Consumer Behavior, Monopolistic Competition, and International Trade: CES Redux?

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Abstract

We argue that monopolistic competition models featuring pro-competitive effects, such as the model in Krugman (1979) or models assuming linear demand systems, build on dubious assumptions about consumer behavior and are therefore potentially misleading. In any case, departing from CES preferences in an otherwise standard Dixit-Stiglitz monopolistic competition setting seems to lead to controversial results. We also consider alternative settings featuring, i.e., consumers’ preference for an ideal variety, strategic interactions and heterogeneous firms, and find that a pro-competitive effect may plausibly arise only in the presence of a small number of firms, an assumption clearly at odds with monopolistic competition. Our results suggest that existing monopolistic competition models are not suitable to study the effects of competition on markups, thus making a point in support of CES preferences, which overlook the latter effects. Moreover, we show that trade-induced selection effects à la Melitz (2003) are generally robust to the assumptions about preferences. This suggests that monopolistic competition is better suited to analyze frameworks in which larger markets select more aggressively on productivity rather than forcing firms to move down their average cost curves.

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Although Dixit-Stiglitz monopolistic competition *cum* CES is still the workhorse of trade and macro economists, it is currently under fire. A recent and influential paper by Arkolakis, Costinot and Rodriguez-Clare (2011) argues that under CES preferences (and provided that some other conditions are satisfied) the number of sources of gains from trade does not affect overall estimates of these gains, conditional on observed trade flows. Another recent paper by Zhelobodko, Kokovin, Parenti and Thisse (2011) argues instead against the plausibility of CES preferences, because these represent a knife-edge between cases yielding opposite results. Finally, a standard critique recently revived by Neary (2004, 2009) among others, is that CES preferences are implausible because of the implied invariance of markups to the trade regime. In this paper we argue instead that, although CES preferences are clearly special, their special features may turn out to be a strength rather than a weakness when compared to the available modeling alternatives.

In a celebrated paper, Krugman (1979) developed a simple model providing an appealing explanation for trade in horizontally differentiated varieties between similar countries, a widespread phenomenon which could not be easily explained by the traditional paradigm. Krugman showed that, with additive preferences and monopolistic competition, a trade-induced expansion in market size leads to higher welfare (as consumers can enjoy a greater variety of goods at a lower price) and lower markups (the pro-competitive effect of trade liberalization). Crucially, Krugman (1979) assumed that \( \varepsilon'(c) < 0 \), where \( c \) is individual consumption of a variety and \( \varepsilon \) is the price elasticity of demand perceived by an individual firm. Under the Dixit and Stiglitz (1977, henceforth DS) assumption that firms ignore price interactions with other firms, \( \varepsilon \) can be shown to equal the elasticity of substitution between any two varieties, \( \sigma(c) \), when they are consumed in the same amount. The inequality \( \varepsilon'(c) < 0 \) can therefore be stated as an assumption that preferences feature an elasticity of substitution that is decreasing in the level of individual consumption (henceforth, DES preferences), i.e., \( \sigma'(c) < 0 \). Krugman claimed (p. 476) that: "This assumption [...] seems plausible. In any case, it seems necessary if this model is to yield reasonable results, and I make the assumption without apology." In this paper we argue, instead, that the assumption \( \sigma'(c) < 0 \) is implausible, and that its implications on the effects of competition on markups are no more reasonable than those of the opposite and possibly more plausible assumption that \( \sigma'(c) > 0 \).

To motivate our analysis, we propose the following exercise of introspection. Consider a situation in which you are endowed with two red pencils and two blue pencils, and compare it with a situation in which you are endowed instead with ten red and ten blue pencils. The key question is: do you perceive a red and a blue pencil as more substitutable
in the former or in the latter situation? If you think, as we do, that varieties become less substitutable when consumption of each shrinks, then the assumption $\sigma'(c) < 0$ is violated. In this case, $\sigma'(c) > 0$ and preferences exhibit an increasing elasticity of substitution in the level of individual consumption (henceforth, IES preferences).\footnote{An example of IES preferences is given by $U = \sum_i u(c_i)$, where $u(c_i) = \gamma \left( \frac{c_i^\rho}{\rho} \right) + c_i$ is a weighted average of a CES and a linear sub-utility function. This formulation is amenable to a simple economic interpretation, according to which consumers perceive differentiated varieties of some product as being midway between heterogenous and homogeneous goods, possibly capturing a specific feature of differentiated goods. A possible rationalization of IES preferences is that, by their very nature, differentiated varieties of some product can be used to perform either generic or more specific tasks. For instance, a blue pencil can be used either to write down a laundry list (for which a red pencil would be equally appropriate) or, jointly with a red pencil, to mark different types of comments on an exam paper. Hence, a fall in the symmetric endowment of varieties, by reducing the opportunity to use varieties to perform specific tasks, may also reduce their substitutability.}

In Section 2, we explore the implications of IES versus DES preferences in a framework à la Krugman (1979). We find that, although IES preferences are plausible, they imply that a trade-induced expansion in market size leads to smaller firms and higher markups, two implications strongly at odds with the conventional wisdom.\footnote{As far as we know, this result was first shown by Bertoletti, Fumagalli and Poletti (2008), and then independently found by Zhelobodko et al. (2011).} DES preferences, however, in addition to building on dubious assumptions about consumer behavior, also yield contrasting implications on the effects of competition on markups. In particular, although a rise in market size due to trade opening is pro-competitive, a rise in market size due to productivity growth is instead anti-competitive. Taken together, these results suggest that departing from CES preferences in an otherwise standard DS setting may turn out to be a move in the wrong direction.

A partial counter-argument in defense of Krugman’s (1979) model is that, although it may build on implausible assumptions, it nonetheless captures, in reduced form, a trade-induced pro-competitive effect arising from other mechanisms. Specifically, in Krugman’s model the number of firms $n$ has no direct impact on the demand elasticity $\varepsilon$, and hence a rise in $n$ can affect $\varepsilon$ only indirectly, through the level of individual consumption. Assuming $\sigma'(c) < 0$ is therefore the only way to obtain a trade-induced pro-competitive effect in this setting. One may argue, however, that a pro-competitive effect may naturally arise when relaxing one of the following assumptions, which characterize Krugman’s model as well as, more generally, DS monopolistic competition: 1) preference additivity; 2) a space of characteristics/varieties equal to the number of firms; 3) the absence of strategic interaction among firms. These assumptions indeed prevent the number of firms in the market to exert a positive direct impact on the demand elasticity.

In Section 3, building on the received literature, we therefore consider three different setups in which $n$ may directly affect $\varepsilon$. We start by relaxing the assumption that prefer-
ences are additive, as additivity implies that the elasticity of substitution between any two varieties (which equals $\varepsilon$ in a DS setting) is independent of $n$ (the Appendix provides a formal discussion of this point). Even with non-additive preferences, however, there is no compelling reason for the elasticity of substitution to be directly increasing in $n$. To make the point, we consider the quasi-linear quadratic preferences used in Melitz and Ottaviano (2008), a prominent example of non-additive preferences yielding pro-competitive effects and thus perceived as an appealing alternative to CES preferences. We show that, perhaps surprisingly, quasi-linear quadratic preferences imply that, for given level of individual consumption, the elasticity of substitution is decreasing in $n$. This means that in Melitz and Ottaviano (2008), just as in Krugman (1979), the trade-induced pro-competitive effect is entirely driven by the indirect negative impact of $n$ on individual consumption, namely, by the assumption that $\sigma'(c) < 0$.

The second reason why the demand elasticity may positively depend on the number of firms is that a trade-induced increase in $n$ tends to "crowd" the variety space, thereby making varieties closer substitutes. This effect cannot be captured by Krugman model's, as it implicitly assumes that the number of characteristics/varieties is the same as the number of firms. We therefore revert to Lancaster’s (1979) ideal variety approach to monopolistic competition, where the space of characteristics is fixed and finite and a trade-induced increase in the number of firms makes available varieties closer to one another in the variety space (represented by a unit circle). We show that, even in this framework, the demand elasticity is not, in general, increasing in $n$: indeed, in order for Lancaster’s framework to deliver a pro-competitive (or an anti-competitive) effect, additional and rather ad hoc assumptions unrelated to the basic framework are required. Otherwise, the ideal variety approach yields no pro- or anti-competitive effects, just like the "love for variety" approach with CES preferences.

Finally, a trade-induced pro-competitive effect may naturally arise by simply relaxing the DS assumption that firms are small enough to ignore their pricing interactions. The reason is that in this case the demand elasticity no longer coincides with the elasticity of substitution, and $n$ has a direct positive impact on $\varepsilon$ for given elasticity of substitution. If preferences are IES, however, the results are in general ambiguous. This is because a trade-induced expansion in market size has now a pro-competitive effect due to the positive direct impact of $n$ on $\varepsilon$, and an anti-competitive effect due to IES preferences. To make the point, we consider a Cournot variant of our basic setup in which we show that the pro-competitive effect vanishes for $n$ large and can only prevail when the number of firms is small, an assumption clearly at odds with monopolistic competition. Interestingly, these results also suggest that the CES may possibly proxy for a richer and more plausible
model in which pro- an anti-competitive effects roughly offset each other.

