The Cyclical Behavior of Housing, Illiquidity and Foreclosures

Aaron Hedlund∗
University of Pennsylvania

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Abstract

I develop a heterogeneous agents model of the macroeconomy with frictional, decentralized trading in the housing market and equilibrium mortgage default. I use this model to quantitatively investigate the effects of housing and mortgage illiquidity on the cyclical behavior of housing, mortgage debt, and foreclosures. Consistent with U.S. data, the model generates procyclical and volatile house prices, sales, and residential investment as well as procyclical mortgage debt. The model also displays countercyclical and volatile foreclosures and average time on the market of houses for sale. Housing booms in the model are protracted and gradual, while housing busts are initially severe and followed by slow recoveries. Consistent with empirical evidence, sellers with higher mortgage leverage choose higher asking prices for their house, wait longer for their property to sell, and transact at a higher price. The model also accounts for the fact that not all borrowers whose homes are foreclosed have underwater mortgages (that is, owe more than the market value of their property). The model highlights an important feedback mechanism in housing markets—trading frictions tighten endogenous credit constraints, and credit constraints magnify trading frictions in the real estate market.

Keywords: Housing; house prices; liquidity, foreclosures; search theory; portfolio choice; business cycles

JEL Classification Numbers: D83, E21, E22, E32, E44, G11, G12, G21, R21, R31

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1 Introduction

Houses are illiquid assets, both in how they are traded and in how they are financed. Brunnermeier and Pedersen (2009) define an asset’s market liquidity as “the ease with which it is traded”, and they define funding liquidity as “the ease with which (traders) can obtain funding.” Housing suffers from market illiquidity because of trading frictions in the housing market that increase the cost and time to buy and sell houses. For example, the National Association of Realtors reports that houses took an average of 4 months to sell during the recent housing boom, compared to an average of 11 months at the trough of the recent recession. Furthermore, housing inventories—the number of houses listed for sale that do not sell in a given time period—are more than twice as volatile as house prices.

Housing also suffers from funding illiquidity. Houses are financed primarily by collateralized loans, namely, mortgages. According to the Survey of Consumer Finances, the median homeowner in 1998 had $49,000 in mortgage debt, which far exceeds all other household debts. Funding illiquidity results from the presence of substantial transaction costs—the Federal Reserve reports mortgage transaction costs of 1 – 3% of the loan amount—and other barriers to credit. I refer to the lack of market liquidity in the housing market as housing illiquidity, and I refer to the lack of funding liquidity in the mortgage market as mortgage illiquidity. This paper asks two questions. First, what are the effects of housing and mortgage illiquidity on the cyclical behavior of housing, mortgage debt, and foreclosures? Second, what is the relationship between housing and mortgage illiquidity?

To answer these questions, I develop a dynamic model of the macroeconomy with housing and mortgage markets that incorporates three frictions. First, there is imperfect risk sharing in the economy resulting from incomplete markets. Second, housing trades are decentralized and subject to search/coordination frictions. Third, mortgages suffer from a one-sided lack of commitment because borrowers can default on their debt obligations.

This paper’s main theoretical contribution is to provide a tractable framework for analyzing housing, mortgage debt, and foreclosure dynamics with frictional, decentralized trading and endogenous credit constraints. Housing trades occur in a directed (competitive) search environment in which real estate firms act as intermediaries between buyers and sellers. Specifically, real estate firms purchase housing from sellers and sell it to buyers. Search is directed because competitive forces are present both for buyers and sellers. Buyers can search for similar houses at lower prices but are less likely to successfully buy. Similarly, sellers can try to sell their house for a higher price but are less likely to successfully sell. Real estate firms act as market makers by hiring enough real estate agents to equate the total flow of housing from sellers to the total flow of housing to buyers. I embed this housing framework
in an incomplete markets economy with equilibrium mortgage default. The structure of the housing market allows the model to be computed using standard techniques because agents can forecast the dynamics of the entire distribution of house prices simply by forecasting the dynamics of one endogenous variable—the shadow price of housing.

This paper has two main quantitative contributions. First, the model matches several stylized facts of housing, mortgage debt, and foreclosures that the literature has failed to account for in a unified model. Second, this paper shows that trading frictions in the housing market have a significant effect on housing and foreclosure dynamics and therefore should not be overlooked in macroeconomic models that include housing. Regarding the first point, the model successfully accounts for the following stylized facts of the United States economy from 1975 – 2010: 1) house prices, sales, and residential investment are procyclical and more volatile than output; 2) the average time to sell a house is countercyclical and volatile; 3) mortgage debt is procyclical; 4) foreclosures are countercyclical and volatile; and 5) housing booms and busts are protracted and prone to overshooting, i.e. house price changes are persistent in the short run and mean reverting in the long run.\(^1\)

To investigate the effects of housing illiquidity, I compare the dynamics of the baseline model to those of a version without trading frictions. I find that housing illiquidity is necessary to explain fluctuating time to sell and to generate significant foreclosure activity. In addition, housing illiquidity amplifies house price fluctuations and generates stronger positive co-movement between house sales and GDP. The baseline model also highlights the interaction between housing illiquidity and mortgage illiquidity—housing illiquidity contributes to

mortgage illiquidity, and mortgage illiquidity contributes to housing illiquidity.

Housing illiquidity affects mortgage illiquidity because it increases homeowners’ exposure to risk. Because of the cost and time involved in selling a house, homeowners cannot easily sell their house in the event of financial distress. Homeowners who try to sell their house quickly are likely to have to accept a substantially lower sales price. As a result, housing is a consumption commitment as in Chetty and Szeidl (2007, 2010). Although financially distressed homeowners can also try to refinance to a larger mortgage, refinancing involves considerable transaction costs in addition to the possibility of a loan rejection.

This increased exposure to risk impacts the cost and availability of mortgage credit, i.e. mortgage liquidity. If a homeowner cannot afford mortgage payments, housing illiquidity makes it more difficult for the homeowner to sell his house quickly at a high enough price to pay off the mortgage, thus increasing the probability of foreclosure. Even if such homeowners do not immediately default, they are forced to draw down their savings or refinance to a larger mortgage, increasing the probability that they eventually default. If they do not resume making payments, the bank initiates foreclosure proceedings to seize the housing collateral, often resulting in losses for the bank. Banks anticipate this behavior when they issue mortgages, making credit more costly and difficult to obtain, particularly for mortgages with high loan-to-value ratios. In this way, housing market illiquidity contributes to greater mortgage market illiquidity.

This reasoning partially explains why mortgage credit is generally tighter during times of declining house prices. While declining house prices make mortgages riskier for financial institutions even without housing illiquidity, borrowers in that situation only present a risk to banks when their house is worth less than their mortgage. As long as homeowners have positive equity, it is always in their best interest to sell before going into foreclosure. However, RealtyTrac reports that less than 50% of homeowners who go into foreclosure have negative equity. In addition, Pennington-Cross (2010) examines subprime mortgage data and finds that 50% of delinquent loans with loan-to-value ratios between 80% and 90% end up being repossessed, compared to 55% of delinquent loans with loan-to-value ratios between 90% and 100%, and 59% of delinquent loans with loan-to-value ratios above 100%. These facts present a challenge to strictly competitive models of housing. However, when trading frictions are present, homeowners can have paper equity—that is, their house is appraised for more than their mortgage—but may be unable to sell quickly, if at all, at the appraised value. This scenario increases the risk that even borrowers with modest positive equity go into foreclosure following an adverse life event. As a result, banks increase mortgage interest rates or tighten

\footnote{Mayer (1998) applies a repeat-sales methodology to analyze auctions in Los Angeles and Dallas and finds quick-sale discounts of 0% – 9% in Los Angeles and 9% – 21% in Dallas.
lending standards during times of high housing illiquidity.

Just as housing illiquidity contributes to mortgage illiquidity, tighter mortgage credit can exacerbate illiquidity in the housing market by creating a debt-lock problem for homeowners. When mortgage credit is tight, financially distressed homeowners may find it impossible to refinance to a larger mortgage, forcing them to put their house up for sale. Because they must pay off their mortgage following a sale, they are compelled to post a high asking price, thereby decreasing their probability of selling and increasing housing illiquidity. In addition, some of these homeowners go into foreclosure if they fail to sell. The resulting influx of foreclosed houses clogs the housing market and further increases housing illiquidity. Empirically, Krainer, Spiegel, and Yamori (2010) find evidence in Japan that debt overhang contributes significantly to housing market illiquidity and slows the recovery of the housing market following a bust.

1.1 Related Literature

This paper’s first main contribution is to construct a unified, computationally tractable theory of housing with trading frictions and endogenous credit constraints. Quantitatively, this paper simultaneously accounts for some of the primary stylized facts of housing, mortgage, debt, and foreclosures—something the literature has struggled to do—and establishes that trading frictions are an important feature of housing markets. Several papers, including Stein (1995), Ortalo-Magné and Rady (2006), and Lamont and Stein (1999), investigate the impact of credit constraints on house prices and sales. These papers show that credit constraints magnify the effect of income shocks on house prices and sales and cause house prices to overshoot in response to income shocks. These papers are qualitative in nature and do not model the interaction of housing with the broader macroeconomy.

Davis and Heathcote (2005) and Kahn (2009) develop representative agent, multisector stochastic growth models to study the business cycle properties of housing. They are able to generate large volatility of residential investment and procyclicality of prices and sales but are unable to match the volatility of house prices; although, Kahn (2009) does somewhat better in this regard. However, because these papers use representative agent models, they do not address the impact of credit constraints on housing fluctuations.

There is also a growing body of literature on stochastic macroeconomic models of housing with incomplete markets, including papers by Iacoviello and Pavan (2010), Favilukis, Ludvigson, and Van Nieuwerburgh (2011), Kiyotaki, Michaelides, and Nikolov (2010), and Rios-Rull and Sánchez-Marcos (2008). These papers treat housing markets as competitive, model mortgages as short-term contracts and do not allow homeowners to default.
Earlier papers on housing with trading frictions use the theory of random search to model housing trades, as in Wheaton (1990), Berkovec and Goodman, Jr. (1996), and Krainer (2001). More recently, Novy-Marx (2009), Burnside, Eichenbaum, and Rebelo (2011), and Caplin and Leahy (2008) study variants of this framework. Novy-Marx (2009) shows that search frictions magnify shocks to fundamentals when seller entry is imperfectly elastic. Burnside et al. (2011) add learning and social dynamics and are able qualitatively to generate protracted housing booms and busts. One drawback to the random search framework is that it prevents homeowners from lowering their asking price to attract more buyers—behavior which is documented by Merlo and Ortalo-Magné (2004) and Merlo, Ortalo-Magné, and Rust (2008). Therefore, my paper models housing using a directed search framework, as in Díaz and Jerez (2010), Albrecht, Gautier, and Vroman (2010), and Head, Lloyd-Ellis, and Sun (2011). Díaz and Jerez (2010) show that directed search can generate additional amplification in house prices by allowing fluctuations in the surplus sharing rule between buyers and sellers. Head et al. (2011) are able to generate some persistence in house prices as a result of directed search but at the cost of reduced volatility.

My paper also fits into the literature on household default. Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) study bankruptcy in an environment in which different loans made to different types of borrowers are separate assets traded in distinct markets. Mitman (2011), Hintermaier and Koeniger (2011), and Jeske, Krueger, and Mitman (2010) apply this approach to the mortgage market. They treat mortgages as one-period contracts, which is equivalent to assuming that homeowners must refinance each period. Chatterjee and Eyigungor (2011b) and Arellano and Ramanarayanan (2010) develop models of long-term sovereign debt, and Chatterjee and Eyigungor (2011a), Corbae and Quintin (2011), and Garriga and Schlagenhauf (2009) investigate foreclosures when mortgages are long-term contracts. My paper extends that work by studying aggregate foreclosure dynamics with long-term mortgages.

The model of housing markets in this paper is a modification of the model I introduced in Hedlund (2010) and is motivated by Lagos and Rocheteau (2009) and Menzio and Shi (2010). Lagos and Rocheteau (2009) explore the impact of trading frictions on over-the-counter markets for financial assets by developing a random search model in which investors must match with intermediaries to adjust their asset holdings. Menzio and Shi (2010) develop a directed search model of the labor market with aggregate productivity shocks and show that the combination of directed search and free entry of job vacancies allows the equilibrium dynamics to be computed without needing to keep track of the distribution of agents. In related work, Karahan and Rhee (2011) use this housing market framework to study the interaction between housing market conditions and labor mobility.
2 The Model

2.1 Households

Households value a composite consumption good $c$ and housing services $c_h$ according to the period utility function $U(c, c_h)$. Households can directly purchase housing services at the competitive price $r_h$, or they can obtain housing services by owning a house $h \in H$. One unit of housing provides one unit of housing services. Households can only occupy one residence at any point in time; thus, an owner of house $h$ who purchases housing services $c_h$ has utility $U(c, \max\{c_h, h\})$. Households who own a house are homeowners, and those that do not are renters. There is a minimum house size $h \in H$, which is also the maximum amount of housing services that households can purchase directly. Thus, homeowners always want to live in their houses.

Households inelastically supply one unit of time to the labor market and are paid a wage $w$ per unit of labor efficiency. Households are heterogeneous and face idiosyncratic shocks, $e \cdot s$, to their labor productivity with transitory component $e$ and persistent component $s$. The persistent shock $s \in S$ follows a finite state Markov chain with transition probabilities $\pi_s(s'|s)$, and the transitory shock $e$ is drawn from the cumulative distribution function $F(e)$ with compact support $E \subset \mathbb{R}_+$. Households initially draw $s$ from the invariant distribution $\Pi_s(s)$.

Households evaluate intertemporal utility using recursive Epstein-Zin preferences, i.e.

$$V = [(1 - \beta) U^{1 - \sigma} + \beta (E V^{1 - \sigma})^{1 - \frac{1}{\psi}}]^{\frac{\eta}{1 - \sigma}}$$

where $\beta$ is the discount factor, $\psi$ is the intertemporal elasticity of substitution, $\sigma$ controls risk aversion, and $\eta = \frac{1 - \sigma}{1 - \frac{1}{\psi}}$. When $\eta = 1$, households have standard time-separable constant relative risk aversion preferences.

