Endogenous Inequality of Nations Through

Financial Asset Market Integration

by

Volker Böhm, and George Vachadze

Department of Economics
Bielefeld University
1 Motivation

Financial globalization - the phenomenon of rising cross-border financial flows

Some empirical evidence on inter-country income inequality:

- Bianchi (JAE 1997), Jones (JEP 1997) and Quah (JEG 1997) ⇒ emerging twin peaks between 1960-1990

- Durlauf and Johnson (JAE 1995) ⇒ dependence on growth rate on initial per capita income

Is there any CAUSAL LINK between financial globalization and inequality of nations?
Conventional wisdom:

- an equalizing force, facilitates rich to poor capital flow
- poor countries increase domestic capital investment
- poor countries achieve faster growth and catch up with the rich

Can foreign capital inflows crowd out domestic investment?

Some empirical evidence: Agosin and Mayer (UNCTAD 2000) and Edison and Levine (JIMEF 2002)
Questions:

- Can financial globalization affect the inequality of nations?
- Can foreign capital inflows crowd out domestic investment?

Financial globalization may have an adverse effect on poor countries:

- Matsuyama (Econometrica 2004) ⇒ wealth constrained investment can be a source of symmetry breaking. Distortion created by the capital market inefficiency is smaller in the richer rather than in poor countries.

- Boyd and Smith (JET 1997) ⇒ asymmetry of information can complicate the allocation of credit. The presence of costly state verification creates an unequalizing force and in case of a negative domestic capital shock creates a downward spiral of low-income/low-investment.
The game plan:

- Build a model of the world economy consisting of structurally identical economies
- Analyze the set of the stable steady states \textit{WITH} and \textit{WITHOUT} the international asset markets

The main result:

- \textit{WITHOUT} the international asset market, the steady state is unique, symmetric and stable

- \textit{WITH} the international financial market, for some parameter values, the unique symmetric steady state loses its stability and the asymmetric steady states appear as only possible outcome
2 The Closed Economy Model

- discrete time OLG model with consumption and production sectors
- market for a single produced commodity usable for consumption and investment
  *plus* markets for capital, labor, and a re tradable asset
- perfect competition in all markets
- neither strategic behavior nor any information asymmetry

The unproductive asset:

- asset can be interpreted broadly to include property, land, or any other asset not directly used in production
- aggregate supply of an asset is constant and normalized to unity
- ownership of an asset means the claim for a random dividend stream
- asset is traded between generations at a competitive market
2.1 The Consumption Sector

- two period lived overlapping generations of consumers
- no population growth
- inelastic labor supply in the first period
- lifetime utility to depend only on the second period consumption
- possibility to invest either in capital market or in the asset market
- young agent faces the budget constraint

\[ b_t + x_t p_t \leq w_t, \text{ with } x_t \geq 0, \ b_t \geq 0 \]

- old agent’s random consumption is

\[ c_{t+1} \leq b_t r_{t+1} + x_t (p_{t+1} + \varepsilon_{t+1}) \]  (2.1)
Assumptions about the random dividend payment and agents’ utility function:

**Assumption 2.1** \( \varepsilon = \{\varepsilon_t\}_{t=1}^{\infty} \) is

- a sequence of i.i.d. random variables
- \( \varepsilon_t \) takes values of \( d > 0 \) and 0 with probabilities \( q \in (0,1) \) and \( 1 - q \) respectively for any \( t = 1, 2, ... \)

**Assumption 2.2** Agent’s preferences over the old age consumption is described by the twice continuously differentiable utility function \( u \), satisfying

- \( u''(c) < 0 < u'(c) \) for all \( c \in \mathbb{R}_+ \)
- absolute risk tolerance function, \( T(c) = -\frac{u''(c)}{u'(c)} \), is non-increasing
For given values of \((w_t, r_{t+1}, p_{t+1}, p_t) \geq 0\), we define the young agent’s asset demand as:

\[
\varphi(w_t, r_{t+1}, p_{t+1}, p_t) := \arg \max_{x_t \in B(w_t, p_t)} \mathbb{E}[u(c_{t+1})],
\]

(2.2)

where \(\mathbb{E}[.]\) denotes the expectation operator and

\[
c_{t+1} := r_{t+1}w_t + x_t(p_{t+1} + \varepsilon_{t+1} - p_tr_{t+1})
\]

