

The Distributional Consequences of Group Procurement: Evidence from a Randomized Trial of a Food Security Program in Rural India*

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Abstract

Food security programs in developing countries often provide transfers in the form of basic staple food goods (to encourage take-up by poor beneficiaries) and incorporate public choice over goods (to make procurement more efficient and transparent). In programs with both features, a model demonstrates that heterogeneity of the staple good makes program take-up a function of both household wealth and the quality of the food chosen. Evidence from a randomized trial of a program in rural India which incorporates public choice to provide in-kind, subsidized loans of food grains shows that villages choose a range of qualities of the staple good, and the quality chosen by the program determines the selection of beneficiaries by wealth. Average treatment effects on consumption which ignore heterogeneity of wealth and quality are close to zero, but estimates that incorporate heterogeneity of wealth show that impacts are positive for the wealthiest households and are negative for moderately poor households, consistent with the model.

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1 Introduction

When designing distributive social programs, policymakers often attempt to balance two goals: 1. improving the welfare of people believed to have the highest return to the program while 2. minimizing total cost. One way that policies have attempted to address the first goal is by offering goods that only households with a high return would want, for example food that only poor households buy or a drug that targets only infected households. Making transfers in the form of physical goods can lead to inefficiencies, however, because of corruption and wastage related to transportation, storage, and distribution, and because program implementers do not know the preferences of potential beneficiaries. To avoid these inefficiencies, programs may want to have communities choose the good themselves and procure the chosen good locally. This paper examines whether using public choice this way is consistent with self-targeting. I study two questions regarding a program that gives villages the choice of which variety of food grain to transfer to beneficiaries. First, will take-up be concentrated among the poor if villages jointly choose the good? Second, how does the quality of grain chosen determine heterogeneity of measured impacts on quantity of grain consumed by intended beneficiaries of varying wealth?

In general, tension often arises between the goal of improving welfare for the poor at the lowest cost, because while policymakers may deem poor households most in need of the program, wealthy households also would like to participate, and participation by relatively wealthy households increases the cost of the program. If program administrators can easily observe poverty or a proxy for poverty, they can allow or disallow participation based on that characteristic. But given the cost and uncertainty in measuring poverty, setting fixed eligibility rules often causes both exclusion errors (poor households are left out) and inclusion errors (wealthy households participate). (Alatas et al, 2013)

Given the challenge of determining exclusion directly on wealth, researchers and poli-

cymakers have attempted to design program features such that households with a particular characteristic favored by policymakers reveal themselves through their choice to participate or drop out. One unique case arises when the program offers a good for whom only the targeted population has a return, for example when a drug affects an easily diagnosable disease and has no benefit for non-infected members. (Cohen, Dupas, Schaner, 2012) In the drug case, patients can be diagnosed, and excluding non-infected members is relatively straightforward, but most goods do not have this unique property. One class of mechanisms recently receiving attention are ordeal mechanisms, which hypothesize that making people complete a costly task can lead to better targeting, because only those with the highest marginal return to participation will choose to pay the cost. (Atalas et al, 2013; Dupas et al, 2013) Implementing ordeal mechanisms has proven to be complicated however, because wealthy and poor members may have differential abilities to circumvent the ordeals, and because non-linear utility functions mean that fixed ordeals may impose bigger burdens on the people who have the highest returns to the program, undermining the potential for targeting. (Atalas et al, 2013)

An alternative example of a self-targeting mechanism was described by Besley and Coate (1991) and demonstrated empirically by Jacoby (1997). These models proposed that programs could focus on providing certain goods that would be taken up only by low income households. If policymakers subsidize only low-quality goods, poor beneficiaries for whom the marginal utility of consumption is high will choose to participate, but relatively wealthy households will willingly opt to consume higher quality goods, even though the subsidy effectively raises the marginal cost of quality. Jacoby demonstrated the concept using a subsidy of a low-cost, bland staple food in Jamaica, finding that take-up of a school feeding program providing the bland food was higher among poorer households and those with more eligible children than among comparatively wealthy households.

Directly observing poverty through expenditure surveys imposes a cost on programs who

must fund expenditure surveys or proxy means tests, and ordeal mechanisms impose costs on beneficiaries that are by definition a burden to the potential beneficiaries. In contrast, in-kind transfers may have features that make them more desirable to both households and policymakers than equivalently valued cash transfers. First, in-kind transfers may be politically appealing if they are more easily portrayed and understood as using public funds only to help disadvantaged households buy the things they really need. (Barrett, 2002) Second, in-kind transfers can have differential consumption impacts than equivalently sized transfers of cash in ways that are useful for policy design. Jacoby (2003) demonstrates that in-kind transfers targeted to specific members have fly-paper effects which influence the consumption of targeted members more than the consumption of co-resident, but non-targeted members. Finally, policymakers might have paternalistic preferences for increasing consumption of food in particular. Currie's (2006) review of the literature cites studies claiming that various food transfers or subsidies may increase food consumption more than equivalently valued cash transfers would.

In-kind transfers can also have impacts which work against policy goals, and the expected effects on consumption of food subsidies can be theoretically ambiguous. Subsidy programs can even cause a reduction in estimated food consumption for certain sub-populations, either because of general equilibrium effects (Cunha, De Giorgi, Jayachandran, 2011) or because of a Giffin effect, wherein income effects of cheaper staple foods outweigh the cross-price effect of the subsidy, causing people to substitute to higher quality foods, which could ultimately reduce caloric intake for some households. (Jensen and Miller, 2011)

Unfortunately, implementing programs to directly transfer staple goods presents challenges for program design. Procurement and distribution of physical goods, especially food goods, is logistically difficult to implement by a centralized agency. Avoiding these logistical costs is especially important to policymakers in India, where the national governments primary food subsidy program, the Public Distribution System (PDS) has been criticized

for leakage and corruption in the procurement and distribution process (Jha and Srinivasan, 2001). Local procurement obviates the need for these systems, since goods are purchased and stored in close proximity to beneficiaries, eliminating the need for transportation and storage. Comparisons of internationally transported food aid with locally or regionally sourced grains have found that buying grain locally is generally cheaper because of savings on transportation and storage, (Lentz, Passarelli, Barrett, 2013) and more preferred by beneficiaries because of differences in local varieties and cultivation methods. (Violette et al, 2013)

If policymakers choose to employ local procurement when implementing an in-kind transfer, additional challenges arise. First, in many contexts, even basic staple goods are heterogeneous, and their price and quality can vary widely. If a centralized policy maker chooses a good that is too costly, poor members may not be able to participate, but if the policy maker chooses a good that is too inferior, even relatively poor members targeted by the program may opt out. Second, program implementers may not have good information about sources of food and prices, causing the program to overspend on grain. An attractive candidate to address both of these challenges is to incorporate a public choice mechanism into the policy design. Public choice can potentially incorporate information about the preferences of beneficiaries that is not easily available to a central planner, increasing satisfaction and take-up. Both theory and experience suggest that communities can leverage information to identify poor households and adapt programs to their preferences, but that doing so makes the program susceptible to elite capture. (See review in Mansuri and Rao, 2012.) In addition, the exercise of public choice has been shown to independently increase program participation and satisfaction. (Olken, 2010)

This paper asks whether the strategy of local procurement and public choice is compatible with the goal of self-targeting through the provision of a good generally preferred by the poorest members of the communities in a context where basic staple goods have some heterogeneity. A simple model shows the mechanisms through which price and quality of

varieties of the staple good can generate separation of potential beneficiaries by wealth, and predicts the relative likelihood of participating over the wealth/income distribution. Households have preferences for quality of the staple grains they consume, and quality is a normal good, meaning that optimal quality is increasing in household wealth/income. The model predicts that the surplus utility of participation in a program is maximized when the program subsidizes a particular quality of food corresponding to household wealth, and that surplus utility is decreasing as quality is increased or decreased from this point. The implication of this model is that households who choose to participate in the program in a given village will come from particular sections of the wealth distribution, i.e. when the village subsidizes high quality grain, only wealthy people participate, and when the village subsidizes low quality grain, only poor people participate. Whether the program is targeted toward the poor then becomes an empirical question of what kind of grain the villages decide to provide. The model further shows that if some villages choose relatively high quality grain, the program may actually lead to decreases in the quantity of grain consumed relative to control villages, because some moderately poor households may decide to use the program to upgrade the quality of the grain they buy, even though the high quality program rice is still more expensive than the low quality grain they would have purchased otherwise.

Using data from a randomized evaluation of a food security program that provides in-kind subsidized loans, I show that communities participating in the program do not always choose the cheapest available staple good. The data suggests that the particular representative choice model for public choice employed by the program leads to village committees being more slanted to choosing higher price/quality goods and to lower participation by less wealthy members. Next, I show evidence that the variation in the price of the good chosen by each participating village predicts whether wealthier or poorer potential beneficiaries are more likely to participate. Finally, the paper turns to heterogeneous consumption impacts. The model predicts that the effects on quantity of rice consumed by the household are only

unambiguously positive for the wealthiest households, since the program for them is subsidizing grain that is already at least as inexpensive as they would have purchased from the market. Moderately poor households who choose to purchase grain which is more expensive than what they would have chosen to buy from the market may actually decrease their consumption. The balance of treatment effects over the distribution of wealth in this particular case leads the program to show no impact on consumption across the distribution of wealth, even though the program: 1. Is at least weakly welfare improving for all households and 2. Does result in consumption increases for some groups, even if not the ones that the program designers might wish to target for consumption increases.

The next section describes the program and the particular context for the model. Section 3 provides data on the characteristics of rice in Bihar to set up the intuition of the model. Section 4 presents the model and demonstrates the results related to targeting. The implications of the model for VO choice and targeting are taken to the data in Section 5 and Section 6 respectively. Section 7 demonstrates the application of the model for the analysis of consumption effects from the program. The consumption predictions are tested in Section 8 using the cluster randomization of the program. Section 9 summarizes the results, draws conclusions, and raises further questions and extensions.

2 The Program and Data

The data used was collected as part of an evaluation of a program run by an organization called the Bihar Rural Livelihoods Project, a joint project of the World Bank and the state government of Bihar in India. The BRLP's primary activity is to organize groups of women called Self-Help Groups (SHGs) which conduct microfinance activities and participate in programs operated by the BRLP which aim to increase the livelihood potential of the women and their families. The SHGs are further grouped together into Village Organizations (VOs)

which participate in programs designed to involve multiple SHGs. Each VO consists of 100-150 women all of whom live within the same village.

The program that was evaluated during the collection of the data used here is a food security intervention called the Food Security Fund (FSF). The FSF is designed to help VO members who are potentially credit constrained access large quantities of food grains in a single purchase without exposing them to high interest rates. The BRLP grants 100,000 rupees (about \$2,000) to the VO, and assigns a staff member to use the funds to organize an in-kind lending program. First, members of the VO are interviewed about their food practices, and encouraged to think about how much food they may need to purchase or borrow over the following three months, based on the amount of grain they expect their family to produce and consume over that period. If members expect to require purchasing or borrowing grain, they are asked if they would like to take a loan from the FSF fund. Once all members have expressed a desired amount to borrow, the BRLP staff adds up the aggregate loan demand. Members of the VO leadership take this amount to nearby grain vendors and solicit bids from each vendor. Samples of vendors grains are taken, along with prices, and the VO leadership votes on which type of grain to purchase. Once the grain is purchased, it is distributed to members, who are given three months to repay their loan with no interest rate charged.

Although this program does not directly subsidize the price of the grain, the project believes that it benefits participants in three ways: 1. Many members rely on loans to purchase food in absence of the program, meaning that the program is a direct subsidy on interest costs for these members, 2. Encouraging members to purchase three months worth of food at one time rather than smaller quantities at regular intervals allows individual members to leverage potential bulk discounts, and 3. Pooling the demands of the whole VO together when bargaining with shop keepers and wholesalers to solicit competitive bids could help them leverage further bulk purchasing discounts. Because of issues with comparing quality,

the exact size of the bulk discount is not measured in this paper. But program participation is voluntary, and high participation rates suggest that there is some benefit to the program relative to purchasing independently from the market. This benefit will be thought of for the remainder of the paper as a price effect.

