Insights from an Estimated Search-Based Monetary Model with Nominal Rigidities

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Abstract

We are developing a search-based monetary dynamic equilibrium model with nominal rigidities. We use Bayesian methods to estimate the model based on quarterly U.S. data and compare it to a money-in-the-utility (MIU) specification. We also compare steady state welfare implications of the estimated search-based and the MIU model.

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and a monetary policy represented by interest rate feedback rules are emerging as the workhorse of applied monetary policy analysis in many central banks. Much of the empirical work with DSGE models, e.g. Smets and Wouters (2003), Del Negro, Schorfheide, Smets, and Wouters (2007), Levin, Onatski, Williams, and Williams (2005), as well as the theoretical work summarized in Woodford (2003) is based on models in which real money balances directly enter the households’ utility function. Such money-in-the-utility-function (MIU) specifications are informally motivated by the insight that money balances reduce transaction costs and therefore increase utility. Once monetary policy is represented by an interest-rate-feedback rule and real money balances enter the utility function in an additively separable fashion, the model becomes block triangular and aggregate outcomes are not affected by the money stock. In fact, it has become common practice to consider cash-less models, which are obtained by letting the weight on real money balances in the utility function converge to zero. Econometric work typically excludes a measure of the money stock from the list of observables and ignores the model implied money demand equation. While the cashless approach appears reasonable if the estimated model is used to study the propagation of structural shocks other than money demand shocks, it is not innocuous for welfare analysis. To the extent that real money balances indeed affect households’ utility, they are relevant for assessing the welfare consequences of changes in monetary policy.

The contribution of our paper is threefold. First, as an alternative to the commonly used MIU model, we develop an estimable DSGE model where money demand arises due to the micro-foundations laid out in the search-based monetary theory stemming from the work of Kiyotaki and Wright (1989). In our model, following the basic structure of in Lagos and Wright (2005, henceforth LW) and Aruoba, Waller, and Wright (2007, henceforth AWW), in every period economic activity takes place in two markets. In a decentralized market (DM), households engage in bilateral trade with a fraction of households producing and a fraction of households consuming. The centralized market (CM) resembles a standard DSGE model with nominal rigidities where production is carried out by firms. Physical capital is a factor of production in both markets. Demand for money arises because the transactions in the decentralized markets are facilitated by a medium of exchange. Our specification adds nominal rigidities in the centralized market, represents monetary policy by an interest rate feedback rule, and introduces stochastic disturbances to technology, preferences, government spending, and monetary policy to make the model amenable to econometric estimation methods. While the structure of our model to a large extent resembles that of a canonical New Keynesian model with capital, the presence of the decentralized market provides a micro-founded motive for holding money and creates a non-separability between consumption and the value of real money balances. Hence, our model differs from an MIU specification both in terms of the resulting money demand equation as well as its welfare implications.
Second, using post 1984 U.S. data on output, inflation, interest rates, and the money stock we use Bayesian techniques surveyed in An and Schorfheide (2007) to estimate our search-based DSGE model. We also fit a standard MIU model with nominal rigidities to the same set of observations. While most of the work on search-based monetary model has been theoretical, our analysis produces formal estimates of the taste and technology parameters that determine the exchange in the decentralized market. We compare the fit of the money demand equations obtained from the two estimated models. We also discuss dynamics by looking at variance decompositions and impulse-response functions.

Finally, we compare the effects of changes in the central bank’s target inflation rate on steady state welfare using the two estimated DSGE models. Our choice of estimation objective function requires the models to fit both the post 1984 average velocity in U.S. data as well as the fluctuations in M2. Since our estimates imply that the households place very little weight on real money balances in the MIU model, its welfare implications essentially resemble that of a cashless New Keynesian model. Since it is costly for firms to choose prices optimally and suboptimal prices lead to a distortion, a target inflation rate near zero maximizes steady state welfare. The welfare implications of the search-based model are driven by two opposing forces: on the one hand, the presence of nominal rigidities makes price changes costly and welfare reducing. On the other hand, the Friedman motive for keeping the nominal rate near zero is present in the model: inflation is a tax on money holdings. If this tax is large, then it depresses economic activity in the decentralized market, which is welfare reducing.

The remainder of the paper is organized as follows. We provide a detailed derivation and discussion of the search-based DSGE model in Section 2. A canonical MIU model with nominal rigidities and capital can be obtained by shutting down the decentralized market in the search-based model and adding a real-money-balance term to the households’ utility function. This MIU model is described in Section 3. The Bayesian estimation results are presented in Section 4 and the welfare analysis is summarized in Section 5. Finally, Section 6 concludes. Detailed derivations for the two DSGE models are provided in the Appendix.

2 The Search-Based Model

The model is an extension of the two-sector model developed in LW. In every period, there is economic activity in two markets, which we label the decentralized market (DM) and the centralized market (CM). In the DM, households engage in decentralized bilateral trade with other households with one party producing and the other consuming, while the CM resembles a standard macro model where production is carried out by firms.

We extend the LW model in two dimensions. First, we include physical capital as a factor of production, following AWW. The only deviation we have from AWW in this regard is that we introduce an adjustment
cost for investment to improve the empirical fit. Second, we replace the neoclassical structure on the firm side with a New Keynesian one. Intermediate goods producing firms sell their differentiated output to final good producers. The intermediate good producers face a downward sloping demand curve for their product and choose prices to maximize their profits. However, in any period only a fraction of these firms is able to re-optimize their prices. The remaining firms either adjust their prices by the lagged inflation rate or not at all. This mechanism of generating nominal rigidity is due to Calvo (1983) and widely used in the literature on New Keynesian DSGE models. Unlike in more elaborate empirical version in Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005), we exclude habit formation, wage stickiness, and variable capital utilization from our model specification. In turn we will describe the firms’ problem in the centralized market (Section 2.1), the households’ decision problems in both the centralized and the decentralized market (Section 2.2). We then characterize the behavior of fiscal and monetary policy (Section 2.3) and derive an aggregate resource constraint (Section 2.4). Our model economy is subject to aggregate disturbances, characterized in Section 2.5. A summary of all the equilibrium conditions is provided in the Appendix.

2.1 Firms in the Centralized Market

The setup of the centralized market resembles that of a New Keynesian DSGE model. Production is carried out by two types of firms in the CM: final good producers combine differentiated intermediate goods. Intermediate goods producing firms hire labor and capital services from the households to produce the inputs for the final good producers. To introduce nominal rigidity we follow Calvo (1983) by assuming that only a constant fraction of the intermediate goods producers is able to re-optimize prices.

2.1.1 Final Good Producers

The final good $Y_t$ in the CM is a composite made of a continuum of intermediate goods $Y_t(i)$:

$$Y_t = \left[ \int_0^1 Y_t(i) \frac{1}{\lambda} di \right]^{1+\lambda}.$$  \hspace{1cm} (1)

Note that the elasticity is $(1 + \lambda)/\lambda$. $\lambda = 0$ corresponds to the linear case and $\lambda \to \infty$ corresponds to the Cobb-Douglas case. We will constrain $\lambda \in (0, \infty)$. The final good producers buy the intermediate goods on the market, package them into $Y_t$ units of the composite good, and resell them to consumers. These firms maximize profits in a perfectly competitive environment. Their problem is:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \text{ s.t. } (1)$$  \hspace{1cm} (2)

taking $P_t(i)$ as given. The first-order condition is:

$$P_t(i) = P_t Y_t^{\frac{1}{1+\lambda}} Y_t(i)^{-\frac{1}{1+\lambda}}.$$  \hspace{1cm} (3)
Therefore,
\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{\frac{1-\alpha}{\alpha}} Y_t. \] (4)
Combining this condition with the zero profit condition one obtains an expression for the price of the composite good:
\[ P_t = \left[ \int_0^1 P_t(i)^{-\frac{1}{2}} di \right]^{-\lambda}. \] (5)

### 2.1.2 Intermediate Goods Producers

Intermediate goods producers, indexed by \( i \), use the following technology:
\[ Y_t(i) = \max \left\{ Z_tK_t(i)^\alpha H_t(i)^{1-\alpha} - \mathcal{F}, 0 \right\}, \] (6)
Firm \( i \)'s profit is given by:
\[ \Pi_t(i) = P_t(i)Y_t(i) - P_tW_tH_t(i) - P_tR_t^kK_t(i). \] (7)
All firms take factor prices \( W_t \) and \( R_t^k \), as well as the prices of the other firms and the aggregate price level as given. We distinguish two types of firms: (i) firms are allowed to re-optimize their price \( P_t(i) \) and (ii) firms that are not able to re-optimize their price. Firms that are not allowed to choose \( P_t(i) \) optimally, satisfy the demand for their differentiated good (4) and choose capital and labor inputs to minimize costs. Firms that are able to change their price in an optimal fashion, maximize future expected profits. The profit maximization problem can be solved in two steps. First, given a desired level of output \( Y_t(i) \) we determine the cost-minimizing choice of factor inputs. Second, we determine the profit maximizing price \( P_t(i) \) and quantity \( Y_t(i) \) that satisfies (4).

Cost minimization subject to (6) yields the conditions:
\[ P_tW_t = \mu_t(i)P_t(i)(1-\alpha)Z_tK_t(i)^\alpha H_t(i)^{-\alpha} \] (8)
\[ P_tR_t^k = \mu_t(i)P_t(i)\alpha Z_tK_t(i)^{\alpha-1}H_t(i)^{1-\alpha} \] (9)
where \( \mu_t(i) \) is the Lagrange multiplier associated with (6). In turn, these conditions imply:
\[ K_t(i) = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} H_t(i). \]
If we integrate both sides of the equation with respect to \( di \) and define \( K_t = \int K_t(i) \, di \) and \( H_t = \int H_t(i) \, di \) we obtain a relationship between aggregate labor and capital:
\[ K_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} H_t. \] (10)
Thus, the aggregate capital labor ratio is a linear function of the ratio of factor prices.
Total variable cost \((VC_t)\) is given by
\[
VC_t(i) = \left( W_t + R_k^t \frac{K_t(i)}{H_t(i)} \right) H_t(i) = \left( W_t + R_k^t \frac{K_t(i)}{H_t(i)} \right) Z_t^{-1} \left( \frac{K_t(i)}{H_t(i)} \right)^{-\alpha} Y_t^v(i),
\]
where \(Y_t^v(i) = Z_t K_t(i)^\alpha H_t(i)^{1-\alpha}\) is the “variable” part of output \(Y_t(i)\). The real marginal cost \(MC_t\) is the same for all firms and equal to:
\[
MC_t = \left( W_t + R_k^t \frac{K_t(i)}{H_t(i)} \right) Z_t^{-1} \left( \frac{K_t(i)}{H_t(i)} \right)^{-\alpha} = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} W_t^{1-\alpha} (R_k^t)^\alpha Z_t^{-1}.
\]
Conditional on the optimal choice of factor inputs, nominal profits as a function of output \(Y_t(i)\) and prices \(P_t(i)\) can then be expressed as
\[
\Pi_t(i) = [P_t(i) - P_t MC_t] Y_t(i) - P_t MC_t \mathcal{F}.
\]
conditions can be reduced to the following set of equations:

\[ F(1)_t = (p_t^0) - \frac{1 + \lambda}{\lambda} Y_t + \zeta \beta \left( \pi_t^i \pi_{t+1} - \frac{1 + \lambda}{\lambda} \right) \left[ E_t \left( \frac{p_t^0}{\pi_t^{i+1} P_t^{i+1}} \right)^{-1 + \frac{\lambda}{\lambda}} | F(1)_{t+1} \right] \] (14)

\[ F(2)_t = (p_t^0) - \frac{1 + \lambda}{\lambda} - \frac{1}{Y_t} MC_t + \zeta \beta \left( \pi_t^i \pi_{t+1} - \frac{1 + \lambda}{\lambda} \right) \left[ E_t \left( \frac{p_t^0}{\pi_t^{i+1} P_t^{i+1}} \right)^{-1 + \frac{\lambda}{\lambda}} | F(2)_{t+1} \right] \] (15)

\[ F(1)_t = (1 + \lambda) F(2)_t \] (16)

Here we are considering only the symmetric equilibrium in which all firms that can readjust prices will choose the same \( P_t^0 (i) \). In the above formula, we dropped the \( i \) index and used the definitions \( p_t^0 = P_t^0 / P_t \) and \( \pi_t = P_t / P_{t-1} \). Equations (14) to (16) essentially determine the optimal price \( p_t^0 \) as a function of marginal costs.

### 2.1.3 Aggregate Price Dynamics in the CM

From (5) it follows that:

\[ P_t = \left[ (1 - \zeta) (P_t^0)^{-\frac{1}{\lambda}} + \zeta (\pi_t^{i-1} \pi_{t-1}^{1-i} P_{t-1})^{-\frac{1}{\lambda}} \right]^{-\lambda} \] (17)

Hence,

\[ \pi_t = \left[ (1 - \zeta) (P_t^0)^{-\frac{1}{\lambda}} + \zeta (\pi_t^{i-1} \pi_{t-1}^{1-i} P_{t-1})^{-\frac{1}{\lambda}} \right]^{-\lambda} \] (18)

The system of equations (14) - (16) and (18) links inflation to real marginal costs and output and hence defines a so-called New Keynesian Phillips curve.

### 2.2 Households

There is a continuum of ex-ante identical households in the economy. These households derive utility from their activities in the two markets. A household that consumes \( q_t \) units of consumption good in the DM gets utility \( \chi_t u(q_t) \) while he gets utility \( U(x_t) \) by consuming \( x_t \) units in the CM. The disutility of effort in the DM for a seller and disutility of labor for a worker in the CM is linear:

\[ U_t = U(x) - Ah_t \begin{cases} +\chi_t u(q_t) & \text{if buyer in DM} \\ -e_t & \text{if seller in DM} \end{cases} \] (19)

Instead of using the disutility of effort \( e_t \) in the DM, we express the disutility as a function of output produced by the seller. To see this, we assume the following structure. For a seller, the output \( q_t \) is obtained using the production function \( q_t = Z_t f(e_t, k_t) \) where \( Z_t \) is a technology shock which is common across the two markets.

---

1This assumption, in particular the linearity of disutility of labor in the CM is a critical assumption that prevents a non-degenerate distribution of money holdings.
This production function can be inverted to get $c_t = \xi(q_t, k_t, Z_t)$. Using the linear disutility in effort, we can define $c(q_t, k_t, Z_t) = \xi(q_t, k_t, Z_t)$ as the utility cost of production for the sellers. We have $c_q > 0$, $c_k < 0$, and $cz < 0$.

In a given period, the households participate in the DM followed by the CM. To characterize the household’s behavior in this economy, we start from the problem of the household in the CM, followed by the DM problem.

