Abstract

Recent work (Hu and Black (2005, 2007, 2008); Dekel and Wolinsky (2012)) has detailed the twin issues of empty voting and hidden/morphable ownership. The former arises when an actor has control rights over a corporation but no economic interest in that corporation, such as when an investor borrows shares. The latter arises when a shareholder has an economic or voting interest in a corporation but avoids regulatory disclosure requirements through derivatives transactions. Both offer significant challenges to financial regulatory systems.

The lack of a framework for analyzing this phenomenon has hindered attempts to respond to these challenges. Most economic models of financial markets do not incorporate control rights. And, once control rights are introduced, the standard competitive equilibrium concept is unsatisfactory: We show that once control rights are introduced, competitive equilibria may fail to exist and may be inefficient if they do exist. Allowing the transfer of control rights, such as through transactions in corporate stock, only exacerbates the problem. We show that if control rights may be traded independently of economic ownership rights—as modern derivatives markets increasingly allow—competitive equilibria essentially never exist.

We propose an alternative equilibrium concept, the Core Outcome (“CO”). A CO consists of an allocation of economic ownership, an allocation of control rights, and a set of voting behaviors such that no group of actors can improve the utility of its members via a mutual deviation. We show that COs have several desirable properties as an equilibrium concept: They are always efficient, and can always be reached through voluntary trading. Our model has strong implications for the regulation of securities markets in general and of derivatives markets in particular. We conclude that there are strong efficiency justifications in favor of an effective mandatory disclosure regime. We also find that, in certain circumstances, empty voting can act as a backstop to public regulation, allowing private actors to block socially inefficient actions that public regulation would permit.

*We are grateful to Anat Admati, Muriel Niederle, and Paul Pfleiderer for helpful discussions. Any comments or suggestions are welcome and may be emailed to jbarry@sandiego.edu.
The paper remains a work in progress. Included for submission please find an introductory section that summarizes our results as well as a fully developed theoretical model that includes all of our main results.
1 Overview of Findings

1.1 The Failings of Competitive Equilibrium as a Solution Concept When Control Rights Are Present

Recently, a great deal of attention has been devoted to the phenomena of hidden and morphable ownership and of empty voting (Martin and Partnoy (2005); Hu and Black (2005, 2007, 2008); Brav and Mathews (2011); Dekel and Wolinsky (2012)). The former arises when an owner of public securities uses derivative transactions to avoid the sweep of federal securities disclosure laws and conceal the size of her voting interest in a particular firm. The latter arises when an actor has voting rights in a corporate election, but either has no economic interest or a negative economic interest in the firm at issue.

Each of these phenomena has profound implications for our securities laws. Yet consideration of these phenomena, and in particular of empty voting, has largely been confined to observational analysis to date. It has been extremely difficult to conduct a more systematic analysis because, in the context of financial markets with control rights, competitive equilibrium—the most natural equilibrium concept for economic analysis and the one on which the most analyses of markets rely—fails.

A competitive equilibrium is essentially a set of prices at which the market clears. That is, at the prices in question, everyone who wishes to purchase or sell an item may do so. Competitive equilibria generally have several positive features that recommend them as a solution concept. Chief among these are efficiency and, under a fairly wide range of assumptions, existence.

Unfortunately, once the notion of control rights is introduced into financial market models, competitive equilibrium ceases to function well as a solution concept. Even if control rights are not transferable, competitive equilibria may be inefficient; worse, they may not exist at all. Example 1, in Section 2.1.1, provides a formal illustration of how competitive equilibrium may not be efficient, and Example 2, in Section 2.1.2, provides a formal
illustration of how a competitive equilibrium may not exist at all.

In real-world securities markets, control rights are tradable. A share of stock conveys both an economic interest in the underlying corporation as well as the right to cast a vote in certain corporate elections. In models that reflect this reality, competitive equilibrium becomes an even more problematic solution concept. As we show, competitive equilibria generally do not exist in these circumstances unless a firm’s decisions only affect its own value. Given that one firm’s actions tend to have implications for other firms—such as its competitors, suppliers, customers, and producers of complimentary products—it is rare for this condition to be satisfied.

Yet the problem is even worse than that. With modern derivative markets, it becomes ever easier to decouple economic ownership and control altogether and trade them separately. In this instance, we show that, so long as results of any election at any firm affect the value of anything that is traded in the financial markets—such as the shares of the firm itself, shares of a competitor, or commodity prices—a competitive equilibrium never exists.

1.2 An Alternative Solution Concept: The Core Outcome

We introduce a different equilibrium concept for modern financial markets: the Core Outcome. To accurately describe the state of a financial market, it is necessary to specify three things:

1. the allocation of economic ownership of publicly traded firms;

2. the allocation of control rights with respect to corporate elections by those firms; and

3. how each actor exercises her control rights.

These three things also induce a final set of stock prices. A Core Outcome is an allocation of economic and ownership rights and a specification of each actor’s voting behavior such that no group of actors can coordinate their behavior in such a way that each of them is strictly better off.
Essentially, a Core Outcome is one that is stable: there is no subgroup of defectors who can unite and improve their lot, so no one has an incentive to deviate from the Core Outcome. Once a financial market reaches a Core Outcome, it should be expected to stay there. On the other hand, if a state of the market is not a Core Outcome, then there are people ("Potential Defectors") who would be better off deviating from it. Thus, a non-Core Outcome is subject to instability and rapid change—including to another non-Core Outcome that itself may rapidly change.

In addition to stability, Core Outcomes have some very desirable properties as a solution concept, including some of those that make competitive equilibrium such a good solution concept in other contexts. First, Core Outcomes always exist: in particular, we show that, given any initial distribution of securities and control rights, it is possible to reach a Core Outcome. Core Outcomes exist regardless of whether control rights are non-transferable, tradable in a package with ownership interests, or separately transferable.

