Offshore Production and Business Cycle Dynamics with Heterogeneous Firms*

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December 23, 2009

Abstract
Cross-country variation in production costs encourages the relocation of production facilities to other countries, a process known as offshoring through vertical foreign direct investment. I examine the effect of offshoring on the international transmission of business cycles. Unlike the existing macroeconomic literature, I distinguish between fluctuations in the number of offshoring firms (the extensive margin) and in the value added per offshoring firm (the intensive margin) as separate transmission mechanisms. In the model, firms are heterogeneous in labor productivity. They face a sunk entry cost in the domestic market and an additional fixed cost to produce offshore. Offshoring increases with the difference between the domestic and foreign cost of effective labor and with the firm-specific productivity. The key results are: (1) The model replicates the procyclical pattern of offshoring, as well as the extensive and intensive margin dynamics that I document using data from Mexico’s maquiladora sector; (2) Offshoring enhances the co-movement of output between the economies involved; and (3) Offshoring reduces the price dispersion across countries, as it dampens the real exchange rate appreciation that follows a productivity increase in the presence of endogenous firm entry. The results are relevant for the study of macroeconomic interdependence between countries separated by persistent wage differences, such as the U.S. and Mexico or the original and the new E.U. member states.

JEL classification: F23, F41

Keywords: offshore production, extensive margin, heterogeneous firms, firm entry, business cycle dynamics, terms of labor, real exchange rate.

*I am grateful to Fabio Ghironi, James Anderson and Susanto Basu for their help during my dissertation work at Boston College. Special thanks to George Alessandria, Richard Arnott, David Arseneau, Marianne Baxter, Bora Durdu, Matteo Iacoviello, Peter Ireland, Federico Mandelman, Joel Rodrigue, Pedro Silos, Vitaliy Strohush, and Christina Wang for insightful discussions on this paper. I would like to recognize participants at the SCIEA 2009 Meeting of the Federal Reserve, the IEFs/ASSA 2009 Meeting, the 4th DYNARE Conference at the Boston Fed, the Green Line Macro Meetings at Boston College/Boston University, as well as the R@BC and Dissertation Workshops at Boston College for important suggestions. Dissertation fellowships at the FRB of Atlanta and the FRB of Boston were extremely useful for the development of this paper. The views expressed here are the author’s and not necessarily those of the Board of Governors of the Federal Reserve System.

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1 Introduction

Firms often follow strategies that involve the fragmentation of production chains and the establishment of foreign affiliates at locations with relatively lower labor costs, an activity known in the international trade literature as offshoring through vertical foreign direct investment (FDI) (Helpman, 1984). Unlike production under horizontal FDI - which means that foreign affiliates aim to gain market access by replicating the operations of their parent firms in the country where final consumption takes place - the type of vertically-integrated production that I model is primarily motivated by lower production costs, as foreign affiliates add value to the final goods consumed in the multinationals’ country of origin or in third countries. The number of offshoring firms (which I refer to as the extensive margin of offshore production) and the real value added per offshoring firm (the intensive margin) fluctuate over the business cycle, and thus affect output, prices and wages in both the parent and the host countries.

This paper contributes to the international macroeconomics literature by analyzing the extensive and intensive margins of offshoring as separate transmission mechanisms of business cycles between the parent and the host country. I model offshoring as an endogenous, firm-level decision that depends on the difference between the domestic and the foreign cost per unit of effective labor, the fixed and trade costs of offshoring, as well as the firm-specific productivity. Fluctuations in the number of offshoring firms are linked to domestic firm entry and to the resulting changes in the relative cost of effective labor. Thus, an increase in aggregate productivity in the parent country encourages domestic firm entry and causes the domestic wage to rise faster than productivity, as labor demand increases to cover firm entry requirements. In turn, the increase in the domestic cost of effective labor causes more firms to relocate production offshore (i.e. an increase in the extensive margin). The increase in the number of offshoring firms is gradual, as it mirrors the gradual appreciation of the cost of effective labor.

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1The term "offshoring" refers to the activity of firms that relocate certain stages of their production to foreign countries. To this end, firms become integrated across borders through vertical or/and horizontal FDI, or purchase intermediate goods and services from unaffiliated foreign suppliers. In contrast, "outsourcing" applies to situations when firms purchase intermediates from unaffiliated suppliers - either at home or abroad - rather than producing them in house. See Helpman (2006) for a discussion of the related literature.


3Bergin, Feenstra and Hanson (2008) analyze the extent to which fluctuations in the extensive margin of offshoring account for variations in Mexico’s maquiladora employment. They show that more than one third of the adjustment in industry-level employment and nearly half of the adjustment in maquiladora’s total employment occur at the extensive margin, i.e. through variation in the number of plants over time.

4I maintain a one-to-one identification between an offshoring firm, an intermediate good variety, and an offshore plant. Under this assumption, the extensive margin of offshoring can also be interpreted as the number of offshore plants every period; the intensive margin can be regarded as the value added per offshore plant.
labor generated by domestic firm entry.

I document a set of stylized facts that characterize the cyclicality of offshoring from U.S. manufacturing to Mexico’s maquiladora sector. Using the number of maquiladora plants (establishments) as an empirical proxy for the extensive margin of offshoring, I show that the total value added and the number of plants in the maquiladora sector are strongly procyclical with U.S. manufacturing output. More, the business cycle dynamics of the maquiladora sector differ across the total value added and the extensive margin (Figure 5 in Section 5.2). In particular, expansions in U.S. output precede increases in the number of plants by at least four quarters, a result which highlights the inter-temporal link between U.S. manufacturing and the extensive margin of offshoring.

Despite the empirical evidence, the theoretical macroeconomic literature does not fully capture the business cycle dynamics of offshoring along its extensive margin. For instance, Burstein, Kurz, and Tesar (2009) examine the role of production sharing in the transmission of business cycles in a two-country model in which the location of plants is fixed over time. Bergin, Feenstra, and Hanson (2007) focus on the importance of offshore production in amplifying the transmission of shocks across countries, in a model in which the number of offshoring firms makes an abrupt shift rather than a gradual adjustment over time - as I find in the data - in response to aggregate shocks.

I address this issue by introducing the endogenous determination of the number of offshoring firms in a two-country (North and South), dynamic stochastic general equilibrium model with firm entry and firm heterogeneity, along the lines of Ghironi and Melitz (2005, henceforth GM2005). In my model, firm entry is subject to a sunk cost reflecting the regulation of starting a business in the parent country. Following entry, each firm can use either domestic or foreign labor in the production of a different variety of intermediate goods. The use of foreign labor involves the establishment of an offshore plant, and is subject to a fixed cost of offshoring every period. Also, offshoring involves the so-called iceberg trade cost that reflects transportation, insurance, and trade barriers, costs which are incurred in the shipping of intermediate goods produced offshore back to the country of origin. Thus, when deciding on where to locate production (domestically vs. offshore), each firm balances the lower

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5Mexico’s maquiladora sector consists of manufacturing plants that import intermediate goods, process them, and export the resulting output (Gruben, 2001). Although only some of the maquiladora plants are U.S.-owned, most of them accommodate the offshoring operations of U.S. firms: They import most of their inputs (82 percent) and send most of their gross output (90 percent) from/to the U.S. (Hausman and Kaytko, 2003; Burstein, Kurtz and Tesar, 2009). The maquiladora sector accounts for 20 percent of Mexico’s manufacturing value added (INEGI), for nearly 50 percent of the manufacturing exports, and for approximately 25 percent of Mexico’s employment (Bergin, Feenstra and Hanson, 2007, 2008).

6In Burstein, Kurtz and Tesar (2008), the low elasticity of substitution between the domestic and foreign varieties in an Armington composite enhances the cross-country co-movement of output.
foreign costs of effective labor against the fixed and trade costs associated with offshore production.\textsuperscript{7} Since firms are heterogeneous in productivity, the decision to produce offshore is firm-specific. Only the more productive firms can afford the fixed costs of offshoring, and their number varies over time. The model also implies that the relocation of production offshore takes place one-way; since the cost of effective labor is relatively lower in the South, only Northern firms have the incentive to relocate production offshore. All Southern firms produce domestically.\textsuperscript{8}

The key results of the paper are as follows. First, the model generates a procyclical pattern of offshoring that is consistent with the stylized facts from Mexico’s maquiladora industry. In particular, following an economic expansion in the country of origin, the value added per offshoring firm (the intensive margin) jumps on impact. Domestic firm entry leads to a gradual appreciation of the terms of labor, which in turn generates a gradual increase in the number of offshoring firms over time (the extensive margin), like it is observed in the maquiladora data.\textsuperscript{9} Second, offshoring enhances the co-movement of output relative to the alternative framework with exports only in GM2005. As firm entry places upward pressure on the domestic effective wage and causes more firms to relocate production offshore, the higher demand for domestic labor (due to firm entry) and sequentially higher demand for offshore labor (due to the relocation of production) enhance the co-movement of wages and aggregate incomes.\textsuperscript{10} The result is consistent with the empirical regularity documented in Burstein, Kurz, and Tesar (2009) that countries with stronger offshoring-related trade links tend to exhibit higher correlations of manufacturing output. Third, offshoring narrows the price dispersion across countries, as it dampens the appreciation of the real exchange rate that follows an increase in aggregate productivity in the parent country. In the absence of offshoring, the framework with firm entry and endogenous number of exporting firms generates the Harrod-Balassa-Samuelson effect (i.e. more productive economies exhibit higher average prices relative to their trade partners) through the appreciation of the terms of labor and the rise in import prices that follow an aggregate productivity increase at home (GM2005). With offshoring, this effect is dampened though several channels, including the transfer of upward

\textsuperscript{7}I define the cost of effective labor as the ratio between the real wage and aggregate productivity \((w_t/Z_t)\) in the North and \((w_t^s/Z_t^s)\) in the South.

\textsuperscript{8}I derive an asymmetric steady state, in which differences in the regulation of firm entry are translated in differences in the real effective wage across countries. In the model, I set the firm entry cost to be higher in the South; since the more regulated economy attracts a relatively smaller number of firms, labor demand and the cost of effective labor are lower in the South.

\textsuperscript{9}The terms of labor is defined as the ratio between the Southern and Northern real cost of effective labor expressed in units of the same consumption basket, \(TOL_t = Q_{w_t/Z_t}/Q_{w_t^s/Z_t^s}\). An increase in the cost of effective labor in the North would cause the terms of labor to appreciate (i.e. the \(TOL\) to decrease).

\textsuperscript{10}In contrast, in the traditional IRBC literature, a domestic increase in aggregate productivity leads to increased production at home but not offshore, as in Backus, Kehoe and Kydland (1992).
pressure from the domestic to the foreign wage, the relatively smaller size of the domestic non-traded sector, and also the decline in import prices that occurs as offshoring crowds out the less productive foreign exporters.

The implications of the model are consistent with the empirical evidence provided by recent studies on the determinants of offshore production. For instance, Hanson, Mataloni, and Slaughter (2005) show that U.S. multinational firms import more intermediate inputs from their foreign affiliates when the latter benefit from lower trade costs and lower wages in the host economies. Kurz (2006) shows that the U.S. plants and firms that use imported components in production are larger and more productive than their domestically-oriented counterparts, as their higher idiosyncratic productivity levels allow them to cover the fixed costs of offshoring.

This paper is related to a growing body of macroeconomic literature that focuses on endogenous firm entry and adjustments along the extensive margin of exports (but not of offshoring).\textsuperscript{11} For example, GM2005 study the export decision of firms in the presence of fixed exporting costs, in a framework with firm entry and firm heterogeneity. Alessandria and Choi (2007) analyze the extensive margin of exports in a model with sunk and continuation fixed costs that explains the "exporter hysteresis" behavior.\textsuperscript{12} Corsetti, Martin, and Pesenti (2007) examine the terms-of-trade implications of productivity improvements in the sectors of firm entry and production, in a model in which the extensive margin of exports is endogenous. Finally, Mejean (2006) emphasizes the implications of endogenous firm entry in the tradable sector for the real exchange rate dynamics and the Harrod-Balassa-Samuelson effect.

The study of the macroeconomic implications of offshoring through vertical FDI is particularly relevant for pairs of countries and for economic areas that are separated by persistent differences in the cost of effective labor. For instance, offshoring through vertical FDI represents an important share of the operations of U.S. multinational affiliates in the NAFTA region, and also for those in Central and South America. As much as 50 percent of the manufacturing sales of the U.S. affiliates in Mexico, and 26 percent of the sales of the U.S. affiliates in Latin America as a whole, were directed towards their U.S. parent firms in 2005. The fraction was only 3 percent for the U.S. affiliates in Europe, and

\textsuperscript{11} Recent empirical literature highlights the role of the extensive margin in international trade in the presence of fixed exporting costs: Baldwin and Harrigan (2007) show that the number of traded goods (the extensive margin) decreases with distance and increases with the size of the importing country. Besedes and Prusa (2006) find that the survival rate of exports for differentiated good varieties increases with the initial transaction size and also with the length of the relationship. Hummels and Klenow (2005) show that larger economies have larger exports, and that the extensive margin accounts for as much as 60 percent of this difference.

\textsuperscript{12} "Exporter hysteresis" refers to the behavior of firms that continue to serve the foreign market even after a real exchange rate appreciation reduces their export competitiveness.
5 percent for those in the Asia-Pacific region (BEA, 2007). A similar pattern exists between Western Europe and the new member countries of the European Union (Marin, 2006; Meyer, 2006).

The rest of this paper is organized as follows: Section 2 introduces a DSGE model of offshoring that allows for fluctuations in offshoring along both the extensive and the intensive margins. Section 3 defines the average productivity levels of the representative firms that produce domestically and offshore. Section 4 discusses the model calibration. Section 5 presents the results, including the business cycle dynamics of offshoring in the presence of aggregate productivity shocks, and a comparison between the empirical moments of offshoring to Mexico and their model counterparts. Section 6 concludes with a summary and possible extensions of the model.

2 Model of Offshoring with Heterogeneous Firms

2.1 Markets and Production Strategies

This section summarizes the model of firm entry and offshore production, which is illustrated in Figure 1. Every period $t$, an unbounded pool of potential entrant firms face a trade-off between the sunk entry cost (reflecting the cost of starting a business in the country of origin), the expected stream of future monopolistic profits, and the probability of exit very period, as in GM2005. After paying the sunk entry cost, each firm is assigned an idiosyncratic labor productivity factor $z$ drawn independently from a common distribution $G(z)$ with support over the interval $[z_{\text{min}}, \infty)$. The firm keeps $z$ until it receives the exogenous exit shock, which occurs with probability $\delta$ every period.

![Figure 1. Destination markets and production strategies of firms.](attachment:image.png)
Following entry, firms are monopolistically competitive and heterogeneous in labor productivity. Every period after entry (from $t+1$ onwards), the existing firms choose the destination market(s) that they serve as well as the location of production, as follows:

1. All firms serve their domestic market. They produce intermediate goods using either domestic or foreign labor. The use of foreign labor (i.e. offshoring) involves the establishment of an offshore production plant, and offers the advantage of a lower production cost. However, offshoring is subject to a per-period fixed cost, and also to an iceberg trade cost that affects the intermediate goods produced offshore and shipped back to the country of origin.

2. Some of the firms also serve the foreign market. They use exclusively domestic labor in production, and export the resulting goods subject to a per-period fixed cost, as in GM2005.

Thus, the extreme case in which the offshoring cost is set prohibitively high - so that all firms produce at home - is equivalent to the framework with endogenous exports in GM2005.$^{13}$

Next I describe in detail the model with firm entry and offshore production.

2.2 Firms Serving the Domestic Market: Domestic vs. Offshore Production

This section outlines the mechanisms of domestic and offshore production as alternative choices for the Northern firms that serve the domestic market. It does not concern the Southern firms, as offshoring takes place one-way, from the Northern economy to the low-wage South.

In the North, a continuum of monopolistically-competitive firms produce intermediate good varieties for the domestic market. Firms are heterogeneous in productivity, with each firm producing a different variety of goods. Since each firm produces one variety, the firm-specific labor productivity $z$ also serves as an index for the existing varieties of intermediate goods. Every period, firms can choose one of the two possible production strategies:

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$^{13}$I abstract from the possibility of offshoring through horizontal FDI as an alternative to exports. Under that scenario, firms serving the foreign market may produce abroad using the local labor (Contessi, 2006). Production under horizontal FDI is motivated by improved access to the foreign market, and involves the simultaneous production of the same intermediate good variety both at home (to be sold in the home market) and offshore (to be sold in the host market). In my model, firms engaging in vertical FDI shift production offshore in order to take advantage of the relatively lower cost of effective labor. They relocate downstream production activities offshore (e.g. manufacturing, assembly and packaging) while continuing to perform the upstream operations (e.g. research, marketing and sales, represented by the sunk cost) at home.
(1) **Domestic production:** The firm with idiosyncratic labor productivity \(z\) employs labor \(l_t\) to produce its intermediate good variety:

\[
y_{D,t}(z) = Z_t z l_t, \tag{1}
\]

where \(Z_t\) is the aggregate productivity of labor in the North and \(z\) is the firm-specific labor productivity;

(2) **Offshore production:** Alternatively, the firm with idiosyncratic labor productivity \(z\) may choose to relocate production offshore using Southern labor \(l_t^s\) as the only input in production:

\[
y_{V,t}(z) = Z_t^s z l_t^s. \tag{2}
\]

Thus, I assume that each offshoring firm becomes subject to the Southern aggregate labor productivity \(Z_t^s\), but is able to carry its own idiosyncratic labor productivity \(z\) to the South.\(^{14}\) Given the demand for varieties produced domestically, \(y_{D,t}(z) = \rho_{D,t}(z)^{-\theta} C_t\), and also the demand for varieties produced offshore by the vertically-integrated firms, \(y_{V,t}(z) = \rho_{V,t}(z)^{-\theta} C_t\), the monopolistically-competitive firms solve their profit-maximization problem:

\[
\begin{align*}
\max_{\{\rho_{D,t}(z)\}} & \quad d_t = \rho_{D,t}(z) y_{D,t}(z) - \frac{w_t}{Z_t z} y_{D,t}(z), \\
\max_{\{\rho_{V,t}(z)\}} & \quad V_t = \rho_{V,t}(z) y_{V,t}(z) - \tau \frac{w_t^s Q_t}{Z_t^s z} y_{V,t}(z) - f_V \frac{w_t^s Q_t}{Z_t^s}.
\end{align*} \tag{3, 4}
\]

The cost of producing one unit of output either domestically or offshore varies not only with the cost of effective labor \(\frac{w_t}{Z_t^d}\) and \(\frac{w_t^s Q_t}{Z_t^s}\) across countries, but also with the level of idiosyncratic labor productivity \(z\) across firms.\(^{15}\) I define the real exchange rate \(Q_t = \frac{P_t^e}{P_t^s}\) as the ratio between the price indexes in the

\(^{14}\)Strategies 1 and 2 are extreme cases of a broader framework of offshoring, in which I allow for the offshoring firm with idiosyncratic labor productivity \(z\) to use a mix of Northern and Southern labor, \(l_t\) and \(l_t^s\). Following the specification in Antras and Helpman (2004), the production of final good variety \(z\) is a Cobb-Douglas function of domestic and foreign inputs:

\[
y_{V,t}(z) = Z_t^s z l_t^s. \tag{2}
\]

In this paper, I explore two extreme scenarios: At one extreme, I set \(\alpha = 1\) to shut down offshoring under vertical FDI, a case in which I revisit GM2005. At the other extreme, I set \(\alpha = 0\) so that the offshoring firms use exclusively foreign inputs. The smaller \(\alpha\), the larger the range of operations that offshoring firms relocate abroad. For the two extreme cases, I use the l’Hôpital rule to obtain: \(\lim_{\alpha \to 0} \frac{1}{\beta} = \lim_{\beta \to \infty} \beta^{1/\beta} = e^{\lim_{\beta \to \infty} \frac{\ln \beta}{\beta}} = e^{\lim_{\beta \to \infty} \frac{\ln \beta}{\beta}} = e^{\lim_{\beta \to \infty} (1/\beta)} = e^{0} = 1\).

