1 Introduction

In most existing models of bargaining it is assumed that traders know how much they value the object they are bargaining over. However, we observe in practice that economic agents often dedicate time and resources into learning the characteristics of goods and services they are considering to trade. For instance, imagine a consulting firm pitching a project to a potential customer. While the deliverables may be clear, the firm may not know exactly how costly they will be to complete; similarly, the buyer may not know exactly how much benefit they will get. Prior to negotiations, both sides may choose to invest time and resources in sharpening their estimate.

This project examines the effect of endogenous private information on bilateral bargaining with independent private values. Specifically, our goal is to analyze how the possibility of acquiring information about own’s valuation affects bargaining outcomes and how the choice of a bargaining mechanism influences information acquisition decisions. While Bergemann and Välimäki (2002) show that, with independent private values, the Vickrey-Clark-Groves mechanism attains both ex post efficiency and optimal information acquisition, it is already known that ex post efficiency is not attainable in bilateral bargaining (Myerson and Satterthwaite (1983)) due to binding budget balance and individual rationality constraints. Given this, important questions arise when the agents have the option of (costly) information acquisition before engaging in bargaining:

1. If info acquisition is covert, what are the properties of incentive compatible (IC) bargaining mechanisms? Can weaker notions of efficiency such as ex ante and interim incentive efficiency (Holmstrom and Myerson (1983)) be attained?
2. Under what conditions IC bargaining mechanisms lead to information over- and under-acquisition?
3. If info acquisition is observable, does an ex ante incentive efficient mechanism lead to optimal information acquisition?

4. What can be achieved if agents can commit to a bargaining mechanism before acquiring information?

Regarding the first question, we find that if the mechanism designer wants to elicit truthful revelation of both expected valuations and information acquisition decisions for any possible report of the opponent (akin to ex post implementation), then ex ante incentive efficient bargaining given traders’ information acquisition decisions is not attainable. The reason is that this type of IC requirement implies that buyer and seller’s expected trade probabilities are essentially invariant to their own reports of how much information they acquired. Given this, we are exploring both weaker notions of IC (i.e. Bayesian implementation) and of efficiency (i.e. interim incentive efficiency). We present below the basic setup and some preliminary results. We expect to have a complete draft of the paper addressing the above questions by the end of the summer.

2 The setup

There are two risk neutral traders bargaining over an indivisible object, a buyer with value $\tilde{v}_b \in [v_b, v_b]$ and a seller with value $\tilde{v}_s \in [v_s, v_s]$. Each trader can independently acquire information (at a cost) by choosing the accuracy of a signal. Let $\eta \in [0, 1]$ denote buyer’s information accuracy and $\theta \in [0, 1]$ be the seller’s accuracy. Accuracies imply different joint distributions $F^\eta_b(\tilde{v}_b, X_b), F^\theta_s(\tilde{v}_s, X_s)$ of buyer and seller signals $(X_b, X_s)$ and valuations, respectively. We assume that these distributions are independent.

When traders cannot commit to a bargaining mechanism before acquiring information we assume that traders report simultaneously their expected value and the information acquisition decision. In this context, a direct bargaining mechanism (DBM) specifies, for any report $(v_b, v_s; \eta, \theta)$ where $v_b, v_s$ represent the expected values conditional on signals, a probability of trade $p(v_b, v_s; \eta, \theta)$ and a transfer from the buyer to the seller $t(v_b, v_s; \eta, \theta)$ (i.e. there is budget balance). Abusing notation, let $F^\eta_b(\cdot), F^\theta_s(\cdot)$ denote the distribution of buyer and seller expected values, respectively.

Let $V^\eta_b = \text{supp}(F^\eta_b)$ and $V^\theta_s = \text{supp}(F^\theta_s)$. We assume that $\forall^\eta_b = [v^\eta_b, \overline{v}^\eta_b]$ and $\forall^\theta_s = [v^\theta_s, \overline{v}^\theta_s]$ and that $F^\eta, F^\theta$ are continuously differentiable for all $\eta, \theta \in [0, 1]$. We also assume that $\forall^\eta_b \cap \forall^\theta_s$ has positive measure for all $(\eta, \theta)$. Define expected payoffs given $(\eta, \theta)$ as

$$U_b(v_b; \eta, \theta) = v_b\overline{p}_b(v_b; \eta, \theta) - \overline{t}_b(v_b; \eta, \theta) \quad \text{and} \quad U_s(v_s; \eta, \theta) = \overline{t}_s(v_s; \eta, \theta) - v_s\overline{p}_s(v_s; \eta, \theta),$$
with
\[ p_i(v_i; \eta, \theta) = \int_{V_{i-1}} p(v_i, y; \eta, \theta) dF_{i-1}^{\gamma_i}(y), \quad i = b, s; \quad \gamma_i = \theta, \eta, \]
and
\[ t_i(v_i; \eta, \theta) = \int_{V_{i-1}} t(v_i, y; \eta, \theta) dF_{i-1}^{\gamma_i}(y), \quad i = b, s; \quad \gamma_i = \theta, \eta. \]