Moreover, we also show that Core Outcomes are always efficient and that they are neither pathological nor otherwise clearly implausible. Nor are they "knife-edge" portfolios—small alterations in the allocation of economic or voting rights, or of actors' voting behavior, does not invalidate or destabilize the Core Outcome. While Core Outcomes are not unique, the Core Outcome framework still allows for significant predictive power: Given a particular specification of a financial market, all of the Core Outcomes for that financial market share a number of features. Each Core Outcome provides the same total social utility and each corporate election generally produces the same result in each Core Outcome.

1.3 The Social Welfare Implications of Empty Voting Under the Core Outcome Framework

Our results have significant implications for understanding the significance of the growth of derivatives markets. Some have thought that derivatives markets simply enable the market for the underlying security to incorporate information about the underlying security better
and faster. Thus, growing the size of the derivatives market could only increase the efficiency of the market for the underlying security.

Some commentators and pundits have rejected this view, particularly in the wake of the recent financial crisis. This school of thought has argued that the growth of a derivatives market could have other, and potentially negative, consequences for the market for the underlying security. However, this group has lacked a formal economic model that supported their position. Our CO framework provides the first formal support for the notion that the size of the derivative markets for a security can have major and, potentially (but not exclusively), negative consequences for the operation of capital markets.

Before the growth of large derivatives markets, ownership and control in the real world were largely aligned. (Though, of course, there have always been exceptions, such as corporations with classes of stock that differ in their voting and economic rights.) If one assumes a well-functioning public and private regulatory environment, such that the decisions that maximize the value of public firms also maximize social welfare, then any allocation of shares, matched with the right share prices, will generally constitute a Core Outcome. It was not necessary for economic analysis to consider the allocation of economic or voting rights, and there was no concern about empty voting; it was generally sufficient to focus on share prices. This state of affairs largely worked well with the prevailing competitive equilibrium solution concept.

But this view of the financial markets is becoming increasingly untenable. With large, modern derivative markets, ownership and control simply are not one-to-one any longer, and this divergence should only be expected to increase over time. A large derivatives market essentially allows people without an ownership interest in the company to influence its decisions. As both a theoretical and a practical matter, the larger the derivatives market is in comparison to the market for the underlying security, the more influence non-owners have. The theory predicts that there will be empty voting and this has been borne out in
practice. It is no longer enough to focus on prices; market participants, regulators, and observers must keep track of how economic interests and control rights are allocated among actors.

If one assumes that whenever a firm is faced with a shareholder decision, the choice that maximizes the value of the firm also maximizes social welfare, one might be concerned about growth in unregulated derivatives markets. A large derivatives market that lacks disclosure rules increases the likelihood that a corporate election will select an inefficient outcome. This is true because, the larger the derivatives market, the easier it is for those who favor a corporate election outcome that does not maximize the value of the corporation to affect the outcome of that election. And since, by assumption, the optimal result for the corporation is also best for society, larger derivative markets can have negative effects on social efficiency.

However, there are other instances in which a large derivatives market may increase efficiency. For example, consider a case in which the choice that is best for the corporation at issue is not best for society. This could easily happen in the event of regulatory failure—for instance, if antitrust regulators erroneously allow two duopolists to merge into a single monopoly firm or environmental regulations do not force a firm to take into account the full amount of the externalities that its actions will impose. In such a situation, a large derivatives market can increase efficiency by making it easier for those actors who would be hurt by the company’s actions to influence the company’s behavior toward a more socially beneficial outcome. Thus, a large derivatives market essentially facilitates a private backstop to public regulation: it can increase private actors’ ability to block socially inefficient actions that public regulation would allow.

One of the instances in which researchers have documented empty voting affecting the outcome of a corporate election fits this mold. A merger had been proposed that offered significant potential and expected efficiency gains. However, the merger required a favorable vote by the acquiring firm’s shareholders, and changes in market conditions since the an-

\footnote{See, for instance, the many examples documented by \textit{Hu and Black} (2005, 2008).}
nouncement of the deal rendered the acquisition price too high. Thus, while the acquisition
remained socially optimal, it was no longer in the interests of the acquiring firm’s shareholders. A shareholder in the target used derivatives market transactions to sway the election in favor of the acquisition, producing the socially optimal result.

There may also be indirect efficiency properties with respect to the allocation of control
and information rights. Shareholders receive signals about economic conditions both within
and outside of the firm, and make decisions based on them. The quantity and quality of
these signals differ across various shareholders. By separating voting and ownership, large derivative markets allow shareholders to allocate decision rights among people in order to maximize social value ex ante, based on the quality and quantity of signals that different shareholders are expected to receive. Similarly, if it is costly to acquire information, large derivatives markets can facilitate information gathering by particular shareholders. The benefits to a shareholder from information gathering increase in proportion to the shareholder’s ownership in the corporation. A large derivatives market can lead to an increase in information gathering because it allows for shareholders to have a larger economic interest in the corporation than would otherwise be possible. For example, if shareholders engage in derivatives transactions that significantly increase their economic ownership in the corporation, there could be multiple shareholders that possess an economic interest equivalent to holding a majority of the company’s outstanding shares.

Empty voting is not necessary to achieve these benefits. In theory, they can all be achieved through other means. Nonetheless, empty voting may offer significant efficiency benefits in practice. Agents’ actions may be constrained by limitations on the size of the portfolios they can hold, credit constraints, the presence of poison pills, or other factors. Empty voting offers another mechanism through which these actors may pursue these goals and avoid transaction costs.
1.4 Implications for Securities Regulation Policy

Viewing financial markets through the Core Outcome analytical framework provides a strong justification for mandating disclosure in derivatives markets. The key intuitive difference between Core Outcomes and competitive equilibria is that, in a competitive equilibrium, anyone is permitted to buy or sell as much of an economic interest (or a control right, to the extent they are transferable) at a fixed unit price. This price is based on actors’ expectations as to how corporate control rights will be exercised. The reason that competitive equilibria do not exist when control rights are transferable is that, even when a financial market reaches an efficient result, a person with control power still has the opportunity to profit by shifting the market away from that result: First, she can engage in trades that take advantage of the difference between the value a security is anticipated to have based on the expected decision and the value it would have if the corporation made a different decision. Then, using her control power, she can change the decision away from what was expected and reap large profits. The greater the extent to which control rights are transferable, the easier it is for any particular actor to acquire a sufficient degree of control rights to make this strategy viable.