### 2.2.1 Household Activity in the Centralized Market

The households take as given the aggregate price level in the CM, $P_t$, the nominal interest rate $R_t$, and the factor prices $W_t$ and $R^*_t$. Using $W_t(m_t, k_t, i_{t-1}, b_t, S_t)$ and $V_t(m_t, k_t, i_{t-1}, b_t, S_t)$ to denote the value functions in the CM and DM of period $t$ where $m_t$ is the money balances of the household entering the CM, the CM problem is

$$W_t(m_t, k_t, i_{t-1}, b_t, S_t) = \max_{x_t, h_t, m_{t+1}, i_t, k_{t+1}, b_{t+1}} \{U(x_t) - Ah_t + \beta E_t[V_{t+1}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1})]\}$$

s.t. $P_t x_t + P_t i_t + b_{t+1} + m_{t+1} \leq P_t W_t h_t + P_t R^*_t k_t + \Pi_t + R_{t-1} b_t + m_t - T_t$

$$k_{t+1} = (1 - \delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right]i_t$$

given the laws of motion for the aggregate shocks, $S_t$. Here $A$ is the disutility of one unit of labor, $R_{t-1}$ is the gross nominal return on a government bond purchased in period $t - 1$, $T_t$ is a nominal lump-sum tax and $\Pi_t$ denotes the total profits the household receives from intermediate good producers. (21) shows how capital is accumulated where the adjustment cost function $S(.)$ satisfies properties $S(1) = 0$, $S'(1) = 0$ and $S''(1) > 0$. Using $\Upsilon_t$ to denote the Lagrange multiplier for (21) and after eliminating $h$ using (20), the FOC are

$$x_t : \ U'(x_t) = \frac{A}{W_t} \quad (22)$$

$$m_{t+1} : \ \frac{U'(x_t)}{P_t} = \beta EV_{t+1, m}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \quad (23)$$

$$i_t : \ \frac{U'(x_t)}{P_t} = \Upsilon_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}}S'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta EV_{t+1, i}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \quad (24)$$

$$k_{t+1} : \ \Upsilon_t = \beta EV_{t+1, k}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \quad (25)$$

$$b_{t+1} : \ \frac{U'(x_t)}{P_t} = \beta EV_{t+1, b}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \quad (26)$$

---

2We could index households with $j$, but we will see that the assumption of complete markets implies that the index will drop out of most of these variables. In equilibrium households will make the same choice of consumption, money demand, and investment. So, we drop this index from the outset.
assuming that an interior solution exists. This leads to two key results. First, since the individual state
variables, \((\hat{m}_t, k_t, i_{t-1}, b_t)\) do not appear in (23)-(26), household’s decisions in the CM do not depend on his
state variables. More specifically, for any distribution of assets \((\hat{m}_t, k_t, b_t)\) across agents entering the CM,
the distribution of \((m_{t+1}, k_{t+1}, b_{t+1})\) is degenerate.\(^3\) Second, we have the following envelope conditions,

\[
\begin{align*}
W_{t,m}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{A}{P_t W_t} \\
W_{t,k}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{AR^k_t}{W_t} + (1 - \delta)Y_t \\
W_{t,i}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= Y_t \left( \frac{i_t}{i_{t-1}} \right)^2 S' \left( \frac{i_t}{i_{t-1}} \right) \\
W_{t,b}(\hat{m}_t, k_t, i_{t-1}, b_t, S_t) &= \frac{AR_{t-1}}{W_t}
\end{align*}
\]

which show that \(W_i(.)\) is linear in \(\hat{m}_t\) which will be important in the DM problem below. Finally, the
Lagrange multiplier associated with the households’ nominal budget constraint (20) is \(U'(x_t)/P_t\). Under the
assumption that households have access to a complete set of state-contingent claims we obtain that

\[
\Xi^{\sigma}_t = \frac{U'(x_{t+1})/P_{t+1}}{U'(x_t)/P_t},
\]

which the firms use to discount the future. We need to specify the details of the DM to characterize the
equilibrium next. Specifically we will find \(V_m, V_k, V_i\) and \(V_b\) to obtain the equilibrium conditions.

### 2.2.2 Household Activity in the Decentralized Market

As we said, the centralized market in this model resembles a standard New Keynesian DSGE model. It
is important to recognize that transactions in the CM take place without requiring a medium of exchange.
Unlike a standard monetary model where money demand is generated by constructs such as cash-in-advance,
money-in-the-utility-function or transaction costs, we follow a search-based approach. The DM is critical in
generating the money demand. All trades take place in bilateral meetings. The agents are anonymous in
the DM which means no household would accept an IOU from another household and any trade must be
quid pro quo. Following AWW, at the start of each DM a measure \(\sigma\) of households receive a taste shock that
make them buyers and another \(\sigma\) measure of households become sellers. Alternatively, we can consider the
setup in LW where each household can produce a measure \(\sigma\) of goods out of a measure one of all possible

\(^3\)This result requires a small qualification for bond holdings. There are two parts of the argument that guarantees the
degeneracy. The first part relies on the observation that \((\hat{m}_t, k_t, b_t)\) does not appear in (26). The second part relies on the
strict concavity of \(V(.)\) or, more specifically, the strict monotonicity of \(V_b(.)\) which means the choice of \(b_{t+1}\) is unique. Both
parts of the argument go through for money and capital in our environment, but only the first part goes through for bonds
since \(V_b(.)\) is constant as we show below. This means that in principle there could be multiple values of \(b_{t+1}\) that households
choose, which can create a distribution of bond holdings. Fortunately, such a distribution of bonds holdings is not important
for any of our results because bond-holdings will not affect the DM problem, as we show below.
goods, and they like consuming another \( \sigma \) measure of goods.\(^4\) When two households meet at random, with \( \sigma \) probability there is a single coincidence where one party likes the good the other party can produce but not vice versa. The literature started by Kiyotaki and Wright (1989) and many papers that follow show that a medium of exchange will emerge in an environment where the agents are anonymous and there is a double-coincidence problem such as the one above. In a monetary equilibrium, in such single-coincidence meetings, the party who likes what the other party has (the buyer), uses money to purchase the good from the seller.\(^5\) The possibility to consume in the DM generate a demand for money in this model.

The value of starting the DM for a household whose taste shock has not been realized yet is given by

\[
V_t(m_t, k_t, i_{t-1}, b_t, S_t) = \sigma V^b_t(m_t, k_t, i_{t-1}, b_t, S_t) + \sigma V^s_t(m_t, k_t, i_{t-1}, b_t, S_t) + (1 - 2\sigma)W_t(m_t, k_t, i_{t-1}, b_t, S_t),
\]

where the values of being a buyer and a seller are

\[
V^b_t(m_t, k_t, i_{t-1}, b_t, S_t) = \chi_t u(q^b_{t}) + W_t(m_t - d^b_t, k_t, i_{t-1}, b_t, S_t)
\]

\[
V^s_t(m_t, k_t, i_{t-1}, b_t, S_t) = -c(q^s_{t}, k_t, Z_t) + W_t(m_t + d^s_t, k_t, i_{t-1}, b_t, S_t)
\]

with \( q^b_{t} \) and \( d^b_t \) (\( q^s_{t} \) and \( d^s_t \)) denoting output and money exchanged when buying (selling) which are determined via bilateral bargaining as describe below. We interpret \( \chi_t \) as a money demand shock as it affects the utility from consuming in the DM and money serves as a medium of exchange. Using (27) we have

\[
V_t(m_t, k_t, i_{t-1}, b_t, S_t) = W_t(m_t, k_t, i_{t-1}, b_t, S_t) + \sigma \left[ \chi_t u(q^b_{t}) - \frac{d^b_t A}{P_t W_t} \right] + \sigma \left[ \frac{d^s_t A}{P_t W_t} - c(q^s_{t}, k_t, Z_t) \right].
\]

To solve (23)-(26), we need:

\[
V_{t,m}(m_t, k_t, i_{t-1}, b_t, S_t) = \frac{A}{P_t W_t} + \sigma \left[ \chi_t u'(q^b_{t}) \frac{\partial q^b_{t}}{\partial m_t} - \frac{A}{P_t W_t} \frac{\partial d^b_t}{\partial m_t} \right] + \sigma \left[ \frac{A}{P_t W_t} \frac{\partial d^b_t}{\partial m_t} - c_q(q^s_{t}, k_t, Z_t) \frac{\partial q^s_{t}}{\partial m_t} \right]
\]

\[
V_{t,k}(m_t, k_t, i_{t-1}, b_t, S_t) = \frac{AR^k_t}{W_t} + (1 - \delta)Y_t + \sigma \left[ \chi_t u'(q^b_{t}) \frac{\partial q^b_{t}}{\partial k_t} - \frac{A}{P_t W_t} \frac{\partial d^b_t}{\partial k_t} \right] + \sigma \left[ \frac{A}{P_t W_t} \frac{\partial d^b_t}{\partial k_t} - c_q(q^s_{t}, k_t, Z_t) \frac{\partial q^s_{t}}{\partial k_t} - c_k(q^s_{t}, k_t, Z_t) \right]
\]

\[
V_{t,b}(m_t, k_t, i_{t-1}, b_t, S_t) = W_t_A(m_t, k_t, i_{t-1}, b_t, S_t)
\]

\[
V_{t,b}(m_t, k_t, i_{t-1}, b_t, S_t) = \frac{A}{P_t W_t} R_{t-1}
\]

\(^4\)As AWW argue, the setup with idiosyncratic taste shocks and the setup with search leads to the same mathematical construct which we describe below.

\(^5\)As with any deep model of money, there is a nonmonetary equilibrium in this model which is dominated by the monetary equilibrium in terms of welfare. We focus on the monetary equilibrium.
It remains to specify how the terms of trade \((q,d)\) are determined, so that we can substitute for their derivatives in (32) and (33), for which we turn to the bargaining problem. We drop the time subscripts since everything is period \(t\). Our bargaining problem is
\[
\max_{q,d} \left[ \chi u(q) - \frac{Ad}{PW} \right]^\theta \left[ \frac{Ad}{PW} - c(q,k,Z) \right]^{1-\theta} \quad \text{s.t.} \quad d \leq m^b.
\]
where \(\theta\) is the bargaining power of the buyer, the first term is the buyer’s surplus and the second term is the seller’s surplus.

Using the insights of LW and AWW, in any monetary equilibrium \(d = m^b\), that is the buyer spends all his money in exchange for some \(q\) that the seller produces using his capital and effort. Inserting \(d = m^b\) and taking the FOC with respect to \(q\), we get
\[
\frac{m^b}{P} = \frac{g(q,k^*,\chi,Z)W}{A} \quad (36)
\]
where
\[
g(q,k,\chi,Z) \equiv \frac{\theta c(q,k,Z)\chi u'(q) + (1 - \theta)\chi u(q)c_q(q,k,Z)}{\theta \chi u(q) + (1 - \theta) c_q(q,k,Z)}.
\]
and the quantity produced will be \(q = q(m^b,k^*,\chi,Z)\), where \(q()\) is given by solving (36) for \(q\) as a function of \((m^b,k^*,\chi,Z)\). Turning to the partial derivatives we need, we get
\[
\frac{\partial d}{\partial m^b} = 1, \quad \frac{\partial q}{\partial m^b} = \frac{A}{PW g_q(q,k,\chi,Z)} > 0, \quad \text{and} \quad \frac{\partial q}{\partial k^*} = -\frac{g_k(q,k,\chi,Z)}{g_q(q,k,\chi,Z)} > 0
\]
while the other derivatives in (32) and (33) are 0.

Now reintroducing the time subscripts and inserting these results, (32) and (33) reduce to
\[
V_{t,m}(m_t,k_t,\iota_{t-1},b_t,S_t) = \frac{(1 - \sigma)A}{P_t W_t} + \frac{\sigma A \chi_t u'(q_t)}{P_t W_t g_q(q_t,k_t,\chi_t,Z_t)} \quad (38)
\]
\[
V_{t,k}(m_t,k_t,\iota_{t-1},b_t,S_t) = \frac{A R^k_t}{W_t} + (1 - \delta) Y_t - \sigma \gamma(q_t,k_t,\chi_t,Z_t) \quad (39)
\]
where
\[
\gamma(q,k,\chi,Z) \equiv c_k + c_q \frac{\partial q}{\partial k} = \frac{c_k(q,k,Z)g_q(q,k,\chi,Z) - c_q(q,k,Z)g_k(q,k,\chi,Z)}{g_q(q,k,\chi,Z)} < 0. \quad (40)
\]
is the marginal return of having capital in the DM when the household is a seller. In particular, having more capital will reduce the seller’s cost for a given quantity produced, which is captured by the \(c_k\) term. However, due to the non-competitive nature of DM, having more capital for the seller will also affect the terms of trade by increasing the output produced and this will increase his cost. This second term will be the source of one of the holdup problems we will discuss.
2.2.3 Household’s Optimality Conditions

We obtain the optimality conditions for the household by simply substituting (34), (35), (38) and (39) into the household’s FOC to get the optimality conditions for the household. We also define \( \mu_t \equiv \Upsilon_t/U'(x_t) \). Formally, taking as given \( \{ P_t, R_t, W_t, R^k_t, \Pi_t, T_t \}_t \) and exogenous aggregate states \( \{ Z_t, \chi_t \}_t \), the household solves for \( \{ q_t, x_t, m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mu_t \}_t \) using the following equations:

\[
W_t = \frac{A}{U'(x_t)} \quad (41)
\]

\[
1 = \beta E_t \left[ \frac{U'(x_{t+1})}{U'(x_t)} \frac{R_t}{\pi_{t+1}} \right] \quad (42)
\]

\[
1 = \mu_t \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) + \frac{i_t}{i_{t-1}} S' \left( \frac{i_t}{i_{t-1}} \right) \right] + \beta E_t \left\{ \mu_{t+1} \frac{U'(x_{t+1})}{U'(x_t)} \left( \frac{i_{t+1}}{i_t} \right)^2 S' \left( \frac{i_{t+1}}{i_t} \right) \right\} \quad (43)
\]

\[
k_{t+1} = (1 - \delta)k_t + \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right] i_t \quad (44)
\]

\[
\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[ R^k_{t+1} + (1 - \delta)\mu_{t+1} \right] - \frac{\sigma}{U'(x_t)} \gamma(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1}) \right\} \quad (45)
\]

\[
m_t = \frac{g(q_t, k_t, \chi_t, Z_t)W_t}{A} \quad (46)
\]

\[
1 = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \pi_{t+1} \left[ \frac{\sigma \chi_{t+1} u'(q_{t+1})}{g_q(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1})} + (1 - \sigma) \right] \right\} \quad (47)
\]

where we used \( \pi_{t+1} \equiv P_{t+1}/P_t \). Equations (41) to (44) resemble the optimality conditions that arise in a standard DSGE model with capital. (41) is a labor supply equation that relates the wage to the marginal rate of substitution between consumption and labor, (42) is the Euler equation for Bond holdings. (43) describes the evolution of the shadow price of installed capital, \( \mu_t \), and (44) is the capital accumulation equation. Equations (45), (46) and (47) reflect the presence of the decentralized market. (45) is the Euler equation for capital stock holdings. The return to capital has two components, namely the return from renting capital to intermediate good producing firms in the centralized market, \( R^k_t \), net of capital depreciation, and the return to capital when producing in the decentralized market which we discussed above. (46) defines the output produced in the DM. Finally, (47) is the Euler equation for holding money where the term in square brackets reflects the additional consumption provided in the DM by holding money. Note that combining (41), (46) and (47) we obtain the following equation that define money demand in this environment.

\[
\frac{m_{t+1}}{P_t} = \frac{\beta}{U'(x_t)} E_t \left\{ g(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1}) \left[ \frac{\sigma \chi_{t+1} u'(q_{t+1})}{g_q(q_{t+1}, k_{t+1}, \chi_{t+1}, Z_{t+1})} + (1 - \sigma) \right] \right\} \quad (48)
\]

The set of equations above determines the path of money balances, given \( m_0 \) which is identical across all households assuming an interior solution. As all households start period \( t \) with the same money balances, \( m_t = M_t \) where \( M_t \) is the aggregate money stock, the buyers in the DM enter the CM with \( \hat{m} = 0 \), the sellers with \( \hat{m} = 2M \) while the remaining \( 1 - 2\sigma \) households carry \( \hat{m} = M \). Looking at (20), this means that
individual labor supply depends on the status of the agent in the previous DM as the money holdings. In particular, we have

\[
h_t = \begin{cases} 
H_t + \frac{(M_t - 0)}{W_t} & \text{buyers} \\
H_t + \frac{(M_t - 2M_t)}{W_t} & \text{sellers} \\
H_t & \text{others}
\end{cases}
\] (49)

where \(H_t\) is aggregate hours which we define below. This shows buyers in the DM work more than others since they have to make up for the money they have spent and sellers work less than others. We only care about total hours \(H_t\) in equilibrium and will not track individual \(h_t\).

