Risk, Returns, and Multinational Production

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(PRELIMINARY AND INCOMPLETE)

Abstract

This paper starts by unveiling a new empirical regularity: multinational corporations tend to exhibit systematically higher earnings-to-price ratios than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings-to-price ratios than firms selling only in their domestic market. To explain this pattern, we develop a real option value model where firms are heterogeneous in productivity, and have to decide whether and how to sell in a foreign market where demand is risky. Firms can serve the foreign market through trade or foreign direct investment, thus becoming multinationals. Multinational firms are more exposed to risk: following a negative shock, they are reluctant to exit the foreign market because they would forgo the option premium (sunk cost) that they paid to become multinationals. The theory provides a complementary explanation for the cross section of returns by exploiting the production side from an international point of view. The model has predictions for trade and FDI dynamics, and can be estimated and used to perform counterfactual exercises.

Keywords: Multinational firms, option value, cross-sectional returns

JEL Classification: F12, F23, G12

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1 Introduction

Multinational corporations tend to have higher earnings-to-price ratios than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings-to-price ratios than firms selling only in their domestic market. Variation in earnings-price ratios across firms is directly related to the variation in the stock returns across firms. Many papers in the new trade literature have focused on firm level features determining selection into foreign markets: exporters and multinational firms tend to be larger, more productive, to employ more workers, and sell more products than firms selling only domestically.\textsuperscript{1} To our knowledge, none of this literature has studied whether the international status of the firm matter for its investors. Conversely, there has been little attention in the financial literature in attempting to explain the features that set multinationals apart from other types of firms.

In this paper we present a real option value model, where firms choose optimally whether to produce only domestically, export, or set up an offshore affiliate to serve a foreign market where aggregate demand is uncertain. Firms in the domestic market effectively purchase an option that allows them to enter into the foreign market through exports or direct investment. Productivity and prospects of growth of the foreign demand for goods will determine the equilibrium choice. In equilibrium, firms will differ in the covariance of their earnings yields with the aggregate economy. The model provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view.

The selection mechanism is modeled following Helpman, Melitz, and Yeaple (2004). Exports are characterized by low fixed costs and high variable costs, due to the necessity of shipping goods every period, while FDI entail high fixed costs of setting a plant abroad, but low variable costs. We adapt this cost structure to a dynamic and stochastic environment, using Dixit (1989) as a benchmark to model entry decision under uncertainty. The dynamic model is able to generate the observed yields’ variation across firms. The main idea is the following: since foreign investment is associated to larger sunk costs compared to trade, it generates more hysteresis in the firm’s strategies over time, and this implies larger losses if the foreign economy is hit by negative shocks. Multinationals are more exposed to aggregate risk since – in case of a negative shock – by exiting they would forgo the option premium (sunk cost) that they paid to become multinationals. Pulling back is a more expensive strategy for multinationals than for exporters, who paid a lower option

\textsuperscript{1}See Bernard, Jensen, and Schott (2009).
premium to serve the foreign market. This framework imperfectly sorts firms by productivity. More productive firms will have more incentives to become multinationals, but the hysteresis induced by fixed costs associated with uncertain demand may generate different dynamics for very similar firms. The difference between per-period profits and the firm’s valuation over time implies that dividend yields depend on the choice of whether and how to serve foreign markets, therefore vary across firms with different international status.

The solution of the model delivers a series of prediction related to a firm’s productivity and to the realization of aggregate demand. First, as expected given the cost structure of the economy, foreign investment is associated to a wider hysteresis (or inaction zone) than trade. Second, more productive firms need smaller positive shocks to enter the foreign market, and larger negative shocks to exit. Moreover, for the same choice of whether and how to serve the foreign market, more productive firms are characterized by less hysteresis than less productive firms. Nonetheless, like in Helpman, Melitz, and Yeaple (2004), they tend to choose the strategy (FDI) that is associated to more hysteresis. This interaction generates an ambiguous result, which gives to the model the degree of freedom to explain the cross-sectional variation we observe in the data.

Why are we interested in the cross section of earnings-to-price ratios? Historically, average returns vary across stocks. Fama and French (1996) is a comprehensive description of the cross-sectional picture of returns. Stocks with high book-to-market ratios have yielded higher average returns than stocks with low book-to-market ratios. Since this variation is not captured by the traditional CAPM (or consumption-based alternatives), it has been called an anomaly, specifically, the “value premium puzzle”. In this paper, we address this anomaly by introducing a production based model in which firms select optimally whether to serve the foreign market at all, and how to do it. The model endogenously determines a cross-sectional difference in earnings-to-price. We focus on the cash flow dynamics of the firm, and how these are determined by endogenous decisions and exogenous risks. The notion of book value is not defined in our model, thus we use earnings-price ratios instead. Fama and French (1996), Table II, Santos and Veronesi (2005), and Lettau and Wachter (2007), Table II, show that sorting by earnings-to-price, cash-flow to price, or dividend-price ratios generates as sizable a value premium as sorting by book-to-market.

The existing financial literature that focuses on cross-sectional differences in price-dividend ratios and returns abstracts from the international organization of the firm. For instance, the answers of the value premium puzzle in the finance literature are based on a decomposition of aggregate risks. Campbell and Vuolteenaho (2004), Santos and Veronesi (2005), and Hansen,
Heaton, and Li (2008) identify two types of risk: long-run and transient. Shocks to expected growth rates are persistent, therefore they affect real variables in the long run as well. Alternatively, i.i.d. risks that impact current period growth vanish in the short run. The different exposure of firms’ cash flows to these two types of shocks determines cross-sectional differences in returns observed in the data. Turns out that value firms are more exposed to the former. Thus, agents require a higher reward for those risks that they fear the most. We contribute to the financial literature by endogenizing the exposure of cash-flows to these types of shocks. Exposure is directly linked to the decision of when and how to serve the foreign market, which is ultimately driven by productivity. The conclusion of the model is that more productive firms will offer better returns in the long run than less productive firms.