2.1 Incentive compatibility

Basically we are dealing with IC for traders with multidimensional signals \(((\eta, v_b)\) and \((\theta, v_s))\) with the special feature that each trader directly picks one of the signals (information accuracy) whereas the other (expected valuation) is not known until after the first one has been acquired. Thus, when talking about IC we distinguish truthful revelation of values conditional on truthful revelation of accuracy, which we refer to as value-IC, and truthful revelation of both values and accuracies, which represents interim IC. Value-IC is the relevant notion when information acquisition decisions are observable.

Definition 1 A DBM \((t, p)\) is IC with respect to \((\eta, \theta)\) if

(IC1) \(U_b(v_b; \eta, \theta) \geq v_b \overline{p}_b(\hat{v}_b; \eta, \theta) - \overline{t}_b(\hat{v}_b; \eta, \theta)\) for all \(v_b\) and \(\hat{v}_b\) in \(V_b^\eta\), and

(IC2) \(U_s(v_s; \eta, \theta) \geq \overline{t}_s(\hat{v}_s; \eta, \theta) - v_s \overline{p}_s(\hat{v}_s; \eta, \theta)\) for all \(v_s\) and \(\hat{v}_s\) in \(V_s^\eta\).

We say \((t, p)\) is value-IC if it is IC with respect to all \((\eta, \theta) \in [0, 1]^2\).

When \((\eta, \theta)\) are not observable, IC depends on traders’ beliefs about information acquisition. Let \(G_s\) and \(G_b\) represent traders’ beliefs about the other trader information acquisition decisions and define interim payoffs as

\[ U_b(v_b; \eta, \hat{G}_s) = \int_0^1 U_b(v_b; \eta, \theta) d\hat{G}_s(\theta), \quad \text{and} \quad U_s(v_s; \theta, \hat{G}_b) = \int_0^1 U_s(v_s; \eta, \theta) d\hat{G}_b(\eta). \]

Definition 2 A DBM \((t, p)\) is said to be IC with respect to \((\hat{G}_s, \hat{G}_b)\) if

(IC3) \(U_b(v_b; \eta, \hat{G}_s) \geq v_b \int_0^1 \overline{p}_b(\hat{v}_b; \hat{\eta}, \theta) d\hat{G}_s(\theta) - \int_0^1 \overline{t}_b(\hat{v}_b; \hat{\eta}, \theta) d\hat{G}_s(\theta)\) for all \(v_b \in V_b^\eta\), all \(\hat{v}_b \in V_b^\eta\) and all \(\hat{\eta}, \hat{\theta} \in [0, 1]\), and

(IC4) \(U_s(v_s; \eta, \hat{G}_b) \geq \int_0^1 \overline{t}_s(\hat{v}_s; \eta, \hat{\theta}) d\hat{G}_b(\eta) - v_s \int_0^1 \overline{p}_s(\hat{v}_s; \eta, \hat{\theta}) d\hat{G}_b(\eta)\) for all \(v_s \in V_s^\eta\), all \(\hat{v}_s \in V_s^\eta\), and all \(\eta, \hat{\theta} \in [0, 1]\).

We say that \((t, p)\) is interim-IC if it is IC with respect to all \((\hat{G}_s, \hat{G}_b) \in \Delta([0, 1]) \times \Delta([0, 1]))\).
Interim-IC implies that a trader has an incentive to truthfully reveal both his valuation and his accuracy irrespective of his beliefs about the opponent’s accuracy. Below we present a series of intermediate results that relate the different notions of IC.

Lemma 1 If \((p, t)\) is interim-IC, then it is value-IC.