### 2.3 Goverment Spending and Fiscal Policy

In period \(t\), the government in this model collects a nominal lump-sum tax \(T_t\), spends \(G_t\) on goods from the centralized market, issues one-period nominal bonds \(B_{t+1}\) that pay \(R_t\) gross interest tomorrow and supplies the money to maintain the interest rate rule. It satisfies the following budget constraint every period

\[
P_t G_t + R_{t-1} B_t + M_t = T_t + B_{t+1} + M_{t+1}.
\] (50)

We assume that government spending \(G_t\) evolves exogenously and will provide further details below.

### 2.4 Aggregate Resource Constraint and National Accounting

We begin by adding the households’ CM budget constraints (remember that there are three types of households as they enter the CM depending on their status in the previous DM) and the government budget constraint to obtain

\[
P_t X_t + P_t I_t + P_t G_t = P_t W_t H_t + P_t R_t^k K_t + \Pi_t.
\] (51)

Now consider firms’ profits in the CM:

\[
\Pi_t = \int P_t(i)Y_t(i) di - P_t W_t \int H_t(i) di - P_t R_t^k \int K_t(i) di
\]

\[
= \int P_t(i)Y_t(i) di - P_t W_t H_t - P_t R_t^k K_t
\]

\[
= P_t Y_t - P_t W_t H_t - P_t R_t^k K_t
\]

where the last equality follows from the zero profit conditions for the final goods producers. Combining the expression for profits with (51) we get

\[
X_t + I_t + G_t = Y_t.
\] (52)
which is the resource constraint in the CM. Since there is no savings in the DM (and goods are perishable), there is a trivial resource constraint that sets consumption equal to output. The relationship between output and the aggregate labor and capital inputs in the CM is given by

\[ \bar{Y}_t = Z_t \int K_t \alpha(i) H_t^{1-\alpha} \, di - \mathcal{F} = Z_t K_t \alpha H_t^{(1-\alpha)} - \mathcal{F}. \]

where \( \bar{Y} \) is the output of the intermediate good producers and the second equality follows from the fact that the optimal capital labor ratio \( K_t(i)/H_t(i) \) only depends on relative factor prices which are common to all firms. The relationship between \( \bar{Y}_t \) and \( Y_t \) is given by

\[ \bar{Y}_t = Y_t \int \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} \, di. \] (53)

using (4) or

\[ Y_tD_t = Z_t K_t \alpha L_t^{(1-\alpha)} - \mathcal{F}. \] (54)

where

\[ D_t \equiv \int \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} \, di \] (55)

measures the extent of price dispersion across firms. Unless \( P_t(i) = P_t \) for all firms, \( D_t \) will be greater than unity, which in turn implies the economy will produce inside its production possibilities frontier. We will refer to this as the price-dispersion distortion in our welfare analysis.

In order to understand the evolution of \( D_t \), we need to determine the distribution of prices \( P_t(i) \) in the CM. A fraction \( 1 - \zeta \) of firms was allowed to re-optimize their prices in period \( t \). For these firms \( P_t(i) = P_{t-1}^{\pi} \). A fraction \( \zeta(1 - \zeta) \) of firms re-set their prices in period \( t - 1 \). Hence, for these firms \( P_t(i) = \pi_{t-1}^{i} \pi_{t-1}^{1-\pi} P_{t-1}^{\pi} \). Overall, we obtain

\[ D_t = (1 - \zeta) \sum_{j=0}^{\infty} \zeta^j \left( \frac{\left( \pi_{t-1} \pi_{t-1-\pi} \cdots \pi_{t-j} \right)^{\pi_{t-1-\pi}^{j(1-\pi)} \pi_{t-1}^{j(1-\pi)}} \pi_{t-j}^{\pi_{t-1}}}{\pi_{t-1} \cdots \pi_{t-j+1}} \right)^{-\frac{1+\lambda}{\lambda}}. \] (56)

We verify in the Appendix that \( D_t \) follows the law of motion:

\[ D_t = \zeta \left[ \left( \frac{\pi_{t-1}}{\pi_t} \right)^{i} \left( \frac{\pi_{t-1-\pi}}{\pi_t} \right)^{(1-\pi)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1 - \zeta) \left[ \frac{P_t^{\pi}}{P_t} \right]^{-\frac{1+\lambda}{\lambda}}. \] (57)

To summarize, aggregate prices and quantities in the CM are given by \( P_t \) and \( Y_t \). As is standard in New-Keynesian DSGE models, if we add an interest rate feedback rule to the system, we will only determine period to period changes in the aggregate price level, that is, inflation \( \pi_t \), but not the level \( P_t \).

Real and nominal output in the DM are given by \( \sigma q_t \) and \( \sigma M_t \), respectively. Hence, we can define the price level in the DM as

\[ P_t^{DM} = M_t/q_t. \] (58)
Total nominal output in our model economy is given by

$$Y_t^{(n)} = Y_t P_t + \sigma M_t.$$  \hspace{1cm} (59)

Using the final good produced in the CM as numeraire, we can express real output as

$$Y_t = Y_t + \sigma M_t/P_t = Y_t + \sigma M_t/\pi_t,$$ \hspace{1cm} (60)

where $M_t = M_t/P_{t-1}$ are real money balances, in terms of the CM output.

To take the model to the data we will now construct a GDP deflator and a measure of real output that is consistent with this GDP deflator. Following NIPA conventions, we use a Fisher price index. However, to simplify the analysis we replace time-varying nominal shares by steady state shares. The DM share of nominal output in the steady state is

$$s^* = \frac{\sigma M*}{Y* \pi* + \sigma M*}.$$  \hspace{1cm} (61)

Define $\pi^{DM}_t = P^{DM}_t/P^{DM}_{t-1}$ and let

$$\pi^{GDP}_t = \ln \frac{P^{GDP}_t}{P^{GDP}_{t-1}} = (1 - s^*) \ln \pi_t + s^* \ln \pi^{DM}_t.$$ \hspace{1cm} (62)

Thus,

$$P^{GDP}_t = P^{GDP}_0 \prod_{\tau=1}^{t} \pi_t^{1-s^*} (\pi^{DM}_\tau)^{s^*}.$$ \hspace{1cm} (63)

We now define real GDP as

$$Y^{GDP}_t = \frac{Y_t^{(n)}}{P^{GDP}_t} = Y_t \frac{P_t}{P^{GDP}_t}.$$ \hspace{1cm} (64)

It can be verified that up to a first-order approximation changes in real GDP evolve according to a Fisher quantity index with fixed (steady state) weights. Let $X^*$ denote the steady state of a variable $X_t$ and $\tilde{X}_t = \ln X_t/X^*$. Log-linearizing and differencing our expression for real output in terms of the CM good yields

$$\Delta \tilde{Y}_t = (1 - s^*) \Delta \tilde{Y}_t + s^* [\Delta \tilde{M}_t - \Delta \tilde{\pi}_t].$$

Here $\Delta$ denotes the temporal difference operator. According to the definition of prices in the DM

$$\tilde{\pi}^{DM}_t = \Delta \tilde{M}_t - \Delta \tilde{q}_t.$$  \hspace{1cm} (65)

Combining the two previous equations leads to:

$$\Delta \tilde{Y}_t = (1 - s^*) \Delta \tilde{Y}_t + s^* [\Delta \tilde{q}_t + \tilde{\pi}^{DM}_t - \tilde{\pi}_t].$$

Thus,

$$\Delta \tilde{Y}^{GDP}_t = \Delta \tilde{Y}_t + \tilde{\pi}_t - (1 - s^*) \tilde{\pi}_t - s^* \tilde{\pi}^{DM}_t = (1 - s^*) \Delta \tilde{Y}_t + s^* \Delta \tilde{q}_t.$$ \hspace{1cm} (65)
Hence, the level of GDP in period $t$ is given by

$$\tilde{Y}^{GDP}_t = (1 - s_*) \tilde{Y}_t + s_* \tilde{q}_t + [\tilde{Y}^{GDP}_0 - (1 - s_*) \tilde{Y}_0 - s_* \tilde{q}_0].$$

Under the normalizations $P^{GDP}_0 = 1$ and $P_0 = 1$ we obtain

$$\tilde{Y}^{GDP}_0 = (1 - s_*) \tilde{Y}_0 + s_* (M_0 - \pi_0).$$

We can therefore further simplify our expression for GDP to

$$\tilde{Y}^{GDP}_t = (1 - s_*) \tilde{Y}_t + s_* \tilde{q}_t + s_* (\tilde{M}_0 - \tilde{\pi}_0 - \tilde{q}_0).$$  (66)

2.5 Monetary Policy

Monetary policy is represented by an interest-rate feedback rule

$$\frac{R_t}{R^*_s} = \left( \frac{R_{t-1}}{R^*_s} \right)^{\rho_R} \left[ \left( \frac{\pi^{GDP}_t}{\pi^*_s} \right)^{\psi_1} \left( \frac{\tilde{Y}_t}{\tilde{Y}^*_s} \right)^{\psi_2} \right]^{1 - \rho_R} \exp (\sigma_R \varepsilon^R_t),$$  (67)

where $R_s$ is the gross steady state nominal interest rate, $\tilde{Y}_s$ is the steady state of real GDP (in terms of the CM good), and $\pi^*_s$ is the steady state (or target) inflation. It can be verified that steady state DM, CM, and GDP inflation are equal. Finally, $\varepsilon_t^R$ is a monetary policy shock.

2.6 Aggregate Shocks

Government expenditures as a fraction of real GDP (both the DM and CM output, appropriately aggregated) $\tilde{Y}_t$, denoted by $g_t$ are assumed to be exogenous:

We consider four aggregate disturbances in our model economy. $Z_t$ is the random productivity term that effects production in both markets and $g_t$ is a shock that shifts government spending according to

$$G_t = (1 - 1/g_t) \tilde{Y}_t.$$  (68)

We assume that although government consumption goods are purchased in the centralized market, the overall amount is a stochastic fraction of total GDP. The shock $\varepsilon^R_t$ captures unanticipated deviations from the systematic part of the monetary policy rule. Finally, we have a money demand shock, $\chi_t$, which we model as a taste shock in the DM. We define

$$\tilde{Z}_t = \ln (Z_t/Z^*_s), \quad \tilde{\chi}_t = \ln (\chi_t/\chi^*_s), \quad \tilde{g}_t = \ln (g_t/g^*_s),$$

where $Z_s, \chi_s$ and $g_s$ are steady state values / means of the respective random variable. We assume that the exogenous disturbances evolve according to AR(1) processes:

$$\tilde{Z}_t = \rho_z \tilde{Z}_{t-1} + \sigma_z \varepsilon^z_t, \quad \tilde{\chi}_t = \rho_{\chi} \tilde{\chi}_{t-1} + \sigma_{\chi} \varepsilon^\chi_t, \quad \tilde{g}_t = \rho_g \tilde{g}_{t-1} + \sigma_g \varepsilon^g_t.$$
and we collect all innovations in \( \varepsilon = [\varepsilon^+_t, \varepsilon^X_t, \varepsilon^q_t, \varepsilon^r_t] \) which follows a multi-variate standard normal distribution.

The law of motion for the exogenous processes completes the specification of the search-based DSGE model. The equilibrium conditions are summarized in the Appendix. We derive the deterministic steady state for this model and use a log-linear approximation to its dynamics to form a state-space representation that is used for the Bayesian estimation.

\section{A Money-in-the-Utility-Function Model}

The specification of the MIU model closely resembles search-theoretic model described in the previous section. The production side of the MIU economy is identical to the production sector in the centralized market. Moreover, fiscal and monetary policy are identical as well and the economy is subject to the same set of stochastic shocks. We only discuss the modifications to the household’s problem.

Since there is no decentralized market, households’ consumption is restricted to \( x_t \). The instantaneous utility function is of the form

\[
U_t = U(x_t) - Ah_t + \frac{\chi_t}{1 - \nu_m} \left( \frac{m_t}{P_t} \frac{A}{Z_1^{1/1-\alpha}} \right)^{1-\nu_m},
\]

The third term on the right-hand-side of (69) captures the value of holding real money balances. The scaling by \( A/Z_1^{1/(1-\alpha)} \) can be interpreted as a re-parameterization of \( \chi_t \), which has the effect that steady state velocity stays constant as we change \( A \) and \( Z \). Here \( m_t \) are the (pre-determined) money holdings at the beginning of the period, and \( P_t \) is the price at which the final good is sold in period \( t \). Using again \( W_t(m_t, k_t, b_t, S_t) \) to denote the value function of the household in the centralized market, the household’s problem is given by

\[
W_t(m_t, k_t, i_{t-1}, b_t, S_t) = \max_{x_t, h_t, m_{t+1}, k_{t+1}, i_t, b_{t+1}} \left\{ U(x_t) - Ah_t + \frac{\chi_t}{1 - \nu_m} \left( \frac{m_t}{P_t} \frac{A}{Z_1^{1/1-\alpha}} \right)^{1-\nu_m} + \beta EW_{t+1}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, S_{t+1}) \right\}
\]

s.t. \( P_t x_t + P_t i_t + b_{t+1} + m_{t+1} \leq P_t W_t h_t + P_t R^k_t k_t + \Pi_t + R_{t-1} b_t + \bar{m}_t - T_t \)

\[
k_{t+1} = (1 - \delta) k_t + \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right] i_t
\]

To a large extent, the optimality conditions for the households resemble the ones derived for the centralized market in the search-based model. In fact, the labor supply function, the Euler equation for Bond holdings, the evolution of the shadow price of installed capital, and the capital accumulation equation are identical to Equations (41) to (44). The Euler equation for capital stock holdings is given by

\[
\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[ R^k_{t+1} + (1 - \delta) \mu_{t+1} \right] \right\},
\]
which is identical to (45) except that the term related to the DM does not appear. Similarly, the Euler equation for money in (46) is replaced by

\[
U_t'(x_t) = \beta E_t \left[ \frac{U_t'(x_{t+1})}{P_{t+1}} + \frac{\chi_{t+1}}{P_{t+1}} \left( \frac{A}{Z_{t+1}^{1/1-\alpha}} \right)^{1-\nu_m} \left( \frac{m_{t+1}}{P_{t+1}} \right)^{-\nu_m} \right]
\]

which implies the money demand equation

\[
\left( \frac{m_{t+1}}{P_t} \right)^{\nu_m} = \frac{\beta R_t}{U_t'(x_t)(R_t - 1)} E_t \left[ \left( \frac{A}{Z_{t+1}^{1/1-\alpha}} \right)^{1-\nu_m} \frac{\chi_{t+1}}{\pi_{t+1}^{1-\nu_m}} \right].
\]

Equations (72) and (74) replace the optimality conditions (45) and (47) in the search-based model. Notice that \(m_{t+1}\) has been chosen in period \(t\) based on the realization of time \(t\) shocks. Hence, we detrend it by \(P_t\) and define \(M_{t+1} = m_{t+1}/P_t\) with the understanding that \(M_{t+1}\) only depends on realizations of shocks dated \(t\) and earlier. Since \(M_{t+1}\) does neither enter in the firms’ problems nor is it included in the interest-rate feedback rule of the central bank, the model has a block-diagonal structure: the determination of output, inflation, and interest rates does not depend on the money stock. Since our MIU model is a one-sector model without a decentralized market, aggregate output and prices are simply \(Y_t = Y_t\) and \(P_t = P_t\).