In Section 4, we study the implications of IES preferences in a framework à la Melitz (2003) allowing for firm heterogeneity in productivity and fixed costs of exporting. We find that the anti-competitive effect of IES preferences holds also in the presence of firm heterogeneity. Moreover, under costless trade integration, the flip side of the anti-competitive effect is an anti-selection effect whereby less productive firms can survive in a larger market, due to higher markups.3 A richer set of results arises instead in the presence of fixed costs of exporting, as in this case the anti-selection effect of IES preferences interacts with the standard selection effect à la Melitz. Importantly, we show that independent of the assumptions about preferences, the selection effect always prevails when fixed costs of exporting induce a partitioning of firms into exporters and non-exporters, i.e., arguably, in the empirically relevant case. These results suggest that Melitz-type selection effects are robust to the assumptions about preferences, and hence that monopolistic competition cum CES is well suited to study frameworks in which, in the presence of trade frictions, larger markets select more aggressively on productivity rather than forcing firms to move down their average cost curves.

Section 5 briefly concludes. Our paper is related to the vast theoretical literature on monopolistic competition and international trade, initiated by Dixit and Stiglitz (1977), Krugman (1979, 1980), Lancaster (1979), Helpman (1981), and whose early contributions are systematized in Helpman and Krugman (1985). More recent contributions include Bertoletti (2006) and Behrens and Murata (2007), which discuss specific functional forms consistent with Krugman’s (1979) type of preferences.4 As mentioned earlier, our paper is also related to Zhelobodko et al. (2011), which also notes that departing from CES preferences in a standard DS setting leads to opposite results. It does not discuss, however, the plausibility of these alternative cases, a key contribution of our paper, and which leads us to reach opposite conclusions. Finally, our paper is related to the recent heterogeneous-firm extensions of the monopolistic competition trade model, and in particular to Melitz (2003), which assumes CES preferences, and Melitz and Ottaviano (2008), which builds instead on quasi-linear quadratic preferences and assumes away the fixed costs of exporting. It is also related to Dhingra and Morrow (2012) and Mrazova and Neary (2011), which also consider a monopolistic competition setting with heterogeneous firms and a variable demand elasticity. Our main contribution to this growing literature is to show

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3 See also Dhingra and Morrow (2012) and Zhelobodko et al. (2011) on this point.
4 See also Neary (2004) for a critical survey of monopolistic competition in international trade theory. He also shares Krugman’s view that the assumption \( \varepsilon'(c) < 0 \) is plausible and argues, instead, that the main reason why most scholars opted for a CES utility function is that preferences embedding the assumption \( \varepsilon'(c) < 0 \) proved hardly tractable.
the robustness of Melitz-type selection effects in the presence of fixed costs of exporting.

2 Monopolistic Competition with IES Preferences

Consider an economy populated by $L$ workers, whose wage is $w = 1$. Consumers share the same additive and symmetric preferences, represented by the following utility function:

$$ U = \sum_{i=1}^{N} u(c_i), \quad (1) $$

where $c_i$ is consumption of variety $i$, defined over a large number $N$ of potential varieties. Only varieties indexed by $i = 1, \ldots, n$, with $n < N$, are actually produced. The subutility function $u(\cdot)$ is strictly increasing and concave, and is at least thrice continuously differentiable. In particular, we assume that $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u(0) = 0$.

Firm $i$ produces a differentiated variety with the total cost function $TC_i = \alpha + \beta q_i$, where $q_i = c_i L$ is its output, and $\alpha$ and $\beta$ are the fixed and marginal cost, both in terms of labor. Firms are symmetric on the cost and demand side, and therefore solve the same problem. In the following, we therefore drop the variety index $i$.

Utility maximization subject to a budget constraint implies

$$ u'(c) = \lambda p, \quad (2) $$

and the price elasticity of demand for an individual firm is

$$ \varepsilon(c) = -\frac{p'(c)}{p(c)c} = -\frac{u'(c)}{u''(c)c}. \quad (3) $$

Note that $\varepsilon(c)$ equals the reciprocal of the elasticity of marginal utility with respect to individual consumption and that, for given $c$, is independent of $\lambda$ and $L$. Important,
as shown in the Appendix, \( \varepsilon(c) \) equals the elasticity of substitution between any two varieties when they are consumed in the same amount \( c \). Krugman (1979) assumes that \( \varepsilon'(c) < 0 \), namely, that the elasticity of substitution between any two varieties is decreasing in the level of consumption (DES preferences).\(^7\) As argued in the Introduction, this assumption seems at odds with introspection. In this paper, we therefore explore the implications of the opposite (and at least equally plausible) assumption that \( \varepsilon'(c) > 0 \), namely, that preferences feature an increasing elasticity of substitution in consumption (IES preferences).

The revenue of an individual firm is:

\[
R(c) = p(c)cL = \frac{u'(c)}{\lambda}cL.
\]

(4)

We denote marginal revenue and the derivative of marginal revenue, respectively, by:

\[
R'(c) = \frac{r'(c)}{\lambda} \quad \text{and} \quad R''(c) = \frac{r''(c)}{\lambda L},
\]

where

\[
\begin{align*}
    r'(c) &= u'(c) + u''(c)c, \\
    r''(c) &= 2u''(c) + u'''(c)c.
\end{align*}
\]

(5) (6)

The first-order condition for profit maximization implies:

\[
r'(c) = \lambda \beta. \tag{7}
\]

To obtain a unique and well-behaved solution to the problem of profit maximization, we assume that the marginal revenue is everywhere positive and decreasing. That is, we assume that \( r'(c) > 0 \) and \( r''(c) < 0 \) for all \( c \), with \( \lim_{c \to -\infty} r'(c) = 0 \) and \( \lim_{c \to 0} r'(c) = \infty \).

Equations (2), (3), (5) and (7) imply that the profit maximizing price can be written as

\[
p = \frac{u'(c)}{r'(c)} \beta = \frac{\varepsilon(c)}{\varepsilon(c) - 1} \beta = m(c) \beta, \tag{8}
\]

where \( m(c) > 1 \) is the price-marginal cost markup, and

\[
m'(c) = -\frac{\varepsilon'(c)}{[\varepsilon(c) - 1]^2}. \tag{9}
\]

Evidently, unlike Krugman’s model, where \( \varepsilon'(c) < 0 \) and the markup is increasing in \( c \), IES preferences imply markups to be decreasing in individual consumption.\(^8\) Differentiating

\(^7\)Note, also, that \( \varepsilon(\cdot) \) equals the reciprocal of the "coefficient of relative risk aversion" of \( u(\cdot) \). Krugman’s assumption is thus formally equivalent to assuming that preferences feature increasing relative risk aversion.

\(^8\)As is well known, markups are instead constant with CES preferences.
(3) and using (5) and (6) yields the following expression for $\varepsilon'(c)$:

$$
\varepsilon'(c) = \frac{u'(c) r''(c) - u''(c) r'(c)}{[u''(c)]^2},
$$

(10)

which implies that

$$
\varepsilon'(c) \geq 0 \Leftrightarrow \frac{r'(c) u''(c)}{r''(c) u'(c)} = \frac{\eta(c)}{\varepsilon(c)} \geq 1,
$$

(11)

where $\eta(c) = -\frac{r'(c)}{r''(c)c} > 0$ is an inverse measure for the curvature of the revenue function.

Free entry implies zero equilibrium profits:

$$
\pi = \pi_v - \alpha = (p - \beta)cL - \alpha = 0,
$$

where $\pi$ and $\pi_v$ denote total and variable profits. Using (5) and (8), the free-entry condition can be written as

$$
\pi_v = (m(c) - 1)\beta Lc = \left[ \frac{u'(c)}{r'(c)} - 1 \right] \beta Lc = -\frac{u''(c)}{r'(c)} c \beta Lc = \alpha.
$$

(12)

Differentiating $\pi_v$ with respect to $c$ yields:

$$
\frac{\partial \pi_v}{\partial c} = \beta L \left[ \frac{u''(c) r'(c) - u'(c) r''(c)}{r'(c)^2} c - \frac{u''(c)}{r'(c)} c \right] = \beta L \frac{m(c)}{\eta(c)} > 0.
$$

(13)

Variable profit is therefore monotonically increasing in individual consumption, which ensures that the equilibrium is unique. Finally, full employment implies that labor demand, $n(\alpha + \beta cL)$, equals labor supply, $L$ (equivalently, equilibrium in the product market implies that $n = 1/pc$):

$$
n = \frac{L}{\alpha + \beta Lc} = \frac{L}{\alpha z(c)} = \frac{1}{m(c)c\beta},
$$

(14)

where the latter equalities follow from (12). Note that (12) implicitly defines the level of individual consumption consistent with profit maximization and free entry as a function $c(L, \beta, \alpha)$ of market size $L$, marginal cost $\beta$, and the fixed cost $\alpha$. The equilibrium number of firms, $n(L, \beta, \alpha)$, is instead determined recursively by (14).