\(^{15}\)Given the domestic and offshore real wages \((w_t)\) and \((w_t^s)\), the marginal cost of producing one unit of variety \(z\) domestically is \(\frac{w_t}{Z_t^d}\), and the marginal cost of producing it offshore is \(\frac{w_t^s Q_t}{Z_t^s}\). The real wage in the North \(w_t = W_t / P_t^d\) is expressed in units of the domestic consumption basket; the real wage in the South \(w_t^s = W_t^s / P_t^s\) is expressed in units of the Southern consumption basket.
South and North expressed in the same currency, where $\varepsilon_t$ is the nominal exchange rate. In addition to the marginal cost, the Northern firms producing offshore incur a period-by-period fixed offshoring cost equal to $f_V$ units of Southern effective labor, a cost which reflects the building and maintenance of the offshore production facility.\footnote{The cost of $f_V$ units of Southern effective labor is equivalent to $f_V w_t^*/Z_t^*$ units of the Southern consumption basket.} They also face an iceberg trade cost ($\tau > 1$) associated with the shipping of goods produced offshore back to the parent country: For every $\tau$ units produced offshore, only one unit arrives in the North for consumption, as the difference is lost due to costs associated with trade barriers, transportation, insurance, and differences in the legal systems, as discussed in Anderson and Wincoop (2004).

The profit-maximization problem under monopolistic competition implies the following equilibrium prices per unit of output produced domestically and offshore:

\begin{align}
\rho_{D,t}(z) &= \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z} \\
\rho_{V,t}(z) &= \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^* z}.
\end{align}

The resulting profits from domestic and offshore production, both expressed in units of the aggregate consumption basket $C_t$ in the North, are:

\begin{align}
d_{D,t}(z) &= \frac{1}{\theta} \rho_{D,t}(z)^{1-\theta} C_t \\
d_{V,t}(z) &= \frac{1}{\theta} \rho_{V,t}(z)^{1-\theta} C_t - f_V \frac{w_t^* Q_t}{Z_t^*}.
\end{align}

To summarize the above, the profits associated with domestic and offshore production depend on the cost of effective labor in the North and South, the fixed offshoring cost, the iceberg trade cost, as well as the firm-specific labor productivity. When deciding upon the location of production every period, the firm with productivity $z$ compares the profit $d_{D,t}(z)$ it would obtain from domestic production with the profit $d_{V,t}(z)$ it would obtain from producing the same variety offshore.

The model implies that only the relatively more productive Northern firms find it profitable to locate production offshore every period. Despite the lower cost of effective labor in the South, only firms with idiosyncratic productivity above a certain cutoff ($z > z_{V,t}$) obtain profits that are large enough to cover the fixed offshoring cost and the iceberg trade cost. This feature of the model is consistent with the empirical evidence provided by Kurz (2006), that the U.S. plants and firms that use imported content in production are larger and more productive than their domestically-oriented
counterparts, as the larger idiosyncratic productivity levels allow them to cover the fixed costs of offshoring.\textsuperscript{17}

As a particular case, the firm with labor productivity equal to the cutoff \( z_{V,t} \) is indifferent between producing domestically or offshore. After accounting for the fixed offshoring cost and the iceberg trade cost, the firm at the cutoff obtains equal profits from domestic and offshore production. Using this property, I derive the endogenous productivity cutoff \( z_{V,t} \) that governs the location decision as:

\[
 z_{V,t} = \{ z \mid d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t}) \}.
\]  

**Existence of the equilibrium productivity cutoff** Next I show that the existence of the equilibrium productivity cutoff \( z_{V,t} \) requires a cross-country asymmetry in the cost of effective labor, so that some of the Northern firms will always have an incentive to produce offshore.

I begin by re-writing the profits obtained from domestic and offshore production as

\[
 d_{D,t}(z) = B_t \left( \frac{w_t}{Z_t} \right)^{1-\theta} z^{\theta-1} \quad \text{and} \quad d_{V,t}(z) = B_t \left( \tau \frac{w_t^* Q_t}{Z_t^*} \right)^{1-\theta} z^{\theta-1},
\]

respectively, where \( B_t \equiv \frac{1}{\beta} \left( \frac{\theta}{1-\theta} \right)^{1-\theta} C_t \) measures the size of the Northern market. In Figure 2, I plot the corresponding profits as functions of the idiosyncratic productivity parameter \( z^{\theta-1} \) over the support interval \([z_{min}, \infty)\). The vertical intercepts represent the annualized value of the sunk entry cost for the case of domestic production \((-\Theta f_E \frac{w_t}{Z_t})\), and the annualized value of the sunk entry cost plus the period-by-period fixed cost for the case of offshore production \((-\Theta f_E \frac{w_t}{Z_t} - f_V \frac{w_t Q_t}{Z_t^*})\), where \( \Theta \equiv \frac{1-\beta(1-\delta)}{\beta(1-\delta)} \).

The existence of the equilibrium productivity cutoff \( z_{V,t} \) in Figure 2 requires that the profit function from offshoring must be steeper than the profit from domestic production, i.e. \( \text{slope} \{ d_{V,t}(z) \} > \text{slope} \{ d_{D,t}(z) \} \). When this condition is met, offshoring generates greater profits than domestic production for the firms with idiosyncratic productivity \( z \) along the upper range of the support interval. The slope inequality is equivalent to:

\[
 \tau \frac{w_t^* Q_t / Z_t^*}{w_t / Z_t} < 1,
\]

which implies that the effective wage in the South must be sufficiently lower than that in the North, so that the difference covers both the fixed cost of offshoring and the iceberg trade cost \( \tau > 1 \), and thus provides an incentive for some of the Northern firms to produce offshore every period.\textsuperscript{18}

\textsuperscript{17}A useful implication of firm heterogeneity is that the more productive firms have larger output and revenue (see Melitz, 2003). Given two firms with idiosyncratic productivity \( z_2 > z_1 \), the ratios of output and profits are

\[
 r(z_2) = \left( \frac{z_2}{z_1} \right)^{\theta-1} > 1, \quad \text{and} \quad r(z_2) = \left( \frac{z_2}{z_1} \right) > 1.
\]

Empirical studies show that firms using imported inputs in production are not only more productive, but also larger and employ more workers (Kurz, 2006).

\textsuperscript{18}In Appendix A.8, I derive a second condition necessary to avoid the corner solution when all firms would produce
2.3 Firms Serving the Foreign Market: Exports

In addition to the offshoring operations of the Northern firms in the South, firms in each economy also have the option to serve the foreign market through exports, as in GM2005. This section describes the problem of the Northern firms that serve the Southern market through exports. The equations for the Southern exporters are similar unless indicated otherwise. Variables for the Southern economy are marked with the (*) superscript.

The Northern exporting firm with idiosyncratic productivity $z$ uses an amount of domestic labor $l_t$ in the production of its intermediate variety $y_{H,t}(z)$ exported the Southern market:\footnote{I view exporting as a special case within a broader framework, in which I allow for firms to serve the foreign market by using a mix of domestic and foreign inputs in production. In this framework, production is described by the following Cobb-Douglas specification:

$$y_{H,t}(z) = \left[ \frac{Z_t z_l}{\eta} \right]^\eta \left[ \frac{Z_t^* z_l^*}{1 - \eta} \right]^{1-\eta},$$

where a larger $\eta$ accounts for a smaller content of Southern inputs used in the production of intermediate goods sold in the South. In this paper I nest the special case with endogenous exports in GM2005 by setting $\eta = 1$. Alternatively, I nest the case in which firms serving the Southern market would produce exclusively through their foreign affiliates (as in Contessi, 2006) by setting $\eta = 0$. In the latter case, production in the South through horizontal FDI allows firms to avoid the trade cost $\tau^*$ by using local inputs. Using the l'Hôpital rule, $\lim_{\eta \to 0} \left( \frac{1}{\eta} \right)^\eta = \lim_{\beta \to -\infty} \beta^{1/\beta} = \lim_{\beta \to -\infty} \frac{\ln \beta}{\beta} = e^{\lim_{\beta \to -\infty} \frac{\ln \beta}{\beta}} = e^0 = 1$. The corresponding price and profit functions are $p_{H,t}(z) = \alpha \left( \tau^* \frac{w_t}{z_{t-1}} \right)^{1-\eta} \left( \frac{w_t^*}{z_{t-1}^*} \right)^{1-\eta}$ and $d_{H,t}(z) = \frac{1}{\eta} \rho_{H,t}(z)^{1-\eta} C_t^* Q_t - f_H \left( \frac{w_t}{z_t} \right)^\eta \left( \frac{w_t^*}{z_t^*} \right)^{1-\eta}.$}

$$y_{H,t}(z) = Z_t z_l.$$  

\footnote{I view exporting as a special case within a broader framework, in which I allow for firms to serve the foreign market by using a mix of domestic and foreign inputs in production. In this framework, production is described by the following Cobb-Douglas specification:

$$y_{H,t}(z) = \left[ \frac{Z_t z_l}{\eta} \right]^\eta \left[ \frac{Z_t^* z_l^*}{1 - \eta} \right]^{1-\eta},$$

where a larger $\eta$ accounts for a smaller content of Southern inputs used in the production of intermediate goods sold in the South. In this paper I nest the special case with endogenous exports in GM2005 by setting $\eta = 1$. Alternatively, I nest the case in which firms serving the Southern market would produce exclusively through their foreign affiliates (as in Contessi, 2006) by setting $\eta = 0$. In the latter case, production in the South through horizontal FDI allows firms to avoid the trade cost $\tau^*$ by using local inputs. Using the l'Hôpital rule, $\lim_{\eta \to 0} \left( \frac{1}{\eta} \right)^\eta = \lim_{\beta \to -\infty} \beta^{1/\beta} = \lim_{\beta \to -\infty} \frac{\ln \beta}{\beta} = e^{\lim_{\beta \to -\infty} \frac{\ln \beta}{\beta}} = e^0 = 1$. The corresponding price and profit functions are $p_{H,t}(z) = \alpha \left( \tau^* \frac{w_t}{z_{t-1}} \right)^{1-\eta} \left( \frac{w_t^*}{z_{t-1}^*} \right)^{1-\eta}$ and $d_{H,t}(z) = \frac{1}{\eta} \rho_{H,t}(z)^{1-\eta} C_t^* Q_t - f_H \left( \frac{w_t}{z_t} \right)^\eta \left( \frac{w_t^*}{z_t^*} \right)^{1-\eta}.$}
Serving the foreign market involves a period-by-period fixed exporting cost equal to $f_H$ units of Northern effective labor, as well as the iceberg trade cost $\tau^*$. The profit maximization problem generates the following price and profit functions:

\[
\rho_{H,t}(z) = \frac{\theta}{\theta - 1} \frac{\tau^* w_t Q_t^{-1}}{Z_t z},
\]

\[
d_{H,t}(z) = \frac{1}{\theta} \rho_{H,t}(z)^{1-\theta} C^*_t Q_t - f_H \frac{w_t}{Z_t}.
\]

The model implies that every period $t$, the Northern firms with idiosyncratic labor productivity above a certain cutoff ($z > z_{H,t}^*$) find it profitable to export to the Southern market at the same time with serving their domestic market (North). They obtain profits that are large enough to cover both the fixed cost and the iceberg trade cost of exporting. As in GM2005, the firm with the idiosyncratic labor productivity equal to the cutoff obtains zero profits from exporting. Thus, the time-variant productivity cutoff $z_{H,t}$ is:

\[
z_{H,t} = \inf \{ z \mid d_{H,t}(z) > 0 \}.
\]

### 2.4 Households

Households in each country maximize the expected lifetime utility (as a function of consumption) and provide labor inelastically:

\[
\max_{\{B_{t+1}, z_{t+1}\}} \left[ E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C^1_{s-\gamma}}{1 - \gamma} \right],
\]

subject to the budget constraint:

\[
(v_t + \tilde{d}_t) N_t x_t + (1 + r_t) B_t + w_t L \geq \bar{v}_t (N_t + N_{E,t}) x_{t+1} + B_{t+1} + C_t,
\]

where $\beta \in (0, 1)$ is the subjective discount factor, $C_t$ is the consumption basket that includes varieties produced both in the North and in the South, and $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution.

The representative Northern household starts every period $t$ with mutual fund share holdings $x_t$ (whose market value is $\bar{v}_t N_t$) and real bond holdings $B_t$. It receives dividend income $\tilde{d}_t N_t$ on the mutual fund shares (equal to the profit of the average firm multiplied by the number of firms) in proportion with its share holdings $x_t$. It also receives interest $r_t B_t$ on bond holdings, and labor income equal to the real wage $w_t$ for the amount of labor $L = 1$ that it supplies inelastically. The representative household purchases two types of assets every period. First, it purchases $x_{t+1}$ shares in a mutual
fund of Northern firms that includes: (i) $N_t$ firms already producing at time $t$, either domestically or offshore, and (ii) $N_{E,t}$ new firms that enter the domestic market in period $t$. Each share is worth its market value $\tilde{v}_t$, equal to the net present value of the expected stream of future profits of the average firm. Second, the household also buys the risk-free bond $B_{t+1}$ denominated in units of the Northern consumption basket. (Bond holdings play a role in the extended model with international trade in bonds, which I present in Appendix A.2.)

In addition, the representative household purchases the consumption basket $C_t$, which includes the intermediate good varieties produced by the Northern firms ($\omega \in \Omega_t^{NN}$) either domestically or offshore; it also includes intermediate varieties produced by the Southern exporters ($\omega \in \Omega_t^{SN}$):

$$C_t = \left[ \int_{z_{\text{min}}^{V,t}}^{z_{V,t}} y_{D,t}(\omega) \frac{1}{\sigma} d\omega + \int_{z_{V,t}}^{\infty} y_{V,t}(\omega) \frac{1}{\sigma} d\omega + \int_{z_{H,t}}^{\infty} y_{H,t}(\omega) \frac{1}{\sigma} d\omega \right]^{\frac{1}{\sigma-1}},$$

(17)

where $\theta > 1$ is the symmetric elasticity of substitution across the different varieties of intermediate goods. I use the home consumption basket $C_t$ as the numeraire good, and define the real price of variety $z$ in units of the Northern consumption basket as $p_t(z) \equiv p_t(z)/P_t$. Thus, the the consumption-based price index in the North is:

$$1 = \left[ \int p_t(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}, \quad \omega \in \Omega_t^{NN} \cup \Omega_t^{SN}.$$ 

(18)

The first-order conditions generate the Euler equations for bonds and stocks:

$$C_t^{-\gamma} = \beta (1 + r_{t+1}) E_t \left[ C_{t+1}^{-\gamma} \right],$$

(19)

$$\tilde{v}_t = \beta (1 - \delta) E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (\tilde{d}_{t+1} + \tilde{v}_{t+1}) \right].$$

(20)

---

\footnotesize

20In the model with complete financial autarky (i.e. stocks in the mutual fund and bonds are not traded across countries), the equilibrium conditions for stock and bond holdings are $x_t = x_{t+1} = 1$ and $B_t = B_{t+1} = 0$.

21If $p_t(z)$ denotes the nominal price of each variety $z$, the price index of the home consumption basket is $P_t = \left[ \int p_t(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}$ for $\omega \in \Omega_t^{NN} \cup \Omega_t^{SN}$. 

---
2.5 Firm Entry and Exit

Firm entry takes place in both countries every period, as in GM2005. In the North, firm entry requires a sunk entry cost equal to $f_E$ units of Northern effective labor, which reflects the cost of starting a business in the firms’ country of origin. Potential entrants become aware of their idiosyncratic labor productivity $z$ only after entering the market. After paying the sunk entry cost, each firm is randomly assigned an idiosyncratic labor productivity $z$ that is drawn independently from a common distribution $G(z)$ with support over the interval $[z_{min}, \infty)$, and which the firm keeps for the entire duration of its life.

The potential entrant firms are forward looking and correctly anticipate their expected post-entry value $\tilde{v}_t$, which is given by the expected stream of future profits $\tilde{d}_t$ and by the exogenous probability $\delta$ with which they receive an exit-inducing shock every period. The forward iteration of the Euler equation for stocks (20) generates the following expression for the expected post-entry value of the potential entrants:

$$\tilde{v}_t = E_t \left\{ \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left( \frac{C_s}{C_t} \right)^{-\gamma} \tilde{d}_s \right\}. \quad (21)$$

In equilibrium, firm entry takes place until the value of the average firm $\tilde{v}_t$ equals the sunk entry cost $f_E \frac{w_t}{Z_t}$, both of which are expressed in units of the Northern consumption basket:

$$\tilde{v}_t = f_E \frac{w_t}{Z_t}. \quad (22)$$

The $N_{E,t}$ firms that enter at time $t$ do not produce until period $t + 1$. Irrespective of their idiosyncratic productivity, all firms - including the new entrants - are subject to a random exit shock that occurs with probability $\delta$ at the end of every period after production has taken place. Thus, the law of motion for the number of producing firms is:

$$N_{t+1} = (1-\delta)(N_t + N_{E,t}), \quad (23)$$

where $N_t = N_{t,D} + N_{t,V}$ is the number of firms that produce either domestically or offshore at period $t$.