The equilibrium conditions for the MIU model are summarized in the Appendix. As we did for the search-based model, we derive the deterministic steady state for the MIU model and use a log-linear approximation to its dynamics to form a state-space representation that is used for the Bayesian estimation.

### 4 Empirical Analysis

We will now turn to the estimation of the search-based and the MIU model. We use a Bayesian approach discussed in detail in An and Schorfheide (2007). We begin by describing the data set (Section 4.1). We then proceed by specifying the functional forms we use (Section 4.2) and the specification of the prior distributions used for the parameters of the two DSGE models (Section 4.3). Finally, we present the parameter estimates as well as implied steady states, (Section 4.4) and discuss dynamics via variance decompositions, and impulse response dynamics (Section 4.5).

#### 4.1 Data

Our empirical analysis focuses on quarterly U.S. postwar data on aggregate output, inflation, interest rates, and (inverse) velocity of money.\(^6\) Unless otherwise noted, the data are obtained from the FRED2 database.

\(^6\)Instead of using real money balances directly, we use velocity, i.e. nominal output divided by nominal money balances as one of our observables.
maintained by the Federal Reserve Bank of St. Louis. Our measure of per capita output is defined as real GDP (GDPC96) divided by civilian noninstitutional population (CNP16OV). The population series is provided at a monthly frequency and converted to quarterly frequency by simple averaging. Since the quarterly flow statistics reported in the National Income and Product Accounts are annualized, we divide real GDP by 4. Inflation is defined as the log difference of the GDP deflator (GDPDEF) and multiplied by 400 to obtain annualized percentages. Our measure of nominal interest rates corresponds to the Federal Funds Rate (FEDFUNDS). The Fed Funds Rate is also provided at monthly frequency and we use simple averaging to convert it to quarterly frequency. As a measure of money we use the sweep-adjusted M2S series provided by Cynamon, Dutkowsky and Jones (2006). This series is provided at monthly frequency without seasonal adjustment. We apply the EVIEWS default version of the X12 filter to perform the seasonal adjustment and then use the observation for the last month of every quarter. Finally, we divide our M2 series by population and the GDP deflator to obtain a measure of per capita real money balances.

The models presented in Sections 2 and 3 are specified to capture stationary fluctuations around a deterministic steady state. Hence, we take the natural log of per capita output and extract a deterministic trend by an OLS regression over the sample period 1959:I to 2006:IV. We scale the deviations from the linear trend by 100 to convert them into percentages. At this point, we have removed information about the average velocity in the postwar data, which we need to pin down some of the parameters in our DSGE model. We therefore compute the sample average of the ratio of money and nominal output, take the natural logarithm, scale it by 100 and add it to our detrended log money series. For the subsequent estimation we restrict the sample period to 1984:I to 2005:IV.

4.2 Functional Forms

We use the following functional forms in our estimation

\[ u(q) = \ln(q + \kappa) - \ln(\kappa), \quad U(x) = B \ln(x), \quad f(e, k) = e^{1-\alpha}k^\alpha \]

where \(\kappa > 0\) is a small constant to make sure \(q_t = 0\) can be handled\(^8\) and \(B\) determines the relative weight of the utility from consuming the CM and DM goods. We use a natural logarithm for both utility functions and use the same Cobb-Douglas production function as the function used by the intermediate good producers in

\(^7\)Our choice of monetary aggregate in the estimation is one of necessity more than a choice. We believe that the ideal measure of money supply for our search-based model is M1. In a nutshell, while not explicitly present in our model, checking accounts can be introduced in our model. However, in our estimation period M1 velocity is displays large swings which are presumably due to factors such as changes in payment technologies which we do not model. M2 velocity, on the other hand, is much more stable.

\(^8\)We use \(\kappa = 0.0001\) in our empirical implementation.
the CM as these are necessary conditions for balanced-growth in this model. Also, using the derivations in Section 2.2, we get
\[
c(q, k, Z) = \frac{1}{Z^{1/(1-\alpha)}} q^{1/(1-\alpha)} k^{-\alpha/(1-\alpha)},
\]

### 4.3 Restricted Parameters and Prior Distributions

The goal of our empirical analysis is to compare the propagation of shocks and the steady state welfare implications of the search-based DSGE model and the MIU model. Hence, it is desirable to restrict a subset of the model parameters prior to estimation. These restrictions, which apply to both models, are reported in Table 1. We fix \( \pi^* \) at the average inflation rate in our sample. Moreover, we let \( r_A \) be equal to the difference of the average Federal Funds Rate and the average inflation rate between 1984 and 2005 and let \( \beta = 1/(1 + r_A/400) \). Using these parameter values for both DSGE models implies that the steady state inflation and nominal interest rates are equal to the post-1984 sample averages. We fix the depreciation rate \( \delta \) at 0.014. This value is obtained as the average ratio of fixed asset depreciation and the stock of fixed assets between 1959 and 2005. We set \( g^* = 1.2 \), which is obtained from the average ratio of government consumption plus investment and GDP. The deviations of output from a linear trend in our post-1984 sample are highly persistent. To capture this persistence we let \( \rho_z = 0.98 \), a value that is consistent with the stationarity assumption embodied in our theoretical models as well as the observed persistence in the data.

We also impose that the estimated models agree on the conduct of post-1984 monetary policy. To this end, we conduct a preliminary estimation of the MIU model with capital without the money series. The mechanics of this estimation are identical to those of the subsequent estimation of the full model. This preliminary analysis yields \( \hat{\psi}_1 = 1.82, \hat{\psi}_2 = 0.18, \) and \( \hat{\rho}_R = 0.78 \). We fix the policy rule coefficients at these estimates for the subsequent analysis with both models. Moreover, we let \( \mathcal{F} = 0 \) (no fixed costs) and \( \pi^{**} = 1 \), meaning that there is no static indexation for the firms who cannot change their prices.

In order to have a fair comparison between the two models, especially in terms of welfare, we normalize three parameters – two arbitrarily at unity and one at the value given by the data. We normalize steady state real GDP in both models to be unity and we normalize the mean of the money demand shocks in both models to be unity. Average labor productivity in our sample is 0.03 and we use this value in both our models for \( H^*/Y^* \). Given the two restrictions that we impose on labor productivity, real GDP and the fact

---

9We use NIPA-FAT11 (current cost net stock) and NIPA-FAT13 (current cost depreciation) for fixed assets and consumer durables.

10Ex ante, we do not want the parameters for the policy rule to be different across the two models as we want to be able to compare them, holding monetary policy fixed. It turns out when we free policy rule parameters to be estimated in both models, the parameter estimates we get are very similar.
that we use inverse velocity in our estimation means we endogenously determine the values of our model parameters $A$, $B$ and $Z_*$.

The marginal prior distributions for the remaining parameters of the MIU model are summarized in Table 2. The prior for inverse velocity is based on pre-sample information. Similarly, the prior for $\alpha$ is chosen so that the implied prior for the labor share is consistent with pre-sample evidence. We use a uniform prior on the indexation parameter $\iota$ and our prior for $\zeta$ is broadly consistent with micro-evidence on the frequency of price changes. The prior distributions for $\rho_g$ and $\rho_\chi$ reflect the belief that the government spending (demand) disturbance and the money demand shock are fairly persistent. The remaining priors were loosely chosen such that the implied distribution of the variability of the endogenous variables is broadly in line with the pre-sample variability of the observed series. We assume that all the parameters listed in Table 2 are a priori independent. Since we fix the policy rule parameters at values that are far away from the boundary of the determinacy region, no further adjustment of the prior is needed.

Table 3 provides information on the prior distribution for the parameters of the search model. The priors for parameters common to both models are identical except for $\lambda$. There are two additional parameters in the search model, probability of single-coincidence meeting $\sigma$ and the bargaining power of the buyer $\theta$.\footnote{Since according to our model $\sigma \in [0,1/2]$ we introduce the transformed parameter $\tilde{\sigma}$, which lives on the unit interval.} While it is difficult to fully disentangle the effect of these parameters, it is instructive to take a look at the share of the decentralized market:

$$sh = \sigma \frac{1}{\pi_s} \frac{M_s}{Y_s}.$$  

Since $\sigma$ relates velocity to the share of the decentralized market, a prior for $\sigma$ is linked to prior beliefs about the DM share. The parameter $\theta$ affects the bargaining power of the seller in the decentralized market and hence the markup and we use a uniform prior over the full range $(0,1]$.

In addition to the normalizations we explained above, we want the two models to be similar in terms of two more steady state implications: investment-output ratio (or equivalently capital-output ratio) and average economy-wide markup. To achieve this we follow the approach proposed in Del Negro and Schorfheide (2007). Specifically, we combine the marginal prior distributions reported in Table 3 and denoted as $\tilde{p}(\vartheta)$ with a function $f(\vartheta)$ that reflects beliefs about the investment output ratio and the economy-wide markup. Here the vector $\vartheta$ stacks the parameters of the DSGE model. The overall prior is given by

$$p(\vartheta) \propto \tilde{p}(\vartheta) \exp \left\{ -\frac{1}{2} \left[ \frac{(I_s(\vartheta)/Y_s(\vartheta) - 0.16)^2}{0.005^2} - \frac{(mu(\vartheta) - 0.15)^2}{0.01^2} \right] \right\} I\{\vartheta \in \Theta_D\},$$

where $\propto$ signifies proportionality, $I_s$ and $Y_s$ denote the steady states of investment and output (as a function of $\vartheta$), $mu(\vartheta)$ is the economy-wide mark-up, and $I\{\vartheta \in \Theta_D\}$ is an indicator function that is one if $\vartheta$ falls in the region of the parameter space in which the linearized search model has a unique stable rational expectations
The adjustment function down-weights parameter combinations for which the investment output ratio deviates from 0.16 and the economy-wide mark-up deviates from 15%. We use \( p(\vartheta) \) as the prior for both models.

### 4.4 Parameter Estimates and the Implied Steady States

We report prior and posterior means and 90% credible intervals for the parameters of the two models in Tables 4 and 5. The posterior is obtained by combining the prior distribution described in the previous subsection with the likelihood function derived from the state-space representations of the linearized DSGE models. We then use a random-walk Metropolis algorithm to generate draws from the posterior distribution of the parameters. To make inference about steady states, impulse responses, and variance decompositions, we convert the parameter draws into the statistics of interest. Further technical details are described in An and Schorfheide (2007).

The parameters \( \zeta \) and \( \iota \) determine the shape of the Phillips curve. According to the estimates of the MIU model, there is virtually no dynamic indexation, \( \hat{\iota} = 0.05 \) and a moderate degree of nominal rigidity. The estimate \( \hat{\zeta} = 0.759 \) implies that the average duration of prices is around four quarters. Adjustment costs are fairly large, \( \hat{S}'' \), reducing the volatility of the return to capital and dampening its effect on the marginal costs that enter the Phillips curve relationship. Of particular interest is the estimate of the elasticity \( \nu_m \). In log-linear form, the money demand equation for the MIU model can be written as

\[
\tilde{M}_{t+1} = \frac{1}{\nu_m} \tilde{X}_t - \frac{1 - \nu_m}{\nu_m} E_t[\tilde{\pi}_{t+1}] - \frac{1}{\nu_m (\tilde{R}_* - 1)} \tilde{R}_t + E_t[\tilde{\chi}_{t+1}]
\]

(76)

The money demand shock is highly persistent, \( \hat{\rho}_\chi = 0.974 \). We will defer a discussion of the estimated shock standard deviation until we examine the impulse response dynamics and variance decompositions. The remaining parameters remain close their prior means.

From the search model we obtain a somewhat lower posterior mean estimate of the Calvo parameter, \( \hat{\zeta} = 0.695 \), find a higher degree of dynamic indexation, \( \hat{\iota} = 0.268 \), than in the MIU model and obtain a similar estimate of the adjustment cost parameter, \( \hat{S}'' = 11.92 \). It is important to note that the posterior means for \( \zeta \) and \( \iota \) for both models lie outside the 90% credible intervals of the other model. Turning to CM-related parameters, \( \hat{\alpha} \) is estimated to be significantly larger in the search-based model and \( \hat{\lambda} \) is lower than the MIU model. We conjecture that the first result is due to the lower value of investment-output ratio

\footnote{For the MIU model, \( mu(\theta) \) is simply equal to \( \lambda \) while in the search-based model it is a weighted average between \( \lambda \) and the markup in the DM.}

\footnote{Draws from this prior can be generated using the random-walk Metropolis algorithm described in An and Schorfheide (2007). The normalization constant can be computed using Geweke’s (1999) harmonic mean estimator.}
that results in the search-based model at the more conventional $\alpha = 0.28$ due to the holdup problems related to investment. On the other hand, because of the nonzero markup in the DM, the CM markup needs to be smaller than 0.15 in order for the economy-wide markup to be close to our target of 0.15. Finally, the two parameters specific to the search-based model are in line with our priors and independent calibration results in AWW.

The implied posterior distribution of the steady states is reported in Table 6. As we explained in Section 4.3, one of our goals in the estimation of our models was to have them display similar long-run characteristics, which is in line with the post-war experience in the US. The first panel of the table reports the distributions for important steady-state variables for the two models. All of these distributions are centered very close to our targets and they are fairly tight. This will enable us to make welfare analysis using these models which will be comparable across models. The second panel of the table reports the distributions of some other endogenous objects of interest. We see that about a third of economic activity takes place in the DM and the average markup in the DM is about 0.26, roughly three times the estimated markup in the CM. Compared to AWW, we find a smaller markup in the DM, which coincides with a fairly large estimate of $\theta$, and obtain a more sizeable DM share. While the calibration in AWW also tries to match an overall markup of 15% in the economy, it focuses on the long-run elasticity of demand for M1 to choose the remaining DM parameters. We conjecture that our larger estimate of the DM share is due to the fact that the likelihood function forces the model to capture the observed business cycle fluctuations in the stock of M2.