We can now show how individual consumption depends on the model’s parameters. Differentiating (12) with respect to $L$, $\beta$, and $\alpha$ yields:

$$
\frac{\partial^{2} \pi_v}{\partial c \partial L} + \frac{\partial^{2} \pi_v}{\partial c \partial \beta} + \frac{\partial^{2} \pi_v}{\partial c \partial \alpha} = 0
$$

and

$$
\frac{\partial^{2} \pi_v}{\partial c \partial \alpha} = 1.
$$

Noting that $\frac{\partial n}{\partial \beta} L = \frac{\partial n}{\partial \beta} \beta = \pi_v = \alpha$, we obtain the following

**Lemma 1** Individual consumption is decreasing in market size $L$ and marginal cost $\beta$,
and increasing in the fixed cost $\alpha$, with:

$$
\frac{\partial \ln c}{\partial \ln \beta} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{r'(c)u''(c)}{r''(c)u'(c)} = -\frac{\eta(c)}{\varepsilon(c)} \lesssim -1 \iff \varepsilon'(c) \gtrless 0. 
$$

(15)

Lemma 1 and (9) immediately imply the following

**Proposition 1** With IES preferences, markups are increasing in market size $L$ and marginal costs $\beta$, and decreasing in the fixed cost $\alpha$. In contrast, with DES preferences, markups are decreasing in $(L=\alpha)$.

Note also that, with IES preferences, $\eta(c)/\varepsilon(c) > 1$ implies that firm size, $q(c) = cL$, is decreasing in market size. Conversely, firm size is instead increasing in market size with DES preferences. Finally, the sign of $\varepsilon'(c)$ affects the welfare effects of a change in the model’s parameters. In a symmetric equilibrium, welfare equals

$$
U(L, \beta, \alpha) = n(L, \beta, \alpha)u(c(L, \beta, \alpha)).
$$

(16)

Differentiating (16) with respect to $L$, $\beta$ and $\alpha$ yields:

$$
\frac{\partial \ln U}{\partial \ln L} = \left[ \phi(c) - 1 + \frac{1}{\varepsilon(c)} - \frac{1}{\eta(c)} \right] \frac{\partial \ln c}{\partial \ln L} = -\frac{\partial \ln U}{\partial \ln \alpha},
$$

(17)

$$
\frac{\partial \ln U}{\partial \ln \beta} = \left[ \phi(c) - \frac{d\ln \varepsilon}{d\ln c} \right] \frac{\partial \ln c}{\partial \ln \beta},
$$

(18)

where $\phi(c) = d\ln u/d\ln c < 1$ due to $u(0) = 0$ and the strict concavity of $u(c)$, and $\frac{\partial \ln c}{\partial \ln L} = \frac{\partial \ln c}{\partial \ln \beta} < 0$ from (15). Note that, for $\varepsilon' \leq 0$, the expression in brackets is negative in (17) and positive in (18), implying that welfare is increasing in market size and decreasing in fixed and marginal costs. Instead, for $\varepsilon' > 0$ the sign of the expressions in brackets is ambiguous. Therefore, with IES preferences the welfare effects of a change in the model’s parameters are in general ambiguous. This is because a rise in $L$ (or a fall in $\alpha$) has a positive welfare effect due to the induced rise in $n$ (the standard love for variety effect), and a negative welfare effect due to the rise in markups. Conversely, a rise in $\beta$ has a negative welfare effect due to the fall in the real wages (also due to the rise in markups), and a positive welfare effect due to the induced rise in $n$.

Interestingly, a sufficient condition for a rise in market size to be welfare increasing is that $\phi' > 0$ [see Dixit and Stiglitz (1977: pp. 303-4)]. In fact, note that $\phi' > 0$ can equivalently be written as $m\phi < 1$ and that the expression in square brackets in (17) is equivalent to $\phi - \frac{1}{m} - \frac{1}{\eta}$. Conversely, a sufficient condition for a rise in marginal cost to be welfare decreasing is that $\phi' < 0$ (note that the expression in square brackets in (18) is equivalent to $\phi - \frac{1}{m} + \frac{u''}{u'}$).

9
2.1 Discussion

With IES preferences, frictionless trade integration between identical countries, which in this model is isomorphic to a rise in market size, leads to higher markups and smaller firms. These results are the opposite of the pro-competitive and de-fragmentation effects delivered by Krugman’s model and by virtually all monopolistic competition trade models departing from CES preferences. Moreover, gains from trade and productivity growth are not ensured in this case. Hence, although IES preferences seem plausible, they lead to controversial results when embedded into the standard Dixit-Stiglitz monopolistic competition framework.

Consider now DES preferences. Aside from the fact that they build on dubious assumptions about consumer behavior, their implications concerning markups are also hard to rationalize. Consider, in particular, the effects of a fall in the marginal cost $\beta$. In the trade literature, it is standard to interpret $1/\beta$ as a productivity measure. In this respect, DES preferences imply that a rise in market size due to productivity growth is anti-competitive. DES preferences also imply, however, that a rise in market size due to trade opening is pro-competitive. Finally, consider the effects of an equiproportional fall in both $\alpha$ and $\beta$. In this case, the number of firms increases proportionally (see 14). Moreover, (12) implies that individual consumption, markups and firm size are unchanged, and hence that the sign of $\varepsilon'(c)$ plays no role. However, there is no obvious reason to expect technical change to affect fixed and marginal costs in exactly the same proportion. For instance, in monopolistic competition trade models endogenizing technology it is standard to assume that a lower marginal cost requires a higher fixed cost, e.g., in terms of R&D expenditures [see, among others, Yeaple (2005), Bustos (2011a, 2011b), Costantini and Melitz (2007)]. Yet, this type of technical change would imply even stronger anti-competitive effects under DES preferences.

To conclude, our results in this section suggest that departing from CES preferences in an otherwise standard monopolistic competition setting seems to involve implausible assumptions about consumer behavior and implausible results.

3 Alternative Environments

Our previous results are derived from a simple setup featuring, as in Krugman (1979), additive and symmetric preferences on the demand side, and Dixit-Stiglitz monopolistic competition on the supply side. A crucial implication of these assumptions is that in a symmetric equilibrium the demand elasticity equals the elasticity of substitution, and the latter does not directly depend on the number of firms. Thus, trade opening can only
affect the demand elasticity and markups by affecting individual consumption, which in turn directly affects the elasticity of substitution. It follows that, if the substitutability across varieties is increasing in the consumption level, as it should, then trade is necessarily anti-competitive.

In this Section, we consider alternative environments. In particular, building on examples drawn from the received trade literature, we discuss the implications of relaxing the assumptions that preferences are additive, that the space of characteristics/varieties is not finite and fixed, and that firms ignore price interactions with other firms. We consider, in particular, the following setups: a) quasi-linear quadratic preferences, as in Melitz and Ottaviano (2008); b) the ideal variety approach to monopolistic competition, as in Lancaster (1979); c) strategic interactions à la Cournot. A common denominator of these different frameworks is that they potentially allow for a trade-induced increase in the number of firms to directly affect the demand elasticity.

3.1 QUASI-LINEAR QUADRATIC PREFERENCES

As discussed in the Appendix, the elasticity of substitution is a key ingredient of demand elasticity. When preferences are additive and symmetric, as in (1), the elasticity of substitution \( \sigma_{ij} \) between varieties \( i \) and \( j \) is independent of the number of varieties \( n \) available for consumption (see the Appendix for a proof). This result is easily understood when recalling that, if \( U(\cdot) \) is additive, the marginal rate of substitution between any two varieties is unaffected by consumption of other varieties.

Matters are different, however, if preferences are non-additive. For instance, at a symmetric consumption pattern \( (c_i = c, i = 1, ..., n) \), consumption of other varieties may affect the elasticity of substitution between varieties \( i \) and \( j \) through a direct impact of \( n \) on \( \sigma_{ij} \) (again see the Appendix). Hence, non-additive preferences potentially allow for a sort of (pro- or anti-competitive) externality of \( n \) on the elasticity of substitution. The sign and the interpretation of this externality are not obvious, however. To illustrate, consider the quasi-linear quadratic preferences \( U(c_0, u(c_{-0})) = c_0 + u(c_{-0}) \), where \( c_0 \) is consumption of a numeraire good, \( c_{-0} = [c_1, c_2, ..., c_n] \) is consumption of \( n \) varieties of

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10 See also Dixit and Stiglitz (1979) on this point.

11 In this respect, it is suggestive that in their discussion of "diversity as a public good", Dixit and Stiglitz (1975: section 4.4) consider a non-additive case in which \( n \) enters the utility function without however affecting the marginal rate of substitution between any two varieties.
some product and
\[
  u(c_{-0}) = \alpha \sum_{j=1}^{n} c_j - \frac{\gamma}{2} \sum_{j=1}^{n} c_j^2 - \frac{\eta}{2} \left( \sum_{j=1}^{n} c_j \right)^2,
\]  
(19)

with \( \alpha, \gamma, \eta > 0 \). The above non-additive preferences have been recently used in an influential paper by Melitz and Ottaviano (2008). Their monopolistic competition framework with heterogeneous firms is widely perceived as an appealing alternative to the original Melitz (2003) model, as it delivers a trade-induced pro-competitive effect.