To our knowledge, Rob and Vettas (2003) is the only other paper that developed a model of trade and FDI with uncertain demand growth. In their framework FDI is irreversible, so it can generate excess capacity, but has lower marginal cost compared to export. The authors show that uncertainty implies existence of an interior equilibrium where export and FDI coexist. Our work generalize their model to one with many heterogeneous firms and a more general process for demand growth, and focuses on the implications for the cross section of earnings. Irrarazabal and Opromolla (2009) model entry and exit into the export market with idiosyncratic productivity shocks and sunk costs: our model is close to their framework for the use of the real option value analogy in solving the firm’s optimization problem. While Irrarazabal and Opromolla (2009) concentrate their attention on the impact of idiosyncratic productivity shocks for firm dynamics, we model aggregate demand shocks that affect firms differently only through their endogenous choice of international status. Moreover, we add the choice of the mode of entry. Roberts and Tybout (1997) and Sanghamitra Das and Tybout (2007) address empirically the issue of market participation for export. Our model has similar predictions for both exports and FDI sales, and can be estimated by using information from both trade/FDI flows and stock market prices. Ramondo and Rappoport (2008) introduce idiosyncratic productivity shocks and aggregate shocks in a model where firms can locate plants both domestically and abroad. Multinational production allows firms to match domestic productivity and foreign shocks, and works as a mechanism for risk sharing. We abstract from the risk sharing/diversification motive: the risks in our model are aggregate, hence not insurable. Moreover, we model both trade and multinational production as different modes of dealing with uncertainty in foreign markets.

The remainder of the paper is organized as follows. Section 2 presents preliminary evidence
on the ranking in earnings-to-price ratios according on the firms’ internationality status. Section 3 develops the model and provides analytical properties of the solution. Section 4 contains the calibration of the model and the computation of the earnings yields. Section 5 uses simulations of the model to perform counterfactual exercises on exports and FDI dynamics. Section 6 concludes.

2 Motivating Evidence

In this section, we provide preliminary evidence on the sorting of earnings-to-price ratios according to firms’ international status. We consider all US-based manufacturing firms in the Compustat database. We define a firm to be a multinational (MN) if it reports the existence of a foreign geographical segment associated with positive sales. Similarly, we define a firm to be an exporter if it reports a positive level of exports.

![Figure 1: Earnings-to-price ratios, portfolios of firms in each group.](image)

Figure 1 shows earnings-to-price ratios over time for three portfolios of firms. Each portfolio is composed by firms with the same international status (only domestic sales, exporters, multinationals).\(^2\) The solid line represents earnings-to-price ratios of multinational firms, the dashed line the ones of exporters, and the dash-dotted line the ones of firms selling only domestically. Multinationals

\(^2\)The portfolios are constructed as follows. For each firm \(i\), determine its status \(S (S = D, X, I)\) at the end of year \(t - 1\), and collect data on earnings \((e_i)\) and market capitalization \((p_i)\) in year \(y\). Aggregate earnings and prices for

5
Table 1: Firm-level regressions of earnings-to-price ratios on multinational and exporter dummies, controlling for capital-labor ratio, productivity, and different measures of size (with year and industry fixed effects).

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Table 1: Firm-level regressions of earnings-to-price ratios on multinational and exporter dummies, controlling for capital-labor ratio, productivity, and different measures of size (with year and industry fixed effects).

Multinational firms exhibit higher earnings-to-price ratios than non-multinational firms, consistently over the entire time period. The ranking between domestic firms and exporters is less clear, from the raw data, but appears when controlling for the effect of other variables. Table 1 displays the results of the following firm-level regression:

$$(e/p)_{it} = \alpha + \beta_1 D_{it}^{MN} + \beta_2 D_{it}^{EXP} + \beta_3 (K/L)_{it} + \beta_4 \text{productivity}_{it} + \beta_5 \text{size}_{it} + \delta_{N\text{AICS}} + \delta_t + \varepsilon_{it}$$

where $D_{it}^{MN}$ and $D_{it}^{EXP}$ are multinationals and exporters’ dummies, respectively, $\delta_{N\text{AICS}}$ and $\delta_t$ are 4-digit industry and year fixed effects, respectively. We use sales per employee as our measure of productivity, and run the regressions for different definitions of size.³

³We use sales (operating revenues), market capitalization, and book equity per share as our measures of size.

Firms in each status are computed by weighting each firm’s observation by its market share:

$$E_t^S = \sum_{i \in S} \omega^i_t e^i_t$$

$$P_t^S = \sum_{i \in S} \omega^i_t p^i_t$$

where $\omega^i_t = \frac{p^i_t}{\sum_{i \in S} p^i_t}$, and $S = D, X, I$. Earnings-to-price ratios for each portfolio are given by $E_t^S / P_t^S$. 

³We use sales (operating revenues), market capitalization, and book equity per share as our measures of size.
Table 2: Mean Excess Return (% per year). Portfolios are formed by sorting firms into deciles on the earnings yield (E/P), ratio of cash flow to prices (C/P), dividend yield (D/P), and book-to-market ratio (B/M). Monthly (annualized) data, from 1952 to 2002. From Lettau and Wachter (2007).