4.5 Dynamics

5 Steady State Welfare Comparisons

[This section is based on parameter estimated from an earlier estimation and is not yet updated but the qualitative results are identical.]

In this section, we consider policy experiments where we change the inflation target of the central bank, $\pi^*$, which is the inflation rate that prevails at the steady state. For now we simply compare steady states, ignoring transitions but at the end of the section we comment on what we expect regarding the transitions. Unless noted otherwise, we fix the parameter values at the posterior mean estimates reported in Section 4 (see Tables 1, 4, and 5).

A priori, we can think about four sources of welfare cost of inflation that are present in the search-based model, two of which are also shared by the MIU model. First, inflation is a tax on money holdings in both models and as inflation rises and hence the net nominal interest rate increases welfare will be reduced. This logic underlies Friedman’s prescription of a zero percent net nominal interest rate which has come to be
known as the Friedman rule. We will label this channel of welfare loss the Friedman channel. This channel will reveal itself as a reduction in real money balances and hence lower utility from the MIU part in the MIU model while we will see a lower $q$ in the search-based model, which will lower utility in the DM. Second, both models display some level of price rigidity given by positive $\zeta$ and the fact that some firms cannot optimally change their prices create a price dispersion. This price dispersion becomes more severe as the steady state inflation level goes away from 0% (in both directions), reducing welfare. We will label this channel the price dispersion channel. The remaining two channels in the search-based model are unique to this model and is explained in detail in AWW. To summarize, the bilateral nature of trade and the fact that the surplus in a meeting is split by the two parties in the DM create two holdup problems: the buyers do not bring in the optimal amount of money (a money demand holdup problem) and the sellers do not bring in the optimal amount of capital (an investment holdup problem). These holdup problems are aggravated as inflation increases as this further reduces the payoffs in the DM by reducing $q$. We will collectively refer to these two sources of welfare loss as the holdup problem channel.

Our estimation results reveal differences between the search-based model and the MIU model in terms of some key parameters that affect welfare. For example, $\chi^*$ is fixed at unity in the search-based model while it is estimated to be $4.52 \times 10^{-10}$ in the MIU model. This makes the MIU model essentially a cashless model and diminishes the importance of the Friedman channel. In fact it is well-known that in the cashless version of such models welfare is maximized at 0% inflation. On the other hand, the two model display somewhat different levels of markup of the monopolistically competitive firms and degree of price stickiness. These will affect the dominance of the price dispersion channel.

Before we turn to the results, a brief discussion about how we compute the welfare loss is in order. In the MIU model, the steady state value up to a constant is given by

$$ W(\pi^*) = U(x^*) - Ah^* + \frac{X^*}{1 - \nu_m} \left( \frac{AM^*_e}{\pi^*_e Z^*_e^{1/(1-\alpha)}} \right)^{1-\nu_m} \quad (77) $$

and we solve for the percentage change required in $x^*$ to make the households indifferent between two economies with different steady state inflation rates. In the search-based model, the reduced-form steady state value up to a constant is given by

$$ V(\pi^*) = \sigma [u(q^*) - c(q^*, k^*, Z^*)] + U(x^*) - Ah^* \quad (78) $$

and we solve for the percentage change required in $x^*$ and consumption in the DM which will be the $q^*$ inside the utility function to make the households indifferent between two economies with different steady state

---

$^{14}$Fixing $\chi^{ast}$ is a normalization in the search-based model as both models have the same implied steady state velocity at the estimated parameters.
inflation rates.\footnote{Note that we will not change the \( q_* \) term inside the cost function as it is a part of production.} Finally, as a technical point, we replace \( \left( 1 - \frac{1}{y_*} \right) y_* \) with simply a constant \( G_* \) obtained from the estimations.

Our main results are provided in Figures 1 and 2. Figure 1 plots the welfare cost of deviating from 0% in both models in terms of consumption on a grid running from the Friedman rule up to 10% inflation.\footnote{As a side remark, note that welfare at the Friedman rule is not defined for the MIU model as unless we put an artificial bound on money holdings, there is no solution to the household's problem. Money is costless to hold and utility is increasing in money balances so the households would like to hold arbitrarily large amounts. This is not the case in the search-based model as households would never want to hold more money than what they need to purchase \( q^* \), the first-best quantity. As a result welfare is well-defined at the Friedman rule. For the figures, we omit the Friedman rule for the MIU model.} We see that the welfare loss for the MIU model has the familiar u-shape which shows that welfare is maximized around 0% and deviations in either direction are costly.\footnote{The maximum is not exactly at 0% as one would expect based on earlier work but it is slightly positive. This is an artifact of having capital in the model, which is an observation shared by other researchers in the area.} The welfare cost of 10% inflation is as high as 12% of consumption. The search-based model tells a very different story. Welfare cost is monotonically increasing, which shows that the optimal inflation is the one implied by the Friedman rule. The welfare cost of 10% is under 6%, which is roughly half of what is implied by the MIU model. We see that the Friedman channel is dominated by the price dispersion channel in the MIU model while the Friedman and holdup problem channels dominate the price dispersion channel in the search-based model.

Figure 2 show some of the steady state allocations and prices in both models as inflation changes, relative to the values at 0% inflation. First, we want to emphasize that there is a crucial difference between the MIU and the search-based model. In the MIU model, the change in welfare due to the Friedman channel is only due to the direct effect of inflation on real money balances and this has no indirect effect. However, in the search-based model, inflation effects \( q \), which might be interpreted in the reduced form as an MIU effect but changes in \( q \) in turn affect all the other allocations and prices in the model. As a result of this, we see that some of the key changes, decreases in \( X, K \) and \( W \) and the increase in \( H \) are more severe in the MIU model. This is why the welfare cost of higher levels of inflation is more pronounced in the MIU model.

To emphasize the finding that at the estimated parameters different channels dominate in the two models, and to establish that both models are in principle able to generate either of the two shapes of welfare loss, we vary the level of price stickiness in both models, holding all other parameters at their estimated levels. The results for a region around 0% inflation are plotted in Figure 3. In the first panel, we start from the estimated \( \zeta \) and reduce it towards zero. Around \( \zeta = 0.5 \) the price dispersion channel loses its dominance and the welfare loss curve becomes monotonically increasing, making Friedman rule the optimal. Similarly, in the second panel we start increasing \( \zeta \) from its estimated value towards unity. Around \( \zeta = 0.9 \) the price dispersion channel overweighs the other two channels and we get the u-shape that shows optimal inflation
is around 0%.

To investigate the contribution of the New Keynesian features of the model, we repeat the exercise in Figure 1 with $\lambda = \zeta = 0$ for both models. Figure 4 plots the results along with the curves in Figure 1 for reference. First, we see that in both models welfare cost is monotonically increasing in the rate of inflation and hence the Friedman rule is optimal. Second, the welfare cost of inflation in the MIU model is about one order of magnitude smaller than that in the search-based model. In the MIU model, the only channel that is in play is the Friedman channel, while in the search model the holdup problem channels are also active. In fact, about 3.5% of consumption steady state welfare loss for 10% annual inflation is consistent with the results of AWW. Turning on the New Keynesian features boosts the welfare cost of 10% by about 1.5% for the search-based model while the increase in the MIU model is over 10%. These results confirm that the Friedman channel is much more important for the search-based model.

As we pointed out, we did not consider transitions in this section. One interpretation of the exercise we conducted is that we compare the welfare of two economies with different inflation rates. If, however, the exercise we want to conduct involves a policy change for a given economy, it is more appropriate to consider the transition path of the economy to the new steady state. Recent results in the literature tell us that these transitions may have quite large welfare implications. For example, AWW reports welfare costs associated with going from a bad steady state (high inflation) to a good one (low inflation) can be as large as half of the steady state gain. Intuitively, the magnitude of the transition depends on the change in the level of capital between the two steady states as resources are temporarily devoted to accumulating this extra capital. Looking at Figure 2, we see that going from 10% inflation to 0% inflation, the level of steady state capital changes by more in the MIU model: capital stock goes up by less than 2% in the search-based model while the change is over 4% in the MIU model. This would mean that welfare loss during the transition will be bigger for the MIU model and the curve for the MIU model in Figure 1 will shift down more so than the curve for the search-based model.

6 Conclusion

As an alternative to the commonly used MIU model, we have developed an estimable DSGE model in which the presence of a decentralized market creates an incentive for households to hold money, because money is needed as a medium of exchange. The model specification is closely tied to the theoretical literature that is developing microfounded models of monetary exchange. In particular, we base our model on recent work by Lagos and Wright (2005), and Aruoba, Waller, and Wright (2007). Using post-1984 U.S. on output, inflation, interest rates, and real money balances, we estimate our search-based DSGE model along with a standard New Keynesian model in which real money balances enter the utility function. We obtain parameter
estimates for the taste and technology parameters that determine the exchange in the decentralized market of the search model. These parameter estimates are potentially useful for the theoretically-oriented literature on microfounded monetary models. We compare the dynamics of the estimated search model and the MIU model. While the decentralized market mechanism of the search model creates a stronger linkage between technology shocks and fluctuations in the stock of money, this linkage comes at a cost in terms of overall fit. Finally, we explore the steady state welfare implications of the two models. The estimated MIU model behaves very much like a New Keynesian DSGE model and a near-zero inflation rate is optimal. According to the search model, which also has embodied some New Keynesian feature, the Friedman motive for keeping the nominal interest rate near zero dominates and negative inflation rates are optimal. This paper is part of a research agenda that tries to link the literatures on microfounded monetary models and estimable New Keynesian DSGE models that are popular at central banks. Many interesting questions are left unanswered and will hopefully be addressed in future research.

References


A The Search-Based Model

We use a slightly more general specification of the utility and production functions in the subsequent exposition:

\[ U(x) = B \frac{x^{1-\gamma}}{1-\gamma}, \quad u(q) = \frac{(q + \kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta}. \]

Moreover, we let \( f(e, k) = e^{\Phi} k^{1-\phi} \).

A.1 Further Details: Intermediate Good Producers

The first-order condition for an intermediate good producing firm is:

\[ F_t = (1 + \lambda) \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \zeta^s \beta^s \mathbb{E}_{t+s} \left( \frac{P^o_t(i) \pi^p_{t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s} \left[ P^o_t(i) \pi^p_{t+s} - (1 + \lambda) P_{t+s} MC_{t+s} \right] \right\} = 0. \]

Define and rewrite

\[ F_t^{(1)} = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \zeta^s \beta^s \mathbb{E}_{t+s} \left( \frac{P^o_t(i) \pi^p_{t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s} \pi^p_{t+s} \right\} \]

\[ = \left( \frac{P^o_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta \beta \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \zeta^s \beta^s \mathbb{E}_{t+s} \left( \frac{P^o_t(i) \pi^p_{t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s} \pi^p_{t+s} \right\} \]

\[ \times \mathbb{E}_t \left[ \left( \frac{P^o_t(i)}{P_{t+1}^o(i)} \right)^{-\frac{1+\lambda}{\lambda}} \mathbb{E}_{t+1} \left( \frac{P^o_t(i) \pi^p_{t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s} \pi^p_{t+s} \right] \]

\[ = \left( \frac{P^o_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta \beta \left( \frac{\pi^p_{t+1(i)}^{1-\xi}}{\pi^p_{t+1(i)}} \right)^{-1/\lambda} \mathbb{E}_t \left[ \left( \frac{P^o_t(i)}{P_{t+1}^o(i)} \right)^{-\frac{1+\lambda}{\lambda}} \mathbb{E}_{t+1} \left( \frac{P^o_t(i) \pi^p_{t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s} \pi^p_{t+s} \right]. \]

Similarly,

\[ F_t^{(2)} = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \zeta^s \beta^s \mathbb{E}_{t+s} \left( \frac{P^o_t(i) \pi^p_{t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s} \pi^p_{t+s} MC_{t+s} \right\} \]

\[ = \left( \frac{P^o_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t \pi^p_{t+1(i)} MC_{t+1} + \zeta \beta \left( \frac{\pi^p_{t+1(i)}^{1-\xi}}{\pi^p_{t+1(i)}} \right)^{-1/\lambda} \mathbb{E}_t \left[ \left( \frac{P^o_t(i)}{P_{t+1}^o(i)} \right)^{-\frac{1+\lambda}{\lambda}} \mathbb{E}_{t+1} \left( \frac{P^o_t(i) \pi^p_{t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s} \pi^p_{t+s} \right]. \]

and the first-order condition becomes

\[ F_t^{(1)} = (1 + \lambda) F_t^{(2)}. \]
A.2 Further Details: Price Dispersion

To capture the evolution of the price distribution we introduced a new variable $D_t$. Its law of motion can be derived as follows:

$$
D_t = (1 - \zeta) \sum_{j=0}^{\infty} \zeta^j \left( \frac{1}{\pi_t \pi_{t-1} \cdots \pi_{t-j}} \frac{P^o_{t-j}}{P_{t-j}} \right)^{-1+\lambda} \\
= (1 - \zeta) \left[ \frac{P^o_t}{P_t} \right]^{-1+\lambda} \\
+ (1 - \zeta) \zeta \left[ \left( \frac{\pi_{t-1}}{\pi_t} \right)^i \left( \frac{\pi^{*-}_t}{\pi_t} \right)^{(1-i)} \frac{P^o_{t-1}}{P_{t-1}} \right]^{-1+\lambda} \\
+ (1 - \zeta) \zeta^2 \left[ \left( \frac{\pi_{t-2}}{\pi_t} \right)^i \left( \frac{\pi^{*-2}_t}{\pi_t \pi_{t-1}} \right)^{(1-i)} \frac{P^o_{t-2}}{P_{t-2}} \right]^{-1+\lambda} \ldots 
$$

Lagging $D_t$ by one period yields

$$
D_{t-1} = (1 - \zeta) \left[ \frac{P^o_{t-1}}{P_{t-1}} \right]^{-1+\lambda} \\
+ (1 - \zeta) \zeta \left[ \left( \frac{\pi_{t-2}}{\pi_{t-1}} \right)^i \left( \frac{\pi^{*-2}_{t-1}}{\pi_{t-1}} \right)^{(1-i)} \frac{P^o_{t-2}}{P_{t-2}} \right]^{-1+\lambda} \\
+ (1 - \zeta) \zeta^2 \left[ \left( \frac{\pi_{t-3}}{\pi_{t-1}} \right)^i \left( \frac{\pi^{*-3}_{t-3}}{\pi_{t-1} \pi_{t-2}} \right)^{(1-i)} \frac{P^o_{t-3}}{P_{t-3}} \right]^{-1+\lambda} \ldots 
$$

Therefore, we obtain the following law of motion for the price dispersion:

$$
D_t = \zeta \left[ \left( \frac{\pi_{t-1}}{\pi_t} \right)^i \left( \frac{\pi^{*-}_t}{\pi_t} \right)^{(1-i)} \right]^{-1+\lambda} D_{t-1} + (1 - \zeta) \left[ \frac{P^o_t}{P_t} \right]^{-1+\lambda}. \quad (83)
$$

A.3 Equilibrium Conditions

We now summarize the equilibrium conditions for the search-based model. The timing is such that all shocks are realized at the beginning of $t$ and $\tilde{S}_t = (Z_t, g_t, \chi_t)$ and $R_t$ are observed. $\tilde{S}_t$ summarizes the exogenous state variables. We define $S_t = (\tilde{S}_t, R_t)$ which will be the aggregate state variables of the household’s problem. In the following definitions, we do not track $h_t$ (individual labor supply) and $B_t$ (the bond supply of the government). We also do not track nominal money balances but instead track $M_t = M_t/P_{t-1}$. Recall that $M_t$ is determined based on $t-1$ information and so is $M_t$. Finally, we use $\pi_t \equiv P_t/P_{t-1}$ and do not track the level of prices.