(2.3)

is the random future consumption, while

\[
B(w_t, p_t) = \{x_t | x_t \geq 0, x_tp_t \leq w_t\}
\]

(2.4)

is the young agent’s current budget constraint.
2.2 The Production Sector

- single, infinitely lived firm, homogeneous commodity from capital and labor
- output is sold and there is no inventory hold by the firm
- capital depreciates fully at each period
- wage and capital rental rates are paid according to the marginal product rule

Assumption 2.3 \( f \) it is twice continuously differentiable and satisfies

- \( f(0) = 0 \) and \( f''(k) < 0 < f'(k) \) for all \( k \in \mathbb{R}^+ \)
- \( \lim_{k \to 0} f'(k) = \infty \) and \( \lim_{k \to \infty} f'(k) = 0 \)
- \( \lim_{k \to 0} w'(k) = \infty \) and \( w''(k) < 0 \) for all \( k \in \mathbb{R}^+ \)
2.3 Steady State Equilibrium

- For any given values of \((k_t, r_{t+1}, p_{t+1}) > 0\) we define a market clearing price \(p_t = S(k_t, r_{t+1}, p_{t+1})\) solving equation

\[
\varphi(w(k_t), r_{t+1}, p_{t+1}, p_t) = 1
\]  
(2.5)

**Definition 2.1** A Steady State Equilibrium (SSE) is a pair \((k, p)\), such that

- for given value of \(k \in \mathbb{R}_+\), there exists an asset price \(p \in \mathbb{R}_+\) clearing the equity market, i.e.

\[
p = S(k, r(k), p)
\]  
(2.6)

- for given value of \(p \in \mathbb{R}_+\), capital stock is a fixed point of the capital accumulation equation

\[
k = A(k, p) := w(k) - p
\]  
(2.7)
• \((k, p) = (0, 0)\) is a corner SSE

• in an interior SSE, agent’s optimality condition implies

\[
\frac{d}{p(r(k) - 1)} = 1 + \frac{1 - qu'(c)}{q u'(\bar{c})}
\]  \hspace{1cm} (2.8)

• where \(c\) and \(\bar{c}\) are consumptions in a good and bad states and

\[
c = w(k)r(k) - (w(k) - k)(r(k) - 1) = f(k) - k
\]
\[
\bar{c} = w(k)r(k) + xd - (w(k) - k)(r(k) - 1) = f(k) - k + xd
\]  \hspace{1cm} (2.9)

• equations (2.8) and (2.9) yield that for a given SSE pair, \((k, x)\), the inverse demand on financial asset is given by

\[
p(k, x) = \frac{d}{r(k) - 1}h(k, x) \quad \text{where} \quad h(k, x) = \frac{qu'(\bar{c})}{qu'(\bar{c}) + (1 - q)u'(\bar{c})}
\]  \hspace{1cm} (2.10)
• $h(k, x) \in [0, 1]$ – risk neutral probability of a good state realization

• Assumption 2.2 $\Rightarrow h_k(k, x) \geq 0$ and $h_x(k, x) \leq 0$

• Assumption 2.3 $\Rightarrow k \in [0, k^*]$ where $k^*$ solves $r(k) = 1$

• equilibrium pair $(k, p)$ solves the system of equations

\[ p(k, x) = \frac{d}{r(k) - 1} h(k, x) \quad \text{and} \quad p(k, x) = \frac{w(k) - k}{x}. \]  

(2.11)

• equation (2.11) $\Rightarrow$ equilibrium $k$ solves the equation

\[ (w(k) - k)(r(k) - 1) = dxh(k, x) \]  

(2.12)

with $x = 1$ (supply of financial asset is normalized to unity)
2.3 Steady State Equilibrium

- If \( E_f(k) = \frac{f'(k)}{f(k)} < 0.5 \Rightarrow \rho(k) := (w(k) - k)(r(k) - 1) \) is a decreasing

- \( \rho(0) = \infty \), and \( \rho(k^*) = 0 \)

- thus \( (w(k) - k)(r(k) - 1) = dxh(k, x) \) has a unique and interior solution for any \( x \in \mathbb{R}_+ \)