---

22The sunk entry cost is equivalent to $f_E w_t / Z_t$ units of the Northern consumption basket.
3 Solving the Model with Firm Heterogeneity

As a necessary step in solving the model with firm heterogeneity, this section derives analytical solutions for the average productivity, prices and profits of the representative Northern firms that produce domestically and offshore. It also provides the expressions for the aggregate accounting and the balance of international payments that close the model with offshoring.

3.1 Average Firm Productivity Levels

Firms serving the domestic market I define two average labor productivity levels: (1) $\bar{z}_{D,t}$ corresponds to the Northern firms producing domestically, and (2) $\bar{z}_{V,t}$ corresponds to the Northern firms producing offshore. I illustrate them in Figure 3, in which I plot the density of the firm-specific labor productivity levels $z$ over the support interval $[z_{\min}, \infty)$.

![Figure 3. Average labor productivity levels of firms serving their home market through domestic ($\bar{z}_{D,t}$) and offshore ($\bar{z}_{V,t}$) production.]

Every period $t$, there are $N_{D,t}$ of the relatively less productive Northern firms ($z < z_{V,t}$) that choose to produce domestically; their average productivity is $\bar{z}_{D,t}$. The remaining $N_{V,t}$ are the relatively more productive Northern firms ($z > z_{V,t}$) that choose to produce offshore;\(^23\) their average productivity is $\bar{z}_{V,t}$.\(^24\) Since the firm-specific labor productivities $z$ are random draws from a common distribution $G(z)$ with density $g(z)$, I write the average idiosyncratic productivities of the Northern firms producing

\(^23\) The total number of Northern firms is $N_t = N_{V,t} + N_{D,t}$.
\(^24\) The difference between $\bar{z}_{V,t}$ and $z_{V,t}$ is that the former is the average productivity of offshoring firms, whereas the latter is the productivity cutoff above which firms produce offshore.
domestically and offshore as:

\[
\bar{z}_{D,t} = \left[ \frac{1}{G(z_{V,t})} \int_{z_{\min}}^{z_{V,t}} z^{\theta-1} g(z)dz \right]^\frac{1}{\theta-1} \quad \text{and} \quad \bar{z}_{V,t} = \left[ \frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta-1} g(z)dz \right]^\frac{1}{\theta-1} . \tag{24}
\]

**Pareto-distributed firm productivity** Following GM2005, I assume that the firm-specific labor productivity draws \( z \) are Pareto-distributed, with p.d.f. \( g(z) = k z_{\min} \theta / z^{k+1} \) and c.d.f. \( G(z) = 1 - (z_{\min} / z)^k \) over the support interval \([z_{\min}, \infty)\). Using this assumption, I derive analytical solutions for the average productivities of the two representative Northern firms producing domestically and offshore as functions of the time-variant productivity cutoff \( z_{V,t} \):⁴⁴

\[
\bar{z}_{D,t} = \nu z_{\min} \bar{z}_{V,t} \left[ \frac{z_{V,t}^{\theta-1} - z_{\min}^{\theta-1}}{z_{V,t}^{k} - z_{\min}^{k}} \right]^\frac{1}{\theta-1} \quad \text{and} \quad \bar{z}_{V,t} = \nu z_{V,t} , \tag{25}
\]

where the productivity cutoff is \( z_{V,t} = z_{\min} (N_{t}/N_{V,t})^{1/k} \), and the parameters are \( \nu \equiv \left[ \frac{k}{k-(\theta-1)} \right]^{1/(\theta-1)} \) and \( k > \theta - 1 \).²⁷ Since offshoring takes place one-way, from North to South, the Southern firms serve their domestic market exclusively through domestic production. Their average productivity is a constant, \( \bar{z}_{D}^* = \nu z_{\min}^* \).

**Exporting firms** Under the assumption of Pareto-distributed productivity draws, I derive the average productivity levels of the exporting firms in each economy as in GM2005:

\[
\bar{z}_{H,t} = \nu z_{\min} \left( \frac{N_{t}}{N_{H,t}} \right)^{1/k} \quad \text{and} \quad \bar{z}_{H,t}^* = \nu z_{\min}^* \left( \frac{N_{D,t}^*}{N_{H,t}^*} \right)^{1/k} . \tag{26}
\]

### 3.2 Average Prices and Profits

Using the average productivity levels derived above, I re-write the model of offshoring in terms of three representative Northern firms: one produces domestically, another produces offshore (each serving the Northern market), while a third firm produces domestically and exports to the Southern market. Due

²⁵I provide their derivation in Appendix A.9.
²⁶I use the functional form for the Pareto c.d.f. in order to derive the productivity cutoff as \( z_{V,t} = z_{\min} (N_{t}/N_{V,t})^{1/k} \). The shares of Northern firms producing domestically and offshore are \( N_{D,t}/N_{t} = G(z_{V,t}) \) and \( N_{V,t}/N_{t} = 1 - G(z_{V,t}) \), and the total number of Northern firms in every period is \( N_{t} = N_{D,t} + N_{V,t} \).
²⁷Parameter \( k \) reflects the dispersion of the productivity draws: A relatively larger \( k \) implies a smaller dispersion and a higher concentration of productivities \( z \) towards the lower productivity bound \( z_{\min} \). Also, the condition \( k > \theta - 1 \) ensures that the variance of firm size is finite.
to the wage asymmetry across countries, the Southern firms do not produce offshore. There are only two representative Southern firms: one produces for the local market, and the other exports to the North.

Finally, I use the average firm productivities defined above to re-write the prices and profits associated with each representative firm, as summarized in Table 1.

<table>
<thead>
<tr>
<th>Production</th>
<th>Destination</th>
<th>Prices</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>Domestic</td>
<td>$\tilde{\rho}<em>{D,t} = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} z</em>{D,t}^{\tilde{D},t}$</td>
<td>$\tilde{d}<em>{D,t} = \frac{1}{\theta} (\tilde{\rho}</em>{D,t})^{1-\theta} C_t$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{\rho}<em>{D,t} = \frac{\theta}{\theta - 1} \frac{w_t^<em>}{Z_t^</em>} z</em>{D,t}^{\tilde{D},t}$</td>
<td>$\tilde{d}<em>{D,t} = \frac{1}{\theta} (\tilde{\rho}</em>{D,t})^{1-\theta} C_{t}^{*}$</td>
</tr>
<tr>
<td>Offshore</td>
<td>Domestic</td>
<td>$\tilde{\rho}<em>{V,t} = \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^*} z</em>{V,t}^{\tilde{V},t}$</td>
<td>$\tilde{d}<em>{V,t} = \frac{1}{\theta} (\tilde{\rho}</em>{V,t})^{1-\theta} C_t - f_V \frac{w_t^* Q_t}{Z_t^*}$</td>
</tr>
<tr>
<td>Domestic</td>
<td>Export</td>
<td>$\tilde{\rho}<em>{H,t} = \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^*} z</em>{H,t}^{\tilde{H},t}$</td>
<td>$\tilde{d}<em>{H,t} = \frac{1}{\theta} (\tilde{\rho}</em>{H,t})^{1-\theta} C_t Q_t - f_H \frac{w_t^<em>}{Z_t^</em>}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{\rho}<em>{H,t} = \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^*} z</em>{H,t}^{\tilde{H},t}$</td>
<td>$\tilde{d}<em>{H,t} = \frac{1}{\theta} (\tilde{\rho}</em>{H,t})^{1-\theta} C_t Q_t - f_H \frac{w_t^<em>}{Z_t^</em>}$</td>
</tr>
</tbody>
</table>

**Endogenous productivity cutoff for offshoring** Using the property that the Northern firm at the productivity cutoff $z_{V,t}$ is indifferent between the two locations of production, I derive the following relationship between the average profits of the two representative firms that serve their domestic market through either domestic and offshore production:

$$\tilde{d}_{V,t} = \frac{k}{k - (\theta - 1)} \left( \frac{z_{V,t}}{z_{D,t}} \right)^{\theta - 1} \tilde{d}_{D,t} + \frac{\theta - 1}{k - (\theta - 1)} f_V \left( \frac{w_t}{Z_t} \right)^{\alpha} \left( \frac{w_t^* Q_t}{Z_t^*} \right)^{1-\alpha}. \tag{27}$$

In addition, using the property that the firm at the productivity cutoff $z_{H,t}$ obtains zero profits from exporting as in GM2005, the average profits from exports are:

$$\tilde{d}_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f_H \frac{w_t}{Z_t}, \quad \text{and} \quad \tilde{d}_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f_H \frac{w_t^*}{Z_t^*}. \tag{28}$$

**Price indexes** The consumption price index in the Northern economy includes the average prices of goods produced domestically and offshore by the Northern firms, as well as the average price of the goods imported from the South:

$$1 = N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} + N_{V,t} (\tilde{\rho}_{V,t})^{1-\theta} + N_{H,t}^* (\tilde{\rho}_{H,t}^*)^{1-\theta}. \tag{29}$$

---

28See Appendix A.10 for the derivation.
In the South, there is no representative firm producing offshore. The consumption price index includes
the average price of goods produced domestically by the Southern firms, and also that of goods
imported from the North:

\[ 1 = N_{D,t}^* \left( \tilde{p}_{D,t} \right)^{1-\theta} + N_{H,t} \left( \tilde{p}_{H,t} \right)^{1-\theta}. \] (30)

**Total profits** The total profits of the Northern firms include the average profits from domestic
production, from offshore production, and from exporting:

\[ N_t \tilde{d}_t = N_{D,t} \tilde{d}_{D,t} + N_{V,t} \tilde{d}_{V,t} + N_{H,t} \tilde{d}_{H,t}. \] (31)

The total profits of the Southern firms combine the profits from domestic production and from exports:

\[ N_{D,t}^* \tilde{d}_t = N_{D,t}^* \tilde{d}_{D,t} + N_{H,t}^* \tilde{d}_{H,t}. \] (32)

### 3.3 Aggregate Accounting and the Balance of International Payments

I measure the aggregate income as the sum of the wage bill and the amount of stock dividends that
households in each economy obtain every period:

\[ Y_t = w_t + N_t \tilde{d}_t \quad \text{and} \quad Y_t^* = w_t^* + N_{D,t}^* \tilde{d}_t^*. \] (33)

The amount of offshore production - measured as the wage bill of the Southern workers employed for
production in the offshoring sector - belongs to the aggregate income of the Southern economy.

Under financial autarky in the markets for both bonds and stocks (i.e. \( B_{t+1} = B_t = 0 \) and
\( x_{t+1} = x_t = 1 \) in equilibrium), aggregate accounting implies that households spend their income from
labor and stock holdings on consumption and investment in new firms:

\[ C_t + N_{E,t} \tilde{v}_t = Y_t \quad \text{and} \quad C_t^* + N_{E,t}^* \tilde{v}_t^* = Y_t^*. \] (34)

The real exchange rate \( Q_t \) is determined by the balanced current account condition for the North:

\[
CA_t^{Autarky} = N_{H,t} \left( \tilde{p}_{H,t} \right)^{1-\theta} C_t^* Q_t + N_{V,t} \tilde{d}_{V,t} - N_{V,t} \left( \tilde{p}_{V,t} \right)^{1-\theta} C_t - N_{H,t}^* \left( \tilde{p}_{H,t} \right)^{1-\theta} C_t^*
\]

(a) Exports  (b) Repatriated profits  (c) Value added offshore  (d) Imports of Southern varieties

Under financial autarky, the balanced current account condition \((CA_t^{Autarky} = 0)\) implies that the sum
of (a) the exports of Northern firms to the South and (b) the repatriated profits of offshore affiliates must be equal to the sum of (c) the value added offshore by Northern firms and (d) the imports of varieties produced by the Southern firms. The case with international trade in bonds in presented in Appendix A.2.

3.4 Model Summary

As shown in Appendix A.1, the baseline model with financial autarky for the Northern economy can be summarized by 16 equations in 16 endogenous variables: \(N_t, N_{D,t}, N_{V,t}, N_{H,t}, N_{E,t}, d_t, d_{D,t}, d_{V,t}, d_{H,t}, \bar{z}_{D,t}, \bar{z}_{V,t}, \bar{z}_{H,t}, \bar{v}_t, \bar{w}_t, \) and \(C_t\). Since the Southern firms do not offshore to the high-wage North, the Southern economy is described by only 11 equations in 11 endogenous variables; there are no Southern counterparts for \(N_t, N_{V,t}, d_{V,t}, z_{D,t}\) and \(z_{V,t}\). In particular, the average labor productivity of the representative Southern firm producing for the domestic market \((\bar{z}_D)\) is constant over time. Variables \(N_{D,t}, r_t, N_t^*\) and \(r_t^*\) are predetermined.

4 Calibration

I use a standard quarterly calibration by setting the subjective rate of time discount \(\beta = 0.99\) to match an average annualized interest rate of 4 percent. The coefficient of relative risk aversion is \(\gamma = 2\). Following GM2005, I set the intra-temporal elasticity of substitution at \(\theta = 3.8\). Although the resulting markup of 35.7 percent over the marginal cost might appear too large compared to the standard macroeconomic literature, its magnitude must be considered in the context of the sunk entry cost that offsets the markup. I also calibrate the probability of firm exit \(\delta = 0.025\) to match the annual 10 percent job destruction in the U.S.

As summarized in Table 2, I calibrate the fixed costs of offshoring \((f_V)\) and exporting \((f_H\) and \(f_H^*\)), as well as the Pareto distribution parameter \((k)\) so that the model matches the importance of offshoring and trade for the Mexican economy, as illustrated by four empirical moments: (1) The maquiladora value added represents approximately 20 percent of Mexico’s manufacturing GDP (INEGI, 2008), compared to 25 percent in the model; (2) The maquiladora exports represent about half of Mexico’s total exports (Bergin, Feenstra, and Hanson, 2008), vs. 60 percent in the model; (3) Employment in the maquiladora sector accounts for approximately 25 percent of Mexico’s total manufacturing employment (Bergin, Feenstra, and Hanson, 2008), compared to 22 percent in the model; (4) Total imports represent the equivalent of 33 percent of Mexico’s GDP (INEGI, 2008), and 32 percent in the
model. To this end, I set $f_V = 0.0057$ (the fixed cost of offshoring for Northern firms), $f_H = 0.032$ and $f_H^* = 0.018$ (the fixed costs of exporting for Northern and Southern firms, respectively), as well as $k = 4.2$ (the Pareto distribution coefficient). Without loss of generality, I set the lower bound of the support interval for firm-specific productivities in the North and the South at $z_{min} = z_{min}^* = 1$.

Table 2. Calibration, baseline offshoring model: parameters and targets

<table>
<thead>
<tr>
<th>Key parameters:</th>
<th>Targets:</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed cost of offshoring</td>
<td>$f_V = 0.0057$</td>
<td>Maquila VA in Mexico’s manuf. GDP</td>
<td>20%</td>
</tr>
<tr>
<td>Fixed cost of exporting, North</td>
<td>$f_H = 0.0320$</td>
<td>Maquila exports in Mexico’s total</td>
<td>50%</td>
</tr>
<tr>
<td>Fixed cost of exporting, South</td>
<td>$f_H^* = 0.0180$</td>
<td>Maquila in Mexico’s manuf. employment</td>
<td>25%</td>
</tr>
<tr>
<td>Pareto distribution coefficient</td>
<td>$k = 4.2$</td>
<td>Mexico’s imports relative to GDP</td>
<td>33%</td>
</tr>
</tbody>
</table>

In order to derive an asymmetric steady state with respect to the relative wage, I set the sunk entry cost - which reflects the regulation of starting a business in the firm’s country of origin - to be larger in the South than in the North ($f_E^* = 4f_E$ and $f_E = 1$). As a result, the steady state number of firms, labor demand and real wage are relatively lower in the South. The calibration reflects the considerable variation in the cost of starting a business across countries. The corresponding monetary cost is 3.3 times higher in Mexico than in the U.S. or Canada. It is 6.2 times higher in Hungary than in the U.K. (World Bank, 2007; see Appendix A.3). The asymmetric sunk entry costs, along with the trade iceberg cost ($\tau = 1.3$) and the values for $f_V$, $f_H$ and $f_H^*$ reported above, generate a steady state value for the terms of labor that is less than one ($TOL = 0.76$). In other words, the steady state cost of effective labor in the South, defined as the real wage divided by the aggregate productivity level, is 76 percent of the corresponding value in the North. The calibration provides an incentive for the Northern firms to produce offshore in steady state.

The resulting steady-state fraction of the Northern firms that use foreign labor ($N_V/N$) is 1.4

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29In the alternative model with exports only, I set $f_H = 0.0260$ and $f_H^* = 0.0226$ so that the fraction of Northern exporting firms (10 percent) and that of Southern exporting firms (63 percent) match the corresponding steady state values from the model with offshoring.

30The sunk entry cost asymmetry is one method that generates a gap in the effective wages across countries. The same result would be obtained with at least two other methods: (1) Introduce a cross-country asymmetry in the size of firms (rather than in their number) through the price elasticity of demand. With identical sunk entry costs and equal average labor productivity levels in the two economies, firms in the economy with the lower price elasticity of demand charge relatively higher markups and produce relatively less output. Therefore, the lower labor demand generates lower wages. (2) Another way to generate different firm sizes across countries, similar to the one I use in this paper, would be to allow for multi-product firms and sunk costs of creating new product varieties. While keeping the sunk firm entry costs equal across countries, there will be fewer varieties per firm and lower demand for labor in the economy with the higher sunk cost of creating a new variety.
percent; the fraction of exporting firms \((N_H/N)\) is 10.1 percent.\(^{31}\) Since I model offshoring in an asymmetric two-country framework that abstracts from exchanges between U.S. firms and the rest of the world (other than Mexico), the steady state values reported above are less than their empirical counterparts. In the data, approximately 14 percent of the U.S. firms (other than domestic wholesalers) used imports both from Mexico and from the rest of the world in 1997 (Bernard, Jensen, Redding and Schott, 2007),\(^{32}\) out of which intra-firm imports (as opposed to arm’s length transactions) represented half of the total (Bardhan and Jafee, 2004). Approximately 21 percent of the U.S. manufacturing plants were exporters in 1992 (Bernard, Eaton, Jensen and Kortum, 2003).