All households trade a one-period risky asset $a$, while homeowners’ balance sheets also consist of a house $h$ and a mortgage $m$. Assets are non-contingent claims to future consumption. Households who purchase $a'$ today receive $(1 + r')a'$ units of the consumption good in the next period. In the aggregate, these assets compose the capital stock, which is rented to firms in the consumption good sector. Households purchase mortgages $m'$ from the mortgage sector, which I describe in detail later.
2.2 Production Sectors

2.2.1 Consumption Good Sector

Firms in the consumption good sector produce the consumption good using capital $K$ and labor $N_c$. Output in the consumption sector is given by

$$Y_c = z_c A_c F_c(K, N_c).$$

The production technology $F_c$ is constant returns to scale, exhibits diminishing marginal product of each input, and satisfies the standard Inada conditions. The productivity shock $z_c$ follows a finite state Markov chain with transition probabilities $\pi_z(z'_c | z_c)$. Firms rent capital from households at rental rate $r$ and pay wage $w$ per unit of labor efficiency. The consumption good can be transformed one-for-one into consumption, structures, or investment in capital, and its price is normalized to 1.

**Housing Services**  Landlords transform the consumption good into housing services and vice-versa at a rate of $A_h$ housing services per unit of consumption good. Landlords sell these housing services competitively at price $r_h$.

**Evolution of the Capital Stock**  Capital depreciates at the rate $\delta_k$; thus, the total capital stock evolves according to

$$K' = (1 - \delta_k)K + I$$

where $I$ is total investment by households in new capital.

2.2.2 Housing Construction Sector

Housing construction firms build housing using land/permits $L$, structures $B$ and labor $N_h$. Output in the housing sector is given by

$$Y_h = F_h(L, B, N_h).$$

The production technology $F_h$ is constant returns to scale, exhibits diminishing marginal product of each input, and satisfies the standard Inada conditions. Firms purchase new land/permits from the government at price $p_l$, pay wage $w$ per unit of labor efficiency, and purchase structures $B$ from the consumption good sector. The government supplies a fixed

\footnote{This construction technology resembles the one in Jeske et al. (2010), except here it refers to the production of housing services, not actual houses.}
amount $\bar{L} > 0$ of new land/permits each period, and all revenues go to wasteful government spending. Housing construction firms sell housing at price $p_h$ directly to real estate firms, who are responsible for selling it to home buyers. Housing built in each period is immediately available for occupation.

**Evolution of the Housing Stock** At the beginning of each period, a homeowner’s house completely depreciates with probability $\delta_h$, which implies that the aggregate housing stock depreciates at the rate $\delta_h$; thus, the total end of period housing stock evolves according to

\[ H' = (1 - \delta_h)H + Y'_h \]

### 2.3 Real Estate Sector

Real estate firms purchase new housing from construction firms at the competitive price $p_h$, but trade between real estate firms and households looking to buy or sell a house is accomplished bilaterally according to a frictional matching process. Real estate firms and households trade housing in two distinct markets—a buying market and a selling market.

Real estate firms send real estate agents into the buying market to sell houses to buyers, and they send real estate agents into the selling market to purchase houses from sellers. I assume that households can only own one house at any point in time; thus, the selling market opens before the buying market to allow for homeowner-to-homeowner transitions. Sellers can first sell their house in the selling market before buying a different house in the buying market, all in the same period.

Real estate firms send a continuum of real estate agents to the buying and selling markets. As a result, the law of large numbers applies, and real estate firms know the exact number of successful matches they will have with buyers and sellers. I assume that real estate firms cannot hold housing inventories, which constrains them to have a zero net flow of housing when choosing how many real estate agents to hire in each market and how much new housing to purchase from construction firms.

#### 2.3.1 Buying Market

The buying market is organized into submarkets indexed by $(x_b, h) \in \mathbb{R}_+ \times H$, where $x_b$ is the price buyers pay to real estate firms, and $H$ is a finite set of house sizes. Real estate firms hire a continuum $\Omega_b(x_b, h)$ of real estate agents to enter each submarket at cost $\kappa_b(x_b, h)$ per agent. Whenever a buyer enters submarket $(x_b, h)$, he commits to paying $x_b$ in exchange for house $h$, conditional on matching with a real estate agent. Buyers can only enter one
submarket in a period, and successful buyers immediately occupy their house and receive housing services.

The ratio of real estate agents to buyers in submarket \((x_b, h)\) is \(\theta_b(x_b, h) \geq 0\) and is determined in equilibrium.\(^4\) I refer to this ratio as the market tightness of submarket \((x_b, h)\). The probability that a buyer finds a real estate agent is \(p_b(\theta_b(x_b, h))\), where \(p_b: \mathbb{R}_+ \rightarrow [0, 1]\) is a continuous, strictly increasing function with \(p(0) = 0\). Similarly, the probability that a real estate agent finds a buyer is \(\alpha_b(\theta_b(x_b, h))\), where \(\alpha_b: \mathbb{R}_+ \rightarrow \mathbb{R}_+\) is a continuous, strictly decreasing function such that \(\alpha_b(\theta_b(x_b, h)) = p_b(\theta_b(x_b, h)) / \theta_b(x_b, h)\) and \(\alpha_b(0) = 1\). I allow \(\alpha_b(\cdot)\) to be larger than one to account for the possibility that real estate agents match with multiple buyers. In this event, the real estate agents sell houses to each buyer they meet. Real estate agents and buyers take \(\theta_b(x_b, h)\) parametrically.

### 2.3.2 Selling Market

The selling market is organized into submarkets indexed by \((x_s, h) \in \mathbb{R}_+ \times H\), where \(x_s\) is the price sellers receive from real estate firms. Sellers pay a utility cost \(\kappa\) to enter the selling market. Real estate firms hire a continuum \(\Omega_s(x_s, h)\) of real estate agents to enter each submarket at cost \(\kappa(x_s, h)\) per agent. Whenever a seller of house size \(h\) enters submarket \((x_s, h)\), he commits to selling his house at price \(x_s\), conditional on matching with a real estate agent. Sellers can only enter one submarket in a period. Successful sellers immediately vacate their house.

The ratio of real estate agents to sellers in submarket \((x_s, h)\) is \(\theta_s(x_s, h) \geq 0\) and is determined in equilibrium. As in the buying market, I refer to \(\theta_s(x_s, h)\) as the market tightness of \((x_s, h)\). The probability that a seller finds a real estate agent is \(p_s(\theta_s(x_s, h))\), and the probability that a real estate agent finds a seller is \(\alpha_s(\theta_s(x_s, h))\). The properties of \(p_s\) and \(\alpha_s\) are the same as those of \(p_b\) and \(\alpha_b\), respectively.

### 2.4 Mortgage Sector

The mortgage sector is populated by a continuum of competitive mortgage companies that sell long-term mortgage contracts \(m \in M \equiv \cup_{h \in H} M(h)\) to homeowners. In addition, mortgage companies trade one-period, risk-free bonds with yield \(i\) in an international bond market. I assume the yield \(i\) is exogenous, implying that there is a completely elastic supply of international funds.

\(^4\)In submarkets that are not visited, \(\theta_b(x_b, h)\) is an out-of-equilibrium belief that helps determine equilibrium behavior.
I model mortgage contracts in the spirit of Chatterjee et al. (2007) and Corbae and Quintin (2011) in that mortgages of different sizes made to borrowers with different characteristics are traded in distinct markets. As a result, perfect competition in the mortgage sector dictates that mortgage companies earn zero expected profits on each contract. In other words, there is no cross-subsidization of mortgage contracts. Mortgage companies are owned by risk-neutral investors who consume all ex-post profits and losses.

I assume that the set of mortgage contracts \( M \) is finite, that households choose their asset holdings from a finite set \( A \), and that there is perfect information. As a result, each mortgage contract \( m' \in M \) made to a borrower with asset holdings \( a' \in A \), house size \( h \in H \), and persistent component of labor efficiency \( s \in S \) has its own price.

### 2.4.1 Mortgage Contracts

A mortgage in this economy is a long-term contract that provides funds upfront and is paid off gradually. When homeowners with labor productivity \( s \), house size \( h \), and assets \( a' \) choose a mortgage of size \( m' \), they receive initial funds \( q_{m}(m', a', h, s)m' \) in exchange for mortgage debt of \( m' \), where \( q_{m} \in [0, 1] \) is the price of the mortgage. In subsequent periods, borrowers choose how much principal to pay down and face the common interest rate \( r_{m} \) on all unpaid balances. If homeowners wish to increase their mortgage debt by refinancing, they must first pay off their entire balance, and then take out a new mortgage. I assume that for each \( h \), \( \max M(h) = \sup \{ x_{s} : p_{s}(\theta_{s}(x_{s}, h)) > 0 \} \), meaning that households cannot take out a mortgage for more than the highest price that they could possibly receive for their house in the selling market.

Mortgages have infinite duration and have no fixed payment schedule, instead giving borrowers the flexibility to choose how quickly to pay down their balance. There are two reasons for these assumptions. First, these mortgage contracts proxy for all forms of mortgage debt because I am not allowing homeowners to take out second mortgages or home equity lines of credit, which normally would give borrowers flexibility in paying down their total mortgage debt. Second, dispensing with a fixed payment schedule reduces the dimension of the state space, eliminating the need to keep track of time left on the mortgage.

Mortgage companies incur a proportional cost \( \zeta \) when originating a mortgage and a proportional servicing cost \( \phi \) over the life of the mortgage. Mortgage companies face two types of risk that cause borrowers not to pay off their mortgages. First, borrowers may default on their mortgage, which triggers foreclosure proceedings. Second, the borrower’s house may fully depreciate, in which case I assume that the mortgage company absorbs the

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\(^5\)Mortgage companies issue a continuum of each mortgage contract; thus, the law of large numbers applies and mortgage companies face no idiosyncratic risk.
loss and the borrower is not penalized. This assumption is needed for simplicity and to avoid inflating the number of foreclosures.

The mortgage interest rate \( r_m \) compensates the mortgage company for its opportunity cost of funds and the servicing cost, plus the risk of housing depreciation. However, all individual default risk is priced at origination into \( q_m^0 \), thereby removing the need to keep track of borrower-specific mortgage rates over time. Front-loading all default risk also ensures that it would never be profitable for one mortgage company to siphon off the highest quality mortgages from another mortgage company. For ease of notation, I define 

\[
q_m = \frac{1}{1 + r_m}.
\]

### 2.5 Foreclosures and Legal Environment

I model the foreclosure process in a way that resembles actual foreclosure proceedings but abstracts from some of the details. Foreclosure laws differ by state, from whether lenders need a court order to initiate foreclosure proceedings or not (judicial vs. non-judicial foreclosure) to whether lenders can go after other assets of the borrower in the event that a foreclosure sale does not cover the entire balance of the loan. I assume that in the event of borrower default, the following occurs:

1. The borrower’s mortgage balance is set to zero, and a foreclosure filing is placed on the borrower’s credit record (\( f = 1 \)).

2. The mortgage company repossesses the borrower’s house, making it an REO (Real Estate Owned, i.e. a foreclosure property), and puts it up for sale in the decentralized selling market.

3. The mortgage company has reduced search efficiency \( \lambda \in (0, 1) \) and, upon successful sale in submarket \( x_s \), loses a fraction \( \chi \) of the sale price.\(^6\)

4. If the foreclosure sale more than covers the balance of the mortgage, all profits are sent to the borrower. Otherwise, the mortgage company absorbs any losses. In other words, mortgages are no recourse loans: mortgage companies cannot seize any other assets of defaulting borrowers.

5. Households with \( f = 1 \) lose access to the mortgage market\(^7\) and the foreclosure flag stays on their record at the beginning of the next period with probability \( \gamma_f \in (0, 1) \).\(^8\)

\(^6\)This proportional loss accounts for various foreclosure costs and foreclosure property degradation.

\(^7\)Fannie Mae and Freddie Mac do not purchase mortgages issued to borrowers with recent foreclosure filings, making it much less appealing to lend to these borrowers.

\(^8\)Foreclosure filings stay on a borrower’s credit record for a finite number of years.
2.6 Decision Problems

2.6.1 Household’s Problem

Each period is divided into three subperiods. At the beginning of subperiod 1, households draw labor efficiency shocks \((e, s)\) and learn the aggregate productivity \(z_c\), and households who previously had a foreclosure flag learn their credit status \(f\). Homeowners decide whether to enter the selling market and choose a selling price \(x_s\), which sends them to submarket \((x_s, h)\). Mortgage holders that fail or choose not to sell their house then decide whether or not to default. In subperiod 2, renters and recent sellers decide whether to enter the buying market and choose a submarket \((x_b, h)\). In subperiod 3, households choose consumption \(c\), housing services \(c_h\), and assets \(a'\), and homeowners with good credit choose a mortgage \(m'\).

The aggregate state of the economy consists of the shock \(z_c\), the distribution \(\Phi_1\) of households at the beginning of the period, the capital stock \(K\), and the stock \(H_{REO}\) of REO housing. Agents must forecast the evolution of the aggregate state, \(Z \equiv (z_c, \Phi_1, K, H_{REO})\). Let \((\Phi'_1, K', H'_{REO}) = G(z_c, \Phi_1, K, H_{REO}, z'_c) = G(Z, z'_c)\) be the law of motion for \((\Phi_1, K, H_{REO})\). In each subperiod, homeowners have individual state \((y, m, h, s, f)\), where \(y\) is cash at hand, \(m\) is the mortgage balance, \(h\) is the house size, \(s\) is the persistent component of labor efficiency, and \(f\) indicates whether the household has a foreclosure flag. Renters have individual state \((y, s, f)\). Let \(V_{own}\) and \(V_{rent}\) be the value functions of owners and renters, respectively, in subperiod 3. Let \(R_b\) be the option value of entering the buying market in subperiod 2. Let \(W\) be the value function of homeowners in subperiod 1 conditional on not entering the selling market. Lastly, let \(R_s\) be the option value of entering the selling market at the beginning of subperiod 1.