Given exogenous states $\{ \tilde{S}_t \}_{t=0}^{\infty}$, a monetary equilibrium is defined as allocations $\{ q_t, X_t, H_t, K_t, I_t, \mu_t, Y_t, M_t, Y_t \}_{t=0}^{\infty}$, policy $\{ R_t \}_{t=0}^{\infty}$ and prices $\{ W_t, R^k_t, p_t, \pi_t, D_t \}_{t=0}^{\infty}$ such that:
Household’s Problem: Given exogenous states, policy and prices, \( \{ q_t, X_t, H_t, K_t, I_t, \mu_t, M_t, \Xi_t^{p} \}_{t=0}^{\infty} \), producers make zero profits and leads to satisfy

\[
W_t = \frac{A}{U'(X_t)}
\]

\[1 = \beta E_t \left[ \frac{U''(X_{t+1})}{U'(X_t)} R_t \right]
\]

\[1 = \mu_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} S' \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left\{ \mu_{t+1} \frac{U'(X_{t+1})}{U'(X_t)} \left( \frac{I_{t+1}}{I_t} \right)^2 S' \left( \frac{i_{t+1}}{i_t} \right) \right\}
\]

\[K_{t+1} = (1 - \delta) K_t + \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t
\]

\[\mu_t = \beta E_t \left\{ \frac{U'(X_{t+1})}{U'(X_t)} \left[ R_t^k + (1 - \delta) \mu_{t+1} \right] - \frac{\sigma}{U'(X_t)} \gamma(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1}) \right\}
\]

\[M_t = \frac{g(q_t, K_t, \chi_t, Z_t) W_t \pi_t}{A}
\]

\[U'(X_t) = \beta E_t \left\{ \frac{U'(X_{t+1})}{\pi_{t+1}} \left[ \frac{\sigma \chi_{t+1} u'(q_{t+1})}{g(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1})} + (1 - \sigma) \right] \right\}
\]

\[\Xi_t^{p} = \frac{U'(X_{t+1})}{U'(X_t) \pi_{t+1}}
\]

Intermediate Goods Producing Firms’ Problem: Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

\[K_t = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^p} H_t.
\]

Firms that are allowed to change prices are choosing a relative price \( p_{t+1}^{u}(i) \) (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level \( P_t \) as well as the prices charged by other firms as given, which leads to

\[MC_t = \alpha^{-\alpha} (1 - \alpha)^{-1 - \alpha} W_t^{1 - \alpha} (R_t^k)^{-\alpha} Z_t^{-1}
\]

\[\mathcal{F}_t^{(1)} = (p_t^{u})^{-\frac{1+\lambda}{2}} Y_t + \zeta \beta \left( \pi_t^{(1-\iota)} \right)^{-\frac{1+\lambda}{2}} E_t \left[ \left( \frac{p_t^{u}}{\pi_t^{1+\iota} p_t^{l+1}} \right)^{-\frac{1+\lambda}{2}} \Xi_t^{p} \mathcal{F}_t^{(1)} \right]
\]

\[\mathcal{F}_t^{(2)} = (p_t^{u})^{-\frac{1+\lambda}{2}} Y_t M C_t + \zeta \beta \left( \pi_t^{(1-\iota)} \right)^{-\frac{1+\lambda}{2}} E_t \left[ \left( \frac{p_t^{u}}{\pi_t^{1+\iota} p_t^{l+1}} \right)^{-\frac{1+\lambda}{2}} \Xi_t^{p} \mathcal{F}_t^{(2)} \right]
\]

\[\mathcal{F}_t^{(1)} = (1 + \lambda) \mathcal{F}_t^{(2)}
\]

Final Good Producing Firms’ Problem: Final goods producers take factor prices and output prices as given and choose inputs \( Y_t(i) \) and output \( Y_t \) to maximize profits. Free entry ensures that final good producers make zero profits and leads to

\[\pi_t = \left[ (1 - \zeta) (\pi_t p_t^{u})^{-\frac{1}{2}} + \zeta (\pi_{t-1}^{(1-\iota)})^{-\frac{1}{2}} \right]^{-\lambda}
\]
Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

\[
\frac{R_t}{R^*} = \left( \frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[ \left( \frac{\pi_t^{GDP}}{\pi^*} \right)^{\psi_1} \left( \frac{\psi_2}{\psi_1} \right)^{1-\rho_R} \exp(\sigma_r \varepsilon_t^r) \right] \tag{98}
\]

Aggregate Resource Constraint for CM is given by

\[
Y_t = D_t^{-1} (Z_t K_t^{\alpha} H_t^{1-\alpha} - \mathcal{F}), \tag{99}
\]

where

\[
D_t = \zeta \left[ \left( \frac{\pi_{t-1}}{\pi_t} \right)^{\epsilon} \left( \frac{\pi^*}{\pi_t} \right)^{(1-\epsilon)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1 - \zeta) (p_t^0)^{-\frac{1+\lambda}{\lambda}}. \tag{100}
\]

Market Clearing: The goods market in the CM clears:

\[
X_t + I_t + \left( 1 - \frac{1}{q_t} \right) Y_t = Y_t \tag{101}
\]

GDP and GDP Deflator: Prices and inflation in the DM are given by

\[
\sigma_t P_t = \frac{\sigma_t M_t P_{t-1}}{q_t}, \quad \pi_t^{DM} = \frac{P_t^{DM}}{P_{t-1}^{DM}} = \frac{M_t q_{t-1}}{M_{t-1} q_t} \pi_{t-1}. \tag{102}
\]

According to our (approximate) Fisher index the GDP deflator evolves according to

\[
\pi_t^{GDP} = (\pi_t)^{(1-s_*)} (\pi_t^{DM})^{s_*}. \tag{103}
\]

Real output in terms of the CM good and GDP are

\[
Y_t = Y_t + \frac{\sigma_t M_t}{\pi_t}, \quad Y_t^{GDP} = Y_t P_t / P_t^{GDP}. \tag{104}
\]

Finally, measured real money balances and (inverse) velocity in the data are given by

\[
\frac{M_{t+1}}{P_t^{GDP}} = M_{t+1} P_t / P_t^{GDP}, \quad \frac{M_{t+1}}{P_t^{GDP} Y_t^{GDP}} = \frac{M_{t+1}}{(P_t^{GDP} / P_t) Y_t^{GDP}} = \frac{M_{t+1}}{Y_t}. \tag{105}
\]

A.4 Steady States

For estimation purposes it is useful to parameterize the model in terms of \(Y_s\), \(H_s\), and \(M_s\) and solve the steady state conditions for \(A\), \(B\), and \(Z_s\). Suppose \(q_s\) and \(K_s\) are given then we can solve for the following
steady states recursively:

\[ R_\ast = \pi_\ast / \beta \]
\[ p_\ast^o = \left[ \frac{1}{1 - \zeta} - \zeta \left( \frac{\pi_{**}}{\pi_*} \right)^{-\frac{1}{1+\lambda}} \right]^{-\lambda} \]
\[ D_\ast = \frac{(1 - \zeta)(p_\ast^o)^{-\frac{1+\lambda}{1+\lambda(1-\zeta)}}}{1 - \zeta \left( \frac{\pi_{**}}{\pi_*} \right)^{-\frac{1}{1+\lambda(1-\zeta)}}} \]
\[ Y_\ast = \gamma_\ast - \sigma M_\ast / \pi_* \]
\[ \bar{Y}_\ast = Y_\ast D_\ast \]
\[ Z_\ast = (\bar{Y}_\ast + \mathcal{F}) / (K_\ast H_\ast^{1-\alpha}) \]
\[ R_k^k = \frac{\alpha Z_\ast p_\ast^o}{1 + \lambda} \left[ \frac{\pi_{**}}{\pi_*} \right]^{-\frac{1}{1+\lambda}} \left[ \frac{\pi_{**}}{\pi_*} \right]^{-\frac{1}{1+\lambda(1+\lambda)}} \left( \frac{H_\ast}{K_\ast} \right)^{1-\alpha} \]
\[ W_\ast = \frac{1 - \alpha}{\alpha} \frac{K_\ast}{H_\ast} R_k^k \]
\[ I_\ast = \delta K_\ast \]
\[ X_\ast = Y_\ast - I_\ast - (1 - 1/g_\ast) \gamma_\ast \]
\[ A = g(q_\ast, K_\ast, \chi_\ast, Z_\ast) W_\ast \pi_* \]
\[ U'_\ast = A / W_\ast \]
\[ B = U'_\ast X_\gamma \]
\[ \pi_{GDP}^D = \pi_\ast \]

To determine \( q_\ast \) and \( K_\ast \) we solve the following equations jointly:

\[ R_\ast = 1 + \sigma \left[ \frac{\chi_\ast U'(q_\ast)}{g_\ast(q_\ast, K_\ast, \chi_\ast, Z_\ast)} - 1 \right] \]
\[ 1 = \beta(1 + R_k^k - \delta) - \sigma \beta \gamma(q_\ast, K_\ast, \chi_\ast, Z_\ast) \frac{U'_\ast}{M_\ast} \]

Note that from the firm’s problem we have

\[ \mathcal{F}_\ast^{(1)} = \left( 1 - \zeta \pi_* \left( \frac{\pi_{**}}{\pi_*} \right)^{-\frac{1}{1+\lambda}} \right)^{-1} (p_\ast^o)^{-\frac{1+\lambda}{1+\lambda}} Y_\ast \]
\[ \mathcal{F}_\ast^{(2)} = \left( 1 - \zeta \pi_* \left( \frac{\pi_{**}}{\pi_*} \right)^{-\frac{1}{1+\lambda(1+\lambda)}} \right)^{-1} (p_\ast^o)^{-\frac{1+\lambda}{1+\lambda}} Y_\ast MC_\ast \]
\[ \mathcal{F}_\ast^{(1)} = (1 + \lambda) \mathcal{F}_\ast^{(2)} \]
\[ MC_\ast = \alpha^{-\alpha}(1 - \alpha)^{-\frac{1}{1-\alpha}} W_\ast^{1-\alpha} R_k^k \]
\[ \pi_* = \left[ (1 - \zeta) (\pi_* p_\ast^o)^{-\frac{1}{\lambda}} + \zeta \left( \pi_* \pi_{**}^{1-\epsilon} \right)^{-\frac{1}{\lambda}} \right]^{-\lambda} \]
which lead to the conditions for \( \rho^o \) above. The term \( D_\ast \) measures the steady state price dispersion. The larger \( \pi_\ast/\pi_{**} \), that is, the faster the price of the non-adjusters is eroding in real terms, the bigger \( D_\ast \). Finally, in steady state the DM share of nominal output and the DM markup are given by

\[
\sigma M_\ast = \frac{\sigma M_\ast + Y_\ast \pi_\ast}{\sigma M_\ast + Y_\ast \pi_\ast + Y_\ast \pi_\ast}
\]

\[
\text{markup (dm)} = \frac{g(q_\ast, K_\ast, \chi_\ast, Z_\ast)}{q_c q(q_\ast, K_\ast, \chi_\ast, Z_\ast)} - 1.
\]

### A.5 Log-Linearizations

In the subsequent presentation of the log-linearized equations we adopt the convention that we abbreviate time \( t \) expectations of a \( t + 1 \) variable simply by a time \( t + 1 \) subscript, omitting the expectation operator.

**Firms’s Problem**: Marginal costs evolve according to

\[
\tilde{MC}_t = (1 - \alpha) \tilde{w}_t + \alpha \tilde{R}^k_t - \tilde{Z}_t.. \tag{106}
\]

Conditional on capital and factor prices, the labor demand is determined according to

\[
\tilde{H}_t = \tilde{K}_t + \tilde{R}^k_t - \tilde{W}_t. \tag{107}
\]

Since \( F^{(1)}_t \) and \( F^{(2)}_t \) are proportional, \( \tilde{F}^{(1)}_t = \tilde{F}^{(2)}_t = \tilde{F}_t \). The remaining optimality conditions can be written as follows.

\[
\tilde{F}_t = (1 - A) \left[ -\frac{1 + \lambda}{\lambda} \tilde{p}^{\rho} + \tilde{\gamma}_t \right] \tag{108}
\]

\[
\begin{align*}
A &= \zeta \beta \left( \frac{\pi_{**}}{\pi_\ast} \right)^{-(1-\iota)/\lambda} \\
\tilde{F}_t &= (1 - A) \left[ -\frac{1 + \lambda}{\lambda} \tilde{p}^{\rho} + \tilde{\gamma}_t + \tilde{M}C_t \right] \\
A &= \zeta \beta \left( \frac{\pi_{**}}{\pi_\ast} \right)^{-(1-\iota)(1+\lambda)/\lambda} \tag{109}
\end{align*}
\]

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

\[
\tilde{p}^{\rho}_t = (A - 1) \tilde{\pi}_t - A \zeta \left( \frac{\pi_{**}}{\pi_\ast} \right)^{-(1-\iota)/\lambda} \tilde{\pi}_{t-1} \tag{110}
\]

\[
A = \frac{(p^o_\ast)^{1/\lambda}}{1 - \zeta}
\]
Equations (108) to (110) determine $\tilde{\pi}_t$, $\tilde{F}_t$, and $\tilde{\pi}_t^o$.

**Household’s Problem:** The optimality conditions for the household can be expressed as

$$\tilde{W}_t = \gamma \tilde{X}_t$$  \hspace{1cm} (111)

$$\tilde{X}_t = \tilde{X}_{t+1} - \frac{1}{\gamma} (\tilde{R}_t - \tilde{\pi}_{t+1})$$  \hspace{1cm} (112)

$$\tilde{i}_t = \frac{1}{1+\beta} \tilde{i}_{t-1} + \frac{\beta}{1+\beta} \tilde{i}_{t+1} + \frac{1}{(1+\beta)S\tilde{\eta}} \tilde{p}_t$$  \hspace{1cm} (113)

$$\tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + \delta \tilde{i}_t$$  \hspace{1cm} (114)

$$\tilde{\mu}_t - \gamma \tilde{X}_t = \beta(1 - \delta) \tilde{\mu}_{t+1} - \gamma \beta(1 - \delta + R_k^*) \tilde{X}_{t+1} + \beta R_{st} \tilde{R}_{t+1}^k$$

$$\hspace{1.5cm} + (1 - \beta(1 - \delta + R_k^*)) \tilde{\Gamma}_{t+1}$$

$$\tilde{\pi}_t = \tilde{g}_t + \tilde{W}_t + \tilde{\pi}_t$$  \hspace{1cm} (116)

$$\tilde{R}_t = \frac{R_s - 1 + \sigma}{R_s} \left[ \tilde{X}_{t+1} - \tilde{g}_{q,t+1} - \eta \left( \frac{q}{q_s + \kappa} \tilde{q}_{t+1} \right) \right]$$  \hspace{1cm} (117)

$$\tilde{\Xi}_{t+1} = -\gamma (\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t$$  \hspace{1cm} (118)

Equations (111) to (118) determine wages, CM consumption, investment, capital, the shadow price of installed capital, the rental rate of capital, real money balances, the stochastic discount factor used in the firms’ problem, and DM consumption.

