A Reappraisal of the Allocation Puzzle through the Portfolio Approach ∗†

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Abstract
Paradoxically, high investment developing countries tend to experience capital outflows. This paper shows that this allocation puzzle can be explained simply by introducing uninsurable idiosyncratic investment risk in the neoclassical growth model. Investment risk introduces a precautionary demand for safe assets, which can be satisfied by external financial markets. However, labor income is deterministic, so human wealth constitutes a safe asset that could also fulfill these precautionary needs. Cross-country differences in capital outflows then result from the relative size of the domestic demand and supply of safe assets, which depend respectively on capital and human wealth. When cross-country differences in investment are not driven by TFP (eg. due to differences in distortions on the return to capital), human wealth varies less than proportionally with investment, which makes capital outflows necessary to satisfy the precautionary asset demand. An important consequence of this channel is to generate a positive cross-country correlation between capital outflows and investment for countries with both positive and negative external positions, through an extended “New Rule”. The model then predicts accurately the direction of capital flows between 1980 and 2003 within a sample of 47 emerging countries. The data also confirms that the observed correlation between capital flows and investment is due to non-TFP-driven investment.

Key Words: Capital flows, Global imbalances, Investment risk.

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1 Introduction

Empirical studies on capital flows to developing countries suggest that the predictions of the standard models are not satisfied in the data.\(^1\) Indeed, while these models predict that capital should flow to countries with high investment rates, the opposite happens in reality: capital flows to countries with low investment rates. This is the allocation puzzle.

To understand why this is a puzzle, consider the neoclassical growth model (Ramsey-Cass-Koopmans) with cross-border trade in bonds. A permanent increase in total factor productivity (TFP) creates investment opportunities in the country and increases future labor income. Therefore, investment and capital inflows should be positively related through both an investment and a consumption smoothing effect. Gourinchas and Jeanne (2007) show that, as a result, capital outflows calibrated inside this model are negatively correlated with the actual ones. Providing an explanation to this puzzle is the objective of the paper.

Explaining the allocation puzzle would also help understand the phenomenon of global imbalances, and especially their structure, that is: why are they originated in Asia and not in Latin America or Africa? To illustrate this, consider panel a of Figure 1, which provides capital outflows between 1980 and 2003 across regions. Only Asia has exported capital, whereas Latin America and Africa have attracted capital. Besides, as panel b of the figure shows, Asia is characterized by a significant increase in its capital-labor ratio between 1980 and 2003, whereas in Latin America and Africa, the capital-labor ratio stagnated. Explaining the positive link between investment and capital outflows would then explain why Asia is special and why it exports capital, as opposed to other developing countries. This issue has been overlooked in the literature on global imbalances.\(^2\) Indeed, to explain global imbalances, most researchers either consider emerging markets as a whole or take Asia (and especially China) in isolation.\(^3\)

This paper shows that the allocation puzzle can be explained simply by introducing uninsurable idiosyncratic investment risk in the neoclassical growth model. Investment risk introduces a precautionary demand for safe assets, which can be satisfied by external financial markets. However, labor income is deterministic, so human wealth constitutes a safe asset that could also fulfill these precautionary needs. Cross-country differences in capital outflows then result from the relative size of the domestic demand and supply of safe assets, which depend respectively on

\(^1\)See Aizenman and Pinto (2007); Prasad et al. (2007); Gourinchas and Jeanne (2007).


\(^3\)Aguiar and Amador (2009) is an exception. They explain why countries that grow rapidly tend to accumulate net foreign assets rather than liabilities by introducing political economy and contracting frictions.
capital and human wealth. When cross-country differences in investment are not driven by TFP (eg. due to differences in distortions on the return to capital), human wealth varies less than proportionally with investment, which makes capital outflows necessary to satisfy the precautionary asset demand. An important consequence of this channel is to generate a positive cross-country correlation between capital outflows and investment for countries with both positive and negative external positions, through an extended “New Rule”. The model then predicts accurately the direction of capital flows between 1980 and 2003 within a sample of 67 emerging countries. The data also confirms that the observed correlation between capital flows and investment is due to non-TFP-driven investment.

Investment risk is crucial to explain the allocation puzzle because it changes radically the relationship between investment and capital flows. In the riskless approach, the composition of the portfolio between external bonds, capital and human wealth is undetermined, because these assets are identical in equilibrium. Changes in the portfolio composition for investment or consumption smoothing motives therefore persist in the long run. In the approach with risk, which we call the portfolio approach, the composition of the portfolio between capital and safe assets is determined in the long run by the level of investment risk. As a result, the investment and consumption smoothing effects are completely wiped out in the long run. In particular, if investment is sufficiently risky, a positive amount of safe assets is needed for precautionary motives, that is to self-insure against investment risk.

However, safe assets are composed of both external and domestic assets. Since labor income is deterministic, the latter are represented by human wealth. The extent to which the precautionary demand for safe assets actually leads to capital outflows thus depends on the comovement between investment and human wealth, and therefore on the factors driving investment. non-TFP-driven investment affects positively human wealth because it increases the capital stock and therefore labor productivity. TFP-driven investment increases the capital stock in the same way, but labor productivity benefits additionally from higher total factor productivity. According to this argument, higher investment is more likely to lead to capital outflows if it is not originated in TFP, because the subsequent increase in human wealth is less likely to fulfil alone the precautionary demand for safe assets.

These effects are studied in Section 2 using Angeletos and Panousi (2009)’s framework, which boils down to a Merton-Samuelson portfolio choice problem between risky capital and safe assets, which include human wealth and external bonds. This model is extended to allow, as in Gourinchas and Jeanne (2007), for TFP growth and a capital wedge, that is a distortion between
the private and social return to capital. The latter is introduced to account for cross-country
differences in capital-labor ratio that are not explained by differences in TFP, and therefore for
non-TFP-driven investment. It is then possible to examine how human wealth and the factors
driving investment affect capital flows for a given amount of risk. As expected, the condition
under which capital flows are positively correlated to investment is more restrictive when invest-
ment is originated in TFP growth then when it is not: in the former case, the long-run external
position should be positive, while this is not necessary in the latter.

To understand these results, consider the case without human wealth. With constant risk
across countries, the predictions of the model would then boil down to a cross-country version
of Kraay and Ventura (2000)'s “New Rule”: capital outflows are equal to investment multiplied
by the ratio of external assets to capital, which is constant across countries. Empirically, this
implies that the cross-country correlation between capital outflows and investment should not
differ systematically from the average cross-country ratio between the net external position and
capital. Since emerging countries have negative external positions on average (see panel c of
Figure 1), the New Rule would imply that higher capital outflows should be negatively correlated
with investment, which does not improve on the riskless approach.

This paper's innovation with regards to the New Rule is to take into account labor income and
therefore human wealth as an additional safe asset. This yields an extended New Rule: capital
outflows are equal to investment multiplied by the ratio of total safe assets to capital, minus the
increase in human wealth. This implies that, empirically, if human wealth does not comove too
much with investment, the cross-country correlation between capital outflows and investment
should be larger than the average cross-country ratio between the net external position and
capital. It can then be positive even in countries with negative external positions. In the case
of TFP-driven investment, human wealth comoves too much with investment for this effect to
hold. More precisely, human wealth varies proportionally with the capital stock through the
TFP channel, so the relationship between capital flows and investment comes down to the New
Rule again. In contrast, in the case of non-TFP-driven investment, human wealth varies less
than proportionally with the capital stock, which implies that a positive external position is not
necessary to explain the puzzle.

According to these arguments, the structure of global imbalances should be explained by
non-TFP-driven investment. As panel d of Figure 1 shows, the change in capital-labor ratio due
to non-TFP-driven investment is positively related to capital outflows across regions. Moreover,
this correlation is even better than with total investment (see panel b), since it accounts for the
difference between Latin America and Africa, and not only for the difference between Asia and other regions. The portfolio approach suggests that the positive capital outflows from Asia are due to an increase in the capital stock, and therefore in the demand for safe assets, that was not compensated by an increase in human wealth. On the opposite, capital inflows in other regions are due to a decrease in the capital stock and demand for safe assets that was not matched by a significant decrease in human wealth. These regions then exported their safe assets. This is especially the case in Africa, in which safe assets were even less needed than in Latin America because the non-TFP-driven decrease in capital stock was more pronounced.

Section 3 goes further in investigating whether the portfolio approach does solve the allocation puzzle in the data. A calibration analysis is performed to compare the relative performances of the riskless and portfolio approaches in predicting the direction of capital flows to emerging countries. Common and country-specific parameters are either measured or calibrated in order to compute the amount of capital flows predicted by the model with and without risk in a sample of 67 developing countries between 1980 and 2003. While, consistently with Gourinchas and Jeanne (2007), predicted capital flows are negatively correlated with the actual ones in the riskless approach, the correlation becomes positive in the portfolio approach. The portfolio approach also reproduces accurately the patterns of capital flows across regions.

Section 4 explores empirically the sources of the puzzle and of its resolution in two steps. First, the portfolio approach predicts accurately the direction of flows because it accounts better than the riskless approach both for the unstable link between capital outflows and TFP growth and for their positive link with the capital wedge. We show this by testing a structural linear approximation of the decomposition of capital flows in the data. Second, this suggests that, since only the TFP-adjusted capital-labor ratio correlates positively with both investment and capital outflows, the capital wedge is the channel of the observed correlation between investment and capital outflows. This hypothesis is tested and validated by introducing the potential correlates of capital flows in a regression including investment.

Finally, Section 5 shows that this analysis is robust to the replacement of the calibrated capital wedge by indirect measures such as the relative price and the share of reproducible capital.
2 The neoclassical growth model with idiosyncratic investment risk

In this section, the implications in terms of capital flows of the neoclassical growth model with investment risk are studied. To achieve this, Angeletos and Panousi (2009)’s model is used. Angeletos and Panousi’s framework generalizes the tractable Merton-Samuelson portfolio choice problem to account for human wealth, which is an important feature of the neoclassical growth model. This model is extended here to account, as in Gourinchas and Jeanne (2007), for TFP growth and a capital wedge, that is a distortion between the private and social return to capital that accounts for non-TFP-driven investment.

Since this model nests both the riskless approach (without risk) and the portfolio approach (with risk), it is possible to explain their differences in terms of implied capital flows between an initial period and the long run. We then examine how the different channels of investment (TFP catch-up and the capital wedge) affect capital flows in the portfolio approach.

2.1 The household’s program

Consider a small open economy with a continuum of length 1 of infinitely-lived households, or families, indexed by $i$. Time is continuous, indexed by $t \in [0, \infty)$. Each household owns a firm which produces a homogeneous good using two factors, labor and capital, according to a Cobb-Douglas technology:

$$Y_i^t = F(K_i^t, A_t N_i^t) = K_i^{\alpha t} (A_t N_i^t)^{1-\alpha}, 0 < \alpha < 1$$

where $K_i^t$ is the amount of capital invested in firm $i$ at date $t$, $N_i^t$ the amount of labor hired in the firm and $A_t$ the deterministic domestic level of technology.

Households can invest only in their own firm and cannot trade equity. $K_i^t$ is therefore the household’s holdings in private capital. They face an idiosyncratic investment risk, but cannot diversify away this risk through equity or any other means. The only freely traded asset, domestically and internationally, is the riskless bond $B_i^t$ whose return is fixed internationally to $R^*$.

Denote household $i$’s net capital income by $dQ_i^t$. It is defined as follows:

$$dQ_i^t = (1 - \tau)[F(K_i^t, A_t N_i^t) - w_t N_i^t]dt - \delta K_i^t dt + \sigma K_i^t dv_i^t$$

where $w_t$ the wage rate, which is not firm-specific since the labor market is competitive. $\delta$ is the depreciation rate.
\( \tau \) is a wedge on the gross capital return, that is, before subtracting capital depreciation. Following Gourinchas and Jeanne (2006, 2007), it is introduced in order to account for the cross-country differences in capital-labor ratios that are not attributable to TFP, or non-TFP-driven investment. This wedge can be interpreted as a tax on capital income, or as the result of other distortions that would introduce a difference between social and private returns. We assume that this tax on capital return is distributed equally among households.\(^4\)

The technology is exactly identical to Gourinchas and Jeanne (2007), except that time is continuous and that investment risk is introduced through a standard Wiener process \( dv_i t \). This process is iid across agents and time and satisfies \( E[dv_i t = 0] \). This risk can be interpreted as a production or a depreciation shock that affects the return on capital. It is assumed that this shock is averaged out across households, that is: \( \int_0^1 dv_i t = 0 \). The parameter \( \sigma \) measures the amount of individual risk. Gourinchas and Jeanne (2007)'s specification is nested when \( \sigma = 0 \).

