td>Model (increase in $\eta_s$)</td>
<td><strong>-0.0125</strong></td>
<td><strong>-0.0129</strong></td>
</tr>
<tr>
<td>Model (increase in $\eta_l$)</td>
<td>0.0064</td>
<td>-0.0062</td>
</tr>
</tbody>
</table>

Notes: $\Delta x$ denotes a change in variable $x$. $l$ and $y$ denote total credit and total output, respectively.
A Appendix

A.1 Household Optimality Conditions

Associate the multiplier $\Lambda_{i,t}$ to the impatient household’s budget constraint, $\Lambda_{o,t}$ to the perceived law of motion for small firm owners, and $\Lambda_{s,t}$ to the perceived law of motion for employment in small firms (from the household’s labor demand perspective). Taking into account that the first-order condition with respect to consumption for impatient households is $u'(c_{i,t}) - \Lambda_{i,t} = 0$, the first-order condition for capital search expenditures, $s_t$, is

$$-\Lambda_{i,t} \kappa'(s_t) + (1 - \rho^o) f_{k,t} \Lambda_{o,t} = 0.$$

For small firm owners, $o_{s,t+1}$:

$$\beta \mathbb{E}_t \Lambda_{i,t+1} \left[ p_{s,t+1} z_{s,t+1} F_{o_{s,t+1}} - r_{s,t+1} - \kappa(v_{s,t+1}) \right] + \beta(1 - \rho^o) \mathbb{E}_t \Lambda_{o,t+1} + \beta(1 - \rho^o)(1 - \rho^s) \mathbb{E}_t \Lambda_{s,t+1} v_{s,t+1} q_{s,t+1} - \Lambda_{o,t} = 0.$$

For the demand for salaried employment in small firms, $n_{s,t+1}$:

$$\beta \mathbb{E}_t \Lambda_{i,t+1} \left[ p_{s,t+1} z_{s,t+1} F_{n_{s,t+1}} - w_{s,t+1} (1 + \lambda_{s,t+1}) \phi \right] + \beta (1 - \rho^o) (1 - \rho^s) \mathbb{E}_t \Lambda_{s,t+1} - \Lambda_{s,t} = 0.$$

And for vacancies posted for a given small firm, $v_{s,t}$:

$$-\Lambda_{i,t} \kappa'(v_{s,t}) + (1 - \rho^o)(1 - \rho^s) q_{s,t} \Lambda_{s,t} = 0.$$

Using the fact that $\Lambda_{i,t} = u'(c_{i,t})$, we have

$$\frac{\Lambda_{o,t}}{u'(c_{i,t})} = \frac{\kappa'(s_t)}{(1 - \rho^o)f_{k,t}}.$$
We can then get
\[
\Lambda_{s,t} = \frac{\kappa'(v_{s,t})}{u'(c_{i,t})} = \frac{\kappa'(v_{s,t})}{(1 - \rho^o)(1 - \rho^s)q_{s,t}}.
\]

Then, to obtain the household’s decision to create a small firm, we can write:
\[
\frac{\Lambda_{o,t}}{\Lambda_{i,t}} = \mathbb{E}_t \beta \frac{\Lambda_{i,t+1}}{\Lambda_{i,t}} \left\{ p_{s,t+1} z_{s,t+1} F_{o,s,t+1} - r_{s,t+1} - \kappa(v_{s,t+1}) + (1 - \rho^o) \frac{\Lambda_{o,t+1}}{\Lambda_{i,t+1}} \right\}.
\]

where: $\Xi^{i}_{t+1|t} = \beta U'(c_{i,t+1})/U'(c_{i,t})$ is the stochastic discount factor. Making use of the previous expressions, we can rewrite the above equation to have:
\[
\frac{\kappa'(s_t)}{f_{k,t}} = (1 - \rho^o) \mathbb{E}_t \Xi^{i}_{t+1|t} \left\{ p_{s,t+1} z_{s,t+1} F_{o,s,t+1} - r_{s,t+1} - \kappa(v_{s,t+1}) + \frac{\kappa'(s_{t+1})}{f_{k,t+1}} \right\}.
\]

This equation defines $s_t$.