While this strategy generates large profits for the person with control, when this strategy shifts the financial market away from an efficient result, those profits come at the expense of that actor’s counterparties. If those counterparties had realized what the actor acquiring control was doing, they would not have agreed to transact. However, this possibility is not captured in the competitive equilibrium concept, which allows unlimited purchases and sales at the relevant price. In the Core Outcome framework, this strategy is severely restricted because the actors that the person acquiring control power needs to join his coalition (as his counterparties) will not be willing to do so because they correctly anticipate that they will be worse off if they do. This strategy is only viable in the Core Outcome context if it moves the market to a more socially efficient result.

This insight strongly supports mandating disclosure. If the would-be holder of control
power must publicly disclose her actions, it will be very difficult for her to able to find the counterparties she needs to execute this strategy. In that case, we can be confident that we will reach and remain at a Core Outcome, a stable and efficient result.

In addition, the theory predicts that, in order for actors to reach a Core Outcome through voluntary trading, they must know what the other shareholders’ economic interests and control rights are. The intuition is that, when actors are aware of others’ holdings, it becomes less likely that actors will be surprised to experience an outcome that is dominated by a more efficient outcome that could have been achieved if they had more fully understood the situation they faced. The most plausible way for actors to be informed about other actors’ holdings is a mandatory disclosure regime. Thus, disclosure is necessary to deal with the issue of hidden/morphable ownership, as well.

If disclosure is not possible or is otherwise imperfect, the Core Outcome framework could also provide support for some sort of substantive regulatory intervention to prevent inefficient outcomes. This could include barring those actors who hold a net negative interest in a corporation from exercising control rights. This approach could make sense if one assumes that the decisions that maximize the firm’s value also maximize social utility. However, this approach would actually encourage inefficient results when the result that is best for the firm at issue is not socially optimal.

2 Model

There is a finite set of agents $I$. There is a set of firms $F$, each of which has a unit measure of stock available; let the (possibly negative) fractional economic ownership of firm $f \in F$ held by agent $i \in I$ be denoted $s^i_f \in \mathbb{R}$.\footnote{Note that we do not place any explicit bounds on share ownership. The analysis is unchanged so long as $s^i_f$ may range across an interval containing $[0,1]$ for all agents $i \in I$ and firms $f \in F$.} Hence, each agent has a portfolio $s^i \equiv (s^i_f)_{f \in F}$, and
a portfolio profile is a vector of agent portfolios \((s^i)_i \in I\) such that
\[
\sum_{i \in I} s^i_f = 1 \text{ for all } f \in F.
\]
We assume that agents are initially endowed with shares, that is, there is an initial endowment \((e^i_f)_i \in I, f \in F\) portfolio profile.

There is also a set of shareholder motions \(M\). Each shareholder motion \(m \in M\) represents a binary decision by a firm \(\delta_m \in \{0, 1\}\). This decision may affect the value of that firm, and also may impact the values of other firms. For instance, a motion \(m\) may represent a firm’s decision to enter a market; if entry occurs, then existing firms in that market face additional competition.

Each agent \(i\) submits a ballot \(\beta^i_m \in [0, 1]\) on each motion \(m\); we call the matrix \(\beta\) a ballot profile. Furthermore, for each \(m \in M\) there exists a profile of control rights \((r^i_m)_i \in I\) such that \(r^i_m \geq 0\) for all \(i \in I\) and \(\sum_{i \in I} r^i_m = 1\).\(^3\) We say that \(m\) passes—\(\delta_m(\beta) = 1\)—if and only if \(\sum_{i \in I} r^i_m \beta^i_m \geq \alpha_m\), where \(\alpha_m \in [0, 1]\) is the minimum percentage of votes required for passage of motion \(m\), and that \(m\) fails—\(\delta_m = 0\)—otherwise. After voting, the value of firm \(f\) is given by \(v_f(\delta(\beta))\).

An outcome \(\langle s, \beta \rangle\) is a portfolio profile \(s\) and ballot profile \(\beta\). The utility of agent \(i\) for an outcome \(\langle s, \beta \rangle\) is given by
\[
 u^i(\langle s, \beta \rangle) \equiv \sum_{f \in F} s^i_f v_f(\delta(\beta)).
\]

An outcome \(\langle s, \beta \rangle\) is efficient if
\[
 \beta \in \arg \max_{\hat{\beta} \in [0, 1]^{I \times M}} \sum_{i \in I} \sum_{f \in F} s^i_f v_f(\delta(\hat{\beta})) = \arg \max_{\hat{\beta} \in [0, 1]^{I \times M}} \sum_{f \in F} v_f(\delta(\hat{\beta})).
\]

We let the column vector \((p_f)_{f \in F}\) denote the prices at which each of the firms trades

\(^3\)For now, we assume that the distribution of control rights is fixed; we relax this assumption in Section 3.
initially. An arrangement \([e; \langle s, \beta \rangle; p]\) is an initial endowment \(e\), an outcome \(\langle s, \beta \rangle\), and a price vector \(p\).

Given an initial endowment \(e\), an arrangement \([e; \langle s, \beta \rangle; p]\) induces a utility for agent \(i\) of

\[
\tilde{u}^i([e; \langle s, \beta \rangle; p]) \equiv \sum_{f \in F} (s^i_f v_f(\delta(\beta)) - p_f(s^i_f - e^i_f)).
\]

An arrangement \([e; \langle s, \beta \rangle; p]\) is efficient if the associated outcome \(\langle s, \beta \rangle\) is efficient.

### 2.1 Competitive Equilibrium

The demand correspondence \(D^i(e; \beta; p)\) for agent \(i\), given the ballot matrix \(\beta \in [0, 1]^{I \times M}\), is given by

\[
D^i(e; \beta; p) \equiv \arg \max_{s \in \mathbb{R}^{I \times F}} \tilde{u}^i([e; \langle s, \beta \rangle; p])
\]

and the demand correspondence for the entire economy, \(D(e; \beta; p)\), is given by

\[
D(e; \beta; p) \equiv \bigcap_{i \in I} D^i(e; \beta; p).
\]

We now define the concept of competitive equilibrium.

**Definition 1.** A competitive equilibrium, given an initial endowment \(e\), is an arrangement \([e; \langle s, \beta \rangle; p]\) such that

1. \(s \in D(e; \beta; p)\);

2. \(\beta\) is a Nash equilibrium in undominated strategies of the induced voting game.\(^4\,^5\)

\(^4\)Technically, the Nash equilibrium under consideration may be mixed; in this case, the expression \(\tilde{u}^i([e; \langle s, \beta \rangle; p])\) must be replaced by the analogous expression in terms of expected utilities. As agents are risk-neutral, this concern is immaterial to the analysis, and so we suppress it for ease of exposition.

\(^5\)This assumption ensures that each agent votes as to maximize the value of his portfolio. As is common in voting games, we impose the restriction that agents vote as-if-pivotal.
3. there is no agent $i$ and $(\hat{s}^i, \hat{\beta}^i)$ such that
\[ \tilde{u}^i([e; \langle (\hat{s}^i, s^{-i}), (\hat{\beta}^i, \beta^{-i}) \rangle; p]) > \tilde{u}^i([e; \langle s, \beta \rangle; p]). \]

Condition 1 states that the demand correspondence is non-empty; that is, each agent is demanding an optimal portfolio given the equilibrium voting behavior. Furthermore, Condition 1 ensures that markets clear, as the demand correspondence is only non-empty if supply of stock for each firm $f$ equals demand. Condition 2 ensures that each agent is voting optimally, given his stock holdings and the ballot profile of other agents. Finally, Condition 3 states that no agent can strictly increase his utility by both changing his portfolio and his ballot. Formally, Condition 1 follows from Condition 3; however, we retain Condition 1 as it encapsulates the standard definition of competitive equilibrium.

### 2.1.1 Inefficiency

We show that competitive equilibria, when they exist, may be inefficient.

**Example 1.** Suppose there are two firms, $F = \{f, g\}$ and four agents $\{i, j, k, \ell\}$. There is one motion $m$ which affects the value of both firms, and at least $\alpha_m = \frac{1}{2}$ of the votes are necessary for passage. Let

<table>
<thead>
<tr>
<th>$\delta_m$</th>
<th>0</th>
<th>1</th>
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<tbody>
<tr>
<td>$v_f(\delta_m)$</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>$v_g(\delta_m)$</td>
<td>6</td>
<td>21</td>
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Now let $r_m = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)$, where we use the convention that vectors’ coordinates are listed in alphabetical order. Let the endowment $e$ be given by

\[ e = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]

We now show that $[e; \langle e, (0, 0, 0, 1) \rangle; (16, 6)^T]$ is a competitive equilibrium. The first
condition for competitive equilibrium is clearly satisfied, as the price of a share of either
firm is equal to its value given the voting profile. Furthermore, each agent is voting so as
to maximize the value of his shares; hence, the second condition for competitive equilibrium
holds. Finally, note that no one agent changing his vote changes $\delta_m$, and so the third
condition for competitive equilibrium is vacuously satisfied given that the first condition is.

However, the outcome $\langle e, (0, 0, 0, 1) \rangle$ associated with this arrangement is not efficient, as

$$v_f(\delta((0, 0, 0, 1))) + v_g(\delta((0, 0, 0, 1))) = 22 < 29 = v_f(\delta((1, 1, 1, 1))) + v_g(\delta((1, 1, 1, 1))).$$

Intuitively, the competitive equilibrium in Example 1 is inefficient since each agent (correctly)
believes that his vote will not be pivotal, and so long as each of $i$, $j$, and $k$ holds his
original portfolio, each of them is both holding an optimal portfolio and voting optimally.
Hence the inefficient arrangement $[e; \langle e, (0, 0, 0, 1) \rangle; (16, 6)^T]$ satisfies the definition of com-
petitive equilibrium given in Definition 1. Switching to the more efficient voting outcome
$\delta_m = 1$ would require coordination amongst the agents, and such coalitional deviations are
not considered by the competitive equilibrium solution concept. However, this problem is
not alleviated by allocating the control rights to a single agent; in that case, competitive
equilbria may not exist at all, as we show in the next section.

2.1.2 Non-Existence

We now show that competitive equilibria may not exist.

Example 2. Let $F = \{f, g\}$ and $I = \{i, j\}$. There is one motion $m$ which affects the value
of both firms, and at least $\alpha_m = \frac{1}{2}$ of the votes are necessary for passage. Let

<table>
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<tbody>
<tr>
<td>$v_f(\delta_m)$</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>$v_g(\delta_m)$</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>
Now let \( r_m = (1, 0) \) where we again use the convention that vectors’ coordinates are listed in alphabetical order. Let the endowment \( e \) be given by

\[
e = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2}
\end{pmatrix}.
\]

Now consider any arrangement of the form \([e; \langle s, \beta \rangle; p]\) where \( \delta_m(\beta) = 0 \). Then, in order for the market for shares to clear, we must have that \( p = (8, 2)^\top \). Now consider the following deviation by agent \( i \). Let \((\hat{s}^i, \hat{\beta}^i) = ((0, \frac{1}{2}), (1))\). Note that \( i \)'s utility under the original arrangement \([e; \langle s, \beta \rangle; p]\) is 5, regardless of \( s \). However, his utility under the arrangement induced by the deviation \([e; \langle (\hat{s}^i, s^{-i}), (\hat{\beta}^i, \beta^{-i}) \rangle; p]\) is 6.

Now consider any arrangement of the form \([e; \langle s, \beta \rangle; p]\) where \( \delta_m(\beta) = 1 \). Then, in order for the market for shares to clear, we must have that \( p = (4, 4)^\top \). Now consider the following deviation by agent \( i \). Let \((\hat{s}^i, \hat{\beta}^i) = ((1, \frac{1}{2}), (0))\). Note that \( i \)'s utility under the original arrangement \([e; \langle s, \beta \rangle; p]\) is 4, regardless of \( s \). However, his utility under the arrangement induced by the deviation \([e; \langle (\hat{s}^i, s^{-i}), (\hat{\beta}^i, \beta^{-i}) \rangle; p]\) is 7.

This example shows that when the control rights are allocated to a single agent, competitive equilibria will typically fail to exist. The central difficulty is that the prices of the various firms must reflect a particular decision by the agent with the control right; otherwise, any agent would be able to generate an arbitrarily large profit by buying or selling an arbitrarily large amount of shares of the firm whose price does not reflect the decision that it is believed will be made. However, under such a scenario, the prices can not reflect the value of the firm should some other decision be made. Hence, the agent with the control right can generate an arbitrarily large profit by both changing the decision which is made and buying or selling shares in order to profit from that decision.

\(^6\)For any other set of prices, the demand correspondence for each agent is empty, as each agent will have unbounded demand. For instance, if \( p_f > 8 \), each agent’s demand for firm \( f \) is unbounded. (This same analysis applies throughout the paper when the price of a share of a firm does not reflect its final value.)
2.2 Core-Compatibility

Definition 2. An outcome \( \langle s, \beta \rangle \) is in the core if there does not exist a set of agents \( J \subseteq I \), and \( \hat{\beta}^J \) such that

\[
\sum_{j \in J} u^j(\langle s, (\hat{\beta}^J, \beta^{-J}) \rangle) > \sum_{j \in J} u^j(\langle s, \beta \rangle).
\]

(1)

The standard definition of the core is that there does not exist a coalition and an action for each member of that coalition such that each member is weakly better off with one member being strictly better off. This definition is equivalent to the standard definition: if a coalition \( J \) and voting profile \( \hat{\beta}^J \) satisfying (1) existed, then they could implement transfers amongst each other and strictly increase the utility of each \( j \in J \).

We say that an initial endowment \( e \) is core-compatible if there exists a core arrangement \([e; \langle s, \beta \rangle; p]\) associated with \( e \), such that the outcome \( \langle s, \beta \rangle \) is in the core and for all \( \hat{\beta} \) and every \( i \in I \),

\[
\tilde{u}^i([e; \langle s, \beta \rangle; p]) \geq \tilde{u}^i([e; \langle e, \hat{\beta} \rangle; p])
\]

holds.\(^7\)

2.2.1 Efficiency

Theorem 1. Core outcomes are efficient.

Proof. Suppose that \( \langle s, \beta \rangle \) is not efficient. Then there exists \( \bar{\beta} \) such that

\[
\sum_{i \in I} u^i(\langle s, \beta \rangle) < \sum_{i \in I} u^i(\langle s, \bar{\beta} \rangle).
\]

But then taking \( J = I \) and \( \hat{\beta} = \bar{\beta} \), we see that (1) is satisfied, and hence \( \langle s, \beta \rangle \) is not in the core.

\[\square\]

Corollary 1. For any core-compatible initial endowment \( e \), the associated core arrangement

\(^7\)Note that we do not impose any restriction on beliefs about balloting, so long as agents' expectations about the balloting are consistent.
2.2.2 Existence

Theorem 2. Every initial endowment is core-compatible.

Proof. Consider an initial endowment vector \( e \). Consider an efficient decision vector \( \delta \) and let \( \beta_m = \delta_m \). Consider some \( \hat{\beta} \) and let \( p_f = v_f(\delta(\hat{\beta})) \). Let \( s \) be any portfolio profile such that \( s^i = s'^i > 0 \) for all \( i \in I \) and \( f, f' \in F \). The utility of agent \( i \) under the initial allocation and voting profile \( \hat{\beta} \) is

\[
\bar{u}^i([e; \langle s, \beta \rangle; p]) = e^i v_f(\delta(\hat{\beta})).
\]

(2)

The utility of agent \( i \) under the arrangement \([e; \langle s, \beta \rangle; p]\) is

\[
\bar{u}^i([e; \langle s, \beta \rangle; p]) = \sum_{f \in F} s^i f v_f(\delta(\beta)) - p_f(s^i f - e^i f)
\]

(3)

\[
= \sum_{f \in F} s^i f v_f(\delta(\beta)) - v_f(\delta(\hat{\beta}))(s^i f - e^i f)
\]

\[
= \sum_{f \in F} s^i f (v_f(\delta(\beta)) - v_f(\delta(\hat{\beta}))) + \sum_{f \in F} v_f(\delta(\hat{\beta})) e^i f.
\]

Hence, the difference between utility under the arrangement \([e; \langle s, \beta \rangle; p]\), given by (3), and the utility under the arrangement \([e; \langle e, \hat{\beta} \rangle; p]\), given by (2), is given by

\[
\sum_{f \in F} s^i f (v_f(\delta(\beta)) - v_f(\delta(\hat{\beta})))
\]

which is non-negative as \( \delta(\beta) \) is efficient and \( s^i f = s'^i f > 0 \) for all \( i \in I \) and \( f, f' \in F \). Hence,

\[
\bar{u}^i([e; \langle s, \beta \rangle; p]) \geq \bar{u}^i([e; \langle e, \hat{\beta} \rangle; p]).
\]

and so \([e; \langle s, \beta \rangle; p]\) is core-compatible.\(^8\)

\(^8\)While in the proof agents always obtain non-negative shares of each firm, there may be (and, in general,
To reach a core outcome, given any endowment, agents may trade stock, taking into account how such trades will change voting incentives. Once at a core outcome, no set of agents may improve their joint utility by changing their voting profile. This analysis illustrates the key distinction between the competitive equilibrium and core-compatibility solution concepts: the competitive equilibrium solution concept does not allow agents to consider how buying or selling stock will affect voting decisions, while core-compatibility explicitly considers such incentives.

To illustrate how an economy with inefficient competitive equilibria progresses to the core, we return to the setting of Example 1. For this example, a core-compatible arrangement is given by

$$
\left[ \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \left\langle \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}, (1, 1, 1, 1) \right\rangle, \begin{pmatrix} 16 \\ 6 \end{pmatrix} \right].
$$

To reach this outcome, agents $i$, $j$, and $k$ each buy $\frac{1}{4}$ of a share of firm $g$ from agent $\ell$ at a price consistent with a vote producing $\delta_m = 0$; agent $\ell$ is happy to engage in such a trade, as doing so changes the voting incentives of agents $i$, $j$, and $k$. Their changed votes increase the value of agent $\ell$’s remaining holdings. Similarly, agent $\ell$ buys $\frac{1}{12}$ of a share of firm $f$ from each of $i$, $j$, and $k$. At this outcome, no agents may profitably deviate, as each agent holds an identical portfolio, and so voting to maximize the surplus of any subset of agents is equivalent to voting to maximize social surplus.\(^9\) Similar analysis identifies equally well-behaved core-compatible arrangements for Example 2.

Although we have heretofore treated each motion as an abstract object, which is not specifically linked to any firm, in practice a motion represents the decision by shareholders of a specific firm; this structure can be modeled in our setting by associating each motion

---

\(^9\)Note that here, for expositional clarity, we have focused on a core outcome in which all agents hold market portfolios. This is not necessary in general—any core outcome can be supported in a core-compatible arrangement.

---

\(^9\)Core arrangements where some agents obtain a negative interest in some firms, and hence empty voting may be an equilibrium phenomenon.
m ∈ M with a specific firm φ(m) ∈ F. With such an identification φ, an outcome ⟨s, β⟩ involves empty voting if there exists an agent i ∈ I and a motion m ∈ M such that

1. r^i_m > 0, and
2. s^i_{φ(m)} ≤ 0.

The first condition states that the agent i has positive control rights over a particular shareholder motion m associated with the firm φ(m), while the second condition states that i has non-positive economic interest in φ(m), which is the notion of empty voting that has been discussed by Hu and Black (2005, 2007, 2008).

For instance, in the context of Example 1, we assume that φ(m) = f, and consider the arrangement

\[
\begin{bmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\begin{bmatrix}
-\frac{1}{8} & -\frac{1}{8} & -\frac{1}{8} & \frac{11}{8} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0
\end{bmatrix},
(1,1,1,1),
\begin{bmatrix}
16 \\
6
\end{bmatrix}.
\]

In this arrangement, all three of i, j, and k engage in empty voting, as they are voting on the shareholder proposal m associated with f while each holding a negative amount of shares in firm f. Furthermore, it can be shown that this is a core-compatible arrangement, and hence this example demonstrates that socially efficient outcomes may be associated with empty voting.

3 Transferrable Control Rights

We now consider the case where control rights are transferrable; that is, where r is an outcome variable. Hence, an outcome now takes the form ⟨s, r, β⟩, where r is a profile of control rights and, as before, s is a portfolio profile and β is a ballot profile. Since r is now endogenous, we explicitly indicate the dependence of the decision vector δ = δ(r, β) on the
profile of control rights \( r \). The utility from an outcome \( \langle s, r, \beta \rangle \) is now given by

\[
    u^i(\langle s, \beta \rangle) \equiv \sum_{f \in F} s^i_f v_f(\delta(r, \beta)).
\]

Agents are now endowed with an initial portfolio of control rights \( (k^i_m)_{i \in I, m \in M} \). We also let the column vector \( (q_m)_{m \in M} \) denote the prices at which control rights for each motion trade. Hence, an arrangement now takes the form \([e, k; \langle s, r, \beta \rangle; p, q]\), where \( e \) and \( k \) are the endowed portfolio profile and profile of control rights, respectively, \( \langle s, r, \beta \rangle \) is an outcome, and \( p \) and \( q \) are the prices for shares and control rights, respectively. Given an initial endowment \( e, k \), an arrangement \([e, k; \langle s, r, \beta \rangle; p, q]\), induces a utility for agent \( i \) of

\[
    \tilde{u}^i([e, k; \langle s, r, \beta \rangle; p, q]) \equiv \sum_{f \in F} (s^i_f v_f(\delta(r, \beta)) - p_f(s^i_f - e^i_f)) - \sum_{m \in M} (q_m(r^i_m - k^i_m)).
\]

### 3.1 Competitive Equilibrium

We now extend our definitions of the demand correspondence and competitive equilibrium to the setting with transferrable control rights. The demand \( D^i(e, k; \beta; p, q) \) for agent \( i \), given a set of ballots \( \beta \in [0, 1]^{I \times M} \), is given by\(^{10}\)

\[
    D^i(e, k; \beta; p, q) \equiv \arg \max_{(s, r) \in \mathbb{R}^{I \times (F \cup M)}} \tilde{u}^i([e, k; \langle s, r, \beta \rangle; p, q])
\]

and the demand correspondence for the entire economy, \( D(e, k; \beta; p, q) \) is given by

\[
    D(e, k; \beta; p, q) \equiv \bigcap_{i \in I} D^i(e, k; \beta; p, q).
\]

We now define the concept of *competitive equilibrium* for the setting where economic control is tradable.

\(^{10}\)Here, the notation "\( \sqcup \)" denotes the disjoint union of two sets.
Definition 3. Now, a *competitive equilibrium*, given an initial endowment \((e, k)\), is an arrangement \([e, k; (s, r, \beta); p, q]\) such that

1. \((s, r) \in D(e, k; \beta; p, q)\),
2. \(\beta\) is a Nash equilibrium in undominated strategies of the induced voting game, and
3. there is no agent \(i\) and \((\hat{s}^i, \hat{r}^i, \hat{\beta}^i)\) such that
   \[
   \tilde{u}^i([e, k; (\hat{s}^i, s^{-i}), (\hat{r}^i, r^{-i}), (\hat{\beta}^i, \beta^{-i})]; p, q]) > \tilde{u}^i([e, k; (s, r, \beta); p, q]).
   \]

Definition 3 is the natural generalization of Definition 1 to the case where control rights are tradable. Condition 1 states that the demand correspondence is non-empty: that is, each agent is demanding an optimal portfolio given the equilibrium voting behavior, and each agent is also demanding an optimal portfolio of control rights given his economic interests. Condition 2 states that each agent is voting optimally. Finally, Condition 3 states that no agent can strictly increase his utility by changing his share portfolio, his control rights, and his ballot.

We now show that, so long as the outcome of at least one motion affects the value of at least one firm, competitive equilibria cannot exist in settings with transferrable control rights.

Theorem 3. Suppose that there exists at least one firm \(\tilde{f} \in F\) and motion \(\tilde{m} \in M\) such that the value of \(\tilde{f}\) depends on the decision regarding \(\tilde{m}\), i.e., such that \(v_f(0, \delta_{-\tilde{m}}) \neq v_f(1, \delta_{-\tilde{m}})\) for all \(\delta_{-\tilde{m}} \in \{0, 1\}^M \setminus \{\tilde{m}\}\). Then no competitive equilibrium exists.

Proof. Consider any arrangement \([e, k; (s, r, \beta); p, q]\) and suppose it is a competitive equilibrium. We first show that for each firm \(f \in F\), \(p_f = v_f(\delta(r, \beta))\). There are two cases to consider:

1. If \(p_f < v_f(\delta(r, \beta))\), then the first condition of Definition 3 is not satisfied—\((s, r) \notin D^i(e, k; \beta; p, q)\), as agent \(i\) is strictly better off with the portfolio \(\hat{s}^i \equiv (s^i_f + 1, s^i_{-f})\) than
with \( s^i \), as

\[
\bar{u}^i([e, k; (\hat{s}^i, s^{-i}), r, \beta]; p, q]) - \bar{u}^i([e, k; (s, r, \beta); p, q]) = v_f(\delta(r, \beta)) - p_f > 0.
\]

2. If \( p_f > v_f(\delta(r, \beta)) \), then the first condition of Definition 3 is not satisfied—\((s, r) \notin D^i(e, k; \beta; p, q)\), as agent \( i \) is strictly better off with the portfolio \( \hat{s}^i \equiv (s^i_f - 1, s^i_{-f}) \) than with \( s^i \), as

\[
\bar{u}^i([e, k; (\hat{s}^i, s^{-i}), r, \beta]; p, q]) - \bar{u}^i([e, k; (s, r, \beta); p, q]) = p_f - v_f(\delta(r, \beta)) > 0.
\]

Hence, \( p_f = v_f(\delta(r, \beta)) \) for all \( f \in F \). Suppose that \( \delta_\tilde{m}(r, \beta) = 0 \).\(^{11}\) Hence, \( p_f \neq v_f(\hat{\delta}) \), where

\[
\hat{\delta}_m = \begin{cases} 1 & m = \tilde{m} \\ (\delta(r, \beta))_m & \text{otherwise.} \end{cases}
\]

Suppose \( v_f(\hat{\delta}) > p_f \).\(^{12}\) Consider a deviation by \( i \) to \((\hat{s}^i, \hat{r}^i, \hat{\beta}^i)\) where

\[
\hat{r}_m^i = \begin{cases} \frac{1 + q_m(\hat{r}_m^i - r_m^i)}{v_f(\delta) - p_f} & f = \tilde{f} \\ r_m^i & \text{otherwise,} \end{cases}
\]

\[
\hat{s}^i_f = \begin{cases} 1 & m = \tilde{m} \\ 0 & \text{otherwise,} \end{cases}
\]

\[
\hat{\beta}_m^i = \begin{cases} 1 & m = \tilde{m} \\ \beta_m^i & \text{otherwise.} \end{cases}
\]

\(^{11}\) The case of \( \delta_\tilde{m}(r, \beta) = 1 \) is analogous.

\(^{12}\) The case where \( v_f(\hat{\delta}) < p_f \) is analogous.
Under this deviation, the utility of $i$ is

$$
\tilde{u}^i([e, k; ((\hat{s}^i, s^{-i}), (\hat{r}^i, r^{-i}), (\hat{\beta}^i, \beta^{-i})); p, q]) = 
\sum_{f \in F} \left( \hat{s}^i f v_f(\hat{s}^i, s^{-i}, \hat{r}^i, r^{-i}, \hat{\beta}^i, \beta^{-i}) - p_f(\hat{s}^i - e^i_f) \right) - \sum_{m \in M} \left( q_m(\hat{r}^i_m - k^i_m) \right).
$$

Noting that $\delta((\hat{r}^i, r^{-i}), (\hat{\beta}^i, \beta^{-i})) = \hat{\delta}$, the deviation utility given in (4) is given by

$$
\sum_{f \in F} \left( \hat{s}^i f v_f(\hat{\delta}) - p_f(\hat{s}^i - e^i_f) \right) - \sum_{m \in M} \left( q_m(\hat{r}^i_m - k^i_m) \right).
$$

Substituting in the values of $\hat{s}^i$ and $\hat{r}^i$ defined above, (5) becomes

$$
\left( 1 + \frac{q_m(\hat{r}^i_m - k^i_m)}{v_f(\hat{\delta}) - p_f} \right) (v_f(\hat{\delta}) - p_f) + \sum_{f \in F} p_f e^i_f - \sum_{m \in M} \left( q_m(\hat{r}^i_m - k^i_m) \right) - q_m(\hat{r}^i_m - r^i_m)
$$

as $\hat{s}^i_f = 0$ for all $f \neq \hat{f}$. Furthermore, noting that $p_f = v_f(\delta(r, \beta))$ from Items 1 and 2 above, we have that

$$
\tilde{u}^i([e, k; (s, r, \beta); p, q]) = \sum_{f \in F} \left( s^i f v_f(\delta(r, \beta)) - p_f(s^i - e^i_f) \right) - \sum_{m \in M} \left( q_m(r^i_m - k^i_m) \right)
= \sum_{f \in F} p_f e^i_f - \sum_{m \in M} \left( q_m(r^i_m - k^i_m) \right).
$$

Combining expressions (6) and (7), we have that

$$
\tilde{u}^i([e, k; ((\hat{s}^i, s^{-i}), (\hat{r}^i, r^{-i}), (\hat{\beta}^i, \beta^{-i})); p, q]) = (v_f(\hat{\delta}) - p_f) + \tilde{u}^i([e, k; (s, r, \beta); p, q])
> \tilde{u}^i([e, k; (s, r, \beta); p, q])
$$

as $v_f(\hat{\delta}) - p_f > 0$ by assumption. Thus $[e, k; (s, r, \beta); p, q]$ is not a competitive equilibrium, a contradiction.

The key idea of the proof is that for any decision vector $\delta$ there is a unique price vector
for shares which may support a competitive equilibrium, given by \( \bar{p}_f = v_f(\delta) \): for any \( p_f \neq v_f(\delta) \), any agent could buy or sell an arbitrarily large amount of stock in firm \( f \) to make an arbitrarily large profit. However, at the price vector \( \bar{p} \) for shares, an agent may buy or sell an arbitrarily large amount of stock in some firm \( f \) such that \( v_f(\delta) \neq \bar{p}_f \), along with control rights sufficient to change the decision vector to \( \hat{\delta} \); by doing so, that agent may make an arbitrarily large profit. Thus, no price vector can support a competitive equilibrium.

### 3.2 Core-Compatible Outcomes

**Definition 4.** An outcome \( \langle s, r, \beta \rangle \) is in the core if there does not exist a set of agents \( J \subseteq I \), and \( \hat{\beta}^J \) such that

\[
\sum_{j \in J} u^j(\langle s, r, (\hat{\beta}^J, \beta^{-J}) \rangle) > \sum_{j \in J} u^j(\langle s, r, \beta \rangle).
\]

We say that an initial endowment \( (e, k) \) is core-compatible if there exists a core arrangement \( [e, k; \langle s, r, \beta \rangle; p, q] \) associated with \( (e, k) \), such that the outcome \( \langle s, r, \beta \rangle \) is in the core and for all \( \hat{\beta} \) and every \( i \in I \),

\[
\tilde{u}^i([e, k; \langle s, r, \beta \rangle; p, q]) \geq \tilde{u}^i([e, k; \langle e, k, \hat{\beta} \rangle; p, q])
\]

holds.\(^{13}\)

#### 3.2.1 Efficiency

**Theorem 4.** Core outcomes are efficient.

*Proof.* The proof follows as the proof of Theorem 1.

#### 3.2.2 Existence

**Theorem 5.** Every initial endowment is core-compatible.

\(^{13}\)Note that, as in Section 2.2, we do not impose any restriction on beliefs about balloting, so long as agents’ expectations about the balloting are consistent.
Proof. Consider an initial endowment \((e, k)\). Consider an efficient decision vector \(\delta\) and let \(\beta_i^m = \delta_m\). Consider some \(\hat{\beta}\) and let \(p_f = v_f(\delta(k, \hat{\beta}))\) and \(q_f = 0\). Let \(s\) be any portfolio profile such that \(s^i_f = s^i_{f'} > 0\) for all \(i \in I\) and \(f, f' \in F\). The utility of agent \(i\) under the initial allocation and voting profile \(\hat{\beta}\) is

\[
\tilde{u}^i([e, k; \langle e, k, \hat{\beta} \rangle; p, q]) = \sum_{f \in F} e^i_f v_f(\delta(k, \hat{\beta})).
\] (9)

The utility of agent \(i\) under the arrangement \([e, k; \langle s, k, \beta \rangle; p, q]\) is

\[
\tilde{u}^i([e, k; \langle s, k, \beta \rangle; p, q]) = \sum_{f \in F} (s^i_f v_f(\delta(k, \beta)) - p_f(s^i_f - e^i_f))
= \sum_{f \in F} \left( s^i_f v_f(\delta(k, \beta)) - v_f(\delta(k, \hat{\beta}))(s^i_f - e^i_f) \right)
= \sum_{f \in F} s^i_f(v_f(\delta(k, \beta)) - v_f(\delta(k, \hat{\beta}))) + \sum_{f \in F} v_f(\delta(k, \hat{\beta}))e^i_f
\] (10)

Hence, the difference between utility under the arrangement \([e, k; \langle s, k, \beta \rangle; p, q]\), given by (10), and the utility under the arrangement \([e, k; \langle e, k, \hat{\beta} \rangle; p, q]\), given by (9), is given by

\[
\sum_{f \in F} s^i_f(v_f(\delta(k, \beta)) - v_f(\delta(k, \hat{\beta})))
\]

which is non-negative as \(\delta(k, \beta)\) is efficient and \(s^i_f = s^i_{f'} > 0\) for all \(i \in I\) and \(f, f' \in F\). Hence,

\[
\tilde{u}^i([e, k; \langle s, k, \beta \rangle; p, q]) \geq \tilde{u}^i([e, k; \langle e, k, \hat{\beta} \rangle; p, q]).
\]

and so \((e, k)\) is core-compatible. \(\square\)

The intuition behind the existence of a core-compatible arrangement for any starting endowment is exactly the same as that presented in Section 2.2.2 for the case without transferrable control rights. To reach a core outcome in this setting, agents may trade both stock and control rights, taking into account how such trades may affect voting outcomes.
After such trades, in any core outcome, the voting incentives of all agents will be roughly aligned with market efficiency; that is, for any set of agents with majority control, their welfare will be maximized by the efficiency-maximizing decision.

We now return to the setting of Example 1, only now we allow for tradable control rights. In this setting, the initial endowment is given by

\[ e = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

and initial control rights are (as in Example 1) given by \( k = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)^\top \). One core-compatible arrangement for this setting is then given by

\[
\begin{bmatrix}
    e, k, \\
    \left(\begin{pmatrix}
    \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
    \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\end{pmatrix}, \begin{pmatrix}
    1 & 1 & 1 & 1 \\
    6 & 6 & 6 & 6 \\
\end{pmatrix}, (1, 1, 1, 1) \right), \left(\begin{pmatrix}
    16 \\
    6 \\
\end{pmatrix}, (0) \right)
\end{bmatrix}.
\]

Note that in this core-compatible arrangement, control rights are traded in the associated outcome, but are priced at zero. Control rights are priced at 0 in this example as all agents vote identically.
References