Since the labor market is competitive and the production function has constant returns to scale, the capital income is linear in \( K_i t \). Denote \( \tilde{k} i t = K_i t / (A_i N_i t) \) the capital per efficient unit of labor and \( \tilde{g} i t = K_i^{\alpha} (A_i N_i t)^{1-\alpha} / (A_i N_i t) = \tilde{k} i t^{\alpha} \) the production per efficient unit of labor. Employment is chosen after the capital stock has been installed and the shock has been observed. Therefore, at each period \( t \), the firm chooses employment in order to maximize \( F(K_i t, A_i N_i t) - w t N_i t \), where \( w t \) is the competitive wage per unit of labor. This yields \( w t = (1 - \alpha) A_i (\tilde{k} i t)^\alpha \).

Because the competitive wage is constant across firms \( \tilde{k} i t \), the ratio of capital to efficient labor, is also constant. Denote \( \hat{k} t = \hat{k}(\hat{w} t) = \hat{w} t^{1/\alpha} / (1 - \alpha) \), where \( \hat{w} = w / A \) is the wage per efficient unit of labor. Using this result, we can write the capital income as follows:

\[
dQ_i t = r t K_i^{\alpha} dt + \sigma K_i^{\alpha} dv_i t
\]

where \( r t = r(\hat{w} t, \tau) = (1 - \tau) \alpha \hat{k}(\hat{w} t)^{\alpha - 1} - \delta \) is the private net return on capital. The net capital income is therefore linear in \( K_i t \), which makes the analysis tractable when \( \sigma > 0 \).

Each household is composed of \( N_t \) members, and each member is endowed with 1 unit of labor which he supplies inelastically in the competitive labor market. Since, additionally, there is no aggregate risk, all aggregate variables are deterministic, including the competitive individual wage \( w t \). There is neither unemployment risk nor any other shock that would introduce an income risk besides investment risk. Therefore, labor income is deterministic. Moreover, the

\(^4\)The parameter \( \tau \) is a wedge on the gross capital return, that is, before subtracting capital depreciation. This is a deviation from Gourinchas and Jeanne (2007), where the wedge is on capital returns net of depreciation. This specification is chosen only for practical reasons (the resulting amount can be expressed as a fraction of production) and does not change the results dramatically.
tax product $Z_t = \int_0^1 \tau [K_t^{\alpha}(A_t N_t)^{1-\alpha} - w_t N_t] dt$ is by assumption redistributed equally among households, and is therefore also deterministic.

As in Barro and Sala-i Martin (1995), the household maximizes the following discounted expected welfare of the family members:

$$U_t^i = E_t \int_t^{\infty} N_s \ln c_s^i e^{-\rho(s-t)} ds$$

where $\rho > 0$ is the discount rate and $c_s^i$ is the individual consumption of the members of household $i$ in period $t$. The population growth rate is supposed to be exogenous and equal to $n$, with $n < \rho$:

$$N_t = N_0 e^{nt}$$

We now turn to the household’s budget constraint. The evolution of the household’s financial wealth $B_t^i + K_t^i$ obeys to:

$$d(B_t^i + K_t^i) = dQ_t^i + [R^* B_t^i + N_t w_t + Z_t - C_t^i] dt$$

However, this expression of the budget constraint misses some aspects of household’s wealth. Indeed, the household’s effective wealth $\Omega_t^i$ includes his financial wealth, but also human wealth, that is the present discounted value of future labor income and tax product, defined as $H_t^i = H_t = \int_t^{\infty} e^{-(s-t)R^*} (N_s w_s + Z_s) ds$. Therefore, $\Omega_t^i = K_t^i + B_t^i + H_t$. In order to include human wealth in the budget constraint, we follow Angeletos and Panousi (2009) and use the definition of human wealth to write:

$$dH_t = (R^* H_t - N_t w_t - Z_t) dt$$

It follows from (1), (4) and (3), that the evolution of effective wealth, in per capita terms, can be described by:

$$d\omega_t^i = [r_t \kappa_t + R^* (b_t^i + h_t) - c_t^i - n \omega_t^i] dt + \sigma \kappa_t^i dv_t^i$$

with $\omega_t^i = k_t^i + b_t^i + h_t$. Lower case letters, except $n$, the population’s growth rate, stand for per capita (i.e. per family member) values. For each variable $X_t^i$ ($X_t$), $x_t^i$ ($x_t$) stands for $X_t^i / N_t$ ($X_t / N_t$).

The household maximizes (2) subject to his budget constraint (5), which states that household’s wealth is increased by the revenues from capital, whose return $r_t + \sigma dv_t^i$ is risky, and from safe assets, $b_t^i$ and $h_t$, whose return is $R^*$, minus consumption. Population growth additionally diminishes the value of wealth per capita. Thanks to the linearity of the budget constraint with respect to wealth, this problem boils down to a tractable Merton-Samuelson portfolio choice problem.
Kraay and Ventura (2000) and Kraay et al. (2005) also applied this portfolio choice approach to the study of capital flows, but in an AK context without human wealth, which is an important variable in this paper. Here, we use a transformation of the budget constraint introduced by Angeletos and Panousi (2009) in order to make human wealth explicitly appear, without suppressing the tractability of the problem.

2.2 Technology

The country has an exogenous, deterministic productivity path \( \{ A_t \}_{t=0}^{\infty} \), which is bounded by the world productivity frontier:

\[
A_t \leq A_t^* = A_0^* e^{g^* t}
\]

The world productivity frontier is assumed to grow at rate \( g^* \). Following Gourinchas and Jeanne (2007), we define the difference between domestic productivity and the productivity conditional on no technological catch-up as follows:

\[
e^{\pi_t} = \frac{A_t}{A_0 e^{g^* t}}
\]  

We assume that \( \pi = \lim_{t \to \infty} \pi_t \) is well defined. Therefore, the growth rate of domestic productivity converges to \( g^* \).

2.3 Household’s behavior

The linearity of the evolution of the budget constraint along with the homotheticity of preferences ensures that the household’s problem reduces to a homothetic problem à la Samuelson and Merton. It follows that the capital stock and consumption choices are linear in wealth.

**Lemma 1:** Define \( \phi_i = k_i^t/\omega_i^t \), the fraction of effective wealth invested in private capital. For a given interest rate \( R^* \) and a given sequence of wages \( \{ W_t \} \), the optimal choices of household \( i \) are given by:

\[
c_i^t = (\rho - n)\omega_i^t
\]

\[
r_t - R^* = \phi_i \sigma^2
\]

Equation (7) shows the familiar result that consumption per capita equals the annualized value of wealth, taking into account population growth. This is a direct consequence of log utility.
Equation (8) states that the excess return to capital \( r_t - R^* \) is equal to the risk premium. With logarithmic households, the coefficient of risk aversion is equal to one and the risk premium is the covariance between the return on capital and the return on the portfolio, which is equal here to the variance of the return to capital multiplied by the share of capital in the portfolio.

Importantly, this equation provides a portfolio choice rule. It says that the risky share of the portfolio is constant across households and depends positively on the excess return and negatively on risk.

Notice that Equation (8) implies a no-arbitrage condition \( r_t = R^* \) between bonds and domestic capital when \( \sigma = 0 \). This no-arbitrage condition is an equilibrium outcome that derives from the infinite elasticity of private capital demand to the return differential between capital and bonds and from decreasing marginal returns.

2.4 Steady state

The saving rule (7) and the portfolio rule \( k^i_t = \phi_t \omega_t^i \), with \( \phi_t \) defined by (8), are linear in wealth and can therefore be written in aggregate terms: \( c_t = (\rho - n)\omega_t \) and \( k_t = \phi_t \omega_t \), where \( \omega_t = \int_0^1 \omega_t^i di \) is the aggregate value for \( \omega_t^i \). By dividing each side by \( A_t \), they can also be written in terms of efficient units of labor, denoted \( \tilde{c}_t, \tilde{k}_t \) and \( \tilde{\omega}_t \).

Capital per efficient units of labor at the firm level, \( K_t^i/A_t N_t^i \) is constant across firms as a result of common wages and constant returns to scale. Since the labor market clears (\( \int N_t^i di = N_t \)), it is equal to its aggregate value \( \tilde{k}_t = K_t/A_t N_t \). This defines the wage per efficient units of labor unambiguously as \( \tilde{w}_t = (1 - \alpha)\tilde{k}_t^\alpha \).

The aggregate and per efficient units of labor versions of Equations (7) and (5), along with the no-Ponzi conditions and the equilibrium values for \( \tilde{w}_t, r_t \) and \( \phi_t \), characterize the dynamics of \( \tilde{c}_t \) and \( \tilde{\omega}_t \). Once the paths of these variables are known, \( \hat{k}_t = \phi_t \tilde{w}_t, \hat{b}_t = \int_0^\infty e^{-(R^*-(n+g^*))s+\pi_s-\pi_t(1-\alpha+\tau\alpha)\hat{k}_{t+s}^\alpha}ds \) and \( \hat{b}_t = \tilde{w}_t - \hat{k}_t - \hat{h}_t \) can be determined.

We assume no domestic supply of bonds, so the aggregate demand for bonds \( \hat{b}_t \) represents the country’s external position.

These equations are used here only to determine steady state. We define the steady state by \( \dot{\hat{c}}/\hat{c} = 0 \) and \( \hat{\pi}_t = 0 \). Under the aggregate Euler condition (21) and budget constraint (22), this implies stationarity in wealth. This condition implies different constraints on the world interest rate depending on \( \sigma \).

Proposition 1:
(i) If $\sigma > 0$, the open economy steady state exists if and only if $n + g^* < R^* < \rho + g^*$ and is defined by:

\[
(1 - \tau)\alpha \tilde{k}^\alpha(\alpha - 1) - \delta - R^* = \sqrt{\sigma^2(\rho + g^* - R^*)}
\]

\[
\tilde{b}^* + \tilde{h}^* = \frac{1 - \phi^*}{\phi^*} k^*
\]

with $\phi^* = \sqrt{\frac{\rho + g^* - R^*}{\sigma^2}}$ and $\tilde{h}^* = \frac{(1 - \alpha + \alpha \tau)k^\alpha}{\rho - n}$.

(ii) If $\sigma = 0$, the open economy steady state exists if and only if $R^* = \rho + g^*$ and is defined by

\[
(1 - \phi^* )/\phi^* > 0,
\]

\[
\tilde{b}^* + \tilde{h}^* + \tilde{k}^* = \left[\tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0\right] e^{-\pi}
\]

with $\tilde{h}^* = \frac{(1 - \alpha + \alpha \tau)k^\alpha}{\rho - n}$ and $\tilde{h}_0 = (1 - \alpha + \alpha \tau)\int_0^\infty e^{-(\rho - n)t + \pi} dt$.

When $\sigma > 0$, the interest rate $R^*$ should be strictly lower than the adjusted discount rate $\rho + g^*$ to insure stationarity. Indeed, because of risk, the return of the portfolio incorporates a risk premium and is therefore strictly greater than the return on bonds. As a result, if $R^*$ was equal to $\rho + g^*$, the return of the portfolio would be always strictly greater than the propensity to consume out of wealth and the economy would accumulate an infinite stock of foreign assets. Assuming that firms in the rest of the world bear some idiosyncratic risk would ensure that we indeed have $R^* < \rho + g^*$.\(^5\)

In that case, the risk premium and therefore the long-run composition between safe and risky assets is defined by the equality between the return of the portfolio and the propensity to consume. This gives Equation (10). In the presence of risk, safe assets (that is bond holdings and human wealth), are a constant share of the portfolio which depends only on the parameters of the model. The higher the amount of risk $\sigma$ and the higher the interest rate on bonds $R^*$, the lower the share of capital in the portfolio. In particular, if $\sigma^2 > \rho + g^* - R^*$, then $(1 - \phi^*)/\phi^* > 0$, which means that a positive amount of safe assets is needed to self-insure against investment risk.

The condition $\sigma^2 > \rho + g^* - R^*$ is satisfied as long as the level of risk in the country is greater than in the rest of the world. In that case, for a sufficiently low interest rate $R^*$, the precautionary savings motive is overcome by the high price of bonds in the rest of the world, while the domestic economy still wants self-insurance. Assuming that the country has a relatively

\(^5\)However, the interest rate should not be too low: $R^* > n + g^*$ (i.e. the world risk should not be too high). Otherwise, the long-run human wealth would not be well defined. When $\sigma = 0$, this condition amounts to $\rho > n$, which is already assumed for intertemporal utility to be well defined.
high level of risk is consistent with the fact that we are dealing with developing countries. We therefore maintain this assumption for the risky case.\textsuperscript{6}

Equation (10) gives also the steady-state external demand for bonds as the difference between the precautionary demand for safe assets, which depends on the capital stock, and the domestic supply, which is given by human wealth:

\[
\tilde{b}^* = \frac{1 - \phi^*}{\phi^*} \tilde{k}^* - \tilde{h}^*
\]

Equation (9) defines the steady state level of capital by the equality between its excess return and the steady-state risk premium. The steady-state capital stock then defines the steady-state human wealth. Since the external position depends only on capital and human wealth, it is therefore determined at steady state.