### A.2 Value Functions

The household’s value of having an additional member in salaried employment in a large firm, $W_{l,t}$, is given by the contemporaneous wage payment plus the expected continuation value:
\[
W_{l,t} = w_{l,t} + \mathbb{E}_t \Xi_{t+1|t} \left\{ (1 - \rho^l) W_{l,t+1} + \rho^l W_{u,t+1} \right\}.
\]

Similarly, the household’s value of having a member in small-firm salaried employment, $W_{s,t}$, is:
\[
W_{s,t} = w_{s,t} + \mathbb{E}_t \Xi_{t+1|t} \left\{ (1 - \rho^o)(1 - \rho^s) W_{s,t+1} + [1 - (1 - \rho^o)(1 - \rho^s)] W_{u,t+1} \right\}.
\]

The household’s value of having an additional small firm owner, $W_{o,t}$, is given by the contemporaneous net return to capital plus the expected continuation value, taking into account the transition of a small firm owner back to self-employment: 
Finally, the household’s values of having a household member in unemployment $W_{u,t}$, is:

$$W_{u,t} = \mathbb{E}_t \Xi_{t+1} \left\{ (1 - \rho^o) W_{o,t+1} + \rho^o W_{u,t+1} \right\},$$

From the firms’ perspective, the value to large firms from having capital relationships with small firms, $J_{s,t}$, is

$$J_{s,t} = r_{s,t} + (\rho^o - \delta) + (1 - \rho^o) \mathbb{E}_t \Xi_{t+1} J_{s,t+1}.$$

The value of having an additional salaried worker in the large firm sector, $J_{o,t}$, and the large firm’s value of having an additional worker, $J_{l,t}$, are, respectively:

$$J_{o,t} = p_{s,t} z_{s,t} F_{n_{s,t}} - w_{s,t} [1 + \lambda_{s,t} \phi] + (1 - \rho^o)(1 - \rho^s) \mathbb{E}_t \Xi_{t+1} J_{o,t+1},$$

and

$$J_{l,t} = p_{l,t} z_{l,t} F_{n_{l,t}} - w_{l,t} [1 + \lambda_{l,t} \phi] + (1 - \rho^l) \mathbb{E}_t \Xi_{t+1} J_{l,t+1}.$$

### A.2.1 Steady State Productivity by Firm Size Category

Following Fernández and Meza (2014) and EFS (2015), we used the facts in Table 10 from Busso, Fazio, and Levy (2012) to compute the productivity differential between small and large firms. Busso et al. (2012) compute productivity differences by formality/legal status,
and also by firm size. From Table 10 in Busso, Fazio, and Levy (2012), we can write

\[
\log \left( \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.959 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}_{\text{LF}}} = 0.3833,
\]

\[
\log \left( \frac{\text{TFP}_{\text{LI}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -1.759 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LI}}}{\text{TFP}_{\text{LF}}} = 0.1722,
\]

\[
\log \left( \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.214 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}_{\text{LF}}} = 0.8073,
\]

\[
\log \left( \frac{\text{TFP}_{\text{II}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.881 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{II}}}{\text{TFP}_{\text{LF}}} = 0.4144,
\]

for firms of size [0-5],

\[
\log \left( \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.910 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}_{\text{LF}}} = 0.4025,
\]

\[
\log \left( \frac{\text{TFP}_{\text{LI}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.688 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LI}}}{\text{TFP}_{\text{LF}}} = 0.5026,
\]

\[
\log \left( \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.085 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}_{\text{LF}}} = 0.9185,
\]

\[
\log \left( \frac{\text{TFP}_{\text{II}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.632 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{II}}}{\text{TFP}_{\text{LF}}} = 0.5315,
\]

for firms of size [6-10], and

\[
\log \left( \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.746 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}_{\text{LF}}} = 0.4743,
\]

\[
\log \left( \frac{\text{TFP}_{\text{LI}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.395 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LI}}}{\text{TFP}_{\text{LF}}} = 0.6737,
\]

\[
\log \left( \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.092 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}_{\text{LF}}} = 0.9121,
\]

\[
\log \left( \frac{\text{TFP}_{\text{II}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.701 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{II}}}{\text{TFP}_{\text{LF}}} = 0.4961,
\]

for firms of size [11-50].