Maximization of (19) with respect to \( c_i \) subject to a budget constraint yields:
\[
  p_i = \frac{\partial u}{\partial c_i} = \alpha - \gamma c_i - \eta \sum_{j=1}^{n} c_j, \quad i = 1, \ldots, n.
\]  
(20)

Thus, summing (20) across varieties:
\[
  \sum_{j=1}^{n} p_j = n\alpha - (\gamma + n\eta) \sum_{j=1}^{n} c_j \Rightarrow \sum_{j=1}^{n} c_j = \frac{n\alpha - np}{\gamma + n\eta},
\]  
(21)

where \( p = \frac{1}{n} \sum_{j=1}^{n} p_j \) is the average price of a variety. Using (21) in (20) yields an expression for the inverse demand function:
\[
  p_i = \frac{\alpha \gamma - \gamma c_i}{\gamma + n\eta} + \frac{n\eta p}{\gamma + n\eta}.
\]

The direct uncompensated demand function for variety \( i, c_i(p) \), which coincides with the compensated demand \( \tilde{c}_i(p) \), is therefore given by:
\[
  \tilde{c}_i(p) = c_i(p) = \frac{\alpha}{\gamma + n\eta} - \frac{p_i}{\gamma} + \frac{n\eta}{\gamma + n\eta} \frac{1}{p}.
\]  
(22)

Note that the demand perceived by firm \( i \), under the assumption that it takes both \( n \) and \( p \) as given, is linear in \( p_i \), with constant slope \( 1/\gamma \); accordingly, the demand elasticity equals \( \frac{1}{\gamma c_i} \), and it is then decreasing with respect to \( c_i \) for any given price \( p_i \).

As detailed in the Appendix, the expression for the elasticity of substitution is \( \sigma_{ij} = \tilde{\varepsilon}_{ji} - \tilde{\varepsilon}_{ii} \), where \( \tilde{\varepsilon}_{ij} = \tilde{c}_{ij}(p_j/\tilde{c}_i) \) and \( \tilde{\varepsilon}_{ii} = \tilde{c}_{ii}(p_i/\tilde{c}_i) \) are the compensated demand-price elasticities, and \( \tilde{c}_{ii} = \partial \tilde{c}_i/\partial p_i, \tilde{c}_{ij} = \partial \tilde{c}_i/\partial p_j \) are the corresponding derivatives, whose

\[\text{12The following discussion assumes that an internal solution arises in equilibrium (i.e., } c_0 > 0).\]
\[\text{13Recall that, with quasi-linear preferences, there are no income effects on the demand for non-numeraire goods.}\]
expression is given by:
\[ e_{ii} = 1 + n \gamma_i - e_{ij}, \quad i, j = 1, 2, \ldots, n \text{ and } i \neq j. \]

Hence, for \( p_i = p_j \) (which implies \( c_i = c_j \)) we have:
\[ \sigma_{ij} = \tilde{z}_{ji} - \tilde{z}_{ii} = \frac{p_i}{c_i} (\tilde{z}_{ji} - \tilde{z}_{ii}) = \frac{1}{\gamma} \left( \alpha - \gamma c_i - \eta \sum_{h=1}^{n} c_h \right). \] (23)

Notice that the third equality in (23) shows that the demand elasticity perceived by firms in Melitz and Ottaviano (2008) is equal to the elasticity of substitution. In a quasi-symmetric equilibrium, i.e., for \( c_h = c, h = 1, \ldots, n, i \neq h \neq j \), we obtain:
\[ \sigma_{ij} = \frac{1}{\gamma} \left[ \frac{\alpha}{c_i} - \gamma - \eta (2 + (n - 2) \frac{c}{c_i}) \right]. \] (24)

Thus, for given consumption levels \( c_i \) and \( c \), the elasticity of substitution is actually decreasing in the number of available varieties \( n \). Also note also that \( \sigma_{ij} \) is decreasing in \( c_i \) and \( c \) and that, for \( c_i = c \) (i.e., in a fully symmetric equilibrium) its expression boils down to
\[ \sigma = \frac{1}{\gamma} \left[ \frac{\alpha}{c} - \gamma - \eta n \right]. \] (25)

The above results show that, in the most popular example of non-additive preferences used in the monopolistic competition trade literature, the number of firms has a negative direct impact on the elasticity of substitution. This implies that the pro-competitive effect delivered by the model in Melitz and Ottaviano (2008) is entirely driven by the linearity of the demand function perceived by the firms, hence by the fact that, just as in Krugman (1979), the elasticity of substitution is decreasing in the level of individual consumption.

3.2 Ideal Variety Approach to Monopolistic Competition

In our baseline setting, borrowed from Krugman (1979), the introduction of new varieties does not crowd the variety space, as the number of characteristics/varieties is the same as the number of firms. One may argue, however, that a pro-competitive effect may naturally arise in a framework in which a trade-induced increase in the number of available varieties reduces their distance in the fixed characteristics space, thereby increasing their substitutability.

In this Section we show that, surprisingly, this needs not be the case. To make the point, we consider Lancaster’s (1979) "ideal variety" approach to monopolistic competition. In this setting, consumer preferences are heterogeneous and the aggregate demand
for each variety arises from diversity of tastes. In particular, each consumer has a most preferred ("ideal") variety. As described in Helpman and Krugman (1985, pp. 120-21), on which we build in this section, ideal variety means that "when the individual is offered a well-defined quantity of the good but is free to choose any potentially possible variety, he will choose the ideal variety independently of the quantity offered and independently of the consumption level of other goods. Moreover, when comparing a given quantity of two different varieties, the individual prefers the variety that is closest to his ideal product".

These assumptions are formalized by assuming that each variety is represented by a point $\omega$ on the unit length circumference $\Omega$ of a circle, and that preferences for the ideal product are uniformly distributed over $\Omega$ across consumers. $L$ is the size (and density) of the population. The utility function of a consumer with ideal variety $\tilde{\omega}$ is assumed to be:

$$U = \sum_{\omega \in \Omega} \frac{c(\omega)}{h(\delta(\omega, \tilde{\omega}))},$$  

(26)

where $\delta(\omega, \tilde{\omega})$ is the shortest arc distance between $\omega$ and $\tilde{\omega}$, and $h(\delta)$ is the so-called Lancaster’s compensation function, assumed to be positive, non decreasing and generally normalized so that $h(0) = 1$ (see Lancaster, 1975). Moreover, it is generally assumed (see, Helpman and Krugman, 1985) that $h(\delta)$ is strictly increasing and convex, and that $h'(0) = 0$.

We now show that the above assumptions are insufficient to deliver a pro-competitive impact of entry in this setting. To this purpose, note first that preferences as in (26) are of the "perfect substitute" type, with constant marginal rate of substitution (MRS) between any two varieties $\omega'$ and $\tilde{\omega}$ given by:

$$MRS(\omega', \tilde{\omega}) = \frac{h(\delta(\omega', \tilde{\omega}))}{h(\delta(\omega', \omega))}.$$  

(27)

The above assumptions on $h(\cdot)$ imply that $MRS(\tilde{\omega}, \omega) = h(\delta(\omega, \tilde{\omega}))$ is an increasing convex function of the arc distance between $\tilde{\omega}$ and $\omega$. Utility maximization then implies:

$$c(\omega) = \begin{cases} \frac{1}{p(\omega)} & \omega = \omega' \\ 0 & \omega \neq \omega' \end{cases},$$

where $\omega' = \arg \min_{\omega \in \Omega} p(\omega)h(\delta(\omega, \tilde{\omega}))$. In the Appendix we derive the aggregate demand function, $q(\omega)$, for a firm selling variety $\omega$ at the price $p(\omega)$, with contiguous competitors $\omega_l$ and $\omega_r$ charging prices $p(\omega_l)$ and $p(\omega_r)$. In a symmetric equilibrium in which $p(\omega) = p(\omega_l) = p(\omega_r)$ and $d = \delta(\omega_l, \omega) = \delta(\omega_r, \omega) = \frac{1}{n}$, the price elasticity of the aggregate

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14See also Helpman (1981) on this point.
demand is given by:

\[
\varepsilon(q(\omega)) = -\frac{\partial \ln q(\omega)}{\partial \ln p(\omega)} = 1 + \frac{h(d/2)}{2(d/2)h'(d/2)} = 1 + \frac{1}{2\varepsilon_h(d/2)},
\]

where \(\varepsilon_h\) is the elasticity of the compensation function, and crucially affects the relationship between \(\varepsilon\) and \(n\). Specifically, if \(\varepsilon'_h > 0\), a rise in \(n\) increases \(\varepsilon\). In this case, the model implies that a trade-induced increase in the number of firms is pro-competitive. Instead, if \(\varepsilon'_h < 0\), a rise in \(n\) decreases \(\varepsilon\) and is therefore anti-competitive. Finally, if \(\varepsilon'_h = 0\) (i.e., if \(h(\cdot)\) is isoelastic), \(\varepsilon\) is independent of \(n\), just as in the "love for variety" approach when preferences are CES. We therefore conclude that a pro-competitive effect is unwarranted even in a framework in which a trade-induced increase in the number of firms crowds the variety space. This is because the ideal variety approach does not impose sufficient restrictions on \(h(\cdot)\) to pin down the properties of \(\varepsilon_h\), and therefore the relationship between \(\varepsilon\) and \(n\).\(^{15}\)

Is the assumption \(\varepsilon'_h(\cdot) > 0\) plausible? Note that \(MRS(\tilde{\omega}, \omega) = h(\delta(\omega, \tilde{\omega}))\) implies that, in order for Lancaster’s model to deliver a pro-competitive effect, consumer preferences must feature an ever increasing distance elasticity of the marginal rate of substitution between \(\tilde{\omega}\) and \(\omega\). It is hard to provide a rationale for this assumption, which seems no more plausible that the opposite assumption of a decreasing distance elasticity. In this latter case, however, Lancaster’s model would deliver an anti-competitive effect. If follows that in this framework the most natural assumption is that is \(h(\cdot)\) is elastic, which implies the absence of any pro- or anti-competitive effect.

3.3 Cournot Competition

In our setting, the assumption that each firm treats \(\lambda\) (the marginal utility of income) as a constant removes a direct channel whereby a trade-induced increase in the number of firms may raise the perceived demand elasticity \(\varepsilon\). Under the alternative assumption that firms properly treat \(\lambda\) as a function of the price vector (i.e., \(\lambda = \lambda(p)\)), the demand elasticity \(\varepsilon\) no longer coincides with the elasticity of substitution \(\sigma\). In the case of symmetric consumption \(c\), its expression is actually given by (see (61) in the Appendix):

\[
\varepsilon(c, n) = \sigma(c) - \frac{\sigma(c) - 1}{n}.
\]

\(^{15}\)Only in the limit case in which \(d\) goes to zero, and due to the (rather ad hoc) assumptions \(h(0) = 1\) and \(h'(0) = 0\), the aggregate demand elasticity is increasing in \(n\) (and goes to infinite). This requires a situation, unfeasible under a positive fixed cost, in which the circumference of the circle is full and the aggregate demand for each firm is infinitesimal.
It follows that a trade-induced increase in the number of firms has a positive direct impact on \( \varepsilon \) and, with IES preferences, an indirect negative impact through \( \sigma \). The net effect is therefore in general ambiguous, possibly making a case for the standard assumption of a constant markup. Note, however, that:

\[
\frac{\partial \ln \varepsilon}{\partial \ln \sigma} = \frac{(n-1)\sigma}{(n-1)\sigma + 1} \implies \lim_{n \to \infty} \frac{\partial \ln \varepsilon}{\partial \ln \sigma} = 1,
\]

\[
\frac{\partial \ln \varepsilon}{\partial \ln n} = -\frac{\sigma - 1}{(n-1)\sigma + 1} \implies \lim_{n \to \infty} \frac{\partial \ln \varepsilon}{\partial \ln n} = 0,
\]

which suggests that the direct pro-competitive impact of \( n \) on \( \varepsilon \) weakens as the number of firms grows larger, and vanishes in the limit. Thus, for \( n \) large enough, the anti-competitive effect induced by IES preferences should always prevail.

The robustness of this conclusion can be confirmed by considering a Cournotian extension of our baseline setting, in which firms correctly perceive the demand functions they face, but strategically interact with their competitors by setting their production levels. Multiplying both sides of the first-order conditions for utility maximization \((u'(c_i) = \lambda p_i)\) by \( c_i \) and adding up yields:

\[
\lambda = \sum_j u'(c_j)c_j.
\]

Using (29) in the first-order conditions yields the following inverse demand system:

\[
p_i(c) = \frac{\sum_j u'(c_j)c_j}{\sum_j u'(c_j)c_j}, \quad i = 1, \ldots, n.
\]

Let \( R_i(c) = p_i(c)c_iL \) be firm \( i \)'s revenue. Marginal revenue is therefore given by:

\[
\frac{\partial R_i(c)}{\partial c_i} \frac{1}{L} = \frac{r'(c_i)\left(\sum_{j \neq i} u'(c_j)c_j\right)}{\left(\sum_j u'(c_j)c_j\right)^2},
\]

and is decreasing in \( c_i \) under our assumptions that \( r'' < 0 \) and \( r' = u''c + u' > 0 \). In a Nash equilibrium, each firm chooses its quantity to satisfy the first-order condition \( \frac{\partial R_i}{\partial c_i} \frac{1}{L} = \beta \) under a correct conjecture about the quantities produced by its competitors. Then, (31) and (30) imply that, in any symmetric Nash equilibrium:

\[
c = \frac{(n-1) r'(c)}{n^2 u'(c) \beta} = \frac{n - 1}{n^2 m(c) \beta},
\]

\[
p = \frac{1}{nc} = \frac{n}{n - 1} m(c) \beta.
\]
Note, from (33), that the markup \( \frac{n}{n-m} m(c) \) depends on both \( n \) and \( c \) (through \( m \)).\(^{16}\) Moreover, (32) uniquely pins down the equilibrium relationship \( c(n) \) between \( c \) and \( n \), with \( c'(n) < 0 \).\(^{17}\) Accordingly, a rise in the number of firms is anti-competitive if and only if the elasticity of \( c(n) \) is greater than one in absolute value (so that \( p = (nc)^{-1} \) increases):

\[
\left| \frac{d \ln c}{d \ln n} \right| = \frac{(n-2) m(c(n))}{(n-1) m'(c(n)) c(n) + m(c(n))},
\]

where \( \frac{m}{m'+m} > 1 \) (as \( m' < 0 \)) and \( \lim_{n \to \infty} \frac{n-2}{n-1} = 1 \). It follows that, for \( n \) large enough, a trade-induced increase in the number of firms can be anti-competitive even when firms interact strategically.

We summarize our main results in this section in the following

**Proposition 2** a) Even when preferences are quasi-linear and quadratic, as in Melitz and Ottaviano (2008), the demand elasticity perceived by the firms coincides with the elasticity of substitution, which negatively depends on the number of firms, so that the pro-competitive effect of a trade-induced expansion in market size is entirely driven by the decrease of the level of consumption; b) When preferences are heterogeneous across consumers and are of the ideal variety type, as in Lancaster (1979), a trade-induced increase in the number of firms is pro-competitive only when the compensation function features an increasing distance elasticity; c) When firms are Cournotian competitors, and preferences are IES, a trade-induced increase in the number of firms can be pro-competitive only when the initial number of competitors is small.

## 4 Heterogeneous Firms

So far, we have only considered setups with homogeneous firms, thereby ignoring the recent literature on heterogeneous firms in international trade. In this section, we therefore complement our previous analysis by studying how IES preferences interact with firm heterogeneity. Our basic setup is the same as in Section 2.1, except that we now allow for a continuum rather than a discrete number of varieties. More importantly, following Melitz (2003), we depart from the model in Section 2 by assuming that, upon paying a fixed

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\(^{16}\) Also note that (33) implies that Cournotian firms behave "as if" they perceived a demand elasticity equal to \( \frac{m}{m'+m} \), which is lower than the expression in (28). This is an implication of the fact that Cournotian firms are quantity setters. In a model with Bertrand competition (and product differentiation), the relevant demand elasticity would be just the one in (28). In both cases, however, the demand elasticity approaches \( \sigma \) for \( n \to \infty \).

\(^{17}\) This relationship would be unaffected by explicitly considering a free-entry condition which, by endogenizing \( n \) (as in our baseline setting), would redefine the equilibrium value of \( c \) as a function of \( L/\alpha \) and \( \beta \).
entry cost $\alpha_r$, firms draw their marginal cost $\beta \in [\underline{\beta}, \infty)$, with $\underline{\beta} > 0$, from a continuous cumulative distribution $G(\beta)$ with density $g(\beta)$. In the following, we index firms by their marginal cost $\beta$ and denote by $c(\beta)$ the individual demand for their products.

**Firm productivity, size and markups** Denote by $p(\beta) = m(c(\beta))\beta$ and $\pi_v(\beta) = [m(c(\beta)) - 1] \beta L c(\beta)$ the price and variable profit of a $\beta$-firm. The first-order and second-order conditions for profit maximization, $r'(c) = \lambda \beta$ and $r''(c) < 0$, imply that:

$$
\frac{d \ln c(\beta)}{d \ln \beta} = \frac{\lambda \beta}{r''(c(\beta))c(\beta)} = -\eta(c(\beta)) < 0, \quad (34)
$$

$$
\pi_v'(\beta) = -c(\beta)L < 0, \quad (35)
$$

$$
\frac{d \ln p(\beta)}{d \ln \beta} = 1 - \frac{1}{c(\beta)} \left(1 - \frac{\eta(c(\beta))}{\varepsilon(c(\beta))}\right) \geq 1 \iff \varepsilon'(c(\beta)) \geq 0, \quad (36)
$$

$$
\frac{-\pi_v''(\beta)\beta}{\pi_v'(\beta)} = \eta(c(\beta)). \quad (37)
$$

Thus, as in Melitz (2003), high-productivity (low-$\beta$) firms are larger and more profitable. Unlike the Melitz model, however, where preferences are CES and markups are constant, with IES preferences larger firms charge lower markups. With DES preferences, larger firms would instead charge lower markups. Hence, in this simple yet quite general setup, the sign of $\varepsilon'(c)$ has clear-cut (and potentially testable) implications for the cross-sectional relationship between firm size and markups. Finally note that, as shown by (37), $\eta$ can also be interpreted as a measure of the variable profit function curvature.

**Zero cutoff profit condition** Denote by $\beta^*$ the marginal cost cutoff, namely, the value of $\beta$ satisfying the zero cutoff profit condition $\pi(\beta^*) = 0$. This condition implicitly defines the individual demand for the cutoff firm, $c^* = c^*(\beta^*, L, \alpha)$. In particular, using (12) we can write:

$$
\pi_v(\beta^*) = [m(c^*) - 1] \beta^* L c^* = \left[-\frac{u''(c^*)}{r'(c^*)} c^* \right] \beta^* L c^* = \alpha. \quad (38)
$$

Differentiating with respect to $c^*$, $L$, $\beta^*$ and $\alpha$ yields:

$$
\frac{\partial \ln c^*}{\partial \ln L} = \frac{\partial \ln c^*}{\partial \ln \beta^*} = \frac{\partial \ln c^*}{\partial \ln \alpha} = -\frac{\eta(c^*)}{\varepsilon(c^*)} < 0. \quad (39)
$$

Evidently, $c^*$ is decreasing in $(L \beta^*/\alpha)$, with an elasticity whose value depends on the sign of $\varepsilon'$.  

18
**Individual demand for a β-firm** Profit maximization implies \( r'(c^*) = \lambda \beta^* \). Solving for \( \lambda \) and substituting into \( r'(c) = \lambda \beta \) yields:

\[
r'(c) = r'(c^*) \frac{\beta}{\beta^*}. \tag{40}
\]

Thus, the marginal revenue of a β-firm is proportional to the marginal revenue of the cutoff firm. Equation (40) is key to the characterization of the equilibrium, as it implicitly defines the individual demand for a β-firm, \( c(\beta) = c(\beta; \beta^*, c^*) \). In fact, having shown how \( c^* \) depends on \( \beta^* \), \( L \) and \( \alpha \), we can now study how \( c(\beta) = c(\beta; \beta^*, c^*(\beta^*, L, \alpha)) = c(\beta; \beta^*, L, \alpha) \) varies with \( \beta^* \), \( L \) and \( \alpha \). Differentiating (40) with respect to \( \beta^* \) yields:

\[
 r''(c) \frac{\partial c}{\partial \beta^*} = \frac{\beta}{\beta^*} \left[ r''(c^*) \frac{\partial c^*}{\partial \beta^*} + r'(c^*) \right].
\]

Using (39) and rearranging terms yields:

\[
 \frac{\partial \ln c}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0.
\]

Hence, generalizing Melitz’s (2003) result, individual consumption is increasing in \( \beta^* \). Similarly, differentiating (40) with respect to \( L \) and using (39) yields:

\[
 \frac{\partial \ln c}{\partial \ln L} = - \frac{\partial \ln c}{\partial \ln \alpha} = - \frac{\eta(c)}{\varepsilon(c^*)} < 0.
\]

Thus, as in the homogeneous-firm setup, individual consumption is decreasing in market size \( L \) and increasing in the fixed cost \( \alpha \) (for given \( \beta^* \)). Finally, note that with IES preferences \( \varepsilon(c^*) < \varepsilon(c) \) for \( c^* < c \); hence, using (11):

\[
 \frac{\partial \ln c}{\partial \ln L} < - \frac{\eta(c)}{\varepsilon(c)} < -1.
\]

Thus firm size, \( q = cL \), is decreasing in \( L \) for given \( \beta^* \). In contrast, with DES preferences \( \varepsilon(c^*) > \varepsilon(c) \) for \( c^* < c \), and hence firm size is increasing in \( L \):

\[
 \frac{\partial \ln c}{\partial \ln L} > - \frac{\eta(c)}{\varepsilon(c)} > -1.
\]

The following lemma summarizes

**Lemma 2** \( c(\beta; \beta^*, L, \alpha) \) is increasing in \( \beta^* \) and \( \alpha \) and decreasing in \( L \), with

\[
 \frac{\partial \ln c}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0, \quad \frac{\partial \ln c}{\partial \ln L} = - \frac{\partial \ln c}{\partial \ln \alpha} = - \frac{\eta(c)}{\varepsilon(c^*)} \leq -1 \iff \varepsilon'(c) \geq 0. \tag{41}
\]

19
Free-entry condition  Free entry implies that expected profits,

\[ \pi^E = \int_{-\infty}^{\beta^*} \pi(\beta) dG(\beta), \]

equal the fixed cost of entry \( \alpha_e \). Integrating \( \pi^E \) by parts yields:

\[ \pi^E = \pi(\beta^*)G(\beta^*) - \pi(\beta)G(\beta) - \int_{\beta}^{\beta^*} \pi'(\beta) G(\beta) d\beta = -\int_{\beta}^{\beta^*} \pi'(\beta) G(\beta) d\beta, \]
since \( \pi(\beta^*) = G(\beta) = 0 \). Using (35), the free-entry condition can therefore be written as:

\[ \pi^E = \int_{-\infty}^{\beta^*} c(\beta) L G(\beta) d\beta = \alpha_e. \]  
(42)

Differentiating \( \pi^E \) with respect to \( \beta^* \) yields:

\[ \frac{\partial \pi^E}{\partial \beta^*} = c(\beta^*) L G(\beta^*) + \int_{\beta}^{\beta^*} \frac{\partial c(\beta)}{\partial \beta^*} L G(\beta) d\beta > 0, \]  
(43)

where the inequality follows from Lemma 2. Hence, as in Melitz (2003), expected profits are increasing in \( \beta^* \) and the free-entry condition (42) uniquely pins down the equilibrium value of \( \beta^* \), thereby defining the equilibrium value of \( c(\beta) = c(\beta; L, \alpha, \alpha_e) \).

Finally, the measure of active firms \( n \) is pinned down by the budget constraint (or, equivalently, by the full employment condition), requiring average expenditure to equal \( 1/n \), and thus:

\[ n = \left[ \int_{-\infty}^{\beta^*} m(c(\beta)) |\beta c(\beta)| \frac{dG(\beta)}{G(\beta^*)} \right]^{-1}. \]  
(44)

4.1 Comparative Statics

Consider first the effect of a rise in market size \( L \). Totally differentiating (42) with respect to \( L \) yields:

\[ \frac{d \pi^E}{dL} = \frac{\partial \pi^E}{\partial L} + \frac{\partial \pi^E}{\partial \beta^*} \frac{d \beta^*}{dL} = 0, \]  
(45)

where \( \frac{\partial \pi^E}{\partial \beta^*} > 0 \) from (43) and, using Lemma 2,

\[ \frac{\partial \pi^E}{\partial L} = \int_{\beta}^{\beta^*} \left[ \frac{\partial c(\beta)}{\partial L} L + c(\beta) \right] G(\beta) d\beta = \int_{\beta}^{\beta^*} \left( 1 - \frac{\eta(c(\beta))}{\varepsilon(c^*)} \right) c(\beta) G(\beta) d\beta. \]  
(46)

It follows that with IES preferences \( \frac{\partial \pi^E}{\partial L} < 0 \), as firm size is decreasing in market size for given \( \beta^* \). Thus, \( d\beta^*/dL > 0 \): a rise in market size leads to a rise in \( \beta^* \) and a consequent
anti-selection effect, i.e., less productive firms can survive in a larger market. Lemma 2 and (46) also imply that, with DES preferences, a rise in market size leads instead to a standard selection effect.

Next, using (43) and (46) in (45) and rearranging yields:

\[ c(\beta^*) LG(\beta^*) \frac{d\beta^*}{dL} + \int_{\beta}^{\beta^*} c(\beta) G(\beta) d\beta + L \int_{\beta}^{\beta^*} \frac{dc}{dL} G(\beta) d\beta = 0. \] (47)

Note that the first two terms in (47) are positive, thereby implying that the last term is negative. Moreover, using Lemma 2:

\[ \frac{d \ln c}{d \ln L} = \left[ \frac{\partial L c}{\partial \beta^*} + \frac{\partial c}{\partial \beta^*} \frac{d \beta^*}{d \ln L} \right] \frac{L}{c} = -\eta(c) \left[ 1 - (\varepsilon(c^*) - 1) \frac{d \ln \beta^*}{d \ln L} \right]. \] (48)

Note that the sign of the term in square brackets is independent of \( \beta \). It follows that the sign of \( dc/dL \) is the same for all firms, and must therefore be negative according to (47).

Thus, as in the baseline model with homogeneous firms, a rise in market size leads to a fall in individual consumption and a consequent rise of markups with IES preferences.\(^{18}\)

Next, consider the effects of a rise in the fixed production cost \( \alpha \). Totally differentiating (42) with respect to \( \alpha \) yields:

\[ \frac{d \pi^E}{d \alpha} = \frac{\partial \pi^E}{\partial \alpha} + \frac{\partial \pi^E}{\partial \beta^*} \frac{d \beta^*}{d \alpha} = 0, \]

where, using Lemma 2,

\[ \frac{\partial \pi^E}{\partial \alpha} = \int_{\beta}^{\beta^*} \frac{dc(\beta)}{\partial \alpha} LG(\beta) d\beta = \frac{L}{\alpha \varepsilon(c^*)} \int_{\beta}^{\beta^*} \eta(c(\beta)) c(\beta) G(\beta) d\beta > 0. \]

Thus, independent of the sign of \( \varepsilon'(c) \), a rise of \( \alpha \) leads to a fall of \( \beta^* \). Moreover, proceeding as above, it is possible to show that \( dc/d\alpha > 0 \) for all firms, implying that a rise in \( \alpha \) is pro-competitive with IES preferences and anti-competitive with DES preferences, just as in the baseline model with homogeneous firms.

Finally, (42) and (43) immediately imply that, independent of the sign of \( \varepsilon'(c) \), a rise in the fixed entry cost \( \alpha_e \) leads to a rise of \( \beta^* \). Moreover, since

\[ \frac{dc}{d \alpha_e} = \frac{\partial c}{\partial \beta^*} \frac{d \beta^*}{d \alpha_e} > 0, \]

it follows that markups decrease with IES preferences and rise with DES preferences.

\(^{18}\) The fall in individual consumption leads instead to a fall of markups when preferences are DES.
The main results are recorded in the following

**Proposition 3** With IES preferences, a rise in market size induces an anti-selection effect, whereby less productive firms survive in a larger market, and an anti-competitive effect, whereby firms charge higher markups. The converse is true with DES preferences, i.e., market size expansion leads to standard selection and pro-competitive effects. A rise in the fixed production cost $\alpha$, or in the fixed entry cost $\alpha_e$, are pro-competitive with IES preferences, and anti-competitive with DES preferences. In both cases, a rise of $\alpha$ leads to a selection effect, whereas a rise of $\alpha_e$ produces an anti-selection effect.

### 4.2 Fixed Costs of Exporting

In our setup, frictionless trade integration between two identical countries is equivalent to a doubling of market size. In this respect, Proposition 3 suggests that, with IES preferences, trade opening leads to anti-competitive and anti-selection effects. We now show how the above results change in the presence of costly trade. Specifically, we assume that exporting to an identical foreign market involves an additional fixed cost $\alpha_x$. Denoting by a subscript $x$ variables related to the export market, and by no subscript those related to the domestic market, we have that profits of a $\beta$-firm active in both markets are given by:

$$\pi(\beta) = \pi_v(\beta) - \alpha, \quad \pi_x(\beta) = \pi_v(\beta) - \alpha_x,$$

where $\pi_v(\beta) = [m(c(\beta)) - 1] \beta c(\beta)L$, and $L$ denotes the size of each country. The marginal cost cutoff for exporters, $\beta^*_x$, is implicitly given by:

$$\pi_x(\beta^*_x) = [m(c(\beta^*_x)) - 1] \beta^*_x c(\beta^*_x)L - \alpha_x = 0,$$

where $c^*_x = c(\beta^*_x)$ is individual foreign demand for the cutoff exporter.

Total expected profits are now given by:

$$\pi^E = \int_0^{\beta^*} \pi(\beta)g(\beta)d\beta + \int_{\beta^*}^{\beta_x} \pi_x(\beta)g(\beta)d\beta.$$  

Integrating (51) by parts and using the envelope theorem, the free-entry condition can be

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Note that a domestic firm producing only for the foreign market would incur an overall fixed cost equal to $\alpha + \alpha_x$, which implies that this case cannot arise in equilibrium. For analytical convenience, we can therefore apportion the fixed cost $\alpha$ to domestic profits, in this following the heterogeneous-firms literature.
written more conveniently as:

$$\pi^E = \int_{\beta}^{\beta^*} c(\beta) LG(\beta) d\beta + \int_{\beta^*}^{\beta} c(\beta) LG(\beta) d\beta = \alpha_e. \quad (52)$$

Finally, the measure of active firms is given by:

$$n = \left[ \int_{\beta}^{\beta^*} m(c(\beta)) \beta c(\beta) \frac{dG(\beta)}{G(\beta^*)} + \int_{\beta^*}^{\beta} m(c(\beta)) \beta c(\beta) \frac{dG(\beta)}{G(\beta^*)} \right]^{-1}. \quad (53)$$

We can now study how trade opening between two identical countries affects the marginal cost cutoff $\beta^*$ in the presence of fixed costs of exporting. Consider first the case $\alpha_x > \alpha$, which implies that $\pi_x(\beta) < \pi(\beta)$ for all $\beta$. In this case, not all active firms can profitably export and hence $\beta^* > \beta_x^*$. For $\alpha_x \to \infty$ (namely, for $\alpha_x$ larger than a threshold $\bar{\alpha}_x$), we are back in autarky; in this case, $\beta_x^* = \beta$ and the second integral in (52) disappears. Next, note that (50) implicitly defines $\beta_x^* = \beta_x^*(\beta^*, L, \alpha_x)$. Thus, differentiating (50) and using (35), (13) and Lemma 2 yields:

$$\frac{\partial \beta_x^*}{\partial \alpha_x} = \frac{1}{\pi_x'(\beta_x^*)} < 0, \quad \frac{\partial \beta_x^*}{\partial \beta^*} = -\frac{m(c(\beta_x^*)) \beta_x^* L}{\pi_x'(\beta_x^*) \eta(c(\beta_x^*))} \frac{\partial c(\beta_x^*)}{\partial \beta^*} = \frac{m(c_x^*)}{m(c^*)} > 0. \quad (54)$$

Thus, once again generalizing the Melitz’s (2003) result for the CES case, a fall in the fixed costs of exporting leads to a rise in the marginal cost cutoff for exporters $\beta_x^*$. It follows that, for $\alpha_x$ sufficiently low (namely, below $\bar{\alpha}_x$), $\beta_x^* > \beta$ and some high-productivity firms can export. In this case, the second integral in (52) is strictly positive and increasing in $\beta^*$ and $\beta_x^*$ according to (54) (recall that $c$ is also increasing in $\beta^*$). It follows that a move from autarky to costly trade shifts the LHS of (52) upwards, thereby leading to a lower equilibrium value of $\beta^*$. Hence, independent of the sign of $c'$, trade opening leads to a selection effect when $\alpha_x > \alpha$, namely, when fixed costs of exporting induce a partitioning of firms into exporters and non-exporters. Moreover, still as in the Melitz model, a fall of $\alpha_x$ in this range, by increasing $\beta_x^*$ requires a decrease in $\beta^*$ according to (52), and therefore yields a selection effect.

Consider, finally, the effects of trade opening when fixed costs of exporting are in the range $0 \leq \alpha_x \leq \alpha$. In this case, $\pi_x(\beta) \geq \pi(\beta)$ and all active firms export, implying that $\beta^* = \beta_x^*$. Thus, the zero cutoff profit condition and the free-entry condition boil down to:

$$\pi_v(\beta^*) = [m(c^*) - 1] \beta^* L c^* = \alpha + \alpha_x, \quad (55)$$
$$\pi^E = \int_{\beta}^{\beta^*} c(\beta) L G(\beta) d\beta = \alpha_e. \quad (56)$$
where $L' = 2L$. Note first that, for $\alpha_x = 0$, we are back in the case of costless trade integration, implying a change of $\beta^*$ relatively to autarchy according to Proposition 3. Instead, for $\alpha_x = \alpha$, (55) is the same as in autarchy, and therefore $c^*$ and $c(\beta)$ are unaffected for given $\beta^*$. It follows that $\beta^*$ must fall relative to autarchy, due to the direct impact of the rise of $L$ in (56). Moreover, note that a rise of $\alpha_x$ in the range $[0, \alpha]$ is equivalent to a rise in the fixed production cost in a closed economy of size $L'$, and therefore implies a fall of $\beta^*$ according to Proposition 3. Notice that this implies that, with heterogeneous firms, a proportional increase of both $L$ and $\alpha$ affects the equilibrium producing a selection effect. Finally, given that with IES preferences $\beta^*$ is higher than in autarchy for $\alpha_x = 0$, less than in autarchy for $\alpha_x = \alpha$, and monotonically decreasing in between, it follows that there must be a value $\alpha_x^* \in (0, \alpha)$ such that $\beta^*$ is the same as in the autarchic equilibrium.

Figure 1 summarizes. It reports the marginal cost cutoff $\beta^*$ on the vertical axis as a function of the fixed cost of exporting $\alpha_x$ on the horizontal axis. For $\alpha_x \geq \bar{\alpha}_x$, fixed costs of exporting are prohibitively high and $\beta^*$ is therefore constant at the autarchic level $\beta^*_A$. For $\alpha_x \in (\alpha_x^*, \bar{\alpha}_x)$, $\beta^* < \beta^*_A$ and thus trade opening between two identical countries leads to a selection effect. Trade has instead an anti-selection effect for $\alpha_x \in [0, \alpha_x^*)$, as $\beta^* > \beta^*_A$.

Figure 1 - Firm Selection with IES Preferences and Fixed Costs of Exporting

24
in this range and reaches a maximum at $\beta^*_F$ for $\alpha_x = 0$, i.e., in free trade. Finally, note that a fall in the fixed costs of exporting has a non-monotonic impact on $\beta^*$, as it leads to a selection effect when the initial value of $\alpha_x$ is high (i.e., for $\alpha_x > \alpha$), and to an anti-selection effect when the initial level of $\alpha_x$ is low (i.e., for $\alpha_x \leq \alpha$). The selection effect is therefore strongest for $\alpha_x = \alpha$, namely, when fixed costs of exporting leave the overall ratio of fixed costs ($\alpha + \alpha_x$) to market size $L'$ unchanged relative to autarchy. Notice that with DES preferences Figure 1 would exhibit a similar profile, but with an intercept on the vertical axis equal to $\beta^*_F < \beta^*_A$. That is, trade opening would produce a selection effect for any value of $\alpha_x < \pi_x$.