The calibration also implies that the steady-state share of Northern expenditure on the varieties produced by the Northern firms domestically (66.0 percent) - firms which are relatively less productive than the average - is less than their fraction in the total number of varieties available in the North (89.2 percent). In contrast, since the offshore varieties are produced by the relatively more productive Northern firms, their market share (21.2 percent) is more than their fraction in the total number of varieties available in the North (1.2 percent).\(^{33}\)

5 Results

5.1 Offshoring Dynamics

I log-linearize the baseline model of offshore production under financial autarky around the steady state, and compute the impulse responses to a transitory one-percent increase in aggregate productivity in the North. I assume that productivity is described by the univariate process \(\log Z_{t+1} = \rho \log Z_t + u_t\), with the persistence parameter \(\rho = 0.9\).

Figure 4 shows the impulse responses of the baseline model of offshoring (the thick solid lines), and contrasts them to those from two alternative frameworks: (i) a model of offshoring in which the productivity cutoff is fixed, so that the fraction of offshoring firms is constant over the business cycle (the thin solid lines)\(^{34}\); and (ii) the extreme case with no offshoring, a case in which I revisit the model

\(^{31}\)In the Southern economy, the ratio of exporting firms in the total is 63 percent.

\(^{32}\)The value would understate the fraction of plants that use imported inputs if the importing firms tend to operate multiple plants that produce multiple product varieties.

\(^{33}\)The market share of Southern varieties - produced by Southern firms that are relatively less productive than the Northern exporters - is 61.66 percent in the South. Consistent with the lower productivity, their market share is less than their fraction in the total number of varieties available in the South (62.77 percent).

\(^{34}\)Although the fraction of offshoring firms is fixed, their number varies over time due to the entry of new firms in the country of origin. During expansions in the North, the new entrants that draw idiosyncratic productivity factors above the cutoff start by producing directly offshore. However, under this version of the model, none of the firms that initially produce at home can relocate offshore when the terms of labor appreciate.
with exports only in GM2005 (the dotted lines). For each variable, the horizontal axis illustrates quarters after the initial shock, and the vertical axis shows the percent deviations from the original steady state in each quarter.

Figure 4. Impulse responses to a transitory 1 percent increase in aggregate productivity in the North, model with variable fraction of offshoring firms (thick solid line), fixed fraction of offshoring firms (thin line), and no offshoring (dotted line).

The intensive margin On impact, the increase in aggregate labor productivity in the North generates an equal increase in the real wage \( w_t \). The rising demand for the intermediate good...
varieties produced both domestically and offshore causes an immediate increase in offshoring along its intensive margin. Since the aggregate productivity gain in the North is not replicated in the South, the excess demand for Southern effective labor causes the real wage in the South \( (w^*_t) \) and the terms of labor \((TOL_t = \frac{Q_t w_t^*/Z_t^*}{w_t/Z_t})\) to increase. As a result, the number of Northern firms that produce offshore \((N_{V,t})\) drops on impact due to: (i) the increase in the cost of effective labor offshore, and (ii) the increase in the fixed cost of offshoring, both of which are sensitive to the effective wage in the South.

The extensive margin  As the aggregate labor productivity in the North persists above its initial steady state, the larger market size encourages firm entry, as shown by the gradual increase in the number of incumbent firms \((N_t)\). As a result, firm entry leads to an increase in the demand for Northern labor, which causes the cost of effective labor to appreciate gradually in the North relative to the South. In Figure 4, the appreciation is visible as the Northern wage persists above aggregate productivity 10 quarters after the initial shock, and thus the terms of labor remain below their initial steady state level. As a result, the number of offshoring firms \((N_{V,t})\) increases, as some of the more productive Northern firms relocate production to the South in the medium run. Notably, the increase in the number of offshoring firms is gradual, as it mirrors the gradual appreciation of the terms of labor.

The total value added offshore increases by more under the baseline model in which firms can relocate production over the business cycle (the thick solid line) than in the alternative model in which the fraction of offshoring firms is fixed (the thin line). Thus, 20 quarters after the shock, more than half of the increase in the total value added offshore \((VAR)\) is due to the adjustment along the extensive margin.

In the South, the initial jump in the real wage - caused by the spike in offshoring along its intensive margin - is followed by an additional increase which occurs gradually over time, as some of the more productive Northern firms relocate production to the South. The adjustment in offshoring along its extensive margin transfers some of the upward pressure from the domestic to the foreign wage in the medium run, and causes the terms of labor to appreciate by less (i.e. \(TOL\) to decline by less) than in the alternative models with no extensive margin relocation (the thin solid line) and no offshoring (the dotted line). The increase in labor demand in the North, caused by firm entry, and subsequently the increase in labor demand in the South, caused by offshoring, enhance the cross-country co-movement of wages and aggregate incomes relative to the alternative frameworks with no extensive margin relocation or no offshoring.
5.2 The Cyclicality of Offshoring: Mexico’s Maquiladora Sector

In this section I provide a set of stylized facts that describe the cyclicality of offshoring from U.S. manufacturing to Mexico’s maquiladora sector. In particular, I describe the extensive margin dynamics of the maquiladora sector relative to fluctuations in U.S. manufacturing. I also document the link between offshore production and fluctuations in the U.S./Mexico relative manufacturing wage. The stylized facts will be useful to examine the theoretical results in the following sections.

Mexico’s maquiladora sector  The maquiladora sector constitutes an appropriate empirical setup to study the cyclicality of offshoring through vertical FDI, due to the absence of local consumption, as well as the dominant share of the U.S. as the country of origin for intermediate inputs and the destination market for its final goods. By definition, plants operating under Mexico’s maquiladora program import inputs, process them, and ship the resulting goods back to the country of origin (Gruben, 2001). Although not all plants in Mexico’s maquiladora sector are owned by U.S. firms, most of the maquiladora’s imported inputs (82 percent in 2001) originate in the U.S., and most of the maquiladora’s value added (roughly 90 percent) is exported to the U.S. (Hausman and Haytko, 2003; Burstein, Kurz, and Tesar, 2009). The value added of the maquiladora sector is part of Mexico’s manufacturing output.

Empirical cross-correlations  Mexico’s total manufacturing output and, in particular, the maquiladora value added are strongly correlated with U.S. manufacturing. In panel 1 of Figure 5, I plot the detrended series for Mexico’s maquiladora value added (the dashed line) and Mexico’s total manufacturing output (the dotted line) against the manufacturing component of the U.S. industrial production (henceforth U.S. IP, the solid line), for the interval 1990:1-2006:4. The chart shows that the U.S. recessions in 1990 and 2001, as well as the expansion throughout the 1990s, were associated with similar developments in the maquiladora value added. Also, during the 1994-95 financial crisis in Mexico, the decline in the maquiladora value added was less pronounced than the drop in Mexico’s total manufacturing, as the production sharing sector benefited from its direct links with U.S. manufacturing. Indeed, the cross-correlations in panel 2 show that Mexico’s maquiladora moves closely

\[35\] The seasonally-adjusted data is expressed in natural logs and HP-filtered. The data for U.S. manufacturing (i.e. industrial production and the nominal hourly wage) is provided by the Federal Reserve Board and the U.S. Bureau of Labor Statistics. Data for Mexico’s manufacturing and maquiladora sectors (real value added, the number of plants, and nominal remuneration per employee-hour) is provided by the Instituto Nacional de Estadística y Geografía (INEGI). I aggregate the maquiladora data (available at monthly frequency and without seasonal adjustment) at the quarterly level, and seasonally adjust it using the X-12-ARIMA method of the U.S. Census Bureau.
together with U.S. manufacturing, and that its correlation with the U.S. IP is even stronger than that of Mexico’s total manufacturing IP.

Figure 5. Business cycle dynamics of offshoring to Mexico.

In panel 3 (middle left), I plot the detrended series for the number of maquiladora plants in Mexico (the dashed line) - which I view as an empirical proxy for the extensive margin of offshoring - against the U.S. IP for manufacturing (the solid line). In panel 4, the cross-correlations between the number of maquiladora plants and the U.S. IP show that U.S. manufacturing leads the number of maquiladora
plants by at least four quarters. The result suggests that the extensive margin of offshoring adjusts gradually over time, whereas the maquiladora value added is almost contemporaneously correlated with the U.S. manufacturing output.

Finally, in panel 5 (bottom left) I plot the number of maquiladora plants (the dashed line) and the maquiladora value added (the dotted line) along with the ratio of hourly wages in U.S. manufacturing and Mexico’s maquiladora sector, expressed in the same currency (the solid line). The corresponding cross-correlations in panel 6 describe an interesting link between fluctuations in the relative wage and offshoring. The correlations between the number of maquiladora plants and the past U.S./Mexico wage ratio are positive, a result which is consistent with the interpretation that past increases in the hourly wage in U.S. manufacturing relative to that in Mexico’s maquiladora sector are followed by the relocation of production from the U.S. to Mexico. Also, the correlations between the maquiladora plants and the future wage ratios are negative: The relocation of production to Mexico is associated with a future decline in the relative wage ratio, which can be caused either by a U.S. wage moderation or by an increase in Mexico’s maquiladora wage. This empirical result is consistent with the model of offshoring developed in this paper, in which the adjustment along the extensive margin is driven by fluctuations in the relative cost of effective labor over the business cycle.

5.3 Offshoring Dynamics: Output and Consumption Co-movements

In this section I outline the cross-country correlations of output, offshore value added, and consumption generated by the baseline model of offshoring, and examine their sensitivity to key model parameters, such as the trade cost and the persistence of the productivity process.

**Offshoring variables and the productivity process** I assume that aggregate productivity follows the bivariate autoregressive process:

\[
\begin{bmatrix}
\log Z_t \\
\log Z_t^*
\end{bmatrix}
= \begin{bmatrix}
\rho_Z & \rho_{ZZ^*} \\
\rho_{Z^*Z} & \rho_{Z^*}
\end{bmatrix}
\begin{bmatrix}
\log Z_{t-1} \\
\log Z_{t-1}^*
\end{bmatrix}
+ \begin{bmatrix}
\xi_t \\
\xi_t^*
\end{bmatrix},
\]

(35)

where the persistence parameters are asymmetric across the two economies (\(\rho_Z = 0.996\) and \(\rho_{Z^*} = 0.951\)), there are no spillovers (\(\rho_{ZZ^*} = \rho_{Z^*Z} = 0\)), and the technology shocks are less volatile in the North than in the South (i.e. variances \(0.0050939^2\) vs. \(0.0139570^2\)), with the covariance \(0.1898 \times 10^{-4}\).

---

36 Although the interval of four quarters may appear too short for the creation of new offshore plants, one must consider that a sizable fraction of the non-U.S. owned maquiladora plants represent arm’s length contractors that have the flexibility to enter into and exit from outsourcing relationships with U.S. firms over the business cycle.
implying a correlation of shocks of 0.27. These assumptions are based on the results in Mandelman and Zlate (2008), in which we estimate the bivariate productivity process using total factor productivity (TFP) data for the U.S. and Mexico.

The co-movement of output, value added offshore, and consumption Table 3 shows the cross-country correlations of output \( \text{Corr}(Y_R, Y_R^*) \), the correlation of offshore value added with output in the country of origin, \( \text{Corr}(Y_R, VA_R) \), and the cross-country correlation of consumption \( \text{Corr}(C_R, C_R^*) \). It also reports the correlations generated by the alternative framework in which I shut down offshoring, and thus revisit the model with endogenous exports in GM2005.

### Table 3. Cross-country contemporaneous correlations

<table>
<thead>
<tr>
<th>Model:</th>
<th>Financial autarky</th>
<th>International bond trading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Offshoring</td>
<td>No offshoring</td>
</tr>
<tr>
<td>( \text{Corr}(Y_R, Y_R^*) )</td>
<td>0.35</td>
<td>0.27</td>
</tr>
<tr>
<td>( \text{Corr}(Y_R, VA_R)   )</td>
<td>0.99</td>
<td>n/a</td>
</tr>
<tr>
<td>( \text{Corr}(C_R, C_R^*) )</td>
<td>0.40</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Two notable results emerge from Table 3. First, the correlation of the value added offshore with Northern output, \( \text{Corr}(Y_R, VA_R) \), is notably higher than the cross-country correlation of total output, \( \text{Corr}(Y_R, Y_R^*) \), a result which is consistent with the stylized facts from Mexico’s maquiladora and total manufacturing described above (i.e. the offshoring sector co-moves more closely with output in the country of origin than does the total output of the economy that hosts offshoring). In the model, the value added offshore is closely related to aggregate productivity, firm entry and the appreciation of the cost of effective labor in the North, and thus is highly correlated with the Northern output. The total output in the South, on one hand, receives the positive contribution of the offshoring sector, which enhances its co-movement with the Northern output. On the other hand, the relative productivity gains in the North dampen firm entry in the South, and thus partially offset the additional co-movement

\[ \text{Corr}(Y_R, Y_R^*) = \frac{\sum_i w_i Y_{R,i}^*}{\sum_i w_i Y_{R,i}} \]

\[ \text{Corr}(Y_R, VA_R) = \frac{\sum_i w_i VA_{R,i}}{\sum_i w_i VA_{R,i}} \]

\[ \text{Corr}(C_R, C_R^*) = \frac{\sum_i w_i C_{R,i}^*}{\sum_i w_i C_{R,i}^*} \]

\[ P_t = \left( \frac{N_V t + N_H}{1 - \theta} \right)^{1-\theta} P_{1t} \]

\[ VA_{R,t} = N_V t \left[ \frac{\pi_{V,t}}{\pi_{V,t}} \right]^{1-\theta} C_t \]

\[ P_V t = \left( \frac{N_V t}{N_V t + N_H t} \right)^{1-\theta} P_{1V,t} \]

\[ VA_{R,t} = \left( N_V t + N_H t \right)^{1-\theta} VA_{R,t} \]

37In order to compute the cross-country correlations, I deflate output and consumption by the average price indices in each country, since the empirical price deflators are best represented by the average price index \( \bar{P}_t \) rather than the welfare-based price index \( P_t \), as discussed in GM2005. For instance, I deflate output in the North by the average price index as \( Y_{R,t} = P_t Y_{1,t} \overline{P}_t \), where \( P_t = (N_D t + N_V t + N_H) \overline{P}_t \).

38The total value added offshore in the model is \( VA_t = N_V t \left[ \frac{\pi_{V,t}}{\pi_{V,t}} \right]^{1-\theta} C_t \), where \( \tilde{z}_{V,t} \) is the average idiosyncratic productivity of the offshoring firms, and \( \theta > 1 \) is the price elasticity of demand. To compute the correlations, I deflate the value added offshore by the average price index of the offshored varieties, \( VA_{R,t} \overline{P}_{V,t} \). To this end, I decompose the price index for the offshore varieties into components reflecting (i) variety and (ii) average price as \( P_t = (N_V t + N_H t) \overline{P}_{V,t} \), to obtain \( VA_{R,t} = (N_V t) \overline{P}_{V,t} VA_{R,t} \).
generated by offshoring. In addition, offshoring reduces the competitiveness of the Southern exporters, as it transfers some of the upward wage pressure from the North to the South through the relocation of production. Nonetheless, in the model with international trade in bonds, the cross-country co-movement of total output is further dampened by the fact that Southern households lend to finance firm entry in the more productive Northern economy.

Second, the model with offshoring enhances the co-movement of output relative to the model with no offshoring in GM2005. Intuitively, in GM2005 the positive shock to aggregate productivity in the North generates an increase in the demand for all intermediate good varieties - both from the North and from the South - as well as an immediate increase in the relative price of varieties from the South. However, the resulting substitution effect is offset by the fact that the increase in aggregate productivity encourages firm entry and leads to a gradual appreciation of the cost of effective labor in the North, which in turn enhances the export competitiveness of the Southern firms. Therefore, the model with no offshoring (GM2005) still generates positive co-movement of output across the two economies.

The model of offshoring - in which the more productive firms relocate production offshore over the business cycle - introduces an additional transmission mechanism that enhances the co-movement of output relative to the model with no offshoring. Following an increase in aggregate productivity in the North, firm entry causes a gradual appreciation of the terms of labor, which in turn provides an incentive for some of the more productive firms from the North to relocate production to the South. The increase in output in the North, generated by the positive shock to aggregate productivity and firm entry, and subsequently in the South, generated by the relocation of production by the Northern firms, enhance the co-movement of total output across the two economies. The effect is dampened in the version with international trade in bonds, as the Southern households lend to the North.

The reminder of this section examines the sensitivity of the cross-country correlations to variations in: (a) the iceberg trade cost $\tau$ (which applies to both offshoring and non-offshoring trade), and (b) the persistence of the bivariate autoregressive productivity process $\rho_Z$. Figures 6 and 7 show that the model with offshoring generates larger cross-country correlations for both output and consumption relative to the extreme case with no offshoring in GM2005, results which hold for a wide range of values for the trade cost $\tau \in [1.21, 1.34]$ and for the persistence parameter $\phi_Z \in [0.9, 1]$. 

28
Sensitivity to the trade cost $\tau$  The cross-country correlation of output decreases with the trade cost (Figure 6, panels 1 and 2). During expansions in the North, a lower trade cost enhances the demand for intermediate good varieties produced in the South either by the Northern offshoring firms or by the Southern exporters. A lower trade cost also facilitates the relocation of production offshore, and thus enhances the cross-country co-movement of output.

The result is consistent with the stylized facts documented in Burstein, Kurz, and Tesar (2009), namely that countries involved in production sharing more intensely tend to display larger correlations
of manufacturing output. As shown in panels 3 and 4, a lower trade cost is associated in steady state with (i) a larger fraction of firms originating in the North that produce offshore; (ii) a larger share of offshoring-related trade in the Southern exports; (iii) a larger share of exports in the Southern output; and (iv) a larger share of the offshoring value added in the Southern output. Thus, a decrease in the trade cost from its baseline calibration value (1.30) to the lower extreme (1.21) - which is associated with an increase in the steady-state fraction of offshoring firms from 1.4 to 22 percent - enhances the cross-country correlation of output by 10 percentage points.