\footnote{In particular, they consider firms that are formal and legal, semi-legal and formal, semi-legal and semi-formal, informal and legal, and informal and illegal. See their paper for more details.}
For firms of size [+50], we have

\[
\log \left( \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.574 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LSF}}}{\text{TFP}_{\text{LF}}} = 0.5633,
\]

\[
\log \left( \frac{\text{TFP}_{\text{LI}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.491 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{LI}}}{\text{TFP}_{\text{LF}}} = 0.6120,
\]

\[
\log \left( \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -0.159 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{SLSF}}}{\text{TFP}_{\text{LF}}} = 0.8530,
\]

\[
\log \left( \frac{\text{TFP}_{\text{II}}}{\text{TFP}} \right) - \log \left( \frac{\text{TFP}_{\text{LF}}}{\text{TFP}} \right) = -1.039 \quad \Rightarrow \quad \frac{\text{TFP}_{\text{II}}}{\text{TFP}_{\text{LF}}} = 0.3538.
\]

We can then use Tables 6 and 10 in Busso, Fazio, and Levy (2012) and create employment-weighted measures of TFP for small and large firms, TFP, and TFP, respectively.

Given our interest in comparing the cyclical dynamics of sectoral employment generated by the model to the data, and the fact that the evidence on cyclical employment dynamics in emerging economies is based on formality status rather than firm size, we must take a stand with regards to the mapping between large and small firms in the model and their empirical counterparts. We assume that large firms in the model comprise firms of size [11-50] and [+50 ]that are formal and legal, semi-formal and legal, and semi-formal and semi-legal.\(^{47}\)

\(^{47}\)This mapping is based on the allocation of employment by firm size based on formality status. In particular, firms of size [0-5] have formality rates ranging from 91 to 98 percent; firms of size [6-10] have formality rates ranging from 60 to 88 percent; firms of size [11-50] have formality rates ranging from 39 to 74 percent, and firms of size [+50] have formality rates ranging from 23 to 62 percent, depending on how we define formality. The upper bounds are based on a definition that considers employment in illegal and informal firms, legal and informal firms, semi-legal and semi-formal firms, and legal and semi-formal firms. The lower bound is based on the two first categories of firms.
Small-firm productivity $TFP_s$ is then given by:

$$TFP_s = \left( \frac{23.6}{61.26} \right) TFP_{[0-5],LI} + \left( \frac{0.79}{61.26} \right) TFP_{[0-5],LSF} + \left( \frac{1.69}{61.26} \right) TFP_{[0-5],SLSF}$$

$$+ \left( \frac{10.94}{61.26} \right) TFP_{[0-5],II} + \left( \frac{0.77}{61.26} \right) TFP_{[0-5],LF} + \left( \frac{1.63}{61.26} \right) TFP_{[6-10],LI}$$

$$+ \left( \frac{0.50}{61.26} \right) TFP_{[6-10],LSF} + \left( \frac{1.92}{61.26} \right) TFP_{[6-10],SLSF} + \left( \frac{1.09}{61.26} \right) TFP_{[6-10],LF}$$

$$+ \left( \frac{3.66}{61.26} \right) TFP_{[6-10],II} + \left( \frac{2.39}{61.26} \right) TFP_{[11-50],LI} + \left( \frac{3.36}{61.26} \right) TFP_{[11-50],II}$$

$$+ \left( \frac{1.18}{61.26} \right) TFP_{[+50],II} + \left( \frac{7.74}{61.26} \right) TFP_{[+50],LI}$$

In turn, large-firm productivity $TFP_l$ is computed as:

$$TFP_l = \left( \frac{3.84}{38.74} \right) TFP_{[11-50],LF} + \left( \frac{0.57}{38.74} \right) TFP_{[11-50],LSF}$$

$$+ \left( \frac{4.72}{38.74} \right) TFP_{[11-50],SLSF} + \left( \frac{14.67}{38.74} \right) TFP_{[+50],LF}$$

$$+ \left( \frac{1.19}{38.74} \right) TFP_{[+50],LSF} + \left( \frac{13.75}{38.74} \right) TFP_{[+50],SLSF}.$$