The coefficients associated to exporters and MN dummies are positive and significant across all specifications. Moreover, the coefficient associated to multinationality appears to be larger than the one associated to exporters, highlighting a further difference between MN firms versus exporters.

As we argued in the Introduction, earnings-to-price ratios carry information about returns on the firms’ stocks. Table 2 is taken from Lettau and Wachter (2007), and shows how portfolios delivering higher earning yields are also associated to higher excess returns.

3 Model

The economy is composed of two countries, a domestic and a foreign one. There are two sectors, a homogeneous sector and a differentiated sector. In both counties, agents have preferences defined by:

$$ U = \int_0^\infty e^{-\rho t} \left[ H(s)^{1-\zeta} \left( \int q_i(s)^{1-1/\eta} di \right)^{\zeta\eta/(\eta-1)} \right] ds $$

where $\rho > 0$ is the discount rate, $\eta$ is the elasticity of substitution across varieties of the differentiated good, $\zeta \in [0, 1]$, $Q = \left( \int q_i(s)^{1-1/\eta} di \right)^{\eta/(\eta-1)}$.

We assume that foreign firms only produce the homogeneous good, while domestic firms produce goods in both sectors and sell differentiated products in both countries. Labor is the only factor of production. The homogeneous sector is perfectly competitive. The homogeneous good is produced with a one-to-one linear technology, and is perfectly tradable. The differentiated sector is characterized by a monopolistically competitive market structure. There is an unbounded mass of firms that may potentially enter the differentiated sector. To enter, they pay a fixed entry cost $F_e$. Upon entry, each firm draws a unit labor requirement $a$ from a distribution $G(a)$. $a$ indicates the number of units of labor that a firm needs in order to produce one unit of a differentiated good.
Differentiated products are imperfect substitutes in the eyes of the consumers ($\eta > 1$).

Once entered the domestic market, a firm starts producing there. Aggregate demand in the domestic market ($Q_D$) is deterministic, so a firm that decides to produce makes profits $\pi_D(a;Q_D)$ from domestic sales with certainty. Domestic firms must decide whether to produce only for the domestic market, or also for the foreign country one. Production in the foreign market involves fixed operating costs, to be paid every period, and sunk costs of entry. If a firm decides to sell in the foreign market as well, it can do so either via exports or via foreign direct investment (henceforth, FDI). We call multinationals those firms that decide to serve the foreign market through FDI sales.

We model the choice between trade and FDI along the lines of Helpman, Melitz, and Yeaple (2004): exports entail a relatively small sunk cost $F_X$, but a per-unit iceberg transportation cost $\tau$ paid every period, while FDI is associated to a larger sunk cost $F_I$ ($F_I > F_X$), but there are no transportation costs to be covered every period, as production happens in the foreign market. Both options also entail fixed operating costs to be paid every period, $(f_I \geq f_x)$. Production in the foreign market is risky, as aggregate demand $Q$ evolves according to a stochastic process. The state of the economy is described by the realization of $Q$. When deciding their international status, firms solve an intertemporal profit maximization problem, which is also affected by the current state of demand in the foreign market.

For a given realization of $Q$, a firm with productivity $1/a$ must choose its optimal status $s$ ($s \in \{D,X,I\}$, i.e. domestic, exporter, or multinational), the current selling price $p_s(a)$ and an updating rule (how to change the optimal price and status following changes in aggregate foreign demand).

Marginal costs of production and optimal prices vary with the status of the firm: the marginal cost of domestic production is given by the labor requirement times the domestic wages, $MC_D = aw$. The marginal cost of exporting is augmented by the iceberg transportation cost: $MC_X = \tau aw$. When the firm serves the foreign market through FDI, domestic productivity is transferred to the foreign country, but the firm employs labor in the foreign country: $MC_I = aw^*$. Given the CES preferences across varieties of the differentiated good, the optimal prices are $p_s(a) = \frac{\eta}{\eta - 1} MC_s(a)$. Notice that, since prices do not depend on the aggregate quantity demanded, they are also independent on the state of the economy $Q$.

$$p = \frac{\eta}{\eta - 1} MC.$$ Let $\pi_D(a;Q_D)$, $\pi_X(a;Q)$ and $\pi_I(a;Q)$ denote the per-period profits from domestic sales, from exports and from FDI sales abroad, respectively, for a firm with productivity
1/a, given a realization of the aggregate quantity demanded in the domestic (foreign) market equal to $Q_D$ ($Q$).

$$\pi_D (a; Q_D) = B (aw)^{1-\eta} P_D^\eta Q_D \quad (1)$$

$$\pi_X (a; Q) = B (\tau aw)^{1-\eta} P^\eta Q - f_X \quad (2)$$

$$\pi_I (a; Q) = B (aw^*)^{1-\eta} P^\eta Q - f_I. \quad (3)$$

where $f_S$, $S = \{D, X, I\}$, denotes the fixed operational cost associated with status $S$, that must be payed every period. $P_D$ ($P$) is the aggregate price of the differentiated good in the domestic (foreign) market, that firms take as given while solving their maximization problem. Finally, $B \equiv \frac{1}{\eta-1} \left( \frac{w^*}{w} \right)^{\eta-1}$. In order to assure the existence of exporters in equilibrium, we assume:\footnote{Condition (4) is the “present discounted value equivalent” of the analogous assumption in Helpman, Melitz, and Yeaple (2004). It is derived imposing that the profit functions of an exporter and of a multinational firm – expressed as functions of the productivity level $a$ – cross at a point associated to positive profits.}

$$\left( \frac{w^*}{w} \right)^{\eta-1} (f_I + \rho F_I) > \tau^{\eta-1} (f_X + \rho F_X). \quad (4)$$

The aggregate quantity $Q$ demanded in the foreign market follows a geometric Brownian motion:

$$\frac{dQ}{Q} = \mu dt + \sigma dz \quad (5)$$

where $\mu > 0$ and $dz$ is the increment of a standard Wiener process, satisfying: $E(dz) = 0$ and $E(dz^2) = dt$. Properties of Brownian motions also imply: $E(Q_t|Q_0) = e^{\mu t}$, hence we need to assume $\mu < \rho$ to assure convergence.

