Unions in a Frictional Labor Market*

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Abstract

We analyze a labor market with search and matching frictions where wage setting is controlled by a monopoly union. We take a benevolent view of the union in assuming it to care equally about employed and unemployed workers and we assume, moreover, that it is fully rational, thus taking job creation into account when making its wage demands. Under these assumptions, if the union is also able to fully commit to future wages it generates an efficient level of long-run unemployment. However, in the short run, it uses its market power to collect surpluses on existing matches by raising current wages above the efficient level. These elements give rise to a time inconsistency. Without commitment, and in a Markov-perfect equilibrium, not only is unemployment well above its efficient level, but its responses to productivity shocks are amplified and in some cases strongly so.

We compare labor-market outcomes arising from different assumptions on the length of commitment over wages. We also analyze related labor-market settings. In one, we look at partial unionization and evaluate the effects of requiring universal coverage or outlawing the union; depending on the parameter values, these can both be welfare-improving relative to partial unionization. In another, we look at “collective bargaining” between a union and an employers’ association representing firms; here, the long-run outcome is also efficient when the parties can commit but, generically, inefficient when they cannot.

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1 Introduction

Labor unions play an important role in many labor markets in many countries. There is also a large literature within labor economics studying how union presence influences labor-market outcomes. Yet there is relatively little work studying the impact of this institution on the aggregate labor market when this market is described as having frictions and featuring unemployment due to these frictions. Since search and matching models have come to play a central role as a workhorse for macroeconomic labor-market analyses, this gap in the literature leaves open important questions. What is the impact of unions on aggregate unemployment, and wages? How do unions affect how strongly unemployment varies over the business cycle? What institutional settings for wage setting are desirable, if one consider implementing rules regarding union membership or when there is central bargaining between a union and employer representatives? A circumstance of particular relevance, especially for many European economies, is that where there is a nationwide union, or cooperation/agreements among unions representing different industries. In these cases, unions directly take on an aggregate role and their behavior therefore matter in a direct way for the macroeconomy. To develop our understanding of the impact of a large union on the aggregate economy, this paper develops a dynamic model of unionized frictional labor markets in discrete time. Using this model, we then examine, in turn, the union impact on wage setting both in the long run, in response to shocks, and in settings where the institutional detail differ.

Our first and arguably most general finding is that the degree to which the union can commit to future wage setting is qualitatively and quantitatively important for outcomes. We start from a rather benevolent presumption about the union: it cares equally about employed and unemployed workers. We take this view chiefly to simplify the analysis. Second, we assume that the union is fully rational, thus taking job creation into account when making its wage demands. Current creation helps currently unemployed workers; future job creation helps workers who are unemployed then, some of whom are employed now. Of course, job creation
comes about by lowering wage demands, and the union thus sets wages trading these effects against each other. Under these general assumptions and the additional assumption that the union is also able to fully commit to future wages, we show that the outcome is an efficient level of long-run unemployment. However, in the short run, unemployment is inefficiently high as the union uses its market power to collect surpluses on existing matches by raising current wages above the efficient level. More precisely, we demonstrate that labor-market tightness is inefficient in the very first period but set efficiently from the second period and on.

These elements give rise to a time inconsistency. That is, if a union had implemented a commitment plan yesterday but had the opportunity to revise it today, it would indeed revise it and lower labor-market tightness relative to the plan, thus benefitting again from the pre-existing matches. What, then, would the outcome be if one simply assumed that unions do not have commitment? We answer this question by analyzing Markov-perfect equilibria. In these equilibria, we show, unemployment is above its efficient level both in the short and in the long run. The longer is the length of commitment, the weaker is this effect. For a yearly commitment length, the effects on unemployment are still quite sizable: unemployment without commitment is almost twice the efficient level, and the output loss is 4% of GDP per period.

Interestingly, economies with large unions (who are not able to commit to future wages) also display amplified responses to productivity shocks. Again the length of commitment is important, but for some parameterizations it is possible to move the basic Pissarides model much closer to the data by considering unionized wage setting, thus significantly helping the model’s performance in light of the issues raised in Shimer (2005).

Throughout the analysis, our analytical work-horse, both for qualitative analysis of the different forces underlying equilibria and for numerical computation, is the Euler equation of the wage-setting union. This equation is readily compared to its efficient equivalent, and the Euler equation under commitment can also be compared to that without.
We also look at economies with less than full unionization of workers. In order to side-step the issue of how union objectives are altered over time as more or less workers are unionized, we look at the case of a constant unionization rate: a subset of workers are simply members, whereas the remainder bargain individually with firms. Moreover, we assume here that firms cannot discriminate workers based on union membership; thus, they search in an indirected manner and may end up being matched either with a unionized worker—where the firm would have no influence on the wage—or with a non-member, and thus Nash-bargain to a wage outcome. A special case of this setting is that where the unionization rate is such that the member and non-member wage are the same. Such an outcome is, namely, possible if individual workers have strong bargaining power. In this case, we can show in some circumstances that a law that requires all workers to be unionized is welfare-enhancing: it would imply lower wages and lower unemployment. However, if individual workers have low bargaining power, union members always earn higher wages than non-union workers, and outlawing unions would improve welfare.

Finally, we examine a Nash bargaining game between a centralized union and an employers’ association: collective bargaining. We do not allow side payments: the two parties simply have to agree on a wage or, in the case of commitment, a wage path. This game leads to the same general conclusion as in our simple monopoly union case: under commitment, outcomes are inefficient only in the short run (tightness is optimal beginning one period from now). Here, the direction of the inefficiency—higher or lower tightness than optimally—depends on the bargaining parameter. We display an illustrative example where a bargaining share for unions close to (but strictly less than) one leads to an efficient outcome.

Within the literature on labor unions, this paper is most closely related to two strands: i) a set of papers considering the dynamic decision problem of a labor union when labor is subject to adjustment costs, and ii) a second set of papers incorporating labor unions into the Mortensen-Pissarides search and matching framework, work which has so far largely focused on static union problems.\(^1\)

\(^1\)Further recent work on labor unions in macroeconomics includes Acemoglu, Aghion, and Violante (2001),
The first set of papers develops the idea that dynamic concerns become important in union decision making when labor markets are frictional. The most directly related papers in this vein are Lockwood and Manning (1989) and Modesto and Thomas (2001). These papers study labor markets where firms face adjustment costs to labor, and forward-looking unions take these adjustment costs into account in deciding on their wage demands. Modesto and Thomas (2001) introduce the idea that the union’s ability to commit to future wage demands matters in this setting and contrast the differences between outcomes with and without union commitment. The simple reduced-form adjustment cost framework allows these authors to derive closed-form results which speak to the level of union wage-demands, as well as the speed of adjustment in employment, both argued to be greater in a unionized labor market than a non-unionized one. We, on the other hand, study dynamic union decision-making within the context of the Mortensen-Pissarides search and matching model—the modern workhorse model of frictional labor markets—where such adjustment costs are endogenous. This allows us to study the impact of unions on equilibrium unemployment, vacancy creation, output, and welfare, something the adjustment-cost framework cannot directly address.

The second set of papers incorporate unions into models of labor markets with search frictions. Perhaps the closest in spirit to our paper in this group is Pissarides (1986), which first introduces a monopoly union into the Pissarides (1985) framework, and studies the impact on equilibrium outcomes in the labor market. Like most of the literature following it, that paper focuses on steady states, side-stepping the dynamic issues we highlight here. Mortensen and Pissarides (1999) proceed to incorporate a notion of wage-compression into the analysis, allowing for worker heterogeneity. Wage compression plays a central role in subsequent work on unions in the Mortensen-Pissarides framework. For example Garibaldi and Violante (2005) and Boeri and Burda (2009) study the effects of employment protection in a

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Alvarez and Shimer (2009), and Greenwood (2010). While these papers adopt varying approaches to modeling the labor market, they all depart from the Mortensen-Pissarides framework.

2 Other papers which feature unions in settings where labor adjustment occurs slowly due to adjustment costs or otherwise, but focus on other issues, include Booth and Schiantarelli (1987), Card (1986), and Kennan (1988). Some authors have also sought to develop dynamic models of unions building on the insider-outsider theory of Lindbeck and Snower (1986), such as Huizinga and Schiantarelli (1992). In order to focus our analysis, we abstract from insider-outsider concerns here.
frictional labor market where a centralized labor union compresses wages. Some papers depart from the assumption of a centralized union, allowing firm-level unions instead, and also speak to the degree of unionization, such as Ebell and Haefke (2006), Acikgoz and Kaymak (2009), and Taschereau-Dumouchel (2011). Finally, Delacroix (2006) extends the framework of Ebell and Haefke (2006) to industry-level unions, illustrating the non-monotonic relationship between the degree of coordination in bargaining and economic performance discussed by Calmfors and Driffield (1988).

Our paper is organized as follows. Section 2 analyzes the benchmark model—first a one-period model to set out notation and introduce the key elements, then an infinite-horizon model with commitment, and lastly the infinite-horizon model without commitment. Section 3 provides the quantitative analysis and Section 4 looks at extensions: partial unionization in Section 4.1 and collective bargaining in Section 4.2. Section 5 concludes.

2 The benchmark model

This section begins by describing the simple Mortensen-Pissarides search and matching environment we base our analysis on. We then introduce a monopoly union into that framework, and characterize its behavior. We consider extensions to partial unionization and collective bargaining later on.3

A frictional labor market Time is discrete and the horizon infinite. The economy is populated by a continuum of measure one identical workers, together with a continuum of identical capitalists who employ these workers. All agents have linear utility, and discount the future at rate $\beta < 1$. Capitalists have access to a linear production technology, producing $z$ units of output per period for each worker employed.

The labor market is frictional, however, requiring capitalists seeking to hire workers to post

3See Section 4.
vacancies, which is costly. The measure of matches in the beginning of the period is denoted by \( n \), leaving \( 1 - n \) workers searching for jobs. Searching workers and posted vacancies are matched according to a constant-returns-to-scale matching function \( m(v, 1 - n) \), where \( v \) is the measure of vacancies. With this, the probability with which an unemployed worker finds a job within a period can be written \( \mu(\theta) = m(\theta, 1) \); the probability with which a vacancy is filled is \( q(\theta) = m(1, \theta^{-1}) \), where \( \theta = v/(1 - n) \) is the labor market tightness. We will assume that \( \mu'(\theta) \) is positive and decreasing and that \( q'(\theta) \) is negative and increasing. With this, employment equals \( n \) plus the number of new matches, \( \mu(\theta)(1 - n) \). Jobs are destroyed each period with probability \( \delta \). Thus, the number of matches evolves over time according to the law of motion

\[
n_{t+1} = (1 - \delta) \left( n_t + \mu(\theta_t)(1 - n_t) \right).
\]  

(1)

Notice that a worker separated after production at \( t \) may be reemployed in \( t + 1 \) and not need to suffer unemployment. In addition to the market production technology, unemployed workers also have access to a home production technology, producing \( b(< z) \) units of output per period.

**Firms** Firms need to post vacancies in order to find workers, at a cost \( \kappa \) per vacancy. Competition on the firm side drives profits from vacancy-creation to zero, with firms taking into account the union wage-setting behavior today and in the future. The zero-profit condition thus determines the current labor-market tightness according to current and future wages as follows:

\[
\kappa = q(\theta) \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s [z - w_{t+s}].
\]

(2)

**The labor union** Wages are set unilaterally by a labor union, with universal coverage. The union sets wages to maximize the welfare of all workers, but cannot discriminate among
workers in doing so. In particular, we rule out favorable treatment of “insiders” versus “outsiders” by assuming that all workers are paid the same wage. The union objective thus becomes

\[
\sum_{t=0}^{\infty} \beta^t \left( \left[ n_t + \mu(\theta_t)(1-n_t) \right] w_t + \left( 1-n_t \right) \left( 1-\mu(\theta_t) \right) b \right)
\]  

The union takes as given the evolution of employment according to equation (1). It also internalizes the effect of its wage-setting decisions on hiring. Therefore, the union’s problem is to choose a sequence of wages \( \{w_t\}_{t=0}^{\infty} \) to maximize the objective (3) subject to the law of motion (1) and zero profit condition (2). Below, we will consider different assumptions regarding the union’s commitment power.

Summarizing the events in period \( t \), we have

- \( n_t \) given
- vacancy posting, \( v_t \)
- production

\[
\text{union sets } w_t \quad v_t \quad \text{and } 1-n_t \text{ search} \quad \text{separations}
\]

Given the path of wages \( \{w_t\}_{t=0}^{\infty} \), then, equation (2) determines the path of the vacancy-unemployment ratio \( \{\theta_t\}_{t=0}^{\infty} \), which in turn determines the evolution of employment \( \{n_t\}_{t=0}^{\infty} \).

### 2.1 A one-period example

To illustrate some of the key forces at play, we first consider the impact of the union in a very simple setting: a one-period version of the above economy. Many of the features present here will be present in the subsequent analysis.

A natural starting point is the efficient benchmark—the output maximizing level of vacancy-
creation a social planner would choose. Here the planner solves the problem

$$\max_{\theta} \left( \underbrace{n + \mu(\theta)(1-n)}_{\text{employed}} \right) z + \left( \underbrace{(1-n)(1-\mu(\theta))}_{\text{unemployed}} \right) b - \theta(1-n) \kappa,$$

taking as given initial matches $n$. The planner’s optimum is characterized by the first-order condition $-\kappa + \mu'(\theta)(z-b) = 0$, which pins down $\theta$ independent of $n$. For concreteness, consider the matching function $m(v, u) = vu/(v + u)$, such that $\mu(\theta) = \theta/(1 + \theta)$. In this case the planner’s optimum is given by $\theta^e = \sqrt{(z-b)/\kappa} - 1$, with labor-market tightness an increasing function of market productivity.

The union instead aims to maximize

$$\left( \underbrace{n + \mu(\theta)(1-n)}_{\text{employed}} \right) w + \left( \underbrace{(1-n)(1-\mu(\theta))}_{\text{unemployed}} \right) b,$$

by choice of $w$ and $\theta$, subject to the free-entry condition of firms: $\kappa = q(\theta)[z - w]$. Using this free entry condition to solve for the wage, as $w = z - \kappa/q(\theta)$, and substituting into the union objective yields a maximization problem in $\theta$ only:

$$\max_{\theta} \left( \underbrace{n + \mu(\theta)(1-n)}_{\text{employed}} \right) (z - \frac{\kappa}{q(\theta)}) + \left( \underbrace{(1-n)(1-\mu(\theta))}_{\text{unemployed}} \right) b$$

$$= \max_{\theta} -\frac{n\kappa}{q(\theta)} + \left( \underbrace{n + \mu(\theta)(1-n)}_{\text{planner’s objective}} \right) z + \left( \underbrace{(1-n)(1-\mu(\theta))}_{\text{planner’s objective}} \right) b - \theta(1-n) \kappa,$$

also taking as given $n$. The first line expresses the tradeoff the union faces in choosing $\theta$: increasing $\theta$ increases employment, but at the cost of the lost wage income on new and existing workers required to raise $\theta$.

Notice that this union objective differs from the planner’s objective only by the initial term, $-\frac{n\kappa}{q(\theta)}$. To understand how the two objectives relate to each other, recall that the planner is concerned about all agents in the economy, while the union only cares about workers. The
union objective thus equals the planner objective less the capitalists’ share of incomes: firm profits from new matches—which are zero due to free entry—and firm profits from existing matches, i.e., \((z - w)n = \frac{m_0}{q(\theta)}\).

The union optimum is characterized by the first-order condition 

\[-\kappa - \kappa \frac{n}{1 - n} \frac{\mu'(\theta)}{q'(\theta)} + \mu'(\theta)(z - b) = 0,\]

which implies that the union’s choice of \(\theta\) depends on \(n\). In our example, the union optimum is given by 

\[\theta = \sqrt{1 - n \sqrt{(z - b) / \kappa}} - 1.\]

Labor-market tightness is thus again an increasing function of market productivity, but now decreases in initial employment. We can see that the union implements the socially optimal level of vacancy creation if initial employment is zero. But if initial employment is positive, the union has an incentive to raise wages above the level consistent with efficient vacancy creation, in order to collect surpluses from firms with existing matches.\(^4\)

The one-period problem captures the essence of why a monopoly union chooses a sub-optimally low level of employment, and production. How does the argument just put forth play out in the infinite-horizon model—what is, for example, the effect on steady-state unemployment? That depends crucially, as we shall see below, on the extent to which the union can commit.

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\(^4\)It should be clear from this example that the outcome would be different if the union could choose different wages for the
2.2 The efficient benchmark and a recursive planner’s problem

To characterize union wage-setting when the time horizon is infinite, we begin with the efficient benchmark. The planner chooses a sequence of \( \{\theta_t\}_{t=0}^{\infty} \) to maximize

\[
\sum_{t=0}^{\infty} \beta^t \left[ (n_t + \mu(\theta_t)(1-n_t)) z + (1-n_t)(1-\mu(\theta_t)) b - \theta_t(1-n_t) \kappa \right]
\]

subject to

\[
n_{t+1} = (1-\delta) (n_t + \mu(\theta_t)(1-n_t)),
\]

with \( n_0 \) given.

For what comes later it will be useful to formulate problems recursively. Thus, we begin by writing the planner’s problem recursively, and discussing the optimal vacancy-creation decisions of the planner. We then compare these to outcomes in the unionized economy.

The recursive form for the planner’s problem reads

\[
V(n) = \max_{\theta} \left( n + \mu(\theta)(1-n) \right) z + (1-n)(1-\mu(\theta)) b - \theta(1-n) \kappa + \beta V(N(n, \theta)), \quad (4)
\]

where \( N(n, \theta) \equiv (1-\delta) (n + \mu(\theta)(1-n)) \). Notice that the state variable is \( n \), the number of matches at the beginning of the period, and that the control variable—labor-market tightness \( \theta \)—determines \( n' \) according to the law of motion \( N(n, \theta) \).

The first-order condition, assuming an interior solution, is

\[
\kappa \begin{aligned}
\text{vacancy cost}
\end{aligned} = \mu'(\theta) \begin{aligned}
\text{increase in matches}
\end{aligned} \left( z - b + \beta (1-\delta) V'(n') \right). \quad (5)
\]

It equalizes the cost of an additional vacancy to its benefits: its immediate match payoff, \( z - b \), and the future value of the matches that do not separate. The envelope condition, or
the marginal value of a beginning-of-period match, is

$$V'(n) = (1 - \mu(\theta) + \theta \mu'(\theta)) \left( z - b + \beta (1 - \delta) V'(n') \right). \tag{6}$$

An increase in employment increases the present discounted value of the total match surpluses by the flow surplus $z - b$ each period, but the resulting decrease in unemployment also affects matching today (note that the derivative of the matching function with respect to unemployment, $m_u(\theta, 1)$, equals $\mu(\theta) - \theta \mu'(\theta)$).

By standard elimination of the derivative of the value function, the Euler equation can be rewritten solely in terms of the evolution of the state, together with the optimal policy function mapping $n$ into a choice $\theta$. This equation becomes

$$\frac{\kappa}{\mu'(\theta)} = z - b + \beta (1 - \delta) \left( 1 - \mu(\theta) + \theta \mu'(\theta) \right) \frac{\kappa}{\mu'(\theta')}, \tag{7}$$

where $\theta$ is short for the optimal choice of $\theta$ given $n$. This equation is a way of stating the standard efficiency condition for the planner’s solution in the Mortensen-Pissarides model. It can be interpreted as a variational calculation: a tradeoff between cost of creating a new job today and its benefits today and tomorrow, keeping the number of matches thereafter constant.

To understand equation (7), note that the cost of creating an additional match in a given period can be broken down into two factors: the cost of creating a vacancy ($\kappa$) times the number of vacancies required to fill one job. Since an increase in vacancies by one unit gives an increase in tightness of $1/(1 - n)$ units and an increase in tightness by one unit gives $(1 - n)\mu'(\theta)$ jobs, one new vacancy creates $\mu'(\theta)$ new jobs—hence the cost of creating one new job is $\kappa/\mu'(\theta)$. The benefits include the market production output net of home production output today, $z - b$, as well as the net saving on vacancy creation costs by having created the additional matches. How much is saved tomorrow? First, the net change in the number
of matches tomorrow is not simply $1 - \delta$. These are the new jobs left tomorrow from one created today, but the smaller unemployment pool tomorrow will leave fewer matches from any pre-existing vacancy creation. Thus there is a decrease in the number of matches, for each worker now out of the unemployment pool, by the amount $m_u(\theta', 1)$, where $m(v, u)$ is the matching function. Since $m_u(\theta, 1) = \mu(\theta) - \theta \mu'(\theta)$, we thus obtain a net increase in the number of matches next period by $(1 - \delta)(1 - \mu(\theta) + \theta \mu'(\theta))$. Finally, each additional match next period saves $\kappa/\mu'(\theta')$ consumption units.

Looking at the Euler equation, we notice the familiar result of the benchmark search/matching model: it does not feature the state variable $n$ explicitly—only market tightness today and tomorrow appear, so that a natural solution is a constant tightness independently of $n$. It is straightforward to show that the Bellman equation is solved by a value function $V$ that is linear in $n$, and that the efficient allocation thus features a constant $\theta^e$, independent of $n$.

### 2.3 A union with commitment

The planner problem and union problem are closely related. To see this, note first that the union can be thought of as choosing a sequence of vacancy-unemployment ratios, $\{\theta_t\}$. This is because the union’s choice of a sequence of wages $\{w_t\}$ determines, at each instant, the present value of wages firms must pay workers over their employment spell: $W_t = \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s w_{t+s}$. The sequence of these present values then pins down the sequence $\{\theta_t\}$ through the firm free-entry conditions. Intuitively, choosing higher wages (in present value, as of the beginning of period $t$) reduces firm profits from vacancy creation, thereby reducing $\theta_t$. For any solution to the union problem—a sequence $\{\theta_t\}$—one can then back out per-period wages by first using the free-entry condition to find the present value of wages $W_t$ each period, and then computing $w_t = W_t - \beta(1 - \delta)W_{t+1}$.

Thus, using the free-entry condition to eliminate the wage sequence, the union objective
becomes

\[- \frac{n_0 \kappa}{q(\theta_0)} + \sum_{t=0}^{\infty} \beta^t \left[ \left( n_t + \mu(\theta_t)(1 - n_t) \right) z + \left( 1 - n_t \right) \left( 1 - \mu(\theta_t) \right) b - \theta_t (1 - n_t) \kappa \right], \]

revealing an identical objective to that of the planner except for the first term. This term—
we recognize it from equation (2)—summarizes the surplus accruing to capitalists. To see
this, note that the capitalists share of the surplus, i.e., the present value of profits to firms,
can be written as

\[
\kappa \left[ \frac{t}{\theta_0} \sum_{s=0}^{\infty} \beta^s \left( 1 - \delta \right)^s \left[ z - w_{t+s} + \kappa \right] - \theta_t (1 - n_t) \kappa \right].
\]

(9)

Here the first term captures the present value of profits to initial matches, and the second
those to new vacancies created in periods \( t = 0, 1, \ldots \). This objective reduces to representing
initial matches only, however, as the free entry condition implies zero present value of profits
to new vacancies.\(^7\) Matches that exist at time zero, however, are due a strictly positive
present value of profit: these firms paid the vacancy cost in the past, anticipating positive
profits in the future to make up for it. Moreover, using the free entry condition, this present
value of profits can be written as \( \kappa/q(\theta_0) \).

Equation (8) reflects the fact that while the planner maximizes the present value of output,
the union only cares about the workers’ share of that present value—not the capitalists’. In
fact, the union will have an incentive to grab some of this present value from capitalists by
raising wages—and this is exactly how the solutions to the two problems will differ. The
union distorts vacancy creation in doing so, however, because these higher wages apply also
to new vacancies.

\(^6\)See Appendix ??.

\(^7\)Formally, can write the second term in (9) as \( \sum_{t=0}^{\infty} \beta^t (1 - n_t) \theta_t q(\theta_t) \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s \left[ z - w_{t+s} - \kappa \right] \),
which equals zero under the free entry condition (2).
Proposition 1. When able to commit to future wages, the union attains efficient vacancy creation after the initial period. In the initial period, vacancy creation is efficient if $n_0 = 0$ and below the efficient level if $n_0 > 0$.

Note that the union effectively solves the planner’s problem in all periods but the first. In the first period the union instead solves the problem

$$
\max_{\theta} -\frac{n\kappa}{q(\theta)} + \left(n + \mu(\theta)(1-n)\right)z + (1-n)(1-\mu(\theta))b - \theta(1-n)\kappa + \beta V(N(n, \theta))
$$

where $N(n, \theta) = (1-\delta)(n + \mu(\theta)(1-n))$, $n$ is given, and $V$ solves the problem (4) above.

Deriving the first-order condition for this first period is straightforward using the same methods as above. It becomes

$$
\frac{\kappa}{\mu'(\theta)} - \frac{n}{1-n} \frac{q'(\theta)}{q(\theta)} \frac{\kappa}{\mu'(\theta)} = z - b + \beta(1-\delta)(1-\mu(\theta) + \theta\mu'(\theta)) \frac{\kappa}{\mu'(\theta)}. \tag{11}
$$

Knowing that the later periods will entail a constant, and efficient, level of market tightness (and no dependence on the number of matches $n_1$), we can thus also write

$$
\frac{1}{\mu'(\theta)} [1 - \frac{n}{1-n} \frac{q'(\theta)}{q(\theta)^2}] = \frac{1}{\mu'(\theta^e)},
$$

where $\theta^e$ refers to the efficient outcome. Because $q' < 0$ and $\mu'$ is decreasing, this equation implies (i) a lower value of $\theta$ for the first period than later on and (ii) the higher is initial employment, the stronger is this effect. Quite intuitively, an increase in vacancy creation now includes the additional cost of giving up the the wage income the union could collect from firms on initial matches. Thus, in this first period market tightness now depends (negatively) on the number of pre-existing matches. This is an important force in the present model, and a central mechanism behind unemployment dynamics when unions do not have commitment.

That the outcome in the initial period of the above problem differs from later ones reflects
a time inconsistency problem in the union wage-setting problem. If the union were to re-optimize wages after the first period, it would face a different objective and thus choose a different path of wages. While the union can thus get relatively close to the efficient outcome when it can commit, this immediate time-inconsistency begs the question: what happens if the union cannot commit to future actions? To study time-consistent union decision-making we next turn to a game-theoretic setting, which will be based on the kind of recursive formulation of the union problem we set up above.

### 2.4 A union without commitment

The union problem (10) suggests that if the union were to re-optimize at any date, its choice of $\theta$ (via its choice of wages) would depend on employment $n$. In particular, higher employment would imply a lower $\theta$. How would outcomes be affected if the union could not commit to not re-optimizing? We study this question by focusing on (differentiable) Markov-perfect equilibria with $n$, the number of matches in the beginning of the period, as a state variable. That $n$ is a payoff- and action-relevant state variable should be clear from the analysis of the problem under commitment, where we argued that a higher $n$ will cause a lower initial union choice for tightness.\footnote{One can add states, representing histories of past behavior, but we do not consider such equilibria here.} In a Markov-perfect equilibrium, the union anticipates its future choices of $\theta$ to be a (decreasing) function of $n$, which we label $\Theta(n)$, and our task is now to characterize $\Theta$.

This function solves a problem similar to (10), namely

$$
\Theta(n) \equiv \arg \max_\theta -\frac{n\kappa}{q(\theta)} + \left( n + \mu(\theta)(1 - n) \right) z + \left( 1 - n \right)(1 - \mu(\theta))b - \theta(1 - n)\kappa + \beta \tilde{V}(N(n, \theta)).
$$

(12)
The continuation value \( \tilde{V} \) satisfies the recursive equation

\[
\tilde{V}(n) = (n + \mu(\Theta(n))(1 - n))z + (1 - n)(1 - \mu(\Theta(n)))b - \Theta(n)(1 - n)\kappa + \beta \tilde{V}(N(n, \Theta(n))).
\]

(13)

Thus, the union recognizes that its future actions will follow \( \Theta(n) \). Since \( \Theta(n) \) will not in general be the efficient level, this means that \( \tilde{V} \) will not equal \( V \), the continuation value under commitment.

A Markov-perfect equilibrium is thus defined as a pair of functions \( \Theta(n) \) and \( \tilde{V}(n) \) solving (12)–(13) for all \( n \). We will assume that these functions are differentiable and characterize equilibria based on this assumption. We discuss issues of existence and uniqueness/multiplicity of equilibria in Section 3 below.

The first-order condition for the choice of tightness reads

\[
\kappa \left( -\frac{n}{1 - n} \frac{q'(\theta)}{q(\theta)^2} \right) = \mu'(\theta) \left( \frac{z - b + \beta(1 - \delta)\tilde{V}'(n')}{PDV} \right)
\]

and the equation paralleling the envelope condition in the commitment case—which now is not an envelope condition since the union does not agree with its future decisions—becomes

\[
\tilde{V}'(n) = (1 - \mu(\theta) + \theta \mu'(\theta)) \left( \frac{z - b + \beta(1 - \delta)\tilde{V}'(n')}{PDV} \right) + \left( \Theta'(n)(1 - n) - \theta \right) \left( -\frac{n}{1 - n} \frac{q'(\theta)}{q(\theta)^2} \kappa \right).
\]

(15)

Equation (15) is derived by first, and straightforwardly, taking derivatives of the equation defining the present value, equation (13), and then using equation (14) so as to arrive at a formulation that is close to the equivalent condition for the planner—equation (6). A feature of equation (15) not present in the planner’s envelope condition is the presence of
terms involving the derivative of \( \Theta \). This, again, is because the envelope theorem does not apply. Nevertheless, we can combine the two equations to eliminate \( \tilde{V}' \): solve for it in equation (14) and insert it into equation (15) on both sides of the equation (evaluated at \( n \) and at \( n' \), respectively). Thus we obtain

\[
\frac{\kappa}{\mu'(\theta)} [1 - \frac{n}{1 - n/q(\theta)^2}] = z - b + \beta(1 - \delta)(1 - \mu(\theta) + \theta \mu'(\theta)) \frac{\kappa}{\mu'(\theta')} [1 - \frac{n'}{1 - n'/q(\theta')^2}]
\]

\[
+ (\Theta'(n')(1 - n') - \theta')(\frac{n'}{1 - n'/q(\theta')^2} \kappa),
\]

which is a generalized Euler equation. It is a functional equation in the unknown policy function \( \Theta \), where the derivative of \( \Theta \) appears. The equation is written in a short-hand way: \( \theta \) is short for \( \Theta(n) \), \( \theta' \) is short for \( \Theta(N(n, \Theta(n))) \), and \( n' \) is short for \( N(n, \Theta(n)) \). Thus, the task is to find a function \( \Theta \) that solves this equation for all \( n \). In contrast to the case of the benevolent planner, or the commitment solution after period 0, \( n \) appears nontrivially in this equation and will generally matter for the tightness outcome—it is easily verified that a guess that \( \Theta \) is constant will not solve this functional equation.

In terms of interpretation, this equation, like the planner’s Euler equation (7), represents the tradeoff between creating matches today versus tomorrow, but now for the union instead of for the planner. To understand it, recall that the union controls vacancy creation through its choice of wages. If the union wishes to increase vacancy creation today, it does so by reducing (the present value of) wages today. The cost of an additional match for the union differs from the cost for the planner, however. In addition to the increase in vacancy costs \( \kappa/\mu'(\theta) \), the union also takes into account the loss of surpluses from existing matches that is associated with reducing wages in order to increase hiring, \( \frac{\kappa}{\mu'(\theta)} \frac{n}{1 - n/q(\theta)^2} \) (in the present and in the future). The present value of an additional worker must therefore, from equation (14), be higher in the unionized economy than what is efficient. This feature is present also in the Euler equation for the union with commitment, equation (11), though here this extra tax
on matches appears in both periods, since the problem of the union today and tomorrow are symmetric, unlike in the commitment solution where tomorrow’s union mechanically carries out the orders of today’s plan.

But beyond this difference, here the union also takes into account its inability to commit to future wages: higher employment tomorrow will reduce vacancy creation, as the union will take advantage of its ability to collect surpluses from existing matches through higher wages. To see this, note that the measure of vacancies can be written as $\Theta(n)(1 - n)$ and its derivative with respect to employment as $\Theta'(n)(1 - n) - \Theta(n)$. Higher employment tomorrow reduces vacancy creation tomorrow, and that is a cost: it appears as the last, negative, term on the right-hand side of (16). It appears, in particular, since the current union regards next period’s union as selecting an employment (or vacancy-creation) level that is too low compared to what it would select were it able to commit.

For the present model it is hard to establish, in general, that $\Theta(n)$ is indeed decreasing. In the one-period example of section 2.1 we saw that $\Theta$ becomes a decreasing function of $n$, and in our numerically solved examples below, this feature is always present.\(^9\) What is possible to show for the infinite-horizon case, however, is that whenever $\Theta$ is decreasing, steady-state unemployment is strictly below its efficient level:

**Proposition 2.** If $\Theta(n)$ is decreasing in $n$, then the steady-state level of market tightness, $\theta$, in the unionized economy (without commitment) is strictly below its efficient level.

**Pf.** Consider a steady state of the unionized economy, where $\Theta'(n) = -c$ for some $c > 0$. Using this fact, steady-state employment can be written as $n = (1 - \delta)\mu(\theta)/(1 - (1 - \delta)(1 - \mu(\theta)))$, equation (16) implies that the steady-state $\theta$ satisfies the equation

$$
1 = \mu'(\theta)\frac{z - b}{\kappa} + \beta(1 - \delta)(1 - \mu(\theta) + \theta\mu'(\theta)) - \Delta(\theta),
$$

(17)

\(^9\)We also have not been able to find an example where $\Theta$ is not decreasing.
where $\Delta(\theta) = 0$ in the efficient outcome, and

$$
\Delta(\theta) = -\frac{1 - \delta}{\delta} \mu(\theta) \frac{q'(\theta)}{q(\theta)^2} \left[ 1 - \beta (1 - \delta) (1 - \mu(\theta)) - \frac{\mu'(\theta) \delta c}{1 - (1 - \delta) (1 - \mu(\theta))} \right]
$$

in the unionized economy. The term $\Delta(\theta)$ thus captures the union distortion. Under efficiency, the right-hand side of equation (17) is strictly decreasing in $\theta$ pinning down a unique steady-state $\theta$ (as long as $\mu'(0) \frac{z - b}{\kappa} + \beta (1 - \delta) > 1$). Because the union distortion $\Delta(\theta)$ is strictly positive for any $\theta > 0$, the unionized economy must have lower steady-state $\theta$. □

It follows that steady-state unemployment in the unionized economy is strictly above its efficient level.

### 3 Quantitative results: comparative statics and comparative dynamics

The previous section shows that the presence of the monopoly union affects the levels of unemployment, wages, and output in the economy. But are these effects quantitatively relevant? In this section we parameterize the model in order to study this question. We will also look at an extension with stochastic shocks to productivity and ask whether, in this model, shock amplification is significantly different than in the standard model.

#### 3.1 Wages, unemployment, and output in steady state

How does the presence of the union in the labor market affect the levels of wages, unemployment, and output? The theory tells us that the answer hinges on the union’s ability to commit to future wages. If the union can commit, the unionized economy attains efficiency in steady state. If the union cannot commit, the theory leads us to expect higher wages and

10Note that $m_u(v, u) = \mu(\theta) - \mu'(\theta) \theta$, an expression which is reasonable to assume to be increasing in $\theta$.\footnote{Note that $m_u(v, u) = \mu(\theta) - \mu'(\theta) \theta$, an expression which is reasonable to assume to be increasing in $\theta$.}
unemployment, and consequently lower output, in the unionized economy than what would be efficient.

**Calibration** We adopt the benchmark parametrization used by Shimer (2005), but use an annual frequency for our calibration, motivated by the annual contracting practices commonly observed (we look at other frequencies below). We first set the time discount rate to correspond to a 5% annual rate of return, with $\beta = 1/1.05$. We normalize labor productivity to $z = 1$ and set $b = 0.4$. We depart from Shimer’s specification slightly by adopting the matching function $m(v, u) = \mu_0 vu / (v + u)$, used by, e.g., den Haan, Ramey, and Watson (2000). This form is better suited for the discrete-time setting than a Cobb-Douglas functional form because it guarantees that matching probabilities remain between zero and one. We set the separation rate $\delta$ and matching function coefficient $\mu_0$ following Shimer, but adjusting his numbers, 0.034 and 0.45, to the annual frequency. This implies $\delta = 0.34$ and $\mu_0 \approx 1$. With these parameters, a vacancy cost of $\kappa = 0.015$ guarantees a steady-state unemployment rate of approximately 5.4%, the calibration target used by Shimer.

**Numerical solution technique** The planner’s problem, as well as the case of a union with commitment, can be solved almost in closed form. But solving for the union’s behavior when it cannot commit is more challenging. There are several issues to bear in mind: On the one hand, there are few results available on equilibrium existence for differentiable Markov-perfect equilibria. Further, differentiable equilibria may not be unique. And moreover, non-differentiable equilibria may exist as well.$^{11}$ Clearly, one needs to proceed with caution and be prepared to use several different solution techniques. The results we present in the tables and figures below use the methods in Krusell, Kuruşçu, and Smith (2002) and rely on approximating the equilibrium function $\Theta$ with polynomials of increasingly high order. However, we have also used two other, very different methods, and they deliver very similar quantitative results. We discuss these issues in more detail in Appendix B.

$^{11}$For examples where no differentiable equilibria exist but there exists a non-differentiable equilibrium see, e.g., Krusell, Martin, and Ríos-Rull, 2010); for cases with a continuum of non-differentiable equilibria along with one or more differentiable equilibrium, see (Krusell and Smith, 2003 or Phelps and Pollak, 1968).
Results Table 1 reports the steady-state levels of the wage $w$, the vacancy-unemployment ratio $\theta$, unemployment, vacancies, and output. The table compares outcomes under efficiency, or in a unionized economy where the union can commit, to outcomes in a unionized economy where the union cannot commit. In the latter case, the union’s incentives to raise wages lead to a 1.5 percent increase in wages, which amount to a 38 percent reduction in firm profits from an employed worker. This reduction in profits is reflected in a 44 percent drop in the vacancy-unemployment ratio, composed of an over 70 percent increase in unemployment, and 40 percent drop in vacancies. The reduction in employment results in a 4 percent drop in per-period output.

<table>
<thead>
<tr>
<th>Level</th>
<th>Efficient</th>
<th>Union</th>
<th>Union impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment</td>
<td>0.05</td>
<td>0.09</td>
<td>+71%</td>
</tr>
<tr>
<td>Vacancies</td>
<td>2.23</td>
<td>1.34</td>
<td>-40%</td>
</tr>
<tr>
<td>V-U ratio</td>
<td>5.93</td>
<td>3.33</td>
<td>-44%</td>
</tr>
<tr>
<td>Output</td>
<td>0.95</td>
<td>0.91</td>
<td>-4%</td>
</tr>
</tbody>
</table>

Notes: The table reports steady-state values.

As a robustness check, we provide results in Table 2 for a higher value of home production, with very similar results.

<table>
<thead>
<tr>
<th>Level</th>
<th>Efficient</th>
<th>Union</th>
<th>Union impact</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>Output</td>
<td>0.95</td>
<td>0.91</td>
<td>-4%</td>
</tr>
</tbody>
</table>

Notes: The table reports steady-state values. Here $b = 0.5$ and, to maintain efficient unemployment at the original level, $\kappa = 0.0125$.

The role of the commitment horizon The recursive formulation assumes the union can commit to current period wages, but not beyond. This implies that limited commitment becomes less of an issue as the period length increases. By adjusting the discount rate $\beta$, the separation rate $\delta$, and the matching function coefficient $\mu_0$, one can examine different
period lengths, and hence different amounts of commitment. Table 3 reports the results. On the one hand, we can see that moving from the annual horizon to a shorter, biannual horizon, exacerbates the negative effects of the commitment problem on labor market outcomes significantly: employment and output fall by as much as four percentage points, with unemployment increasing by about the same, from 9.2 to 12.9 percent. On the other, moving to the infinite horizon limit would increase employment and output by four percentage points, with unemployment falling from 9.2 to 5.4 percent.\textsuperscript{12}

<table>
<thead>
<tr>
<th>Table 3: Role of Commitment Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
</tr>
<tr>
<td>-------------------------------------</td>
</tr>
<tr>
<td>Unemployment</td>
</tr>
<tr>
<td>Vacancies</td>
</tr>
<tr>
<td>V-U ratio</td>
</tr>
<tr>
<td>Output</td>
</tr>
</tbody>
</table>

Notes: The table reports steady-state values. First columns correspond to the baseline parameters, the third column has $\delta = 0.187$ and adjusts $\kappa = 0.047$ (to maintain unemployment fixed).

### 3.2 Welfare comparisons

The union maximizes the welfare of all workers in the economy, thus internalizing the general equilibrium effects of its wage demands. Nevertheless, the unionized economy generally departs from efficiency. Even in the simple one-period example, there is a source of inefficiency: there is less than fully efficient job creation because the union also cares about redistributing resources from firms to workers, and given that it only has one instrument—the wage on old as well as new jobs—full efficiency could not be achieved. In the dynamic model, another source of inefficiency appears whenever the union lacks commitment: there is a long-run loss from suboptimal job creation.

\textsuperscript{12}It is intuitive that if the period length approaches zero, the lack of union commitment will lead to 100% unemployment. It is not exactly true that if the period length approaches infinity, the commitment solution obtains, however, as the commitment solution generally involves a time-varying policy, while here policy is fixed within a period.
How large are the welfare losses resulting from the labor union presence? To shed light on this question, we study the transitional dynamics of an economy with a labor union which cannot commit to future wages. Starting from steady state, we ask: 1) what would happen if the union gained commitment, and 2) how do these outcomes differ from the efficient response? Figure 1 illustrates the responses of the vacancy-unemployment ratio and employment in the two cases. As is clear from the pictures, the dynamics of market tightness $\theta$ reflect our analytical results above: with sudden commitment the union would maintain a low $\theta$ the first period—it is slightly above that of the no-commitment, steady-state starting point—but then have a fully efficient $\theta$ beginning in the consecutive period.\textsuperscript{13} Consequently, the dynamics are rather fast, in the sense that in a few periods the efficient and commitment-union economies both have unemployment very close to the efficient steady-state rate. The figure contrasts outcomes in the annual calibration (on the right) with those in a monthly calibration (on the left). The difference between the efficient response and the commitment-union response is naturally greater in the annual calibration, where also the first period is longer.

How large are the effects on welfare? The present value of output at time zero on these three transition paths are as follows: in the annual calibration, the planner’s response yields present value 2.54, attaining commitment gives present value 2.51, while remaining at no commitment gives 2.45. In terms of the per-period increase in output from attaining efficiency, these figures translate to 4.07%, while the increase from attaining commitment is 2.61%. For comparison, in the monthly calibration these numbers are 32.7% and 31.9%, respectively. Attaining commitment leads to non-trivial welfare gains in both cases, but the gains are much larger in the monthly calibration because the no-commitment outcome is substantially worse in that case. The difference between attaining commitment and attaining efficiency is larger in the annual calibration, however, as the initial adjustment period is longer in that case.

\textsuperscript{13}From the Euler equations for the commitment and no-commitment cases, a comparison reveals that it is not clear that the dynamics will be monotone. It turns out to be the case in the graph but it could have turned out instead that initial tightness would decrease slightly the first month before jumping up to the steady-state level.
Figure 1: Adjustment Dynamics when Union Gains Commitment versus Efficient Response

Notes: The figure plots adjustment dynamics starting from the steady state where the union cannot commit. The figure shows: how the planner would respond, how the union would respond if it gained commitment, and how the union would respond if it did not.

### 3.3 Aggregate shocks

The dynamics of the model under efficiency are well known, but how do these dynamics change when the labor market has a monopoly union? To answer this question, one can study deterministic transition to steady state. However, it appears more empirically interesting to compare economies that actually feature recurring fluctuations. The standard way of conducting this kind of analysis is that pioneered in Pissarides (1985) and revisited in Shimer (2005). A question of interest here is whether the amplification of productivity shocks on the labor market is stronger in the monopoly-union economy than in the basic model as calibrated in Shimer (2005). As we shall see, it is.

One could think of various kinds of shocks perturbing the economy over time. For purposes of illustration, the most obvious shock to consider is one to productivity $z$. It is straightforward to extend the setup above to allow $z$ to follow a Markov process. A union that cannot commit to future wage setting in this environment will, as in the analysis above, play a dynamic game with its future counterparts, though the game here will be stochastic. As before, it is natural
to focus on Markov-perfect equilibria. Thus, \( \Theta(n, z) \) now depends on productivity, as

\[
\Theta(n, z) \equiv \arg \max_\theta -\frac{n\kappa}{q(\theta)} + (n + \mu(\theta)(1 - n))z + (1 - n)(1 - \mu(\theta))b - \theta(1 - n)\kappa \\
+ \beta E_z \tilde{V}(N(n, \theta), z'),
\]

where the continuation value \( \tilde{V} \) satisfies the recursive equation

\[
\tilde{V}(n, z) = (n + \mu(\Theta(n, z))(1 - n))z + (1 - n)(1 - \mu(\Theta(n, z)))b - \Theta(n, z)(1 - n)\kappa \\
+ \beta E_z \tilde{V}(N(n, \Theta(n, z)), z').
\]

It is straightforward, along the lines above, to derive the generalized Euler equation for this case as well. It reads

\[
\frac{\kappa}{\mu'(\theta)}(1 - \frac{n}{1 - n} \frac{q'(\theta)}{q(\theta)^2}) = z - b + \beta(1 - \delta)E_z \left[ (1 - \mu(\theta) + \theta\mu'(\theta)) \frac{\kappa}{\mu'(\theta')} (1 - \frac{n'}{1 - n'} \frac{q'(\theta')}{q(\theta')^2}) \right. \\
+ \left. \left( \Theta_n(n', z')(1 - n') - \theta' \right) \left( -\frac{n'}{1 - n'} \frac{q'(\theta')}{q(\theta')^2} \right) \right],
\]

thus differing only in that there is an expectations operator in front of all the marginal future payoffs.

The model is calibrated as the deterministic economy, and our numerical method easily extended to cover the shock case.\(^{14}\)

We first look at impulse responses. Figure 2 plots the impulse responses of wages, \( \theta \), unemployment and output, comparing the unionized economy (solid line), to the efficient outcome (dashed line). Note that the wage response in the efficient outcome refers to the wage which would implement the efficient allocation, i.e., which gives firms exactly the amount of surplus in matching to induce them to create the efficient amount of vacancies. Again, the right panel displays the annual calibration, while the left displays a monthly calibration meant to

\(^{14}\)See the appendix for details. In brief, the numerical solution is recursive: one can first solve for deterministic dynamics in the state \( n \) and, as a function of that, for responses to \( z \).
highlight the effects.

Figure 2: Impulse responses: efficient versus union
Notes: The figure plots impulse responses to a one percent positive productivity shock, when labor productivity follows an AR(1) process.

As can be observed, in the short run, the union acts so as to introduce “real wage stickiness” into the dynamics. This occurs because a positive productivity shock leaves the level of employment too low compared to what the higher productivity would imply. Although employment soon rises due to increased vacancy creation, the low employment level curbs the union’s distortionary motive to raise wages. Wages thus appear sticky in the very short run, while vacancy creation consequently overshoots. We see, in line with insights in the literature following Shimer (2005), that the response of tightness to a wage change can be substantial. As a result, the percentage responses to a shock are generally to amplify responses (apart from wages, where the effects are roughly zero).\(^{15}\) Generally speaking, the effect of the monopoly union on dynamics is stronger when employment is higher, because a large number of pre-existing matches gives the union incentives that are different from those in the efficient allocation.

Table 4 below reports simulated moments. As we can see, the dynamics are different from the

\(^{15}\)The absolute responses vary, however, as there is amplification in unemployment and output but a certain dampening in \(\theta\) and \(v\).
efficient allocation and in a direction that helps us understand the data: productivity shocks are propagated throughout the economy with stronger amplification for market tightness and unemployment.

Table 4: Effect of Union on Volatility of Aggregate Variables

<table>
<thead>
<tr>
<th>Volatility</th>
<th>1-month horizon</th>
<th>1-year horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Efficient</td>
<td>Union</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.45%</td>
<td>0.91%</td>
</tr>
<tr>
<td>Vacancies</td>
<td>0.67%</td>
<td>3.15%</td>
</tr>
<tr>
<td>V-U ratio</td>
<td>0.87%</td>
<td>3.30%</td>
</tr>
<tr>
<td>Output</td>
<td>1.02%</td>
<td>1.37%</td>
</tr>
</tbody>
</table>

Notes: The table reports standard deviations of model variables relative to the standard deviation of labor productivity, based on simulated data from the model, logged and filtered. Productivity process is an AR(1), with the aggregated, logged and filtered series having standard deviation 1.62% and persistence 0.47 with an annual horizon, and 1.30% and 0.76 with a monthly horizon.

4 Extensions

Two extensions of the setting above are particularly relevant for understanding labor-market institutions: one where some workers are not unionized, and one where employers also act as an entity. We study partial unionization in Section 4.1 and collective bargaining in Section 4.2.

4.1 Partial unionization

In most countries, workers are free to decide on becoming union members. The analysis above does not allow for such a choice. One difficulty in incorporating endogenous membership into the model has to do with the formulation of the union’s objective function. How does membership evolve over time, and how does the endogeneity of the unionization rate affect the incentives of the union when setting wages? The evolution of the unionization rate over time is a focus of some recent work, e.g., Greenwood (2010). Here, we stay short of a full analysis but
nevertheless examine how the key labor-market variables depend on the unionization rate. This analysis offers some preliminary insights into the welfare consequences of policies such as forbidding

In the analysis below, we use $\alpha$ to denote the fraction of workers who belong to the union. We treat $\alpha$ as exogenous, and assume that a worker’s membership status is constant over time. We assume that the union’s objective is to maximize the utility of its members. We also confine attention to steady states. In general, union workers may or may not earn higher wages than non-union workers. A particularly interesting steady state is a case which would make workers indifferent between being unionized and not, because this steady state can be interpreted as allowing workers to choose whether or not to become union members. As we will show, such a steady state exists for some parameter values.

Of course, we need to make clear how wages are determined for non-union workers. It is most natural here to simply adopt the standard assumption in the literature, i.e., one of decentralized Nash bargaining. We let the worker’s bargaining share be denoted $\gamma$. We also need to make an assumption about whether the labor market is segmented by worker type—union vs. non-union—since firms in general are not indifferent about whom to meet. Our assumption is that the worker’s union status is not observable ex ante so that the matching is undirected.

Suppose, thus, that only a share $\alpha$ of workers are unionized, while the rest bargain their wages bilaterally with firms. Both workers search in the same labor market, and firms learn the union status of workers only upon matching. At that time, bargaining occurs (also for previously matched firm-worker pairs).

\[
\begin{align*}
n_t & \text{ given} \\
\text{union sets } w_t & \quad \text{vacancy posting, } v_t & \text{bargaining} & \quad \text{separations} \\
\text{production} & \quad \text{search} & \quad 1 - n_t & \quad v_t 
\end{align*}
\]

In this labor market, the union recognizes the presence of the non-union workers when deciding on wage demands.
4.1.1 Analytical characterization

Beginning with a one-period example to gain intuition, we have that the union maximizes utility per member:

\[
(n + \mu(\theta)(1 - n)) w + (1 - n)(1 - \mu(\theta)) b,
\]

by choice of \( w \) and \( \theta \), subject to the free-entry condition of firms: \( \kappa = q(\theta)[\alpha(z - w) + (1 - \alpha)(1 - \gamma)S] \). Here \( S \equiv z - b \) is the total surplus from a match between a firm and a non-union worker.

To see how the analysis compares to that with full unionization, we again use the free-entry condition to substitute out the wage, \( w = z - \frac{\kappa}{\alpha q(\theta)} + \frac{1 - \alpha}{\alpha}(1 - \gamma)S \), in the union objective, obtaining a maximization problem in \( \theta \) only:

\[
\max_{\theta} \left( n + \mu(\theta)(1 - n) \right) \left( z - \frac{\kappa}{\alpha q(\theta)} + \frac{1 - \alpha}{\alpha}(1 - \gamma)S \right) + (1 - n)(1 - \mu(\theta)) b,
\]

also taking as given \( n \). As before, increasing \( \theta \) increases employment, but at the cost of the lost wage income on new and existing workers required to raise \( \theta \). The expression differs from the one before for two reasons. First, the union wage now has a more limited impact on vacancy-creation, because of the non-union workers among the pool of unemployed. Raising \( \theta \) through the union wage thus requires giving up more wage income: \( \frac{\kappa}{\alpha q(\theta)} \) is greater with \( \alpha < 1 \). This works to reduce \( \theta \) and raise the union wage, compared to the fully unionized case. Second, the tradeoff between the union wage and \( \theta \) also depends on the firms’ surplus from matching with non-union workers, \((1 - \gamma)S = (1 - \gamma)(z - b)\), in proportion with their prevalence among the unemployed, \((1 - \alpha) / \alpha\). If the non-union surplus is large (relative to the union surplus), the union can target a higher \( \theta \) without giving up as much in wages, which works to raise both \( \theta \) and the union wage. In the next section, we illustrate how these effects manifest themselves in labor market outcomes.

In a fully dynamic setting, non-union workers and firms operate according to the usual
Bellman equations:

\[
U_t = \mu(\theta_t)E_t + (1 - \mu(\theta_t))(b + \beta U_{t+1}),
\]

\[
E_t = w^n_t + \beta\delta U_{t+1} + \beta(1 - \delta)E_{t+1},
\]

\[
J_t = z - w^n_t + \beta(1 - \delta)J_{t+1},
\]

where \( U_t \) is the value of an unemployed worker, \( E_t \) the value of an employed worker, \( J_t \) the value of a filled job, and \( w^n_t \) the wage of a non-union worker. Based on these equations, the non-union worker-firm match surplus, defined as \( S_t = E_t + J_t - b - \beta U_{t+1} \), satisfies

\[
S_t = z - b + \beta(1 - \delta)(1 - \mu(\theta_{t+1})\gamma)S_{t+1},
\]

where bilateral wage bargains imply \( J_t = (1 - \gamma)S_t \), and \( E_t - b - \beta U_{t+1} = \gamma S_t \).

The zero-profit condition for firms reads

\[
\kappa = q(\theta_t)[\alpha \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s z - W_t] + (1 - \alpha)(1 - \gamma)S_t.
\]

Clearly, firms realize that union workers require a present value of wages of \( W_t \), while non-union workers yield the firm a net profit of \( (1 - \gamma)S_t \).

Using the free-entry condition to substitute out wages in the union objective yields

\[
\sum_{t=0}^{\infty} \beta^t [(n_t + \mu(\theta_t)(1 - n_t))z + (1 - \mu(\theta_t))(1 - n_t)b - \theta_t(1 - n_t)\frac{\kappa}{\alpha} + \frac{1 - \alpha}{\alpha}(1 - \gamma)\mu(\theta_t)(1 - n_t)S_t] - \frac{n_0\kappa}{\alpha q(\theta_0)} + \frac{1 - \alpha}{\alpha}(1 - \gamma)n_0S_0.
\]

The objective here is the dynamic extension of the objective in the one-period example above, with terms slightly reordered. Also, \( S_t \) is of course not exogenous here (it was equal to \( z - b \) in the one-period model) because it depends on future surpluses as well.
We consider the case of no commitment again and extend the Markov-perfect equilibrium
definition to cover a general value of $\alpha$. The equilibrium will have the functions $\Theta(n)$, for
market tightness, $\tilde{V}(n)$, for the indirect utility of union members, and $S(n)$, the total surplus
of the match between a firm and a non-union worker. Note that no new state variable is
needed. The functions satisfy the following functional equations:

$$S(n) = z - b + \beta(1 - \delta)(1 - \mu(\Theta(n)))\gamma S(N(n, \Theta(n))),$$

$$\tilde{V}(n) = (n + \mu(\Theta(n))(1 - n))z + (1 - \mu(\Theta(n)))(1 - n)b - \Theta(n)(1 - n)\frac{\kappa}{\alpha}$$
$$+ \frac{1 - \alpha}{\alpha}(1 - \gamma)\mu(\Theta(n))(1 - n)S(N(n, \Theta(n))) + \beta\tilde{V}(N(n, \Theta(n))),$$

and

$$\Theta(n) = \arg \max_{\theta}(n + \mu(\theta)(1 - n))z + (1 - \mu(\theta))(1 - n)b - \theta(1 - n)\frac{\kappa}{\alpha} - \frac{n\kappa}{\alpha q(\theta)}$$
$$+ \frac{1 - \alpha}{\alpha}(1 - \gamma)(n + \mu(\theta)(1 - n))S(N(n, \theta)) + \beta\tilde{V}(N(n, \theta)).$$

It is straightforward to derive the functional first-order condition for the union here, but
it is more complex since it contains both $S(N(n, \theta))$ and $S'(N(n, \theta))$, which cannot be
eliminated with simple substitution as in the case $\alpha = 1$. We therefore proceed directly to
the quantitative analysis.

4.1.2 Quantitative results

We calibrate as in the benchmark case and vary $\alpha$ and $\gamma$ to illustrate the workings of the
model. The numerical analysis uses the same methods as above, with the mere difference
that there is an additional unknown function $S$.\textsuperscript{16}

\textsuperscript{16}For details, see the Appendix.
Figure 3 plots the vacancy-unemployment ratio, wages, unemployment, and output as a function of the unionization rate $\alpha$ when worker bargaining power $\gamma$ has a relatively low value.\textsuperscript{17} In this case, firms pay non-union workers lower wages than union workers, making non-union workers more profitable to firms. The figure contrasts outcomes with the model calibrated to a monthly, versus annual, frequency. The annual calibration, on the right, illustrates the first mechanism discussed: higher unionization increases employment and output, as the union internalizes the effects of its wage demands on the labor market. In this case greater unionization brings the economy closer to efficiency.

In the monthly calibration, on the left, the union’s commitment problem is more severe (reflected in lower vacancy creation than on the left). Here the second mechanism discussed becomes dominant for vacancy creation: the presence of non-union workers in the pool of unemployed helps mitigate the adverse effects of the commitment problem. As a result, greater unionization reduces employment and output, taking the economy farther away from efficiency.

Figure 3: Labor markets when non-members are poor bargainers

\textit{Notes:} The figure plots outcomes as a function of the unionization rate $\alpha$ for $\gamma = 0.7$.

Figure 4 turns to the case where workers are good bargainers on their own, showing that a

\textsuperscript{17}The specific value is 0.7; qualitatively, the graphs do not change if $\gamma$ is lowered further.
higher unionization rate now globally encourages vacancy-creation, leading to lower unemployment and greater output.

1-month horizon

1-year horizon

Figure 4: Labor markets when non-members are good bargainers

Notes: The figure plots outcomes as a function of the unionization rate $\alpha$ for $\gamma = 0.95$.

Interestingly, the figure shows that there is a level of unionization such that workers earn the same wages whether unionized or not. That is, if given the choice between becoming a union member or not, they would be indifferent. This steady state can be interpreted as the equilibrium outcome when workers can choose, at time zero, whether or not to be unionized. For these parameter values, the steady-state union wage response to an increase in $\alpha$ can be non-monotonic. For very low unionization rates the union wage is high, and falling in $\alpha$ (for the same reason explained above), but eventually starting to rise again. This last part can be understood by noting that, when non-union workers earn high wages (relative to union workers), the union may find it optimal to moderate its wage claims to prevent employment from falling. As the unionization rate rises, these non-union workers become less important for vacancy creation, however, allowing the union to raise its wage.

The previous example demonstrates that requiring all workers to be unionized—or covered by the union wage—can be welfare improving. Interpreting the intermediate value of $\alpha$ where wages are the same for union and non-union workers as an equilibrium where workers can
choose membership status: for that value of \( \alpha \), the outcome with forced union membership would be better (and outlawing unions worse), in a steady-state sense. For the economy depicted first, the situation is of course the reverse. There, workers would all choose to become unionized, leading to \( \alpha = 1 \), but outlawing unions would be a good idea.

4.2 Collective bargaining

We can generalize the monopoly union framework to collective bargaining between a labor union and an employers’ association, using a “right-to-manage” approach. Right-to-manage refers to firms having the right to decide on hiring independently, taking as given wages that are centrally bargained between the labor union and the employers’ association. In the Mortensen-Pissarides framework this translates to hiring being determined by the usual free entry condition, based on the bargained wages. Proceeding directly to the fully dynamic model, we adopt the same union objective in equation (3), and assume the employers’ association maximizes the present value of profits accruing to firms in equation (9). We look at both (joint) commitment and lack thereof.

4.2.1 Commitment

We now denote the bargaining power of the labor union vis a vis the employers’ association by \( \gamma \). With commitment to future wages, the collective bargaining problem solves the problem

\[
\max_{\{w_t, \theta_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t [ (n_t + \mu(\theta_t)(1 - n_t))w_t + (1 - n_t)(1 - \mu(\theta_t))b] \right\}^{\gamma} \{ n_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t[z - w_t] \}^{1 - \gamma}
\]

subject to the law of motion (1) and the free-entry condition (2). Note that, as before, the free-entry condition implies that the employers’ association objective reduces to representing initial matches only.\(^{18}\)

\(^{18}\)One could go into more detail in specifying alternative threat points in this bargaining problem, but we refrain to simply outlining the broader approach here.
To simplify, this bargaining problem can then be rewritten as a choice of a sequence of $\theta_t$'s. Using the free-entry condition, we arrive at

$$\max_{\{\theta_t\}_{t=0}^{\infty}} \left\{ -\frac{n_0\kappa}{q(\theta_0)} + \sum_{t=0}^{\infty} \beta^t \left[ (n_t + \mu(\theta_t)(1-n_t))z + (1-n_t)(1-\mu(\theta_t))b - \theta_t(1-n_t)\kappa \right] \right\}^{\gamma} \frac{n_0\kappa}{q(\theta_0)}^{1-\gamma}$$

subject to the law of motion (1).

For thinking about how the solution differs from the monopoly union case, it is useful to note that future values of $\theta_t$ only enter the union objective, not the employers’ association objective. Given this, one could equally well follow the earlier approach of reformulating the objective as

$$\left\{ -\frac{n_0\kappa}{q(\theta_0)} + (n_0 + \mu(\theta_0)(1-n_0))z + (1-n_0)(1-\mu(\theta_0))b - \theta_0(1-n_0)\kappa + \beta V(\cdot) \right\}^{\gamma} \frac{n_0\kappa}{q(\theta_0)}^{1-\gamma}$$

where $V(n)$ solves the recursive form of the planner’s problem in equation (4). The solution to this planner’s problem has $\theta$ constant at the efficient level, with $V(n)$ linear and increasing in $n$. The bargaining problem gives a different $\theta_0$ in the initial period, however, depending on the bargaining power of the union vis a vis the employers’ association. The employers’ association moderates union wage demands, which translates into increased hiring. In fact, one can show that as union power $\gamma$ declines, $\theta_0$ increases from the monopoly union level.

**Proposition 3.** When able to commit to future wages, collective bargaining attains efficient vacancy creation after the initial period. In the initial period, vacancy creation is efficient if $n_0 = 0$. If $n_0 > 0$, vacancy creation increases as union power falls, and is generically inefficient.
4.2.2 Without commitment

We can adapt the right-to-manage formulation to the case of no commitment to future wages as follows. As before, we have an accounting equation for the continuation value

\[
\tilde{V}(n) = (n + \mu(\Theta(n))(1 - n))z + (1 - n)(1 - \mu(\Theta(n)))b - \Theta(n)(1 - n)\kappa + \beta\tilde{V}(N(n, \Theta(n)));
\]

where

\[
\Theta(n) := \arg\max_{\theta} \left\{-\frac{n\kappa}{q(\theta)} + (n + \mu(\theta)(1 - n))z + (1 - n)(1 - \mu(\theta))b \right.
\]

\[
- \theta(1 - n)\kappa + \beta\tilde{V}((1 - \delta)(n + \mu(\theta)(1 - n)))\right\}^\gamma \left\{\frac{n\kappa}{q(\theta)}\right\}^{1-\gamma}.
\]

This differs from the monopoly union case only in that the choice of \(\Theta(n)\) is now determined based on the bargaining problem instead of maximizing the union objective alone.

We proceed immediately to a numerical illustration, computed as in Section 4.1. Figure 5 plots the outcomes for key labor-market variables as a function of the labor union’s bargaining power \(\gamma\), over a range where steady-state unemployment takes on values both above and below the efficient level. As the the figure shows, the stronger is the bargaining power of the labor union, the higher are union wages, leading to higher unemployment and lower output. Moreover, the collective bargaining outcome is efficient for an intermediate value of \(\gamma\).

5 Conclusions

[TO BE WRITTEN]
Figure 5: Labor market with collective bargaining

Notes: The figure plots outcomes as a function of the labor union bargaining power $\gamma$.

References


