Endogenous Public Information and Welfare

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Abstract

In this paper we perform a welfare analysis of economies with private information and in particular on the role of public information, where public information is endogenously generated and agents can condition on public statistics when making their choices in the rational expectations tradition. We show that rational expectations equilibria will not be team-efficient even when the allowed allocations share similar properties as the market equilibrium (i.e. linear in information). The reason is that the market in general does not internalize the informational externality when public statistics (e.g. prices) convey information. Rational expectations equilibria will be team-efficient only in exceptional circumstances (when the information externality vanishes). Under strategic substitutability equilibrium prices will tend to convey “too little” information when the “informational” role of prices prevails over its “index-of-scarcity” role and “too much” in the opposite situation. Under strategic complementarity prices convey too little information. Results extend to the internal team profit benchmark and received results on the relative weights to private and public information (when the latter is exogenous) may be overturned.

Keywords: information externality, strategic complementarity and substitutability, welfare, team solution, rational expectations, schedule competition

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1. Introduction
There is a recent surge of interest in the welfare analysis of economies with private information and in particular on the role of public information (see for example work by Morris and Shin (2002)). Agents may not put the welfare-optimal weights on private and public information. The reasons are payoff and information externalities. In this paper we examine the issue in a context where public information is endogenously generated and agents can condition on public statistics when making their choices.

If we take as a benchmark a pure prediction model where there are no payoff externalities then typically agents will rely too much on public information. The reason is that agents do not take into account that their reaction to private information affects the informativeness of public statistics and general welfare. That is, agents do not internalize an information externality. The appropriate benchmark to measure the inefficiency at the market solution is the team solution where agents internalize collective welfare but still have to rely on private information for their decisions (Radner (1979), Vives (1988)). Pure information externalities will make agents respond too little to their private information. Payoff externalities complicate welfare analysis and may rebalance the weights in the other direction.

In this paper we consider a tractable linear-quadratic-Gaussian model which allows us to shed light on the issue when public information is endogenously generated and influenced by the action of agents. We use a model which has a rational expectations flavor but in the context of a well-specified game where agents compete in schedules. See Hellwig and Veldkamp (2009) and Amador and Weill (2008) for related work.

We show that rational expectations equilibria will not be team-efficient even when the allowed allocations share similar properties as the market equilibrium (i.e. linear in information). The reason is that the market in general does not internalize the informational externality when public statistics (e.g. prices) convey information. Rational expectations equilibria will be team-efficient only in exceptional circumstances (when the
information externality vanishes). Under strategic substitutability equilibrium prices will tend to convey “too little” information when the “informational” role of prices prevails over its “index-of-scarcity” role and “too much” in the opposite situation. Under strategic complementarity prices convey too little information.

Results can be extended to the internal team profit benchmark and we find that endogenous public information may overturn conclusions reached using exogenous information models (e.g. Angeletos and Pavan (2007)). In particular we find that in the presence of information externalities and with strategic substitutability in payoffs it may happen that agents rely too much on private information making prices too informative. With strategic complementarity in payoffs agents with always rely too little on private information. This latter result is in stark contrast with the exogenous information case where agents rely too much on private information under strategic complementarity (Angeletos and Pavan (2007)).

The plan of the paper is as follows. Section 2 presents the model and possible interpretations. Section 3 characterizes the equilibrium and comparative statics properties. Section 4 performs a welfare analysis of the case of a homogenous product while Section 5 studies an internal team welfare benchmark.

2. The model and interpretations

We consider a quadratic payoff game with a continuum of players indexed in the interval $[0,1]$. Player $i$ has the payoff

$$\pi_i(x_i, \bar{x}) = (\alpha + u - \beta \bar{x}) x_i - \left( \theta x_i + \frac{\lambda}{2} x_i^2 \right)$$

$$= (\alpha - \theta + u) x_i - \beta \bar{x} x_i - \frac{\lambda}{2} x_i^2$$

$$= \left( \alpha - \theta + u - \beta \bar{x} - \frac{\lambda}{2} x_i \right) x_i$$
where \( x_i \) is the individual action, \( \bar{x} = \int_0^1 x_i \, di \) is the aggregate action and \( \alpha, \beta, \lambda \) are positive parameters. We have thus that 
\[
\frac{\partial^2 \pi}{\partial x_i} = -\lambda < 0 \quad \text{and} \quad \frac{\partial^2 \pi}{\partial x_i \partial \bar{x}} = -\beta
\]
and the slope of the best reply of a player is 
\[
\left( \frac{\partial^2 \pi}{\partial x_i \partial \bar{x}} \right) / \left( -\frac{\partial^2 \pi}{\partial x_i} \right) = -\frac{\beta}{\lambda}.
\]
We have thus strategic substitutability (complementarity) for \( \beta > (\beta < 0) \). We assume \( -\beta / \lambda < 1 \) or \( \beta + \lambda > 0 \) limiting the extent of strategic complementarity. Note that in the case of \( \beta = 0 \) we have no payoff externalities among players.