The data collected for the project was collected as part of a cluster-randomized control trial of this in-kind lending program. Six blocks¹ were selected for the study, chosen because in these locations the project had formed enough VOs to be included in the study but had not yet started the FSF program. Each block is divided by the project into three units which the project calls clusters. Random assignment to either receive the FSF treatment or not was done at the cluster level, so that within a block all of the VOs within one or two clusters were assigned to start the program, while the remaining cluster or clusters were assigned to wait until completion of the study.

After randomization, baseline data was collected from five VOs in each of the eighteen evaluation clusters (90 VOs: 45 treatment, 45 control) in July 2012 prior to initiation of the lending program in any VOs. A VO is comprised of 10-15 smaller units called Self Help Groups (SHGs), which each have 10-15 members, including three leaders (designated as President, Secretary, or Treasurer). To generate the sample of households, three SHGs were randomly chosen in each VO and seventeen members were selected from these SHGs. In each of the selected SHGs, two out of three leaders were selected, as well as either three or four non-leaders. If a respondent could not be located, a substitute was randomly selected from among the remaining members. Upon completion of the baseline, the lending program began in September 2012. A post-intervention survey was collected in February-March, 2013. The surveys covered detailed monthly food expenditure and weekly consumption, coarser non-food expenditure, experiences of food insecurity, agricultural production, income, and child health measures. Data was collected from a total of 1,529 SHG members about

¹A block is an Indian administrative unit similar to a county.

their households in the baseline. In the post-intervention survey, the same members were resurveyed with 1,449 being relocated and agreeing to participate again. Attrition was evenly balanced across treatment and control ($p=.77$). A second post-intervention survey was conducted in September to October 2013, and the data will be used when available to assess further rounds and seasonality issues.

3 Rice as a Staple Good in Bihar

The literature on using in-kind provision of services as a second-best method of targeting (Besley and Coate, 1991; Jacoby, 1997) suggests that choosing an inferior staple good can ensure that program participation is concentrated among disadvantaged members. Jacoby (1997) showed that in Jamaica where a bland, undifferentiated food staple was provided through school lunches, the children of poor families participated, while wealthier children were more likely to opt out. This line of reasoning has encouraged programs to focus on providing staple grains, but neglects potential heterogeneity of quality even among staple goods.

Table 1 shows some summary statistics for the households of the members surveyed. The general pattern is of disadvantage, with the average household earning little more than 1000 rupees per month per capita in the month before the baseline (about \$20), and the average z-score for length-for-age and weight-for-age is near or below the WHO's cutoffs for stunting and underweight. The incidence of poverty and related health indicators reflect both the circumstances of rural Bihar, as well as the project's intentional focus on recruiting disadvantaged households as beneficiaries. Since later results comparing consumption impacts will rely on comparing treatment and control villages, the means here are reported separately for these two groups, with a t-statistic for the significance of the difference. The t-statistics reported in this table are calculated from a regression of each variable on a treat-

ment dummy, clustering at the unit of randomization, and including block fixed effects to account for the stratification procedure used during randomization. The last column shows p-values calculated using Cameron, Gelbach, and Miller's wild cluster bootstrap procedure to account for the small number of clusters. Only two out of the 20 variables are significant at the 5% level, consistent with successful randomization.

In Bihar, the staple foods are mostly rice, wheat, or maize. These three grains account for 33% of food expenditures in the households surveyed, and 30% of households cultivate at least one of these grains. Out of the three available staple grains, rice accounts for the largest share of expenditure and consumption. Including home produced goods valued at the median market rate in the village, rice alone accounts for 25% of expenditures on food. In addition to its importance in the diet of the population of rural Bihar, rice is an inferior good. Table 2 shows the relationship between the share of food expenditure including consumption from home production and the log of monthly household expenditure. The correlation is negative, indicating that wealthier households spend a lower percentage of their total expenditure specifically on rice.

Given the importance of rice in meeting the food needs of households and its relative importance for poor households in particular, rice seems to meet the criteria to be the focus of a program which intends to increase availability of food and to most benefit the poorest potential beneficiaries.

Other characteristics of rice, however, make it a less ideal candidate to choose for a program with these goals. The main issue is that rice is not a homogenous good. Even within a market or a single shop, there is often a diversity of varieties and qualities of rice available. Rice varies according to color, taste, texture, presence of impurities, whether grains are broken or whole, and other dimensions. It is reasonable to expect that some of this variation in quality and variety is associated with prices paid for rice. Figure 1 shows the distribution of prices paid by households at the baseline for 1 kg of rice at the market in

the baseline. The prices range from around 10 rupees per kg to over 20 rupees, suggesting that some members pay nearly twice as much for rice as others. Some of this variation is undoubtedly explained by geography, bulk discounting, and negotiating ability, but some is also likely explained by preferences for quality.

If there is a quality component underlying the price variation of market purchases and quality is positively associated with price, we would expect that wealth will be positively associated with quality choice, and consequently with the price paid. Table 3 shows the linear relationship between the unit price paid at the market for rice in the month prior to the baseline survey with wealth measured in various ways. Three out of the four measures of wealth show a positive relationship between the price paid at the market and wealth. The two which are the best measured in the survey, household expenditures and expenditure on food (excluding rice) show the most significant relationship.

Another way to view the relationship between the price of rice and wealth is shown in Figure 2. The figure shows the kernel density plots of prices paid for rice in the month prior to the baseline survey for the lowest quartile of wealth and the highest quartile. The plots reveal that the sorting based on wealth is not perfect, but it does seem that those paying lower prices are more likely to be from the poorest quartile, and those paying higher prices seem more likely to be from the highest.

4 A Simple Model of Take-up Under Heterogeneity of the Staple Good

The descriptive evidence in the previous section suggests that in Bihar, rice is an important staple good, and is particularly important as a component of the consumption basket for the poorest members. These characteristics suggest that in this context, a program which attempts to target participation and benefits toward the poorest members should consider

providing rice. However, the evidence also suggests that rice is not a homogeneous good. Although rice when defined broadly seems to be an inferior good, meaning that the relative share of rice consumed out of the total consumption basket is decreasing, the quality component of rice is likely a normal good, meaning that household demand for quality is increasing in household wealth. To explain why heterogeneity of the staple good may be problematic for targeting, the model below extends a model of take-up similar to those of Jacoby (1997) and Besley and Coate (1991). The model will demonstrate that a tendency for wealthier households to prefer purchasing higher quality foods causes programs which provide lower quality, less costly food items to attract beneficiaries which are relatively poor, while programs which offer higher quality goods attract wealthier participants. To determine which types of participants will choose to participate, we will compare the utility that a household of a certain income would receive from participating in the program with the utility they would receive from choosing to forgo the program and purchase their food from the market. The household consumes two goods, a staple food good c and a composite good n . The staple food has a quality component q . Both of the goods and the quality component can be chosen continuously, but q is bounded above and below, so that there is a minimum quality available and a maximum. The utility function is:

$$U = \alpha \ln(c - a) + \beta \ln(q) + \delta \ln(n)$$

where α , β , and δ are all in the range (0,1). This utility function captures a few key intuitions about the choice of consumption and quality. First, the MRS between quality and the non-staple consumption good n does not depend on the amount of staple good consumed c (weak separability). In other words, the appeal of tastiness, appearance, ease of cooking, etc for the staple good relative to consuming more non-staple goods does not depend on the amount of staple good consumed. Second, food consumption is subject to a subsistence constraint,

which captures the intuition that as food consumption falls to levels needed for survival, the marginal utility of consuming food becomes very high. The budget constraint is simply:

$$y = pqc + nz$$

where y is income (or wealth in this single period model) z is the unit price of n . The key feature of this model is that the price of c is assumed to increasing in q , and q is endogenously chosen within the model along with c and n , with the choice of q directly determining the price paid. To keep the model simple, we assume that the unit price of c is a simple linear function of q :

$$price = pq$$

This function captures the idea that higher quality goods cost more per unit to buy. q is restricted to be in the range $q_{min} \leq q \leq q_{max}$.

Guaranteeing that optimally chosen consumption is positive requires one assumption on parameters, namely that $\beta > \alpha$, which we will assume for the remainder of the paper. In this simple setup, n and q are both increasing in y , but c is a constant in preferences.² Households consume a fixed quantity of the staple good but as their wealth increases, they both upgrade the quality of the staple good and consume more of the other good, which may include other non-staple foods.

Introducing a subsidy or transfer into the above model reveals the mechanism through which targeting can result from the choice of good subsidized. If the program offers a cash grant, the program would simply expand y in the budget constraint. Households would respond by increasing both q , and n , and would experience an increase in utility. A program

²Details of the optimal choices are in Appendix 1. Assuming that prices could be a more flexible function of quality could generate a solution where the optimal c is also increasing in wealth, but this is not necessary for the intuition of the results, but would admit the possibility of the giffen effects observed by Jensen and Miller. (2011)

which offers a voucher not restricted to one particular type of food, could be modeled as a decrease in p for at least up to a maximum quantity for all possible qualities. In the voucher case in this simple model, households will increase q , the quality of rice they desire.³ Again, the vouchers would be a welfare improvement for households of all wealth levels. Equivalently valued vouchers will create a higher welfare surplus to poor households who have a higher marginal utility of quality or consumption, but this alone would not lead to differential participation on the wealth dimension.⁴ In either case, all households would be willing to take up the program, so if the policy maker wants to target the program to lower income households, exogenous rules would have to be put in place to restrict higher income households from participating or to encourage lower income households to take-up the benefits.

In contrast to flexible voucher or subsidy programs, a program which subsidizes the cost of only one particular type of food will have different welfare implications for different types of households. If the program only subsidizes one good, then for participants in the program, q is exogenously chosen and fixed at the level $q = q_p$.⁵ For this quantity, households receive a discount, so that the price of a unit of c for the quality q_p is $\phi p q_p$ where $\phi < 1$ and the difference between ϕ and 1 represents the per unit value of the subsidy. Other than this price discount for the exogenously chosen q , the budget constraint and utility function for a program participant is the same as before, and households optimally choose n and c .⁶

To ensure that the optimal choices are non-negative, we will assume throughout that

³In a more sophisticated model where the price is a more general function of quality, households could also choose to change their consumption of c or n in response to a voucher program.

⁴It is possible that vouchers could lead to separation in take-up between rich and poor if vouchers are tied to specific shops that are more difficult to access than the ones households otherwise normally use. This variety of ordeal mechanism is explored by Dupas, Hoffman, Kremer, and Zwane (2013). But if vouchers are flexible enough to be used in the same places where households generally buy grain, they should be appealing to rich and poor alike.

⁵ Q may be endogenously chosen at the village level in that the village choice is a function of the wealth of the residents of that village. For now, we assume that individual households cannot influence the village choice, so that for a single household, the quality choice is exogenous.

⁶The exact expressions for c and n are shown in Appendix 1.

$y > a\phi pq_{max}$ for all households, which means that every household is able to afford buy a quantity of the staple good at least as high as the subsistence constraint a at the prices the program would offer for the highest quality grain q_{max} . In contrast to the market case, where c was chosen endogenously, if a household participates in the program, optimal levels of both c and n will be increasing in income, y . To determine whether a given household would choose to participate or not, we can define two indirect utilities. Define $u(c_p^*, n_p^*, q_p)$ as the indirect utility a household receives from taking the quality of their staple food as exogenously given by the program and $u(c_m^*, n_m^*, q_m^*)$ as the indirect utility they would receive from purchasing all their goods from the market without the subsidy and endogenously choosing q . Assuming that the optimal q is in the range (q_{min}, q_{max}) ,⁷ the surplus utility from participating in the program relative to not participating can then be given by the following quantity:

$$W = u(c_p^*, n_p^*, q_p) - u(c_m^*, n_m^*, q_m^*) = \alpha \ln \left(\frac{y - a\phi q_p}{\phi q_p} \right) + \beta \ln \left(\frac{q_p}{y} \right) + \delta \ln \left(\frac{y - a\phi q_p}{y} \right) + K$$

where K is a function of preference parameters and is constant in q_p and y .

