Intertemporal Choice with Liquidity Constraints

vs. Hyperbolic Discounting

by

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ABSTRACT

Since Thaler (1981), we have lived with the uncomfortable stylized fact that many humans choose strictly dominated actions in intertemporal choice experiments. Hyperbolic discounting has been advanced as the promising successor to discounted utility to explain observed behavior. We designed an experiment to probe the reasons for the apparently suboptimal behavior, and we find that the classic Fisher (1930) intertemporal choice theory with perceived transaction costs and liquidity constraints is perfectly consistent with our experimental data, whereas hyperbolic discounting is not.

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1. Introduction.

A quarter century ago, Richard Thaler (1981) published experimental results refuting the standard discounted utility model of time preference. This seminal contribution has been confirmed by numerous follow-up experiments.\(^1\) A typical choice task is of the form: “Which would you prefer? (A) $100 at time t, or (B) $105 at time t+\Delta t,” where t is today or say one year from today, and \(\Delta t\) is one day, week or month. The stylized fact is that many subjects choose A when t is today, and choose B when t is much later.

More disturbing than being a violation of constant discounted utility is the fact that the choice of $100 today is strictly dominated if the individual has at least $105 above the minimum required balance in his/her checking account, because they could choose B and lend themselves $105 at 0% interest to consume today knowing it will be paid off at t+\(\Delta t\).\(^2\) Of course, choosing A could be rationalized if there are liquidity constraints, transaction costs, and/or the risk of broken promises for future payments. Another possible explanation for the choice of A is the tendency not to “think outside the box”. Subjects may simply ignore the possibility of self-financing or infer that since self-financing is not explicitly mentioned in the experiment they are supposed to assume it is not allowed.

Recent studies have questioned the validity of experiments that suggest the presence of hyperbolic discounting. For example, Fernández-Villaverde and Mukherji (2006) demonstrate that when we control for uncertainty, the results support exponential discounting. Coller and Williams (1999) and Harrison, Lau, and Williams (2002) show that uncontrolled factors (such as transaction costs, lab and field investment opportunities, and socio-demographic characteristics) can affect intertemporal choice. Benhabib, Bisin and Schotter (2005), while rejecting exponential discounting, find evidence for a fixed-cost “present bias” which is consistent with perceived transaction costs and inconsistent with hyperbolic discounting. However, none of these experiments account for the potential role of liquidity constraints (as we do in this study).

In the last 25 years of economic experiments, two major stylized facts are the occurrence of purely random behavior by subjects who simply do not understand the task, and the heterogeneity of behavior in the subject population. Therefore, before we can conclude that hyperbolic discounting is a better model of behavior than classic theories, we need to dissect the causes of the apparently irrational behavior in these experiments, and control for the possible explanations. We designed an experiment to probe these issues.

\(^1\) See Frederick, et al. (2002) for a literature review.
\(^2\) This observation has been made by Meyers (1976) and Fuchs (1982).
In section 2, we describe the experimental design. In section 3, we present the data, summary statistics and simple hypotheses tests. The data exhibit the usual result that many subjects accept a one-week delay in payoff when the contract date is one year from now, but reject a one-week delay when the contract date is now. However, we also find persuasive evidence that this apparent anomaly is due to perceived transaction costs and liquidity constraints, and is consistent with the classic Fisher (1930) intertemporal choice theory. Section 4 presents an econometric model of the data and finds maximum-likelihood estimates of the parameters. Conclusions are drawn in Section 5.

2. Experimental Design.

The experiments entail questions about the subject’s preferences over the timing of hypothetical payments. We realize that answers to hypothetical questions have to be interpreted carefully because the subjects have no incentive to answer truthfully and even if they intend to answer truthfully, they may not attend to the details of their choice options to the degree they would if the choices were real. Accordingly, we expect more random behavior (or errors) in our data, and we will incorporate this likelihood in our modeling.