**Budget Sets** The lower bound of the budget set for homeowners with good credit entering subperiod 3 is \(y = y(m, h, s, Z)\), which accounts for the fact that homeowners must make a mortgage payment but can also take out a new mortgage. In all other cases the lower bound of the budget set is \(y = 0\).
Subperiod 3

- Renters with good credit solve

\[
V_{\text{rent}}(y, s, 0, Z) = \max_{a' \in A, c \geq 0, c_h \in [0, h]} \left[ (1 - \beta)U(c, c_h) \frac{1 - \sigma}{\sigma} \right. \\
+ \beta(E(e', s', z'_c)(V_{\text{rent}} + R_b)^{1-\sigma}(y', s', 0, Z'))^{\frac{1}{\eta}} \right]^{\frac{\eta}{1 - \sigma}} \\
\text{subject to} \\
c + a' + r_h c_h \leq y, \text{ where} \\
y' = e' \cdot w(Z') + (1 + r(Z')) a' \\
Z' = (z'_c, G(Z, z'_c))
\]

(1)

- Renters with bad credit solve

\[
V_{\text{rent}}(y, s, 1, Z) = \max_{a' \in A, c \geq 0, c_h \in [0, h]} \left[ (1 - \beta)U(c, c_h) \frac{1 - \sigma}{\sigma} \right. \\
+ \beta(E(e', s', f', z'_c)(V_{\text{rent}} + R_b)^{1-\sigma}(y', s', f', Z'))^{\frac{1}{\eta}} \right]^{\frac{\eta}{1 - \sigma}} \\
\text{subject to} \\
c + a' + r_h c_h \leq y, \text{ where} \\
y' = e' \cdot w(Z') + (1 + r(Z')) a' \\
Z' = (z'_c, G(Z, z'_c))
\]

(2)

- Homeowners with good credit solve

\[
V_{\text{own}}(y, m, h, s, 0, Z) = \max_{m' \in M, a' \in A, c \geq 0} \left[ (1 - \beta)U(c, h) \frac{1 - \sigma}{\sigma} + \beta(E(e', s', z'_c) \\
[(1 - \delta_h)(W + R_s)^{1-\sigma}(y', m', h, s', 0, Z') + \delta_h(V_{\text{rent}} + R_b)^{1-\sigma}(y', s', 0, Z'))]^{\frac{1}{\eta}} \right]^{\frac{\eta}{1 - \sigma}} \\
\text{subject to} \\
c + a' + m - q(Z)m' \leq y, \text{ where} \\
q(Z) = \begin{cases} 
q'_m(m', a', h, s, Z) & \text{if } m' > m \\
q_m & \text{if } m' \leq m
\end{cases} \\
y' = e' \cdot w(Z') + (1 + r(Z')) a' \\
Z' = (z'_c, G(Z, z'_c))
\]

(3)
• Homeowners with bad credit solve

\[
V_{own}(y, 0, h, s, 1, Z) = \max_{a' \in A, c \geq 0} \left[(1 - \beta)U(c, h)\frac{1 - \sigma}{\sigma} + \beta(\mathbb{E}_{(e', s', f', Z')}(1 - \delta_h)(W + R_a)^{1-\sigma}(y', 0, h, s', f', Z') + \delta_h(V_{rent} + R_b)^{1-\sigma}(y', s', f', Z'))^{\frac{1}{\gamma}}\right]^{\frac{\gamma}{1-\sigma}}
\]

subject to

\[c + a' \leq y, \quad \text{where} \]
\[y' = e' \cdot w(Z') + (1 + r(Z'))a'
\]
\[Z' = (z', G(Z, z'))\]  

Subperiod 2

• The option value of entering the buying market with good credit is

\[
R_b(y, s, 0, Z) = \max\{0, \max_{(x_b, h)} p_b(\theta_b(x_b, h, Z))(V_{own}(y - x_b, 0, h, s, 0, Z) - V_{rent}(y, s, 0, Z))\}
\]

subject to

\[y - x_b \geq y(0, h, s, Z)\]  

• The option value of entering the buying market with bad credit is

\[
R_b(y, s, 1, Z) = \max\{0, \max_{(x_b, h)} p_b(\theta_b(x_b, h, Z))(V_{own}(y - x_b, 0, h, s, 1) - V_{rent}(y, s, 1))\}
\]

subject to

\[y - x_b \geq 0\]  

Subperiod 1

• Homeowners with good credit who do not enter the selling market have utility

\[
W(y, m, h, s, 0, Z) = \max\{V_{own}(y, m, h, s, 0, Z), V_{rent}(y + \max\{0, J_{REO}(h, Z) - m\}, s, 1, Z)\}
\]

• Homeowners with bad credit who do not enter the selling market have utility

\[
W(y, 0, h, s, 1, Z) = V_{own}(y, 0, h, s, 1, Z)
\]
• The option value of entering the selling market with good credit is

\[
R_s(y, m, h, s, 0, Z) = \max\{0, \max_{x_s \geq 0} p_s(\theta_s(x_s, h, Z))((V_{rent} + R_b)(y + x_s - m, s, 0, Z) - W(y, m, h, s, 0, Z)) - \kappa\}
\]

subject to

\[
y + x_s - m \geq 0
\]

(9)

where the constraint \(y + x_s - m \geq 0\) says that homeowners must pay off their mortgage when they sell their house.

• The option value of entering the selling market with bad credit is

\[
R_b(y, 0, h, s, 1, Z) = \max\{0, \max_{x_s \geq 0} p_s(\theta_s(x_s, h, Z))((V_{rent} + R_b)(y + x_s, s, 1, Z) - W(y, 0, h, s, 1, Z)) - \kappa\}
\]

(10)

2.6.2 Consumption Good Firm’s Problem

Consumption good firms choose capital \(K\) and labor \(N_c\) to solve

\[
\max_{K \geq 0, N_c \geq 0} z_c A_c F_c(K, N_c) - (r(Z) + \delta_k)K - w(Z)N_c
\]

(11)

The necessary and sufficient conditions for profit maximization are

\[
r(Z) = z_c A_c \frac{\partial F_c(K(Z), N_c(Z))}{\partial K} - \delta_k
\]

(12)

\[
w(Z) = z_c A_c \frac{\partial F_c(K(Z), N_c(Z))}{\partial N_c}.
\]

(13)

**Landlord’s Problem** Landlords choose how many housing services \(C_h\) to produce by solving

\[
\max_{C_h \geq 0} r_h C_h - \frac{1}{A_h} C_h
\]

(14)

The necessary and sufficient condition for profit maximization is

\[
r_h = \frac{1}{A_h}.
\]

(15)
2.6.3 Construction Firm’s Problem

Construction firms choose land/permits $L$, structures $B$, and labor $N_h$ to solve

$$
\max_{L \geq 0, B \geq 0, N_h \geq 0} p_h(Z)F_h(L, B, N_h) - p_l(Z)L - B - w(Z)N_h
$$

(16)

The necessary and sufficient conditions for profit maximization are

$$
p_l(Z) = p_h(Z) \frac{\partial F_h(L(Z), B(Z), N_h(Z))}{\partial L}
$$

(17)

$$
1 = p_h(Z) \frac{\partial F_h(L(Z), B(Z), N_h(Z))}{\partial B}
$$

(18)

$$
w(Z) = p_h(Z) \frac{\partial F_h(L(Z), B(Z), N_h(Z))}{\partial N_h}
$$

(19)

2.6.4 Real Estate Firm’s Problem

Real estate firms hire a continuum $\Omega_b(\cdot, \cdot)$ of real estate agents to enter the buying market and a continuum $\Omega_s(\cdot, \cdot)$ to enter the selling market, and they purchase new housing $Y_h$ from construction firms to solve

$$
\max_{\Omega_b(x_b, h), \Omega_s(x_s, h) \geq 0, Y_h \geq 0} \int [\kappa_b(x_b, h) + \alpha_b(\theta_b(x_b, h, Z))x_b] \Omega_b(dx_b, dh)
$$

$$
- \int [\kappa_s(x_s, h) + \alpha_s(\theta_s(x_s, h, Z))x_s] \Omega_s(dx_s, dh) - p_h(Z)Y_h
$$

subject to

$$
Y_h + \int h\alpha_s(\theta_s(x_s, h, Z)) \Omega_s(dx_s, dh) \geq \int h\alpha_b(\theta_b(x_b, h, Z)) \Omega_b(dx_b, dh)
$$

(20)

The constraint states that the total amount of housing sold to buyers cannot exceed the total amount of housing purchased from sellers and construction firms. Let $\mu(Z)$ be the multiplier on this constraint. The static nature of the objective function rules out the opposite scenario, namely, that the real estate firm wants to accumulates housing inventories. Note that real estate firms pool the total amount of housing traded in each submarket (including across different house sizes $h$) when satisfying this constraint. In other words, housing is fungible to real estate firms.\footnote{An alternative would be to have one constraint for each $h$ at the cost of increased computational complexity.}
The necessary and sufficient conditions for profit maximization are

\[ p_h(Z) \geq \mu(Z), \]  
\[ \kappa_b(x_b, h) \geq \alpha_b(\theta_b(x_b, h, Z))(x_b - \mu(Z)h), \]  
\[ \kappa_s(x_s, h) \geq \alpha_s(\theta_s(x_s, h, Z))(\mu(Z)h - x_s), \]  
\[ Y_h \geq 0, \]  
\[ \Omega_b(x_b, h) \geq 0, \]  
\[ \Omega_s(x_s, h) \geq 0. \]

Recall that the market tightness is the ratio of real estate agents to buyers or sellers; hence, the following profit maximization conditions are equivalent:

\[ p_h(Z) \geq \mu(Z) \text{ and } Y_h \geq 0 \text{ with complementary slackness,} \]  
\[ \kappa_b(x_b, h) \geq \alpha_b(\theta_b(x_b, h, Z))(x_b - \mu(Z)h) \text{ and } \theta_b(x_b, h, Z) \geq 0 \text{ with comp. slackness,} \]  
\[ \kappa_s(x_s, h) \geq \alpha_s(\theta_s(x_s, h, Z))(\mu(Z)h - x_s) \text{ and } \theta_s(x_s, h, Z) \geq 0 \text{ with comp. slackness.} \]

The cost of hiring real estate agents in submarket \((x_b, h)\) of the buying market is \(\kappa_b(x_b, h)\), and the benefit to the firm is that a fraction \(\alpha_b(x_b, h)\) of real estate agents match with buyers and are paid \(x_b\). However, hiring more real estate agents in the buying market also tightens the real estate firm’s housing flows constraint.

In the selling market, the cost of hiring real estate agents in submarket \((x_s, h)\) is \(\kappa_s(x_s, h)\), and the firm also pays \(x_s\) to sellers for the fraction \(\alpha_s(x_s, h)\) of real estate agents that successfully match. However, the benefit of hiring real estate agents in the selling market is that successful purchases from sellers loosen the housing flows constraint.

### 2.6.5 Mortgage Company’s Problem

Mortgage companies choose the number of type - \((m', a', h, s)\) mortgage contracts to sell to maximize present value profits, discounted at the rate \(i\).

Profit maximization implies the following recursive relationship:

\[
q^0_m(m', a', h, s, Z) = \frac{1 - \delta_h}{(1 + \zeta)(1 + i + \phi)} \mathbb{E}_{(a', s', z_c')}(p_s(\theta_s(x_s', h, Z')) + (1 - p_s(\theta_s(x_s', h, Z')))) \times \left[ d_s' \min \left\{ 1, \frac{J_{REO}(h, Z')}{m'} \right\} + (1 - d_s')(1 + (1 + \zeta)q^0_m(m'', a'', h, s', Z') - q^0_m(m'', h, s', Z') - m''_s) \right],
\]

\[ Z' = (z_c', G(Z, z_c')) \]  

(30)
for all \((m', a', h, s)\), where \(J_{REO}\) is the value to repossessing a house, \(x^*_s = x_s(X', Z')\), \(d' = d(X', Z')\), \(m'^* = m'(X', Z')\), \(a'^* = a'_{own}(X', Z')\), and

\[
X' = (e' \cdot w(Z') + (1 + r(Z'))a', m', h, s, 0).
\]

Recall that \(q_m = \frac{1}{1 + r_m}\), where \(r_m\) is the continuation interest rate faced by all borrowers. This interest rate compensates the mortgage company for its opportunity cost of funds, its servicing cost, and for the risk that the borrower’s house fully depreciates. Therefore,

\[
q_m = \frac{1 - \delta_h}{1 + i + \phi},
\]

which implies that absent default risk, \(q_m^0(m', a', h, s) = \frac{q_m}{1 + \phi}\) for all \((m', a', h, s)\), where \(\phi\) is the origination cost.

**REO Optimization**  Mortgage companies manage their REO inventory by deciding which submarket to enter to sell their housing.

The value to a mortgage company of repossessing and selling a house of size \(h\) is

\[
J_{REO}(h, Z) = R_{REO}(h, Z) + \frac{1 - \delta_h}{1 + i} E z' J_{REO}(h, Z'),
\]

where

\[
R_{REO}(h, Z) = \max\{0, \max_{x_s \geq 0} \lambda p_s(\theta_s(x_s, h, Z))((1 - \chi)x_s - \frac{1 - \delta_h}{1 + i} E z' J_{REO}(h, Z'))\}
\]

(31)

**REO Inventories**  Let \(\{H_{REO}(h) : h \in H\}\) be the stock of REO housing and \(\Phi_1\) be the distribution of households over individual states at the beginning of subperiod 1.

The stock of REO housing evolves according to

\[
H'_{REO}(h) = [H_{REO}(h) + \int (1 - p_s(\theta_s(x^*_s, h, Z)))d^* \Phi_1(dy, dm, h, ds, 0)]
\times (1 - \lambda p_s(\theta_s(x^*_{REO}(h, Z), h, Z)))(1 - \delta_h)
\]

\[
(32)
\]

where \(x^*_s = x_s(y, m, h, s, 0, Z)\) is the homeowner’s optimal selling submarket, \(d^* = d(y, m, h, s, 0, Z)\) is the homeowner’s optimal default choice, and \(x^*_{REO}(h, Z)\) is the mortgage company’s optimal selling submarket.