Having defined the surplus utility from participating in the program, we can use this quantity to show that wealthier households will opt to participate when the program chooses a higher quality grain to provide. The relationship between household wealth and program can be stated in the following proposition:

Proposition 1: For a fixed quality chosen by the program q_p , there exist $y_L(q_p)$ and $y_H(q_p)$ with $y_L(q_p) < y_H(q_p)$ such that $W(q_p, y) > 0$ for all y in (y_L, y_H) , and $W < 0$ for all other y .

Before demonstrating Proposition 1 for the general case, Figure 3 depicts the proposition graphically by showing $W(q_p, y)$ plotted over y , choosing representative values for the

⁷The optimal q may be a corner solution as shown in Appendix 1, however the function assuming the interior solution is used in most of the proofs that follow. The specific cases where corner solutions for quality apply are handled separately in the proofs where these cases apply.

preference and price parameters keeping q_p fixed at various levels. The solid line shows the surplus utility from participating in the program relative to the market, $W(q_p, y)$, assuming that the program chooses the minimum available quality. For such a program, $W(q_{min}, y)$, the surplus value of participating in the program rather than the market, is highest for low-income households. Intuitively, this is because low-income households have the highest marginal utility of consumption (c), and they already optimally choose to purchase the lowest quality good available in the market without the subsidy. When the program is introduced, they opt into the program and benefit from moving away from the subsistence constraint. As consumers move up the income distribution, a gap opens up between q_m^* , the quality households would choose to purchase from the market, and q_{min} , the quality they would have to purchase through the program. For moderately low-income households, the gain in utility from the extra consumption they can purchase at the lower prices makes it worthwhile for them to participate, and they are willing to switch from slightly above-minimum quality food to minimum quality food in order to participate. But as income increases, the marginal utility of consumption falls, while the utility gap between and widens. For higher-income households, the extra consumption they get by switching to q_{min} and participating in the program does not compensate them from having to consume a substantially lower quality good. When the program offers the minimum quality good, households with income in the range $[0, B]$ shown in the graph are willing to participate, and households with income greater than B opt out.

Comparing households with wealth lower than B in the graph with households with wealth above B demonstrates the result described by Jacoby (1997). By choosing the lowest quality good, low-income households self-select into the program, and high-income households self-select out without the policymaker otherwise needing to fix rules for who is allowed to participate.

The above shows how focusing a program on inferior goods can self-target the program

toward relatively poor households. But given the heterogeneity in cost and quality of rice shown above, a program which allows choice over the good provided by the program is not guaranteed to produce this result. The dashed line in Figure 3 again shows $W(q_p, y)$, the surplus utility from participating in the program and buying the rice supplied by the program rather than their optimally chosen variety of market priced rice, over the income axis, this time under the supposition that the program supplies the highest quality rice available in the market, ie $W(q_{max}, y)$. The graph shown assumes that the subsidy is not so large as to make the highest quality rice under the program prices cheaper than the lowest quality rice. Choosing the highest quality rice available reverses the targeting result shown above. Now, households with wealth below the point marked C in the graph would choose not to participate, since the utility they would get from consuming lower quality rice at a lower price would be more than the utility they would get from upgrading to the higher quality rice from the program. For wealth levels above C , households experience a utility premium from buying the discounted program rice rather than their optimal basket at market prices, and choose to participate.

Comparing the solid line and the dashed line demonstrates that at least in the simplified case where the program can only buy either the highest or the lowest quality/price rice available, the choice of rice totally determines whether the program is targeted toward the poor or captured by relatively wealthy households. Since there is no reason to assume that markets will only make two kinds of rice available, the model assumes that grain varieties and prices are available from the market on a continuum between the highest and lowest available quality of rice, and it is reasonable that the program could choose one of these intermediate varieties.

The dotted line in Figure 3 shows the surplus utility from program participation for the choice of an intermediate good. Not surprisingly, the take-up implications of this choice are halfway between what we see from the choice of the lowest or the highest quality good. For

values of wealth below the point A , households choose not to participate. Above the level of wealth D , wealthy households also self-select out. This curve demonstrates that there may be excluded populations at both the upper and lower ends of the income distribution, but such regions need not exist. If the income distribution in a village is bounded within the range $[A - D]$, the middle quality of grain shown by the dotted line would appeal to both low and high income types.

Proposition 1 says that when q is fixed at some q_p , the function $W(q_p, y)$ defines a range of $y = (y_L, y_H)$ such that $W(q_p, y) > 0$ for all q_p in this range where y_L, y_H are functions of q_p . In Figure 3, $(y_L(q_{min}), y_H(q_{min})) = (0, B)$, $(y_L(q_{max}), y_H(q_{max})) = (C, \infty)$, and $(y_L(q_p), y_H(q_p)) = (A, D)$ for another q_p for which $q_{min} < q_p < q_{max}$.

Figure 3 plots $W(q_p, y)$ for representative values of the preference parameters and prices, but Proposition 1 is true for general values of these parameters. If $W(q_p, y)$ were a concave function with respect to y for fixed q_p , demonstrating Proposition 1 for the general case would only require that $W(q_p, y) > 0$ for some y and Proposition 1 would follow directly, but $W(q_p, y)$ is not concave over the entire range of y . So proving Proposition 1 proceeds in four steps. First, we show that for any fixed income $y = \bar{y}$, there exists at least one value of q_p such that $W(q_p, \bar{y}) > 0$. Second, we show that for a fixed value of income $y = \bar{y}$, there exist $q_L(\bar{y})$ and $q_H(\bar{y})$ with $q_L(\bar{y}) < q_H(\bar{y})$ such that $W(q_p, \bar{y}) > 0$ for all q_p in (q_L, q_H) , and $W(q_p, \bar{y}) < 0$ for all other q_p by showing that $W(q_p, \bar{y})$ is a concave function with respect to q_p . Third, we show $q_L(\bar{y}), q_H(\bar{y})$ are increasing functions of y . Finally, we use the second and third step to show the existence of the y_L, y_H described in Proposition 1.

The first step in proving Proposition 1 is to show the following Lemma:

Lemma 1: For a fixed $y = \bar{y}$, $W(q_p, \bar{y})$ is positive for at least one value of q_p .

Lemma 1 states that no matter which income is chosen, there is at least one quality of grain that the program could choose to offer and the household would be willing to participate in the program. To prove Lemma 1, imagine that the program chooses to subsidize the same

quality of grain that the household would have chosen to consume at the market, so that the household consumes the same quality grain, but gets it a cheaper price if the grain is purchased from the program. In this case:

$$W(q_m^*, \bar{y}) = (\alpha + \delta) \ln\left(\frac{\phi\alpha + \beta(1 - \phi) + \delta}{\alpha + \delta}\right) + \alpha \ln\left(\frac{1}{\phi}\right) > 0$$

since $\phi < 1$ and we have assumed that $\beta > \alpha$, which proves the lemma.

The next step in proving Proposition 1 is to demonstrate the following:

Lemma 2: For a fixed value of income y , there exist $q_L(y)$ and $q_H(y)$ with $q_L(y) < q_H(y)$ such that $W(q_p, y) > 0$ for all q_p in $(q_L(y), q_H(y))$, and $W(q_p, y) < 0$ for all other q_p .

Lemma 2 can be proven by showing that $W(q_p, \bar{y})$ is concave. Concavity of $W(q_p, \bar{y})$ and the existence of a global maximum with respect to q_p and Lemma 1 implies that for any choice of y there exist $q_1(y)$, $q_2(y)$ such that $W(q_1, y) = W(q_2, y) = 0$.⁸ Define $q_H = \max(q_1, q_{min})$ and $q_L = \max(q_2, q_{max})$. The concavity of $W(q_p, \bar{y})$ implies that $W(q_p, \bar{y}) > 0$ for all q_p in (q_L, q_H) and $W(q_p) \leq 0$ for all q_p not in (q_L, q_H) , which proves Lemma 2.

Next, we show that $q_L(y)$, $q_H(y)$ are increasing functions of y , by demonstrating the following:

Lemma 3:

Define y_H, y_L such that $y_H > y_L$.

3.a. The lowest quality offered by the program for which a household of income y_L is willing to participate is weakly lower than the lowest quality that would induce a household of income y_H to participate. The inequality is strict if the lowest quality for which a household of income y_H would be willing to participate is greater than q_{min} .

3.b. Similarly, the highest quality offered by the program for which a household of income y_L would be willing to participate is weakly lower than the highest quality which would induce

⁸A detailed proof of this statement is in Appendix 2.

a household of income y_H to participate, with strict inequality when the highest quality for which a household of income y_L would be willing to participate is less than q_{max} .⁹

Lemma 3 can be proven by focusing on three points on the range $q_p : q_1(y), q_2(y)$ and a point $q_p^* - \epsilon$ just to the left of the q_p where $W(q_p, y)$ attains its maximum and where $W(q_p - \epsilon, y) > 0$. Define y_L, y_H with $y_L < y_H$ and suppose that $q_1(y_L) > q_1(y_H)$. Then it would have to be the case that $\frac{\partial W(q_p, y_H)}{\partial q_p} < \frac{\partial W(q_p, y_L)}{\partial q_p}$, which contradicts the fact that $\frac{\partial W(q_p, y)}{\partial q_p \partial y} > 0$. A more detailed proof is in Appendix 3.

With Lemmas 1-3, we are ready to prove Proposition 1. Suppose $q_p = q_{min}$. By Lemma 2, $W(q_{min}, y) \geq 0$ for any y such that $q_1(y) \leq q_{min}$. By Lemma 3 and the continuity of $W(q_p, y)$ w.r.t. y , there exists y such that $W(q_{min}, y) = 0$. Define this y as $y_H(q_{min})$. For all $y < y_H(q_{min})$, $W(q_{min}, y) > 0$, and for all $y \geq y_H(q_{min})$, $W \leq 0$.

Suppose $q_p = q_{max}$. By Lemma 2, $W(q_{max}, y) \geq 0$ for any y where $q_2(y) \geq q_{max}$. By Lemma 3 and the continuity of $W(q_p, y)$ w.r.t. y , there exists y such that $W(q_{max}, y) = 0$. Define this y as $y_L(q_{max})$. For all $y > y_L(q_{max})$, $W(q_{max}, y) > 0$, and for all $y \leq y_L(q_{max})$, $W \leq 0$.

For q_p in (q_{min}, q_{max}) , fix $q_p = \bar{q}$. Following from the proof of Lemma 2, we can define functions $q_1(y)$ and $q_2(y)$ for which we must have $W(q_1(y), y) = W(q_2(y), y) = 0$. By Lemma 3, both functions $q_1(y)$ and $q_2(y)$ are continuous and monotonically increasing in y , which implies that the inverse of each function exists. Define $y_H(q_p)$ as the inverse of $q_1(y)$ and $y_L(q_p)$ as the inverse of $q_2(y)$. By Lemma 3, $y_L(q) < y_H(q)$. $W(y_L(\bar{q}), \bar{q}) = W(y_H(\bar{q}), \bar{q}) = 0$, which proves that if q_p is in (q_{min}, q_{max}) , there are at least two roots to the function $W(y, \bar{q})$.

Now suppose there are three points $y_1 < y_2 < y_3$ such that $W(\bar{q}, y_1) = W(\bar{q}, y_2) = W(\bar{q}, y_3) = 0$. By Lemma 2, we have that for fixed y , we can define $q_L(y)$ and $q_H(y)$ such that

⁹Lemma 3 can also be expressed in terms of the notation already introduced. For any y_H, y_L such that $y_H > y_L$:

- 3.a. $q_L(y_L) \leq q_L(y_H)$ and $q_L(y_L) = q_L(y_H)$ iff $q_L(y_L) = q_L(y_H) = q_{min}$
- 3.b. $q_H(y_L) \leq q_H(y_H)$ and $q_H(y_L) = q_H(y_H)$ iff $q_H(y_L) = q_H(y_H) = q_{max}$.

$W(q_L(y), y) = W(q_H(y), y) = 0$, and by Lemma 3, it must be the case that $q_L(y_1) < q_L(y_2) < q_L(y_3)$ and $q_H(y_1) < q_H(y_2) < q_H(y_3)$. Suppose that $\bar{q} = q_H(y_1)$. Then $q_H(y_2) > \bar{q}$ and $q_H(y_3) > \bar{q}$. Additionally, suppose that $\bar{q} = q_L(y_2)$. Then $q_L(y_3) > \bar{q}$. But by Lemma 2, $q_L(y_3)$ and $q_H(y_3)$ are the only values of q_p for which $W(q_p, y_3) = 0$, which contradicts that $W(\bar{q}, y_3) = 0$. Therefore, there cannot be more than two points in y such that $W(\bar{q}, y) = 0$.

All that remains to be shown is that for y in (y_L, y_H) , $W(\bar{q}, y) > 0$. The above argument shows that there cannot be more than two roots of the function $W(\bar{q}, y)$, which implies that either $W(\bar{q}, y) < 0$ or $W(\bar{q}, y) > 0$ in the region (y_L, y_H) . Therefore, if we can find one point y in the region where $W(\bar{q}, y) > 0$, the function must be positive over the entire region. In order to have two roots, the function must have a stationary point between the two roots. Solving $\frac{\partial W(q_p, y)}{\partial y}$ for 0 gives:

$$y^* = \frac{a\phi q_p(\beta + \delta)}{\beta - \alpha}$$

Since there is only one point where the derivative of the function is 0, this must be the stationary point, and therefore y^* must be in the region bounded by the two roots (y_L, y_H) . Substituting y^* into $W(q_p, y)$ gives:

$$W = \beta \ln\left(\frac{1}{\phi}\right)$$

Which is > 0 since $\phi < 1$.

We have shown: 1. That for a fixed $q_p = \bar{q}$ there exist $y_L(\bar{q}), y_H(\bar{q})$ with $y_L(\bar{q}) < y_H(\bar{q})$ such that $W(\bar{q}, y_L(\bar{q})) \geq 0$ and $W(\bar{q}, y_H(\bar{q})) \geq 0$ and 2. for all y in (y_L, y_H) , $W(\bar{q}, y) > 0$, which proves the Proposition.

Having demonstrated that fixing the program quality q_p , defines a range of income such that people within the range are willing to participate, we now to turn to the relationship between this income region and the quality of the good provided. In Figure 3, we saw the surplus utility of program participation plotted for three different qualities selected by the

program, q_{min} , q_{max} , and a quality q_p such that $q_{min} < q_p < q_{max}$. In the figure, the range of income such that people would be willing to participate in a program that offers grain of quality q_p is $[0, B]$. When the program offers a higher quality of grain q_p , the lower bound of the income range of participants goes up to A and the upper bound goes up to D , suggesting that the range of income of participants is shifted to the right. As the program increases the quality offered even further to q_{max} , the lower bound of participants income shifts up to C , and the upper bound extends to ∞ . The plot suggests that the as the quality of grain offered by the program increases, the range of income of participants will shift to the left. This feature is generally true for the assumed parameters of the utility function and can be stated more formally in the proposition below:

Proposition 2: $\frac{\partial(y_L)}{\partial(q_p)} > 0$ and $\frac{\partial(y_H)}{\partial(q_p)} > 0$

The proof for Proposition 2 proceeds exactly as in the proof of Lemma 3.

Propositions 1 and 2 allow us make predictions about the choice of q_p and the targeting implications for the program. Typically, improving targeting refers to one or both of two goals: 1. Increasing the percentage of people below some cutoff who participate (desired inclusion) and 2. Decreasing the percentage of people above that cutoff who participate (desired exclusion). Focusing on this definition of targeting can obscure the relationship between quality choice and takeup however, because for a particular choice of program quality q_p , the income distribution between the income distribution between $y_L(q_p)$ and $y_H(q_p)$ determines the percentage of people above and below any particular cutoff who participate.

To see how the relationship between q_p and the traditional measures of targeting, suppose that the program sets an income cutoff to assess targeting at the point marked C in Figure 3, so that the program intends to include households with income below C and exclude households with income above C . Then either a program that provides the lowest quality grain or one that offers the intermediate quality shown on the graph would be strictly better targeted than the one that provides the highest quality grain, because only the high-

est quality program out of the three would attract participants with income above C , and only the two lower quality programs attract any participants with income below C . But the comparison between the low quality and the intermediate one is ambiguous, because only the low quality program would attract those with income below A , while only the intermediate quality program would attract those with income between B and C . Which of the two programs attracts a higher percentage of those below the cutoff C will depend entirely on the number of households with income below A relative to the number of households whose income is between B and C .

More formally, the percentage of households below an income cutoff y^* can be written as:

$$I = \int_0^{y^*} \pi(y)f(y)dy$$

where $\pi(y)$ is the probability that a household with wealth y will choose to participate in the program. The distribution of income $f(y)$ is a feature of the data sampled or the population of interest, and y^* can be chosen to either by the policy maker or the econometrician. Assessing the simple comparative static $\frac{\partial I}{\partial q_p}$ depends on the shape of $f(y)$, the y^* chosen, and the function $\pi(y)$. The fact that the income distribution interacts with the quality choice when determining the percentage of means that the standard way of measuring targeting by choosing a cutoff and defining targeting as increasing the percentage of members below that cutoff participating a bad way of empirically assessing whether quality choice in the program influences take-up, because any relationship between quality choice and percentage of members participating is going to depend crucially on both the specific cutoff chosen and the distribution of income in the sample. To avoid confusion about whether results on targeting are driven by $f(y)$ or y^* , the remainder of the paper will focus on the quantity $\pi(y)$. This quantity can be thought of as expressing the proportion of people with income y who participate, or alternatively, what would be the representation of households with income if

income were uniformly distributed.

5 VO Choice of Quality as a Function of VO Characteristics

In the BRLP program, participation in VOs has a hierarchical structure. Each VO is composed of members of 10-15 self help groups (SHGs), who all participate together in VO level programs like the food security loan. To choose the quality and price of grains which will be provided through the program, members of the VO leadership collect bids from local merchants to supply the estimated amount of grain needed for the program. At a meeting of VO leadership, samples of rice are taken which represent each of the bids and are assessed for quality and appearance. In some cases, the rice samples are cooked to allow members to taste the food before choosing which bid to accept. Although any member of an SHG would be allowed to participate, the VO committees are formed from leaders appointed by each SHG to represent them in VO decisions, and it is likely the case that that these members have more influence on the choice of program characteristics, both because they are expected to attend the VO committee meetings, and through their influence as official members. Such a representative model is a common way in which programs in developing countries attempt to aggregate preferences and use local information in deciding policies and program allocations. (Olken, 2010)

In this type of representative choice allocation, the preferences of the chosen officials are more likely to be represented in the choice of goods, including the choice over the quality of grain selected in this context. In the model, wealth is correlated with the price paid for rice is through a quality preference channel, and empirical results in Section 3 showed at least a weak relationship between household expenditure and the price of paid for rice at market rates prior to the baseline survey. Together, these results suggest that VO representation

may underlie part of the choice of quality and explain some of the variation in the prices offered by the program.

Table 4 shows the relationship between VO committee status and household expenditure at baseline. An extra log point of household expenditure is associated with a 7% increase in the likelihood of being a VO committee member, suggesting that wealthier members are disproportionately represented on the VO committees making the choice about which quality of rice to provide through the program. Columns 2 and 3 show that VO committee members are also more likely to participate than non-committee members. If VO committees are often made up of relatively wealthy members of the SHGs which elect the committees, we might expect these members preferences to influence the program prices. Table 5 shows the relationship between the median VO committee members' wealth and the price of rice selected by the VO. The coefficients on household expenditure for the VO committee members show that the VO having a committee with higher baseline expenditure predicts that the VO will choose to supply rice of higher cost to its beneficiaries through the program, though with only 82 round-VO observations, this correlation is not significant.

6 Empirical Results on Targeting and Take-up

The model described above outlines the mechanism through which quality and price of the good provided interact with wealth to determine which segments of the income distribution choose to participate. When communities choose their own grains, it is not clear which type of grain will be chosen, and which types of members will choose to participate as a result. Figure 4 shows the distribution of prices paid for grain by VO members purchasing through the community lending program against the distribution of prices paid in by households from the market in the control villages. Comparing the prices paid by the VOs with prices paid by the households confounds the price differentials associated with quality with

any discounts members receive by buying through the food security program rather than the market. This confounding means that the apparent leftward shift of the VO distribution relative to the household/market distribution cannot be necessarily attributed only to either a bulk discount or to VOs systematically choosing lower qualities.¹⁰ The main feature of the comparison is that there is significant variation in the prices that VOs pay. The VOs offering the most expensive grains offer the rice at a price about 50% higher than the VOs offering the grains at the lowest price. There seem to be VOs who offer grain at prices equivalent to those experienced by members purchasing the most expensive grains and other VOs offering rice at prices equal to the cheapest prices available in the market. Some of the variation may be spatial or temporal, so that all the VOs are in fact buying the same rice, which happens to be available at different prices in different times and places, but given the correlation between wealth and market price seen earlier and the correspondence between market and VO prices, it seems reasonable to speculate that some of the difference in prices offered by VOs relates to the quality of grain chosen. It therefore seems unlikely that public choice of the staple good in this context will have led to either systematic targeting of the poor or of the wealthy, but that different VOs will have different participant profiles based on the wealth/quality choice made by that VO.

To assess whether variation in take-up is correlated with the variation in prices of the grains offered by the VOs as predicted by the model, we attempt to estimate the quantity $\pi(y)$. Starting from the simplest case, under the assumption that there are only two quality/price combinations of rice available in a particular village, the model predicts that take-up should vary based on whether the high or low quality rice is chosen. The model predicts that the surplus utility from participation should be positive for low levels of wealth and negative for high levels of wealth when the low quality of rice is chosen and the opposite

¹⁰There is also the issue of heaping at 20 rupees per kg in the market prices. It is not clear whether this heaping comes from stickiness of prices around a round number, measurement error, or both.

when the high quality rice is chosen. If the probability of participation is an increasing function of the surplus utility from participating in the program relative to buying only from the market, the quantity $\pi(y)$ can be estimated by the following simple logit model:

$$Y = \text{logit}(\alpha + \beta_1 * \ln_expenditure + \beta_2 * vo_price + \beta_3 * \ln_expenditure * vo_price) + \epsilon$$

The model predicts that $\beta_1 < 0, \beta_2 < 0, \beta_3 > 0$, capturing the idea that when the VO chooses the low price good, wealth would be negatively correlated with the probability of participation. Table 6 shows the results of estimating the above logit equation for each round that the food security program was offered between the baseline survey and the post-intervention survey. The first two columns show the two rounds separately, and the third column shows the regression pooling the two rounds and including a dummy for the round. The signs of the coefficients confirm the expectations of the model, but the coefficients are not significant.

In the above regressions, the price chosen and offered by the VO is not randomly assigned and may be correlated with characteristics of the VO that are related to both wealth as measured by expenditure and take-up. To avoid this source of bias, we will instead implement a village fixed effects version of the above estimation equation. Since the program price only varies at the village level, we cannot separately identify β_2 , the effect of raising the VO price for the low-wealth households.

Table 7 shows the results of the village fixed effects procedure. Once again, the first two columns show the results for each round separately, with the third column showing the regression pooling both rounds and including a dummy for the round. As predicted by the model, the coefficient on $\ln_expenditure$ is negative, suggesting that when the price of the grain chosen by the VO is low, the probability of take-up is highest for the lowest income members. This is a confirmation of the results predicted by Besley and Coate (1991) and demonstrated elsewhere by Jacoby (1997). But also consistent with the model allowing

for the choice of a higher quality is the fact that the coefficient on $vo_price * \ln_expenditure$ is negative. Solving the following equation for VO prices allows us to estimate the price at which household expenditure switches from being negatively correlated with take-up to being positively correlated.:

$$\beta_1 * \ln_expenditure + \beta_3 * \ln_expenditure * vo_price = 0$$

The inflection point ranges from 18.7 to 19.2 depending on the rounds of lending whose data is included. The interpretation of this fact is that the Jacoby result of self-selection into the program of poorer members only appears when VOs select a quality of grain whose price is less than a value roughly equal to the median price chosen by VOs. When VOs choose a variety of grain whose value is above 19.2 rupees, wealthier members are actually more likely to participate than poorer members. In the higher cost VOs, even when the program provides rice, a staple food which is an especially important component of food expenditure for the poorest members, wealthier members are more likely to participate, reflecting that they perceive an additional benefit from participating in the program that the poorer members do not experience.

The above results restricted the probability of participation to be monotonically increasing or decreasing in wealth. In a version of the model where there are only two varieties of rice available in the local market, this restriction is consistent with the model. But when the model allows for a bounded continuum of quality choices, the model predicts that for a particular quality choice between the minimum and the maximum available quality, there can be regions at both the bottom and the top of the income distribution where members choose not to participate. If probability of participation is increasing in the surplus utility from participation, a more realistic representation based on the model would be to allow the probability of participation as a function of income to have both increasing and decreasing

regions. To allow this loosening of restrictions, Figure 5 shows local polynomial plots of participation over the log expenditure distribution. The dependent axis is a dummy for participation removing VO and round fixed effects. The log expenditure is trimmed at the top and bottom 2 percent of expenditure.

The figure shows the relationship between wealth and participation for three different wealth categories of prices of grains chosen by the VO, corresponding to terciles of the distribution of VO prices. The plots confirm the main features of the model and the logit results. Comparing the relationship for the third of VOs with the lowest cost grains (the solid line) with the line for the third of VOs with the highest cost grains (the dashed line), the data show the same crossing pattern predicted by the model. Lower wealth households are more likely to participate when the cost of grain chosen is the lowest, and higher wealth people are more likely to participate when the highest cost grains are chosen.

7 Implications of the Model for Average Treatment Effects on Consumption of the Staple Good

Until now, the targeting of the program has been assessed by referring in the model to the wealth range of households who would choose to participate, and empirically referring to the probability of a household of a given wealth taking up the program. But another way to view whether a program is well targeted or not is to assess the regions of the income distribution where the program changes the consumption patterns of households. A goal of policy makers might be not only that the program should have higher take-up among relatively poor households, but that that the program should increase the consumption of relatively poor households, and that the increase among poor households should be more than the increase among wealthy households.

Under heterogeneity of the good that can be selected for provision through the pro-

gram, whether consumption is increased by the program and whether these increases are concentrated among the poor will depend on the quality of the good selected for provision. Returning to the model, the optimal choice of consumption in the simple model when buying the staple good only from the market is:

$$c_m^* = \frac{\beta\alpha}{\beta - \alpha}$$

If they participate in the program, households optimally choose to consume:

$$c_p^* = \frac{\alpha\delta p\phi q_p + \alpha y}{(\alpha + \delta)p\phi q_p}$$

The role of income in determining optimally consumption is different depending on whether the household participates in the program or purchases their staple food from the market. This suggests that the difference between optimal consumption when purchasing from the market and consumption when purchasing from the program will be a function of income.

For an individual, the programs effect on consumption is 0 if the household chooses not to participate in the program and $c_p^* - c_m^*$ if the household does choose to participate.

Proposition 3: For a fixed choice of program quality q_p , among households who choose to participate, the size of the program effect on an individual households consumption of rice is increasing in wealth.

Assuming that a household is willing to participate, the program effect is:

$$c_p^* - c_m^* = \frac{\alpha\delta p\phi q_p + \alpha y}{(\alpha + \delta)p\phi q_p} - \frac{\beta\alpha}{\beta - \alpha}$$

The derivative of this quantity with respect to income is :

$$\frac{\alpha}{(\alpha + \delta)p\phi q_p} > 0$$

which implies that if everyone were required to participate in the program, for a fixed program quality q_p , the size of the program effect is increasing in wealth.¹¹

Proposition 3 says that the program effect in terms of increased consumption will be bigger for wealthier households than for poorer ones, which likely runs counter to the goals of policymakers interested in targeting programs to the poor. But even the policy goal of increasing consumption may not hold under the heterogeneity of the program good explored here. Others, in particular Jensen and Miller (2011), have already demonstrated that consumption effects of a subsidy on inferior goods need not be positive over the income distribution, and can even be negative because of a Giffen effect. But the model of heterogeneity in staple goods presented above also introduces the possibility of heterogeneity of consumption impacts because of a cross price effect within the choice of available goods. When the program affects the price of only one variety out of a continuum of varieties of rice available, some members may choose to upgrade to a higher quality rice (if the program offers a higher quality than they choose to consume at market rates) or lower the quality of their grain in order to take advantage of the program discount and purchase more grain. This effect is stated in the proposition below:

Proposition 4: For a fixed program quality q_p , where $\phi p q_p > p q_{min}$, within the range of households with income such that the household is willing to participate, there is a level of income such that all households with income below this level have lower consumption when participating in the program than when buying at market rates, and all households with income above the range will consume more of the staple good when participating in the program than from market

¹¹It may appear that this result is only a product of the fact that at market rates consumption of the staple good is not a function of income, while in program participating households, income does influence optimal consumption. The fact that consumption at market rates results from the assumption that the price of the staple good is simply the quality times a constant (Price = Pq). If the price is a slightly more complicated function of the quality (Price = $P_0 + P_1q$), then c_m^* is an increasing function of y . Using these functional forms precludes closed form solutions, but numerical solutions suggest that the results are robust to such an extension.

rates.

To show Proposition 4, first assume that all households are willing to participate. Solving $c_p^* - c_m^* = 0$ for y gives:

$$y_0 = \frac{a(\beta + \delta)p\phi q_p}{\beta - \alpha}$$

Given that the first derivative of the program effect with respect to income is positive (Proposition 3), for any income less than y_0 , the program effect on consumption would be negative and for any $y > y_0$, the program effect would be positive.

To prove the proposition, we have only to show that $W(y_0, q_p) > 0$, so that there are participants with income just above and just below y_0 . Substituting y_0 into W gives:

$$W = \beta * \ln\left(\frac{1}{\phi}\right)$$

which is > 0 since $\phi < 1$. At the point where the program effect is 0, households receive a level of surplus utility from program participation which is strictly positive, implying that they are willing to participate in the program. Since W is a continuous function, there exists $\epsilon > 0$ such that $W(q_p, y_0 - \epsilon) > 0$ and $W(q_p, y_0 + \epsilon) > 0$. By Proposition 3, at $y = y_0 - \epsilon$, $c_p^* - c_m^* < 0$ and at $y_0 + \epsilon$, $c_p^* - c_m^* > 0$, which proves Proposition 4.

Proposition 4 states that for any choice of q_p such that the program price is higher than market price of the lowest available quality of grain,¹² the lowest income households who choose to participate will respond to the program by lowering their consumption of the staple good, and only the highest income participants will show an increase in consumption.

The average treatment effect on consumption is the average of the individual program

¹²If the program offers grain of a quality higher than the minimum available at the market, but at a price lower than the market price available of the lowest quality grain, even the poorest households would respond by (weakly) increasing their consumption of the staple food. The reduction in consumption when participating in the program comes from households who respond to the program by choosing to upgrade the quality of the grain they buy, even though the program grain is more expensive than the market grain they would have bought (though not as much more expensive as it would have been without the program). If the program rice is cheaper than any market rice, no one faces higher prices by participating in the program.

effects on consumption over the entire range of income. Proposition 4 then yields an interesting result with regard to consumption: the average treatment effects from a program that subsidizes only one quality of rice need not be positive. If the program does not subsidize the grain below the market price of the minimum available quality of rice, some households will respond by using the program to upgrade the quality of their consumption while actually reducing their consumption. Whether the average treatment effect is positive or negative will depend on the distribution of income in the village and the size of the treatment effects at each income level, since the average treatment effect for a fixed q_p can be defined as:

$$\int_0^{\infty} (c_p(y) - c_m(y))dy$$

Where $c_p(y) = c_m^*$ if a household chooses not to participate, $c_p(y) = c_p^*$ if the household does participate, and $c_m(y) = c_m^*$.

These propositions demonstrate several important implications of the model: 1. Consumption impacts are heterogenous on the income distribution, 2. The average treatment effect is only unambiguously positive in the case where the VO chooses to provide the lowest quality of rice available in the local market, 3. If a quality of rice higher than the lowest possible quality is chosen, treatment effects are only unambiguously positive for the wealthiest households, but may be only weakly positive for these households, 4. If higher than the minimum quality of rice is chosen, negative treatment effects are possible and are mostly likely to be observed for households who are not the wealthiest in the village, but only weakly negative for the very poorest.

8 Empirical Results on Consumption Impacts

Table 8 shows the average treatment effects (ATE) on consumption estimated separately by quintiles of the expenditure distribution. The results show that positive treatment effects are observed only in the highest section of the wealth distribution, and that negative impacts of consumption are observed for low to moderate incomes in the sample. This pattern of heterogeneity of impacts is consistent with the model presented as predicting if high prices of rice are at least sometimes chosen, positive impacts are only expected for wealthy households, and some households may actually reduce their consumption in response to the program. It is important to note however, that the program is welfare enhancing for all households. Even the households who reduce their consumption view the program as a net benefit, because the quality improvement in the grain they consume compensates them for effectively facing a higher price and reducing their consumption in response to the program.

9 Conclusion

Although rice seems to be a staple food and an inferior good, the fact that the program under study here focused on providing rice did not seem to guarantee relatively high take-up by the poorest eligible beneficiaries. This paper suggests that one reason that take-up is not uniformly concentrated among the poor could be that rice is actually a heterogeneous good, and some villages seem to have opted to buy high quality rice. Empirical results from the program evaluation show that the choice of rice seems to predict participation of relatively poor or wealthy households, consistent with the model. These results suggest that if programs want to self-target by offering inferior goods, and also allow for public choice, they need to have an additional mechanism to ensure that the preferences of the poorest members are given more weight than occurs in a representative candidate model like the one employed by the program under study. Understanding the processes by which some VOs

seem to successfully select goods that are preferred by the poorest members and designing mechanisms to encourage this choice is the next step for this research.

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Table 1:

Variable	Mean in Treatment	Mean in Control	Difference T-C	t-test for Difference	CGM Wild Cluster BS p-value
Number of HH members	5.68	5.69	-0.01	-0.21	0.838
Dummy for Household is a General Cast Hindu	0.06	0.07	-0.01	-0.46	0.663
Dummy for Head Literate	0.32	0.38	-0.05	-0.79	0.790
Per Cap. Monthly Income	1258.66	1281.25	-22.59	-0.77	0.314
Monthly HH Expenditure on Non-Food Items	4965.12	4790.10	175.03	1.62	0.058
Monthly HH Expenditure on Food	3492.25	3385.38	106.88	0.03	0.726
Total Monthly HH Expenditure	8457.38	8175.47	281.91	1.56	0.094
Number of Assets Owned	6.42	6.70	-0.28	-2.60	0.030
Total Household Debt	13733.14	28174.50	-14441.36	-1.00	0.352
Acres of Land Cultivated in Past 1 Year	1.47	1.94	-0.47	-2.29	0.058
Dummy for Household Cultivates Grains	0.25	0.36	-0.11	-4.08	0.014
Kg. of Rice Purchased in Last 1 Month	54.22	51.88	2.34	0.82	0.446
Kg. of Rice Consumed in Last 1 Week	10.15	9.99	0.15	0.75	0.382
Price per Kg. of Rice purchased from Market	17.45	17.66	-0.21	-0.31	0.878
Kg. of Wheat Purchased in Last 1 Month	58.14	61.14	-3.00	-0.59	0.452
Kg. of Wheat Consumed in Last 1 Week	9.86	10.07	-0.22	0.52	0.310
Price per Kg. of Wheat purchased from Market	13.76	16.81	-3.05	-0.04	0.740
Z-score Length for Age for children < 5 years old	-3.27	-3.32	0.05	-0.88	0.125
Z-score Weight for Age for children < 5 years old	-1.96	-2.19	0.23	0.52	0.979
Z-score Weight for Length for children < 5 years old	-0.50	-0.96	0.46	2.86	0.107

Table 2:

VARIABLES	(1) rice_share	(2) rice_share
Log of Total HH Expenditure	-0.0191** (0.00662)	-0.0178*** (0.00538)
Block FE	NO	YES
Constant	0.417*** (0.0590)	0.403*** (0.0497)
Observations	1528	1528
R-squared	0.005	0.019

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3:

VARIABLES	(1) Log Rice Mkt Price	(2) Log Rice Mkt Price	(3) Log Rice Mkt Price	(4) Log Rice Mkt Price
Log of total HH Expenditure	0.0129** (0.00633)			
Log of food expenditure (excluding wheat and rice)		0.0188** (0.00851)		
Asset Index			0.00311 (0.00562)	
Owned Land Area Under Cultivation				-0.00124 (0.000937)
Constant	2.754*** (0.0558)	2.724*** (0.0650)	2.868*** (0.000644)	2.870*** (0.000827)
Observations	1039	1039	1039	1032
R-squared	0.003	0.005	0.000	0.001
Number of VOs	90	90	90	90

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 4:

VARIABLES	(1) Dummy for VO Committee	(2) Dummy for Participated in Round 2	(3) Dummy for Participated in Round 1
Log Tot. HH Exp	0.0738*** (0.0202)		
VO Committee		0.0998*** (0.0353)	0.0757* (0.0381)
Constant	-0.370** (0.173)	0.403*** (0.0433)	0.547*** (0.0353)
Observations	1,468	730	730
R-squared	0.014	0.007	0.004

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 5:

VARIABLES	(3) VO Price
Median HH Expenditure among VO committee members	0.302 (0.713)
Block FE	YES
Round FE	YES
Constant	15.38** (6.330)
Observations	82
R-squared	0.136

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 6:

VARIABLES	(1) Round 1	(2) Round 2	(3) Both Rounds
Log Total HH Expenditure	-1.250 (1.674)	-3.644 (2.797)	-2.138 (1.528)
Log Total HH Expenditure X VO Price	0.0614 (0.0892)	0.196 (0.149)	0.113 (0.0816)
VO Price	-0.456 (0.766)	-1.638 (1.329)	-0.906 (0.701)
Round Dummies	N/A	N/A	YES
Constant	9.697 (14.30)	30.32 (24.95)	17.50 (13.11)
Observations	703	605	1308

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 7:

VARIABLES	(1) Round 1	(2) Round 2	(3) Both Rounds
Log Total HH Expenditure	-2.026 (1.977)	-5.422* (3.040)	-0.701*** (0.234)
Log Total HH Expenditure X VO Price	0.106 (0.107)	0.289* (0.161)	0.0366*** (0.0111)
Round FE	N/A	N/A	YES
Village FE	YES	YES	YES
Observations	658	590	1308
Number of Q1_8_VOCODE	42	38	45

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 8:

Consumption impacts by wealth quintile

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
	Change in KG	Change in KG	Change in KG	Change in KG	Change in KG	Change in KG
	All HHs	1 st Quintile	2 nd Quintile	3 rd Quintile	4 th Quintile	5 th Quintile
Treatment	-0.240 (0.271) [0.315]	0.224 (0.791) [0.894]	-1.499** (0.529) [0.024]	-0.613 (0.416) [0.108]	-0.353 (0.573) [0.576]	1.146** (0.514) [0.028]
Constant	-1.199*** (0.222)	-0.995 (0.787)	-0.585 (0.715)	-1.034 (0.949)	-1.031* (0.502)	-3.632*** (0.547)
Block FE	YES	YES	YES	YES	YES	YES
Observations	1385	264	259	250	248	252
R-squared	0.019	0.023	0.044	0.075	0.047	0.035

Robust standard errors in parentheses

P-values from Cameron, Gelbach, Miller's (2008) wild cluster bootstrap in brackets

*** p<0.01, ** p<0.05, * p<0.1

Figure 1:

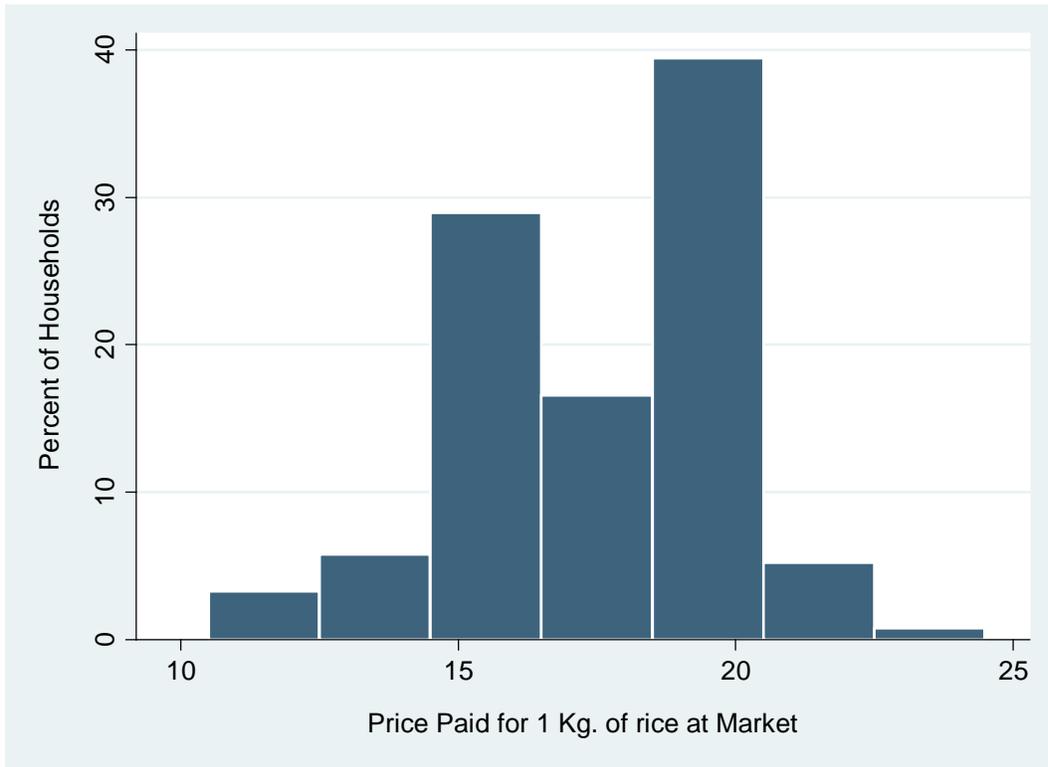


Figure 2:

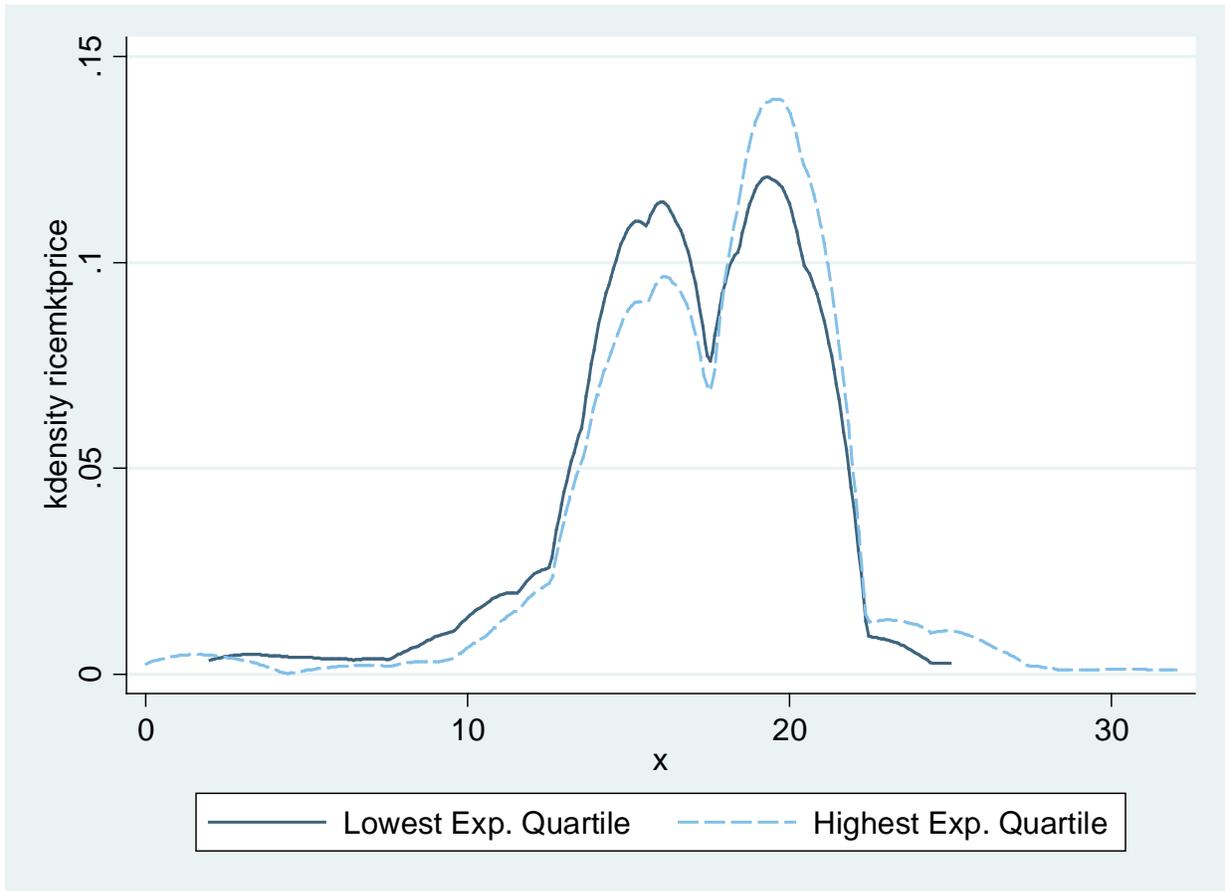


Figure 3:

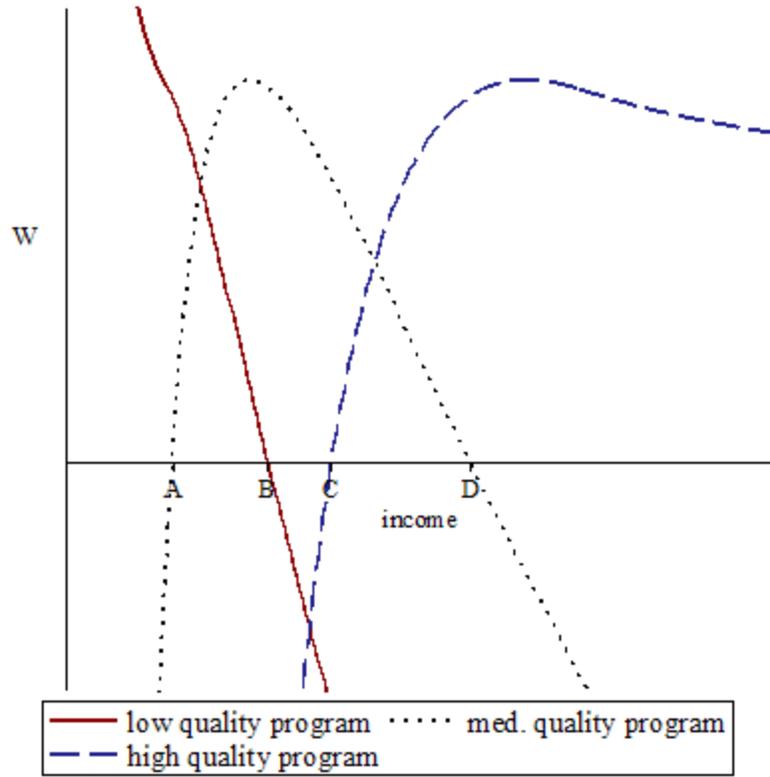


Figure 4:

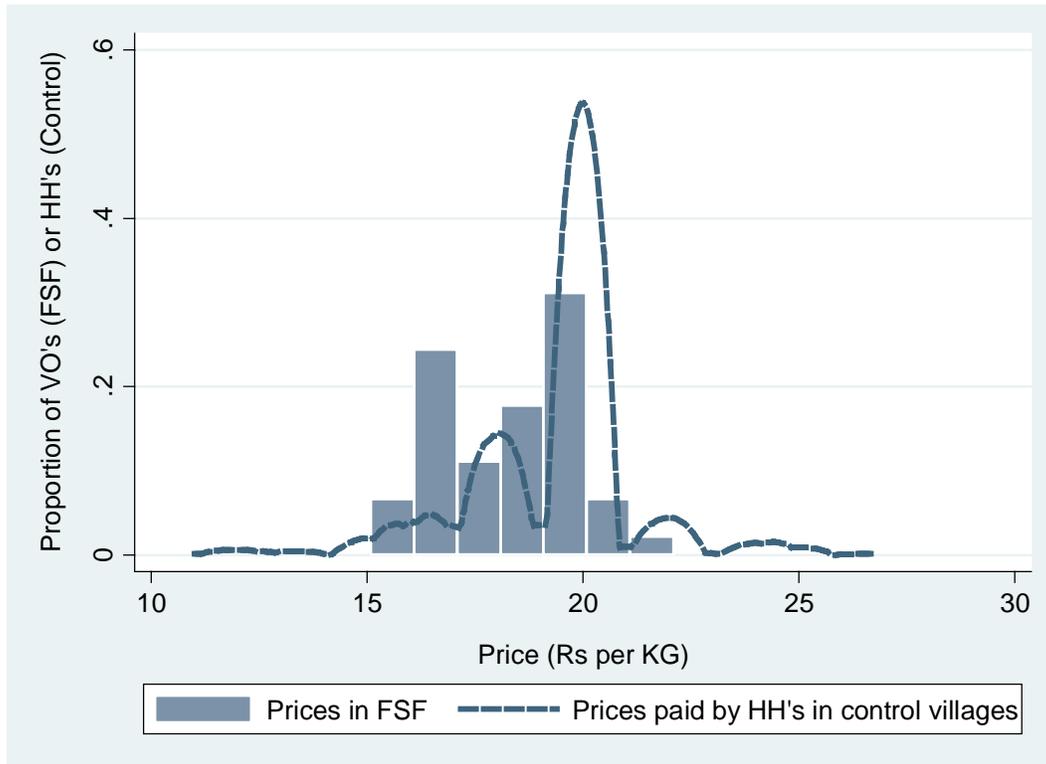
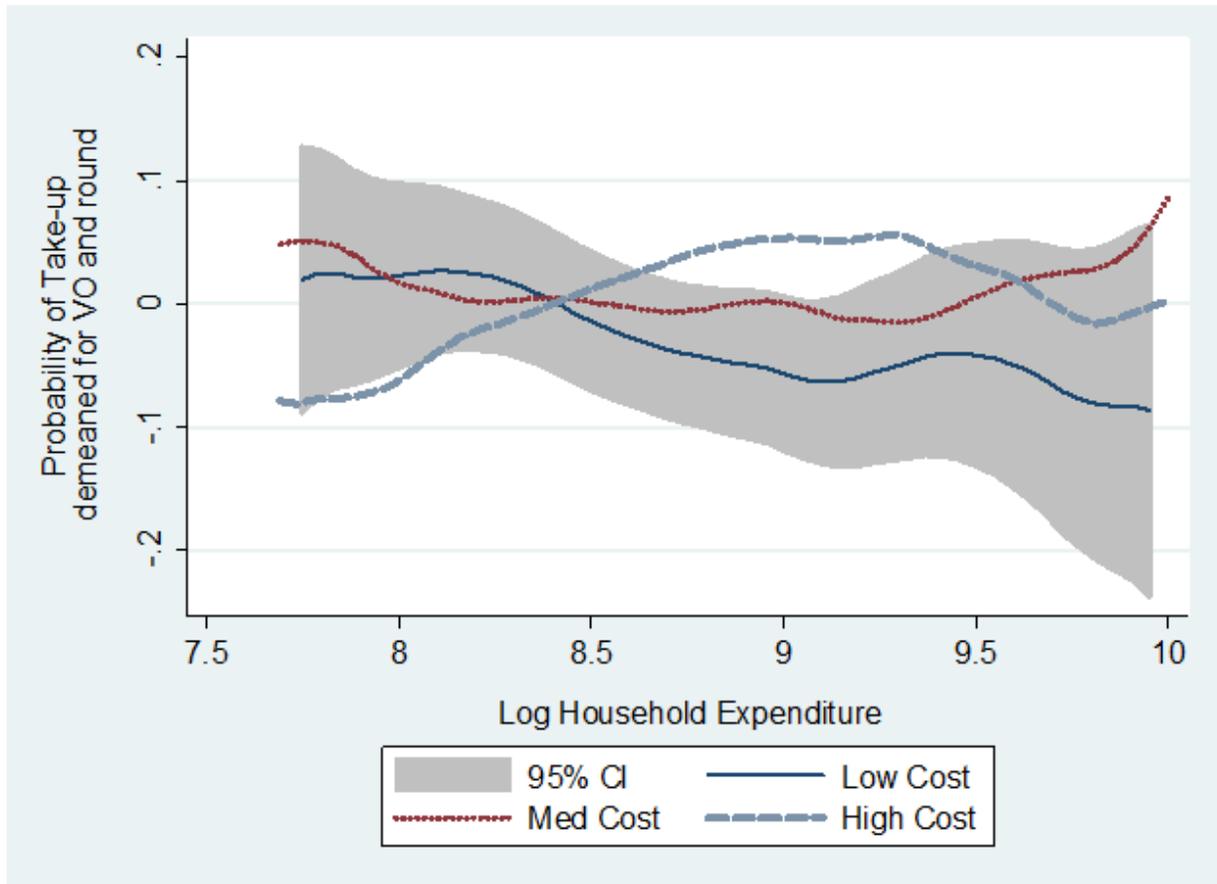


Figure 5:



Appendices

Appendix 1: Optimal Choices of q , c , n in the Market and the Program

In the market, where q is optimally chosen by the household and the price of a unit of the staple good is pq , the optimal choices of q , c , and n are:

$$q_m^* = \frac{y(\beta - \alpha)}{ap(\beta + \delta)}, \quad c_m^* = \frac{\beta a}{\beta - \alpha}, \quad n_m^* = \frac{y\delta}{z(\beta + \delta)}$$

unless $q_m^* < q_{min}$ or $q_m^* > q_{max}$. If $q_m^* < q_{min}$, the optimal choices of q , c , and n are:

$$q_m^* = q_{min}, \quad c_m^* = \frac{a\delta pq_{min} + \alpha y}{pq_{min}(\alpha + \delta)}, \quad n_m^* = \frac{\delta(y - apq_{min})}{z(\alpha + \delta)}$$

Finally, if $q_m^* > q_{max}$, the optimal choices of q , c , and n are:

$$q_m^* = q_{max}, \quad c_m^* = \frac{a\delta pq_{max} + \alpha y}{pq_{max}(\alpha + \delta)}, \quad n_m^* = \frac{\delta(y - apq_{max})}{z(\alpha + \delta)}$$

If the household chooses to buy from the program, they take q as exogenously given as q_p . They get a discount on each unit of c purchased, so that the price of a unit of c through the program is ϕpq where $\phi < 1$. For households who purchase through the program, the optimal c and n are:

$$c_p^* = \frac{a\delta q\theta + \alpha y}{\theta q(\alpha + \delta)}, \quad n_p^* = \frac{\delta(y - aq\theta)}{z(\alpha + \delta)}$$

Appendix 2: Proof that a continuous, concave function $y = f(x)$ with $y > 0$ at the functions global maximum has exactly two roots over the range $x = (-\infty, \infty)$.

By the fact that $f(x)$ is a concave function, we have that:

$$f(z) \leq f(x) + f'(x)(z - x)$$

Define x^* as the value of x such that $f(x)$ takes a global maximum. By assumption, $f(x^*) > 0$. Then there exists $\epsilon > 0$ such that $f(x^* + \epsilon) > 0$ and $f(x^* - \epsilon) > 0$.

$$\text{Define } x_1 = x^* - \epsilon \text{ and } z_1 = \frac{f'(x_1)x_1 - f(x_1)}{f'(x_1)}.$$

$$\text{Then } f(z_1) \leq f(x_1) + f'(x_1) \frac{f'(x_1)x_1 - f(x_1)}{f'(x_1)x_1}$$

The concavity of $f(x)$ implies that $f'(x_1) < 0$, since $f'(x^*) = 0$ and $x_1 < x^*$. Using this fact, we can simplify the above to:

$$f(z_1) \leq 0$$

Using again the fact that $f'(x_1) < 0$, we have that $z_1 < x_1 < x^*$, which proves that there is a point $z_1 < x^*$, such that $f(z_1) \leq 0$. If $f(z_1) = 0$, z_1 is a root. If $f(z_1) < 0$, then the fact that $f(x)$ is continuous and $f(x^*) > 0$ implies that there must exist a point x_{lbar} in the range (z_1, x^*) such that $f(x) = 0$, which proves the existence of one root where $x_{lbar} < x^*$.

To prove that there is a second root $x_{hbar} > x^*$, define $x_2 = x^* + \epsilon$ and the proof proceeds exactly as above.

Therefore there are at least 2 roots, x_{lbar} and x_{hbar} . To prove that these are the only roots, suppose that q_1, q_2, q_3 are all roots of W , so that $W(q_1) = W(q_2) = W(q_3) = 0$, and $q_1 < q_2 < q_3$. By concavity of W , $W(z) > W(q_1)$ and $W(z) > W(q_3)$ for all z in (q_1, q_3) , which contradicts that $W(q_2) = 0$. Therefore, q_2 cannot be a root of W , and there are at most 2 roots.

To apply this proof to Lemma 2, we have only to show that $W(q_p, y) > 0$ at the global

maximum. Setting the derivative of $W(q_p, y)$ w.r.t. q_p equal to 0 and solving for q_p gives q^* , the maximand of $W(q_p, y)$. Substituting q^* into $W(q_p, y)$ gives:

$$W(q^*, y) = \beta \ln \left(\frac{1}{\phi} \right) > 0$$

where the last inequality is true since $\phi < 1$.

Appendix 3: Detailed Proof of Lemma 3

We will do the proof only for 3.a. concerning $q_L(y)$. The proof for 3.b. concerning $q_H(y)$ is similar. The proof follows two cases: 1. $q_1(y_L) \leq q_{min}$ and 2. $q_1(y_L) > q_{min}$.

Case 1: $q_1(y_L) > q_{min}$

The proof of Proposition 3 uses relies on characterizing four points on the q_p axis:

1. q_p^* : The q_p which maximizes $W(q_p, y_L)$ for some fixed $y = y_L$
2. $q_p^* - \epsilon$: A point just to the left of q_p^*
3. $q_1(y_L)$: The point where $W(q_p, y_L) = 0$
4. $q_1(y_H)$: The point where $W(q_p, y_H) = 0$ with $y_H > y_L$

To clarify the interpretation of each point described in the proof, $W(q_p, y)$ is graphed below for two values of y . In the graph, the blue line is W plotted over q_p for $y = y_L$, and the red line is W plotted over q_p for $y = y_H$.

To avoid clutter on the graph, the 4 key points are labeled as:

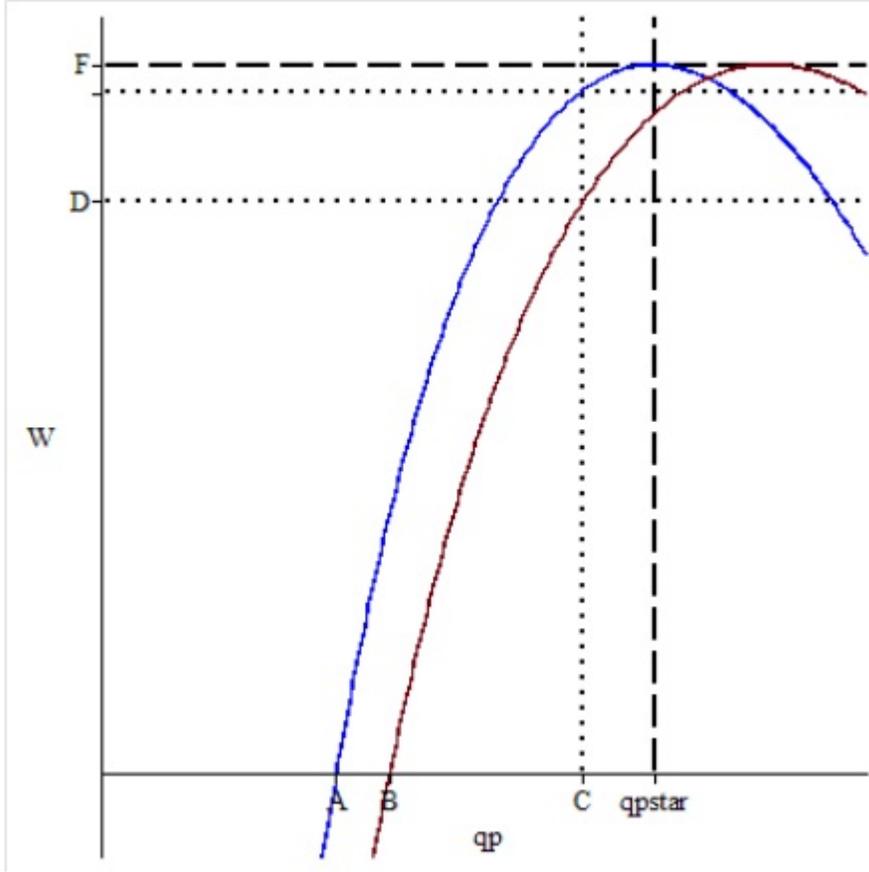
$$q_p^{**} = \text{qpstar}$$

$$q_p^{**} - \epsilon = C$$

$$q_1(y_L) = A$$

$$q_1(y_H) = B$$

Figure 1:



The proof follows three steps:

Step 1: Characterize the point $q_p^* - \epsilon$ which will be used in the proof. ϵ is chosen so that:

1. $q_1(y_L) < q_p^* - \epsilon < q_p^*$
2. $W(y_L, q_p - \epsilon) > 0$

Lemma 5 then shows that $W(q_p^* - \epsilon, y_L) > W(q_p^* - \epsilon, y_H)$.

Step 2: Assume that $q_1(y_L) > q_1(y_H)$. This assumption implies that $W(q_p, y_H)$ and $W(q_p, y_L)$ must cross at some q_p between the point $q_1(y_L)$ and the point $q_p^* - \epsilon$. We will use the crossing point in Step 3 to prove a contradiction that negates the assumption.

Step 3: At the crossing point described in step 2, it would have to be the case that $\frac{\partial W(q_p, y_H)}{\partial q_p} < \frac{\partial W(q_p, y_L)}{\partial q_p}$

. Show that this result contradicts the fact that the following cross partial derivative is pos-

itive: $\frac{\partial W(q_p, y)}{\partial q_p \partial y}$

Step 1: Characterize the point $q_p^* - \epsilon$ that we will use in the proof of proposition 3.

For fixed y , there exists at least one point $z_1 > q_1(y)$ such that $0 < W(z_1, y) < W(q_p^*(y), y)$ and $\frac{\partial W(z_1, y)}{\partial y} < 0$.

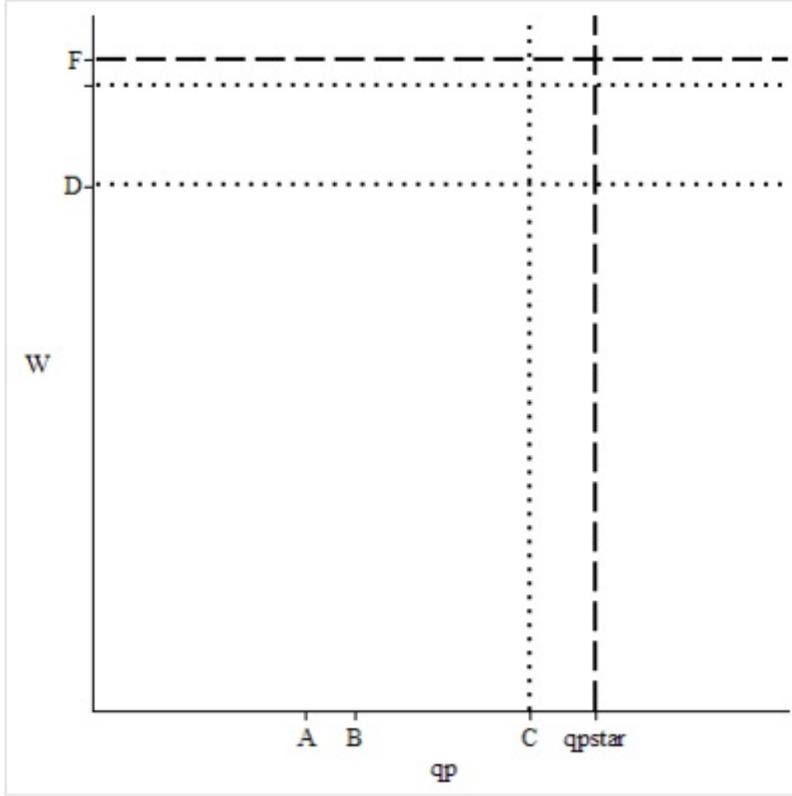
Proof: We have shown that there exists $q_1(y)$ such that $W(q_1(y), y) = 0$ and that $W(q_p^*(y), y) = \beta \ln\left(\frac{1}{\phi}\right) > 0$. Since q_p^* is the only stationary point in $W(q_p, y)$ (Lemma 3) and $q_1(y) < q_2(y)$, the fact that $W(q_p^*(y), y) > 0$ implies that $q_1 < q_p^*$. By the fact that $W(q_p, y)$ is continuous in q_p , there must exist $\epsilon > 0$ such that $q_p^* - \epsilon$ is in the range (q_1, q_p^*) and $W(q_p^* - \epsilon) > 0$. Evaluating $\frac{\delta W(q_p, y)}{\delta y}$ at the point $q_p = q_p^* - \epsilon$ gives:

$$-\frac{a\phi\epsilon(\beta^2 + 2\beta\delta + \delta^2)}{(a\beta\epsilon\phi + a\delta\epsilon\phi + \alpha y + \delta y)y} < 0$$

The negative sign on the above derivative implies that $W(q_p^* - \epsilon, y_L) > W(q_p^* - \epsilon, y_H)$ at the point $q_p^* - \epsilon$.

The graph below demonstrates the proof. q_p^* in the graph is the point where $W(q_p, y_L)$ attains its maximum of F . At $q_p = q_p^* - \epsilon$ (labeled C in the graph), $W(q_p^* - \epsilon, y_L) = E$, where $0 < E < F$. The fact that $E < F$ follows from the fact that $W(q_p, y)$ is a local maximum. The negative sign on the partial derivative at the end of the proof above implies that $W(q_p^* - \epsilon, y_H) < W(q_p^* - \epsilon, y_L)$ as shown by the fact that $D < E$ in the graph.

Figure 2:



Step 2: Assume that $q_1(y_L) > q_1(y_H)$. This assumption implies that $W(y_H, q_p)$ and $W(y_L, q_p)$ must cross at some q_p between the point $q_1(y_L)$ and the point $q_p^* - \epsilon$.

To obtain the contradiction that proves Lemma 3, suppose in the graph that $A = q_1(y_H)$ and $B = q_1(y_L)$ so that $q_1(y_H) < q_1(y_L)$. Step 1 demonstrated that $W(C, y_L) > W(C, y_H)$. Under the assumption, $W(q_p, y_H)$ must intersect both $(A, 0)$ and (C, D) and $W(y_L, q_p)$ must intersect $(B, 0)$ and (C, E) . Now imagine two lines that meet both conditions, there must be a point where both lines cross. This crossing must happen for any two continuous functions between these points, including the particular convex function $W(q_p, y)$. Therefore, the assumption that $q_1(y_L) > q_1(y_H)$ implies that there must be a point where $W(q_p, y_H) = W(q_p, y_L)$ in the range of q_p in (B, C) .

Step 3: Show that the result from step 2 contradicts the fact that the cross partial derivative of $W(y, q_p)$ w.r.t. y, q_p is positive.

Focus on the point where $W(q_p, y_H) = W(q_p, y_L)$ in the range of q_p in (B, C) . Since $W(y_H, q_p) > W(y_L, q_p)$ for any point to the left of this crossing point and $W(q_p, y_H) < W(q_p, y_L)$ to any point to the right, it must be the case that:

$$\frac{\partial W(q_p, y_H)}{\partial q_p} < \frac{\partial W(q_p, y_L)}{\partial q_p}$$

at the crossing point.

All that is left is to argue why the above is a contradiction of what we already know about $W(q_p, y)$. It is in fact a contradiction of Lemma 4, which states:

$$\frac{\partial W(q_p, y)}{\partial q_p \partial y} > 0$$

Lemma 4 implies that for all q_p , $\frac{\partial W(q_p, y_L)}{\partial q_p} < \frac{\partial W(q_p, y_H)}{\partial q_p}$ for all $y_H > y_L$, which contradicts

$$\frac{\partial W(q_p, y_H)}{\partial q_p} < \frac{\partial W(q_p, y_L)}{\partial q_p}$$

The assumption that $q_1(y_H) < q_1(y_L)$ leads to a contradiction of a known feature of W , namely that the cross partial derivative of W w.r.t. y, q_p is positive, so we must reject the assumption. It therefore follows that $q_1(y_H) > q_1(y_L)$, which proves proposition 3.

Case 2: $q_1(y_L) \leq q_{min}$

We showed in Proposition 2 that $q_L(y_L) = q_{min}$.

Suppose that $q_1(y_H) \leq q_{min}$, then again from proposition 2, $q_L(y_H) = q_{min}$, and we have that $q_L(y_H)$.

Now suppose that $q_1(y_H) > q_{min}$. Then by Proposition 2, $q_L(y_H) = q_1(y_H) > q_{min}$.

Since these are the only two possible cases for $q_1(y_H)$, we have shown that $q_L(y_L) = q_L(y_H)$ iff $q_L(y_L) = q_L(y_H) = q_{min}$.