**Decentralized Market:** We now determine the law of motion for $\tilde{g}_{q,t}$, $\tilde{\Gamma}_t$, and $\tilde{g}_t$. In addition, we are introducing some auxiliary variables. We begin with (omitting $t$ subscripts),

$$u = \frac{(q + \kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta}$$

$$u' = (q + \kappa)^{-\eta}$$

$$u'' = -\eta (q + \kappa)^{-\eta-1}$$

$$c = \exp\{-\bar{Z}\} q^{\psi} k^{1-\psi}$$

$$c_q = \psi \exp\{-\bar{Z}\} q^{\psi-1} k^{1-\psi}$$

$$c_k = (1 - \psi) \exp\{-\bar{Z}\} q^{\psi} k^{-\psi}$$

$$c_{qq} = \psi(\psi - 1) \exp\{-\bar{Z}\} q^{\psi-2} k^{1-\psi}$$

$$c_{kk} = \psi(\psi - 1) \exp\{-\bar{Z}\} q^{\psi} k^{-\psi-1}$$

$$c_{qk} = \psi (1 - \psi) \exp\{-\bar{Z}\} q^{\psi-1} k^{-\psi}$$
which can be log-linearized as follows

\[ \ddot{u}_* = \frac{q_*}{(q_* + \kappa)\eta} \dot{q} \]
\[ \dot{u}' = -\eta \frac{q_*}{q_* + \kappa} \dot{q} \]
\[ \ddot{u}'' = -(\eta + 1) \frac{q_*}{q_* + \kappa} \dot{q} \]
\[ \ddot{c} = -\psi \ddot{Z} + \psi \dot{q} + (1 - \psi) \ddot{k} \]
\[ \ddot{c}_q = -\psi \ddot{Z} + (\psi - 1) \dot{q} + (1 - \psi) \ddot{k} \]
\[ \ddot{c}_k = -\psi \ddot{Z} + \psi \dot{q} - (1 + \psi) \ddot{k} \]
\[ \ddot{c}_{qq} = -\psi \ddot{Z} + (\psi - 2) \dot{q} + (1 - \psi) \ddot{k} \]
\[ \ddot{c}_{kk} = -\psi \ddot{Z} + \psi \dot{q} - (1 + \psi) \ddot{k} \]
\[ \ddot{c}_{qk} = -\psi \ddot{Z} + (\psi - 1) \dot{q} - \psi \ddot{k} \]

Recall that

\[ \Gamma_t = \frac{c_{k,t} g_{q,t} - c_{q,t} g_{k,t}}{g_{q,t}} \]

which implies that \( \hat{\Gamma}_t \) evolves according to

\[ \hat{g}_{q,t} + \hat{\Gamma}_t = \frac{c_{k,t} g_{q,t}}{c_{k,t} g_{q,t} - c_{q,t} g_{k,t}} [\hat{c}_{k,t} + \hat{g}_{q,t}] - \frac{c_{q,t} g_{k,t} - c_{k,t} g_{q,t}}{c_{k,t} g_{q,t} - c_{q,t} g_{k,t}} [\hat{c}_{q,t} + \hat{g}_{k,t}]. \] (119)

Now consider the equation

\[ g_t (\theta \chi u' + (1 - \theta) c_{q,t}) = \theta \chi c_t u'_t + (1 - \theta) \chi c_{q,t} u_t, \]

which can be written in log-linear form as

\[ [\theta \chi u'_* + (1 - \theta) c_{q,t}] g_* \dot{g}_t \]
\[ = \theta \chi u'_* (c_* - g_* \dot{u}_t) + (1 - \theta) \chi c_{q,t} u_* \ddot{u}_t + (1 - \theta) c_{q,t} (\chi u_* - g_* \dot{c}_t) \dot{c}_{q,t} \]
\[ + \theta \chi c_* u'_* \ddot{c}_t + [-\theta \chi g_* u'_* + \theta \chi c_* u'_* + (1 - \theta) \chi c_{q,t} u_*] \ddot{\chi}_t \] (120)

and determines \( \ddot{g}_t. \)

Now consider

\[ g_q = \frac{\chi u' c_q [\theta \chi u' + (1 - \theta) c_q] + \theta (1 - \theta) (\chi u - c) (\chi u' c_{qq} - c_q \chi u'')}{[\theta \chi u' + (1 - \theta) c_q]^2} \]
In log-linear form, the equation can be rewritten as

\[
g_q \left[ \theta \chi \sigma' + (1 - \theta) c_q \right]^2 \tilde{g}_{q,t} = 0
\]

Furthermore,

\[
g_k = \frac{\theta \chi \sigma' c_k \left[ \theta \chi \sigma' + (1 - \theta) c_q \right] + \theta (1 - \theta) \chi u - c}{\left[ \theta \chi \sigma' + (1 - \theta) c_q \right]^2},
\]

which leads to an equation for \( \tilde{g}_{k,t} \):

\[
g_k \left[ \theta \chi \sigma' + (1 - \theta) c_q \right]^2 \tilde{g}_{k,t} = 0
\]

To summarize, Equations (119) to (123) determine \( \tilde{F}_t, \tilde{g}_t, \tilde{g}_{q,t}, \) and \( \tilde{g}_{k,t} \). The first three variables appear in the characterization of the households’ problem above.

**Resource Constraint, Market Clearing Conditions in the CM:** Aggregate output across evolves according to

\[
\tilde{Y}_t = \tilde{Y}_t + \tilde{D}_t = (1 + F/\tilde{Y}_s)[\tilde{Z}_t + \alpha \tilde{K}_t + (1 - \alpha)\tilde{H}_t].
\]

and the steady state price dispersion follows

\[
\tilde{D}_t = c \left( \frac{\pi_{ss}}{\pi_s} \right)^{-1} \left[ \tilde{D}_{t-1} + \frac{(1 + \lambda) \tilde{p}_{t-1}}{\lambda} - \frac{\lambda}{\lambda} \tilde{p}_{t-1} \right] - \frac{\lambda}{\lambda} \tilde{p}_{t-1} \]

The goods market clearing condition is of the form

\[
\tilde{Y}_t = X_s \tilde{X}_t + I_s \tilde{I}_t + \left( 1 - \frac{1}{g_t} \right) \tilde{Y}_s \tilde{Y}_t + \tilde{Y}_s \tilde{g}_t \]

and determines investment.

**Aggregate Output and Prices, Measured Real Money Balances**
In log-linear terms, inflation in the DM evolves according to
\begin{equation}
\tilde{\pi}_t^{DM} = \tilde{M}_t - \tilde{M}_{t-1} - (\tilde{q}_t - \tilde{q}_{t-1}) + \tilde{\pi}_{t-1}.
\end{equation}
(127)

Since all inflation rates share the same steady state, changes in the GDP deflator are given by
\begin{equation}
\tilde{\pi}_t^{GDP} = (1 - s_\ast)\tilde{\pi}_t + s_\ast \tilde{\pi}_t^{DM}.
\end{equation}
(128)

Real output in terms of the CM final good evolves according to
\begin{equation}
\tilde{Y}_t = (1 - s_\ast)\tilde{Y}_t + s_\ast (\tilde{M}_t - \tilde{\pi}_t).
\end{equation}
(129)

As we showed in the main text, real GDP can be expressed as
\begin{equation}
\tilde{Y}_t^{GDP} = (1 - s_\ast)\tilde{Y}_t + s_\ast \tilde{q}_t + s_\ast (\tilde{M}_0 - \tilde{\pi}_0 - \tilde{q}_0).
\end{equation}
(130)

Finally, inverse velocity evolves according to
\begin{equation}
\frac{\tilde{M}_{t+1}}{\tilde{Y}_t} = \tilde{M}_{t+1} - \tilde{Y}_t.
\end{equation}
(131)

**Government Policies:** The monetary policy rule can be written as
\begin{equation}
\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R) [\psi_1 \tilde{\pi}_t^{GDP} + \psi_2 \tilde{Y}_t] + \epsilon_{R,t}.
\end{equation}
(132)
B The MIU Model

The subsequent exposition is based on a slightly more general utility function:

\[ U(x) = B \frac{x^{1-\gamma}}{1-\gamma}. \]

B.1 Equilibrium Conditions

**Household’s Problem:** Given exogenous states, policy and prices,

\[ U'(x_t) = \frac{A}{W_t} \]

\[ 1 = \beta E_t \left[ \frac{U'(x_{t+1})}{U'(x_t)} \frac{R_t}{\pi_{t+1}} \right] \]

\[ 1 = \mu_t \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) + \frac{i_t}{i_{t-1}} S' \left( \frac{i_t}{i_{t-1}} \right) \right] + \beta E_t \left\{ \mu_{t+1} \frac{U'(x_{t+1})}{U'(x_t)} \left( \frac{i_{t+1}}{i_t} \right)^2 S' \left( \frac{i_{t+1}}{i_t} \right) \right\} \]

\[ k_{t+1} = (1-\delta)k_t + \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right] \]

\[ \mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left[ R_{t+1} + (1-\delta)\mu_{t+1} \right] \right\} \]

\[ \frac{U'(x_t)}{P_t} = \beta E_t \left[ \frac{U'(x_{t+1})}{P_{t+1}} + \frac{A}{Z_{t+1}^{1-\alpha}} \left( \frac{m_{t+1}}{P_{t+1}} \right)^{-\nu_m} \right] \]

\[ \Xi^p_{t+1|t} = \frac{U'(x_{t+1})}{U'(x_t)\pi_{t+1}} \]

**Intermediate Goods Producing Firms’ Problem:** Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

\[ K_t = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^\alpha} H_t. \]

Firms that are allowed to change prices are choosing a relative price \( p^*_t(i) \) (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level \( P_t \) as well as the prices charged by other firms as given, which leads to

\[ MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R_k^k)^{\alpha}Z_t^{-1} \]

\[ \mathcal{F}^{(1)}_t = (p_t^\alpha)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta \beta \left( \frac{m_{t+1}}{P_{t+1}^{1-\alpha}} \right)^{-1/\lambda} \left( \frac{P_t^\alpha}{P_{t+1}^\alpha} \right)^{-\frac{1+\lambda}{\lambda}} \Xi^p_{t+1|t} \mathcal{F}^{(1)}_{t+1} \]

\[ \mathcal{F}^{(2)}_t = (p_t^\alpha)^{-\frac{1+\lambda}{\lambda}-1} Y_t MC_t + \zeta \beta \left( \frac{m_{t+1}}{P_{t+1}^{1-\alpha}} \right)^{-\frac{1+\lambda}{\lambda}} \Xi^p_{t+1|t} \mathcal{F}^{(2)}_{t+1} \]

\[ \mathcal{F}^{(1)}_t = (1+\lambda) \mathcal{F}^{(2)}_t \]
Final Good Producing Firms’ Problem: Final goods producers take factor prices and output prices as given and choose inputs $Y_t(i)$ and output $Y_t$ to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$\pi_t = \left[ (1 - \zeta) \left( \pi_t \mu_t \right)^{-\frac{1}{\lambda}} + \zeta \left( \pi_t^{1-\alpha} \pi_t^{1-1} \right)^{-\frac{1}{\lambda}} \right]^{-\lambda}$$

(145)

Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$\frac{R_t}{R_s} = \left( \frac{R_{t-1}}{R_s} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi_s} \right)^{\psi_1} \left( \frac{Y_t}{Y_s} \right)^{\psi_2} \right]^{1-\rho_R} \exp(\sigma_r \varepsilon_t^r)$$

(146)

Aggregate Resource Constraint: is given by

$$Y_t = D_t^{-1} (Z_t K_t H_t^{(1-\alpha)} - \mathcal{F})$$

(147)

where

$$D_t = \zeta \left[ \left( \frac{\pi_t - 1}{\pi_t} \right)^{\psi_1} \left( \frac{\pi_s}{\pi_t} \right)^{(1-\psi_1)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1 - \zeta) \left( \mu_t \right)^{-\frac{1+\lambda}{\lambda}}.$$  

(148)

The gross domestic product of this economy is given by

$$\mathcal{Y}_t = Y_t$$

(149)

Market Clearing: The goods market in the CM clears:

$$X_t + I_t + \left( 1 - \frac{1}{g_t} \right) \mathcal{Y}_t = Y_t$$

(150)
B.2 Steady States

For estimation purposes it is useful to parameterize the model in terms of $Y^* = Y_*, H_*,$ and $M_*$ and solve the steady state conditions for $A, B,$ and $Z_*.$

\[
\begin{align*}
R_* &= \pi_*/\beta \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta} \left(\frac{\pi_*}{\pi_+}\right)^{-\frac{1}{1+\lambda}}\right]^{-\lambda} \\
R^k_* &= \frac{1}{\beta} + \delta - 1 \\
D_* &= \frac{(1 - \zeta)(p_*^o)^{-\frac{1}{1+\lambda}}}{\left(1 - \zeta \left(\frac{\pi_*}{\pi_+}\right)^{-\frac{1}{1+\lambda}} - 1\right)} \\
Y_* &= Y_0 D_* \\
Z_* &= (\bar{Y}_* + \mathcal{F})/(K^*_a H_*^{1-a}) \\
K_* &= \frac{\alpha(\bar{Y}_* + \mathcal{F})p_*^o}{(1 + \lambda)R^k_*} \left[\frac{1 - \zeta \beta \left(\frac{\pi_*}{\pi_+}\right)^{(1-a)/(1+\lambda)}}{1 - \zeta \beta \left(\frac{\pi_*}{\pi_+}\right)^{(1-a)/(1+\lambda)}} \right]^{-1} \\
W_* &= \frac{1 - \alpha K_* R^k_*}{\alpha H_*} \\
I_* &= \delta K_* \\
X_* &= Y_* - I_* - (1 - 1/g_*)Y_* \\
A &= \frac{1}{M_*} \left[\frac{\chi_* \pi_*^{\nu_m} W_*}{(R_* - 1) Z_*^{(1-\nu_m)/(1-a)}}\right]^{1/\nu_m} \\
U_0' &= A/W_* \\
B &= U_0' X_0'
\end{align*}
\]

B.3 Log-Linearizations

We will frequently use equation-specific constants, such as $\mathcal{A}$ and $\mathcal{B}.$ Variables dated $t + 1$ refer to time $t$ conditional expectations.