The parameter \( \theta \) is uncertain with prior normal distribution with mean \( \bar{\theta} \) and variance \( \sigma^2_0 \) (we write \( \theta \sim N(\bar{\theta}, \sigma^2_0) \)). Player \( i \) receives a signal \( s_i = \theta + \varepsilon_i \) with \( \varepsilon_i \sim N(0, \sigma_i^2) \). Error terms are uncorrelated across players. We make the convention that error terms cancel in the aggregate: \( \int_0^1 \varepsilon_i \, di = 0 \) (a.s.). The aggregation of all individual signals will reveal the underlying uncertainty: \( \int_0^1 s_i \, di = \theta + \int_0^1 \varepsilon_i = \theta \).

Players have access to the (endogenous) public statistic \( p = \alpha + u - \beta \bar{x} \) where \( u \sim N(0, \sigma_u^2) \) which can be interpreted as the marginal benefit of taking action level \( x_i \) which has cost \( \theta x_i + \frac{\lambda}{2} x_i^2 \). When \( \beta = 0 \) there is no informational externality either.

The strategies of the players are schedules that map private signals and the public statistic into an action. For player \( i \), \( x_i = X_i(s_i, p) \). This has a rational expectations flavor but in the context of a well specified game.

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1 That is, we make the convention that the Strong Law of Large Numbers (S.L.L.N) holds for a continuum of independent random variables with uniformly bounded variances. Suppose that \( \{q_i\}_{i=1}^\infty \) is a process of independent random variables with means \( E[q_i] \) and uniformly bounded variances \( \text{var}[q_i] \). Then we let \( \int_0^1 q_i \, di = \int_0^1 E[q_i] \, di \) almost surely (a.s.). This convention will be used, taking as given the usual linearity property of integral. Equality of random variables has to be understood to hold almost surely.
We will be interested in linear Bayesian equilibria of the schedule game for which the public statistic functional is of the type \( P(\theta, u) \). These, in fact, are the equilibria with bounded means and uniformly (across players) bounded variances. Positing strategies

\[
x_i = \hat{b}_i + \hat{c}_p - a_is_i,
\]

the aggregate action is given by

\[
\bar{x} = \int_0^1 x_i di = \hat{b} + \hat{c}p - a\theta - \int_0^1 a_i \epsilon_i di = \hat{b} + \hat{c}p - a\theta
\]

where \( \hat{b} = \int_0^1 \hat{b}_i di \), \( \hat{c} = \int_0^1 \hat{c}_i di \), \( a = \int_0^1 a_i di \), all assumed to be well defined, and since, according to our convention on the average error terms of the signals, \( \int_0^1 a_i \epsilon_i di = 0 \) (almost surely) -provided that \( \text{var}[a_i, \epsilon_i] \) is uniformly bounded across agents. Given that \( \text{var} [\epsilon_i] = \sigma^2 \) it is sufficient that \( a_i \) be uniformly bounded. In equilibrium this will be the case. Therefore we are restricting attention to candidate linear equilibria with parameters \( a_i \), uniformly bounded in \( i \) and with well defined average parameters \( \hat{b} \) and \( \hat{c} \).

Given the symmetric structure of payoffs and information structure, and that payoffs are strictly concave there is no loss of generality in restricting attention to symmetric equilibria. Indeed, the solution to the problem of player \( i \)

\[
\max_{x_i} \mathbb{E} \left[ \left( p - \frac{\lambda}{2} x_i \right) x_i | s_i, p \right]
\]

is unique (given strict concavity of profits) and symmetric across firms (since the cost function and signal structure is symmetric across firms):

\[
X(s_i, p) = \lambda^{-1} \left[ p - \mathbb{E} \left[ \theta | s_i, p \right] \right]
\]

where \( p = P(\theta, u) \).

The model admits several interpretations:
**Homogenous product market with quadratic production costs.** In this case \( p = \alpha + u - \beta \tilde{x} \) is the inverse demand for the homogenous product and the cost function of firm \( i \) is given therefore by \( C(x_i) = \theta x_i + \frac{\lambda}{2} x_i^2 \). Firms use supply functions as strategies and markets clear \( p = \alpha + u - \beta \tilde{x} = \alpha + u - \beta \left( \int_0^1 X(s_i, p) \, ds \right) \). When \( \beta > 0 \) demand is downward sloping, and we have strategic substitutability in the usual partial equilibrium market. When \( \beta < 0 \) we have strategic complementarity and demand is upward sloping. The situation may arise in the case of a network good with compatibility.

**Investment complementarities.** In this case \( \beta < 0 \) and we have strategic complementarity among investment decisions of the agents. The marginal benefit of investing is \( p = \alpha + u - \beta \tilde{x} \) and cost is \( C(x_i) = \theta x_i + \frac{\lambda}{2} x_i^2 \). Agents can condition decisions on the marginal benefit of investment. This need not be understood literally but may be the reduced form of a dynamic process.

**Monopolistic competition.** Finally, the model can represent a monopolistically competitive market with quantity setting firms. In the quantity setting mode suppose that \( \beta > 0 \) (goods are substitutes) or \( \beta < 0 \) (goods are complements), firm \( i \) faces the inverse demand for its product \( p_i = \alpha + u - \beta \tilde{x} - \frac{\lambda}{2} x_i \) and has costs \( \theta x_i \). Firms use supply functions contingent on its own price: \( X(s_i, p_i) \) for firm \( i \). We have then that observing the price is informationally equivalent for firm \( i \) to observe \( p = \alpha + u - \beta \tilde{x} \).

In all interpretations we can interpret \( p \equiv \alpha + u - \beta \tilde{x} \) as the marginal benefit or “price” to take an action and \( MC(x_i) \equiv \theta + \lambda x_i \) as the marginal cost.
3. Equilibrium

In order to characterize a symmetric LBE let us conjecture then that players use strategies of the following form:

\[ x_i = X(s_i, p) = \hat{b} + \hat{c}p - a s_i. \]

The aggregate action, according to our convention on the average error terms of the signals, is given by

\[ x = \int_0^1 x_j dj = \hat{b} + \hat{c}p - a \theta. \]

Using the public statistic \( p = \alpha + u - \beta x \) we obtain that, provided \( \hat{c} > -1/\beta \)

\[ p = (1 + \beta \hat{c})^{-1} \left( \alpha - \beta \hat{b} + z \right), \]

where the random variable \( z = u + \beta a \theta \) is informationally equivalent to the price.

Note that \( z \) (and the price) will provide in general a noisy signal of the unknown parameter \( \theta \) because \( u \) is random. We can write the information available to firm \( i \) as \( \Omega_i = (s_i, z) \). We can posit strategies of the form

\[ x_i = b - a s_i + c z, \]

and obtain that \( p = \alpha - \beta b + (1 - \beta c)z \). If \( 1 + \beta \hat{c} > 0 \) then \( 1 - \beta c > 0 \) and \( p \) and \( z \) will move together.

We can write the information available to firm \( i \) as \( G_i = \{s_i, z\} \) or \( G_i = \{s_i, p\} \) and the strategy of firm \( i \) as

\[ X(s_i, z) = \lambda^{-1} \left[ \alpha - \beta b + (1 - \beta c)z - E[\theta | s_i, z] \right]. \]
We can solve for the linear equilibrium in the usual way by identifying coefficients with the candidate linear strategy \( x_i = b - as_i + cz \) by calculating \( E[\theta | s_i, z] \). An alternative way of characterizing the LBE is the following. Note that the F.O.C. for firm \( i \),

\[
E[p - MC(x_i) | s_i, z] = 0
\]

has to hold on average given the private signal of the firm. That is,

\[
E\left[E\left[p - MC\left(X\left(s_i, p\right)\right) \middle| s_i, p\right] \middle| s_i \right] = E[p - MC\left(X\left(s_i, p\right)\right) \middle| s_i] = 0.
\]

To ease notation and without loss of generality let \( \theta = 0 \). Given normality, we obtain immediately (recalling that \( E[s_i] = \theta = 0 \) by assumption):

\[
E[p - MC(x_i) | s_i] = E[p - MC(x_i)] + \frac{\text{cov}[p - MC(x_i), s_i]}{\text{var}[s_i]}s_i = 0
\]

Since the equation has to hold for all possible signals \( s_i \) and \( \text{var}[s_i] > 0 \) we must have

\[
E[p - MC(x_i)] = 0 \quad \text{and} \quad \text{cov}[p - MC(x_i), s_i] = 0.
\]

Some algebra yields that the first equality is equivalent to \( b = \alpha / (\beta + \lambda) \) and the second to

\[
c = \frac{a\left(\lambda(\tau_e + \tau_0) + \beta \tau_e\right) - \tau_e}{a\beta \tau_e (\lambda + \beta)}.
\]

At a LBE firms make efficient private use of the signals and this yields in particular the elimination of the covariation between signals and the “margin” \( E[(p - MC(x_i)) | s_i] = 0 \).

Furthermore, using a similar reasoning (i.e. the properties of normal distributions) we would obtain that at a LBE agents also make efficient use of public information, eliminating the covariation between the margin and public information:
\[ E\left[ (p - MC(x))z \right] = 0. \]

This equality can be seen equivalent to \( c = \frac{1}{\lambda + \beta} \cdot \frac{\beta a \tau_u (1 - \lambda a)}{\tau (\lambda + \beta)} \)

where \( \tau \equiv (\text{var}[\theta | p])^{-1} = \tau_0 + \beta^2 a^2 \tau_u \) is the precision of public information about \( \theta \). In the relevant range this equation yields \( c \) as a decreasing function of \( a \). In equilibrium the parameters \( a \) and \( c \) are determined by the intersection of the (privately) efficient use of private information and of the efficient use of public information. The efficient use of private information yields \( c \) as an increasing function of \( a \). From the two curves in \( (a, c) \) space (see Figure 1 for the case \( \beta > 0 \)) it follows that

\[
\begin{align*}
a &= \frac{\tau_e}{\lambda (\tau_e + \tau)} \quad \text{and} \quad c &= \frac{1}{\beta + \lambda} - \frac{\beta a \tau_u}{\beta + \lambda (\tau_e + \tau)}. \\
\end{align*}
\]

From the second equality it is immediate that \( c < 1/ (\beta + \lambda) \) provided that \( a \geq 0 \). The equilibrium parameter \( a \) is determined as the unique (real) solution of the cubic equation

\[
a = \frac{\tau_e}{\lambda (\tau_e + \tau_0 + \tau_u \beta^2 a^2)}. 
\]

**Comparative statics**

- As the degree of complementarity \( m \equiv -\beta / \lambda \) increases \( c \) also increases and
  \[
  \text{sign} \left( \frac{\partial a}{\partial m} \right) = \text{sign}(\beta). 
  \]
  Therefore, an increase in the degree of strategic complementarity makes agents rely more on public information while it makes agents rely more on private information only in the strategic substitutes case (\( \beta > 0 \)).

- \( a \) decreases from \( \tau_e / \lambda (\tau_0 + \tau_e) \) to \( 0 \), and \( c \) from \( (\beta + \lambda)^{-1} \) to \( -\infty \), as \( \tau_u \) ranges from \( 0 \) to \( \infty \) (the latter follows since in equilibrium
The strategy of firm $i$ is of the form $x_i = \alpha (\beta + \lambda)^{-1} - a_i + cz$ or, recalling that $p = \alpha - \beta b + (1 - \beta c) z$, $\beta c < 1$, the supply function is given by

$$X(s_i, p) = \hat{b} - a_i + \hat{c}p$$

where it easily checked that $\hat{b} = b(1 - \lambda \hat{c})$ where $b = \frac{\alpha}{\lambda + \beta}$ and $\hat{c} = \frac{c}{1 - \beta c}$. We have that $\hat{c} > 0$ if $\beta < 0$. This is immediate from

$$c = \frac{1}{(\beta + \lambda)} - \frac{\beta a \tau_u}{(\beta + \lambda)(\tau_i + \tau)} = \frac{1}{(\beta + \lambda)} - \frac{\beta \lambda \tau_u a^2}{(\beta + \lambda) \tau_i} > 0.$$  

A high price conveys the good news that average output tends to be high and therefore costs low.

It is worth noting that the slope $\hat{c}$ of the supply function may be negative if $\beta > 0$.

Then we have that $c > (\langle)0$ if and only if $a > (\langle)\tau_c / \left( (\lambda (\tau_c + \tau_o) + \beta \tau_c \right)$ and $c = 0$ if and only if $a = \tau_c / \left( (\lambda (\tau_c + \tau_o) + \beta \tau_c \right)$. (See Figure 1) Given that $a$ is strictly decreasing in $\tau_u$ we have that $\hat{c} > 0$, and supply functions are increasing, for $\tau_u$ small and $\hat{c} < 0$, and supply functions are decreasing for $\tau_u$ large.  

2 C. Wilson (1979, 1980) finds an upward-sloping demand schedule in a market with asymmetric information with quality known only to the sellers.
The price serves a dual role as index of scarcity and as conveyor of information. Indeed, a high price has a direct effect to increase the competitive supply of a firm, but also conveys news about costs: if $\beta > 0$ ($\beta < 0$) that costs are high (low). In the case $\beta < 0$ with upward sloping demand this is so because we assume that $\lambda > -\beta$. If $\tau_u = 0$ then demand is so noisy that the price conveys no information on costs and $\hat{c} = 1/\lambda$ from the supply function $X(s, p) = \lambda^{-1}(p - E[\theta | s])$. Let us consider first the case $\beta > 0$.

As $\tau_u$ increases then the slope $\hat{c} = (1 - \tau_u \beta a/ (\tau + \tau)) / (\lambda + \tau_u \beta^2 a/ (\tau + \tau))$ decreases because of the informational component of price, it becomes negative at some point and as $\tau_u$ tends to $\infty$, $\hat{c}$ tends to $-1/\beta$ (since $\tau_u a/ (\tau + \tau_u + \beta \tau a^2)$ tends to infinity from the fact that as $\tau_u \to \infty$ both $a \to 0$ and $\tau_u a^2 \to \infty$ and therefore $\tau_u a \to \infty$). In the particular case where the scarcity and informational effects balance, firms set a zero weight ($c_0 = 0$) on public information. In this case firms do not condition on the price and the model reduces to the Cournot model where firms compete in quantities and $\hat{c} = 0$. However, in this particular case, when supply functions are allowed, not reacting to the price (public information) is optimal. When $\beta < 0$ then a high price conveys goods news both from the point of view of the scarcity and informational effects and supply is always upward sloping.
Figure 1. Determination of the LBE (labeled REE) parameters (a, c) as the intersection of the efficient use of private information (Private) and of the efficient use of public information (Public) for the case $\beta > 0$—illustrated for cases of $\tau_u$ high, intermediate and low. T stands for the team solution, REE for the rational expectations equilibrium and C for Cournot.
Figure 2. Determination of the LBE (labeled REE) parameters (a, c) as the intersection of the efficient use of private information (Private) and of the efficient use of public information (Public) for the case $\beta < 0$—illustrated for cases of $\tau_u$ high, intermediate and low. T stands for the team solution and REE for the rational expectations equilibrium.

The equilibrium is partially revealing (with $0 < \tau_u < \infty$ and $0 < \tau_c < \infty$) and therefore expected total surplus is strictly greater under first best production (full information) than in the REE. The reason is that firms produce under uncertainty and rely on estimation which is costly because of errors and costs are strictly convex.
Firms take public information $z$, or $p$, as given and use it to form probabilistic beliefs about the underlying uncertain cost parameter $\theta$. This in turn determines coefficients $a$ and $c$ for private and public information, respectively. At the same time, the informativeness of public information $z$ depends on the sensitivity of strategies to private information $a$. In the LBE firms behave as information takers and thus, from the viewpoint of an individual firm, public information is perceived as exogenous. This lies at the root of the informational externality present at the LBE. Firms do not take into account their impact on public information and therefore on other firms.

In three particular cases the information externality vanishes.

The first is when signals are perfectly informative ($\tau_e = \infty$). Then we are back to a full-information competitive equilibrium. (As we know this is Pareto optimal.) We have then that $c = 1/(\beta + \lambda)$, $a = \hat{c} = 1/\lambda$, and $X(s, p) = \lambda^{-1}(p - \theta)$. In this case firms do not have anything to learn from the price.

The second is when signals are uninformative about the common cost parameter $\theta$ ($\tau_e = 0$). Then $a = 0$, $c = 1/(\beta + \lambda)$, $\hat{c} = 1/\lambda$, and $X(s, p) = \lambda^{-1}(p - \tilde{\theta}) = \lambda^{-1}p$ (since by assumption $\tilde{\theta} = 0$). In this case the price has no information to convey.

The third is when demand is extremely noisy ($\tau_u = 0$). Then $c = 1/(\beta + \lambda)$, $\hat{c} = 1/\lambda$, and $a = \frac{\tau_e}{\lambda(\tau_e + \tau_0)}$, with $X(s_i, p) = \lambda^{-1}(p - E[\theta|s_i])$. In this case public information is pure noise. The same happens when $\beta = 0$ (then on top there is no payoff externality): $c = \hat{c} = 1/\lambda$ and $a = \frac{\tau_e}{\lambda(\tau_e + \tau_0)}$. 
4. Welfare analysis of the homogenous product market

The benchmark we will use is the team solution that maximizes expected total surplus subject to the use of linear decentralized strategies (as in Vives (1988) and Angeletos and Pavan (2007)). This is in the spirit of Hayek where the private signals of agents cannot be communicated to a center. The team efficient solution internalizes the information externalities of the actions of agents and is restricted to use the same type of strategies that the market (decentralized and linear). Indeed, an agent when reacting to his information does not take into account the contribution he makes to public information by influencing public statistics with his action.

Total surplus is given as follows:

\[ TS = (\alpha + u - \beta \bar{x}/2)\bar{x} - \int_0^1 (\theta x_i + \frac{\lambda}{2} x_i^2) \, di = \\
(\alpha + u - \theta - \beta \bar{x}/2)\bar{x} - \frac{\lambda}{2} \int_0^1 x_i^2 \, di = (\alpha + u - \theta)\bar{x} - \left( \beta \bar{x}^2 + \lambda \int_0^1 x_i^2 \, di \right)/2 \]

Note that under the assumptions \((\beta + \lambda > 0)\) the TS function is strictly concave for symmetric solutions.

At the team efficient solution expected total surplus \(E[TS]\) is maximized under the constraint that firms use decentralized linear production strategies.

That is,

\[
\max_{a,b,c} E[TS] \\
\text{subject to } x_i = b - a_s_i + cz, \text{ with } x = b - a\theta + cz \text{ and } z = u + a\beta\theta.
\]

Given that \(\frac{\partial x_i}{\partial a} = -s_i + c\beta\theta, \frac{\partial x_i}{\partial b} = 1, \frac{\partial x_i}{\partial c} = z\), the team solution is characterized by the following F.O.C.:
\[
\frac{\partial \text{ETS}}{\partial a} = E\left[ (p - MC(x_i))(-s_i + c\beta \theta) \right] = 0 \\
\frac{\partial \text{ETS}}{\partial b} = E\left[ (p - MC(x_i)) \right] = 0 \\
\frac{\partial \text{ETS}}{\partial c} = E\left[ (p - MC(x_i))z \right] = 0
\]

with \( p = \alpha + u - \beta x \) and \( MC(x_i) = \theta - \lambda x_i \).

We know that the constraint \( E[p - MC(x_i)] = 0 \) is equivalent to \( b = \alpha / (\beta + \lambda) \), and \( E[(p - MC(x_i))z] = 0 \) to \( c(a) = \frac{1}{\beta + \lambda} - \frac{\beta a \tau}{\beta + \lambda} \) where \( \tau = \tau_0 + \beta^2 a^2 \tau_u \). Replacing \( c \) by \( c(a) \) in the expression for ETS it can be shown that ETS is strictly concave in \( a \) (at least when \( c(\cdot) \) is strictly decreasing). Evaluating \( \frac{\partial \text{ETS}}{\partial a} \) at the LBE, where \( E[(p - MC(x_i))s_i] = 0 \), we obtain that \( \frac{\partial \text{ETS}}{\partial a} = c\beta E[(p - MC(x_i))\theta] \) and note that at the LBE we have that \( c(\cdot) \) is strictly decreasing (see Figures 1 and 2). We know that
\[
E[(p - MC(x_i))s_i] = E[(p - MC(x_i))\theta] + E[(p - MC(x_i))\varepsilon_i] = 0
\]
and therefore
\[
E[(p - MC(x_i))\theta] = -E[(p - MC(x_i))\varepsilon_i] = E[MC(x_i)\varepsilon_i] = E[(\theta + \lambda x_i)\varepsilon_i] = -\lambda a \sigma^2 < 0
\]

since \( \varepsilon_i \) is independent of all other random variables of the model. We conclude that at the LBE
\[
\text{sign}\left(\frac{\partial \text{ETS}}{\partial a}\right) = \text{sign}\left(-\beta c^{\text{REE}}\right).
\]

When \( \beta = 0 \) there is neither a payoff nor an informational externality and the tema and market solutions coincide. For \( \beta \neq 0 \), \( \tau_\varepsilon > 0 \) and \( \tau_\theta > 0 \), the solutions coincide only
if $c^{\text{REE}} = 0$. This is the case if at the LBE $c = \frac{a(\lambda(\tau_\epsilon + \tau_\theta) + \beta \tau_\epsilon) - \tau_\epsilon}{\tau_\epsilon(\beta + \lambda)a\beta} = 0$ or when $a = \tau_\epsilon \frac{1}{\lambda(\tau_\epsilon + \tau_\theta) + \beta \tau_\epsilon}$ (the intermediate case in Figure 1). When firms do not respond to the price ($c = 0$) the model reduces to a Cournot model with private information. Recall that in Vives (1988) it is shown that a Cournot market with private information and a continuum of firms solves a team problem with expected total surplus as objective function. For $c^{\text{REE}} < 0$ the distortion is positive and $a$ should be increased while the contrary is true for $c^{\text{REE}} > 0$. The team optimal solution uses public information efficiently but is not bound by the privately efficient use of information. At the REE with strategic substitutability ($\beta > 0$) there is excessive (insufficient) weight to private information whenever $\tau_\epsilon$ is small (large) and supply functions are increasing (decreasing) (See Figure 1).

With strategic complementarity ($\beta < 0$) we have that $c^{\text{REE}} > 0$ and $\text{sign}\left(\frac{\partial \text{ETS}}{\partial a}\right) = \text{sign}\left(-\beta a^{\text{REE}}\right) > 0$ always, agents put too little weight on private information. (See Figure 2)

There is no information externality when firms have perfect information ($\tau_\epsilon = \infty$) and the full information first best is obtained (and price equals marginal cost); when the price contains no information ($\tau_\epsilon = 0$) and when signals are uninformative ($\tau_\epsilon = 0$). In the latter two cases the team and the market solution coincide. Both for the team and the market solutions, when $\tau_\epsilon = 0$, $E\left[(p-MC_i)z\right] = 0$ implies that $c = \frac{1}{\beta + \lambda}$ and $a = \frac{\tau_\epsilon}{\lambda(\tau_\epsilon + \tau_\theta)}$, and when $\tau_\epsilon = 0$ we have that $a = 0$ and $c = \frac{1}{\beta + \lambda}$.

The conclusion is that with strategic substitutability team-efficiency requires to increase (decrease) $c$ when $c^{\text{REE}}$ is negative (positive). For $c^{\text{REE}} < 0$ the informational role of the
price dominates and the price reveals too little information. In this case more weight should be given to private signals so that public information becomes more revealing. On the contrary, when the price is mainly an index of scarcity, \( c_{REE} > 0 \), then the price reveals too much information and \( a \) should be decreased. Only in the knife-edge (Cournot) case, where \( c_{REE} = 0 \), the REE is team-efficient. With strategic complementarity agents put too little weight on private information. When \( \beta < 0 \) the informational externality is aligned with the price scarcity effect and it always pays to induce agents to rely more on their private information.

Remark: In the monopolistic competition case the total surplus function (consistent with the differentiated demand system) is slightly different:

\[
TS = (\alpha + u - \theta) \tilde{x} - (\beta \tilde{x}^2 + (\lambda / 2) \int_{0}^{1} x_i^2 \, \text{di}) / 2
\]

In this case the market is not efficient under complete information because price does not equal marginal cost. Each firm has some residual market power. We could proceed with a similar welfare analysis but instead in the next section we provide a welfare benchmark which only depends on the payoffs of the players in the game (i.e. in the monopolistic competition case welfare is evaluated from the perspective of the firms) and therefore is good for any interpretation of the model.)

5. Internal welfare benchmark

Now at the team efficient solution expected profit is maximized under the constraint that firms use decentralized linear production strategies. That is,

\[
\max_{a,b,c} E\pi_i \\
\text{subject to } \pi_i = px_i - C(x_i) \\
p = \alpha - \beta b + (1 - \beta c) z, x_i = b - a s_i + c z, \text{ with } x = b - a \theta + c z, \text{ and } z = u + a \beta \theta.
\]

Given that \( \frac{\partial x_i}{\partial a} = -s_i + c \beta \theta, \frac{\partial x_i}{\partial b} = 1, \frac{\partial x_i}{\partial c} = z \), the team solution is characterized by the following F.O.C.:
\[
\begin{align*}
\frac{\partial E\pi_i}{\partial a} &= E\left[ (p-MC(x_i))(-s_i + c\theta) - \beta \theta (c\beta - 1)x_i \right] = 0 \\
\frac{\partial E\pi_i}{\partial b} &= E\left[ (p-MC(x_i)) - \beta x_i \right] = 0 \\
\frac{\partial E\pi_i}{\partial c} &= E\left[ (p-MC(x_i))z - \beta x_z \right] = 0
\end{align*}
\]

It is clear now that the LBE will not efficient in relation to the internal team benchmark if \( \beta \neq 0 \) since the latter internalizes the payoff externalities at the LBE. At the internal team benchmark joint profits are maximized. Indeed, at the LBE we have that \( E[p-MC(x_i)] = 0 \) while at the internal team solution we have that \( E[(p-MC(x_i)) - \beta x_i] = 0 \) or \( E[(p-MC(x_i))] = \beta E[x_i] \). The question is whether the LBE gives the right weights to private and public information. We will see that the answer to the question is qualitatively similar to the analysis with the TS team benchmark.

We know that the constraint \( E[p-MC(x_i)] = 0 \) is equivalent to \( b = \alpha/(-\beta + \lambda) \) and we can check that \( E[(p-MC(x_i))z - \beta x_z] = 0 \) is equivalent to

\[
c(a) = \frac{1}{2\beta + \lambda} \beta \tau \mu \left( \frac{1 - (\lambda + \beta)a}{\tau (2\beta + \lambda)} \right)
\]

where \( \tau = \tau_0 + \beta \tau \mu \). It is possible to show that replacing \( c \) by \( c(a) \) in the expression for \( E\pi_i \) that \( E\pi_i \) is strictly concave in \( a \) (at least when \( c(\cdot) \) is strictly decreasing).

Evaluating \( \frac{\partial E\pi_i}{\partial a} \) at the LBE, where \( E[(p-MC(x_i))s_i] = 0 \), we obtain that

\[
\frac{\partial E\pi_i}{\partial a} = c\beta E\left[ (p-MC(x_i))\theta - \beta x_i \right]
\]

and note that at the LBE we have that \( c(\cdot) \) is strictly decreasing (see Figures 1 and 2). We know that

\[
\frac{\partial E\pi_i}{\partial a} = c\beta E\left[ (p-MC(x_i))\theta - \beta x_i \right]
\]

Indeed, when \( \beta = 0 \) there are no externalities (payoff or informational) and the profit team and market solutions coincide.
\[ E \left[ (p - MC(x_i))s_i \right] = E \left[ (p - MC(x_i))\theta \right] + E \left[ (p - MC(x_i))\varepsilon_i \right] = 0 \]

and therefore

\[ E \left[ (p - MC(x_i))\theta \right] = -E \left[ (p - MC(x_i))\varepsilon_i \right] = E \left[ MC(x_i)\varepsilon_i \right] = E \left[ (\theta + \lambda x_i)\varepsilon_i \right] = -\lambda \alpha \sigma^2 < 0 \]

since \( \varepsilon_i \) is independent of all other random variables of the model.

At the LBE we have that \( (\beta - c^{-1}) < 0 \) and \( E[\theta x_i] = aE[\theta] (c\beta - 1) < 0 \). We conclude that at the LBE

\[ \text{sign} \left( \frac{\partial E\pi}{\partial a} \right) = \text{sign} \left( -\beta c^{\text{REE}} \right). \]

This is the same qualitative result than with the TS team benchmark with regard to the weight to private information. Therefore, for \( \tau > 0 \) and when \( \beta > 0 \), for \( c^{\text{REE}} < 0 \) the distortion is positive, there is too little reaction to private information, and \( a \) should be increased while the contrary is true for \( c^{\text{REE}} > 0 \). When \( \beta < 0 \) we have that \( c^{\text{REE}} > 0 \) and \( -\beta c^{\text{REE}} > 0 \). Players always react too little to private information at the REE.

It is interesting to note that when agents cannot use contingent strategies and there is no information externality issue (Cournot or Bertrand competition cases, for example), then according to Angeletos and Pavan (2007) in the strategic complementarity case we would have over-reliance in private information (the opposite as in the endogenous public information case) and in the strategic substitutability case under-reliance in private information (while in the endogenous public information case we may get under- or over-reliance on private information).
References