**Sensitivity to the aggregate productivity persistence** $\rho_Z$  
The co-movement of output increases with the persistence of the productivity process.$^{39}$ The result is visible particularly under financial autarky (panel 1 in Figure 7), as the additional co-movement generated by offshoring relative to GM2005 increases with the persistence parameter (from six to 12 additional percentage points from $\rho_Z = 0.900$ to $\rho_Z = 0.995$). A larger persistence of the aggregate productivity shock generates a relatively stronger firm entry and a larger appreciation of the terms of labor, which in turn provides a greater incentive for firms to relocate production offshore. The effect is dampened by the introduction of international trade in bonds (panel 2), when Southern households finance firm entry in the North.

1. Cross-country correlations, autarky
2. Cross-country correlations, trade in bonds

![Figure 7. Cross-country correlations, sensitivity to $\rho_Z$.](image)

$^{39}$I assume that the persistence parameter is symmetric across countries, that there are zero spillovers, and maintain the variance-covariance matrix of shocks from Mandelman and Zlate (2009).
5.4 Offshoring Dynamics: the Extensive and Intensive Margins

In this section I provide additional evidence in support of the offshoring model in which firms make extensive margin decisions by relocating production offshore over the business cycle. To this end, I provide unconditional cross-correlations between lags and leads of the U.S. manufacturing output and two empirical indicators of offshore production in Mexico: (i) the number of maquiladora establishments, as an empirical proxy for the extensive margin; and (ii) the value added per establishment, as a proxy for the intensive margin. Then I compare the empirical correlations to their theoretical counterparts, focusing on the dynamics of the extensive and intensive margins of offshoring implied by the model.

**Offshoring margin variables** In the model, the total value added offshore is a function of the number of offshoring firms, their average idiosyncratic productivity, the foreign cost of effective labor, and the domestic aggregate consumption, \( VA_t = N_{V,t} \left[ \tau^{\frac{\theta}{\theta-1}} \frac{w^*_t Q_t^*}{Z^*_t Z_{V,t}} \right]^{1-\theta} C_t \). The number of offshoring firms \((N_{V,t})\) measures the extensive margin of offshore production, and constitutes the theoretical counterpart for the number of maquiladora plants in Mexico. The real value added per offshoring firm \((VA_t/N_{V,t})\) represents the extensive margin of offshoring; it is the theoretical counterpart for the value added per maquiladora plant.\(^{40}\)

**Theoretical vs. empirical cross-correlations** In study the theoretical implications of the baseline model of offshoring under financial autarky. I also assume that aggregate productivity in the North and the South follow the bivariate autoregressive process described by equation (35), where the persistence parameters are \( \rho_Z = \rho_Z^* = 0.906 \), and the spillovers are \( \rho_{ZZ^*} = \rho_{Z^*Z} = 0.088 \), as in Backus, Kehoe, and Kydland (1992). The variance of the shocks is 0.00852\(^2\) and the covariance is 0.18728 \(\times 10^{-4}\), values which correspond to a correlation of innovations of 0.258.

Figure 8 shows the empirical correlations between U.S. manufacturing and the two maquiladora margin indicators (the black line), as well as their 95 percent confidence interval.\(^{41}\) It also shows the corresponding theoretical correlations generated by the baseline model of offshoring (the red line), and also the correlations generated by the baseline model augmented with elastic labor supply (the dotted line).

---

\(^{40}\) I deflate the value added offshore by the average price index of offshore varieties, as \( VA_{R,t} = (N_{V,t}) \frac{1}{VA_t} \).

\(^{41}\) The 95 percent interval is computed using the "corrcoef" command in Matlab.
Regarding the extensive margin (panel 1), the data shows a strong and positive correlation between the number of maquiladora plants and the past U.S. manufacturing output. As discussed, expansions in U.S. manufacturing tend to lead the number of offshore plants by at least four quarters. The model is successful in capturing this pattern: the correlation between the number of offshoring firms and the past output in the North is positive, and reaches a peak for the Northern output lagged by four quarters. The result is explained by the fact that, following a productivity improvement in the North, the increase in the number of offshoring firms is gradual, as it mirrors the appreciation of the terms of labor caused by domestic firm entry. Although the contemporaneous correlation between the number of offshoring firms and Northern output is slightly negative (rather than positive as in the data) - on impact, the increased demand for Southern varieties causes an immediate jump in the Southern wage, and thus an immediate decline in the number of offshoring firms - the model replicates the inter-temporal dynamics of the extensive margin.\footnote{The initial decline in the number of offshoring firms is later followed by a gradual increase as the terms of labor appreciate over time.}

Turning towards the intensive margin (panel 2), the empirical correlation between the maquiladora value added per plant and the past U.S. manufacturing output is negative and statistically significant. The model is successful in replicating this pattern as well. Following a positive technology shock in the North, the number of offshoring firms increases faster than the total value added offshore due to
the appreciation of the terms of labor. As a result, the value added per offshoring firm declines below its initial level several quarters after the shock, and the theoretical correlation between the intensive margin of offshoring and past output in the North is negative.

**Estimated impulse responses** To provide additional evidence on the extensive margin dynamics in Mexico’s maquiladora sector, Appendix A.4 shows the empirical impulse responses of the maquiladora indicators (total value added, number of plants, and value added per plant) to permanent technology shocks in U.S. manufacturing, which I identify using long-run restrictions in a structural vector-autoregressive (VAR) framework. The estimated impulse responses show that, following a permanent technology shock in U.S. manufacturing, the value added per maquiladora plant increases on impact, whereas the number of plants increases gradually over time.

5.5 Real Exchange Rate Dynamics

**Average prices and product variety** In this section I analyze the model’s predictions for the relative price dynamics in response to aggregate shocks. Due to the existence of endogenous product variety in the model, I use the consumer price index (CPI)-based real exchange rate \( \tilde{Q}_t = \varepsilon_t \tilde{P}_t^*/\tilde{P}_t \) as the theoretical counterpart to the empirical real exchange rate. As discussed in Broda and Weinstein (2003) and GM2005, the average prices \( \tilde{P}_t \) and \( \tilde{P}_t^* \) best represent the corresponding empirical CPI levels. Therefore, I break down the welfare-based price indexes \( P_t \) and \( P_t^* \) into (a) components reflecting product variety, and (b) components reflecting average prices (\( \tilde{P}_t \) and \( \tilde{P}_t^* \)):

\[
Q_t = \left( N_{D,t} + N_{V,t} + N_{H,t}^* \right)^{1-\theta} \tilde{P}_t \quad \text{and} \quad P_t^* = \left( N_{D,t} + N_{H,t}^* \right)^{1-\theta} \tilde{P}_t^* \tag{36}
\]

Then I write the CPI-based real exchange rate as:

\[
\tilde{Q}_t^{1-\theta} = \left( \frac{N_{D,t} + N_{V,t} + N_{H,t}^*}{N_{D,t} + N_{H,t}} \right) \frac{N_{D,t}^* \left( \frac{\text{TOL}_{D,t}}{z_{D,t}} \right)^{1-\theta} + N_{H,t}^* \left( \frac{\tau_{TOL}}{z_{H,t}} \right)^{1-\theta}}{N_{D,t} \left( \frac{\tau_{D,t}}{z_{D,t}} \right)^{1-\theta} + N_{V,t} \left( \frac{\tau_{TOL}}{z_{V,t}} \right)^{1-\theta} + N_{H,t}^* \left[ \frac{\tau_{TOL}}{z_{H,t}} \right]^{1-\theta}} \tag{37}
\]

\[43\]Variable \( N_{D,t}^* \) represents the number of intermediate good varieties produced by Northern firms domestically and sold in the North; \( N_{V,t}^* \) represents varieties produced by Northern firms offshore and sold in the North; and \( N_{H,t}^* \) reflects varieties produced by Southern firms and exported to the North. It follows that \( \tilde{Q}_t^{1-\theta} = \left( \frac{N_{D,t} + N_{V,t} + N_{H,t}^*}{N_{D,t} + N_{H,t}} \right) Q_t^{1-\theta} \).

\[44\]The CPI-based real exchange rate \( \tilde{Q}_t \) deviates from the welfare-based real exchange rate \( Q_t = \varepsilon_t P_t^*/P_t \) due to cross-country differences in product variety. As discussed in GM2005, an appreciation of the CPI-based real exchange rate \( \tilde{Q}_t \) (i.e. an increase in the CPI in North relative to that in South) may be offset by the increase in product variety in the North \( (N_{D,t} + N_{V,t} + N_{H,t}^*) \) relative to the South \( (N_{D,t}^* + N_{H,t}) \), so that the welfare-based real exchange rate \( Q_t \) depreciates (i.e. despite the increase in average prices, consumers derive higher utility due to the larger product variety).

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where the terms of labor $TOL_t = \frac{Q_L w_l^t Z_t^*}{w_l^t Z_t}$ measures the cost of effective labor in the South relative to the North; the iceberg trade costs $\tau_t$ and $\tau_t^*$ (which I allow to vary over time) affect the imports of the North and the South, respectively. The expression nests the model with endogenous exports of GM2005; I shut down offshoring and revisit the GM2005 case when $N_{V,t} = 0$.

**Analytical results**  The log-linearized version of (37) is:

\[
\hat{Q}_t = [s_D - s_V + s_{D*} - 1]TOL_t + \\
+ (s_D - s_V)\hat{\bar{Z}}_{D,t} + s_V\hat{\bar{Z}}_{V,t} - (1-\alpha)s_V\hat{\tau}_t + \\
+ (1 - s_D)(\hat{\bar{Z}}_{H,t} - \hat{\tau}_t) - (1 - s_{D*})(\hat{\bar{Z}}_{H,t} - \hat{\tau}_t^*) + \\
+ \frac{1}{\theta - 1} \left( s_V - \frac{N_V}{N_D + N_V + N_H} \right) \left( \hat{N}_{V,t} - \hat{N}_{H,t}^* \right) + \\
+ \frac{1}{\theta - 1} \left[ - \left( \frac{N^*_{D}}{N_D + N_V + N_H} - s_{D*} \right) \left( \hat{N}_{D,t} - \hat{N}_{H,t}^* \right) - \\
\left( s_V - \frac{N_V}{N_D + N_V + N_H} \right) \left( \hat{N}_{D,t} - \hat{N}_{H,t}^* \right) \right],
\]

where parameter $s_D$ is the steady-state share of spending in the North on goods produced by Northern firms both domestically and offshore; $s_V$ is the steady-state share of spending in the North only on goods produced by Northern firms offshore (I revisit GM2005 when $s_V = 0$); $s_{D*}$ is the steady-state share of spending in the South on goods produced by Southern firms. The calibration ensures that:

(a) $(s_D - s_V) + s_{D*} > 1$, as the domestically-produced varieties represent more than 50 percent of the consumption spending in each country; (b) $\left( \frac{N_D}{N_D + N_V + N_H} - (s_D - s_V) \right) > 0$ and $\left( \frac{N^*_{D}}{N_D + N_V + N_H} - s_{D*} \right) > 0$, i.e. the market shares of varieties produced domestically by the less productive firms are smaller than their fraction in the total number of varieties; and (c) finally, $\left( s_V - \frac{N_V}{N_D + N_V + N_H} \right) > 0$, i.e. the market share of varieties produced offshore by the more productive Northern firms is larger than their fraction in the total number of varieties available in the North. The model implies that the relatively more productive offshoring firms have larger market shares than their less productive domestic counterparts, which is in line with the empirical evidence in Kurz (2006).

The log-linearized form of (37) outlines five channels (labeled C1-C5 in the log-linearized expression above) through which the CPI-based real exchange rate is affected by: (1) changes in the price of non-traded goods induced by fluctuations in the terms of labor ($TOL_t$); (2) changes in the price of offshored goods reflecting fluctuations in the average productivity of offshoring firms ($\hat{\bar{Z}}_{V,t}$) and in the magnitude of trade costs ($\hat{\tau}_t$); (3) changes in the relative import prices triggered by fluctuations in the average

\[\text{See Appendix A.11 for the derivation.}\]
productivity of Northern exporters \( \left( \tilde{z}_{H,t} \right) \) relative to that of their Southern counterparts \( \left( \tilde{z}^*_{H,t} \right) \); (4) changes in the relative availability of varieties produced by Northern offshoring firms \( \left( \tilde{N}_{V,t} \right) \) relative to that of Southern exported varieties \( \left( \tilde{N}^*_{H,t} \right) \); and (5) changes in the relative availability of domestic varieties \( \left( \tilde{N}_{D,t} \right) \) relative to that of Southern exported varieties \( \left( \tilde{N}^*_{H,t} \right) \).

Figure 9. Impulse responses to a transitory 1 percent increase in aggregate productivity in the North, model with variable fraction of offshoring firms (thick solid line), fixed fraction of offshoring firms (thin line), and no offshoring (dotted line).
Impulse responses I find that, relative to the alternative model with exports only, offshoring dampens the appreciation of the real exchange rate that follows an aggregate productivity improvement in the North. As described in the introduction, the alternative framework with firm entry and endogenous exports in GM2005 generates a Harrod-Balassa-Samuelson (HBS) effect, as an increase in aggregate productivity causes an appreciation of the real exchange rate in the medium run. Specifically, offshoring dampens the HBS effect through channels C1 (the price of non-traded goods), C3 (the relative import prices) and C4 (the availability of offshored varieties vs. Southern imported varieties). The effect of each channel on the real exchange rate is described next.

Figure 9 provides the impulse responses of the key variables of the offshoring model under financial autarky to a transitory one-percent increase in aggregate productivity in the North, when productivity follows the univariate process \[ \log Z_{t+1} = \rho \log Z_t + u_t \] with \( \rho = 0.9 \).

(C1) Changes in the price of non-traded goods. In the model with endogenous exports and no offshoring, a productivity increase in the North encourages firm entry and leads to the appreciation of the terms of labor in the medium run (i.e. \( TOL_t \) decreases). This causes the average price of non-traded goods in the North to increase relative to that in the South, and thus leads to the appreciation of the real exchange rate (i.e. \( fQ_t \) decreases).

Offshoring dampens the appreciation of the real exchange rate through this channel in two ways: (a) It dampens the appreciation of the terms of labor (i.e. causes \( TOL_t \) to decrease by less) relative to the alternative model with a fixed fraction of offshoring firms, and also relative to the model with exports only, because the relocation of production offshore transfers upward pressure from the domestic to the foreign wage. (b) Offshoring also reduces the share of non-traded goods in total spending (i.e. \( s_D - s_V \), where \( s_V > 0 \) is the share of offshored varieties in total spending), and thus reduces the impact of the terms of labor appreciation on the real exchange rate.

(C2) Changes in the price of offshored goods. On impact, the increase in offshoring along its intensive margin and the resulting spike in the Southern wage cause the number of offshoring firms to drop, and their average productivity to rise. In the medium run, however, offshoring becomes an increasingly profitable option due to the gradual appreciation of the terms of labor. The average productivity \( \bar{z}_{V,t} \) of the offshoring firms declines, and their average price increases over time. Therefore, offshoring contributes to the appreciation of the real exchange rate through this channel.

(C3) Changes in relative import prices. In the absence of offshoring, the appreciation of the

\footnote{Exogenous policy changes can also affect the price of goods produced offshore. For instance, tariff cuts for the varieties of final goods produced offshore (i.e. a policy measure reflected by a decrease in \( \tau_t \)) would dampen the appreciation of the CPI-based real exchange rate.}
terms of labor reduces the export profitability of the Northern firms relative to that of their Southern counterparts. Therefore, the average productivity of the surviving Northern exporters \((\tilde{z}_{H,t})\) increases relative to that of the Southern exporters \((\tilde{z}_{H,t}')\), and their relative average price declines. This causes a decline in the average price of imports in the South relative to the North, which results in the appreciation of the real exchange rate.

Offshoring reverses this effect. The upwards pressure on the Southern wage causes the competitiveness of the Southern exporters to decline, and thus the productivity of the surviving Southern exporters to increase relative to that of their Northern counterparts. In contrast to the model with exports only, offshoring causes the average price of imports in the North to decline relative to that of imports in the South, a result which dampens the appreciation of the real exchange rate through import prices.

(C4) Expenditure switching from imports towards offshored goods. Following an increase in aggregate productivity, offshoring puts upward pressure on the Southern wage and reduces the competitiveness of the Southern exports. Thus, Northern consumers switch their expenditure away from the increasingly less competitive Southern varieties \((N_{H,t}')\) decreases) and towards the relatively cheaper varieties produced offshore \((N_{V,t}\) increases). The result dampens the appreciation of the real exchange rate through import prices in the medium run. It is consistent with the empirical evidence that FDI inflows in Mexico between 1994 and 2002 were associated with the crowding out of domestic investment, particularly in manufacturing (Gallagher and Zarsky, 2007).

(C5) Expenditure switching from imports towards domestic goods. Firm entry in the North generates an increase in the number of domestic varieties \((N_{D,t})\) relative to foreign imported varieties \((N_{H,t}')\) available to Northern consumers. Therefore, consumers switch their expenditures from imports towards the good varieties produced domestically by the relatively less productive firms, which are available at a relatively higher average price. As in the model with no offshoring, this channel works towards the appreciation of the real exchange rate.

5.6 Theoretical Moments: Offshoring and the Macroeconomy

Table 4 compares the theoretical moments generated by the model with offshore production (panel A) and those generated by the model with exports only and no offshoring (panel B), under the baseline framework of offshoring augmented with international trade in bonds. (The equations are in Appendix A.2.) I assume that aggregate productivity follows the bivariate productivity process in equation (35), with the persistence parameters \(\rho_Z = 0.996\) and \(\rho_{Z'} = 0.951\), zero spillovers, the variance of shocks
smaller in the North than in the South \((0.0050939^2 \text{ vs. } 0.0139570^2)\), and covariance \(0.1898 \times 10^{-4}\) as in Mandelman and Zlate (2008).

One notable result in the model with offshoring is that the trade balance in the North is more countercyclical than in the framework with exports only. Thus, the correlation between the trade balance \((NX/Y)\) and output in the North is more negative with offshoring \((-0.33)\) than with exports only \((-0.28)\). The result is generated by the fact that, given the procyclical nature of offshoring, the imports of varieties produced offshore contribute to the expanding trade deficit that follows a productivity increase in the North. Consistent with this result, the correlation between the trade balance and the value added offshore is negative \((-0.24)\).