Now, we use the values for $TFP$ relative to $TFP_{LF}$ by firm size and legality/formality status from Busso, Fazio, and Levy (2012) documented above, normalizing $TFP_{LF}$ to 1. This yields

$$TFP_s = \left( \frac{23.6}{61.26} \right) (0.1722) + \left( \frac{0.79}{61.26} \right) (0.3833) + \left( \frac{1.69}{61.26} \right) (0.8073)$$

$$+ \left( \frac{10.94}{61.26} \right) (0.4144) + \left( \frac{0.77}{61.26} \right) (1) + \left( \frac{1.63}{61.26} \right) (0.5026)$$

$$+ \left( \frac{0.50}{61.26} \right) (0.4025) + \left( \frac{1.92}{61.26} \right) (0.9185) + \left( \frac{1.09}{61.26} \right) (1)$$

$$+ \left( \frac{3.66}{61.26} \right) (0.5315) + \left( \frac{2.39}{61.26} \right) (0.6737) + \left( \frac{3.36}{61.26} \right) (0.4961)$$

$$+ \left( \frac{1.18}{61.26} \right) (0.3538) + \left( \frac{7.74}{61.26} \right) (0.6120)$$
\[
T\!F\!P_l = \left( \frac{3.84}{38.74} \right) (1) + \left( \frac{0.57}{38.74} \right) (0.4743) \\
+ \left( \frac{4.72}{38.74} \right) (0.9121) + \left( \frac{14.67}{38.74} \right) (1) \\
+ \left( \frac{1.19}{38.74} \right) (0.5633) + \left( \frac{13.75}{38.74} \right) (0.8530).
\]

We then obtain \( T\!F\!P_s = 0.4128 \), and \( T\!F\!P_l = 0.9160 \). Normalizing \( T\!F\!P_s \) to 1 and given the mapping \( T\!F\!P_l = z_l \) and \( T\!F\!P_s = z_s \), we have

\[ z_l = 2.2189, \quad z_s = 1. \]

A.3 Robustness Checks

A.3.1 Introducing Capacity Utilization for Small Firms

In the benchmark model, large firms can vary their capital usage but small firms cannot. As shown in Table A1, allowing for capacity utilization for small firms does not change the main conclusions of the paper and makes the changes in volatility due to financial development stronger (in the same direction as in the benchmark model).

<table>
<thead>
<tr>
<th>Benchmark Specification:</th>
<th>( \sigma_{w_{1,t}} )</th>
<th>( \sigma_{w_{s,t}} )</th>
<th>( \sigma_{w_{t}} )</th>
<th>( \sigma_{u_{t}} )</th>
<th>( \sigma_{c_{t}} )</th>
<th>( \sigma_{c_{t}^{h}} )</th>
<th>( \sigma_{t_{t}} )</th>
<th>( \sigma_{y_{t}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Economy</td>
<td>3.585</td>
<td>3.776</td>
<td><strong>3.673</strong></td>
<td><strong>2.052</strong></td>
<td>2.713</td>
<td>2.978</td>
<td>6.646</td>
<td>2.390</td>
</tr>
<tr>
<td>Change in ( \bar{\eta}_l )</td>
<td>3.514</td>
<td>3.502</td>
<td><strong>3.491</strong></td>
<td><strong>2.446</strong></td>
<td>2.609</td>
<td>2.974</td>
<td>5.030</td>
<td>2.371</td>
</tr>
<tr>
<td>Change in ( \bar{\eta}_s )</td>
<td>3.202</td>
<td>3.275</td>
<td><strong>3.239</strong></td>
<td><strong>1.639</strong></td>
<td>2.576</td>
<td>3.040</td>
<td>4.155</td>
<td>2.342</td>
</tr>
</tbody>
</table>

Note: \( \sigma_{c_{t}^{h}} \) and \( \sigma_{c_{t}} \) denote the volatility of household consumption and total consumption, respectively.
A.3.2 Including Hiring Costs in Collateral Constraints

In the benchmark model, the collateral constraint takes into account the wage bill. Introducing the hiring bill into the collateral constraint does not change the main conclusions of the paper. In fact, the quantitative changes in volatility are virtually the same as those in the benchmark model.

A.3.3 Introducing Interest Rate Spreads

Economies with a higher share of domestic credit to the private sector (as a percent of GDP) also tend to have lower financial intermediation costs, proxied by banks’ overhead costs, as well as lower interest rate spreads (defined as the spread between lending and deposit rates). To determine whether the inclusion of costly financial intermediation matters for our results, we follow Cúrdia and Woodford (2010) and assume that, when financial intermediaries extend loans $l_l$ and $l_s$ to large and small firms, they incur additional costs $\Phi_j (l_{j,t}) = \tau_j (l_{j,t})^\zeta_j$, with $\tau_j, \zeta_j > 1$ for $j \in \{l, s\}$, where $\Phi'_j (l_{j,t}) > 0$ and $\Phi''_j (l_{j,t}) \geq 0$. From the the first-order conditions for financial intermediaries, we obtain a spread between lending and deposit rates that depends on the cost of financial intermediation.