**Firms's Problem:** Marginal costs evolve according to

\[
\hat{MC}_t = (1 - \alpha)\hat{w}_t + \alpha \hat{R}^k_t - \hat{Z}_t. \tag{151}
\]

Conditional on capital, the labor demand is determined according to

\[
\hat{H}_t = \hat{K}_t + \hat{R}^k_t - \hat{W}_t \tag{152}
\]
Since \( \tilde{F}_t^{(1)} \) and \( \tilde{F}_t^{(2)} \) are proportional, \( \tilde{F}_t^{(1)} = \tilde{F}_t^{(2)} = \tilde{F}_t \). The remaining optimality conditions can be written as follows.

\[
\begin{align*}
\tilde{F}_t &= (1 - A) \left[ \frac{1 + \lambda}{\lambda} \tilde{p}_t^0 + \tilde{Y}_t \right] \\
+ A \left[ -\frac{1 + \lambda}{\lambda} \tilde{p}_t^3 - \frac{1 + \lambda}{\lambda} \tilde{p}_t^0 + \frac{1 + \lambda}{\lambda} \tilde{p}_{t+1}^0 + \tilde{F}_{t+1} + \tilde{p}_{t+1}^p \right]
\end{align*}
\]

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

\[
\tilde{p}_t^0 = (A - 1) \tilde{\pi}_t - A \zeta \left( \frac{\pi_{ss}}{\pi_s} \right)^{(1 - \iota)/\lambda} \tilde{p}_{t-1}^0
\]

Equations (153) to (155) determine \( \tilde{\pi}_t, \tilde{F}_t, \) and \( \tilde{p}_t^0 \).

**Household’s Problem** The optimality conditions for the household can be expressed as

\[
\begin{align*}
\tilde{W}_t &= \frac{1}{\gamma} \tilde{X}_t \\
-\gamma \tilde{X}_t &= -\gamma \tilde{X}_{t+1} + (\tilde{R}_t - \tilde{\pi}_{t+1}) \\
\tilde{\pi}_t &= \frac{1}{1 + \beta} \tilde{\pi}_{t-1} + \frac{\beta}{1 + \beta} \tilde{\pi}_{t+1} + \frac{1}{(1 + \beta)S} \tilde{\pi}_t^0 \\
\tilde{K}_{t+1} &= (1 - \delta) \tilde{K}_t + \delta \tilde{\pi}_t \\
\tilde{\pi}_t - \gamma \tilde{X}_t &= \beta(1 - \delta) \tilde{\pi}_{t+1} - \gamma \tilde{X}_{t+1} + \beta \tilde{R}_t^k \tilde{R}_{t+1}^k \\
\nu_m \tilde{M}_{t+1} &= \gamma \tilde{X}_t + \nu_m \tilde{X}_{t+1} - (1 - \nu_m) \tilde{\pi}_{t+1} - \frac{1}{R_s - 1} \tilde{R}_t \\
\tilde{p}_{t+1}^p &= -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t.
\end{align*}
\]

Equations (156) to (162) determine wages, consumption, investment, capital, the shadow value of installed capital, the rental rate of capital, real money balances, and the stochastic discount factor.

**Resource Constraint, Market Clearing Conditions** Aggregate output across evolves according to

\[
\tilde{Y}_t = \tilde{Y}_t + \tilde{D}_t = (1 + \tilde{F}/\tilde{Y}_t) [\tilde{Z}_t + \alpha \tilde{K}_t + (1 - \alpha) \tilde{H}_t].
\]
and the steady state price dispersion follows

$$\tilde{D}_t = \zeta \left( \frac{\pi_{ss}}{\pi_s} \right)^{-\frac{(1+\lambda)(1-\gamma)}{\lambda}} \left[ \tilde{D}_{t-1} + \frac{(1+\lambda)}{\lambda} \tilde{\pi}_t - \frac{\iota(1+\lambda)}{\lambda} \tilde{\pi}_{t-1} \right] - \frac{\rho_o^o(1+\lambda)(1-\zeta)}{\lambda D_s} \bar{p}_t^o$$  \hspace{1cm} (164)

The goods market clearing condition is of the form

$$\dot{\bar{y}}_t = \frac{X_s}{X_s + I_s} \dot{\bar{x}}_t + \frac{I_s}{X_s + I_s} \dot{\bar{I}}_t + \bar{g}_t.$$  \hspace{1cm} (165)

**Government Policies** The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R) [\psi_1 \tilde{\pi}_t + \psi_2 \tilde{Y}_t] + \epsilon_{R,t}.$$  \hspace{1cm} (166)
### Table 1: Parameters Fixed During Estimation

<table>
<thead>
<tr>
<th>Name</th>
<th>MIU Model</th>
<th>Search Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation Rate $\delta$</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td>Persistence of TFP $\rho_z$</td>
<td>0.980</td>
<td>0.980</td>
</tr>
<tr>
<td>Fixed Costs $F$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Indexation $\pi_*$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Steady State GDP $Y_*$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Steady State $\ln(H_<em>/Y_</em>)$</td>
<td>-3.50</td>
<td>-3.50</td>
</tr>
<tr>
<td>Preference Parameter $\chi_*$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Preference Parameter $\kappa$</td>
<td>N/A</td>
<td>0.0001</td>
</tr>
<tr>
<td>Steady State Real Rate $r_A$</td>
<td>2.840</td>
<td>2.840</td>
</tr>
<tr>
<td>Steady State Inflation Rate $\pi_A$</td>
<td>2.500</td>
<td>2.500</td>
</tr>
<tr>
<td>Policy Rule $\psi_1$</td>
<td>1.820</td>
<td>1.820</td>
</tr>
<tr>
<td>Policy Rule $\psi_2$</td>
<td>0.180</td>
<td>0.180</td>
</tr>
<tr>
<td>Policy Rule $\rho_R$</td>
<td>0.780</td>
<td>0.780</td>
</tr>
<tr>
<td>Share of Government Spending $g_*$</td>
<td>1.200</td>
<td>1.200</td>
</tr>
</tbody>
</table>

*Notes:* We use the following transformations: $\beta = 1/(1 + r_A/400)$, $\pi_* = 1 + \pi_A/400$. 
Table 2: Prior Distribution for MIU Model

<table>
<thead>
<tr>
<th>Name</th>
<th>Domain</th>
<th>Density</th>
<th>Para (1)</th>
<th>Para (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(M_<em>/Y_</em>)$</td>
<td>$\mathbb{R}$</td>
<td>Normal</td>
<td>0.75</td>
<td>0.30</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>20.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Firms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.30</td>
<td>0.03</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>[0, 1]</td>
<td>Uniform</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.60</td>
<td>0.15</td>
</tr>
<tr>
<td>$\iota$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>$S''$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>5.00</td>
<td>2.50</td>
</tr>
<tr>
<td>Shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td>$\rho_{\chi}$</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGamma</td>
<td>1.00</td>
<td>4.00</td>
</tr>
<tr>
<td>$\sigma_{\chi}$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGamma</td>
<td>1.00</td>
<td>4.00</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGamma</td>
<td>0.50</td>
<td>4.00</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>$\mathbb{R}^+$</td>
<td>InvGamma</td>
<td>1.00</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; $s$ and $\nu$ for the Inverse Gamma distribution, where $p_{\text{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. We multiply the product of the marginal densities reported in the table with the function $f(\cdot) = -0.5(I_*/Y_*) - 0.16)^2/0.005^2 - 0.5(\lambda - 0.15)^2/0.01^2$ and truncate the effective prior at the boundary of the determinacy region.
Table 3: Prior Distribution for Search Model [Updated]

<table>
<thead>
<tr>
<th>Name</th>
<th>Domain</th>
<th>Density</th>
<th>Para (1)</th>
<th>Para (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\ln(M_s/Y_s))</td>
<td>(IR)</td>
<td>Normal</td>
<td>0.75</td>
<td>0.50</td>
</tr>
<tr>
<td>(\theta)</td>
<td>[0, 1]</td>
<td>Uniform</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>(\tilde{\sigma})</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Firms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.30</td>
<td>0.03</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.60</td>
<td>0.15</td>
</tr>
<tr>
<td>(\iota)</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>(IR^+)</td>
<td>Gamma</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>(S'')</td>
<td>(IR^+)</td>
<td>Gamma</td>
<td>5.00</td>
<td>2.50</td>
</tr>
<tr>
<td>Shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\rho_g)</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td>(\rho_\chi)</td>
<td>[0, 1]</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td>(\sigma_g)</td>
<td>(IR^+)</td>
<td>InvGamma</td>
<td>1.00</td>
<td>4.00</td>
</tr>
<tr>
<td>(\sigma_\chi)</td>
<td>(IR^+)</td>
<td>InvGamma</td>
<td>1.00</td>
<td>4.00</td>
</tr>
<tr>
<td>(\sigma_R)</td>
<td>(IR^+)</td>
<td>InvGamma</td>
<td>0.50</td>
<td>4.00</td>
</tr>
<tr>
<td>(\sigma_Z)</td>
<td>(IR^+)</td>
<td>InvGamma</td>
<td>1.00</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; \(s\) and \(\nu\) for the Inverse Gamma distribution, where \(p_{\text{IG}}(\sigma|\nu, s) \propto \sigma^{\nu-1}e^{-\nu s^2/2\sigma^2}\). We multiply the product of the marginal densities reported in the table with the function \(f(\cdot) = -0.5(I_s/Y_s - 0.16)^2/0.005^2 - 0.5(mu - 0.15)^2/0.01^2\) where \(mu\) is the economy-wide markup and truncate the effective prior at the boundary of the determinacy region.
Table 4: Prior and Posterior Moments for MIU Model

<table>
<thead>
<tr>
<th>Name</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean 90% Intv</td>
<td>Mean 90% Intv</td>
</tr>
<tr>
<td>Household</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(M_<em>/Y_</em>)$</td>
<td>0.743 [-0.098, 1.547]</td>
<td>0.779 [0.729, 0.827]</td>
</tr>
<tr>
<td>Firms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.280 [0.266, 0.294]</td>
<td>0.282 [0.269, 0.296]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.151 [0.134, 0.167]</td>
<td>0.150 [0.133, 0.166]</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.595 [0.362, 0.833]</td>
<td>0.759 [0.709, 0.809]</td>
</tr>
<tr>
<td>$\iota$</td>
<td>0.509 [0.094, 0.897]</td>
<td>0.050 [0.000, 0.101]</td>
</tr>
<tr>
<td>Shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.799 [0.646, 0.953]</td>
<td>0.886 [0.850, 0.920]</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>0.800 [0.652, 0.960]</td>
<td>0.974 [0.958, 0.992]</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>1.267 [0.521, 2.065]</td>
<td>1.227 [1.062, 1.388]</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>1.202 [0.546, 1.871]</td>
<td>0.865 [0.757, 0.972]</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.583 [0.273, 0.897]</td>
<td>0.199 [0.175, 0.223]</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>1.194 [0.555, 1.874]</td>
<td>0.557 [0.471, 0.639]</td>
</tr>
</tbody>
</table>

Notes: The log marginal likelihood for this specification is $-441.1$. 
Table 5: **Prior and Posterior Moments for Search Model**

<table>
<thead>
<tr>
<th>Name</th>
<th>Prior Mean</th>
<th>90% Intv</th>
<th>Posterior Mean</th>
<th>90% Intv</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln(M_<em>/Y_</em>))</td>
<td>0.427</td>
<td>[-0.524, 1.157]</td>
<td>0.768</td>
<td>[0.742, 0.793]</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.938</td>
<td>[0.897, 0.998]</td>
<td>0.939</td>
<td>[0.931, 0.948]</td>
</tr>
<tr>
<td>(\tilde{\sigma})</td>
<td>0.141</td>
<td>[0.019, 0.284]</td>
<td>0.305</td>
<td>[0.271, 0.339]</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.305</td>
<td>[0.275, 0.334]</td>
<td>0.372</td>
<td>[0.352, 0.390]</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0.136</td>
<td>[0.086, 0.189]</td>
<td>0.094</td>
<td>[0.049, 0.127]</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>0.629</td>
<td>[0.448, 0.810]</td>
<td>0.695</td>
<td>[0.643, 0.753]</td>
</tr>
<tr>
<td>(\iota)</td>
<td>0.703</td>
<td>[0.426, 0.995]</td>
<td>0.268</td>
<td>[0.123, 0.426]</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\rho_g)</td>
<td>0.808</td>
<td>[0.677, 0.962]</td>
<td>0.859</td>
<td>[0.827, 0.896]</td>
</tr>
<tr>
<td>(\rho_x)</td>
<td>0.829</td>
<td>[0.692, 0.956]</td>
<td>0.933</td>
<td>[0.914, 0.953]</td>
</tr>
<tr>
<td>(\sigma_g)</td>
<td>1.344</td>
<td>[0.657, 2.008]</td>
<td>0.936</td>
<td>[0.816, 1.048]</td>
</tr>
<tr>
<td>(\sigma_x)</td>
<td>1.168</td>
<td>[0.572, 1.787]</td>
<td>1.619</td>
<td>[1.400, 1.815]</td>
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<tr>
<td>(\sigma_R)</td>
<td>1.027</td>
<td>[0.287, 1.890]</td>
<td>0.246</td>
<td>[0.215, 0.278]</td>
</tr>
<tr>
<td>(\sigma_Z)</td>
<td>1.162</td>
<td>[0.551, 1.880]</td>
<td>0.393</td>
<td>[0.327, 0.455]</td>
</tr>
</tbody>
</table>

**Notes:** The log marginal likelihood for this specification is \(-503.0\).
### Table 6: Posterior Steady States

<table>
<thead>
<tr>
<th>Shock</th>
<th>Search Model</th>
<th>MIU Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>90% Intv</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$I_*/\gamma_0$</td>
<td>0.154 [0.147, 0.161]</td>
<td>0.163 [0.155, 0.171]</td>
</tr>
<tr>
<td>$M_*/\gamma_0$</td>
<td>2.155 [2.099, 2.207]</td>
<td>2.179 [2.072, 2.289]</td>
</tr>
<tr>
<td>$H_<em>/Y_</em>$</td>
<td>0.030</td>
<td>0.030</td>
</tr>
<tr>
<td>$W_* H_<em>/Y_</em>$</td>
<td>0.576 [0.546, 0.604]</td>
<td>0.625 [0.608, 0.642]</td>
</tr>
<tr>
<td>Overall Markup</td>
<td>0.149 [0.134, 0.167]</td>
<td>0.150 [0.133, 0.166]</td>
</tr>
<tr>
<td>DM Share</td>
<td>0.326 [0.289, 0.359]</td>
<td>N/A</td>
</tr>
<tr>
<td>DM Markup</td>
<td>0.267 [0.172, 0.356]</td>
<td>N/A</td>
</tr>
<tr>
<td>$B$</td>
<td>0.095 [0.085, 0.105]</td>
<td>0.229 [0.206, 0.252]</td>
</tr>
</tbody>
</table>
Figure 1: WELFARE COMPARISON

Notes:
Figure 2: Allocations in Search-Based and MIU Model

Notes:
Figure 3: Nominal Rigidities and Welfare

MIU Model

Search-Based Model

Notes:
Figure 4: Welfare and the New Keynesian Mechanism

MIU Model

Search-Based Model

Notes: