Moral Hazard and Debt Maturity

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VERY PRELIMINARY

Comments welcome!
Motivation

Although core deposits continue to be a key liability funding source, many insured depository institutions are increasingly looking to wholesale funding sources…

The use of wholesale funding, in and of itself, is not viewed negatively. Active and effective risk management can mitigate the added risks…

*FDIC Risk Management Manual of Examination Policies*
Motivation

→ J. Tirole (AEA 2003), *Inefficient Foreign Borrowing*

Should “dangerous forms of finance” be discouraged?

Borrowers design their financial structures to their own benefit, and one cannot presuppose that dangerous forms of debt constitute suboptimal liability structures.
Introduction

Issue

- Use of short-term wholesale financing → liquidity risk
- What is the upside?

This paper

- Costs and benefits of short-term debt
  (leaving aside liquidity insurance à la Diamond-Dybvig)
- Disciplining device vs. inefficient liquidation
Preview of model

• Single firm + large number of wholesale investors
• Firm’s assets: Long-term risky investment
• Firm’s liabilities: Short-term (ST) or long-term (LT) debt
• Moral hazard problem in choice of risk
• Public signal on investment return at rollover date of ST debt
Preview of results

• Risky ST debt may be the only way to secure funding
  → because of positive incentive effects

• ST debt may dominate LT debt (when the latter is feasible)
  → even when it involves inefficient liquidation

• ST debt appears to be “cheaper” that LT debt
  → because of positive incentive effects, not by assumption
Overview

• Model setup
• LT debt
• Safe ST debt
• Risky ST debt
• Numerical results
• Conclusion
Model setup

• Three dates ($t = 0, 1, 2$)

• Large number of risk-neutral wholesale investors (the lenders)
  → Required return normalized to 0

• Single risk-neutral firm
  → Indivisible unit investment with random return at $t = 2$
  → Funded with debt (no capital)
  → LT debt (maturing at $t = 2$) or ST debt (rolled over at $t = 1$)
Model setup

• Return of investment at $t = 2$

$$R = \begin{cases} R_1 & \text{with probability } p \\ R_0 & \text{with probability } 1 - p \end{cases}$$

→ $p \in [0,1]$ is a risk parameter chosen by the firm at $t = 0$

→ Assume that $R_0 = 0$ and $R_1 = R(p)$

→ Assume that $R(p)$ is decreasing and concave, with $R(1) \geq 1$

• Liquidation value of investment at $t = 1$: $L \in [0,1]$
Model setup

• First-best level of risk

\[ \max_p[pR(p)] \]

→ First-order condition: \([pR(p)]' = 0\)

→ Since \(pR(p)\) is concave a solution \(\hat{p}\) exists, with \(\hat{p}R(\hat{p}) > 1\)
Model setup

\[ pR(p) \]

relevant range

0 \quad \hat{p} \quad 1

p
Long-term debt

• Suppose that firm is funded at \( t = 0 \) with long-term debt

\[ \rightarrow B \text{ is face value of debt payable at } t = 2 \]
Long-term debt

• Firm shareholders’ choice of risk

$$\max_p p[R(p) - B]$$

→ First-order condition: $$[pR(p)]' = B \implies \frac{dp}{dB} < 0$$

→ For any $$B > 0$$ the firm chooses

→ Standard risk-shifting effect $$p < \hat{p}$$
Model setup

$p_R(p)$

$p_B$

$p(B)$

$p$
Long-term debt

Definition 1  An equilibrium is a pair \((p^*, B^*)\) that satisfies

1. Firm’s optimal choice of risk

\[
p^* = \arg\max_p p[R(p) - B^*]
\]

2. Lenders’ participation constraint

\[
p^* B^* = 1
\]
Long-term debt

• Characterization of equilibrium

\[ \begin{align*}
\text{FOC: } & \quad [p^* R(p^*)]' = B^* \\
\text{PC: } & \quad p^* B^* = 1
\end{align*} \rightarrow H(p^*) = 1 \]

where

\[ H(p) = p(pR(p))' \]

**Proposition 1** If there exists \( p^* \) with \( H(p^*) = 1 \), then \( (p^*, 1/p^*) \) is an equilibrium with LT debt.
Long-term debt

- Three possible cases
  - No equilibrium
  - Unique equilibrium
  - Multiple equilibria → choose the one with highest $p^*$

- Example: $R(p) = a(2 - p)$

  $p[pR(p)]' = 1 \rightarrow p^* = \frac{1}{2}(1 + \sqrt{1 - 2a^{-1}})$

  → Equilibrium requires $a \geq 2$ (unique if $a = 2$)
Long-term debt

\[ H(p) \]

1

\[ p_{LT} \]

\[ \hat{p} \]

\[ p \]
Short-term debt

• At $t = 1$ lenders observe signal $s \in \{s_0, s_1\}$

• Assume

$$\Pr(s_0 | R_0) = \Pr(s_1 | R_1) = q \in [1/2, 1]$$

$\rightarrow q$ describes the quality of the lenders’ information
Short-term debt

• By Bayes’ law

\[
\Pr(R_1 | s_0) = \frac{p(1-q)}{p+q-2pq} \quad \text{and} \quad \Pr(R_1 | s_1) = \frac{pq}{1-p-q+2pq}
\]

• For \( q = 1/2 \) \quad \Pr(R_1 | s_0) = \Pr(R_1 | s_1) = p

\rightarrow \text{Signal uninformative}

• For \( q = 1 \) \quad \Pr(R_1 | s_0) = 0 \quad \text{and} \quad \Pr(R_1 | s_1) = 1

\rightarrow \text{Signal completely reveals future return of investment}

• For \( 1/2 < q < 1 \) \quad \Pr(R_1 | s_0) < p < \Pr(R_1 | s_1)

\rightarrow s_0 \text{ is bad state and } s_1 \text{ is good state}
Short-term debt

• Suppose firm is funded at $t = 0$ with ST debt
  $\rightarrow M$ is face value of debt payable at $t = 1$
  $\rightarrow N_s$ is face value of debt payable at $t = 2$
  when lenders decide to refinance in state $s$

$t = 0$  \(\rightarrow\)  \(t = 1\)  \(\rightarrow\)  \(t = 2\)

- ST debt $M$
- Choice of $p$
- Signal $s$
- Refinance $N_s$
- or liquidate

Final payoffs
Short-term debt

• Lenders’ participation constraint in state $s$

$$\text{Pr}(R_1|s)N_s = M$$

→ Initial debt refinanced in state $s$ if

$$N_s = \frac{M}{\text{Pr}(R_1|s)} \leq R_1 \rightarrow \text{Pr}(R_1|s)R_1 \geq M$$

→ Inefficient liquidation when

$$L < \text{Pr}(R_1|s)R_1 < M$$
Short-term debt

• Decision to refinance initial debt in state $s$

\[ I\left( \Pr(R_1|s)R_1 - M \right) = \begin{cases} 1, & \text{if } \Pr(R_1|s)R_1 - M \geq 0 \\ 0, & \text{otherwise} \end{cases} \]

• Initial lenders’ participation constraint

\[ g(p, M) = \sum_{i=0,1} \Pr(s_i)\left[ L + (M - L)I\left( \Pr(R_1|s)R_1 - M \right) \right] = 1 \]
Short-term debt

• Firm’s payoff for a given $M$ conditional on $s$

$$\Pr(R_1|s) \max \{R_1 - N_s, 0\} = \Pr(R_1|s) \max \left\{ R_1 - \frac{M}{\Pr(R_1|s)}, 0 \right\}$$

• Firm’s payoff for a given $M$

$$\pi(p, M) = \sum_{i=0,1} \Pr(s_i) \Pr(R_1|s_i) \max \left\{ R_1 - \frac{M}{\Pr(R_1|s_i)}, 0 \right\}$$
Short-term debt

Definition 2 An equilibrium is a pair \((p^*, M^*)\) that satisfies

(1) Firm’s optimal choice of risk

\[
p^* = \arg \max_p \pi(p, M^*)
\]

(2) Initial lenders’ participation constraint

\[
g(p^*, M^*) = 1
\]

(Second period lenders’ PC implicit in \(\pi\) and \(g\))
Short-term debt

- Two possible types of equilibrium
  - Safe ST debt (no liquidation in bad state $s_0$)
  - Risky ST debt (liquidation in bad state $s_0$)
Safe short-term debt

- Lenders’ participation constraint becomes
  
  \[ g(p^*, M^*) = M^* = 1 \]

- Firm’s shareholders payoff becomes
  
  \[ \pi(p, M^*) = p \left[ R(p) - \frac{1}{p^*} \right] \]

- Characterization of equilibrium

  \[ H(p^*) = 1 \quad \text{(as in the case of LT debt)} \]
  
  \[ \Pr(R_1|s_0)R_1 \geq 1 \quad \text{(no liquidation in bad state } s_0 \text{)} \]
Safe short-term debt

Special cases

• $q = 1/2$ \( \Pr(R_1 | s_0) = \Pr(R_1 | s_1) = p \)
  \[ \rightarrow \text{Constraint } \Pr(R_1 | s_0) R_1 \geq 1 \text{ is always satisfied} \]

• $q = 1$ \( \Pr(R_1 | s_0) = 0 \)
  \[ \rightarrow \text{Constraint } \Pr(R_1 | s_0) R_1 \geq 1 \text{ is never satisfied} \]
  \[ \rightarrow \text{Efficient liquidation (as long as } L > 0) \]
  \[ \rightarrow \text{Possible role for risky ST debt?} \]
Safe short-term debt

Proposition 2 If there exists $p^*$ with $H(p^*) = 1$ and $q \leq q(p^*)$, then $(p^*, 1)$ is an equilibrium with safe ST debt.

• Comments
  - Eq. with safe ST debt $\Rightarrow$ Eq. with LT debt (same $p^*$)
  - Eq. with LT debt $\not\Rightarrow$ Eq. with safe ST debt
  - Safe ST debt does not add anything relative to LT debt
Risky short-term debt

• Lenders’ participation constraint becomes

\[ g(p^*, M^*) = \Pr(s_0)L + \Pr(s_1)M^* = 1 \]

• Firm’s shareholders payoff becomes

\[ \pi(p, M^*) = \Pr(R_1, s_1) \left[ R(p) - \frac{M^*}{\Pr(R_1|s_1)} \right] \]
Risky short-term debt

• Characterization of equilibrium

\[ H(p^*) = F(p^*, q, L) \]

\[ \Pr(R_1 | s_0) R_1 < M^* \quad \text{(liquidation in bad state } s_0) \]

where

\[ F(p, q, L) = \frac{1 - \Pr(s_0) L}{q} \quad \rightarrow \quad \text{linearly increasing in } p \]
Risky short-term debt

Proposition 3  *If there exists* $p^*$ *with* $H(p^*) = F(p^*, q, L)$ *and* $L < L(p^*, q)$, *then* $(p^*, M^*)$ *is an equilibrium with risky ST debt.*

- Three possible cases
  - No equilibrium
  - Unique equilibrium
  - Multiple equilibria

- Liquidation in bad state $s_0$ may be inefficient (if $\Pr(R_1|s_0)R_1 > L)$
Risky short-term debt

\[ H(p) \]

\[ F(p, q, L) \]

\[ p_{ST} \]

\[ \hat{p} \]

\[ p \]
Numerical results

Parameterization

• Suppose that $R(p) = a(2 - p)$
  
  $\rightarrow a$ measures the profitability of the investment project

• First-best level of risk
  
  $[pR(p)]' = 2a(1 - p) = 0 \rightarrow \hat{p} = 1$

• Equilibrium with LT debt
  
  $p[pR(p)]' = 1 \rightarrow p_{LT} = \frac{1}{2} \left( 1 + \sqrt{1 - 2a^{-1}} \right)$

  $\rightarrow$ Equilibrium requires $a \geq 2$
Numerical results

• Solve for equilibrium with risky ST debt for
  - Quality of the lenders’ information $q \in [1/2, 1]$
  - Liquidation value of the investment $L \in [0, 1]$

• Two cases
  - $a = 1.9 < 2$ (there is no equilibrium with LT debt)
  - $a = 2.1 > 2$ (there is an equilibrium with LT debt)
Figure 1: Only ST debt viable
Figure 2: Both ST and LT viable

- ST debt (efficient liquidation)
- ST debt (inefficient liquidation)
- LT debt dominates ST debt
- Only LT debt

a=2.1
Summing up

• Risky ST debt may be the only way to secure funding
  → when investment profitability $a$ is low

• Risky ST debt may dominate LT debt
  → when quality of lenders’ information $q$ is high

• Risky ST debt may be optimal despite inefficient liquidation
  → because of positive incentive effects
What is the intuition?

• The source of the positive incentive effects of ST debt?
  - Choice of risk under LT debt
    \[
    \max_p \ p[R(p) - B^*] \rightarrow \text{FOC: } [pR(p)]' = B^* = \frac{1}{p_{LT}}
    \]
  - Choice of risk under risky ST debt
    \[
    \max_p \ pq \left[ R(p) - \frac{M^*}{\Pr(R_1|s_1)} \right] \rightarrow \text{FOC: } [pR(p)]' = \frac{M^*}{\Pr(R_1|s_1)}
    \]
What is the intuition?

• Since $[pR(p)]'$ is decreasing we have

$$p_{ST} > p_{LT} \iff \frac{M^*}{\Pr(R_1|s_1)} < \frac{1}{p_{LT}}$$

• In special case of $q \to 1$ we have

$$\frac{M^*}{\Pr(R_1|s_1)} \to \frac{1-(1-p_{ST})L}{p_{ST}} < \frac{1}{p_{LT}}$$

→ Lower payment to lenders implies lower risk
Conclusion

• Cost of risky ST debt: Inefficient liquidation

• Benefit of risky ST debt: Discipline device to lower risk-taking

• Benefits may be greater than costs
  → When transparency $q$ is high
  → When liquidation value $L$ is high

• Future research
  - Mixed debt finance: ST + LT
  - LT debt with covenants forcing liquidation in bad state
  - Equity finance