We then analyze how financial development, as reflected in joint improvements in firms’ borrowing capacity and a reduction in the cost of financial intermediation, affect aggregate dynamics. This joint change generates patterns that broadly characterize financial development across economies in the data: higher aggregate credit-output ratios are, on average, also accompanied by lower interest rate spreads. More specifically, we consider experiments that improve access to formal credit for either large or small firms, but that generate the same change in aggregate credit shares and average interest rate spreads and discipline the changes in borrowing capacity and the spread based on the sample of countries for which we have data on unemployment and wage volatility. The main results of the paper do not change when financial development is reflected in a joint change in borrowing capacity the spread. 48

48 Regressing unemployment or wage volatility on interest rate spreads delivers a statistically significant coefficient (with the caveat that we have a small number of observations given data limitations). However, this significance disappears once we control for our measure of financial development (credit to the private sector as a share of GDP), where the coefficient on the latter becomes significant.
A.3.4 Allowing for Input Credit as a Pledgable Asset in the Small Firms’ Borrowing Constraint

Campello and Larrain (2014) document how a number of countries implemented reforms that expanded the set of pleasurable assets acceptable as collateral. This set included movable assets. Allowing for small firms to use the capital obtained via input credit relationships as additional collateral does not change the main conclusions.

A.3.5 Joint Changes in Sectoral Productivity and Access to Formal Credit

Recent studies document a positive link between firm productivity and financial development.49

Assuming that an increase in borrowing capacity for small firms reduces the sectoral productivity gap between large and small firms does not change the main conclusions, though the quantitative change in unemployment and wage volatility is smaller relative to the benchmark calibration.

A.4 Data Sources and Details

The following data sources were used to construct the plots in Figure 1. All data files are available upon request.

A.4.1 Credit to the Private Sector (Percent of GDP)

World Bank World Development Indicators (WDI). The sample includes both developed and developing countries with a total of 161 economies. The credit share for each country is constructed as an average of annual data from 1999 to 2011. Restricting the last year of the sample for both series to 2007 does not change the main patterns.

49See Arizala, Cavallo, and Galindo (2009), Greenwood, Sánchez, and Wang (2013), and Levine and Warusawitharana (2014), among others.
A.4.2 Share of Firms with a Bank Loan or Line of Credit, Share of Firms Using Banks to Meet their Working Capital Needs, Value of Collateral

World Bank Enterprise Surveys. The year used varies by country. Small firms are defined as firms with 5-19 workers. Large firms are defined as firms with more than 100 workers. We establish year 2011 (included) as a ceiling and take the most recent year available for each country. The sample is comprised of 123 countries (we exclude Zimbabwe as it represents a clear outlier), 5 of which are classified as developed economies (Germany, Greece, Ireland, Portugal, Spain; these countries do not generally have data on the share of firms with a bank loan or line of credit, but do have data on the percent of firms that finance working capital using banks).

A.4.3 Unemployment and Wage Volatility

To stay close to the international business cycle literature, we borrow the cross-country data on unemployment and wage volatility from Boz, Durdu, and Li (2015), where the country sample is restricted by data availability on unemployment and wages at a high frequency. As stated by the authors, the data used to compute these second moments is at a quarterly frequency, logged (when applicable) and HP-filtered with smoothing parameter 1600. The countries in the sample are: Australia, Austria, Belgium, Brazil, Canada, Chile, Denmark, Ecuador, Finland, Hungary, Ireland, Israel, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Portugal, Spain, Sweden, and Turkey.

A.4.4 Share of SME Loans in Total Loans

Consultative Group to Assist the Poor (CGAP) Financial Access 2010. The countries with available data are: Botswana, Cape Verde, Liberia, South Africa, China, Chinese Taipei, Hong Kong, Mongolia, Albania, Armenia, Georgia, Kazakhstan, Latvia, Russia, Tajikistan, Uzbekistan, Australia, Belgium, Estonia, France, Hungary, Italy, Japan, Netherlands, Poland, Portugal, South Korea, Turkey, United States, Iran, Jordan, Morocco, Afghanistan, Bangladesh, India, Indonesia, Malaysia, Pakistan, Singapore, Thailand, Guatemala, Panama, Peru, Costa Rica, Argentina, Brazil, El Salvador, Ecuador, and Uruguay.