When $\sigma = 0$, the return on capital does not incorporate any risk premium, so the return of the portfolio is exactly equal to the return on bonds. Stationarity then implies that the world interest rate $R^*$ is exactly equal to the growth-adjusted psychological discount rate $\rho + g^*$. If it is slightly lower (higher), then the country would accumulate an infinite level of liabilities (assets). This condition is satisfied if we assume that the rest of the world is also characterized by the absence of risk, since $\rho + g^*$ would be the long-run autarky interest rate in an economy without risk.

Consider now the portfolio composition in the absence of risk, given by Equation (11). It states that the steady-state wealth is equal to initial wealth. The equality between the interest rate and the adjusted discount factor ($R^* = \rho + g^*$) makes the household completely indifferent between consumption and investment, hence the persistence of initial wealth. This equation also defines the steady-state external demand for bonds as the domestic excess demand for assets:

\[
\tilde{b}^* = \left[\tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0\right] e^{-\pi} - (\tilde{h}^* + \tilde{k}^*)
\]

\textsuperscript{6}Formally, this argument runs as follows. The constraint that $\phi^*/(1 - \phi^*) > 0$ and equivalently that $\phi^* < 1$ can be rationalized by the general equilibrium argument. The world as a whole is in autarky, that is $\tilde{b}_W = 0$. According to (10), we have:

\[
\tilde{k}_W^* = \frac{\phi_W^*}{1 - \phi_W^*} \frac{(1 - \alpha + \alpha \sigma_W)(\tilde{k}_W^*)^\alpha}{R^* - g^* - n_W}
\]

Since $\tilde{k}_W^*$ is positive, $\phi_W^*/(1 - \phi_W^*)$ is necessarily positive and therefore $\phi_W^* < 1$. This is made possible by the adjustment of the world interest rate $R^*$ in order to clear the world bond market. If we assume, as in the calibration section, that $\sigma > \sigma_W$, then $\phi^* < \phi_W^*$. It is therefore consistent with the portfolio approach to assume that $\phi^* < 1$. 

12
Notice that (13) is equivalent to (12) when $\sigma$ goes to zero (i.e. when $(1 - \phi^*)/\phi^*$ goes to -1), plus an additional term that depends on initial wealth $\left[\tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0\right]e^{-\pi}$.

First, because households want to maintain their wealth constant, $\left[\tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0\right]e^{-\pi}$ is the long-run demand for assets. On the opposite, in the portfolio approach, because the aggregate risk premium on the portfolio depends on its composition, the household is indifferent between investment and consumption only for a particular portfolio structure. As a consequence, the long-run portfolio composition, and hence bond holdings, are independent of initial wealth, contrary to the riskless approach.

Second, as in the portfolio approach, human wealth $\tilde{h}^*$ constitutes a supply of assets. However, $\tilde{k}^*$ defines a supply, and not a demand of assets, because the precautionary motive is absent.

So far, the portfolio approach exhibits two main differences with regards to the riskless one. First, the relationship between safe assets and capital in the long run is positive, whereas it is negative in the riskless approach. Second, the composition of the portfolio is determined in the long run. These two elements contribute to alter the relationship between investment and capital flows.

### 2.5 Capital flows: from the riskless to the portfolio approach

Following the method of Gourinchas and Jeanne (2007), the model is confronted with the data observed over a finite period $[0,T]$. Before deriving the level of bonds predicted by the model, some assumptions must be made. First, we abstract from unobserved future developments in productivity by assuming that all countries have the same productivity growth rate $g^*$ after $T$.

**Assumption 1:** $\pi_t = \pi$ for all $t \geq T$.

When $\sigma = 0$, $\tilde{k}_t = \tilde{k}^*$ for all $t$. The steady state is reached immediately. However, when $\sigma > 0$, $\tilde{k}_t$ is contingent on time, which makes it impossible to abstract from future $\tilde{k}_t$, except if $\tilde{k}_T$ is sufficiently close to the steady state. In the remainder of the analysis, it is therefore assumed that $T$ is sufficiently large to be able to make the following approximation: $\tilde{k}_t = \tilde{k}^*$ for all $t \geq T$.

Denote by $\Delta B/Y_0 = (B_T - B_0)/Y_0$ the amount of capital outflows between 0 and $T$. Under Assumption 1 and for $T$ large, it can be written as follows:

$$\frac{\Delta B}{Y_0} = e^{\pi + (n + g^*)T} \frac{\tilde{b}^*}{y_0} - \frac{\tilde{b}_0}{y_0}$$

(14)
In order to distinguish the predicted capital flows according to the riskless and portfolio approaches, denote the former $\Delta B^R/Y_0$ and the latter $\Delta B^P/Y_0$. By replacing $b^*$ as given by equations (11) and (10) in equation (14), we get the following proposition:

**Proposition 2:** Under Assumption 1 and the stationarity conditions of Proposition 1, and for $T$ sufficiently large, the ratio of cumulated capital inflows to initial input is given by:

(i) If $\sigma = 0$:

$$\frac{\Delta B^R}{Y_0} = \frac{1}{k_0^\alpha} \left[ -e^{\pi(n+g^*)T} \frac{k^*}{k_0^\alpha} - e^{\pi+\pi(n+g^*)T} \frac{(1 - \alpha + \alpha \tau)k^*}{k_0^\alpha} \int_0^T e^{-(\rho-n)t} (1 - e^{\pi(n+\pi)\tau}) dt \right] \text{ Effect of change in capital stock}$$

$$+ (e^{\pi(n+g^*)T} - 1) \frac{\tilde{b}_0}{k_0^\alpha} \text{ Effect of change in human wealth}$$

(ii) If $\sigma > 0$:

$$\frac{\Delta B^P}{Y_0} = \frac{1}{\phi^* \phi^*} \left[ e^{\pi(n+g^*)T} \frac{k^*}{k_0^\alpha} - e^{\pi+\pi(n+g^*)T} \frac{(1 - \alpha + \alpha \tau)k^*}{k_0^\alpha} \right] \frac{\tilde{b}_0}{k_0^\alpha} \text{ Effect of capital stock}$$

$$- e^{\pi+\pi(n+g^*)T} \frac{(R^* - g^* - \pi)k_0^\alpha}{k_0^\alpha} \text{ Effect of human wealth}$$

Equations (15) and (16) give the predicted capital outflows as a function of $n, g^*, \rho, R^*, \pi, \tau$, the sequence of productivity catch-up $\{\pi_t\}_{t=1,..,T}$ and initial values $\tilde{b}_0$ and $\tilde{k}_0$. Note that $\tilde{k}^*$ is also a function of $\tau$. They decompose predicted flows into a capital and human wealth component, plus a term depending on initial bond holdings, according respectively to the riskless and portfolio approaches. Notice that these components are similar, except that they are in level in the portfolio approach whereas they are in difference in the riskless approach. This is because, as we have seen, initial wealth is persistent in the riskless approach but not in the portfolio approach.

Consider first Equation (15), which presents the decomposition of capital flows in the absence of risk. The first term reflects an investment effect. It represents the impact of the change in capital stock. The difference $\tilde{k}^* - \tilde{k}_0$ is the amount immediately borrowed by the country to
equalize its private return to capital to the world’s interest rate. Then the country increases its investment as TFP grows, always through borrowing, in order to maintain this equality.

The second term reflects consumption smoothing. It is equal to the change in human wealth. Indeed, if the household borrows if he expects higher labor revenues in the future. This consumption smoothing term depends in a complicated way on $\pi_t$. In order to simplify the problem, as in Gourinchas and Jeanne, the following assumption is made:

**Assumption 3:** $\pi_t = f(t)\pi$ where $f(.)$ is common across countries and satisfies $0 \leq f(t) \leq 1$ and $\lim_{t \to \infty} f(t) = 1$.

Under Assumption 3, we can rewrite the human wealth term in the riskless approach as:

$$-e^{\pi + (n + g^*)T} \left( \frac{1 - \alpha + \alpha \tau}{\bar{k}_0} \right) \bar{k} \alpha \int_0^T e^{-(\rho - n)t} (1 - e^{\pi(f(t) - 1)}) dt$$

In the Appendix, I show that this term depends negatively on the long-run productivity catch-up $\pi$ as long as $\pi > -100\%$, which is a weak assumption. Faster relative productivity growth implies higher future income, leading to an increase in consumption and a decrease in savings. As a result, the external position deteriorates, including in the long run.

The third term is a trend component. It reflects the amount of capital flows necessary to maintain the ratio of bonds to production constant.

Gourinchas and Jeanne (2007) used these arguments to show that, according to the neoclassical growth model (without risk), capital outflows should be correlated negatively with TFP growth, both through the consumption smoothing and investment effects. This prediction of the model is at odds with the data, which contributes to the failure of the standard model in predicting accurately the direction of capital flows. However, as we will show, the portfolio approach differs in several aspects from the riskless approach.

Consider now Equation (16), which provides the decomposition of capital flows in the portfolio approach. The first term represents the effect of the capital stock on the demand for safe assets. As argued above, with a reasonable amount of investment risk, this term is positive, reflecting a precautionary saving motive. This term depends on the TFP-adjusted per-capita stock of capital $\bar{k}$ and on TFP catch-up $\pi$. Indeed, risk exposure is proportional to the total capital stock, which depends positively on these terms.

The second term reflects the effect of the domestic supply of safe assets. Countries with high investment levels have higher capital outflows only to the extent that their domestic agents do not
provide sufficient safe assets, which are represented by human wealth. As a result, TFP catch-up \( \pi \) or from higher TFP-adjusted per-capita stock of capital \( \hat{k}^* \) will have a negative impact on capital outflows because they affect positively future revenues and therefore human wealth.

Capital flows can actually also be decomposed into an investment, a consumption smoothing and a trend component, just as in the riskless approach, but with an additional term, which reflects portfolio adjustment:

\[
\frac{\Delta B^P}{Y_0} = \underbrace{\frac{1 - \phi^* e^{(n+g^*)T}}{\phi^*} e^{\pi \hat{k}^* - \hat{k}_0} \left( 1 - \frac{k^0}{k_0^{\alpha}} \right)}_{\text{Effect of change in capital stock}} - \underbrace{e^{\pi + (n+g^*)T} \left( 1 - \alpha + \alpha \tau \right) \frac{k^0}{k_0^{\alpha}} \int_0^T e^{-(R^*-(n+g^*))t} (1 - \frac{\hat{k}^0}{k^{\alpha}} e^{\pi \hat{k}^* - \pi}) dt}_{\text{Effect of change in human wealth}} \\
\underbrace{\left. (e^{(n+g^*)T} - 1) \frac{b_0}{k^0} \left( \frac{1}{\phi^*} - \frac{1}{\phi_0} \right) e^{(n+g^*)T} \frac{k_0}{k^0^{\alpha}} \right)}_{\text{Portfolio adjustment}}
\]

See the Appendix for the derivation. The three terms that are common to both approaches are completely offset by the adjustment of the portfolio towards the desired composition, which is determined. In the riskless approach, because the composition of the portfolio is undetermined, this last term is absent. The first three terms then persist in the long run. Besides, the investment term is positive in the portfolio approach. It corresponds to the need for safe assets necessary to self-insure against the additional capital stock.

To sum up, in the portfolio approach, the investment and consumption smoothing effects are completely offset in the long run by a precautionary motive to hold safe assets. Investment has therefore a positive effect on capital outflows through the need for precautionary savings. A positive correlation between growth and capital outflows could then emerge, as opposed to the riskless approach. However, the impact of the demand for safe assets can be counteracted by a human wealth effect. Indeed, human wealth is a domestic asset that could satisfy these precautionary needs. The remainder of the analysis examines what is the resulting effect of investment on capital flows, depending on the factors driving investment: differences in TFP or differences in the capital wedge.