The choice questions described in the Introduction (“Which would you prefer? (A) $100 at time t, or (B) $105 at time t+Δt”) are not specific about how the hypothetical payments would be made. In particular, when t = now, the subject might assume that under (A) he/she would leave with cash, while under (B) he/she would have to return later to receive the cash with some risk of a broken promise. We will lump the perceived transportation cost, time cost and risk of a broken promise all into the category of transaction costs.

In order to control for some of the transaction costs that arise with future promises, we use two main treatments. Treatment I leaves the method of payment unspecified. Treatment II attempts to minimize perceived transaction costs in two ways. It is stated that all payments will be made by an electronic transfer to the subject’s checking account, so no one would leave with cash, nor would anyone have to return to receive payment. Further, a legally binding Promissory Note was presented as the form for hypothetical payments (see below).

While these are hypothetical payments so no one really has to return, if the subject takes the question seriously, the perceived possibility of having to return under option B could have an affect on the subject’s choice.
The two treatments were presented to subjects sequentially with a questionnaire after each treatment. In each treatment, the subjects were presented with the same four choice tasks.

1. Which of the following two options would you prefer?
   A) $100 today (within the hour), or  
   B) $105 one week from now.

2. Which of the following two options would you prefer?
   C) $100 one year from now, or  
   D) $105 one year plus one week from now.

3. Suppose you return here exactly one year from now, and are asked Question 1. 
   What do you think your answer will be then? A or B.

4. Which of the following two options would you prefer?
   E) $100 on December 23, 2006 (two days before Christmas), or  
   F) $105 on December 30, 2006.
The fourth question was intended to probe the impact of liquidity constraints. It was anticipated (and confirmed by a post-experiment questionnaire) that most students (whether of the Christian faith or not) purchase Christmas presents for family and friends. Coming at the end of the first semester, we presumed that many more students would be liquidity constrained two days before Christmas than at most any other time of the year (e.g. in the middle of the spring semester). Hence, even though the Christmas date was 6-10 months later, we anticipated at least as many E choices as A choices and significantly more E choices than C choices.

After these four choice tasks, we asked one survey-style question before going to the second treatment: namely, whether or not they thought they would have to return to receive their payment when they contemplated choosing B.

The second treatment came next and was preceded by an explanation of the Promissory Note and that all payments would be by electronic transfer. After the four choice tasks, we asked five survey-type questions, which we will discuss later.

Four sessions of 24 subjects each were run at the University of Texas at Austin on April 13, April 20, June 15 and July 27, 2006, all recruited from the general student population. The subjects were given a $5 payment for the 15 minutes it took to complete these questions, and afterwards they participated in a game theory experiment, so overall the students averaged $20-25 for a one hour and 15 minute experience.

3. The Experiment Data.

Table 1 shows the aggregate choices of the four tasks under Treatments I and II, as well as the Chi-square statistic for the treatment effect of each task. We find no treatment effect for the distant tasks (Q2 and Q4), as one would hope since transaction costs should not affect the choice between the distant options (C vs. D, and E vs. F). For the immediate choice task (Q1), we find fewer choices of A when all payments are by electronic transfers secured by a Promissory Note than when the method of payment is unspecified, consistent with our transaction cost hypothesis. However, the difference has a p-value of 0.105, which is barely significant.

\begin{table}
\centering
\caption{Aggregate Choices for Q1-Q4.}
\begin{tabular}{lccc}
& Treatment I & Treatment II & Fisher exact test \\
Q1 & 44 A & 52 B & 33 A & 63 B & 0.141 \\
Q2 & 17 C & 79 D & 19 C & 77 D & 0.854 \\
Q3 & 46 A & 50 B & 32 A & 64 B & 0.056 \\
Q4 & 39 E & 57 F & 36 E & 60 F & 0.767 \\
\end{tabular}
\end{table}
The power of the previous test is diminished by random behavior by some subjects. In particular, for Q1, some individuals chose B under Treatment I and chose A under treatment II. Similarly, for Q2-4. Clearly, these choices cannot be rationalized by any rational preferences with or without transaction costs. Eliminating these irrational responses, we are left with the following rationalizable choices:

<table>
<thead>
<tr>
<th></th>
<th>Treatment I</th>
<th>Treatment II</th>
<th>Fisher exact test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>41 A</td>
<td>43 B</td>
<td>22 A 62 B</td>
</tr>
<tr>
<td>Q2</td>
<td>14 C</td>
<td>70 D</td>
<td>10 C 74 D</td>
</tr>
<tr>
<td>Q3</td>
<td>45 A</td>
<td>42 B</td>
<td>24 A 63 B</td>
</tr>
<tr>
<td>Q4</td>
<td>39 E</td>
<td>48 F</td>
<td>30 E 57 F</td>
</tr>
</tbody>
</table>

Using only the rationalizable choices, the treatment effect for the immediate choice (Q1) is statistically significant (p-value = 0.0025), while there still is no treatment effect for the distant choice (Q2). Similarly, the treatment effect for Q3 is highly significant, while the treatment effect for the Christmas question (Q4) is not significant, consistent with the transaction cost hypothesis.

Within treatments, the difference between Q1 and Q2 responses is highly significant, confirming the stylized fact that more participants prefer the early payment when it is immediate than when it is delayed one year. The difference between Q1 and Q3 responses is not significant. Therefore, we cannot reject the hypothesis that the participants correctly foresee their preference for the early payment when it is immediate. In other words, the participants’ time preferences do not display this form of time inconsistency.

The within treatment comparisons for the Christmas question (Q4) can shed light on the impact of liquidity constraints, under the assumption that most participants will be more liquidity constrained two days before Christmas than in late spring or summer. The difference between Q2 and Q4 responses is highly significant, consistent with this assumption. On the other hand, the difference between Q1 and Q4 is not statistically significant, indicating that liquidity constraints (along with transaction costs) might be a major factor in explaining Q1 responses.

Another way to test the transaction cost hypothesis is to sort the participants by their answer to Q5.

5. When considering option B ($105 one week from today), did you think you would have to return here one week from today to get the $105? YES or NO
Over half of the participants (49/96) answered Yes. It is reasonable to expect those participants to be more affected by the transaction cost treatment. Table 3 shows the rationalizable choices for Q1 and Q2 sorted by the answer for Q5. Not surprisingly, the treatment effect is statistically significant only for the participants who thought they would have to return (Q5=y).

Table 3. Aggregate Rationalizable Choices for Q1-Q2 Sorted by Q5.

<table>
<thead>
<tr>
<th></th>
<th>Treatment I</th>
<th>Treatment II</th>
<th>Fisher exact test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q5=y</td>
<td>Q1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q1</td>
<td>22 A</td>
<td>8 A</td>
<td>0.003</td>
</tr>
<tr>
<td>Q5=n</td>
<td>19 A</td>
<td>14 A</td>
<td>0.368</td>
</tr>
<tr>
<td>Q5=y</td>
<td>Q2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q2</td>
<td>9 C</td>
<td>5 C</td>
<td>0.382</td>
</tr>
<tr>
<td>Q5=n</td>
<td>5 C</td>
<td>5 C</td>
<td>1</td>
</tr>
</tbody>
</table>

Surprisingly, the answer to Q5 seems to matter only under Treatment II. There is no significant difference in Q1 choices under the ambiguous payment treatment as one might expect. Rather the effect materializes in a significant difference (p-value = 0.014) to the Q1 choices under treatment II (electronic transfers with a Promissory Note). It is as if reading Q5 and answering it brought transaction costs to their attention, but coming after the choice under treatment I was made, the only way for the participants who answered yes to produce responses consistent with their answer was through their choices under treatment II. Consistent with this conjecture, we find that under treatment II those who answered yes to Q5, adjusted their choices of A downward closer to their choices of C, and those who answered no, make no significant downward adjustment consistent with their claim that transaction costs were not an important consideration. It is also quite possible that if they are not conscious of transaction cost considerations during treatment I, their answer to Q5 could be uncorrelated with their subconscious assumptions about how payments in the future would be made.

Following the second treatment, we asked these questions:

6. When considering option B ($105 one week from now) in Question 1, did you realize that as long as you have a sufficient checking account balance now, if you wanted to spend any part of the $105 today, you could simply withdraw funds from your checking account today knowing that your balance will be automatically restored in one week when the $105 transfer arrives, and you would not have to pay any interest for this loan to yourself? YES or NO

That is, adjusted from what they would have been had they not read Q5. Participants were not allowed to change any choices already made.
7. Given what you’ve been reminded of in Question 6 above, suppose the minimum balance you expect in your checking account between now and one week from now is at least $250. Which of the following two options would you choose? [circle one]

A) $100 today (within the hour), or

B) $105 one week from now.

For Q6, 58.3% answered Yes, so 41.7% admitted to being unaware of the self-borrowing possibility. After being appraised of this possibility, 89.6% answered B to Q7. In contrast, only 54.1% chose B for Q1 under treatment I, and 65.6% chose B under treatment II. These responses support our hypothesis that a significant fraction of subjects were liquidity constrained or perceived that self-borrowing was prohibited.5

We also asked the participants to state the minimum balance they expected in their checking account over the next week. The categories were:

- $0,
- $1 to $50,
- $51 to $150,
- $150 to $250,
- $251 to $500,
- over $500.

While all responses were completely anonymous, we recognize that the participants have no incentive to report truthfully; hence, we expect considerable noise in the responses. Nonetheless, it is fair to ask whether their choices in Q1 are correlated with this self-reported liquidity variable in the manner predicted by Fisher’s intertemporal choice model. We divided the data into two groups based on two questions. The Unconstrained (UC) group consists of those participants who answered Yes to Q6 and reported a minimum balance of $150 or more. The remaining participants are labeled the Liquidity Constrained (LC) group. Table 4 presents the choices of these two groups for Q1.

<table>
<thead>
<tr>
<th>Group</th>
<th>A</th>
<th>B</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC</td>
<td>3</td>
<td>29</td>
<td>32</td>
</tr>
<tr>
<td>LC</td>
<td>19</td>
<td>33</td>
<td>52</td>
</tr>
<tr>
<td>Sum</td>
<td>22</td>
<td>62</td>
<td>84</td>
</tr>
</tbody>
</table>

5 In a previous pilot run, subjects were apprised of the self-borrowing possibility before treatment I, and 19 of 20 subjects chose B for Q1.
Consistent with our liquidity hypothesis, the LC group chose A more frequently than the UC group; the Fisher exact test has a p-value of 0.01.


The aggregate analysis of the data presented in the previous section suggests that transaction costs and liquidity constraints might be a major factor in explaining Q1 and Q2 responses. Figure 1 is a graphical presentation of Fisher’s intertemporal choice model with liquidity constraints. The smaller budget set represents the status quo. We assume that while the status-quo consumption plan may have entailed borrowing or lending at the market rate of interest, very short-term adjustments from those plans are possible on a cash-only basis at a zero interest rate. Option A of Q1 shifts the status quo budget set to the right by $100, while option B shifts the budget set upward by $105. Two cases are depicted in Figure 1a. The first is for an individual for whom the borrowing constraint does not bind, and the second is for an individual for whom the liquidity constraint is binding for option B. Since the indifference curve intersects the corner of the B budget set with a slope less than -1, the maximum utility is less with this liquidity constraint than it would be without it (point O in Figure 1a). Figure 1b depicts a case in which option B is inferior to option A. Similar figures can be drawn for options C and D of Q2.

Figure 1. Fisher’s Model of Intertemporal Choice

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6 We really only need to assume that this interest rate is less than 260%, so the discounted present value of option B is greater than $100.
Let $V_{BA}^{I}(q_0)$ and $V_{DC}^{I}(q_1)$ denote the difference of the maximized utilities for the options of Q1 and Q2 respectively under treatment $j \in \{I, II\}$, where $q_0$ and $q_1$ are the perceived borrowing limits now and one year from now respectively. For the distant task (Q2), since transaction costs are the same for options C and D, we assume (consistent with the data) that there is no treatment effect. To simplify notation, we will henceforth let $v(q_1) \equiv V_{DC}^{I}(q_1) = V_{DC}^{II}(q_1)$.

Next, for the immediate task (Q1), it is reasonable to assume that individuals are more likely to face liquidity constraints now than they perceive in the future. That is, $q_0 = q_1 - b$; for some $b \in [0,q_1]$. Since treatment II entails no transaction cost difference, $V_{BA}^{II}(q_1) = V_{DC}^{II}(q_1 - b)$. Equivalently, let $L$ denote the utility cost of the liquidity constraint, so $V_{BA}^{II}(q_1) = v(q_1) - L$.

Under treatment I with an unspecified payment method for option B, the perception of having to make a return trip and/or the risk of default can be represented as a cost $\tau_c$ that is subtracted from the $105$ payment. Thus, to represent this in Figure 1, we would shift the status-quo budget upward by only $105 - \tau_c$. Since all we could hope to identify is the difference in transaction costs between treatment I and treatment II (electronic transfers with a Promissory Note), we will interpret $\tau_c$ as this difference. Equivalently, let $T$ denote the utility cost associated with $\tau_c$, so $V_{BA}^{I}(q_1) = V_{BA}^{II}(q_1) - T = v(q_1) - L - T$. 
In summary, if \( v(q_1) > L+T \), the optional choices are BDBD (where the first pair denotes the choices to Q1 and Q2 under treatment I respectively, and the second pair denotes the choices to Q1 and Q2 under treatment II). If \( L < v(q_1) < L+T \), the optional choices are ADBD. If \( 0 < v(q_1) < L \), the optional choices are ADAD. Finally, if \( v(q_1) < 0 \), the optional choices are ACAC.

In other words, only four of the sixteen possible choice combinations are consistent with the standard Fisher intertemporal choice model with liquidity constraints.

Table 5 displays the empirical frequency of the sixteen possible choice combinations. Clearly, the data is consistent with this prediction. Indeed, the only cells with at least 5% of the choices are those predicted: ACAC, ADAD, ADBD and BDBD. Of all choices, 79.2% fall into these four cells, and the other 21.8% appear to be uniformly random over the other 12 cells.\(^7\)

<table>
<thead>
<tr>
<th>Table 5. Q1 and Q2 Choices under Treatments I and II.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment II</td>
</tr>
<tr>
<td>Treatment I</td>
</tr>
<tr>
<td>AC</td>
</tr>
<tr>
<td>AD</td>
</tr>
<tr>
<td>BC</td>
</tr>
<tr>
<td>BD</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

A simple econometric model can describe this data extremely well. Suppose that with probability \( \alpha_0 \), individual i is uniformly random over the 16 cells, and with probability \( 1 - \alpha_0 \), i behaves according to the Fisher intertemporal model.\(^8\) Further, suppose that \( v_i(q_{i1}) \) is distributed according to a cumulative distribution \( F(v_i) \). Then, the predicted choice probabilities are

\[
\begin{align*}
\text{Pr}(ACAC) &= \frac{\alpha_0}{16} + (1 - \alpha_0)F(0), \\
\text{Pr}(ADAD) &= \frac{\alpha_0}{16} + (1 - \alpha_0)[F(L) - F(0)], \\
\text{Pr}(ADBD) &= \frac{\alpha_0}{16} + (1 - \alpha_0)[F(L+T) - F(L)], \\
\text{Pr}(BDBD) &= \frac{\alpha_0}{16} + (1 - \alpha_0)[1 - F(L+T)], \\
\text{and Pr}(wxyz) &= \frac{\alpha_0}{16} \text{ for each of the other 12 cells.}
\end{align*}
\]

\(^7\) The Chi-square test has a p-value of 0.749.

\(^8\) We also tested an alternative model with trembles from the four predicted behaviors, which generates a non-uniform distribution of “errors” over the 16 cells, and we found no improvement in the fit.
To complete the econometric specification, we need to specify $F()$. In the absence of prior information about $F$, a natural trial specification is a normal distribution $N(5, \sigma^2)$. Because the choice model is scale invariant, we cannot identify both the mean and standard deviation of $F$ along with the other parameters. Hence, we arbitrarily fix the mean of $F()$, and since we are assessing the utility value of an extra $5 (with a one week delay), we fix the mean to be 5, so we can easily interpret the magnitude of the other parameter estimates as relative to the $5 bonus.

This specification yields the parameter estimates (and standard errors) shown in column 2 of Table 6. We have measured the log-likelihood relative to the entropy of the data; twice this is distributed Chi-square with 16 degrees of freedom, and so has a p-value of 0.938. While the Root Mean Squared Error (RMSE) is less than 1%, it is noteworthy that the choice frequencies for the four cells predicted by the intertemporal choice model are fit exactly.\(^9\)

### Table 6. Parameter Estimates.

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Expon.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.278 (0.053)</td>
<td>0.278 (0.054)</td>
</tr>
<tr>
<td>$L$</td>
<td>2.995 (0.516)</td>
<td>2.634 (0.451)</td>
</tr>
<tr>
<td>$T$</td>
<td>1.572 (0.464)</td>
<td>0.915 (0.271)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>3.259 (0.675)</td>
<td>n/a</td>
</tr>
<tr>
<td>$\rho$</td>
<td>n/a</td>
<td>0.555 (0.142)</td>
</tr>
<tr>
<td>LL-Ent.</td>
<td>-4.17</td>
<td>-4.17</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0096</td>
<td>0.0096</td>
</tr>
</tbody>
</table>

To assess the robustness of the estimates to the specification of $F$, we tried the alternative specification of an exponential distribution: $e^{\alpha_0(v-5)}$. These estimates are given in column 3 of Table 6. Again, the choice frequencies for the four cells predicted by the intertemporal choice model are fit exactly. The estimates of $\alpha_0$ are identical because this is just the number of observations outside the four predicted cells divided by $12/16$ths of the total number of observations. This estimate might seem high, but the experiment entails hypothetical choices, so it is more likely that subjects do not put as much effort (if any) into making their choices than they would for non-hypothetical choices.

\(^9\) Obviously, only four parameters are needed to fit these four cells exactly, so the power of the test is based on the hypothesis of uniformly random behavior from those not described by the Fisher model.
The estimates for L indicate that on average over half of the potential gains from waiting one week are diminished by liquidity constraints. The estimates for T indicate that on average the perceived transaction cost is between $0.91 and $1.57. The reason for the larger estimate when F is specified as a normal distribution is that it declines less rapidly near the mean than does the exponential distribution. We would not put much faith in these estimates since we have no a priori justification for the distributional assumptions. On the other hand, it is clear that the observed behavior is consistent with the Fisher intertemporal choice theory, whatever the distribution F.

5. Conclusions.

We designed an experiment to probe the reasons why so many experimental subjects make dominated choices in typical time-preference experiments. This apparent anomalous behavior occurs in our experiment as well. However, we included questions that allowed us to explain this behavior. In particular, we hypothesized that transactions costs (either transportation and time to return to collect a delayed payment, or the risk of broken promises), and perceived or real liquidity constraints could rationalize the observed behavior.

Indeed, we find that both transaction costs and liquidity constraints are consistent with the pattern of choices in our experiment. In contrast, hyperbolic discounting, which includes neither transaction costs nor liquidity constraints, predicts no treatment effect, and no “Christmas” effect (i.e. Q4); hence, hyperbolic discounting is inconsistent with our data. Moreover, of the 16 possible choice patterns (Q1 and Q2 under Treatments I and II), only 4 have frequencies more than 5%, and those are exactly the four predicted by the classic Fisher intertemporal choice model with liquidity constraints and transaction costs.

Therefore, we believe that choice experiments of this sort cannot be used to reject rational choice models in favor of hyperbolic discounting. We hasten to add, however, that other experiments may be more favorable to the hyperbolic-discounting hypothesis. It is not our purpose to thoroughly review all experimental evidence for hyperbolic discounting. On the other hand, whatever theory supersedes will have to explain our results as well as other data.
References