We solve the model along the lines of Dixit (1989). Let $V_S (a; Q)$ denote the expected net present value for a firm with productivity $1/a$ in status $S$ ($S = D, X, I$) starting with aggregate demand $Q$ in the foreign market and following optimal policy. As we assume no uncertainty in the domestic market, firms in all statuses $S$ have positive profits $\pi_D (a; Q)$ from domestic sales. We abstract from those profits and interpret the value functions as net of domestic profits.

Over a generic time interval $\Delta t$, the Bellman equation for a firm that is currently selling only in the domestic market is:

$$V_D (a, Q) = \max \left\{ \frac{1}{1 + \rho \Delta t} E[V_D (a, Q')|Q] ; \ V_X (a, Q) - F_X ; \ V_I (a, Q) - F_I \right\}. \quad (6)$$
The right-hand side of the Bellman equation expresses the firm’s possibilities. If it remains domestic, it gets the continuation value from not changing status, equal to the expected value of the firm conditional on the demand realization $Q$. If it decides to switch to export (FDI) it gets the value of being an exporter, $V_X$ (multinational, $V_I$) minus the sunk cost of entry $F_X$ ($F_I$). Similarly, the Bellman equation for an exporter is:

$$V_X(a, Q) = \max \left\{ \pi_X(a, Q)\Delta t + \frac{1}{1 + \rho\Delta t} E[V_X(a, Q')|Q] ; \ V_D(a, Q) ; \ V_I(a, Q) - F_I \right\}$$  \hspace{1cm} (7)$$

and for a multinational:

$$V_I(a, Q) = \max \left\{ \pi_I(a, Q)\Delta t + \frac{1}{1 + \rho\Delta t} E[V_I(a, Q')|Q] ; \ V_D(a, Q) ; \ V_X(a, Q) - F_X \right\}.$$  \hspace{1cm} (8)$$

Notice that the continuation value of an exporter (a multinational) also includes the flow profits from sales in the foreign market $\pi_X(a, Q)\Delta t (\pi_I(a, Q)\Delta t)$.

In the continuation region, the Bellman equation of a domestic firm reduces to: $\rho V_D(a, Q) = \frac{E(dV_D(a, Q))}{dt}$ (the instantaneous return must be equal to the expected capital gain). By applying Ito’s lemma, we obtain:

$$\rho V_D(a, Q) = V_D'(a, Q)\hat{\mu}Q + \frac{1}{2} V_D''(a, Q)\sigma^2 Q^2.$$  \hspace{1cm} (9)$$

The solution takes the form $V_D(Q) = Q^\xi$, where:

$$\xi = \frac{(1 - m) \pm \sqrt{(1 - m)^2 + 4r}}{2}$$

and $m = 2\hat{\mu}/\sigma^2$, $r = 2\rho/\sigma^2$. Hence the value function for a domestic firm takes the form:

$$V_D(a, Q) = A_D(a)Q^\alpha + B_D(a)Q^\beta$$  \hspace{1cm} (10)$$

where $\alpha$ and $\beta$ are the negative and positive values of $\xi$, respectively, and $A_D(a)$ and $B_D(a)$ are parameters to be determined.

To compute the value functions of exporters and multinationals, we need to take into account also the current profits from sales in the foreign market. In the continuation region, the value of a
firm starting as an exporter with aggregate demand $Q$ satisfies:

$$\rho V_X(a, Q) = B(\tau aw)^{1-\eta} P^n(a)Q - f_X + V'_X(a, Q)\mu Q + \frac{1}{2} V''_X(a, Q)\sigma^2 Q^2$$  \hspace{1cm} (11)$$

and the value of a firm starting as a multinational with aggregate demand $Q$ satisfies:

$$\rho V_I(a, Q) = B(aw^*)^{1-\eta} P^n(a)Q - f_I + V'_I(a, Q)\mu Q + \frac{1}{2} V''_I(a, Q)\sigma^2 Q^2. \hspace{1cm} (12)$$

For $S = X, I$, the value function takes the affine form $V_S(Q) = Q^\xi + c_{s0} + c_{s1}Q$. By substituting it in the expressions above, the value functions for an exporter and a multinational firm can be written as:

$$V_X(a, Q) = A_X(a)Q^\alpha + B_X(a)Q^\beta + \frac{B(\tau aw)^{1-\eta} P^n Q}{\rho - \mu} - \frac{f_X}{\rho} \hspace{1cm} (13)$$

$$V_I(a, Q) = A_I(a)Q^\alpha + B_I(a)Q^\beta + \frac{B(aw^*)^{1-\eta} P^n Q}{\rho - \mu} - \frac{f_I}{\rho} \hspace{1cm} (14)$$

where $A_X(a), B_X(a), A_I(a)$ and $B_I(a)$ are parameters to be determined.

In order to find the quantity thresholds that make a firm switch across statuses, we impose the following value-matching conditions, where we denote with $Q_{SR}$ the quantity threshold that makes a firm switch from status $S$ to status $R$, for $S, R \in \{D, X, I\}$.

$$V_D(a, Q_{DX}) = V_X(a, Q_{DX}) - F_X \hspace{1cm} (15)$$

$$V_D(a, Q_{DI}) = V_I(a, Q_{DI}) - F_I \hspace{1cm} (16)$$

$$V_X(a, Q_{XD}) = V_D(a, Q_{XD}) \hspace{1cm} (17)$$

$$V_X(a, Q_{XI}) = V_I(a, Q_{XI}) - F_I \hspace{1cm} (18)$$

$$V_I(a, Q_{ID}) = V_D(a, Q_{ID}) \hspace{1cm} (19)$$

$$V_I(a, Q_{IX}) = V_X(a, Q_{IX}) - F_X. \hspace{1cm} (20)$$

Similarly, we impose the smooth-pasting conditions:

$$V'_D(a, Q_{DX}) = V'_X(a, Q_{DX}) \hspace{1cm} (21)$$

$$V'_D(a, Q_{DI}) = V'_I(a, Q_{DI}) \hspace{1cm} (22)$$

$$V'_X(a, Q_{XD}) = V'_D(a, Q_{XD}) \hspace{1cm} (23)$$
\[ V'_X(a, Q_{XI}) = V'_I(a, Q_{XI}) \] (24)
\[ V'_I(a, Q_{ID}) = V'_D(a, Q_{ID}) \] (25)
\[ V'_I(a, Q_{IX}) = V_X(a, Q_{IX}) \] (26)

For each \( a \), equations (15)-(26) are a system of 12 equations in the 12 unknowns given by the quantity thresholds \( Q_{SR} \) and by the value functions parameters \( A_S, B_S \), for \( S, R \in \{ D, X, I \} \). The system is highly nonlinear, and as such is associated to multiple solutions. To get an economically sensible solution, we follow Dixit (1989) and impose \( A_D = 0 \): the option of entering a foreign market is nearly worthless for a domestic firm experiencing a very low \( Q \). Consistently, it must be that \( B_D \geq 0 \) to insure non-negativity of \( V_D(Q) \). The option of quitting FDI for another strategy is nearly worthless for a multinational firm experiencing an extremely high \( Q \), hence \( B_I = 0 \). Similarly, a multinational firm has expected value \( \frac{B(\tau_{aw})^{1-\eta} P^{\eta} Q}{\rho - \mu} - \frac{f_I}{\rho} \) from the strategy of never changing status, hence the optimal strategy must yield a no lesser value: \( A_I \geq 0 \). Finally, an exporter has expected value \( \frac{B(\tau_{aw})^{1-\eta} P^{\eta} Q}{\rho - \mu} - \frac{f_X}{\rho} \) from the strategy of never changing status, hence its optimal strategy must yield a no lesser value for any realization of \( Q \): \( A_X, B_X \geq 0 \).

### 3.1 Qualitative Properties of the Solution

Figure 2 displays a plot of the value functions for a firm with unit labor requirement \( a \), for an arbitrary set of parameters.\(^5\) The value function of a domestic firm \( V_D \) is increasing on the entire domain, indicating the fact that, as the realized aggregate demand in the foreign market \( Q \) increases, the value of the option of entering the foreign market (either through trade or FDI) increases. The value functions of an exporter and of a multinational (\( V_X \) and \( V_I \) respectively) are U-shaped: for low levels of \( Q \), the value is high thanks to the option of leaving the market, while for high levels of \( Q \) the value is high due to the profit stream that the firm derives from staying in the market.

The quantity thresholds that induce a firm to switch its status are also indicated in Figure 2. A higher quantity demanded \( Q \) is needed to induce a firm to enter a foreign market through FDI with respect to the quantity necessary to induce the firm to export: \( Q_{DI} > Q_{DX} \). Similarly, a larger negative shock is needed to induce a multinational to exit the foreign market with respect

\(^5\)The picture represents the value functions of a firm with productivity \( 1/a = 1 \). The elasticity of substitution across goods is set to \( \eta = 2 \), wages are equal across countries, and production has the following cost parameters: \( \tau = 1.2, f_X = f_I = 1, F_X = 4, F_I = 20 \). The interest rate is set at \( \rho = 0.025 \) and the Brownian motion for \( Q \) has parameters \( \mu = .02 \) and \( \sigma = 0.1 \).
to the shock needed to induce an exporter to exit: $Q_{ID} < Q_{XD}$. For the set of parameters used, a multinational that decides to divest always goes back to domestic sales only, and does not become an exporter: $Q_{IX} < 0$. Finally, while an extremely large quantity demanded ($Q_{XI} > Q_{DI}$) is needed to induce an exporter to undertake the sunk cost of becoming multinational since he is already serving the foreign market with exports. These properties are summarized by the following ordering:

$$Q_{IX} < Q_{ID} < Q_{XD} < Q_{DX} < Q_{DI} < Q_{XI}. \quad (27)$$

### 3.2 Comparative Statics: Value and Productivity

In this section we show how the solution of the model depends on a firm’s productivity level $a$.

Figure 3 shows the value function of a domestic firm as a function of aggregate quantity demanded in the domestic market $Q$ and of the productivity measure $a^{1-\eta}$. By looking at the behavior of

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6This happens because in this parameterization $f_X = f_I$ and $w = w^*$. Whenever $f_I \leq f_X + \rho F_X$ and wages are equalized, the per-period profits from FDI are higher than the per-period profits from exports: $\pi_I(a; Q) > \pi_X(a; Q), \forall Q$. Hence a negative shock to $Q$ may induce a multinational firm to exit the market and sell only domestically (if it drives $\pi_I(a; Q)$ significantly below zero), but does not induce the firm to stay in the market as an exporter, since under this scenario profits would be even more negative.

7Results about hysteresis, together with the analytical proof of the ordering of the thresholds $Q_{RS}$, for $R,S \in \{D, X, I\}$, is contained in the Appendix.
the value function along the two horizontal axes, it is evident how the effect of demand combines with different productivity levels. As observed in the previous section, $V_D$ is increasing in $Q$, as the option value of entering the foreign market is increasing in the quantity demanded. $V_D$ is also increasing in firm’s productivity, as more productive firms can get higher profits from entering the foreign market. The value function is proportional to our theoretical measure of returns ($\rho V_D(a;Q)$), hence returns of domestic firms are increasing in both quantity demanded and firm’s productivity.

Figure 4 shows the value functions of an exporter and of a multinational firm as functions of $Q$ and $a^{1-\eta}$. As observed in the previous section, $V_X$ and $V_I$ are U-shaped functions of $Q$, expressing the high option value of exiting for low realizations of $Q$ and the high option value of not changing status for high realizations of $Q$. The behavior of the value functions for $Q \to 0$ does not vary across the productivity dimension: when $Q$ is low, the value is high as firms of all productivity level associate a high value to the option of exiting. The behavior of the value functions when $Q$ is “large”, conversely, varies with individual productivity: the value function is steeper for higher productivity firms, indicating that more productive firms obtain higher returns from staying in the foreign market when the realized aggregate demand is high.

From Figure 4, the qualitative behavior of $V_X$ and $V_I$ appears very similar. Figure 5 plots the difference between the value functions of firms serving the foreign market and domestic firms,
Figure 4: Value functions of an exporter and of a multinational firm.

Figure 5: Difference between the value functions of exporters and multinationals and the value function of domestic firms.

$V_X - V_D$ and $V_I - V_D$. For each productivity level $a^{1-\eta}$, each plot has two stationary points, a local maximum and a local minimum. By observing the value matching and smooth pasting conditions, we argue that the local maxima correspond to the “entry” thresholds ($Q_{DX}$ and $Q_{DI}$ in the left and right plot respectively), while the local minima correspond to the “exit” thresholds ($Q_{XD}$ and $Q_{ID}$). The picture shows that both entry and exit thresholds are decreasing in $a^{1-\eta}$, indicating that more productive firms enter the foreign market for lower realizations of aggregate demand $Q$ with respect to less productive firms. Similarly, more productive firms need larger negative shocks to demand to be induced to exit the foreign market with respect to less productive firms.

Notice that for $Q \to 0$, $V_X - V_D$ and $V_I - V_D$ tend to infinity, because the option value of
Figure 6: Difference between the value functions of an exporter and of a multinational firm.

Exiting the foreign market is extremely high for very low realizations of $Q$ (and irrespective of firm’s productivity). Conversely, for $Q \to \infty$, $V_X - V_D$ and $V_I - V_D$ tend to negative infinity, because the domestic firms’ option value of entering the foreign market is extremely high, compared to the flow profits of staying for firms that are already serving that market.

Figure 6 plots the difference between the value functions of a multinational firm and of an exporter, $V_I - V_X$, as a function of $Q$ and $a_1 - \eta$. In this picture, for each value of $a_1 - \eta$, the peak of the surface represents the quantity threshold where the firm switches from being an exporter to being a multinational, $Q_{XI}$. The figure shows that also the threshold $Q_{XI}$ is decreasing in $a_1 - \eta$, consistent with the prediction of Helpman, Melitz, and Yeaple (2004), according to which the most productive firms are more likely to become multinationals than exporters.

Notice also that, for constant $Q$, the excess value $V_I - V_X$ decreases as productivity $a_1 - \eta$ increases: for the same level of $Q$, a more productive exporter has a higher option value (of switching to FDI) compared to a less productive one.

Figure 7 shows the quantity thresholds $Q_{RS}$, for $R, S, \in \{D, X, I\}$ as functions of firm’s productivity $a_1 - \eta$. As already noticed by inspecting the previous figures, all thresholds $Q_{RS}$ are decreasing in productivity, indicating that more productive firms need smaller positive shocks to demand to
enter the foreign market, and larger negative shocks to exit.\textsuperscript{8}

How does hysteresis combines with productivity? Keeping fixed the status, I would expect that more productive firms exhibit less hysteresis (need to check on that). But we know that more productive firms self-select into the status (I) that is associated with more hysteresis. This generates an imperfect sorting of productivities into status, which (hopefully) will generate the variation in $d/P$ ratios that we observe in the data.

3.3 Earnings-to-Price Ratios

The data reported in Section 2 show that the ranking in earnings-to-price ratios holds controlling for measured productivity. Hence we expect the model to generate a ranking whithin groups of firms of the same size (with the same $a$). Due to the imperfect sorting of firms into statuses according to productivity, it is possible that firms with the same productivity have different statuses, hence it is meaningful to compare dividend-price ratios of firms that have the same productivity level and different statuses.

Our earnings yields measure in the model is given by the ratio $\frac{\pi_t}{V_t}$, where $\pi_t$ represents the per-period profits and $V_t$ is the (market) value of the firm. In a static model, $\frac{\pi_t}{V_t} = 1$ is constant and

\textsuperscript{8}The proof of monotonicity of $Q_{RS}(a)$, for $R,S \in \{D,X,I\}$, is contained in the Appendix.
independent on the firm’s status, since per-period profits and value of the firm coincide. Dynamics and uncertainty introduce a wedge between these two magnitudes, which reflects the option value.

Assuming zero trend growth ($\mu = 0$) and common productivities $a^{1-\eta}$, dividend-price ratios in the model are given by:

$$\frac{\pi_D}{V_D} = 0 \quad \frac{\pi_X}{V_X} = \frac{\pi_X(Q)}{A_X Q^\alpha + B_X Q^\beta + \pi_X(Q) / \rho}$$

(28) \hspace{1cm} (29)

$$\frac{\pi_I}{V_I} = \frac{\pi_I(Q)}{A_I Q^\alpha + \pi_I(Q) / \rho}$$

(30)

Hence (by default) the model predicts that exporters and multinational have higher earnings-yields than domestic firms, provided that their profits from foreign sales are positive. To establish the ranking between exporters and multinationals, notice that $\frac{\pi_I}{V_I} > \frac{\pi_X}{V_X}$ if:

$$\frac{\pi_I(Q)}{A_I Q^\alpha + \pi_I(Q) / \rho} > \frac{\pi_X(Q)}{A_X Q^\alpha + B_X Q^\beta + \pi_X(Q) / \rho}$$

$$\left[ \frac{A_I Q^\alpha}{\pi_I(Q)} + \frac{1}{\rho} \right]^{-1} > \left[ \frac{A_X Q^\alpha + B_X Q^\beta + \pi_X(Q) / \rho}{\pi_X(Q)} + \frac{1}{\rho} \right]^{-1}$$

$$\frac{A_I Q^\alpha}{\pi_I(Q)} < \frac{A_X Q^\alpha + B_X Q^\beta + \pi_X(Q) / \rho}{\pi_X(Q)}$$

if $\pi_I(Q) > \pi_X(Q) \geq 0$. Hence a sufficient condition to obtain the desired result is:

$$A_I Q^\alpha \leq A_X Q^\alpha + B_X Q^\beta.$$  \hspace{1cm} (31)

Notice that this condition is very likely to be satisfied. As observed while inspecting the shape of $V_X$ and $V_I$, for $Q \to 0$ (hence when the coefficients $A$ are more important) the two value functions are almost indistinguishable. This happens because for very low realizations of $Q$, the option value of exiting the foreign market is similarly high for both exporters and multinationals. Hence $|A_I - A_X|$ is “small”. Conversely, for $Q \to \infty$ (hence when the coefficients $B$ are more important), $V_X > V_I$, because the option value of becoming a multinational is high, so $B_X$ is “large”.

Unfortunately, when $\pi_I(Q) < 0$ or $\pi_X(Q) < 0$, the ranking appears to be reversed. Figure 8 shows which types of firms coexist across realizations of $Q$. With the exception of the red intervals in the picture, the model delivers the observed ranking. In the area for $Q \in (Q_{ID}, W_{ID})$ (where
Figure 8: Ranking in Earnings Yields Predicted by the Model.

\( W_{ID} \) is defined as the quantity threshold at which a multinational makes zero profits), unprofitable multinational coexist with domestic firms (which by definition have zero profits from the foreign market), hence the ranking is reversed. Similarly, for \( Q \in (Q_{XD}, W_{XD}) \), unprofitable exporters coexist with domestic firms. The results of our regressions show that the ranking in earnings yields holds on average, hence we need to find a suitable parameterization that – while being consistent with the trade data on trade dynamics – also generates the observed ranking in earnings yields on average (i.e., a parameterization for which the areas in which the ranking holds dominate the areas in which it does not). Section 4 will deal with this issue.

### 3.4 Closing the Model

Before moving to the numerical analysis, we need to close the model. Under our assumption, whereby firms from the foreign country only produce the homogeneous good, the aggregate price of the differentiated good in the two countries is given by:

\[
\begin{align*}
P_{D}^{1-\eta} &= n \int \left( \frac{\eta aw}{\eta - 1} \right)^{1-\eta} dG(a) \\
P^{1-\eta} &= n \left[ \int_{\Omega_X} \left( \frac{\eta aw}{\eta - 1} \right)^{1-\eta} dG(a) + \int_{\Omega_I} \left( \frac{\eta aw}{\eta - 1} \right)^{1-\eta} dG(a) \right]
\end{align*}
\]

(32)

(33)

where \( n \) is the equilibrium number of firms and \( \Omega_X (\Omega_I) \) is the set of firms that are currently exporting (doing FDI). Notice that the sets \( \Omega_X \) and \( \Omega_I \) vary with the realization of \( Q \), as firms switch status, but only depend on the firms’ status in the previous period, due to the Markov property of Brownian motions.
We solve for the equilibrium number of firms $n$ by imposing a free-entry condition:

$$F_E = \int \pi_d(a)dG(a) + \int_{\Omega_D} V_D(a)dG(a) + \int_{\Omega_X} [V_X(a) - F_X]dG(a) + \int_{\Omega_I} [V_I(a) - F_I]dG(a). \quad (34)$$

The aggregate quantities $Q_D$ and $Q$ are taken as given, and perfect tradeability of the homogeneous good implies: $w = w^* = 1$.