Lemma 2 If \((p, t)\) is value-IC and we have that \(\mathcal{V}_b^\eta = \mathcal{V}_b\) and \(\mathcal{V}_s^\theta = \mathcal{V}_s\) for all \(\eta, \theta \in [0, 1]\), then it is IC with respect to \((\hat{\mathcal{G}}_s, \hat{\mathcal{G}}_b)\) if

\((IC5)\) \(U_b(v_b; \eta, \hat{\mathcal{G}}_s) \geq U_b(v_b; \hat{\eta}, \hat{\mathcal{G}}_s)\) for all \(\eta, \hat{\eta} \in [0, 1]\) and all \(v_b \in \mathcal{V}_b^\eta\); and

\((IC6)\) \(U_s(v_s; \theta, \hat{\mathcal{G}}_b) \geq U_s(v_s; \hat{\theta}, \hat{\mathcal{G}}_b)\) for all \(\theta, \hat{\theta} \in [0, 1]\) and all \(v_s \in \mathcal{V}_s^\theta\).

This result implies that to check for interim-IC, we can engage in a two step process, checking value-IC first and then looking at the incentives to reveal accuracies when traders truthfully report values.

The following results characterize expected trade probabilities. Value-IC implies monotonicity of expected trade probabilities with respect to trader’s expected value, whereas interim-IC implies that a trader’s expected trade probabilities are essentially invariant to his accuracy report and transfers have to be designed so as to make traders indifferent between any accuracy report.

Lemma 3 If \((p, t)\) is value-IC then \(\overline{p}_b(\cdot, \eta, \theta)\) is increasing and \(\overline{p}_s(\cdot, \eta, \theta)\) is decreasing for all \(\eta, \theta \in [0, 1]\)

Lemma 4 If \((p, t)\) is interim-IC then

(i) \(\overline{p}_b(v, \eta, \theta) \geq \overline{p}_b(\hat{v}, \hat{\eta}, \theta)\) for all \(v \in \mathcal{V}_b^\eta, \hat{v} \in \mathcal{V}_b^\hat{\eta}\) such that \(v > \hat{v}\) and all \(\eta, \hat{\eta}, \theta \in [0, 1]\); and \(\overline{p}_s(v, \eta, \theta) \leq \overline{p}_s(\hat{v}, \hat{\eta}, \hat{\theta})\) for all \(v \in \mathcal{V}_s^\eta, \hat{v} \in \mathcal{V}_s^\eta\) such that \(v > \hat{v}\) and all \(\eta, \hat{\eta}, \hat{\theta} \in [0, 1]\);

(ii) \(\overline{t}_b(v, \eta, \theta) - \overline{t}_b(v, \hat{\eta}, \theta) = v(\overline{p}_b(v, \eta, \theta) - \overline{p}_b(v, \hat{\eta}, \theta))\) for all \(v \in \mathcal{V}_b^\eta \cap \mathcal{V}_b^\hat{\eta}\) and all \(\eta, \hat{\eta}, \theta \in [0, 1]\); and \(\overline{t}_s(v, \eta, \theta) - \overline{t}_s(v, \eta, \hat{\theta}) = v(\overline{p}_s(v, \eta, \theta) - \overline{p}_s(v, \eta, \hat{\theta}))\) for all \(v \in \mathcal{V}_s^\eta \cap \mathcal{V}_s^\hat{\theta}\) and all \(\theta, \hat{\theta}, \eta \in [0, 1]\).

Loosely speaking, the first condition in this lemma requires trade probabilities to exhibit some sort of lexicographic behavior: higher buyer values imply higher expected trade probabilities regardless of the accuracy reported. The second condition makes traders indifferent between any alternative accuracy reports when they are restricted to report their true values (i.e. value-IC).

\(^1\)Notice that this definition has the flavor of ex post IC, i.e. belief independence, but it is still an interim condition since each agent takes expectations over opponent’s expected values for each precision.
Lemma 5  Assume that $V^\eta_b = V_b$ and $V^\theta_s = V_s$ for all $\eta, \theta \in [0, 1]$. If $(p, t)$ is interim-IC then $\bar p_b(v_b, \cdot, \theta)$ is constant in $[0, 1]$ for almost all $v_b \in V_b$ and all $\theta \in [0, 1]$; and $\bar p_s(v_s, \eta, \cdot)$ is constant in $[0, 1]$ for almost all $v_s \in V_s$ and all $\eta \in [0, 1]$.

The invariance of trade probabilities implied by Lemma 5 greatly narrows the class of DBM and provides a justification for the use of simple detail-free bargaining mechanisms. This result will potentially help us to compare alternative mechanisms in terms of the incentives to acquire information they provide.

2.2 Individual Rationality

Given our assumption that traders cannot commit to a mechanism before acquiring information, the relevant notions of individual rationality correspond to those in which the trader expects non-negative payoffs conditional on his expected value and his beliefs about the other trader information acquisition decision. The first definition of IR corresponds to the case of observable $(\eta, \theta)$ whereas the second refers to unobservable information acquisition.

Definition 3  A DBM $(t, p)$ is IR with respect to $(\eta, \theta)$ if

$(IR1) \ U_b(v_b; \eta, \theta) \geq 0$ for all $v_b \in V^\eta_b$, and

$(IR2) \ U_s(v_s; \eta, \theta) \geq 0$ for all $v_s \in V^\theta_s$.

We say $(t, p)$ is interim-IR if it is IR with respect to all $(\eta, \theta) \in [0, 1]^2$.

Definition 4  A DBM $(t, p)$ is said to be IR with respect to $(\hat G_s, \hat G_b)$ if

$(IR3) \ U_b(v_b; \eta, \hat G_s) \geq 0$ for all $v_b \in V^\eta_b$, and all $\eta \in [0, 1]$, and

$(IR4) \ U_s(v_s; \eta, \hat G_b) \geq 0$ for all $v_s \in V^\theta_s$ and all $\theta \in [0, 1]$.

Lemma 6  If $(p, t)$ is interim IR, then it is IR with respect to $(\hat G_s, \hat G_b)$.

3 Preliminary Results

3.1 Efficiency

With transferable utility and observable information acquisition, the strongest notion of efficient bargaining involves maximizing of expected gains from trade given $(\eta, \theta)$ subject to IC and IR with respect to $(\eta, \theta)$.
**Definition 5** A DBM \((p, t)\) is ex ante incentive efficient with respect to \((\eta, \theta)\) if it maximizes expected gains from trade and satisfies IC and IR with respect to \((\eta, \theta)\). That is, \((p, t)\) solves

\[
\max_{(p, t)} \int_{\mathcal{V}_b} \int_{\mathcal{V}_s} (v_b - v_s)p(v_b, v_s, \eta, \theta)dF^\eta_b(v_b)dF^\theta_s(v_s)
\]

s.t. IC1-IC2 and IR1-IR2

Theorem 2 in Myerson and Satterthwaite (1983) characterizes ex ante incentive efficient DBMs. Let

\[c_b(v_b, \alpha) = v_b - \alpha \frac{1 - F^\eta_b(v_b)}{f^\eta_b(v_b)}, \quad c_s(v_s, \alpha) = v_s + \alpha \frac{F^\theta_s(v_s)}{f^\theta_s(v_s)}\]

Define \(p^\alpha\) as

\[p^\alpha(v_b, v_s) = \begin{cases} 1 & \text{if } c_s(v_s, \alpha) \leq c_b(v_b, \alpha) \\ 0 & \text{if } c_s(v_s, \alpha) > c_b(v_b, \alpha) \end{cases} = \begin{cases} 1 & \text{if } v_b - v_s \geq \alpha \left( \frac{F^\theta_s(v_s)}{f^\theta_s(v_s)} + \frac{1 - F^\eta_b(v_b)}{f^\eta_b(v_b)} \right) \\ 0 & \text{otherwise.} \end{cases}\]

A mechanism with trade probabilities given by \(p^\alpha\) for some \(\alpha \in [0, 1]\) such that \(U_b(v^\eta_b; \eta, \theta) = U_s(v^\theta_s; \eta, \theta) = 0\) maximizes the expected gains from trade given IC-IR with respect to \((\eta, \theta)\).

The next result shows that unobservable information acquisition generally rules out the implementation of this strong form of efficiency.

**Theorem 1** Assume \(\mathcal{V}^\eta_b = \mathcal{V}_b\) and \(\mathcal{V}^\theta_s = \mathcal{V}_s\) for all \((\eta, \theta) \in [0, 1]^2\). If a DBM is ex ante incentive efficient with respect to all \((\eta, \theta) \in [0, 1]^2\), then it is not interim-IC.

### 3.2 Next steps

We are currently working on several directions. First, we intend to characterize the class of interim incentive efficient DBMs that are interim-IC. Second, we are using the information acquisition framework in Bergemann and Välimäki (2002) and Persico (2000) to look at Bayesian implementation of efficient bargaining outcomes and at traders’ optimal information acquisition decisions.
References