2.6 The role of productivity

In the portfolio approach, the effect of \( \pi \) is ambiguous. The precautionary demand for safe assets depends positively on \( \pi \) through the capital stock, but the domestic supply is negatively related to it through human wealth. However, the following proposition can be stated:
Proposition 3: Suppose that $\sigma > \rho - R^* + g^*$, that $T$ is sufficiently large, that the stationarity conditions of Proposition 1 and Assumption 1 are satisfied. Consider two countries A and B, identical except for their long-run productivity catch-up $\pi$. Then there are two cases:

(i) If $A$ and $B$ have a positive long-run external position ($\tilde{b}^* > 0$), then country A sends more capital outflows than country B if and only if country A catches up faster than country B towards the world technology frontier:

$$\Delta B^A > \Delta B^B \iff \pi^A > \pi^B$$

(ii) If $A$ and $B$ have a negative long-run external position ($\tilde{b}^* < 0$), then the opposite holds:

$$\Delta B^A > \Delta B^B \iff \pi^A < \pi^B$$

This result comes from the definition of capital outflows (14) and the fact that $\tilde{b}^*$ does not depend on $\pi$ in the portfolio approach (see Proposition 1). Therefore, for given parameters, $\frac{\Delta B}{Y_0}$ is increasing in $\pi$ if $\tilde{b}^*$ is positive and decreasing otherwise. This result is the cross-country, growth-accounting counterpart of the “New Rule” introduced by Kraay and Ventura (2000).

Regarding the role of productivity, the portfolio approach can help explain the puzzle only to the extent that countries with positive long-run external positions drive the observed positive correlation between growth and capital outflows.

2.7 The role of the capital wedge

In this subsection we show that the TFP-adjusted capital-labor ratio affects capital flows differently than TFP does, which could contribute to solve the puzzle. In our model, this variable is isomorphic to the capital wedge $\tau$, for given risk $\sigma$ (see equation(9)). The capital wedge explains precisely the amount of capital per head that is not accounted for by TFP. We now examine therefore the effect of a variation in $\tau$ on capital flows.

Proposition 4: Suppose that $\sigma > \rho - R^* + g^*$, that $T$ is sufficiently large, that the stationarity conditions of Proposition 1 and Assumption 1 are satisfied. Consider two countries A and B, identical except for their capital wedge $\tau$, then there are two cases:

(i) If $A$ and $B$ satisfy $\tilde{b}^* \geq - (1 - \alpha) \frac{\tilde{k}^{\alpha}}{n - g^*}$, then country A sends more capital outflows than country B ($\Delta B^A > \Delta B^B$) if and only if country A has a lower capital wedge than country B ($\tau^A < \tau^B$):

$$\Delta B^A > \Delta B^B \iff \tau^A < \tau^B$$
(ii) If $A$ and $B$ do not satisfy $\tilde{b}^* \geq -(1 - \alpha) \frac{k^*}{R - n - g}$, the opposite holds.

$$\Delta B^A > \Delta B^B \iff \tau^A > \tau^B$$

The proof is provided in the appendix. The way capital outflows are related to non-TFP-driven investment (i.e. to the capital wedge) depends on a condition on $\tilde{b}^*$, that is the long-run external position, as in the case of TFP-driven investment. However, the condition for non-TFP-driven investment to generate capital outflows is less restrictive than the condition for TFP-driven investment to generate capital outflows (see Proposition 3).

### 2.8 An extended “New Rule”

To understand the different impacts of the TFP and capital wedge margins of investment on capital flows, consider the long-run capital-labor ratio $k^*$. This capital-labor ratio can be decomposed into TFP $A$ and TFP-adjusted capital-labor ratio $\tilde{k}^*$ (which depends on $\tau$): $k^* = A\tilde{k}^*$. A variation in $k^*$ can either be originated in changes in $A$ or changes in $\tilde{k}^*$: $\partial k^* = k^* \partial A + A \partial \tilde{k}^*$.

According to Proposition 1, the long-run ratio between safe and risky assets $\frac{b^* + \tilde{h}^*}{k^*}$ is constant across countries with identical amounts of risk. Denote by $\partial k^*$, $\partial b^*$ and $\partial h^*$ the difference in capital, external bond holdings and human wealth per capita in the long-run between two countries. The following relationship can be derived:

$$\frac{\partial b^*}{\partial k^*} = \frac{\tilde{b}^* + \tilde{h}^*}{\tilde{k}^*} - \frac{\partial h^*}{\partial k^*}$$ (18)

where $\frac{\tilde{b}^* + \tilde{h}^*}{\tilde{k}^*}$ is a constant.

Without human wealth, Equation (18) boils down to the “New Rule” (Kraay and Ventura, 2000):

$$\frac{\partial b^*}{\partial k^*} = \frac{\tilde{b}^*}{\tilde{k}^*}$$

Differences in long-run external assets are equal to the ratio of external assets to capital multiplied by the differences in long-run capital stock. If countries have the same initial external position, this New Rule applies also to capital outflows. Therefore, the correlation between capital outflows and investment across countries should not differ systematically from the average long-run ratio between the net external position and the capital stock. The New Rule does not hold as an explanation of the allocation puzzle because a positive correlation between capital outflows and investment is not compatible with the average negative external position that we observe in developing countries.
With human wealth, Equation (18) gives an extended New Rule: differences in long-run external assets are equal to the ratio of total safe assets to capital multiplied by the differences in long-run capital stock, minus differences in human wealth. If we abstract from the correlation between investment and human wealth, the correlation between capital outflows and investment across countries should then be higher than the average long-run ratio between the net external position and the capital stock. If the correlation between capital outflows and investment is not too strong, then the extended New Rule could provide an explanation to the allocation puzzle. Indeed, the assumption of higher risk in developing countries, which implies a positive correlation between capital outflows and investment, is not incompatible with the observed negative external positions. On the opposite, because it ignores human wealth, the New Rule underestimates the need for safe assets in developing countries.

However, the extended New Rule is indeed a consistent explanation of the puzzle only if human wealth does not comove too much with investment. The ways investment interacts with human wealth through the TFP and capital wedge margins explains the different results of Propositions 3 and 4. Human wealth can actually be written as \( h^* = A\hat{h}^* = A\chi\hat{k}^{*\alpha} \), with \( \chi \) a constant.\(^{10}\)

Consider first the effect of investment on \( b^* \) through the TFP margin (i.e. the effect of TFP-driven investment): \( \partial k^* = \hat{k}^* \partial A \). Then human wealth reacts proportionally to investment: \( \frac{\partial h^*}{k^* \partial A} = \frac{\hat{h}^*}{k^*} \). As a consequence, the external position also reacts proportionally to investment:

\[
\frac{\partial b^*}{\partial k^*} = \frac{\hat{b}^* + \hat{h}^*}{k^*} - \frac{\partial h^*}{k^* \partial A} = \frac{\hat{b}^* + \hat{h}^*}{k^*} - \frac{\hat{h}^*}{k^*} = \frac{\hat{b}^*}{k^*}
\]

The portfolio composition between capital, external assets and human wealth is identical across countries. We then recover the New Rule, despite of the presence of human wealth.

Consider now the effect of investment on \( b^* \) through the capital wedge margin (i.e. the effect of non-TFP-driven investment): \( \partial k^* = A\hat{k}^* \). Then human wealth reacts less than proportionally to investment: \( \frac{\partial h^*}{\partial k^*} = \frac{\hat{h}^*}{k^*} = \frac{\hat{h}^*}{k^*} \). As a consequence, the external position reacts more than proportionally to investment:

\[
\frac{\partial b^*}{\partial k^*} = \frac{\hat{b}^* + \hat{h}^*}{k^*} - \frac{\partial h^*}{A \partial k^*} = \frac{\hat{b}^* + \hat{h}^*}{k^*} - \frac{\hat{h}^*}{k^*} = \frac{\hat{b}^* + (1 - \alpha)\hat{h}^*}{k^*} \geq \frac{\hat{b}^*}{k^*}
\]

The correlation between investment and capital outflows can be positive for negative external positions. The portfolio composition between capital and safe assets is identical across countries, as for TFP-driven investment, but not the composition of safe assets between human wealth and external assets. Indeed, because of marginal returns to capital, production and therefore labor

\(^{10}\)For expositional clarity, we neglect the effect of \( \tau \) on \( \chi \).
income and human wealth react less than proportionally to investment through the capital wedge margin. Countries with high investment originated in lower capital wedge will then increase their demand for safe assets without increasing their supply proportionally. Therefore, the condition for investment leading to capital outflows is less restrictive than in the TFP case.

3 The riskless versus the portfolio approach

In this section, the model is calibrated in order to compute the amount of capital outflows predicted by the riskless and portfolio approaches for a sample of developing countries between 1980 and 2003. We assess whether each approach accurately allocate capital flows between regions and then between countries.

3.1 Calibration

In order to construct the estimated capital flows, we need either to measure or calibrate the parameters on the right-hand side of (15) and (16).

3.1.1 Data and calibration method

In order to facilitate the comparison with Gourinchas and Jeanne (2007), the same sample of 69 emerging countries is used. However, Jordan and Angola are removed from this sample because their working-age population does not satisfy \( n < \rho \). The sample is therefore reduced to 67 countries.

The parameters which are common across countries also follow their paper: the depreciation rate \( \delta \) is set to 6\%, the capital share of output \( \alpha \) to 0.3 and the growth rate of world productivity \( g^* \) to 1.7\%. Only the discount rate \( \rho \) is set to a higher value of 5\% (instead of 4\%) in order to accommodate high growth rates of labor in the data.\(^\text{12}\) The time span is extended to 1980-2003.

\(^{11}\)This sample includes: Angola, Argentina, Bangladesh, Benin, Bolivia, Botswana, Brazil, Burkina Faso, Cameroon, Chile, China, Colombia, the Republic of Congo, Costa Rica, Cyprus, Côte d’Ivoire, Dominican Republic, Ecuador, Egypt, Arab Republic, El Salvador, Ethiopia, Fiji, Gabon, Ghana, Guatemala, Haiti, Honduras, Hong Kong, India, Indonesia, Iran, Israel, Jamaica, Jordan, Kenya, Republic of Korea, Madagascar, Malawi, Malaysia, Mali, Mauritius, Mexico, Morocco, Mozambique, Nepal, Niger, Nigeria, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Senegal, Singapore, South Africa, Sri Lanka, Syrian Arab Republic, Taiwan, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Uganda, Uruguay and Venezuela.

\(^{12}\)Indeed, in the portfolio approach, the world interest rate (i.e. \( R^* \)) is lower than \( \rho + g^* \). If \( \rho \) is too small, then we could have \( R^* - (g^* + n) \) negative or very close to zero. In the first case, capital flows are not well defined; in the second, their variance goes to infinity.
(instead of 1980-2000) in order to encompass the recent surge in capital outflows from developing countries and to shed some light on the “global imbalances” debate.

In order to determine the exogenous interest rate, we make the hypothesis that each country is too small to influence the world’s demand for bonds. We also assume that the world is composed exclusively of developed countries with zero labor growth and no distortions to the marginal capital return. The world interest rate then corresponds to the autarky steady-state interest rate with \( n = 0 \) and \( \tau = 0 \). In the portfolio approach, the amount of risk \( \sigma \) in developed countries is set to 0, which is the amount of entrepreneurial risk commonly reported by empirical studies in the US and the Euro area (Campbell et al., 2001; Kearney and Poti, 2006). This gives \( \phi^* = 13.1\% \) and \( R^* = 6.55\% \). If we extend this calibration approach to the case without risk, we get \( R^* = \rho + g^* = 6.7\% \), which is the long-run value of the autarky interest rate when \( \sigma = 0 \).

The amount of risk in emerging countries is set to \( \sigma = 0.6 \), in line with Koren and Tenreyro (2007), who find that the amount of both macroeconomic and idiosyncratic (sectoral) risk is roughly twice as large in developing countries as in industrial ones.

The country-specific data are the paths for output, capital, productivity and working-age population. These data come from Version 6.2 of the Penn World Tables (Heston et al., 2006). Following Gourinchas and Jeanne (2007) and Caselli (2004), the capital stock is constructed with the perpetual inventory method from time series data on real investment. The level of productivity \( A_t \) is calculated as \( (y_t/k_t)^{1/(1-\alpha)} \) and the level of capital per efficient unit of labor \( \tilde{k}_t \) as \( (y_t/k_t)^{1/(1-\alpha)} \). The level of TFP \( A_t \) and the capital per efficient unit of labor \( \tilde{k}_t \) are filtered using the Hodrick-Prescott method in order to suppress business cycles. The parameter \( n \) is measured as the annual growth rate of the working-age population. Under Assumption 1, the long-term catch-up \( \pi \) can be measured as \( \ln(A_T) - \ln(A_0) - Tg^* \).

The capital wedge \( \tau \) is calibrated in order to account exactly for the steady-state TFP-adjusted capital-labor ratio. We use the fact that, assuming that \( T = 20 \) is a sufficiently large number, \( \tilde{k}^* \) is approximately equal to \( \tilde{k}_T \). We thus take \( \tilde{k}^* = \tilde{k}_T \). Then \( \tau \) is used to adjust the private marginal return on capital to the world interest rate, which shuts down the Lucas paradox: \( \tau = 1 - \tilde{k}_T^{(1-\alpha)} \frac{R^*+\delta+\sqrt{\sigma^2(\rho+g^*-R^*)}}{\alpha} \). This method assigns a high capital wedge to countries with low end-of-period TFP-adjusted capital-labor ratio.