The first term in the braces represents REOs at the beginning of the period and the second term represents housing repossessed from homeowners that defaulted this period. A fraction \(\lambda p_s(\theta_s(x^*_{REO}(h, Z), h, Z))\) of these houses is sold by the mortgage company, and a fraction \(\delta_h\) of the unsold REO stock fully depreciates.
2.7 Equilibrium

2.7.1 Market Tightnesses

Recall from (27) that \( p_h(Z) \geq \mu(Z) \) and \( Y_h \geq 0 \) with complementary slackness. The real estate firm’s constraint binds because it is always profitable for the real estate firm to sell all of the housing that it purchases. Therefore, \( \mu(Z) > 0 \), which implies that \( p_h(Z) > 0 \). Conditions (17) – (19) from the construction firm’s maximization problem, combined with the assumptions on \( F_h \), imply that \( Y_h > 0 \). Therefore, \( p_h(Z) = \mu(Z) \). Substituting this result into (28) – (29) gives

\[
\begin{align*}
\kappa_b(x_b, h) &\geq \alpha_b(\theta_b(x_b, h, Z))(x_b - p_h(Z)h) \text{ and } \theta_b(x_b, h, Z) \geq 0 \text{ with comp. slackness}, \\
\kappa_s(x_s, h) &\geq \alpha_s(\theta_s(x_s, h, Z))(p_h(Z)h - x_s) \text{ and } \theta_s(x_s, h, Z) \geq 0 \text{ with comp. slackness}.
\end{align*}
\]

These conditions state that submarket tightnesses depend on the aggregate state \( Z \) only through \( p_h(Z) \), which I call the shadow housing price. In particular, the market tightnesses do not directly depend on the distribution of households over individual states \( (y, m, h, s, f) \). As a result, households only need to forecast the evolution of \( p_h(Z) \) to know the dynamics of all submarket tightnesses.

Directed search and free entry of real estate agents are responsible for this result. Directed search fixes the terms of trade before matching takes place, and free entry renders the measure of buyers or sellers in each submarket irrelevant because the ratio of real estate agents to buyers or sellers adjusts until the real estate firm experiences no gains from trade. This result is similar to the block recursivity result obtained by Menzio and Shi (2010), except here the distribution of households over individual states does indirectly affect the market tightnesses through its impact on the shadow housing price \( p_h(Z) \).

2.7.2 Housing Market Clearing

To determine \( p_h(Z) \), call the left side of the real estate firm’s constraint housing supply and call the right side housing demand. Housing demand is the total amount of housing sold to buyers in the buying market, and housing supply is the total amount of housing purchased from construction firms and sellers in the selling market. Let \( \Phi_1 \) be the distribution of households over individual states in subperiod 1; let \( \Phi_2 \) be the distribution in subperiod 2; and let \( \Phi_3 \) be the distribution in subperiod 3. In the following, the notation \( (\cdot; p_h) \) indicates implicit dependence on \( p_h \).

Because the number of matched buyers equals the number of matched real estate agents
in the buying market, housing demand is given by

\[ D_h(p_h; Z) = \int h^* p_b(\theta_b(x_b^*, h^*; p_h)) \Phi_2(dy, 0, 0, ds, df) \]

where \( h^* = h(y, s, f, Z; p_h) \) and \( x_b^* = x_b(y, s, f, Z; p_h) \).

Similarly, because the number of matched homeowners and mortgage companies equals the number of matched real estate agents in the selling market, housing supply is given by

\[ S_h(p_h; Z) = Y_h(p_h; Z) + S_{REO}(p_h; Z) + \int h p_s(\theta_s(x_s^*, h; p_h)) \Phi_1(dy, dm, dh, ds, df) \]

where \( x_s^* = x_s(y, m, h, s, f, Z; p_h) \) and \( Y_h(p_h) \) is optimal construction, as given by (17) – (19). The first term is the supply of new housing, the second term is the supply of REO housing, and the third term is the supply of existing housing by homeowners.

The supply of REO housing is given by

\[ S_{REO}(p_h; Z) = \sum_{h \in H} h [H_{REO}(h) + \int (1 - p_s(\theta_s(x_s^*, h; p_h))) d^* \Phi_1(dy, dm, h, ds, 0)] \times \lambda p_s(\theta_s(x_s^{REO}(h, Z; p_h), h; p_h)) \]  

where \( d^* = d(y, m, h, s, 0, Z; p_h) \).

The equilibrium price \( p_h(Z) \) satisfies

\[ D_h(p_h(Z), Z) = S_h(p_h(Z), Z) \]  

2.7.3 Recursive Equilibrium

**Definition 1** Given international bond yield \( i \), a recursive equilibrium is

- Value and policy functions for homeowners \( V_{own}, W, R_s, c_{own}, a'_{own}, m', x_s, \) and \( d \);
  - value and policy functions for renters \( V_{rent}, R_b, c_{rent}, c_h, a'_{rent}, x_b, \) and \( h \)
- REO value and policy functions \( J_{REO} \) and \( x_s^{REO} \)
- Mortgage prices \( q_0^m \)
- Market tightness functions \( \theta_b \) and \( \theta_s \)
- Policy functions for production firms \( K(Z), N_c(Z), L(Z), B(Z), N_h(Z) \)
- Prices \( p_h(Z), p_l(Z), r(Z), w(Z), r_h \)
• An aggregate law of motion $G = (G_{\Phi_1}, G_K, G_{\text{REO}})$ such that

1. **Household Optimization:** The household value and policy functions solve the household’s problem.

2. **REO Optimization:** The REO value and policy functions solve the mortgage company’s REO problem.

3. **Firm Optimization:** $K(Z), N_c(Z), L(Z), B(Z), \text{ and } N_h(Z)$ solve, given $p_h(Z), p_l(Z), r(Z), \text{ and } w(Z),$

   \[
   r(Z) = z_cA_c \frac{\partial F_c(K(Z), N_c(Z))}{\partial K} - \delta_k \\
   w(Z) = z_cA_c \frac{\partial F_c(K(Z), N_c(Z))}{\partial N_c} = p_h(Z) \frac{\partial F_h(L(Z), B(Z), N_h(Z))}{\partial N_h} \\
   1 = p_h(Z) \frac{\partial F_h(L(Z), B(Z), N_h(Z))}{\partial B} \\
   p_l(Z) = p_h(Z) \frac{\partial F_h(L(Z), B(Z), N_h(Z))}{\partial L}
   \]

4. **Market for Land/Permits Clears:**

   \[L(Z) = \bar{L}\]

5. **Labor Market Clears:**

   \[N_c(Z) + N_h(Z) = \sum_{s \in S} \int_E c \cdot sF(de)\Pi_s(s)\]

6. **Capital Market Clears:**

   \[K(Z) = K\]

7. **Market for Housing Services Clears:**

   \[r_h = \frac{1}{A_h}\]

8. **Market Tightnesses in the Buying Market:**

   \[\kappa_b(x_b, h) \geq \alpha_b(\theta_b(x_b, h, Z))(x_b - p_h(Z)h) \text{ and } \theta_b(x_b, h, Z) \geq 0 \text{ with comp. slackness.}\]
9. **Market Tightnesses in the Selling Market:**

\[ \kappa_s(x_s, h) \geq \alpha_s(\theta_s(x_s, h, Z))(p_h(Z)h - x_s) \text{ and } \theta_s(x_s, h, Z) \geq 0 \text{ with comp. slackness.} \]

10. **Housing Market Clears:**

\[ D_h(p_h(Z), Z) = S_h(p_h(Z), Z) \]

11. **Mortgage Market Clears:** \( q_m^0 \) satisfies (30) for all \((m', a', h, s)\).

12. **Law of Motion for the Distribution of Households:** \( \Phi_1' = G_\Phi_1(Z, z'_c) \) is consistent with the Markov process induced by the exogenous processes \( \pi_z, \pi_s, \) and \( F \), and all relevant policy functions.

13. **Law of Motion for Capital:**

\[ K' = G_K(Z, z'_c) = \int a'_\text{rent}(y, s, f, Z)\Phi_3(dy, ds, df) \]
\[ + \int a'_\text{own}(y, m, h, s, f, Z)\Phi_3(dy, dm, dh, ds, df) \]

14. **Law of Motion for REO Housing Stock:**

\[ H_{REO}'(h) = G_{H_{REO}}(Z, z'_c)(h) = [H_{REO}(h) + \int (1 - p_s(\theta_s(x^*_s, h, Z))))d^*\Phi_1(dy, dm, h, ds, 0)] \]
\[ \times (1 - \lambda p_s(\theta_s(x^*_{REO}(h, Z), h)))(1 - \delta_h) \]

#### 2.8 Equilibrium with Bounded Rationality

The block recursive structure of the housing market means that computing the equilibrium is not made any more difficult by the presence of search frictions. Nevertheless, households still need to keep track of the entire distribution \( \Phi_1 \) as well as the REO housing stock vector \( H_{REO} \in \mathbb{R}_+^H \) to forecast \( p_h(Z), w(Z), \) and \( r(Z) \).

To deal with this difficulty, I follow the approach first introduced by Krusell and Smith (1998). In their model, agents approximate the distribution using a finite collection of moments that are sufficient statistics for current prices. Agents then form approximating forecasting functions to predict the evolution of these moments. This setup is a form of bounded rationality that has proven quite useful in a variety of settings.
As in Ríos-Rull and Sánchez-Marcos (2008) and Favilukis et al. (2011), I use the capital stock $K$ and the shadow price of housing $p_h$ as the endogenous aggregate state variables. Therefore, the aggregate state in this bounded rationality economy is $(z_c, p_h, K)$, and I posit the following forecasting functions:

$$p_h'(z_c, p_h, K, z'_c) = a^p_0(z_c, z'_c) + a^p_1(z_c, z'_c)p_h + a^p_2(z_c, z'_c)K$$  \hfill (37)

$$K'(z_c, p_h, K, z'_c) = a^K_0(z_c) + a^K_1(z_c)p_h + a^K_2(z_c)K$$  \hfill (38)

An approximate equilibrium is then a choice of coefficients that maximizes the predictive accuracy of the forecasting functions relative to simulated time series of $p_h'$ and $K'$.

3 Calibration

The model is calibrated to match selected aggregate and cross-sectional facts of the U.S. economy from 1975 - 2000. Some parameters are chosen externally from the literature or from a priori information. The remaining parameters are calibrated internally to make the steady state version of the model—that is, the model without aggregate shocks—jointly match targets from U.S. data. The starting year is 1975 because it is the first year of the Freddie Mac House Price Index (FMHPI) data. The ending year is 2000 to exclude the most recent housing boom and bust, which has produced historic swings in house prices, homeownership rates, and housing wealth relative to income. Prior to this housing cycle, these variables were fairly stable. For variables that were not stable from 1975 – 2000, I target their average values during the 1990s.

3.1 Model Specification

3.1.1 Households

Preferences  Household have CES period utility, which is given by

$$U(c, c_h) = [\omega c^{\frac{1}{\nu-1}} + (1 - \omega) c_h^{\frac{1}{\nu-1}}]^{\frac{\nu}{\nu-1}}$$

where $\omega$ is the consumption good’s share of utility and $\nu$ is the intratemporal elasticity of substitution between the consumption good and housing services. I set $\nu = 0.13$ following Flavin and Nakagawa (2008). They use PSID data to estimate the intratemporal elasticity of substitution between housing and non-housing consumption, taking into account lumpy adjustment costs. I choose $\omega$ during the joint calibration.
Recall that households have Epstein-Zin preferences, namely,

\[ V = [(1 - \beta)U^{\frac{1-\sigma}{\sigma}} + \beta(\mathbb{E}V^{1-\sigma})^{\frac{1}{\sigma}}]^{\frac{\sigma}{1-\sigma}} \]

I choose the discount factor, \( \beta \), in the joint calibration. I set the risk aversion parameter to \( \sigma = 8 \) as in Favilukis et al. (2011). Although this value is on the high end of values reported in the literature, high risk aversion enables the model to match the fact that agents simultaneously hold large amounts of assets and debt. With standard constant relative risk aversion preferences, high risk aversion implies a low intertemporal elasticity of substitution. However, the Epstein-Zin specification disentangles agents’ preferences toward risk and toward intertemporal substitution. I choose \( \psi = 1.75 \) for the intertemporal elasticity of substitution, which is close to the value of 1.73 estimated in van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2010).

**Labor Productivity** I assume that the log of labor efficiency follows

\[
\begin{align*}
\ln(e \cdot s) &= \ln(s) + \ln(e) \\
\ln(s') &= \rho \ln(s) + \varepsilon' \\
\varepsilon &\sim \mathcal{N}(0, \sigma_\varepsilon^2) \\
\ln(e) &\sim \mathcal{N}(0, \sigma_e^2)
\end{align*}
\]

where I truncate \( \ln(e) \) to have compact support and I approximate \( \ln(s) \) with a finite state Markov chain with transition probabilities \( \pi_s(s'|s) \). To calibrate \( \rho, \sigma_\varepsilon, \) and \( \sigma_e \), I follow Storesletten, Telmer, and Yaron (2004), with some modifications. They use PSID data from 1968 – 1993 to estimate the idiosyncratic component of household earnings using a specification similar to the one above. However, they also include a permanent shock that agents receive at birth, and they allow for the variance of the persistent shock to differ in expansions and in recessions. They report \( \rho = 0.952, \sigma_e = 0.255, \) and a frequency-weighted average (over expansions and recessions) \( \sigma_\varepsilon = 0.17. \)

I cannot directly use these estimates, however, because they were estimated on annual data. In the appendix I explain in detail how I deal with this issue. The result is that I set \( \sigma_e^2 = 0.49 \) and calibrate \( s \) using a two state Markov chain following the Rouwenhorst (1995) method. The persistent shock takes on the values \( \{s_1, s_2\} = \{0.5739, 1.7426\} \) and has transition matrix

\[
\pi_s(\cdot, \cdot) = \begin{pmatrix}
0.994 & 0.006 \\
0.006 & 0.994
\end{pmatrix}
\]
3.1.2 Production Sectors

Consumption Good Sector Output in the production sector is given by

\[ Y_c = z_c A_c F_c(K, N_c) = z_c A_c K^\alpha_k N_c^{1-\alpha_k} \]

Capital in the consumption good sector translates to non-residential capital in the data, which is the same definition of capital used by Díaz and Luengo-Prado (2010). I use their value for the capital share, \( \alpha_k = 0.26 \). I set the depreciation rate \( \delta_k = 0.025 \), which corresponds to an annual depreciation rate of 10%. Nakajima (2010) uses similar values for the capital share and depreciation—0.247 and 0.109, respectively. I choose \( A_c \) to normalize mean quarterly earnings to 0.25.

To calibrate the aggregate TFP shocks, I assume that

\[ \ln(z'_c) = \rho_z \ln(z_c) + \varepsilon'_z \]
\[ \varepsilon_z \sim \mathcal{N}(0, \sigma^2_{\varepsilon_z}) \]

with \( \rho_z = 0.95 \) and 0.007, which are standard values. I approximate this process with a two state Markov chain using the Rouwenhorst method, which gives the values \( \{z_{cl}, z_{ch}\} = \{0.978, 1.0227\} \) and transition matrix

\[ \pi_z(\cdot, \cdot) = \begin{pmatrix} 0.975 & 0.025 \\ 0.025 & 0.975 \end{pmatrix} \]

Housing Services I set \( A_h \) in the joint calibration.

Construction Sector Output in the construction sector is given by

\[ Y_h = F_h(L, B, N_h) = L^{\alpha_l}(B^{\alpha_b} N_h^{1-\alpha_b})^{1-\alpha_l} \]

Using the FHFA-based price index data from the Lincoln Institute of Land Policy, \(^{10}\) I calculate that land accounts for an average 33% of the market value of housing during 1975–2000. Therefore, I set \( \alpha_l = 0.33 \). Davis and Heathcote (2007) report a similar value of 36% for the period 1975–2006. For the share of structures in construction, I follow Favilukis et al. (2011) and set \( \alpha_b = 0.3 \).

The literature reports different values for housing depreciation, which range from 1.14%

\(^{10}\)http://www.lincolniest.edu/subcenters/land-values/price-and-quantity.asp
from BEA estimates\(^{11}\) to 2.5% in Harding, Rosenthal, and Sirmans (2007). I follow Davis and Heathcote (2005) and use a 1.4% depreciation rate, implying a quarterly rate of 0.351%. I choose a number on the small end of the range because houses either stay the same or fully depreciate in the model. I view having a large probability that houses fully depreciate as being implausible.

### 3.1.3 Real Estate Sector

I determine the minimum house size \(h\) in the joint calibration, and I set the maximum house size to be \(5h\). Let the matching function in the buying and selling markets be of the form

\[
M_j(\Omega_j, \Lambda_j) = \min\{\Omega_j^{\gamma_j} \Lambda_j^{1-\gamma_j}, \Lambda_j\} \text{ for } j = B, S
\]

where \(\Omega_j\) is the measure of real estate agents and \(\Lambda_j\) is the measure of buyers (if \(j = B\)) or sellers (if \(j = S\)).

The probability that a buyer or seller matches with a real estate is then

\[
\frac{M_j(\Omega_j, \Lambda_j)}{\Lambda_j} = M_j(\frac{\Omega_j}{\Lambda_j}, 1) = M_j(\theta_j, 1) = \min\{\theta_j^{\gamma_j}, 1\} = p_j(\theta_j)
\]

Similarly, the probability that a real estate agent matches with a buyer or seller is

\[
\frac{M_j(\Omega_j, \Lambda_j)}{\Omega_j} = \frac{M_j(\theta_j, 1)}{\theta_j} = \frac{p(\theta_j)}{\theta_j} = \alpha_j(\theta_j)
\]

With a slight abuse of notation, rewriting \(p_j\) as a function of \(\alpha_j\) gives

\[
p_j(\alpha_j) = \min \left\{ \left( \frac{1}{\alpha_j} \right)^{\frac{\gamma_j}{1-\gamma_j}}, 1 \right\}
\]

**Buying Market**  Let the cost of hiring a real estate agent in buying submarket \((x_b, h)\) be \(\kappa_b(x_b, h) = \kappa_b h\). Assuming that submarket \((x_b, h)\) is actively visited, its equilibrium market tightness satisfies

\[
\kappa_b h = \alpha_b(\theta_b(x_b, h))(x_b - p_h h)
\]

which implies that

\[
\alpha_b(\theta_b(x_b, h)) = \frac{\kappa_b h}{x_b - p_h h}
\]

\(^{11}\)http://www.bea.gov/national/FA2004/Tablecandtext.pdf
The probability that a buyer successfully trades in submarket \((x_b, h)\) is therefore

\[
p_b(\theta_b(x_b, h)) = \min \left\{ \left( \frac{x_b - p_h h}{\kappa_b h} \right)^{\gamma_b}, 1 \right\}
\]

To summarize,

\[
p_b(\theta_b(x_b, h)) = \begin{cases} 
0 & \text{if } x_b < p_h h \\
\left( \frac{x_b - p_h h}{\kappa_b h} \right)^{\gamma_b} & \text{if } p_h h \leq x_b \leq (p_h + \kappa_b)h \\
1 & \text{if } x_b > (p_h + \kappa_b)h
\end{cases}
\]

This equation states that the search frictions in the buying market are a form of endogenous adjustment cost. If buyers pay the full adjustment cost \(\kappa_b h\) on top of the shadow housing value \(p_h h\), they receive the house with probability 1. Otherwise, if they choose a submarket such that they pay an adjustment cost somewhere in between 0 and \(\kappa_b h\), they receive the house with probability less than 1. I determine \(\gamma_b\) and \(\kappa_b\) as part of the joint calibration.

**Selling Market** Let the cost of hiring a real estate agent in selling submarket \((x_s, h)\) be \(\kappa_s(x_s, h) = \kappa_s h\). Using similar reasoning to that in the buying market, it can be shown that the probability that a seller successfully trades in submarket \((x_s, h)\) is

\[
p_s(\theta_s(x_s, h)) = \begin{cases} 
1 & \text{if } x_s < (p_h - \kappa_s)h \\
\left( \frac{p_h h - x_s}{\kappa_s h} \right)^{\gamma_s} & \text{if } (p_h - \kappa_s)h \leq x_s \leq p_h h \\
0 & \text{if } x_s > p_h h
\end{cases}
\]

Just as in the buying market, search frictions in the selling market act as an endogenous adjustment cost. If sellers post a price equal to the shadow value of housing \(p_h h\) minus the adjustment cost \(\kappa_s h\), their house sells with probability 1. Otherwise, if they post a higher price (which is equivalent to a smaller adjustment cost), their house sells with probability less than 1. Homeowners cannot sell their house for more than \(p_h h\). I determine \(\gamma_s\), \(\kappa_s\), and the utility cost of entry \(\kappa\) as part of the joint calibration.

### 3.1.4 Mortgage Sector

I let the yield on international bonds be 4% annually. The Federal Reserve reports mortgage costs of 1% – 3% of the loan balance, and the Federal Housing Finance Board reports closing
costs of approximately 1% of the loan value during the 1990s; thus, I set the mortgage origination cost to 2%, i.e. \( \zeta = 0.02 \).

I calibrate the mortgage servicing cost \( \phi \) to match the average contract rate on conventional mortgages during the 1990s. The Federal Reserve reports an average interest rate on 30-year conventional mortgages of 8.1% during the 1990s. In addition, the American Housing Survey in 1997 reports that the median mortgage interest rate was 8.1%. Recall that in the model

\[
q_m = \frac{1}{1 + r_m} = \frac{1 - \delta_h}{1 + i + \phi}.
\]

With \( \delta_h = 0.00351 \), a bond rate of 4% (quarterly \( i = 0.00985 \)), and a mortgage interest rate of 8.1% (quarterly \( r_m = 0.0197 \)), the implied quarterly mortgage servicing cost is \( \phi = 0.00623 \). The largest mortgage that a homeowner can take out is the highest price they could receive for their house in the selling market, i.e. \( \max M(h) = \sup \{ x_s : p_s(\theta_s(x_s, h)) > 0 \} = p_h h \), which comes from the equilibrium selling probabilities above.

### 3.2 Foreclosures and Legal Environment

I target a waiting period of 5 years after a foreclosure before a borrower can take out a new mortgage. This time frame is consistent with the policies of Fannie Mae and Freddie Mac, which guarantee most U.S. mortgages. In the model, the waiting period is stochastic and depends on the probability \( \gamma_f \) that a foreclosure flag stays on a borrower’s credit record. I set \( \gamma_f = 0.95 \) to give an expected duration of 5 years.

The remaining parameters—the foreclosure sale loss \( \chi \) and REO search efficiency \( \lambda \)—I determine as part of the joint calibration.

### 3.3 Joint Calibration Procedure

I calibrate the remaining 11 parameters by targeting 11 steady-state statistics using the model without aggregate shocks. The parameters and targets are given below:

**Parameters**

1. The discount factor \( \beta \)
2. The utility share of the consumption good \( \omega \).
3. The technology parameter \( A_h \) for housing services production.
4. The minimum house size \( h \)
5. The cost of hiring real estate agents in the buying market $\kappa_b$
6. The cost of hiring real estate agents in the selling market $\kappa_s$
7. The curvature of the matching function in the buying market $\gamma_b$
8. The curvature of the matching function in the selling market $\gamma_s$
9. The utility cost of entering the selling market $\kappa$
10. The foreclosure sale loss $\chi$
11. REO search efficiency $\lambda$

**Targets**

1. Mean net worth divided by mean income of 3.609.
2. Mean housing wealth divided by mean homeowner income of 2.375.
3. A rent-to-price ratio of housing of 5%.
4. A homeownership rate of 64%.
5. A maximum premium of 2.5% of house value in the buying market.
6. A maximum discount of 20% of house value in the selling market.
7. An average buyer search time of 10 weeks.
8. An annual foreclosure (repossession) rate of 0.352%.
9. An average months supply of housing inventories of 6 months.
10. A foreclosure discount of 22%.
11. An average time on the market for REOs of 52 weeks.

Now I explain the choices of targets and how they are measured in the model. I use the 1998 Survey of Consumer Finances to calculate selected income and wealth statistics. I restrict the sample to all households under the age of 65 because the model does not include Social Security or retirement. The first target from the SCF is the ratio of mean housing wealth to mean homeowner income, which is 2.375. I use the shadow value of housing to determine housing wealth in the model, i.e. an owner of house $h$ has housing wealth $p_h h$. The
second target from the SCF is the ratio of mean net worth to mean annual income, which is 3.609. In the model, I define net worth as assets plus housing wealth minus mortgage debt.

For the rent-to-price ratio, I use data from the Lincoln Institute of Land Policy. They estimate annual rents for owner-occupied units based on data from the Census Bureau on annual rents paid to rental units. They divide these estimated rents by the average self-reported value of owner-occupied units to obtain the rent to price ratio. Prior to the recent housing boom this number was fairly stable and hovered around 5%. I define the rent to price ratio in the model as $r_h/p_h$, and I target a quarterly value of 1.25%.

I target a homeownership rate of 64%, which is near the average reported by the Census Bureau for the period 1975 – 2010.

For the foreclosure rate target, I use the foreclosure starts data from the National Delinquency Survey released by the Mortgage Banker’s Association (MBAA). The quarterly rate of foreclosure starts averaged 0.352% of outstanding loans during the 1990s, implying an annual rate of 1.408%. However, Jeske et al. (2010), citing the MBAA, report that only 25% of foreclosures end in repossession. In the model, all foreclosures end in repossession; thus, I target an annual foreclosure (repossession) rate of 0.352%.

I target two facts related to foreclosure sales. Pennington-Cross (2006) finds that foreclosed houses sell at a substantial discount relative to their appraised value. He calculates this discount to be 22%, so I target a foreclosure discount of 22% relative to the shadow housing value $p_h^s$. The second target is an average time on the market for REO houses of 52 weeks. In the model there is no delay in processing foreclosures—as soon as a borrower misses a payment, the mortgage company repossesses his house—thus, this target implicitly includes both processing time and actual selling time. The total foreclosure duration in the data varies substantially from state to state and over time—from 90 days to multiple years—so I choose 52 weeks as a reasonable middle ground.

The first target related to the buying market is a buyer search time of 10 weeks, which is in the middle of available estimates. For example, the National Association of Realtors reports that the average number of weeks searched by buyers varied between 7 and 12 weeks during the past decade. I also target a maximum buying premium of 2.5%. Recall from the formulation of the matching function that buyers can purchase a house with probability 1 if they choose submarket $x_b = (p_h + \kappa_b)h$. Gruber and Martin (2003) report that the median household spends 2.5% of a house’s value to purchase it; thus, I target $\kappa_b = 0.025p_h$.

In the model buyers follow buying strategies that depend non-trivially on their individual state, which can change over time. As a result, buyers that fail to buy in one period may choose a different submarket—and hence a different buying probability—in the next period. When computing the average buyer search duration I ignore these dynamic buying strategies.
Instead, I take the cross-sectional average of each buyer’s expected search duration based on their current submarket choice. Because the model is a quarterly model rather than a weekly model, I follow the same timing convention as in Caplin and Leahy (2008). They assume that housing trades that occur in period $t$ are uniformly distributed between period $t$ and period $t+1$. Therefore, an immediate purchase corresponds to an average search time of 0.5 periods, or 1.5 months (6 weeks). A purchase after $n$ periods corresponds to an average search time of $(n + 0.5) \times 12$ weeks. Therefore, I target an average search duration of $n = 0.33$, i.e.

$$\int \frac{1 - p_b(\theta_b(x_b(y, s, f), h(y, s, f)))}{p_b(\theta_b(x_b(y, s, f), h(y, s, f)))} \Phi_2(dy, ds, df) = 0.33$$

The last two targets involve the selling market. The first target is a 20% maximum discount in the selling market, which is the price at which sellers can sell their house immediately with probability 1. Garriga and Schlagenhauf (2009) use data from the American Housing Survey to measure what they call an idiosyncratic capital gains shock to housing, which can be seen as a deviation between the selling price of a house and its market value. They calculate that 90% of capital gains shocks fall in the range from -9.7% to 12.2%, which is a range of 21.9%. In the model, it is not possible to sell at a higher price than $p_h$; thus, I target a 20% maximum selling discount instead. As further support for this choice, RealtyTrac provides information for average pre-foreclosure discounts. As stated previously, foreclosed houses sell at an average 22% discount, but a significant portion of this discount is likely due to neglect of the property while it was going through the foreclosure process. Pre-foreclosure sales, however, are less likely to manifest as much physical degradation because they have not been vacant for an extended period of time. RealtyTrac reports an average pre-foreclosure discount in the U.S. of 9.49%, but it ranges from 1.28% to 34.94% throughout the different states in the U.S.\textsuperscript{12} Among other reasons, these houses are likely to sell at a discount because the owners are motivated to sell and willing to accept a lower price.\textsuperscript{13}

I also target a months supply of 6 months. The National Association of Realtors releases housing sales and inventory data for existing houses, and the Census releases these data for new houses. Housing inventories are defined as houses that are listed for sale but do not sell by the end of the period. The months supply is defined as housing inventories divided by the annual sales rate multiplied by 12 months. Loosely speaking, it measures how long it would take for the housing inventory to be sold at the current sales rate. A balanced housing

\textsuperscript{12}Montana, New Mexico, and Utah, however, show negative pre-foreclosure discounts.

\textsuperscript{13}This is especially true if the sale is a short-sale, where neither the owners nor the bank deem it possible to make a profit by waiting longer for a higher bid.
market (neither a housing boom nor a housing bust) is usually characterized by a months supply of approximately 6 months. In the model I define housing inventories as the quantity of housing put up for sale that does not sell and housing sales as the quantity of housing that is put up for sale and successfully sells. I summarize the calibration in Table 6.

4 Results

This section presents the results of the model. First, I discuss the steady state results of the model regarding housing market behavior, portfolio choice, and foreclosure decisions. Then, I discuss the cyclical properties of the model and compare them to those of the U.S. data. Lastly, I present results for a version of the model without housing illiquidity, and I discuss the effects of illiquidity on the behavior of housing, debt, and foreclosures.

4.1 Steady State Results

4.1.1 Housing Market Behavior

Table 1 reports some statistics related to housing sales for the model and the U.S. data. I divide housing sales into three categories: existing home sales, new home sales, and foreclosure (REO) sales. RealtyTrac reports that REO sales typically account for 5% of total sales. The model has a similar REO share of 4.35%. However, the model understates the share of existing sales—and therefore overstates the share of new housing sales—because of fewer homeowner-to-homeowner transitions. Dieleman, Clark, and Deurloo (2000) report an annual turnover rate of 8%, compared to 6.92% turnover in the model. Lower turnover in the model is not surprising, given that the model does not include any preference or relocation shocks that might motivate homeowners to move for non-financial reasons.

Buying Behavior  In the buying market, buyers choose a house size and a purchase price. Buyers increase their expected spending on housing when they have more cash at hand, either by choosing a larger purchase premium—that is, a larger $x_b$ relative to $p_h h$—or by choosing
a larger house size. With discrete house sizes, buyers’ optimal house choice is an increasing step function of cash at hand. For a given house size choice, buyers increase their purchase price as their cash at hand increases, thus raising their probability of trade. However, when deciding at the margin between two house sizes, buyers who choose the larger house also choose a smaller purchase premium, leading to a lower probability of trade. In other words, buyers choosing at the margin between two house sizes can increase their purchase price for the smaller house to increase their probability of buying, or they can try to buy the larger house with a lower probability.

**Selling Behavior** There are two types of sellers in the model. Sellers with high net worth relative to their current house size want to upgrade. Those with low net worth relative to their current house size want to downsize. Both types of sellers must pay off their mortgage upon selling their house. Thus, sellers must either set a sufficiently high selling price to pay off their mortgage or they must have enough liquid assets to cover the difference. Because downgrading sellers typically have high mortgage debt and low assets, they are constrained to post a high selling price. Upgrading sellers are typically not debt-constrained. On average, sellers post a selling price discount of 7.44%. By comparison, Gruber and Martin (2003) report that the median household spends 7% of a house’s value to sell it.

For upgrading sellers, the optimal selling price is a decreasing function of cash at hand and an increasing function of mortgage debt. As homeowners become wealthier, they are willing to accept a larger selling price discount to move to a larger house more quickly. The selling behavior of homeowners with low assets depends on their level of mortgage debt. Homeowners with low assets and low debt do not try to sell their house; they simply take out a larger mortgage. However, homeowners with low liquid assets and high mortgage debt find it difficult and costly to refinance to a larger mortgage, mostly because they must pay a large default premium to mortgage companies. As a result, many of these homeowners are forced to try to sell their house.

The optimal selling price for downgrading sellers is non-monotonic in mortgage debt. It is approximately flat for low to medium values of leverage, is decreasing for medium to high values of leverage, and is increasing for high values of leverage. Thus, the probability of selling is approximately flat for low to medium values of leverage, is increasing for medium to high values of leverage, and is decreasing for high values of leverage. This behavior confirms the findings of Genesove and Mayer (1997, 2001), who show that sellers with high leverage post higher asking prices, wait longer on the market, and sell at higher prices. As a function of liquid assets, the optimal selling price is typically increasing for downsizing sellers until they no longer want to downsize.
4.1.2 Portfolio Choice

Households in the model and in the data hold substantial amounts of housing wealth, mortgage debt, and liquid assets. I calibrate the steady state version of the model to match mean net worth and mean housing wealth relative to income, but the model also matches the average amount of mortgage debt and liquid assets, as reported in Table 2. The model also replicates the leverage distribution of homeowners well, which is important because of the role of indebtedness in the interaction of housing and mortgage illiquidity. The model approximately matches the mean leverage of homeowners and the percentage of homeowners with more than 80% leverage, while it overstates the percentage of homeowners with mortgage debt and the percentage of homeowners with more than 90% leverage.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid assets</td>
<td>2.575</td>
<td>2.547</td>
</tr>
<tr>
<td>Homeowner net worth</td>
<td>4.291</td>
<td>4.297</td>
</tr>
<tr>
<td>Mortgage debt</td>
<td>1.08</td>
<td>1.078</td>
</tr>
<tr>
<td>Percent with mortgage debt</td>
<td>79.1%</td>
<td>97.15%</td>
</tr>
<tr>
<td>Unconditional mean leverage</td>
<td>49.6%</td>
<td>46.7%</td>
</tr>
<tr>
<td>Percent with leverage &gt; 80%</td>
<td>17.6%</td>
<td>19.75%</td>
</tr>
<tr>
<td>Percent with leverage &gt; 90%</td>
<td>10.2%</td>
<td>15.8%</td>
</tr>
</tbody>
</table>

These results provide a challenge to models of housing and portfolio allocation that combine debt and savings into one net worth variable, as in Iacoviello and Pavan (2010) and Favilukis et al. (2011). Here, a household’s net financial position is not a sufficient statistic for their mortgage and asset holdings. Homeowners simultaneously hold substantial amounts of mortgage debt and assets even though the mortgage interest rate is higher than the return to liquid assets.

There are two reasons why homeowners simultaneously hold debt and assets. First, liquid assets are more efficient insurance against transitory risk than mortgage debt. Taking out a new mortgage requires paying an origination cost and a default premium to the mortgage company. The default premium is especially costly to households taking out a larger mortgage in response to a negative income shock. Mortgage companies view these households as riskier, and therefore the households receive fewer resources $q_{m}^{0}m'$ due to a lower $q_{m}^{0}$. Instead, households can take out a larger mortgage initially and use some of those resources to purchase liquid assets. If a household experiences a negative income shock in the future, it can simply consume some of its liquid assets. In this way, the long-term nature of the mortgage contract is important because the terms of the contract do not change if a borrower’s
financial situation deteriorates or his house depreciates.

The second reason why households hold both mortgage debt and liquid assets is because of the insurance provided by the no-recourse foreclosure process. Whenever a household defaults on its mortgage, its house is repossessed, but its liquid assets are left intact. Because households are excluded from the mortgage market after a foreclosure, their only form of insurance becomes the liquid asset, thus providing an additional incentive for households to accumulate liquid assets even while holding mortgage debt.

4.1.3 Foreclosure Decisions

Homeowners default when they are unable or unwilling to continue making mortgage payments and are unable to sell their house in a timely manner. It is never beneficial for a homeowner to default when they can sell their house and make a profit after paying off their mortgage. As a result, homeowners who go into foreclosure tend to have high leverage and low liquid assets. When selling, these homeowners are forced to post a high selling price to pay off their mortgage, leading to a low probability of trade.

Table 3: Foreclosure Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum loan to value</td>
<td>82.5%</td>
</tr>
<tr>
<td>Mean loan to value</td>
<td>97.97%</td>
</tr>
<tr>
<td>Mean annual earnings*</td>
<td>0.368</td>
</tr>
<tr>
<td>Mean liquid assets*</td>
<td>0.163</td>
</tr>
</tbody>
</table>

*Reported as a fraction of economy-wide mean annual earnings.

Table 3 reports the mean leverage, mean annual earnings, and mean liquid assets of homeowners who go into foreclosure. Because the foreclosure process is no recourse, even homeowners with strictly positive liquid assets go into foreclosure. In addition to having high leverage and low assets, the average defaulting homeowner has low earnings. Homeowners with higher earnings have a stronger incentive not to default. They tend to have higher persistent labor productivity; thus, even if they have low net worth, their expected future income is high. As a result, it is more valuable to them to maintain access to borrowing, making them less likely to default.

4.2 Cyclical Results

This section presents the cyclical results of the model. First, I present standard business cycle statistics for output, consumption, and investment for the model and the data. Then,
I discuss the cyclical behavior of housing, debt, and foreclosures. The model output and the data are detrended by taking logs and running an HP filter with a large smoothing parameter of $10^8$ to include both high and low frequency fluctuations.

### 4.2.1 Output, Consumption, and Investment Dynamics

Table 8 summarizes the standard business cycle statistics of the model and the data for output, consumption, and investment. Both consumption and investment include housing and non-housing components. Output is defined as the sum of consumption and investment. The model matches the stylized fact that housing investment is procyclical and volatile relative to output. The relative volatility of housing investment in the data from 1975 – 2010 is 5.15, compared to 4.64 in the model. Furthermore, the correlation between housing investment and output is 0.92 in the data and 0.90 in the model.

In addition, the model matches the volatility and procyclicality of non-housing investment. Non-housing investment has a relative volatility of 2.69 in the data compared to 3.37 in the model, and the correlations with output in the data and in the model are 0.78 and 0.86, respectively. If attention is restricted to the period 1975 – 1998—before the recent housing boom and bust—then the model almost exactly replicates the relative volatilities of both types of investment. The relative volatility of non-housing investment in this period is 3.35 versus 3.37 in the model, and the relative volatility of housing investment is 4.48 versus 4.64 in the model.

The model also does a good job matching the aggregate volatility of non-housing consumption, which is 0.64 in the model and 0.72 in the data from 1975 – 2010. Moreover, the model replicates the fact that housing consumption is less volatile than non-housing consumption, though the model under predicts that volatility at 0.23, versus 0.60 in the data.

The model performs favorably when compared to the literature, which tends to overstate the volatility of non-housing investment while understating the volatilities of consumption and housing investment, as in Iacoviello and Pavan (2010). One exception is Favilukis et al. (2011), who come closer to matching the relative volatilities of both types of consumption and investment. However, they generate substantially less annual autocorrelation in consumption and investment than in the data. Here, the model nearly matches the autocorrelation of both types of investment, and slightly overstates the autocorrelation of consumption.
4.2.2 Housing and Foreclosure Dynamics

Table 10 reports the relative volatilities of selected housing market variables and their lagged correlations with output. The main stylized facts are that house prices and sales are volatile and procyclical, while inventories, months supply, and foreclosures are volatile and countercyclical. The model is consistent with these facts.

The relative volatility of the Freddie Mac House Price Index is 2.07 from 1975 – 2010, and the relative volatility of the shadow housing price in the model is 1.99. In addition, the contemporaneous correlation of house prices with output is 0.50 in the data and 0.92 in the model. In other words, house prices in the data and in the model are volatile and procyclical, though the model exaggerates the degree of procyclicality. Matching the relative volatility of house prices is a particular success of the model because the literature has had difficulty generating volatile house prices. For example, house prices in Davis and Heathcote (2005) are less than one third as volatile as in the data, and house prices in Head et al. (2011) are only one half as volatile as in the data. Like the current paper, Favilukis et al. (2011) also successfully generate volatile house prices.

The model also matches the fact that house sales are procyclical and more volatile than output. The relative volatility of total sales is 4.13 in the data and 1.60 in the model. The correlation between total sales and output is 0.76 in the data and 0.54 in the model. For existing sales in the data, the relative volatility and correlation with output are 3.93 and 0.73, respectively. In the model, the corresponding figures are 1.52 and 0.28. For new sales in the data, the relative volatility and correlation with output are 6.90 and 0.78, respectively. In the model, the corresponding figures are 2.66 and 0.89. Therefore, new and existing sales are procyclical and more volatile than output, though not by as much as in the data.

The model also generates volatile and countercyclical inventories and months supply. In the data, the relative volatility of inventories and its correlation with output are 5.14 and -0.11, respectively. Inventories are not as volatile in the model, with a volatility of 1.89, but they are even more countercyclical, with a correlation with output of -0.82. For months supply—a proxy for average time on the market—the relative volatility in the data is 6.11, and its correlation with output is -0.44. Months supply in the model has a lower volatility of 2.23 but is more countercyclical with a correlation with output of -0.89. To summarize, inventories and months supply are more volatile than output—though not by as much as in the data—and are even more countercyclical than in the data.

Lastly, the foreclosure rate in the model is volatile and countercyclical, as in the data. The relative volatility of the foreclosure rate in the data is 4.96, and its correlation with output is -0.64. Foreclosures are even more volatile and countercyclical in the model, with a
Buying and Selling Behavior  To understand why house prices co-move positively with sales and co-move negatively with months supply, consider the optimal behavior of buyers and sellers when house prices are high and when they are low. When house prices are above trend, they are expected to fall in the future. As a result, it is a good time to sell and a bad time to buy. Therefore, for given trade probability schedules \( \{p_b(\theta_b(x_b, h))\} \) and \( \{p_s(\theta_s(x_s, h))\} \), buyers choose a lower \( x_b \) to wait for prices to come down while sellers choose a lower \( x_s \) to sell quickly before prices fall. Call this the substitution effect.

However, changes in the shadow price of housing \( p_h \) cause shifts in the trade probability schedules. From the optimality conditions (33) – (34) for real estate companies, an increase in \( p_h \) decreases the return to hiring real estate agents in the buying market and increases the return to hiring real estate agents in the selling market. Therefore, fewer real estate agents enter the buying market when \( p_h \) is high, while more real estate agents enter the selling market. As a result, buying probabilities decrease, and selling probabilities increase. Buyers respond to the decrease in \( \{p_b(\theta_b(x_b, h))\} \) by increasing their purchase price \( x_b \) but not by enough to fully offset the decrease in their buying probability. In other words, buyers absorb part of the deterioration in their terms of trade by paying more, and they absorb the rest by trading at a lower probability. Sellers, on the other hand, experience an improvement in their terms of trade. They respond by increasing their selling price \( x_s \) but not by so much as to offset the increase in their selling probability. Call this the price schedule effect.

Taking into account both effects, buyers pay a higher purchase price and trade at a lower probability when house prices are high, while homeowners sell at a higher price and with a higher probability, as shown in figures 3 and 4. Higher selling probabilities mean higher sales and lower time on the market. Therefore, the co-movement between house prices and sales is positive, and the co-movement between house prices and months supply is negative.

Foreclosure Behavior  Foreclosures are countercyclical in the model for two reasons. First, as house prices increase, it becomes easier for homeowners to sell their house. This reduces the risk that a financially distressed homeowner goes into foreclosure because he fails to sell his house. Second, higher house prices increase home equity and decrease mortgage leverage. Increased home equity loosens borrowing constraints, making it easier for homeowners to refinance and making them less likely to default.
4.2.3 Portfolio Dynamics

Table 11 reports the relative volatilities of net worth, liquid assets, housing wealth, and mortgage debt, as well as their lagged correlations with output. The model matches the stylized fact that mortgage debt is procyclical. In the data, the contemporaneous correlation between mortgage debt and output is 0.22, and in the model it is 0.41. The relative volatility of mortgage debt is 1.64 in the data and 0.96 in the model.

The model also matches other cyclical properties of household portfolios. The relative volatility of net worth is 1.84 in the data and 1.72 in the model. The correlation of net worth with output is 0.77 in the data and 0.82 in the model. For liquid assets, the relative volatility and correlation with output in the data are 1.76 and 0.65, respectively, and the corresponding figures in the model are 0.98 and 0.74. Lastly, for housing wealth, the relative volatility and correlation with output in the data are 2.61 and 0.54, respectively. In the model, the corresponding figures are 2.09 and 0.90. To summarize, in addition to matching the procyclicality of mortgage debt, the model also replicates the fact that net worth, liquid assets, and housing wealth are procyclical. Furthermore, the model does a good job matching the relative volatilities of net worth and housing wealth.

4.3 The Effects of Housing Illiquidity

This section investigates the effects of housing market illiquidity by comparing the results of the baseline model to those of a version of the model without housing market frictions. All other aspects of this “frictionless” model are the same as those of the baseline model, and it is calibrated to match all applicable targets from the baseline calibration.\textsuperscript{14} The calibration is summarized in Table 7.

To investigate the effects of housing market illiquidity, I look primarily at the differences between the cyclical properties of the models. However, differences between some of the steady state features of the models help shed light on the mechanisms by which housing illiquidity affects the behavior of housing, debt, and foreclosures.

4.3.1 Steady State Effects of Housing Illiquidity

Housing and Foreclosures Table 4 compares the steady state behavior of housing sales in the baseline model and in the frictionless model. The first major difference is that there are no REO sales in the frictionless model because without housing market frictions, negative equity is a necessary condition for mortgage default. Homeowners with positive equity

\textsuperscript{14}There are some targets that the frictionless model cannot possibly hit, such as average months supply.
always prefer to sell their house and use the proceeds to pay off their mortgage. However, with stationary prices and the requirement that all new mortgages be smaller than the value of the house, negative equity never arises in the steady state equilibrium.

The second major difference is that turnover is noticeably higher in the frictionless model. As a result, there are more homeowner-to-homeowner transitions, which increase the share of existing house sales. Without housing frictions, homeowners change houses more frequently in response to changes in their financial situation. In fact, turnover would be even higher in the frictionless model if house sizes were continuous rather than discrete. In the baseline model, adding more house sizes has little impact on turnover because housing frictions are the limiting factor for mobility.

Table 4: Housing Sales Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Baseline</th>
<th>Frictionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing housing share of sales</td>
<td>79.17%</td>
<td>72.38%</td>
<td>86.82%</td>
</tr>
<tr>
<td>New housing share of sales</td>
<td>15.83%</td>
<td>23.27%</td>
<td>13.18%</td>
</tr>
<tr>
<td>REO share of sales</td>
<td>5%</td>
<td>4.35%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Turnover</td>
<td>8%</td>
<td>6.92%</td>
<td>12.54%</td>
</tr>
</tbody>
</table>

Portfolio Choice  Table 5 compares steady state average portfolio holdings in the baseline model and in the frictionless model. I calibrate the frictionless model to match mean net worth and mean housing wealth relative to income, just as in the baseline model. However, homeowners in the frictionless model have less assets and less mortgage debt than homeowners in the baseline model. Furthermore, there are no homeowners with more than 80% leverage in the frictionless model, compared to almost 20% of homeowners in the baseline model.

Table 5: Portfolio Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Baseline</th>
<th>Frictionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid assets</td>
<td>2.575</td>
<td>2.547</td>
<td>2.442</td>
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<tr>
<td>Homeowner net worth</td>
<td>4.291</td>
<td>4.297</td>
<td>4.297</td>
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<tr>
<td>Mortgage debt</td>
<td>1.08</td>
<td>1.078</td>
<td>0.956</td>
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<tr>
<td>Percent with mortgage debt</td>
<td>79.1%</td>
<td>97.15%</td>
<td>99.92%</td>
</tr>
<tr>
<td>Mean leverage</td>
<td>49.6%</td>
<td>46.7%</td>
<td>41.29%</td>
</tr>
<tr>
<td>Percent with leverage &gt; 80%</td>
<td>17.6%</td>
<td>19.75%</td>
<td>0.00%</td>
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<tr>
<td>Percent with leverage &gt; 90%</td>
<td>10.2%</td>
<td>15.8%</td>
<td>0.00%</td>
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</table>

There are two reasons for the differences in portfolio composition. First, because housing frictions increase homeowners’ exposure to risk, homeowners respond by taking out larger
mortgages and using those funds to increase their precautionary saving in liquid assets. Second, if households suffer an income drop in the model with housing illiquidity, they cannot simply sell their house immediately to purchase more consumption. Doing so would be costly, and there is no guarantee that they would successfully sell. These households are then forced to either go into foreclosure or refinance to a larger mortgage. This forced borrowing causes an increase in overall mortgage debt and in the number of homeowners with high leverage.

4.3.2 Cyclical Effects of Housing Illiquidity

Output, Consumption, and Investment Dynamics There are two main differences between the baseline model and the frictionless model regarding the dynamics of output, consumption, and investment. First, housing investment is less volatile and non-housing investment is more volatile in the frictionless model. The relative volatilities are 3.95 and 3.89, respectively. In the baseline model, the corresponding relative volatilities are 4.64 and 3.37, which are closer to those in the data.

The second difference is that housing services in the frictionless model are even less volatile and less correlated with output than they are in the baseline model. The relative volatility of housing services is 0.16 in the frictionless model and 0.23 in the baseline model, and the respective correlations with output are 0.15 and 0.25. Again, the baseline results are closer to those in the data. Figure 9 plots the responses of output, consumption, and investment in both models to a permanent positive shock to non-housing TFP. Figure 12 plots the responses to a permanent negative shock.

Housing and Foreclosure Dynamics The most obvious difference between housing dynamics in the two models is that there are no inventories in the frictionless model. Houses always sell instantly in the frictionless model, so there is no notion of fluctuating time on the market. In addition, even with house price fluctuations, foreclosures are almost nonexistent in the frictionless model. As in the steady state equilibrium, negative equity is a necessary precondition for foreclosure in the frictionless model. However, in the steady state, there are no homeowners with more than 80% leverage, which means that house price fluctuations would have to be very large to put any homeowners underwater on their mortgage—something that does not happen in the model simulations.

There are also differences in the behavior of house prices and sales in the baseline and frictionless models. House prices are more volatile in the baseline model than in the frictionless model, with relative volatilities of 1.99 and 1.76, respectively. However, existing house sales are more volatile in the frictionless model than in the baseline model, with relative
volatilities of 3.52 and 1.52, respectively. Existing house sales are moderately procyclical in
the baseline model, but their correlation with output in the frictionless model is only -0.01.
House sales are less volatile and more procyclical in the baseline model because trading
delays in the housing market intertemporally smooth the impact of aggregate shocks.

Figure 10 plots the responses of house prices, sales, inventories, months supply, and
foreclosures in both models to a permanent positive shock to non-housing TFP. Figure
13 plots the responses to a permanent negative shock. The baseline model, in particular,
generates house price booms and busts that are protracted and exhibit bubble-like behavior.
Specifically, house prices change slowly and overshoot in response to a shock before partially
mean-reverting. The model accomplishes this without any of the social learning dynamics
present in Burnside et al. (2011).

**Portfolio Dynamics**  The baseline model does a better job than the frictionless model at
matching the dynamics of mortgage debt. The relative volatility of mortgage debt is 1.64 in
the data, while it is 0.96 in the baseline model and 0.80 in the frictionless model. In addition,
the correlation of mortgage debt with output is 0.22 in the data, whereas it is 0.41 in the
baseline model and 0.86 in the frictionless model.

The frictionless model also does a better job at matching the relative volatilities of net
worth and housing wealth, which are 1.84 and 2.61 in the data, respectively. In the baseline
model, the corresponding figures are 1.72 and 2.09; in the frictionless model, they are 1.53
and 1.81. Figure 11 plots the responses of net worth, liquid assets, housing wealth, and
mortgage debt in both models to a permanent positive shock to non-housing TFP. Figure
14 plots the responses to a permanent negative shock.

4.3.3 The Interaction of Housing and Mortgage Illiquidity

The baseline model highlights an important interaction between housing illiquidity and mort-
gage illiquidity, namely, that they are mutually reinforcing. This positive feedback is similar
to the interaction of market and funding liquidity in Brunnermeier and Pedersen (2009).
Higher housing illiquidity, as manifested by lower selling probabilities \( p_s(\theta_s(x_s, h)) \), in-
creases the probability of default for financially distressed homeowners. Mortgage companies
anticipate the increased default risk and respond by requiring a larger default premium when
issuing new mortgages. The result is lower mortgage prices \( q_m^0(\cdot) \) and tighter endogenous
borrowing constraints. In other words, higher housing illiquidity causes higher mortgage
illiquidity.

Conversely, higher mortgage illiquidity causes higher housing illiquidity. Higher mortgage
illiquidity, as manifested by lower mortgage prices \( q_m^0(\cdot) \), forces some financially distressed
homeowners to try to sell their house rather than refinance to a larger mortgage. Because these homeowners tend to have high mortgage leverage and low assets, they must post a high selling price to pay off their mortgage, leading to a low selling probability and increased housing illiquidity. In addition, some of these homeowners go into foreclosure after failing to sell. Their house is then repossessed by the mortgage company and sold as an REO. However, mortgage companies take longer to sell houses than typical homeowners. Foreclosed houses clog the housing market and further increase housing illiquidity.

This interaction between housing illiquidity and mortgage illiquidity explains the added volatility of house prices in the baseline model compared to the frictionless model. In the baseline model, rising house prices increase housing liquidity, thereby increasing mortgage liquidity by lowering endogenous borrowing constraints. Looser credit further stimulates housing demand, leading to higher house prices.

5 Conclusions

This paper develops a tractable model of the macroeconomy with illiquid housing and mortgage markets to analyze the dynamics of housing, debt, and foreclosures. The model successfully generates procyclical and volatile house prices, sales, and investment; procyclical mortgage debt; and countercyclical and volatile time on the market and foreclosures. The model is also consistent with empirical evidence on selling and foreclosure behavior. In addition, the model highlights how housing illiquidity and mortgage illiquidity are mutually reinforcing.

The framework in the model lends itself to other applications. Future work on housing markets includes analyzing the impact of foreclosure modification programs and home buyer tax credits on housing and mortgage liquidity. Housing illiquidity also has implications for the optimal design of unemployment insurance because it makes unemployment riskier by turning housing into a consumption commitment that cannot be easily adjusted. Another avenue of research is to investigate the effects of monetary policy on housing and mortgage illiquidity and to determine optimal policy in such an environment. More broadly, this framework can be used to analyze the dynamics of prices and liquidity in other markets in which trader heterogeneity is important, such as in financial markets.
References


## 6 Tables

Table 6: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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Parameters determined independently

Parameters determined jointly
Table 7: Frictionless Model Calibration

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<th>Target</th>
<th>Data</th>
<th>Model</th>
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Parameters determined independently

Parameters determined jointly
## Table 8: Standard Business Cycle Statistics

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Output, consumption, and investment data come from NIPA table 1.5.5.
Table 9: Lagged Correlations—Output, Consumption, and Investment

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**Data: 1975 - 2010**

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<td>0.83</td>
<td>0.94</td>
<td>1.00</td>
<td>0.94</td>
<td>0.83</td>
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<td>0.79</td>
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<td>0.56</td>
<td>0.66</td>
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<td>0.81</td>
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<td>0.77</td>
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**Data: 1975 - 1998**

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<td>1.00</td>
<td>0.86</td>
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<td>0.73</td>
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**Baseline Model**

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<td>1.00</td>
<td>0.85</td>
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<td>0.72</td>
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<td>0.80</td>
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**Frictionless Model**
Table 10: Lagged Correlations—House Prices, Sales, Inventories, and Foreclosures

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<td>0.79</td>
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<tr>
<td>Output (Y)</td>
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<td>0.73</td>
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<td>0.86</td>
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<td>0.82</td>
<td>0.92</td>
<td>0.90</td>
<td>0.87</td>
<td>0.86</td>
<td>0.83</td>
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<td>0.41</td>
<td>0.44</td>
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<td>0.70</td>
<td>0.72</td>
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<td>0.86</td>
<td>0.85</td>
<td>0.83</td>
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<tr>
<td>Existing sales</td>
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<td>0.22</td>
<td>0.24</td>
<td>0.27</td>
<td>0.29</td>
<td>0.28</td>
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<td>Output (Y)</td>
<td>1.00</td>
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<td>0.72</td>
<td>0.76</td>
<td>0.85</td>
<td>1.00</td>
<td>0.85</td>
<td>0.76</td>
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<td>1.76</td>
<td>0.66</td>
<td>0.71</td>
<td>0.75</td>
<td>0.82</td>
<td>0.92</td>
<td>0.89</td>
<td>0.86</td>
<td>0.85</td>
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<td>0.90</td>
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</tr>
<tr>
<td>Existing sales</td>
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<td>-0.07</td>
<td>-0.05</td>
<td>-0.03</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.07</td>
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<tr>
<td>Inventories</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Months Supply</td>
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<td></td>
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<td></td>
<td></td>
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<td>Foreclosure rate</td>
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<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Existing housing sales, inventories and months supply come from data spanning 1982 - 2010 from the National Association of Realtors. New housing sales come from the Census. Foreclosure data is from the Mortgage Bankers' Association and covers 1979 - 2010. For house prices I use the Freddie Mac House Price Index.
Table 11: Lagged Correlations—Household Portfolios

<table>
<thead>
<tr>
<th>x</th>
<th>$\sigma_x/\sigma_Y$</th>
<th>$x(-4)$</th>
<th>$x(-3)$</th>
<th>$x(-2)$</th>
<th>$x(-1)$</th>
<th>$x$</th>
<th>$x(+1)$</th>
<th>$x(+2)$</th>
<th>$x(+3)$</th>
<th>$x(+4)$</th>
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<td>1.00</td>
<td>0.97</td>
<td>0.90</td>
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<tr>
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<td>0.77</td>
<td>0.77</td>
<td>0.75</td>
<td>0.71</td>
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<tr>
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<td>0.49</td>
<td>0.58</td>
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<td>0.67</td>
<td>0.65</td>
<td>0.62</td>
<td>0.59</td>
<td>0.55</td>
<td>0.51</td>
</tr>
<tr>
<td>Housing Wealth</td>
<td>2.61</td>
<td>0.16</td>
<td>0.28</td>
<td>0.38</td>
<td>0.47</td>
<td>0.54</td>
<td>0.59</td>
<td>0.63</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
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<td>0.13</td>
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**Data: 1975 - 2010**

<table>
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<th>$x(-1)$</th>
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<th>$x(+2)$</th>
<th>$x(+3)$</th>
<th>$x(+4)$</th>
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<td>0.94</td>
<td>1.00</td>
<td>0.94</td>
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<td>0.42</td>
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<td>0.42</td>
<td>0.43</td>
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<td>Housing Wealth</td>
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<td>0.17</td>
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<td>0.27</td>
<td>0.37</td>
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**Data: 1975 - 1998**

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<td>0.86</td>
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<td>0.83</td>
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**Baseline Model**

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<td>1.00</td>
<td>0.85</td>
<td>0.76</td>
<td>0.72</td>
<td>0.66</td>
</tr>
<tr>
<td>Net worth</td>
<td>1.53</td>
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<td>0.64</td>
<td>0.68</td>
<td>0.74</td>
<td>0.82</td>
<td>0.82</td>
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<tr>
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<td>0.56</td>
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<td>0.71</td>
<td>0.75</td>
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<td>0.89</td>
<td>0.88</td>
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</table>

**Frictionless Model**

Net worth, liquid assets, housing wealth, and mortgage debt data come from table B.100 of the Federal Reserve Flow of Funds Accounts. Liquid assets are defined as total financial assets plus consumer durables minus non-mortgage credit market liabilities. Housing wealth is defined as owner-occupied real estate at market values, and I use the home mortgages series for the definition of mortgage debt. I then define net worth as the sum of liquid assets and housing wealth minus mortgage debt.
7 Figures

7.1 Policy Functions

Figure 2: Optimal mortgage and asset choices. The left panel is for homeowners taking out a mortgage immediately after purchasing their house, and the right panel is for homeowners with an existing mortgage.

Figure 3: Buyers’ optimal purchasing price and probability of buying as a function of cash at hand. When prices are high, buyers increase their purchase price but trade with lower probabilities.
Figure 4: Sellers’ optimal selling price relative to the shadow price of housing as well as their probability of selling. The top plots are a function of mortgage leverage, and the bottom plots are a function of cash at hand. Sellers increase their asking price in absolute terms when \( p_h \) is high, but they lower their price in relative terms, thereby selling with a higher probability.
Figure 5: Homeowners' foreclosure sets as a function of cash at hand and mortgage leverage. The foreclosure sets lie to the bottom right of each curve. The top grouping of foreclosure contours corresponds to homeowners with low persistent labor productivity, and the bottom grouping corresponds to high productivity homeowners.
7.2 U.S. Data 1975 - 2010

Figure 6: Consumption and investment in the U.S. from 1975 - 2010. Each series is the log deviation from an HP-filtered trend. For reference, each series is plotted against output, which is the thin, blue curve.
Figure 7: House prices, sales, inventories, months supply and foreclosures in the U.S. from 1975 - 2010. Each series is the log deviation from an HP-filtered trend. For reference, each series is plotted against output, which is the thin, blue curve.
Figure 8: Net worth, liquid assets, housing wealth, and mortgage debt in the U.S. from 1975 - 2010. Each series is the log deviation from an HP-filtered trend. For reference, each series is plotted against output, which is the thin, blue curve.
7.3 Impulse Response Functions—Housing Boom

Figure 9: The response of consumption, investment, and output to a permanent, positive productivity shock in the baseline and frictionless models.
Figure 10: The response of house prices, sales, inventories, months supply, and foreclosures to a permanent, positive productivity shock in the baseline and frictionless models.
Figure 11: The response of net worth, liquid assets, housing wealth, and mortgage debt to a permanent, positive productivity shock in the baseline and frictionless models.
7.4 Impulse Response Functions—Housing Bust

Figure 12: The response of consumption, investment, and output to a permanent, negative productivity shock in the baseline and frictionless models.
Figure 13: The response of house prices, sales, inventories, months supply, and foreclosures to a permanent, negative productivity shock in the baseline and frictionless models.
Figure 14: The response of net worth, liquid assets, housing wealth, and mortgage debt to a permanent, negative productivity shock in the baseline and frictionless models.
A Calibrating the Labor Income Process

As explained in the calibration section, it is not possible to estimate quarterly income processes from PSID data because the PSID is only conducted annually. Instead, I start by specifying a labor process like that in Storesletten et al. (2004), except without life cycle effects or a permanent shock at birth. I adopt their values for the annual autocorrelation of the persistent shock and for the variances of the persistent and transitory shocks, and I transform them to quarterly values.

Persistent Shocks I assume that in each period households play a lottery in which, with probability 3/4, they receive the same persistent shock as they did in the previous period, and with probability 1/4, they draw a new shock from a transition matrix calibrated to the persistent process in Storesletten et al. (2004) (in which case they still might receive the same persistent labor shock). This is equivalent to choosing transition probabilities that match the expected amount of time that households expect to keep their current shock. Storesletten et al. (2004) report an annual autocorrelation coefficient of 0.952 and a frequency-weighted average standard deviation over expansions and recessions of 0.17. I use the Rouwenhorst method to calibrate this process, which gives the following transition matrix:

\[ \tilde{\pi}(\cdot, \cdot) = \begin{pmatrix} 0.976 & 0.024 \\ 0.024 & 0.976 \end{pmatrix} \]

As a result, the final transition matrix is

\[ \pi(\cdot, \cdot) = 0.75 \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 0.25 \times \begin{pmatrix} 0.976 & 0.024 \\ 0.024 & 0.976 \end{pmatrix} = \begin{pmatrix} 0.994 & 0.006 \\ 0.006 & 0.994 \end{pmatrix} \]

Transitory Shocks Storesletten et al. (2004) report a standard deviation of the transitory shock of 0.255. To replicate this, I assume that the annual transitory shock is actually the sum of four, independent quarterly transitory shocks. I make use of the same identifying assumption that Storesletten et al. (2004) use, namely, that all households receive the same initial persistent shock. Any variance in initial labor income is then due to different draws of the transitory shock. Recall that the labor productivity process is given by

\[ \ln(e \cdot s) = \ln(s) + \ln(e) \]

Therefore, total labor productivity (which, when multiplied by the wage \( w \), is total wage
income) over a year in which \( s \) stays constant is

\[
(e \cdot s)_{\text{year 1}} = \exp(s_0)[\exp(e_1) + \exp(e_2) + \exp(e_3) + \exp(e_4)]
\]

For different variances of the transitory shock, I simulate total annual labor productivity for many individuals, take logs, and compute the variance of the annual transitory shock. It turns out that quarterly transitory shocks with a standard deviation of 0.49 give the desired standard deviation of annual transitory shocks of 0.255.

### B Computing the Stochastic Model

I solve for the equilibrium using a hybrid of methods based off of Krusell and Smith (1998) and those used in the literature on equilibrium default. The algorithm is outlined below:

1. Solve for equilibrium submarket tightnesses \( \{\theta_b(x_b, h; p_h)\} \) and \( \{\theta_s(x_s, h; p_h)\} \) for each value of the housing shadow price, \( p_h \), using (33) – (34).

2. **Loop 1** – Make an initial guess of coefficients \( \bar{\alpha}^{p,0} = \{(a_0^p(z_c, z_c'), a_1^p(z_c, z_c'), a_2^p(z_c, z_c')\}^0 \) for the evolution of the housing shadow price and coefficients \( \bar{\alpha}^{K,0} = \{(a_0^K(z_c), a_1^K(z_c), a_2^K(z_c)\}^0 \) for the evolution of the capital stock.

   (a) Solve for tomorrow’s equilibrium wage and rental rate, \( w(Z') \) and \( r(Z') \), using the aggregate laws of motion implied by the coefficients above, the equilibrium conditions for the firm’s problem, (12) – (13) and (18) – (19), and the market clearing conditions for land/permits and labor. In practice, this procedure involves solving a simple fixed point problem in the amount of labor employed in the consumption good sector and then substituting to calculate the remaining quantities and factor prices.

   (b) **Loop 2** – Make an initial guess of mortgage prices \( q_m^{0,n}(m', b', h, s, Z) \) for \( n = 0 \).

   i. Calculate the lower bound of the budget set for homeowners with good credit entering subperiod 3, \( y(m, h, s, Z) \), by solving

\[
y(m, h, s, Z) = \min_{m', a'}[a' + m - q(Z)m'], \text{ where }
q(Z) = \begin{cases} 
q_m^0(m', a', h, s, Z) & \text{if } m' > m \\
q_m & \text{if } m' \leq m 
\end{cases}
\]
ii. **Loop 3** – Make an initial guess for the mortgage company’s REO value function, $J^0_{REO}(h, Z)$.
   A. Substitute $J^0_{REO}$ into the right hand side of (31) and solve for $R_{REO}(h, Z)$ and $J_{REO}(h, Z)$.
   B. If $\sup(|J_{REO} - J^0_{REO}|) < \epsilon_J$, then exit the loop. Otherwise, set $J^0_{REO} = J_{REO}$ and return to A.

iii. **Loop 4** – Make an initial guess for $V^0_{rent}(y, s, f, Z)$ and $V^0_{own}(y, m, h, s, f, Z)$.
   A. Substitute $V^0_{rent}$ and $V^0_{own}$ into the right hand side of (5) – (6) and solve for $R_b$.
   B. Substitute $V^1_{rent}$, $V^1_{own}$, and $R_b$ into the right hand side of (7) – (8) and solve for $W$.
   C. Substitute $W$, $V^1_{rent}$, and $R_b$ into the right hand side of (9) – (10) and solve for $V_{rent}$ and $V_{own}$.
   D. Substitute $W$, $V_{rent}$, $R_s$, and $R_b$ into the right hand side of (1) – (4) and solve for $V_{rent}$ and $V_{own}$.
   E. If $\sup(|V_{rent} - V^0_{rent}|) + \sup(|V_{own} - V^0_{own}|) < \epsilon_V$, then exit the loop. Otherwise, set $V^0_{rent} = V_{rent}$ and $V^0_{own} = V_{own}$ and return to A.

iv. Substitute $q^{0,n}_m$, $J_{REO}$, and the household’s policy functions for asset choice, mortgage choice and selling and default decisions into the right hand side of (30) and solve for $q^0_m$.

v. If $\sup(q^0_m - q^{0,n}_m) < \epsilon_q$, then exit the loop. Otherwise, set $q^{0,n+1}_m = (1 - \lambda_q)q^{0,n}_m + \lambda_q q^0_m$ and return to (i).

(c) **Loop 5** Initialize the distribution of households in subperiod 1, $\Phi^0_1$, and the distribution of REO houses, $H^0_{REO}$.

i. Draw an initial shock $z_{c,0}$ from the stationary distribution $\Pi_z$ followed by a sequence of aggregate shocks $\{z_{c,t}\}^T_1$ using the Markov transition matrix $\pi_z(z'_{c}|z_c)$.

ii. Simulate the economy for $T$ periods\(^{15}\) using the decision rules of the households and mortgage companies. In each period, compute aggregate asset holdings (which equals tomorrow’s capital stock, $K_{t+1}$) and solve for the equilibrium housing shadow price $p_{h,t}$ that satisfies (36).

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\(^{15}\)I simulate the economy for 1,600 periods and ignore the first 600 periods for the regressions.
(d) Run the following regressions:

\[ p_{h,t+1} = a_p^0(z_{c,t}, z_{c,t+1}) + a_p^1(z_{c,t}, z_{c,t+1})p_{h,t} + a_p^2(z_{c,t}, z_{c,t+1})K_t \]
\[ K_{t+1} = a_K^0(z_{c,t}) + a_K^1(z_{c,t})p_{h,t} + a_K^2(z_{c,t})K_t \]

which gives new coefficients \( \vec{a}_p \) and \( \vec{a}_K \).

(e) If \( \sup(|\vec{a}_p - \vec{a}_p^n|) + \sup(|\vec{a}_K - \vec{a}_K^n|) < \epsilon_a \), then the algorithm is complete. Otherwise, set \( \vec{a}_p^{n+1} = (1 - \lambda_a)\vec{a}_p^n + \lambda_a \vec{a}_p \) and \( \vec{a}_K^{n+1} = (1 - \lambda_a)\vec{a}_K^n + \lambda_a \vec{a}_K \) and go to (a).

C Accuracy of Approximate Equilibrium

I try several modifications of the regressions explained above. In particular, I impose various identifying restrictions on the coefficients to see if doing so affects the \( R^2 \) of the regressions or the stability of the computational algorithm. Eliminating the cross terms of the regressions, i.e. imposing \( a_p^2 = 0 \) and \( a_K^1 = 0 \), gives the most accurate and stable solution. The converged results are reported below, both for the baseline model and the frictionless model.
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