4 Calibration

To be added: use moments of trade data to calibrate the model and reproduce the ranking in earnings-to-price ratios.

5 Simulations

5.1 Returns

Earnings yields matter in connection with returns: use CRSP data to compute returns at the firm level and analyze model’s predictions on returns.

5.2 Counterfactual exercises

Testable predictions of the model: compare exporters and multinational firms’ dynamics. The model predicts that – following a negative shock – firms serving a foreign market through export should exit before/faster than MN firms.

(To be added)

6 Conclusions

To be added.
Appendix

A Proofs

A.1 Hysteresis

In this section we extend the analytical results about the nature of hysteresis in Dixit (1989) to our model, where firms also choose the mode of entry in a (foreign) market.

Let \( G_{SR}(Q) \equiv V_S(Q) - V_R(Q) \), for \( S, R \in \{D, X, I\} \). We omit the dependence of the value functions on the productivity parameter \( a \), as the proof carries over \( \forall a \).

Let us consider the value matching and smooth pasting conditions for a firm switching from being domestic only to also export and vice versa. Value matching implies: \( G_{DX}(Q_{DX}) = F_X \) and \( G_{DX}(Q_{XD}) = 0 \). Similarly, smooth pasting implies: \( G'_{DX}(Q_{DX}) = G'_{DX}(Q_{XD}) = 0 \).

By using the expressions for the value functions in equations (10), (13), we can compute the limits of \( G_{DX}(\cdot) \):

\[
G_{DX}(Q) = A_X Q^\alpha + B_X Q^\beta + \frac{B(\tau a)^{1-\eta} P^\eta Q}{\rho - \mu} - \frac{f_X}{\rho} - B_D Q^\beta
\]

which implies:

\[
\lim_{Q \to 0} G_{DX}(Q) = +\infty
\]

since \( \alpha < 0 \) and \( \beta > 1 \). Similarly:

\[
\lim_{Q \to +\infty} G_{DX}(Q) = \pm \infty
\]

(according to whether \( B_X \geq B_D \) or \( B_X < B_D \)). Hence \( G''_{DX}(Q_{DX}) \leq 0 \) and \( G''_{DX}(Q_{XD}) > 0 \).

From Ito’s Lemma (combining equations (9) and (11)):

\[
\rho G_{DX}(Q) = G'_{DX}(Q) \mu Q + \frac{1}{2} G''_{DX}(Q) \sigma^2 Q^2 + B(\tau a)^{1-\eta} P^\eta Q - f_X
\]

which, evaluated at \( Q_{DX} \) reduces to:

\[
\rho F_X - B(\tau a)^{1-\eta} P^\eta Q_{DX} + f_X \leq 0
\]
since $G'_{DX}(Q_{DX}) = 0$ and $G''_{DX}(Q_{DX}) \leq 0$. Hence:

$$Q_{DX} \geq \frac{\rho F_X + f_X}{B(\tau aw)^{1-\eta} P^n} \equiv W_{DX}$$

where $W_{DX}$ is the corresponding threshold in absence of uncertainty. By evaluating the Ito’s lemma condition at $Q_{XD}$ one can also show that:

$$Q_{XD} < \frac{f_X}{B(\tau aw)^{1-\eta} P^n} \equiv W_{XD}.$$  

With the same procedure, the result follows for the other four thresholds as well:

$$Q_{DI} > \frac{\rho F_I + f_I}{B(aw^*)^{1-\eta} P^n} \equiv W_{DI}$$

$$Q_{ID} < \frac{f_I}{B(aw^*)^{1-\eta} P^n} \equiv W_{ID}$$

$$Q_{XI} > \frac{\rho F_I + (f_I - f_X)}{B(aw^*)^{1-\eta} P^n - B(\tau aw)^{1-\eta} P^n} \equiv W_{XI}$$

$$Q_{IX} \leq \frac{-\rho F_X + (f_I - f_X)}{B(aw^*)^{1-\eta} P^n - B(\tau aw)^{1-\eta} P^n} \equiv W_{IX}.$$  

Notice also that for $f_X = f_I$, like in the numerical example in Section 3, $W_{IX} < 0$ and hence $Q_{IX} < 0$.

### A.2 Ordering of the quantity thresholds

It is easy to prove that, if $F_I > F_X$, $f_I \geq f_x$, and $w^* < \tau w$, the following order of the deterministic thresholds holds:

$$W_{IX} < W_{ID} < W_{XD} < W_{DX} < W_{DI} < W_{XI}.$$  

Dixit and Pindyck (1994) show that hysteresis, here defined as the difference $Q_{RS} - W_{RS}$, for $R,S \in \{D,X,I\}$ is increasing in sunk costs. Hence: $Q_{DI} - W_{DI} > Q_{DX} - W_{DX}$. Since $W_{DI} > W_{DX}$, we have $Q_{DI} > Q_{DX}$. By applying the same reasoning, one can also show that $Q_{XI} > Q_{DI}$. Symmetrically, one can also show the ordering of the exit thresholds, so that:

$$Q_{IX} < Q_{ID} < Q_{XD} < Q_{DX} < Q_{DI} < Q_{XI}.$$  

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A.3 Monotonicity of the quantity thresholds

Everything else constant, a higher productivity $a^{1-\eta}$ is equivalent to a lower operating cost. Dixit and Pindyck (1994), pp. 221-223, show that entry and exit thresholds are decreasing in operating costs, hence $\frac{\partial Q}{\partial a^{1-\eta}} < 0$.

References