Finally, actual capital flows are taken from Lane and Milesi-Ferretti (2006)’s net foreign asset data. They provide estimates for the net external position in current US dollars.\(^{13}\) Actual capital flows

\(^{13}\)These estimates are calculated using the cumulated current account data and are adjusted for valuation effects. In order to be consistent with the PPP-adjusted data used here, a PPP deflator is extracted from the Penn World Table and is used to calculate a PPP-adjusted measure of net external position.
outflows during the period, as a share of initial output, are denoted $\frac{\Delta B}{Y_0}$. These estimates are confronted with the predicted values given by the riskless and portfolio approaches, respectively $\frac{\Delta B^R}{Y_0}$ and $\frac{\Delta B^P}{Y_0}$.

### 3.1.2 Some key parameters

Table 1 sums up some key parameters given by the calibration method. Countries are classified by region. For robustness checks, China is excluded from the Asian group.

Consider the long-run productivity catch-up $\pi$ in column (1) of Table 1. On average, non-OECD countries have fallen behind the world frontier in terms of productivity. When looking into details, only high income economies and Asia have caught up with the world productivity. Both Africa and Latin America fell behind.

Column (3) presents the average growth rate of capital $\Delta \tilde{k}$. The main observation is that the capital stock decreased on average. Consistently with Gourinchas and Jeanne (2006, 2007), emerging countries were not capital-scarce but capital-abundant. Among regions, only Asia increased its capital per efficient unit of labor.

Consider now column (4). When calibrated inside the riskless framework, the average wedge $\tau$ on the net capital return is equal to 38%, which is consistent with the average wedge on gross capital return of 12% found in Gourinchas and Jeanne (2007). Africa, which has the smallest end-of-period capital level, has therefore the highest estimated capital wedge, while Asia’s estimated capital wedge is the smallest, since it benefits from a high end-of-period capital level. The capital wedge calibrated inside the portfolio framework is given by column (5). It is lower than the one reported in column (4) because the risk premium accounts partially for the low levels of capital. This leaves unchanged the regions’ ranking.

### 3.2 The riskless approach

In this subsection we investigate whether our findings are consistent with those of Gourinchas and Jeanne. That is: without risk, do we have an allocation puzzle? Figure 2 summarizes the outcome of the riskless approach across regions. The upper panel reports the actual net capital outflows as a share of initial output $\Delta B/Y_0$: their size is $-46\%$ on average, which means that emerging countries have received net capital inflows during the period. The middle panels report the predicted capital outflows based on equation (15). These estimates are constructed under the hypothesis that the productivity catch-up follows a linear trend: $\pi_t = \pi \min\{t/T, 1\}$. Our results are in line with Gourinchas and Jeanne (2007). According to the model, non-OECD countries
should have received capital inflows on average, which is the case. Moreover, average predicted flows in non-OECD countries are of the same order of magnitude as the actual ones.

However, while satisfying in terms of global trends, the picture is more contrasted when considering the direction and magnitude of flows inside the sample. According to the predictions, Africa and Latin America should have exported capital while Asia should have received capital inflows (middle panel of Figure 2). This is because Asia has benefited from high TFP catch-up during the period while the other regions have fallen behind the world frontier (see Table1). As a consequence, the former have negative capital and human wealth effects, while these effects are positive in the latter. Asia should have borrowed from the rest of the world both to take advantage of local investment opportunities and to smooth consumption in the face of rising revenues. Africa and Latin America should have exported capital to seek better returns and to smooth consumption in the face of decreasing revenues (relatively to the rest of the world). Actually, the opposite happens (upper panel of the same figure).

However, if we abstract from the sign of capital flows, Latin America and Africa are correctly ranked by the model. Latin America experienced slower TFP growth than Africa and should then have exported (imported) more (less) capital, which is consistent with the data. Therefore, Asia appears as an outlier inside the riskless approach.

On the whole, capital seems to flow in the right direction, except for Asia. The same idea emerges from Figure 3, which shows the scatter plot of actual versus predicted flows. The correlation appears as negative on the whole, which represents the allocation puzzle. However, this negative correlation is driven by a set of countries dominated by Asia, in the upper left of the graph. If we excluded these countries, the correlation would be positive. Explaining the allocation puzzle therefore entails to explain why Asian countries do not behave as the others.

### 3.3 The portfolio approach

We have shown that the model without risk reproduces the allocation puzzle, and that this allocation puzzle is mainly driven by Asia. We now turn to the extension with risk, and examine whether it solves this anomaly. It appears that, contrary to the riskless approach, Asia is not an outlier in the portfolio approach. More importantly, this is not at the expense of another outlier, since Latin America and Africa are still correctly ranked. This is because the portfolio approach does not simply reverse the link between growth and capital flows. It also focuses on another source of investment, namely the capital wedge and not TFP.

The lower panel of Figure 2 reports the estimated predicted net outflows according to equation
The estimates are computed under the assumption that the productivity catch-up follows a linear path, as in the riskless approach. The path of capital per efficient unit of labor $\tilde{k}_t$ implied by the model is approximated by the following formula: $\tilde{k}_t = \tilde{k}_0 e^{\ln(\tilde{k}^*/\tilde{k}_0)(1-\lambda)}$, where $1-\lambda$ is the convergence rate estimated from the data (that is $1-\lambda = 0.3$).\textsuperscript{14}

Note first that the magnitude of predicted flows is above the actual ones (upper panel) by several orders of magnitude. This is a shortcoming of the portfolio approach that has been already highlighted in Kraay et al. (2005). It can also be related to the home bias in portfolio (Lewis, 1999). But this shortcoming is accentuated here by the presence of potentially huge human wealth effects, due to labor and productivity growth. Besides, the human wealth and investment effects have a higher magnitude in the portfolio than in the riskless approach because, in the latter, the change in bonds is a function of the change in capital stock and of the change in human wealth; while in the former, it is a function of the levels of capital and human wealth (see Equations (14) and (15)).

When abstracting from the magnitude issue, it appears that the predicted outflows in the lower panel of Figure 2 exhibit the right sign, which is negative, and, contrary to the riskless approach, the right ranking between regions. According to the model’s predictions, Africa and Latin America should receive capital inflows while Asia should export capital, as in the data. Additionally, the model predicts accurately that Africa should receive more capital inflows than Latin America.

The capital and human wealth terms, in other words the demand and supply for assets, interact in a subtle way to yield these results. For example, consider Asia and Latin America. Asia has both a larger capital term and a larger human wealth term than Latin America. However, the difference between their capital terms is proportionally larger than the difference between their human wealth terms. As a result, Asia exports while Latin America imports capital. This is due to the fact that, besides a higher TFP catch-up, Asia has a lower capital wedge than Latin America (see Table 1). The difference in TFP catch-up affects proportionally the capital and human wealth terms. If only TFP catch-up was at play, then Asia and Latin America would have had the same sign of external position. However, in Asia, a lower wedge (equivalently, a larger stock of capital) increases more than proportionally the capital term than it increases the human wealth term. The same argument applies when comparing Asia to Africa.

This analysis sheds some light on a neglected issue in the studies on global imbalances. Namely: why are these imbalances originated in Asia and not in other developing regions. An

\textsuperscript{14}The assumed trend is a good proxy for the capital dynamics since the theory predicts that it moves smoothly from $\tilde{k}_0$ to $\tilde{k}^*$.\n
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answer suggested here is that Asia has been accumulating capital in a large scale with limited benefits to labor, because this capital accumulation was only partially originated in TFP growth. The subsequent demand for safe assets has then not been compensated by a matching domestic supply. This resulted in capital outflows. On the opposite, when adjusting for TFP growth, Africa and Latin America have accumulated limited amounts of capital, which implies that their human wealth is large relative to their capital stock. Their demand for safe assets is then low relative to their domestic supply, leading to capital inflows.

Not only does the portfolio approach rationalize the behavior of Asia relative to other regions, but it also explains the difference between Africa and Latin America. Both regions have low TFP-adjusted capital-labor ratios, but this is especially true for Africa, which has a particularly large capital wedge. Human wealth is then particularly disproportionate as compared to the capital term in Africa (see lower panel of Figure 2). This results in larger capital inflows in Africa than in Latin America. Interestingly, Latin America and Africa have differing capital inflows while having a comparable capital stock (see the investment terms in the lower panel of Figure 2). But what is crucial to explain capital flows here is the source of investment (TFP versus capital wedge), not its level.

The portfolio approach seems therefore to be a better predictor, if not of the magnitude of flows, at least of their direction. Figure 4 sums up this idea by plotting predicted flows according to the portfolio approach against the actual ones. A positive correlation appears. Besides, this correlation is not driven by Asian countries, contrary to the riskless approach.

4 Empirical investigation

The main objective of this empirical section is to understand the sources of the puzzle and of its resolution in the data.

First, the portfolio approach predicts accurately the direction of flows because it accounts better than the riskless approach both for the unstable link between TFP growth and capital outflows and for the positive link between the TFP-adjusted capital-labor ratio (equivalently the capital wedge) and capital outflows. We show this by testing a structural linear approximation of the decomposition of capital flows in the data.

Second, this suggests that, since only the TFP-adjusted capital-labor ratio correlates positively with both investment and capital outflows, the capital wedge is the channel of the observed correlation between investment and capital outflows. This hypothesis is tested and validated by introducing the potential correlates of capital flows in a regression including investment.
4.1 Determinants of predicted and observed flows

Here we show that the portfolio approach accounts better both for the relation of capital flows with TFP growth and with the TFP-adjusted capital-labor ratio. We achieve this by showing that these variables correlate with the observed flows just as the portfolio approach predicts.

The decomposition of capital flows in the portfolio approach (16) can be log-linearized as follows around the cross-country averages $\pi, (n + g^*)T, \ln(\bar{y}_0)$ and $\ln(\bar{k}^*)$:

$$\frac{\Delta B^P}{Y_0} = \frac{B}{Y_0} + \frac{B}{Y_0} \left[ \left( \pi + (n + g^*)T - \ln(\bar{y}_0) \right) - \left( \pi + (n + g^*)T - \ln(\bar{y}_0) \right) \right] + \frac{B + H}{Y_0} \left( \ln(k^*) - \ln(\bar{k}^*) \right) \frac{B_0}{Y_0}$$  \hspace{1cm} (19)

where $\frac{B}{Y_0}$ and $\frac{B + H}{Y_0}$ are respectively the implied end-of-period net foreign assets and safe assets evaluated at the country averages. Based on this approximation, the following structural equation can be tested in our sample:

$$\frac{\Delta B}{Y_0} = \gamma_1 + \gamma_2 \pi_i + \gamma_3 \ln(\bar{k}^*)_i + \gamma_4 (n + g^*)T_i + \gamma_5 \ln(\bar{y}_0)_i + \gamma_6 \frac{B_0}{Y_0} + \epsilon_i$$  \hspace{1cm} (20)

with $i$ the country index, $\gamma_1 - \gamma_6$ the coefficients to estimate and $\epsilon_i$ an error term. Table 2 estimates this equation with successively the predicted flows in the riskless and portfolio approaches as dependent variables then the observed flows between 1980 and 2003 for the same sample as the one used for the calibration analysis.

First, consider the impact of TFP growth $\pi$ on capital outflows. According to the riskless approach, it should be negative, whereas, according to the portfolio approach (Proposition 3), it should follow the New rule, that is: it should depend on the sign of the average net foreign asset position, as suggested also by Equation (19). Given that this sign can be both positive and negative, this coefficient should be sample-dependent. These predictions are illustrated by columns (1) to (3) of Table 2. In the riskless approach, the coefficient of $\pi$ is significantly negative (column (1)). In the portfolio approach, the coefficient is positive and not significant when estimated on the whole sample (column (2)), but it becomes significantly negative when restricting the sample to countries with predicted negative external positions (column (3)).

Turning to the impact of $\pi$ on the observed flows, it appears that, as predicted by the portfolio approach, it is sample-dependent. When abstracting from the other correlates, it is significantly positive at the 10% level when estimated on the whole sample but it turns significantly negative at the same level when restricting the sample to countries with negative external positions (respectively, columns (4) and (5)). The same happens when we control for the other determinants, only the coefficients are not significant (columns (6) and (7)).

Since the predicted outflows flows in the riskless approach are negatively correlated with TFP
growth and are mainly driven by it (it explains 80% of their variance), this instability suggests that their negative correlation with actual flows was mainly driven by countries with positive end-of-period external positions, among which Asia is well represented. On the opposite, it is in line with the New Rule, as predicted by the portfolio approach, which contributes to its ability to account for the allocation of flows.