- Dynamics of the economy is described by the pair of functions \((P, G)\)

\[
k_1 = G(k) := w(k) - P(k), \quad (2.13)
\]

and

\[
P(k) = S(w(k), r(G(k)), P(G(k))) \quad (2.14)
\]
3 A Two Country Model

• world economy with two economies

• $h$ and $f$ denote the home and foreign countries of the above type

• countries are inhabited by homogeneous consumers and homogeneous firms

• factors of production, capital and labor, are immobile across countries

• assets exist in both economies, they pay stochastically the same dividends, and the asset market operates internationally

• the only difference between the countries is in their initial capital stock
3.1 Steady State Equilibrium: Two County

For any given values of \((k^h_t, k^f_t, r^h_{t+1}, r^f_{t+1}, p_{t+1}) \geq 0\) we define a market clearing price \(p_t = S(k^h_t, k^f_t, r^h_{t+1}, r^f_{t+1}, p_{t+1})\) solving equation

\[
\varphi(w(k^h_t), r^h_{t+1}, p_{t+1}, p_t) + \varphi(w(k^f_t), r^f_{t+1}, p_{t+1}, p_t) := 2.
\]  \hspace{1cm} (3.1)

**Definition 3.1** A **SSE in the two country economy is a triple** \((k^h, k^f, p)\), **such that**

- for given \((k^h, k^f) \in \mathbb{R}^2_+\), asset price \(p \in \mathbb{R}_+\) solves \(p = S(k^h, k^f, r(k^h), r(k^f), p)\);
- for given \(p \in \mathbb{R}_+\), \((k^h, k^f)\) solves \(k^h = A(k^h, p)\) and \(k^f = A(k^f, p)\) where \(x(k^i, p) := \varphi(w(k^i), r(k^i), p, p)\) and \(A(k^i, p) := w(k^i) - x(k^i, p)p\).
• closed economy has two SSE – 0 and $\hat{k}$

• open economy has two symmetric, $(0,0)$ and $(\hat{k},\hat{k})$ steady states

• open economy has two asymmetric, $(0,\tilde{k})$ and $(\tilde{k},0)$ steady states, where $\tilde{k}$ solves $(w(k) - k)(r(k) - 1) = d_x h(k, x)$ with $x = 2$

• **The main question:** are there any asymmetric and interior steady states?
Example

- production function is Cobb-Douglas, \( f(k) := Ak^\alpha \), with \( \alpha < 0.5 \)

- utility function \( u \) is such that

\[
u'(c) := e^{\exp \left( -a \frac{c^{1-b}}{1 - b} \right)}.
\]  
(3.2)

- parameter \( a > 0 \) measures the degree of absolute risk aversion and \( b \geq 0 \) measures the curvature of the absolute risk tolerance

- agents’ absolute risk tolerance function in this case is

\[
T(c) = -\frac{u''(c)}{u'(c)} = ac^{-b},
\]  
(3.3)

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<td>( \theta )</td>
<td>( d )</td>
<td>( q )</td>
<td>( a )</td>
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<tr>
<td>0.80</td>
<td>0.40</td>
<td>4.00</td>
<td>0.90</td>
<td>1.40</td>
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</table>
3.1 Steady State Equilibrium: Two County

Figure 3.1: Distribution of Capital Implied by the \((x, 2 - x)\) Asset Holding

- let for a given \(x \in (0, 2)\), \(k = \phi(x)\) denotes the unique and interior solution of

\[
(w(k) - k)(r(k) - 1) = d x h(k, x)
\]  

(3.4)

- for any \(x \in (0, 2)\), \(k_h = \phi(x)\) and \(k_f = \phi(2 - x)\), denote the distribution of capital stock in an open economy
3.1 Steady State Equilibrium: Two County

Figure 3.2: Asset prices Implied by the \((x, 2 - x)\) Distribution of Asset Holding

- \((p_h, p_f) = (\Phi(x), \Phi(2 - x))\), where \(\Phi\) function is defined as \(\Phi(x) := p(\phi(x), x)\), clears the asset market in each country.