In addition, the volatilities of output, investment and firm entry in the South are slightly greater in the model with offshoring than in the framework with exports only, a result which reflects the effect that the relocation of production has on the volatility of economic activity in the South.

Table 4. Offshoring with trade in bonds: theoretical moments

<table>
<thead>
<tr>
<th></th>
<th>Std. deviations relative to:</th>
<th>Correlations with output in:</th>
<th>Other correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean (%)</td>
<td>output (%)</td>
<td></td>
</tr>
<tr>
<td>(A) Offshoring</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>North 0.60 South 1.55</td>
<td>North - -</td>
<td>South - -</td>
</tr>
<tr>
<td>Consumption</td>
<td>North 0.52 South 0.65</td>
<td>North 0.86 South 0.42</td>
<td>North 0.99 South 0.99</td>
</tr>
<tr>
<td>Investment</td>
<td>North 2.02 South 9.09</td>
<td>North 3.34 South 5.88</td>
<td>North 0.75 South 0.94</td>
</tr>
<tr>
<td>Firm entry</td>
<td>North 2.05 South 9.07</td>
<td>North 3.39 South 5.87</td>
<td>North 0.76 South 0.94</td>
</tr>
<tr>
<td>NX/Y</td>
<td>North 0.25 South 0.30</td>
<td>North 0.41 South 0.19</td>
<td>North -0.33 South 0.20</td>
</tr>
<tr>
<td>RER(_{CP})</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B) No offshoring (GM2005)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>North 0.60 South 1.51</td>
<td>North - -</td>
<td>South - -</td>
</tr>
<tr>
<td>Consumption</td>
<td>North 0.51 South 0.69</td>
<td>North 0.84 South 0.45</td>
<td>North 0.99 South 0.99</td>
</tr>
<tr>
<td>Investment</td>
<td>North 2.04 South 7.46</td>
<td>North 3.39 South 4.93</td>
<td>North 0.76 South 0.96</td>
</tr>
<tr>
<td>Firm entry</td>
<td>North 2.02 South 7.64</td>
<td>North 3.36 South 5.05</td>
<td>North 0.76 South 0.95</td>
</tr>
<tr>
<td>NX/Y</td>
<td>North 0.22 South 0.23</td>
<td>North 0.37 South 0.15</td>
<td>North -0.28 South -0.36</td>
</tr>
<tr>
<td>RER(_{CP})</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6 Conclusion

I study the way in which the relocation of production across borders alters the cross-country transmission of business cycles. In particular, I focus on the fluctuations of offshore production along its extensive and intensive margins (the number of offshoring firms and the value added per firm) as separate transmission mechanisms.

The key results of the paper are as follows. First, the implications of the model are consistent with the procyclical pattern of offshoring, as well as with the extensive and intensive margin dynamics that I document using data from U.S. manufacturing and Mexico’s maquiladora sector. Following an aggregate productivity increase in the North, the value added per offshoring firm (the intensive margin) jumps on impact and then returns to its initial steady state. However, domestic firm entry causes a gradual appreciation of the cost of effective labor, which in turn generates a gradual increase in the number of offshoring firms (the extensive margin).

Second, offshoring enhances the cross-country co-movement of output relative to the model with endogenous exports. As firm entry in the parent country leads to the appreciation of the terms of labor, the increasing demand for domestic labor (due to firm entry) and sequentially the increasing demand for labor offshore (due to the relocation of production) enhance the co-movement of wages and aggregate incomes.

Third, offshoring reduces the price level gap between the countries involved, because it dampens the appreciation of the real exchange rate that follows an aggregate productivity improvement in the parent country. In particular, the relocation of production transfers some of the upward pressure from the domestic to the foreign wage, and thus dampens the appreciation of the terms of labor. Offshoring also crowds out the less competitive foreign exporters, and therefore leads to a decrease in the average import prices.

I suggest several possible extensions to this paper. First, the model provides a useful framework to analyze the impact of offshore production on employment in the parent and the host countries. (A preliminary analysis is provided in Appendix A.5). Second, one extension with rich policy implications would involve the study of interactions between offshore production and labor migration within an integrated framework, in which both offshoring and labor mobility are driven by fluctuations in the relative wage. (For the study of labor migration, see Mandelman and Zlate, 2008). Third, while this paper studies the business cycle fluctuations of offshoring, further research should address the long-run developments in offshoring to Mexico and its implications for U.S. manufacturing.
References


A Appendix

A.1 Offshoring with Financial Autarky

Table A.1. Model Summary

<table>
<thead>
<tr>
<th>Equation</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler equation, bonds</td>
<td>$C_t^{-\gamma} = \beta \left(1 + \tau_{t+1}\right) E_t \left[ C_{t+1}^{-\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$C_t^{<em>^{-\gamma}} = \beta \left(1 + \tau_{t+1}^</em>\right) E_t \left[ C_{t+1}^{*^{-\gamma}} \right]$</td>
</tr>
<tr>
<td>Euler equation, stocks</td>
<td>$\tilde{v}<em>t = \beta(1 - \delta) E_t \left( \frac{C</em>{t+1}}{C_t} \right)^{-\gamma} (\tilde{d}_{t+1} + \tilde{v}_t)$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{v}<em>t^* = \beta^<em>(1 - \delta^</em>) E_t \left( \frac{C</em>{t+1}^<em>}{C_t^</em>} \right)^{-\gamma} (\tilde{d}_{t+1}^* + \tilde{v}_t^*)$</td>
</tr>
<tr>
<td>Free entry</td>
<td>$\tilde{v}<em>t = f</em>{\tilde{E}w_t} Z_t$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{v}<em>t^* = f</em>{\tilde{E}w_t} Z_t^*$</td>
</tr>
<tr>
<td>Rule of motion, # firms</td>
<td>$N_{t+1} = (1 - \delta)(N_t + N_{E,t})$</td>
</tr>
<tr>
<td></td>
<td>$N_{D,t+1} = (1 - \delta)(N_{D,t} + N_{E,t}^*)$</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>$C_t + N_{E,t}\tilde{v}_t = w_t L + N_t\tilde{d}_t$</td>
</tr>
<tr>
<td></td>
<td>$C_t^* + N_{E,t}^<em>\tilde{v}_t^</em> = w_t^* L^* + N_{D,t}^<em>\tilde{d}_t^</em>$</td>
</tr>
<tr>
<td>Consumption price index</td>
<td>$1 = N_{D,t} (\tilde{\rho}<em>{D,t})^{1-\theta} + N</em>{V,t} (\tilde{\rho}<em>{V,t})^{1-\theta} + N</em>{H,t}^* (\tilde{\rho}_{H,t}^*)^{1-\theta}$</td>
</tr>
<tr>
<td></td>
<td>$1 = N_{D,t}^* (\tilde{\rho}<em>{D,t}^*)^{1-\theta} + N</em>{H,t}^* (\tilde{\rho}_{H,t}^*)^{1-\theta}$</td>
</tr>
<tr>
<td>Total profits</td>
<td>$N_t\tilde{d}<em>t = N</em>{D,t}\tilde{d}<em>{D,t} + N</em>{V,t}\tilde{d}<em>{V,t} + N</em>{H,t}\tilde{d}_{H,t}$</td>
</tr>
<tr>
<td></td>
<td>$N_{D,t}\tilde{d}<em>t^* = N</em>{D,t}^<em>\tilde{d}_{D,t}^</em> + N_{H,t}^<em>\tilde{d}_{H,t}^</em>$</td>
</tr>
<tr>
<td>Number of firms (Home)</td>
<td>$N_t = N_{D,t} + N_{V,t}$</td>
</tr>
<tr>
<td>VFDI profits link (Home)</td>
<td>$\tilde{d}<em>{V,t} = \frac{k}{k-(\theta-1)} \left( \frac{z</em>{V,t}}{z_{D,t}} \right)^{\theta-1} \tilde{d}_{D,t} + \frac{\theta-1}{k-(\theta-1)} f_V w_t^* Q_t$</td>
</tr>
<tr>
<td>HFDI profits link</td>
<td>$\tilde{d}_{H,t} = \frac{\theta-1}{k-(\theta-1)} f_H \frac{w_t^<em>}{Z_t^</em>}$</td>
</tr>
<tr>
<td>Dom. productivity (Home)</td>
<td>$z_{D,t} = \nu z_{\min} z_{V,t} \left[ \frac{k-(\theta-1)}{z_{V,t}} \frac{w_t^*}{z_{\min}} \right]^{1/k}$</td>
</tr>
<tr>
<td>VFDI productivity (Home)</td>
<td>$\tilde{z}<em>{V,t} = \nu z</em>{\min} \left( \frac{N_t}{N_{V,t}} \right)^{1/k}$</td>
</tr>
<tr>
<td>HFDI productivity</td>
<td>$\tilde{z}<em>{H,t} = \nu z</em>{\min} \left( \frac{N_t}{N_{H,t}} \right)^{1/k}$</td>
</tr>
<tr>
<td>Balanced trade</td>
<td>$N_{H,t} (\tilde{\rho}<em>{H,t})^{1-\theta} C_t Q_t + N</em>{V,t} \tilde{d}_{V,t} =$</td>
</tr>
<tr>
<td></td>
<td>$= N_{V,t} (\tilde{\rho}<em>{V,t})^{1-\theta} C_t + N</em>{H,t}^* (\tilde{\rho}_{H,t}^*)^{1-\theta} C_t$</td>
</tr>
</tbody>
</table>

The baseline model with financial autarky for the Northern economy is summarized by 16 equations in 16 endogenous variables: $N_t, N_{D,t}, N_{V,t}, N_{H,t}, N_{E,t}, \tilde{d}_t, \tilde{d}_{D,t}, \tilde{d}_{V,t}, \tilde{d}_{H,t}, z_{D,t}, z_{V,t}, z_{H,t}, \tilde{v}_t, r_t, w_t$ and
As the Southern firms do not offshore to the high-wage North, the Southern economy is described by only 11 equations in 11 endogenous variables: There are no Southern counterparts for $N_t$, $N_{V,t}$, $\bar{z}_{D,t}$ and $\bar{z}_{V,t}$. In particular, the average labor productivity of the representative domestic Southern firm ($\bar{z}_D^*$) is constant over time. Variables $N_{D,t}$, $r_t$, $N_t^*$ and $r_t^*$ are predetermined.

A.2 Offshoring with Financial Integration

I introduce international bond trading in the model with offshoring. Following GM2005, I assume that:

1. International asset markets are incomplete, as households in each country issue risk-free bonds denominated in their own currency.
2. Nominal returns are indexed to inflation in each economy, so that each type of bonds provides a real return denominated in units of that country’s consumption basket.
3. Quadratic costs of adjustment for bond holdings ensure stationarity for the net foreign assets in the presence of temporary shocks.

The infinitely-lived representative household in the North maximizes the inter-temporal utility subject to the constraint:

\[
(d_t + \tilde{v}_t)N_t x_t + w_t L + (1 + r_t) B_{h,t} + (1 + r_t^*) Q_t B_{f,t} + T_t \geq C_t + \tilde{v}_t (N_t + N_{E,t}) x_{t+1} + B_{h,t+1} + \frac{\pi}{2} (B_{h,t+1})^2 + Q_t B_{f,t+1} + \frac{\pi}{2} Q_t (B_{f,t+1})^2 ,
\]

where $r_t$ and $r_t^*$ are the rates of return of the North and South-specific bonds; $(1 + r_t)B_{h,t}$ and $(1 + r_t^*)Q_t B_{f,t}$ denote the principal and interest income from each type of bonds; $\frac{\pi}{2} (B_{h,t+1})^2$ and $\frac{\pi}{2} Q_t (B_{f,t+1})^2$ are the adjustment costs for each type of bond holdings; $T_t$ is the fee rebate. Setting $\pi = 0.005$, I add the two Euler equations for bonds to the baseline model:

\[
1 + \pi B_{h,t+1} = \beta (1 + r_{t+1}) E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma},
\]

\[
1 + \pi B_{f,t+1} = \beta (1 + r_{t+1}^*) E_t \frac{Q_{t+1}}{Q_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}.
\]

The budget constraint of the Southern household is similar, and the corresponding Euler equations for bonds are:
The market clearing conditions for bonds are:

\[ B_{h,t+1} + B_{h,t+1}^* = 0, \]  
\[ B_{f,t+1} + B_{f,t+1}^* = 0. \]  

Thus, financial integration through trade in bonds adds four new variables \((B_{h,t}, B_{f,t}, B_{h,t}^*, B_{f,t}^*)\) and six new equations \((39, 40, 41, 42, 43, 44)\) while removing the original two Euler equations from the baseline model with financial autarky. Trade in bonds also involves changes in the aggregate accounting equations and in the balanced current account condition. I re-write the expressions for aggregate accounting in the North and the South as:

\[ C_t + N_{E,t}v_t + B_{h,t+1} + Q_tB_{f,t+1} = w_tL + N_t \dd_t + (1 + r_t) B_{h,t} + (1 + r_t^*) Q_t B_{f,t}, \]  
\[ C_t^* + N_{E,t}^* v_t^* + Q_t^{-1} B_{h,t+1}^* + B_{f,t+1}^* = u_t^* L^* + N_{D,t}^* \dd_t^* + (1 + r_t) Q_t^{-1} B_{h,t}^* + (1 + r_t^*) B_{f,t}^*. \]

I also replace the balanced current account condition from the model with financial autarky with the expression for the balance of international payments:

\[ TB_t + \underbrace{N_{V,t} \dd_{V,t}}_{\text{Repatriated profits}} + r_t B_{h,t} + r_t^* Q_t B_{f,t} = (B_{h,t+1} - B_{h,t}) - Q_t (B_{f,t+1} - B_{f,t}), \]

which shows that the current account balance (trade balance plus repatriated profits of foreign affiliates plus investment income) must equal the negative of the financial account balance (the change in bond holdings).

### A.3 Asymmetric Firm Entry Costs

The World Bank’s *Doing Business* report outlines the large variation in the regulation of starting a business across countries at different levels of economic development (Table A.2). For instance, the monetary cost is 3.3 times higher in Mexico than in the U.S. or Canada; it is 6.2 times higher in
Hungary than in the U.K.

<table>
<thead>
<tr>
<th>Economy</th>
<th>Procedures (number)</th>
<th>Duration (days)</th>
<th>Monetary Cost (USD)</th>
<th>Relative Cost (U.S.=1.0)</th>
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<tbody>
<tr>
<td>U.S.</td>
<td>6</td>
<td>6</td>
<td>314.79</td>
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<tr>
<td>Canada</td>
<td>2</td>
<td>3</td>
<td>325.53</td>
<td>1.0</td>
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<td>Mexico</td>
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<td>27</td>
<td>1,046.71</td>
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<td>Germany</td>
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<td>18</td>
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<tr>
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</table>


A.4 VAR Estimation of Impulse Responses for Mexico’s Maquiladora

I estimate empirical impulse responses for the key maquiladora variables (total value added, number of plants, and the value added per plant) to permanent technology shocks in U.S. manufacturing. The estimation details are discussed in Zlate (2009). I estimate a structural VAR model with five variables: (i) labor productivity in U.S. manufacturing, (ii) labor productivity in Mexico’s maquiladora, (iii) value added per plant and (iv) the number of plants in Mexico’s maquiladora, as well as (v) hours worked in U.S. manufacturing. With the exception of the intensive margin, all variables have a unit root and therefore enter the VAR model in first differences. My identification strategy assumes that long-run labor productivity in U.S. manufacturing responds exclusively to U.S. technology shocks. Conversely, long-run labor productivity in Mexico’s maquiladora sector - which uses production machinery received on loan from U.S. firms - responds to both the U.S. and Mexico-specific permanent technology shocks.

In Figure A.1, I plot the estimated impulse responses of Mexico’s maquiladora indicators, together with the +/- 2 standard error confidence intervals. Following a one-standard-deviation, positive, permanent technology shock to U.S. manufacturing, the number of maquiladora plants (the extensive margin) does not react on impact, but increases gradually over time. The value added per maquiladora plant (the intensive margin) exhibits an immediate jump, followed by an additional increase until it
reaches a peak two quarters after the shock. The intensive margin then declines below its initial level, and returns to it over time.

The medium-run predictions of the theoretical model of offshoring are consistent with the empirical impulse responses. In the model, following a positive, transitory technology shock to aggregate productivity in the North, the extensive margin of offshoring increases gradually over time, despite the initial decline. The intensive margin jumps on impact, then drops below and returns to the original steady state, as in the data.

Figure A.1. Estimated impulse responses of the maquiladora indicators in Mexico to a permanent U.S. technology shock

A.5 Offshoring with Elastic Labor Supply: Employment Dynamics

Theoretical measures of sectoral employment In this section I study the effect of offshoring on employment in the North and the South. To this end, I focus on the offshoring sector in the Southern economy, in addition to the three employment sectors in each economy (firm entry, domestic and exporting). In the South, the representative offshoring firm hires local labor every period to cover the fixed cost of offshoring \((\frac{1}{Z_t}V_t)\) units of Southern labor and also to produce \((\frac{1}{Z_t}V_t^N = \frac{d_V}{w} + f_H \frac{1}{Z_t})\).

In each economy, the representative firm serving the domestic market hires domestic labor for production \((\frac{1}{Z_t}V_t^D = \frac{1}{w} + f_D V_t + f_H \frac{1}{Z_t})\). The representative firm serving the foreign market through exports also hires labor, both for production \((\frac{1}{Z_t}V_t^H = \frac{1}{w} + f_H \frac{1}{Z_t})\) and for covering the fixed cost of exporting \((\frac{1}{Z_t}U_t)\) units of labor. In addition, the new firms that enter at period \(t\) hire labor to satisfy the entry cost requirements \((\frac{1}{Z_t}E_t)\) units of labor per new entrant every period. Thus, the amount of labor hired by each of the three sectors in the North (firm entry, domestic, and export) every period is \(L_{E,t} = N_{E,t} \frac{1}{2} \frac{1}{Z_t}\), \(L_{D,t} = N_{D,t} d_{D,t}\) and \(L_{H,t} = N_{H,t} \left(\frac{1}{Z_t} + \frac{1}{U_t}\right)\), respectively. Similar equations hold for the three sectors in the South.
The total employment in the offshoring sector in the South is:

\[ L_{V,t}^* = N_{V,t} \left( \frac{V_{V,t}}{Z_t} + f_{V,t} \right). \] (48)

The log-linearized expressions for total employment in each economy are:

\[ \begin{align*}
\hat{L}_t &= \frac{L_E}{L} \hat{L}_{E,t} + \frac{L_D}{L} \hat{L}_{D,t} + \frac{L_H}{L} \hat{L}_{H,t}, \\
\hat{L}_t^* &= \frac{L_{V}^*}{L^*} \hat{L}_{V,t} + \frac{L_{E}^*}{L^*} \hat{L}_{E,t} + \frac{L_{D}^*}{L^*} \hat{L}_{D,t} + \frac{L_{H}^*}{L^*} \hat{L}_{H,t},
\end{align*} \] (49)

\[ \text{and} \]

\[ \begin{align*}
\hat{L}_t &= \frac{L_E}{L} \hat{L}_{E,t} + \frac{L_D}{L} \hat{L}_{D,t} + \frac{L_H}{L} \hat{L}_{H,t}, \\
\hat{L}_t^* &= \frac{L_{V}^*}{L^*} \hat{L}_{V,t} + \frac{L_{E}^*}{L^*} \hat{L}_{E,t} + \frac{L_{D}^*}{L^*} \hat{L}_{D,t} + \frac{L_{H}^*}{L^*} \hat{L}_{H,t},
\end{align*} \] (50)

where the calibration implies that the steady state shares of employment in the North are 22, 53 and 25 percent for the firm entry, domestic and exporting sectors. In the South, they are 15, 48 and 15 percent respectively, plus the remaining 22 percent in the offshoring sector.

**Impulse responses for a productivity increase in the North** Figure A.2 illustrates the employment dynamics from the offshoring model in response to a positive productivity shock in the North, when productivity follows the autoregressive univariate process \( \log Z_{t+1} = \rho \log Z_t + u_t \) with persistence parameter \( \rho = 0.9 \). In order to analyze the employment dynamics, I add elastic labor supply to the framework with offshoring under financial autarky.48

In the North, on impact, employment rises in the firm entry sector and declines in the domestic and exporting sectors. The reallocation of labor across sectors supports the creation of new product varieties following the productivity improvement in the Northern economy, both in the model with offshoring and in the model with exports only.

Important differences in employment dynamics across the two models become visible in the medium run. First, as the option to produce offshore improves the average profitability of prospective entrants, firm entry is more persistent and employment in the firm entry sector \( (L_E) \) declines by less (after its

48The representative household maximizes the expected inter-temporal utility 
\[ \max_{\{x_t, \omega_t, L_t\}} \left[ E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \ln C_s - \chi^{\frac{1}{\gamma + \psi}} \right) \right]. \] It consumes \( C_t \) and supplies \( L_t \) working hours elastically, subject to the budget constraint \( B_{t+1} + \bar{v}_t (N_t + N_{E,t}) x_{t+1} + C_t = (1 + r_t)B_t + (\bar{d}_t + \bar{v}_t)N_t x_t + w_2 L_t \), where \( \chi > 0 \) is the weight on the disutility from labor in the period utility function, and \( \psi \geq 0 \) is the Frisch elasticity of labor supply to wages and the inter-temporal elasticity of substitution in labor supply. Following the discussions in King, Plosser and Rebello (1988), Campbell (1994) and Bibbie et al. (2007), I use log utility for consumption (which is equivalent to setting \( \gamma = 1 \) in the baseline model) in order to obtain constant steady state labor supply in a model in which utility is additively separable over consumption and hours. I incorporate the usual first order conditions with respect to hours worked into the model, \( \chi (L_t) = w_3 C_t^{-1} \) and \( \chi^* (L_t) = w_3^* C_t^{-1} \). Using the baseline model calibration, I set the weight parameter \( \chi = 0.9188 \) and \( \chi^* = 0.9458 \), so that the steady-state level of hours worked is equal to unit, \( L = \left( \frac{1}{\frac{1}{\beta}} \right)^{1/\lambda} = 1 \). The wage elasticity of labor supply in North and South is \( \psi = 3 \).
initial spike) in the model with offshoring than in the alternative model with exports only. Second, in the medium run, employment in the Northern exporting sector \((L_H)\) increases above its original steady state in the presence of offshoring - whereas it remains below it in the model with no offshoring - as the dampened appreciation of the terms of labor enhances the competitiveness of the Northern exports. Third, offshoring keeps employment in the Northern domestic sector \((L_D)\) below its original steady state, partly due to the relocation of production to the South, and partly due to the within-country reallocation of employment towards the firm entry and exporting sectors. Overall in the North, the employment loss in the domestic sector is compensated by the employment gains in the firm entry \((L_E)\) and exporting sectors \((L_H)\).

In the South, the employment gains in the offshoring sector \((L_V)\) partially offsets the employment loss in the firm entry, domestic and exporting sectors \((L^*_H, L^*_D\) and \(L^*_H\)). The result is consistent with the empirical evidence that, due to the crowding out of domestic investment, most of the new jobs in Mexico’s manufacturing (96 percent) during 1994-2002 were in the maquiladora sector (Gallagher and Zarsky, 2007, Chapter 2).

Figure A.2. Employment dynamics, impulse responses to a transitory 1 percent productivity shock in North.
A.6 Welfare-Based Price Index and Demand Functions with Offshoring

The Northern representative household minimizes the total expenditure associated with the consumption basket $C_t$, which includes varieties produced by the Northern firms domestically ($y_{D,t}$), offshore ($y_{V,t}$), as well as varieties produced by the Southern firms ($y_{H,t}$):

$$\min \left\{ y_{D,t}(z), y_{V,t}(z), y_{H,t}(z) \right\} \quad P_t C_t = \int_{z_{\text{min}}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz + \int_{z_{H,t}}^{\infty} p_{H,t}(z) y_{H,t}(z) dz,$$

subject to $C_t = \left[ \int_{z_{\text{min}}}^{z_{V,t}} y_{D,t}(z) \frac{\theta-1}{\sigma} dz + \int_{z_{V,t}}^{\infty} y_{V,t}(z) \frac{\theta-1}{\sigma} dz + \int_{z_{H,t}}^{\infty} y_{H,t}(z) \frac{\theta-1}{\sigma} dz \right]^{\frac{\sigma}{\theta-1}}.$

The first-order conditions with respect to $y_{D,t}(z)$, $y_{V,t}(z)$ and $y_{H,t}(z)$ imply:

$$p_{D,t}(z) = \lambda_t C_t^{\frac{1}{\theta}} y_{D,t}(z)^{-\frac{1}{\theta}} \quad p_{V,t}(z) = \lambda_t C_t^{\frac{1}{\theta}} y_{V,t}(z)^{-\frac{1}{\theta}} \quad p_{H,t}(z) = \lambda_t C_t^{\frac{1}{\theta}} y_{H,t}(z)^{-\frac{1}{\theta}},$$

which I use to re-write the total expenditure amount:

$$P_t C_t = \int_{z_{\text{min}}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz + \int_{z_{H,t}}^{\infty} p_{H,t}(z) y_{H,t}(z) dz =$$

$$= \lambda_t C_t^{\frac{1}{\theta}} \left[ \int_{z_{\text{min}}}^{z_{V,t}} y_{D,t}(z) \frac{\theta-1}{\sigma} dz + \int_{z_{V,t}}^{\infty} y_{V,t}(z) \frac{\theta-1}{\sigma} dz + \int_{z_{H,t}}^{\infty} y_{H,t}(z) \frac{\theta-1}{\sigma} dz \right] = \lambda_t C_t.$$

Next I insert the resulting identity $\lambda_t = P_t$ and the demand functions $y_{D,t}(z) = (p_{D,t}(z)/P_t)^{-\theta} C_t$, $y_{V,t}(z) = (p_{V,t}(z)/P_t)^{-\theta} C_t$, $y_{H,t}(z) = (p_{H,t}(z)/P_t)^{-\theta} C_t$ into the expression for total expenditure, $P_t C_t = \int_{z_{\text{min}}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz + \int_{z_{H,t}}^{\infty} p_{H,t}(z) y_{H,t}(z) dz$, in order to derive the price index:

$$P_t = \left[ \int_{z_{\text{min}}}^{z_{V,t}} p_{D,t}(z)^{1-\theta} dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z)^{1-\theta} dz + \int_{z_{H,t}}^{\infty} p_{H,t}(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

Throughout the model I use the consumption basket as the numeraire good in each economy. Thus,
the real prices expressed in units of the Northern consumption basket are:

\[ \rho_{D,t}(z) = \frac{p_{D,t}(z)}{P_t}, \quad \rho_{V,t}(z) = \frac{p_{V,t}(z)}{P_t} \quad \text{and} \quad \rho_{H,t}(z) = \frac{p_{H,t}^*(z)}{P_t}, \]  

and the demand functions are:

\[ y_{D,t}(z) = \rho_{D,t}(z)^{-\theta} C_t, \quad y_{V,t}(z) = \rho_{V,t}(z)^{-\theta} C_t, \quad \text{and} \quad y_{H,t}(z) = \rho_{H,t}^*(z)^{-\theta} C_t. \]  

A.7 Profit Maximization with Offshoring

**Northern firms producing domestically** Firms set optimal prices by solving the profit maximization problem:

\[
\max_{\{\rho_{D,t}(z)\}} \rho_{D,t}(z) y_{D,t}(z) - \frac{w_t}{Z_t z} y_{D,t}(z). \]  

The first order condition with respect to the price implies:

\[
y_{D,t}(z) + \rho_{D,t}(z) \frac{\partial y_{D,t}(z)}{\partial \rho_{D,t}(z)} - \frac{w_t}{Z_t z} \frac{\partial y_{D,t}(z)}{\partial \rho_{D,t}(z)} = 0. \]  

Using the demand function \( y_{D,t}(z) = \rho_{D,t}(z)^{-\theta} C_t \) in the previous expression, it follows that the optimal price is equal to the marginal cost plus a markup (i.e. \( \frac{\theta}{\theta - 1} > 1 \)): 

\[
\rho_{D,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z}. \]  

**Northern firms producing offshore** The firm with idiosyncratic labor productivity \( z \) that produces its good variety using a mix of domestic and offshore inputs solves the following profit maximizing problem:\(^{49}\)

\[
\max_{\{\rho_{V,t}\}} \rho_{V,t}(z) y_{V,t}(z) - \left( \frac{w_t}{Z_t z} \right)^{\alpha} \left( \frac{\tau w_t^* Q_t^*}{Z_t^* z} \right)^{1-\alpha} y_{D,t}(z) - \beta V \left( \frac{w_t}{Z_t} \right)^{\alpha} \left( \frac{w_t^* Q_t^*}{Z_t^*} \right)^{1-\alpha}. \]  

Introducing the demand \( y_{V,t}(z) = \rho_{V,t}(z)^{-\theta} C_t \) in the profit function, the resulting price expression is:

\[
\rho_{V,t}(z) = \frac{\theta}{\theta - 1} \left( \frac{w_t}{Z_t z} \right)^{\alpha} \left( \frac{\tau w_t^* Q_t^*}{Z_t^* z} \right)^{1-\alpha}. \]  

\(^{49}\)The cost minimization problem in the broader framework of offshoring, \( \min_{\{l_t, l_t^*\}} w_t l_t + \tau w_t^* l_t^* \) so that \( y_{V,t}(z) = \left[ \frac{Z_t^* l_t^*}{\alpha} \right]^{\alpha} \left[ \frac{Z_t^* l_t^*}{\alpha} \right]^{1-\alpha}, \) leads to the following expression for the marginal cost: \( MC_t = \left( \frac{w_t}{Z_t z} \right)^{\alpha} \left( \frac{\tau w_t^* Q_t^*}{Z_t^* z} \right)^{1-\alpha}. \)
**Firms serving the foreign market**  The pricing formulas for firms originating in the North and the South, each serving the foreign market through either exports ($\eta = \eta^* = 1$) or horizontal FDI ($\eta = \eta^* = 0$), are obtained in a similar way:

$$
\rho_{H,t}(z) = \frac{\theta}{\theta - 1} \left( \tau^* \frac{w_t Q_t}{Z_t z} \right)^{\eta} \left( \frac{w_t^*}{Z_t^* z} \right)^{1-\eta} \quad \text{and} \quad \rho_{H,t}^*(z) = \frac{\theta}{\theta - 1} \left( \tau^* \frac{w_t^* Q_t}{Z_t^* z} \right)^{\eta^*} \left( \frac{w_t}{Z_t z} \right)^{1-\eta^*}. \quad (63)
$$

**A.8 Existence of Equilibrium for the Offshoring Productivity Cutoff**

As discussed in the paper, two conditions must hold every period in order to ensure the existence of the equilibrium productivity cutoff $z_{V,t}$: (1) $d_{V,t}(z)$ must be steeper than $d_{D,t}(z)$; and (2) $z_{\min} < z_{V,t}$.

The first condition implies that the effective wage in the South must be low enough relative to the effective wage in the North ($TOL_t < 1$), so that the more productive Northern firms find it profitable to relocate production offshore despite the iceberg trade cost ($\tau > 1$):

$$
\tau \frac{w_t^* Q_t}{Z_t^*} < \frac{w_t}{Z_t z} \iff \tau TOL_t < 1. \quad (64)
$$

The second condition, $z_{\min} < z_{V,t}$, requires that:

$$
\text{Slope}(d_{V,t}(z)) < \frac{\Theta_f E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*}}{z_{\min}^{\theta-1}},
$$

$$
\frac{z_{\min}^{\theta-1}}{\theta} \left( \frac{w_t}{\theta - 1 Z_t} \right)^{1-\theta} C_t < \Theta_f E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*},
$$

$$
\frac{1}{\theta} \left( \frac{w_t}{\theta - 1 Z_t z_{\min}} \right)^{1-\theta} C_t < \Theta_f E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*},
$$

$$
d_{D,t}(z_{\min}) < \Theta_f E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*}, \quad (65)
$$

where $\Theta = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}$. 

53
A.9 Average Productivity Factors under the Pareto Assumption

Northern firms producing offshore

\[
\bar{z}_{V,t} = \left[ \frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta - 1} g(z) dz \right]^{\frac{1}{\theta - 1}} = \\
= \left[ \left( \frac{z_{V,t}}{z_{\min}} \right)^k \int_{z_{V,t}}^{z_{\min}} z^{\theta - 1} k z_{\min}^{k - 1} \frac{dz}{z^{k+1}} \right]^{\frac{1}{\theta - 1}} = \\
= \left[ \left( \frac{z_{V,t}}{z_{\min}} \right)^k \frac{k z_{\min}^{k - 1} z_{V,t}^{\theta - 1 - k}}{k - (\theta - 1) z_{V,t}^{\theta - 1 - k}} \right]^{\frac{1}{\theta - 1}} = \\
= \nu z_{V,t}, 
\]
(66)

where \( \nu \equiv \left[ \frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta - 1}} \).

Northern firms producing domestically

\[
\bar{z}_{D,t} = \left[ \frac{z_{V,t}}{G(z_{V,t})} \int_{z_{\min}}^{z_{V,t}} z^{\theta - 1} g(z) dz \right]^{\frac{1}{\theta - 1}} = \\
= \left[ \frac{z_{V,t}}{z_{V,t}^{k - 1} - z_{\min}^{k - 1}} \int_{z_{\min}}^{z_{V,t}} z^{\theta - 1} k z_{\min}^{k - 1} \frac{dz}{z^{k+1}} \right]^{\frac{1}{\theta - 1}} = \\
= \left[ \frac{z_{V,t}^{k - 1} - z_{\min}^{k - 1}}{z_{V,t}^{k - 1} - z_{\min}^{k - 1}} (\theta - k - 1) \left( z_{V,t}^{\theta - 1 - k} - z_{V,t}^{\theta - 1 - k} \right) \right]^{\frac{1}{\theta - 1}} = \\
= \nu \left[ \left( \frac{z_{\min} z_{V,t}^{k - 1}}{z_{V,t}^{k} - z_{\min}^{k}} \right)^k \left( \frac{1}{z_{V,t}^{k - 1} - z_{\min}^{k - 1}} - \frac{1}{z_{V,t}^{k - 1} - z_{\min}^{k - 1}} \right) \right]^{\frac{1}{\theta - 1}} = \\
= \nu z_{\min} z_{V,t} \left[ \frac{k - (\theta - 1)}{z_{V,t}^{k - 1} - z_{\min}^{k - 1}} \right]^{\frac{1}{\theta - 1}}. 
\]
(67)
Appendix

A.10 Average Profits: Domestic and Offshore Production

The average profit of the Northern firms producing domestically is:

$$
\tilde{d}_{D,t} = d_{D,t}(z_{D,t}) = \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \right] \tilde{C}_t = \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \right] \tilde{C}_t z_{D,t}^{\theta-1} =
$$

$$
= \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \right]^{1-\theta} C_t \left( \nu z_{\text{min}} z_{V,t} \right)^{\theta-1} \left[ \frac{k-(\theta-1)}{z_{V,t} - z_{\text{min}}} \right]^{-1} \left[ \frac{k-(\theta-1)}{z_{V,t} - z_{\text{min}}} \right] =
$$

$$
= \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \right]^{1-\theta} C_t \left( \nu z_{\text{min}} z_{V,t} \right)^{\theta-1} \left[ \frac{k-(\theta-1)}{z_{V,t} - z_{\text{min}}} \right]^{-1} \left[ \frac{k-(\theta-1)}{z_{V,t} - z_{\text{min}}} \right] =
$$

$$
= d_{D,t}(z_{V,t}) \left( \nu z_{\text{min}} \right)^{\theta-1} \left[ \frac{k-(\theta-1)}{z_{V,t} - z_{\text{min}}} \right].
$$

The average profit of the Northern firms producing offshore is:

$$
\tilde{d}_{V,t} = d_{V,t}(z_{V,t}) = \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^* z_{V,t}} \right] \tilde{C}_t - f_V \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{w_t^* Q_t}{Z_t^*} \right)^{1-\alpha} =
$$

$$
= \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^* z_{V,t}} \right]^{1-\theta} C_t \tilde{z}_{V,t}^{\theta-1} - f_V \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{w_t^* Q_t}{Z_t^*} \right)^{1-\alpha} =
$$

$$
= \left\{ \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^* z_{V,t}} \right] \right\}^{1-\theta} C_t - f_V \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{w_t^* Q_t}{Z_t^*} \right)^{1-\alpha} \nu^{\theta-1} +
$$

$$
+ \left( \nu^{\theta-1} - 1 \right) f_V \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{w_t^* Q_t}{Z_t^*} \right)^{1-\alpha} =
$$

$$
= d_{V,t}(z_{V,t}) \nu^{\theta-1} + \frac{\theta - 1}{k-(\theta-1)} f_V \left( \frac{w_t}{Z_t} \right)^\alpha \left( \frac{w_t^* Q_t}{Z_t^*} \right)^{1-\alpha} .
$$

The Northern firm with productivity equal to the cutoff $z_{V,t}$ is indifferent between locating production domestically or offshore. Thus, I use the equality of the corresponding profits at the productivity cutoff, i.e. $d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})$, along with the expressions 68 and 69 above, to derive the link
between the two average profits as:

\[
\tilde{d}_{V,t} = \left( \frac{1}{\nu_{z_{\min}}} \right)^{\theta-1} \left[ \frac{k-(\theta-1)}{z_{V,t}} - \frac{k-(\theta-1)}{z_{\min}} \right]^{-1} \tilde{d}_{D,t} \theta^{\theta-1} + \frac{\theta - 1}{k-(\theta-1)} f_V \left( \frac{w_t}{Z_t} \right)^{\alpha} \left( \frac{w^*_V Q_t}{Z^*_t} \right)^{1-\alpha} = \frac{z_{1-\theta}}{z_{\min}} \left[ \frac{k-(\theta-1)}{z_{V,t}} - \frac{k-(\theta-1)}{z_{\min}} \right]^{-1} \tilde{d}_{D,t} \theta^{\theta-1} + \frac{\theta - 1}{k-(\theta-1)} f_V \left( \frac{w_t}{Z_t} \right)^{\alpha} \left( \frac{w^*_V Q_t}{Z^*_t} \right)^{1-\alpha} \equiv \tilde{d}_{D,t}(z_{V,t})
\]

\[
= \frac{k}{k-(\theta-1)} \left( \frac{z_{V,t}}{z_{D,t}} \right)^{\theta-1} \tilde{d}_{D,t} + \frac{\theta - 1}{k-(\theta-1)} f_V \left( \frac{w_t}{Z_t} \right)^{\alpha} \left( \frac{w^*_V Q_t}{Z^*_t} \right)^{1-\alpha}.
\]

(70)

A.11 Real Exchange Rate

Using the definition \(\tilde{Q}_t^{1-\theta} = \left( \frac{N_{D,t} + N_{V,t} + N^*_{H,t}}{N_{D,t} + N_{V,t} + N^*_H} \right) Q_t^{1-\theta} \) and the notation \(\tilde{N}_t \equiv N_{D,t} + N_{V,t} + N^*_H \), \(\tilde{N}_t \equiv N_{D,t} + N^*_H \), I re-write the CPI-based real exchange rate as:

\[
\tilde{Q}_t^{1-\theta} = \frac{\tilde{N}_t}{N_{D,t}} N_{D,t}^* \left( \tilde{\rho}_{D,t} P_t^* \right)^{1-\theta} + N_{V,t} (\tilde{\rho}_{V,t} P_t) \left( \tilde{\rho}_{H,t} P_t^* \right)^{1-\theta}
\]

\[
= \frac{\tilde{N}_t}{N_{D,t}} N_{D,t}^* \left( \frac{w^*_V P_t^*}{Z_t z_{D,t}} \right)^{1-\theta} + N_{V,t} \left[ \left( \frac{\tau T O L}{Z_t z_{V,t}} \right)^{1-\theta} \right] + N_{H,t} \left[ \left( \frac{\tau T O L}{Z_t z_{H,t}} \right)^{1-\theta} \right] = \frac{N_{D,t}}{N_t} \left( \frac{1}{z_{D,t}} \right)^{1-\theta} + \frac{N_{V,t}}{N_t} \left[ \left( \frac{\tau T O L}{z_{V,t}} \right)^{1-\theta} \right] + \frac{N_{H,t}}{N_t} \left[ \left( \frac{\tau T O L}{z_{H,t}} \right)^{1-\theta} \right].
\]

(71)

In what follows I use the notation \(s_D \equiv N_D \left( \frac{w}{Z_{D}} \right)^{1-\theta} + N_V \left( \frac{w}{Z_{D}} \right)^{1-\theta} \) to denote the steady-state share of spending in the North on goods produced by the Northern firms both domestically and offshore. Expression \(s_V \equiv N_V \left( \frac{w}{Z_{D}} \right)^{1-\theta} \) denotes the steady-state share of spending in the North on goods produced by the Northern firms offshore only. (Therefore, \(s_V < s_D \).) Expression \(s^*_D \equiv N_D^* \left( \frac{w^*_D}{Z_{D}} \right)^{1-\theta} \) denotes the steady-state share of spending in the South on goods produced by the Southern firms domestically. I also take into account the fact that the average productivity of the Southern firms producing domestically \(z^*_D \) is constant over time. Using all of the above, I log-linearize
the CPI-based real exchange rate:

\[(1 - \theta)\tilde{Q}_t = s_D^* \left[ \tilde{N}^*_{D,t} - \tilde{N}_t + (1 - \theta) \tilde{TOL}_t \right] + \\
+ (1 - s_D^*) \left[ \tilde{N}^*_{H,t} - \tilde{N}_t + (1 - \theta) \left( \eta \tilde{r}_t^* + (1 - \eta) \tilde{TOL}_t - \tilde{z}_{H,t} \right) \right] - \\
- (s_D - s_V) \left[ \tilde{N}^*_{D,t} - \tilde{N}_t - (1 - \theta) \tilde{z}_{D,t} \right] - \\
- s_V \left[ \tilde{N}^*_{V,t} - \tilde{N}_t + (1 - \theta) \left( (1 - \alpha) \tilde{r}_t + \tilde{TOL}_t - \tilde{z}_{V,t} \right) \right] - \\
- (1 - s_D) \left[ \tilde{N}^*_{H,t} - \tilde{N}_t + (1 - \theta) \left( \eta^* \tilde{r}_t + \tilde{TOL}_t - \tilde{z}^*_{H,t} \right) \right]. \tag{72} \]

Setting \( \eta = \eta^* = 1 \) so that my model of offshoring nests the model with endogenous exports in GM2005 (i.e. in addition to offshoring taking place from North to South, firms in each economy serve the foreign markets through exports rather than horizontal FDI), the log-linearized expression for the CPI-based real exchange rate becomes:

\[\tilde{Q}_t = [s_D - (1 - \alpha)s_V + s_D^* - 1] \tilde{TOL}_t + \\
+ (s_D - s_V) \tilde{z}^*_{D,t} + s_V \tilde{z}^*_{V,t} - (1 - \alpha)s_V \tilde{r}_t + \\
+ (1 - s_D) \left( \tilde{z}^*_{H,t} - \tilde{r}_t \right) - (1 - s^*) \tilde{z}_{H,t} - \tilde{z}_t^* + \\
+ \frac{1}{\theta - 1} \left( s_V - \frac{N_V}{N} \right) \left( \tilde{N}^*_{V,t} - \tilde{N}^*_{H,t} \right) + \\
+ \frac{1}{\theta - 1} \left[ \left( \frac{N_D}{N} - s^*_D \right) \left( \tilde{N}^*_{D,t} - \tilde{N}^*_{H,t} \right) - \left( \frac{N_D}{N} - (s_D - s_V) \right) \left( \tilde{N}^*_{D,t} - \tilde{N}^*_{H,t} \right) \right]. \tag{73} \]

\section*{A.12 Solution for the Asymmetric Steady State (TOL < 1)}

In this section I provide the steady state solution for the model of offshoring in the presence of cross-country differences in the cost of effective labor (TOL < 1). To this end, I use an integrated framework that nests both the baseline model of offshoring (for calibration \( \alpha = 0, \eta = 1 \)) and the benchmark model with exports only and no offshoring in GM2005 (for \( \alpha = 1, \eta = 1 \)), as described in footnotes 14 and 19 above.

I obtain a numerical solution for the unique steady state using a non-linear system of 12 equations in 12 unknowns, listed below. The unknowns are the steady state values of: \( z_V \) (the offshoring cutoff productivity), \( z_H \) (the exporting cutoff productivity in North), TOL, \( \frac{C}{C^*} \), \( Q \), \( \tilde{p}_D \), \( \tilde{d}_V \), \( \tilde{d}_H \), \( z_D^* \) (the exporting cutoff productivity in South), \( \tilde{r}_H \), \( \tilde{p}_H^* \), and \( \frac{N}{N_D} \). Subsequently, I use the numerical solution
for the initial 12 variables to compute the steady state values of the remaining variables of the model. A technical appendix providing their complete derivation is available upon request.

I use the following pricing and profit formulas (in which $Z = Z^* = 1$) in order to derive the steady state solution:

### Table A.3

<table>
<thead>
<tr>
<th>Average Prices</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Domestic production, North $\tilde{\rho}_D = \frac{\theta}{\theta - 1} w_D$</td>
<td></td>
</tr>
<tr>
<td>2. Domestic production, South $\tilde{\rho}_D^* = \frac{\theta}{\theta - 1} w_D^*$</td>
<td></td>
</tr>
<tr>
<td>3. Offshore production (vertical FDI, $\alpha = 0$) $\tilde{\rho}_V = \frac{\theta}{\theta - 1} w_T (\tau TOL)^{1-\alpha}$</td>
<td></td>
</tr>
<tr>
<td>4. Exports ($\eta = 1$) or horizontal FDI ($\eta = 0$), North $\tilde{\rho}_H = \frac{\theta}{\theta - 1} w^* Q (1 - TOL)^{1-\eta}$</td>
<td></td>
</tr>
<tr>
<td>5. Exports ($\eta^* = 1$) or horizontal FDI ($\eta^* = 0$), South $\tilde{\rho}_H^* = \frac{\theta}{\theta - 1} w^* Q (1 - TOL)^{1-\eta}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Profits</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Domestic production, North $\tilde{d}^*_{D,t} = \frac{1}{\theta} (\tilde{\rho}<em>D</em>{t,1})^{1-\theta} C_t$</td>
<td></td>
</tr>
<tr>
<td>2. Domestic production, South $\tilde{d}^<em>_{D,t} = \frac{1}{\theta} (\tilde{\rho}<em>D</em>{t,1})^{1-\theta} C_t^</em>$</td>
<td></td>
</tr>
<tr>
<td>3. Offshore production (vertical FDI, $\alpha = 0$) $\tilde{d}^*<em>{V,t} = \frac{1}{\theta} (\tilde{\rho}<em>V</em>{t,1})^{1-\theta} C_t - f</em>{Vw}TOL^{1-\alpha}$</td>
<td></td>
</tr>
<tr>
<td>4. Exports ($\eta = 1$) or horiz. FDI ($\eta = 0$), North $\tilde{d}^<em>_{H,t} = \frac{1}{\theta} (\tilde{\rho}<em>H</em>{t,1})^{1-\theta} C_t^</em> Q_t - f_{Hw}TOL^{1-\eta}$</td>
<td></td>
</tr>
<tr>
<td>5. Exports ($\eta^* = 1$) or horiz. FDI ($\eta^* = 0$), South $\tilde{d}^<em>_{H,t} = \frac{1}{\theta} (\tilde{\rho}<em>H</em>{t,1})^{1-\theta} C_t^</em> Q_t - f_{Hw}TOL^{1-\eta}$</td>
<td></td>
</tr>
</tbody>
</table>

In addition, using that $v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)} d$, $N_E = \frac{\delta}{1-\delta} N$, and $v = f_{e,w}$ in the expression for total profits in the Northern economy, the first equation in the system is:

$$
1 - \frac{\beta(1-\delta)}{\beta(1-\delta)} f_E = \frac{N_D}{N} \tilde{d}_D \frac{1}{w} + \frac{N_V}{N} \tilde{d}_V \frac{1}{w} + \frac{N_H}{N} \tilde{d}_H \frac{1}{w}.
$$

where $\frac{N_H}{N} = \left(\frac{1}{z_H}\right)^k$, $\frac{N_D}{N} = 1 - \left(\frac{1}{z_D}\right)^k$, $\frac{N_V}{N} = \left(\frac{1}{z_V}\right)^k$.  

58
The expression for the real exchange rate in steady state is:

\[
\left(\frac{z_{V}}{z_{H}}\right)^{\theta - 1} \frac{z_{V}^{k-	heta - 1}}{z_{V}^{k}} - 1,
\]

\[
\left(\frac{z_{V}^{k}}{z_{V}^{k-	heta - 1}} - 1\right)
\]

Using the balanced current account condition, I obtain:

\[
\frac{\tilde{d}_{D}}{w} = \frac{k}{k - (\theta - 1)} f_{H} T O L^{\theta(1-\eta)} C^{*} Q^{1-\theta} \tau^{s(\theta-1)\eta} \left(\frac{z_{V}}{z_{H}}\right)^{\theta - 1} \frac{z_{V}^{k-(\theta-1)}}{z_{V}^{k}} - 1,
\]

\[
\frac{\tilde{d}_{V}}{w} = \frac{k}{k - (\theta - 1)} f_{H} T O L^{1-\alpha + \theta(\alpha - \eta)} C^{*} Q^{1-\theta} \tau^{s(\theta-1)\eta} \left(\frac{z_{V}}{z_{H}}\right)^{\theta - 1} \frac{\tau^{s(\theta-1)\eta}}{\tau^{(1-\alpha)(\theta-1)}} - f_{V} T O L^{1-\alpha},
\]

\[
\frac{\tilde{d}_{H}}{w} = \frac{(\theta - 1)}{k - (\theta - 1)} f_{H} w T O L^{1-\eta},
\]

\[
\frac{\tilde{d}_{V}}{w} = \frac{z_{V}^{k-1}}{z_{V}^{k-(\theta-1)} - 1} - \frac{\tilde{d}_{D}}{w} + \frac{(\theta - 1)}{k - (\theta - 1)} f_{V} w T O L^{1-\alpha}.
\]

The expression for total profits in the Southern economy implies:

\[
\frac{1 - \beta^{*}(1 - \delta^{*})}{\beta^{*}(1 - \delta^{*})} f_{E} = \frac{k}{k - (\theta - 1)} f_{H} T O L^{\theta(\eta^{*} - 1)} \tau^{s(\theta-1)} Q^{\theta - 1} \frac{z_{H}^{1-\theta} C^{*} Q}{C} + \frac{(1 - \eta^{*})}{k - (\theta - 1)} \left(\frac{1}{z_{H}^{*}}\right) f_{H} T O L^{\eta^{*} - 1},
\]

\[
\frac{C}{C^{*} Q} = \frac{f_{H}}{f_{H}} T O L^{\theta(\eta + \eta^{*} - 1)} Q^{\theta - 1} \left(\frac{z_{H}^{\tau^{\eta^{*}}}}{z_{H}^{\tau^{\eta^{*}}}}\right)^{\theta - 1}.
\]

Using the balanced current account condition, I obtain:

\[
(1-\alpha) z_{V}^{k} T O L^{-\alpha} \left[\frac{(\theta - 1) k}{k - (\theta - 1)} f_{H} \left(\frac{z_{V}}{z_{H}}\right)^{\theta - 1} \frac{T O L^{\theta(\alpha + \eta^{*} - 1)}}{\tau^{(\theta-1)(1-\alpha-\eta^{*})}} + f_{V}\right]
\]

\[
= \Lambda_{1} f_{H} \left(\frac{N}{N^{*}_{D}}\right)^{-1} T O L^{-\eta} - \Lambda f_{H} \left(\frac{N}{N^{*}_{D}}\right)^{-1} T O L^{\eta^{*} - 1},
\]

where \(\Lambda = \left(\eta + \frac{(1-\eta)}{\theta}\right) \frac{k \theta}{k - (\theta - 1)} - (1 - \eta)\) and \(\Lambda^{*} = \left(\eta^{*} + \frac{(1-\eta^{*})}{\theta}\right) \frac{k \theta}{k - (\theta - 1)} - (1 - \eta^{*})\).

The expression for the real exchange rate in steady state implies:

\[
Q^{1-\theta} = \frac{T O L^{1-\theta} + (\tau^{\eta} T O L^{1-\eta})^{1-\theta} \frac{z_{H}^{\theta-1-k} N}{N^{*}_{D}}}{(1 - z_{V}^{k}) \frac{z_{V}^{\theta-1-k} N}{z_{V}^{(\theta-1) - 1} N^{*}_{D}} + \frac{\tau^{\theta-1-k}}{z_{V}^{k} N^{*}_{D}} (\tau T O L)^{(1-\alpha)(1-\theta)} + \frac{\tau^{\theta-1-k}}{N^{*}_{D}} (\tau T O L) \eta^{(1-\theta)}}.
\]
The remaining equations are:

\[
\frac{\theta_k}{k - (\theta - 1)} f_H \frac{\rho_H^{\theta-1}}{TOL^n} = 1 + \frac{1 - \beta^s}{\beta^s (1 - \delta^s)} f_E \frac{\rho_H^{\theta-1}}{\Xi_t},
\]  

(83)

\[
\frac{\theta_k}{k - (\theta - 1)} f_H^{s*} TOL^n \rho_H^{\theta-1} = 1 + \frac{1 - \beta}{\beta (1 - \delta)} f_E \frac{\rho_H^{\theta-1}}{\Omega_t},
\]  

(84)

\[
N \frac{N_D}{N_D^*} \left( \frac{\rho_H}{\rho_H^*} \right)^{\theta-1} = \frac{\Xi_t}{\Omega_t},
\]  

(85)

where:

\[
\Xi_t = \left[ \frac{1}{z_H} \left( \frac{\tau^*}{TOL} \right)^{\eta} \right]^{\theta-1} + z_{H^{-k}} \frac{N}{N_D^*},
\]  

(86)

\[
\Omega_t = \left( 1 - z_{V^{-k}} \right) \left( \frac{z_V}{z_H^*} \right)^{\theta-1} \frac{z_{V^{-k}} - 1}{1 - \left( \tau_{TOL} \right)^{\eta (\theta-1)}}
\]  

\[
+ z_{V^{-k}} \left[ \frac{z_V}{z_H^*} \left( \tau_{TOL} \right)^{\eta + \alpha} \right]^{\theta-1} + z_{H^{-k}} \left( \frac{N}{N_D^*} \right)^{-1}.
\]  

(87)