Second, consider the impact of the TFP-adjusted capital-labor ratio $\ln(\tilde{k}^*)$, which is isomorphic to the capital wedge $\tau$, on capital outflows. In the riskless approach, its effect is ambiguous, but, as column (1) of Table 2 shows, it is negative in our sample. In the portfolio approach, it should follow the extended New Rule, that is: it should depend on the sign of the average safe assets, as suggested also by Equation (19). This means that this impact is sample-dependent in general, but a positive correlation is possible even for negative external positions. Indeed, the coefficient of $\ln(\tilde{k}^*)$ is positive, both when estimated on the whole sample and when restricting to countries with negative predicted external positions (respectively, column (2) and (3)).

Turning to the impact of $\ln(\tilde{k}^*)$ on observed flows, it is significantly positive (at the 10% level) whether we estimate it on the whole sample or on the sample of countries with negative end-of-period external positions. This is at odds with the riskless approach, which predicts the opposite, but in line with the portfolio approach. The portfolio approach therefore predicts accurately the direction of flows not only because because it accounts for the observed relationship of capital outflows with TFP growth, but also for their relationship with the TFP-adjusted capital-labor ratio.

4.2 The channels of the correlation between investment and capital outflows

Both TFP catch-up $\pi$ and TFP-adjusted capital-labor ratio $\ln(\tilde{k}^*)$ are potential channels of investment. Indeed, the latter can be TFP-driven (through $\pi$) or non-TFP-driven (through $\ln(\tilde{k}^*)$). However, according to the above empirical analysis and consistently with the portfolio approach, only $\ln(\tilde{k}^*)$ correlates robustly with capital outflows. This suggests that the observed correlation that has been documented in the empirical literature between capital outflows and investment is channelled mainly through non-TFP-driven investment.

However, the fact that both investment and capital outflows are related separately to non-TFP-driven investment is not sufficient to explain the allocation puzzle. Indeed, the positive correlation between investment and capital outflows could be driven by other common factors. In that case, the allocation puzzle would not be solved. To test this, we estimate the cross-country

\footnote{These countries are: Botswana, China, Hong-Kong, Iran, Mauritius, Singapore, Taiwan and Venezuela.}
correlation between capital outflows and investment as follows:

\[ \frac{\Delta B}{Y_0} = \frac{\Delta K}{Y_0} + \nu_i \]

where \( \frac{\Delta K}{Y_0} = \frac{K_{T_i} - K_0}{Y_0} \) refers to investment between 1980 and 2003, \( \gamma \) is the coefficient to estimate and \( \nu_i \) is the error term. We then add the determinants of \( \frac{\Delta B}{Y_0} \) given by the structural decomposition (19) and ask if any of these variables soaks the significance of \( \gamma \). The results are provided by Table 3.

As column (1) of Table 3 shows, the unconditional correlation between \( \frac{\Delta B}{Y_0} \) and \( \frac{\Delta K}{Y_0} \) is positive and significant at the 10% level. This is the allocation puzzle. When we add \( \pi \), this coefficient remains stable and significant (column (2)). This means that TFP-driven investment is not at the source of the allocation puzzle. However, when \( \ln(\tilde{k}^*) \) is added, both with and without \( \pi \) (respectively columns (4) and (3)), the significance of \( \gamma \) is soaked up. This suggests that non-TFP-driven investment is at the origin of the positive correlation between capital outflows and investment across countries.

However, the other determinants of capital outflows in (19) can also be simultaneously correlated with investment. Column (5) adds these other determinants, namely: \( \gamma_4(n + g^*)T, \ln(\tilde{y}_0) \) and \( \frac{B_0}{Y_0} \). \( \ln(\tilde{k}^*) \) is not significant anymore, but no other variable is. This result could be driven by multicollinearity. Importantly, as columns (6) to (8) show, no other variable taken separately captures the correlation between capital outflows and investment. This confirms that non-TFP-driven investment is the channel of the allocation puzzle.

5 Robustness: alternative measures of the capital wedge

As we have seen, the variations in non-TFP-driven investment \( \tilde{k}^* \) across countries are crucial to explain capital flows. In our model, these variations are accounted for by a capital wedge \( \tau \). It is set in order to equalize the private marginal returns to capital across countries, thus shutting down the Lucas (1990) puzzle. However, this capital wedge is a black box: it does not say anything about the sources of the differences in TFP-adjusted capital-labor ratios across countries and thus about the sources of the allocation puzzle. Besides, it might be sensitive to the calibration procedure and therefore subject to measurement errors. In this section, we use as alternative measures of the capital wedge the relative price of capital and its share in production. We then investigate whether the puzzle is still solved, that is: does the model still predicts accurately the cross-country direction of flows and do these measures capture the correlation between capital flows and investment?
Following Caselli and Feyrer (2007), we allow for differing prices of capital relative to output and introduce non-reproducible capital as a production factor to measure the share of capital in production more accurately. Caselli and Feyrer show in their paper that these two extensions of the neoclassical growth model are sufficient to account for the cross-country differences in capital-labor ratio. Indeed, in that case, the social marginal return to capital is 

\[ \alpha_k \frac{Y_t}{P_k K} \]

with \( P_k \) the price of investment goods in terms of final goods and \( \alpha_k \) the share of reproducible capital in production, where production is defined as follows:

\[ Y_t = A_t^{1-\alpha_k} K_t^{\alpha_k} X^{\alpha_x} N_t^{1-\alpha_k-\alpha_x} \]

with \( X \) denoting non-reproducible capital and \( \alpha_x \) its share in production. Equalization between the private marginal return to capital and the risk-adjusted return to capital implies then:

\[ (1 - \tau) \alpha_k \frac{Y}{P_k K} = R^* + \delta + \sqrt{\sigma^2(\rho + g^* - R^*)} \]

In our baseline approach, we assumed that \( \alpha_x = 0 \), that \( \alpha_k \) was constant and equal to 0.3 in all countries and that \( P_k = 1 \). Differences in output-capital ratios (equivalently, in TFP-driven investment \( \tilde{k} \)) translated in differences in capital wedge \( \tau \). In this more general setting, differences in output-capital ratios can be accounted for by differences in \( P_k \) and \( \alpha_k \). In particular, a lower share of reproducible capital and a higher relative price of investment lead to a lower private return to capital and therefore require a lower \( \tau \) to match the risk-adjusted interest rate. Caselli and Feyrer demonstrate that when we take \( P_k \) and \( \alpha_k \) into account, \( \alpha_k \frac{Y}{P_k K} \) is practically equalized across countries.

First, we first need measures of \( P_k \) and \( \alpha_k \). We estimate the relative price of capital \( P_k \) as the ratio of the investment price index to the GDP price index from the Penn World Tables 6.2 (Heston et al., 2006). We use the data of 1996, which is the year where the broadest number of countries were surveyed, and which therefore provides the more accurate measures.

To calibrate \( \alpha_k \), following Caselli and Feyrer (2007), we obtain the labor share \( 1 - \alpha_k - \alpha_x \) from Bernanke and Gurkaynak (2001) and derive the total share of capital (reproducible and non-reproducible) \( \alpha_w = \alpha_k + \alpha_x \). We then use the equality between the steady-state perceived private returns on reproducible capital and on non-reproducible capital. Assuming that \( \tau = 0 \), the former is is equal to \( \alpha_k \frac{Y}{P_k K} - \delta - \sqrt{\sigma^2(\rho + g^* - R^*)} \). To infer the private return on non-reproducible capital, we must make assumptions about its risk. For simplicity, we assume that non-reproducible capital is not risky. In that case, its private return is \( (\alpha_x \frac{Y}{X} + \frac{\partial P_x}{\partial t}) / P_x \), where \( P_x \) is the price of non-reproducible capital in terms of final goods. In steady state, the rate of price appreciation for non-reproducible capital should be equal to \( g^* + n \frac{1-\alpha_w}{\alpha_w} \). Equality between
returns then yields:

$$\alpha_k = \left[ \alpha_w + \left( g^* + \frac{1 - \alpha_w}{\alpha_w} + \delta + \sqrt{\sigma^2(\rho + g^* - R^*)} \right) \frac{P_x}{Y} \right] \frac{P_k K}{P_x X + P_k K}$$

By considering Table 2, we can see how these measures relate to the capital wedge across regions. Latin America benefits both from a lower price of reproducible capital $P_k$ and from a higher share $\alpha_k$ than Africa, which explains its relatively lower capital wedge. Asia has the same share as Latin America, but faces a lower price of capital, which explains why Asia has the lowest capital wedge. As a result, if we calculate the capital wedge $\tau$ that would equalize the new private return to capital to the world’s interest rate, we find that the new capital wedge (column (8)) is much smaller (8% on average) than in the baseline calibration. This is in line with the findings of Caselli and Feyrer (2007).

Capital flows are then computed under the hypothesis that $\tau = 0$ but with $P_k$ and $\alpha_k$ taken into account. Figure 5 plots actual capital outflows against these predicted outflows. According to the figure, the correlation between predicted and observed capital flows is still positive. The variations in investment driven by the relative price and by the share of reproducible capital can then explain some of the variations in capital flows across emerging countries.

We then investigate whether $P_k$ and $\alpha_k$ do capture the correlation between capital outflows and investment in the data. We adopt the same approach as in section 4.2. The results are provided by Table 4. In columns (1) to (3), we use our own estimates of $\alpha_k$. In columns (4) to (6), we use the estimates of Gourinchas and Jeanne (2007) for further robustness checks. It appears that the correlation between capital outflows and investment disappears when we introduce $\alpha_k$ and $P_k$, whether we introduce them alone (columns (1) and (4)), with $\pi$ (columns (2) and (5)) or with all the potential determinants of capital flows (columns (3) and (6)). The coefficient of $\alpha_k$ is always positive, as expected, and significant at the 5% or 10% level. The coefficient of $P_k$ is negative, as expected, but significant (at the 10% level) only when we use our own estimates of $\alpha_k$.

This analysis shows that the resolution of the allocation puzzle is robust to the use of alternative measures of the capital wedge. Besides, it provides an intuitive interpretation of the allocation puzzle: both high shares of capital in production and low relative prices distort the domestic portfolio towards capital and away from human wealth, because they both increase the return to capital relative to labor income. This means that they increase the demand for safe assets at the expense of the supply, leading to capital outflows.
6 Conclusion

This paper develops an extension of the traditional neoclassical growth model to risky investment that contributes to match the actual capital flows and to solve the allocation puzzle. It also provides an explanation to global imbalances and to their regional structure. Namely, it explains why capital flows to the North come from Asia and not from other regions with poor financial development as Africa and Latin America. The advantage of this approach is that it does not constitute a great departure from the textbook model and therefore allows the adoption of a development accounting approach.

The next step of this line of research consists in studying whether the portfolio approach can also account for the composition of flows. Extending the model to the possibility to trade equity could lead to predictions in terms of equity holdings. This would entail to change our framework from a small open economy to a multi-country analysis, which is beyond the scope of this paper. However, according to portfolio choice models, the more productive assets should constitute a higher share both in the domestic and foreign portfolio. Investment risk is therefore a good candidate to explain the fact that direct foreign investment is positively correlated with growth, while securities are not, as shown by Prasad et al. (2007).

This study also opens some new empirical questions. First, it would be interesting to reassess the empirical evidence on the allocation puzzle of Aizenman and Pinto (2007) and Prasad et al. (2007), in particular by taking into account the extended New Rule and the factors driving investment.

References


Kraay, AArt and Jaume Ventura (2000), ‘Current accounts in debtor and creditor countries’, 

Kraay, AArt, Norman Loayza, Luis Serven and Jaume Ventura (2005), ‘Country portfolios’, 

Lane, Philip R. and Gian Maria Milesi-Ferretti (2006), ‘The external wealth of nations mark ii: 
Revised and extended estimates of foreign assets and liabilities for industrial and developing 

Lewis, K. (1999), ‘Trying to explain home bias in equities and consumption’, *Journal of Economic 

Lucas, R. E. Jr. (1990), ‘Why doesn’t capital flow from rich to poor countries?’, *American 

516.

Mendoza, E. G., V. Quadrini and J.-V. Rios-Rull (2007), ‘Financial integration, financial deep-
ness and global imbalances’, NBER Working Paper No 12909.

Prasad, E., R. Rajan and A. Subramanian (2007), ‘Foreign capital and economic growth’, *Brook-

Sandri, D. (2008), ‘Growth and capital flows with risky entrepreneurship’, Mimeo, JHU Depart-
ment of Economics.
7 Appendix

Proof of Lemma 1

Maximizing $U_t^i$ is equivalent to maximizing $V_t^i = U_t^i / N_t = E_t \int_{t}^{\infty} e^{-(r-n)(s-t)} \ln(c_t^i)ds$.

Indexes are now dropped for simplicity. Define \( \phi \) such that \( \phi = k/\omega \). The constraint of the maximization problem is therefore: 
\[
d\omega = [(r - R^*)\phi + R^*)\omega - c - n\omega]dt + \sigma\phi\omega dz.
\]

The Bellman equation for this problem is:
\[
(\rho - n)V_t = \max_{c,\phi}\left\{ \ln(c) + \frac{\partial V_t}{\partial t} \right\}
\]

Then, applying Ito’s Lemma, we obtain:
\[
(\rho - n)V(\omega, t) = \max_{c,\phi}\left\{ \ln(c) + \frac{\partial V_t}{\partial t} \left[ (r - R^*)\phi + R^*)\omega - c - n\omega \right] + \frac{\partial^2 V_t}{\partial \omega^2} \frac{1}{2} \phi^2 \omega^2 \sigma^2 \right\}
\]

The first-order conditions of this problem are:
\[
\frac{1}{c} - \frac{\partial V_t}{\partial \omega} = 0
\]
\[
\frac{\partial V_t}{\partial \omega} (r - R^*) + \frac{\partial^2 V_t}{\partial \omega^2} \phi \sigma^2 = 0
\]

An educated guess for the general form of the value function is:
\[
V(\omega, t) = \frac{\ln(\omega)}{\chi} + \psi
\]

where \( \chi \) and \( \psi \) have to be determined.

Substituting the derivatives of the value function into the first order conditions yields the solutions:
\[
c = \chi \omega \quad \text{and} \quad \phi = \frac{r - R^*}{\sigma^2}
\]

Plugging these expressions into the Bellman equation yields \( \chi = \rho - n \) and \( \psi = \ln(\rho - n) + \frac{(r - R^*)^2}{2\sigma^2} + R^* - \rho \). This gives Equations (7) and (8).

Proof of Proposition 1

In order to prove Proposition 1, we first establish the following Lemma:

Lemma 2: Let \( \bar{x}_t = X_t / (A_t N_t) \) denote the value of \( X_t \) in efficient labor terms at the aggregate level. For a given interest rate \( R^* \), the aggregate dynamics of the economy is characterized by:
\[
\bar{c}_t = (\rho - n)\bar{\omega}_t
\]
\[
\dot{\bar{\omega}}_t = r_t \phi_t + R^*(1 - \phi_t) - (\rho + g^* + \bar{n})
\]

where \( r_t = r(\phi_t \bar{\omega}_t) = (1 - \tau)\alpha(\phi_t \bar{\omega}_t)^{\alpha - 1} - \delta \) and \( \phi_t \) defined by equation (8).

Equation (21) is the counterpart of Equation (7) in terms of efficient units of labor.

Equation (22) is obtained from the aggregation of the individual budget constraints (5) written in terms of efficient units of labor and where the wage clears the labor market.
Equations (21) and (22), along with the no-Ponzi conditions and the definitions of \(r_t\) and \(\phi_t\), characterize the dynamics of \(\dot{c}_t\) and \(\dot{h}_t\). Once the paths of these variables are known, \(\dot{h}_t = \phi_t \dot{\omega}_t\), \(\dot{h}_t = \int_0^\infty e^{-(R^*-(n+g^*))s+\pi_t-\pi_t(1-\alpha+\tau\alpha)\tilde{k}^\alpha_t}ds\) and \(b_t = \dot{\omega}_t - \dot{k}_t - \dot{h}_t\) can be determined. However, these equations are used here only to determine steady state.

We now characterize stationarity.

Since, according to Equation (21), consumption is proportional to wealth, the stationarity of consumption \(\dot{c}/c = 0\) implies the stationarity of wealth \(\dot{\omega}/\omega = 0\). Additionally, the stationarity of catch-up \(\pi = 0\), along with the aggregate budget constraint, implies that the long-run share of capital \(\phi^*\) satisfies:

\[
\dot{r}^* \phi^* + R^*(1 - \phi^*) - (\rho + g^*) = 0
\]

which is equivalent to:

\[
\frac{(\dot{r}^* - R^*)^2}{\sigma^2} = \rho + g^* - R^*
\]

(23)

Therefore, for the steady state to exist, we must have \(R^* \leq \rho + n\).

Since the share of capital in wealth \(\phi\) is necessarily non-negative, then \(r - R^*\) is also non-negative. Equation (23) therefore yields the expression of the return differential:

\[
\dot{r}^* - R^* = \sqrt{\sigma^2(\rho + g^* - R^*)}
\]

This equation is equivalent to Equation (9).

**Proof of (i):**

Equation (10) derives from the definition of \(\phi\) at steady state:

\[
\phi^* = \frac{\dot{k}^*}{(\dot{k}^* + \dot{\theta}^* + \dot{h}^*)}
\]

(24)

To establish Equation (10), we must then derive the value of the steady-state share of capital \(\phi^*\) and the steady-state human wealth \(\dot{h}^*\).

First, when \(\sigma > 0\), we must have necessarily \(R^* < \rho + n\). This can be shown by noticing that Equation (23) can be rewritten as follows:

\[
\sigma^2 \phi^* = \rho + g^* - R^*
\]

Suppose that \(R^* = \rho + g^*\), this would imply, when \(\sigma > 0\), that \(\phi^* = 0\). If wealth is stationary, this means that the stock of capital converges towards zero. Given the Cobb-Douglas specification, this implies that the return to capital would tend to infinity, which contradicts the fact that the return differential is constant, as suggested by Equation (9). Since, as shown above, \(R^* \leq \rho + g^*\) for the steady state to exist, then we must have \(R^* < \rho + g^*\). As a consequence, Equation (23) implies that:

\[
\phi^* = \sqrt{\frac{\rho + g^* - R^*}{\sigma^2}}
\]

Second, to complete Equation (10), we have to determine \(\dot{h}\) at steady state. \(\dot{h}_t = \int_0^\infty e^{-(R^*-(n+g^*))s+\pi_t-\pi_t(1-\alpha+\tau\alpha)\tilde{k}^\alpha_t}ds = \int_0^\infty e^{-(R^*-(n+g^*))s+\pi_t-\pi_t(1-\alpha+\tau\alpha)\tilde{k}^\alpha_t}ds\). Equation (9) gives \(\dot{k}^*\), the steady-state value of \(\dot{k}\). We have also \(\pi_t = \pi\) in the long run. Therefore,

\[
\dot{h}^* = \frac{(1-\alpha+\tau\alpha)\dot{k}^\alpha}{R^* - (n + g^*)}
\]

Replacing \(\dot{h}^*\) in Equation (24) with the steady-state \(\phi^*\) yields Equation (10).
Proof of (ii):
When \( \sigma = 0 \), the no-arbitrage condition \( r_t = R^* \) is necessarily satisfied for all \( t \). Otherwise, according to the expression of \( \phi_t \) (8), the stock of capital would be infinite. Therefore, the stationarity of wealth implies:

\[
R^* = \rho + g^*
\]

Using (21) and (22) in per capita terms, along with the fact that \( r_t = R^* \) and \( R^* = \rho + g^* \), we obtain the following Euler condition:

\[
\frac{\dot{c}_t}{c_t} = g^*
\]  

(25)

Therefore, \( c_t = c_0e^{\alpha t} \), and \( \bar{c}_t = \tilde{c}_0e^{\alpha t}A_0/A_t = \tilde{c}_0e^{-\eta t} \). As a consequence, we obtain at steady state: \( \tilde{\omega}^* = \tilde{c}_0e^{-\eta} \). We know also that \( \tilde{k}_t = k^* \) always because of the no-arbitrage condition. The stationarity of wealth therefore implies that \( \tilde{b}^* \) is also constant in the long run and satisfies:

\[
\tilde{\omega}^* = \tilde{k}^* + \tilde{b}^* + \tilde{h}^*
\]

Since \( \tilde{\omega}^* = (\rho - n)\tilde{\omega}^* \) and, as we have shown above, \( \tilde{h}^* = (1 - \alpha + \tau \alpha)\tilde{k}^{*\alpha}/(R^* - (n + g^*)) \), this equation can be rewritten as follows:

\[
\tilde{b}^* = -\tilde{k}^* - \frac{(1 - \alpha + \tau \alpha)\tilde{k}^{*\alpha}}{\rho - n} + \frac{\tilde{c}0e^{-\eta}}{\rho - n}
\]

Equation (11) is recovered by replacing \( \tilde{c}_0 \) using \( \tilde{c}_0 = (\rho - n)\tilde{\omega}_0 \).

Proof of Proposition 2
Proof of (i):
Replacing the expression for \( \tilde{b}^*(11) \) in Equation (14) and substituting for \( \tilde{h}_0 \), we obtain:

\[
\frac{\Delta B^R}{Y_0} = -\frac{e^{\pi}\tilde{k}^* - \tilde{k}_0}{k^*_0}e^{(n+g^*)T} - e^{\pi + (n + g^*)T}\frac{(1 - \alpha + \alpha \tau)\tilde{k}^{*\alpha}}{\tilde{k}^*_0}\left(1 - \frac{1}{\rho - n} - \int_0^\infty e^{-(\rho - n)t + \pi t} dt\right)\]

\[
+ \left(e^{(n + g^*)T} - 1\right)\frac{\tilde{h}_0}{k^*_0}
\]

Since \( 1/(\rho - n) = \int_0^\infty e^{-(\rho - n)t} dt \), this expression leads to Equation (15).

Proof of (ii):
We obtain Equation (16) simply by replacing the expression for \( \tilde{b}^*(10) \) in Equation (14):

\[
\frac{\Delta B}{Y_0} = e^{\pi + (n + g^*)T}\frac{1 - \phi^*}{\phi^*}\frac{\tilde{k}^*}{k^*_0} - e^{\pi + (n + g^*)T}\frac{(1 - \alpha + \alpha \tau)\tilde{k}^{*\alpha}}{(R^* - (n + g^*)k^*_0)} - \frac{\tilde{h}_0}{k^*_0}
\]

Another decomposition of capital flows in the portfolio approach
Replacing \( \tilde{h}_0 = \frac{k_0}{\phi^*}\tilde{h}_0 = \tilde{h}_0 \) in (16) with \( \frac{\tilde{h}_0}{k^*_0}\left(e^{(n + g^*)T} - 1\right)\frac{k^*_0}{\phi^*} - e^{(n + g^*)T}\right) \), and rearranging using \( \tilde{h}_0 = \frac{1 - \phi^*}{\phi^*}\tilde{k}_0 - \tilde{h}_0 \), we get:

\[
\frac{\Delta B}{Y_0} =\frac{1 - \phi^*}{\phi^*}e^{(n + g^*)T}\frac{\tilde{k}^* - \tilde{k}_0}{k^*_0} + \left(e^{(n + g^*)T} - 1\right)\frac{\tilde{b}_0}{k^*_0} + \left(1 - \frac{1}{\phi^*}\right)e^{(n + g^*)T}\frac{\tilde{h}_0}{k^*_0}
\]
\[-e^{\pi+(n+g^*)\tau} \frac{1}{k_0^c} \left(1 - \alpha + \alpha \tau \right) f(\tilde{k}^*) \left( \frac{1}{R^* - (n + g^*)} - \int_0^\infty e^{-(R^* - (n + g^*))t + \pi \tau - \pi} dt \right) f(\tilde{k}^*) \]

Since \(1/(R^* - (n + g^*)) = \int_0^\infty e^{-(R^* - (n + g^*))t} dt\), this expression leads to Equation (17).

**Productivity and consumption smoothing**

If \(\sigma = 0\), under Assumption 3, \(\frac{\Delta B^s}{Y_0}\) can be written as follows:

\[
\frac{\Delta B^s}{Y_0} = -e^{(n+g^*)T} \frac{1 - \alpha + \alpha \tau}{k_0^c} \int_0^T e^{-(\rho-n)t} \left( e^{\pi} - e^{\pi f(t)} \right) dt
\]

A sufficient condition for \(\frac{\Delta B^s}{Y_0}\) to be decreasing in \(\pi\) is that \(e^{\pi} - e^{\pi f(t)}\) is increasing in \(\pi\). We have:

\[
\frac{\partial \left( e^{\pi} - e^{\pi f(t)} \right)}{\partial \pi} = e^{\pi} \left[ 1 - f(t)e^{\pi f(t)-1} \right]
\]

Consider the term between brackets:

\[
\frac{\partial \left[ 1 - f(t)e^{\pi f(t)-1} \right]}{\partial f(t)} = -e^{\pi(1-f(t))[1+\pi f(t)]}
\]

A sufficient condition for this derivative to be negative is \(\pi \geq -1\). If it is the case, then for \(0 \leq f(t) \leq 1\):

\[
1 - f(t)e^{\pi f(t)-1} \geq 0
\]

which implies that \(\frac{\partial [e^{\pi} - e^{\pi f(t)}]}{\partial \pi} \geq 0\). As a consequence, if \(\pi \geq -1\), then \(\frac{\Delta B^s}{Y_0}\) is decreasing in \(\pi\).

**Proof of Proposition 4**

Notice that the predicted capital flows must satisfy (14). According to this equation the derivative of \(\frac{\Delta B}{Y_0}\) with respect to \(\tau\) depends only on the derivative of \(\tilde{b}^*\).

Differentiating (10) with respect to \(\tau\), we obtain:

\[
\frac{\partial \tilde{b}^*}{\partial \tau} = \frac{\partial \tilde{k}^*}{\partial \tau} \left[ \frac{1 - \phi^*}{\phi^*} - \frac{(1 - \alpha + \tau \alpha) \alpha k^{\pi(\alpha-1)}}{R^* - n - g^*} \right] - \frac{\alpha k^{\pi \alpha}}{R^* - n - g^*}
\]

In order to infer \(\frac{\partial \tilde{k}^*}{\partial \tau}\), we differentiate Equation (9) with respect to \(\tau\) and get:

\[
\frac{\partial \tilde{k}^*}{\partial \tau} = \frac{\tilde{k}^*}{(1 - \tau)(1 - \alpha)}
\]

Replacing in \(\frac{\partial \tilde{b}^*}{\partial \tau}\), we can show that:

\[
\frac{\partial \tilde{b}^*}{\partial \tau} = \frac{1}{(1 - \tau)(1 - \alpha)} \left[ -\tilde{b}^* (1 - \alpha) \frac{\tilde{k}^{\pi \alpha}}{R^* - n - g^*} \right]
\]

Therefore, \(\frac{\partial \tilde{b}^*}{\partial \tau} \leq 0\) if and only if \(\tilde{b}^* \geq -(1 - \alpha) \frac{\tilde{k}^{\pi \alpha}}{R^* - n - g^*}\).
Table 1: Long-term capital per efficient unit of labor, capital wedge and potential determinants of capital flows

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>π</td>
<td>(\delta_k)</td>
<td>(\Delta\kappa_{t_0})</td>
<td>(\tau^3)</td>
<td>(\tau^2)</td>
<td>(P_k)</td>
<td>(\alpha_k)</td>
<td>(\tau^f)</td>
<td>Obs.</td>
</tr>
<tr>
<td>Non-OECD(^\dagger)</td>
<td>-10%</td>
<td>-30%</td>
<td>-0.67%</td>
<td>38%</td>
<td>28%</td>
<td>2.61</td>
<td>0.34</td>
<td>8%</td>
<td>67</td>
</tr>
<tr>
<td>Sub-Saharan Africa</td>
<td>-16%</td>
<td>-39%</td>
<td>-1.33%</td>
<td>55%</td>
<td>47%</td>
<td>3.43</td>
<td>0.32</td>
<td>9%</td>
<td>28</td>
</tr>
<tr>
<td>Latin America</td>
<td>-38%</td>
<td>-33%</td>
<td>-0.79%</td>
<td>32%</td>
<td>20%</td>
<td>2.13</td>
<td>0.36</td>
<td>13%</td>
<td>22</td>
</tr>
<tr>
<td>Asia</td>
<td>35%</td>
<td>-11%</td>
<td>0.57%</td>
<td>19%</td>
<td>5%</td>
<td>1.89</td>
<td>0.36</td>
<td>3%</td>
<td>17</td>
</tr>
<tr>
<td>Asia (Excluding China)</td>
<td>25%</td>
<td>-12%</td>
<td>0.71%</td>
<td>19%</td>
<td>5%</td>
<td>1.91</td>
<td>0.36</td>
<td>5%</td>
<td>16</td>
</tr>
</tbody>
</table>

The figures are unweighted country averages.
\(^\dagger\): Includes also Korea, Mexico and Turkey.
\(^\circ\): calculated within the riskless approach.
\(^\natural\): calculated within the portfolio approach.
\(^\#\): calculated within the portfolio approach and with the adjustment for \(P_k\) and \(\alpha_k\).
Table 2: Determinants of observed and predicted capital outflows

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Predicted capital outflows</th>
<th>Portfolio approach</th>
<th>Observed capital outflows</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
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<tr>
<td></td>
<td>Riskless approach</td>
<td>Portfolio approach</td>
<td></td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆B/Y₀</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td>β</td>
<td>-30.419***</td>
<td>5.466</td>
<td>-29.800***</td>
<td>0.565*</td>
<td>-0.417*</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>(6.562)</td>
<td>(12.177)</td>
<td>(7.969)</td>
<td>(0.313)</td>
<td>(0.220)</td>
<td>(0.355)</td>
</tr>
<tr>
<td>ln(β₀)</td>
<td>-6.541***</td>
<td>62.275***</td>
<td>32.246***</td>
<td>0.831*</td>
<td>0.385*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.724)</td>
<td>(12.693)</td>
<td>(6.230)</td>
<td>(0.432)</td>
<td>(0.224)</td>
<td></td>
</tr>
<tr>
<td>ln(Y₀)</td>
<td>3.494*</td>
<td>4.934</td>
<td>5.587</td>
<td>0.898</td>
<td>-0.135</td>
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</tr>
<tr>
<td></td>
<td>(1.800)</td>
<td>(10.585)</td>
<td>(7.690)</td>
<td>(0.735)</td>
<td>(0.257)</td>
<td></td>
</tr>
<tr>
<td>ln(n + g)*T</td>
<td>21.840***</td>
<td>-47.238</td>
<td>23.117</td>
<td>-0.779</td>
<td>-0.426</td>
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<td></td>
<td>(5.685)</td>
<td>(35.768)</td>
<td>(18.866)</td>
<td>(1.660)</td>
<td>(0.678)</td>
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<tr>
<td>Constant</td>
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<td>-104.926***</td>
<td>-134.512***</td>
<td>-0.422</td>
<td>-0.266</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.222)</td>
<td>(18.530)</td>
<td>(24.926)</td>
<td>(0.835)</td>
<td>(0.507)</td>
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</tr>
<tr>
<td></td>
<td>-5.562</td>
<td>57.156***</td>
<td>75.994***</td>
<td>-0.420**</td>
<td>-0.928***</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(5.257)</td>
<td>(17.969)</td>
<td>(22.041)</td>
<td>(0.200)</td>
<td>(0.114)</td>
<td>(0.769)</td>
</tr>
<tr>
<td>Observations</td>
<td>67</td>
<td>67</td>
<td>53</td>
<td>67</td>
<td>59</td>
<td>67</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.794</td>
<td>0.784</td>
<td>0.820</td>
<td>0.037</td>
<td>0.057</td>
<td>0.228</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

*** p < 0.01, ** p < 0.05, * p < 0.1

♭: β is calibrated inside the portfolio approach.
Table 3: The channels of the allocation puzzle

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$\Delta \frac{K}{Y_0}$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \frac{K}{Y_0}$</td>
<td></td>
<td>0.122*</td>
<td>0.131*</td>
<td>0.066</td>
<td>0.036</td>
<td>-0.013</td>
<td>0.101*</td>
<td>0.098*</td>
<td>0.111*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.063)</td>
<td>(0.072)</td>
<td>(0.047)</td>
<td>(0.060)</td>
<td>(0.076)</td>
<td>(0.056)</td>
<td>(0.058)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>$\pi$</td>
<td></td>
<td>-0.092</td>
<td>0.274</td>
<td>0.414</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.386)</td>
<td>(0.419)</td>
<td>(0.662)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\ln \left( \frac{\tilde{b}}{\tilde{y}_0} \right)$</td>
<td></td>
<td>0.544***</td>
<td>0.584***</td>
<td>0.891</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.182)</td>
<td>(0.202)</td>
<td>(0.638)</td>
<td></td>
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</tr>
<tr>
<td>$\frac{\ln(\tilde{y}_0)}{\tilde{y}_0}$</td>
<td></td>
<td>0.988</td>
<td>0.980</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.740)</td>
<td>(0.762)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(\hat{y}_0)$</td>
<td></td>
<td>-0.923</td>
<td></td>
<td></td>
<td>1.141**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.599)</td>
<td></td>
<td></td>
<td>(0.443)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(n + g^*) \times T$</td>
<td></td>
<td>-0.377</td>
<td></td>
<td>-1.201</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.879)</td>
<td></td>
<td>(0.729)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>-0.755***</td>
<td>-0.805***</td>
<td>-0.722***</td>
<td>0.047</td>
<td>-0.405</td>
<td>-0.844***</td>
<td>0.392</td>
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<tr>
<td></td>
<td></td>
<td>(0.141)</td>
<td>(0.214)</td>
<td>(0.773)</td>
<td>(0.330)</td>
<td>(0.133)</td>
<td>(0.729)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Observations | 67 | 67 | 67 | 67 | 67 | 67 | 67 | 67 |
| R-squared    | 0.091 | 0.092 | 0.167 | 0.171 | 0.228 | 0.136 | 0.132 | 0.110 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Table 4: The channels of the allocation puzzle: alternative measures of the capital wedge

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \frac{K}{Y_0}$</td>
<td>0.066</td>
<td>0.071</td>
<td>0.042</td>
<td>0.047</td>
<td>0.033</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.059)</td>
<td>(0.049)</td>
<td>(0.037)</td>
<td>(0.055)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>-0.064</td>
<td>-0.174</td>
<td>0.144</td>
<td>0.132</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.450)</td>
<td>(0.414)</td>
<td>(0.396)</td>
<td>(0.435)</td>
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</tr>
<tr>
<td>$P_k$</td>
<td>-0.259*</td>
<td>-0.262*</td>
<td>-0.257*</td>
<td>-0.095</td>
<td>-0.100</td>
<td>-0.180</td>
</tr>
<tr>
<td></td>
<td>(0.137)</td>
<td>(0.140)</td>
<td>(0.141)</td>
<td>(0.117)</td>
<td>(0.114)</td>
<td>(0.134)</td>
</tr>
<tr>
<td>$\alpha_k^1$</td>
<td>2.903*</td>
<td>2.933*</td>
<td>5.117**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.521)</td>
<td>(1.564)</td>
<td>(2.419)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_k^2$</td>
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<td></td>
<td></td>
<td>5.537**</td>
<td>5.522**</td>
<td>5.945*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2.538)</td>
<td>(2.505)</td>
<td>(3.246)</td>
</tr>
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<tr>
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<td>(0.926)</td>
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<td></td>
<td>(0.945)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(\tilde{g}_0)$</td>
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<td></td>
<td>-0.290</td>
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<td></td>
</tr>
<tr>
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<td></td>
<td>(0.873)</td>
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</tr>
<tr>
<td>$(n + g^*) * T$</td>
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<td>-0.951</td>
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<tr>
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<td>(0.845)</td>
<td></td>
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<td>(0.817)</td>
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</tr>
<tr>
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<td>-1.660**</td>
<td>-1.590**</td>
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<td>(0.559)</td>
<td>(0.575)</td>
<td>(1.182)</td>
<td>(0.762)</td>
<td>(0.680)</td>
<td>(1.181)</td>
</tr>
<tr>
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<td>62</td>
<td>62</td>
<td>62</td>
<td>61</td>
<td>61</td>
<td>61</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.182</td>
<td>0.182</td>
<td>0.276</td>
<td>0.221</td>
<td>0.222</td>
<td>0.293</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

$\tilde{b}_0$: $\alpha_k$ is calculated within the portfolio approach.

$\tilde{b}_k$: $\alpha_k$ is calculated within the riskless approach (Gourinchas and Jeanne, 2007).
Figure 1: An illustration of the allocation puzzle: Capital flows and investment across regions, 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.
Figure 2: Observed and predicted capital flows by region

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.

Observed capital flows are the observed ratio of net capital outflows to initial output, predicted capital flows in the riskless and portfolio approaches are respectively $\Delta B_R$ as defined by Equation (15) and $\Delta B_P$ as given by Equation (14), as well as their components.

The figures are unweighted country averages.

Non-OECD countries include also Korea, Mexico and Turkey.
Figure 3: Actual capital outflows (as a share of initial GDP) against their predicted value, according to the riskless approach, 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author's calculations. Note: China is excluded.

Figure 4: Actual capital outflows (as a share of initial GDP) against their predicted value, according to the portfolio approach, 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.
Figure 5. Actual capital outflows (as a share of initial GDP) against their predicted value, according to the portfolio approach and after adjusting for $\alpha_k$ and $P_k$, 1980-2003

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), Bernanke and Gurkaynak (2001), author’s calculations.