- Asset market integration requires each \(p_h = p_f\), i.e., in equilibrium \(x\) should solve the equation \(\Phi(x) = \Phi(2 - x)\)
Figure 3.3: All Possible Configurations of \( p(x) \) and \( p(2 - x) \) Functions
3.1 Steady State Equilibrium: Two County

Figure 3.4: Existence of Asymmetric Steady States
3.2 Stability of Symmetric Rational Expectations Equilibria

The dynamic evolution of the two country

\[ p_t = S(k^h_t, k^f_t, r^h_{t+1}, r^f_{t+1}, p_{t+1}) \]

\[ k^h_{t+1} = w(k^h_t) - p_t \varphi(r^h_{t+1}, p_{t+1}, p_t) \] (3.5)

\[ k^f_{t+1} = w(k^f_t) - p_t \varphi(r^f_{t+1}, p_{t+1}, p_t) \]

Suppose \( G : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+ \) and \( P : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+ \) satisfy

\[
P(x, y) = S(r(G(x, y)), r(G(y, x)), P(G(x, y), G(y, x)))
\]

\[
G(x, y) = w(x) - \varphi(r(G(x, y)), P(G(x, y), G(y, x)), P(x, y)) P(x, y).
\]

Then, one obtains a time one map \( H : \mathbb{R}^2_+ \rightarrow \mathbb{R}^2_+ \) of the economy given by

\[
k^h_{t+1} = G(k^h_t, k^f_t)
\]

\[
k^f_{t+1} = G(k^f_t, k^h_t)
\] (3.6)
• Symmetric steady state \((0, 0)\) is a source

• Asymmetric steady states \((\tilde{k}, 0)\) and \((0, \tilde{k})\) are unstable saddles.

• Symmetric steady state \((k^*, k^*)\) can be unstable saddle or can be stable node

Figure 3.5: Stability of the Symmetric SSE
Let for a particular value of $k_h$, $k_f = \Pi(k_h)$ satisfying the following equation

$$k_f = A(k_f, p(k_f, k_h)). \quad (3.7)$$

Due to the symmetry of the problem $k_h = \Pi^{-1}(k_f)$, where $\Pi^{-1}$ is the inverse function, will satisfy the equation

$$k_h = A(k_h, p(k_h, k_f)). \quad (3.8)$$

![Three Possible Phase Portraits](image-url)

Figure 3.6: Three Possible Phase Portraits
4 The Role of Assumptions Made

- role of uncertainty in symmetry breaking If there is no uncertainty, i.e., if $q = 1$, the inverse demand function becomes

$$p(k, x) = \frac{d}{r(k) - 1}.$$  \hfill (4.1)

- role of risk aversion in symmetry breaking If agents are risk neutral then $h(k, x) = q$, the inverse demand function becomes

$$p(k, x) = \frac{dq}{r(k) - 1}.$$  \hfill (4.2)

- after the asset market integration return on capital is equalized with risk adjusted return on financial asset within each country.

- risk adjusted returns can differ in equilibrium, which implies non convergence
### Numerical Example

<table>
<thead>
<tr>
<th></th>
<th>Domestic investment</th>
<th>Spending on asset market</th>
<th>Capital return</th>
<th>Asset return</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Closed Economy:</strong></td>
<td>9%</td>
<td>91%</td>
<td>6%</td>
<td>10.5%</td>
</tr>
<tr>
<td><strong>World Economy:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initially Poor Country:</strong></td>
<td>4.9%</td>
<td>95.9%</td>
<td>7.90%</td>
<td>10.40%</td>
</tr>
<tr>
<td><strong>Initially Rich Country:</strong></td>
<td>21.2%</td>
<td>78.8%</td>
<td>3.50%</td>
<td>10.40%</td>
</tr>
</tbody>
</table>
5 Summary and Conclusions

What this paper DOES

• examines the effects of asset market integration on the inequality of nations

• identifies two sided spill over effects between real markets and asset markets induced by portfolio behavior of rational consumers as a possible un-equalizing force between incomes of otherwise identical economies

• shows that asset market globalization sometimes can magnify the inequality of nations

This paper does NOT argue that

• the world economy has become increasingly unequal

• the inequality of nations should be blamed for the international asset market

• other sources on the inequality are unimportant
Endogenous inequality does NOT mean that exogenous inequality is not important. Instead it suggests that

- a small amount of exogenous inequality can be magnified to generate huge inequality

- possible endogeneity of observed exogenous heterogeneities that are treated as exogenous in the growth literature