Consider, finally, how markups are affected by trade opening in the presence of fixed costs of exporting. We know that markups equal the autarchic level for $\alpha_x = \pi_x$, the free trade level for $\alpha_x = 0$, and that with IES (DES) preferences they are higher (lower) in free trade than in autarchy according to Proposition 3. Moreover, for $\alpha_x \in [0, \alpha]$, a rise in the fixed costs of exporting is equivalent to a rise in the fixed production cost in a closed economy of size $L' = 2L$. It is therefore pro-competitive (anti-competitive) according to Proposition 3 with IES (DES) preferences. Finally, when $\alpha_x \in [\alpha, \pi_x]$, a rise in the fixed costs of exporting affects individual consumption only by increasing $\beta^*$. Therefore, according to Lemma 2, individual consumption rises and this leads to lower (higher) markups with IES (DES) preferences. The model thus implies that markups are monotonic in the fixed costs of exporting.

We record our main results in the following

**Proposition 4** a) In the presence of fixed costs of exporting $\alpha_x$, trade opening between two identical countries induces an anti-selection effect for low values of $\alpha_x$ and IES preferences, and a selection effect otherwise; b) A sufficient condition for the selection effect to hold is that fixed costs of exporting induce a partitioning of firms by export status; c) The marginal cost cutoff $\beta^*$ is non-monotonically related to fixed costs of exporting, as it is first decreasing and then increasing in $\alpha_x$; d) With IES (DES) preferences, markups are monotonically decreasing (increasing) with respect to $\alpha_x$.

To summarize, in this section we have shown that, with heterogenous firms, IES preferences imply a trade-induced anti-selection effect, in addition to the anti-competitive effect discussed earlier. Although this anti-selection mechanism always weakens the selection effect à la Melitz, in our setup it prevails only for low (or zero) fixed costs of exporting, i.e., when the latter are insufficient to induce a partitioning of firms according to their export status. Yet, such a partitioning seems empirically omnipervasive, and hence our results

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20 With CES preferences $\beta^*_F = \beta^*_A$ (i.e., $\alpha^*_x = 0$).
are not inconsistent with the evidence in support of Melitz-type trade-induced selection effects.\footnote{See, e.g., the works cited in Section 2 of Redding (2011)'s recent survey on the heterogeneous-firms literature.}

5 Conclusion

In this paper we have first shown that, relaxing the assumption of a constant elasticity of substitution (CES preferences) in an otherwise standard Dixit-Stiglitz monopolistic competition framework raises the following dilemma: a pro-competitive effect of trade opening holds only if the substitutability across varieties is decreasing in the level of individual consumption (DES preferences), an assumption at odds with introspection. Yet, under the more plausible assumption that the elasticity of substitution is increasing in individual consumption (IES preferences), the model yields that trade opening is anti-competitive, an implication at odds with the conventional wisdom. Moreover, DES preferences yield that market size expansion is anti-competitive when due to productivity growth, an implication that is hard to rationalize (IES preferences imply just the opposite).

Next, we have explored competitive effects in alternative monopolistic competition settings and found the following results. First, we have shown that when preferences are quasi-linear and quadratic, a prominent example of non-additive preferences recently popularized by Melitz and Ottaviano (2008), a trade-induced increase in the number of firms is ceteris paribus anti-competitive, which implies that the pro-competitive effect of trade opening delivered by this model is entirely driven by the implicit assumption that the substitutability across varieties is decreasing in individual consumption. Second, we have shown a pro-competitive effect does not naturally arise, either, in Lancaster’s (1979) ideal variety approach to monopolistic competition, in which a trade-induced increase in the number of firms, by crowding the variety space, makes varieties closer to one another. Third, we have shown that a pro-competitive effect can plausibly arise in the presence of strategic interactions, but only when the number of firms is small, which is clearly at odds with monopolistic competition. Finally, we have shown how competitive effects interact with selection effects in the presence of heterogeneous firms. Here our most important result is that, provided that fixed costs of exporting are large enough to induce a partitioning of firms by export status, selection effects à la Melitz (2003) hold independent of whether preferences are DES, IES or CES.

The main conclusion we draw from our theoretical analysis is that, given our current ignorance of how to plausibly and parsimoniously model the effects of competition on markups in a monopolistic competition setting, departing from CES preferences, which
overlook these effects, may turn out to be a move in the wrong direction. We view this warning as the most important message of our paper. On the positive side, however, the main message from our analysis is that trade-induced Melitz-type selection effects are robust to the assumptions about preferences, possibly suggesting that in monopolistic competition the latter are first order relative to pro-competitive effects.

6 Appendix

6.1 Demand Elasticity and Elasticity of Substitution

Consider a setting in which consumers share the same preferences, represented by the utility function \( U(c_0, u(c_0)) \), where \( c_0 \) is consumption of a numeraire good, \( c_{-0} = [c_1, c_2, ..., c_n] \) is consumption of \( n \) varieties of some good, and \( u(\cdot) \) is symmetric and concave in its arguments. Denote the associated system of compensated (Hicksian) demand by \( \tilde{c} = [\tilde{c}_0, \tilde{c}_{-0}] = \tilde{c}(p, U) \), where \( p = [p_0, p_{-0}] \) is the price vector. Moreover, denote by \( \tilde{c}_{ij} = \partial \tilde{c}_i / \partial p_j \) \((i, j = 0, 1, ..., n)\) the compensated demand-price derivatives, and by \( \tilde{e}_{ij} = \partial \ln \tilde{c}_i / \partial \ln p_j \) the corresponding elasticities. The Morishima’s\(^{22}\) elasticity of substitution between goods \( i \) and \( j \) \((i \neq j)\) is given by

\[
\sigma_{ij} = \tilde{e}_{ji} - \tilde{e}_{ii},
\]

and measures the substitutability between any two goods at a given consumption vector \( c = [c_0, c_{-0}] \). Specifically, \( \sigma_{ij} \) measures how the marginal rate of substitution between \( i \) and \( j \), \( MRS_{ij} = \frac{\partial U / \partial c_i}{\partial U / \partial c_j} \), varies with the consumption ratio \( c_i / c_j \).

That the elasticity of substitution is a key ingredient of the demand elasticity can be seen by manipulating the Slutsky equation, which decomposes the price effect on demand into a substitution and an income effect:

\[
c_{ij} = \tilde{c}_{ij} - c_{iY}c_j,
\]

where \( c_i(p, Y) \) is the Marshallian (uncompensated) demand for commodity \( i \), \( Y \) is income, \( c_{ij} = \partial c_i / \partial p_j \) and \( c_{iY} = \partial c_i / \partial Y \). When expressed in elasticity terms, (58) implies:

\[
\varepsilon_{ii} = \tilde{e}_{ii} - \varepsilon_{iY} \theta_i, \quad \varepsilon_{ji} = \tilde{e}_{ji} - \varepsilon_{jY} \theta_i,
\]

\(^{22}\)See Blackorby and Russell (1989) for a discussion of the elasticity-of-substitution alternative definitions.
where $\varepsilon_{ii} = \partial \ln c_i / \partial \ln p_i$ is the (own-price) demand elasticity of commodity $i$, $\varepsilon_{ij} = \partial \ln c_i / \partial \ln p_j$ is the cross-price demand elasticity of commodity $i$ with respect to commodity $j$, $\theta_i = p_i c_i / Y$ is good $i$’s expenditure share and $\varepsilon_{iY} = \partial \ln c_i / \partial \ln Y$ is good $i$’s income elasticity. Using (57) and (58) yields:

$$\varepsilon_{ii} = -\sigma_{ij} + \varepsilon_{ji} + (\varepsilon_{jY} - \varepsilon_{iY}) \theta_i, \quad i,j = 1, \ldots, n; i \neq j. \quad (60)$$

Thus, the demand elasticity for good $i$ can be written as a function of the elasticity of substitution and the cross-price elasticity with respect to another good $j$ and the difference between the corresponding income effects, $(\varepsilon_{jY} - \varepsilon_{iY}) \theta_i$. This latter term disappears if $U(\cdot)$ is homothetic (as in this case $\varepsilon_{jY} = \varepsilon_{iY} = 1$) or quasi-linear with respect to the numeraire (as in this case $\varepsilon_{jY} = \varepsilon_{iY} = 0$), or if $p_i = p_j$ (as in this case $\varepsilon_{jY} = \varepsilon_{iY}$ by symmetry of the preferences over $\mathbf{c}_{-0}$). More generally, unless either $c_i$ or $c_j$ is disproportionate, $(\varepsilon_{jY} - \varepsilon_{iY}) \theta_i$ is an order of magnitude smaller than $1/n$.

The expression in (60) drastically simplifies when there is no numeraire (as in Krugman (1979) and in our baseline setting) and $p_i = p$ for any $i$, which implies a symmetric consumption pattern (i.e., $c_i = c$ for any $i$). In this case, $\bar{\varepsilon}_{ii} = -(n - 1)\bar{\varepsilon}_{ij}$, $\bar{\varepsilon}_{ij} = \sigma_{ij}/n$, $\varepsilon_{ji} = (\sigma_{ij} - 1)/n$ and

$$\varepsilon_{ii} = -\frac{n - 1}{n} \sigma_{ij} - \frac{1}{n}. \quad (61)$$

Note that $-1/n$ in (61) accounts for the income effect, which vanishes for $n$ large. The same simple expression holds in the presence of a numeraire, provided that preferences are Cobb-Douglas with respect to the numeraire, i.e., $U(c_0, u(c_{-0})) = c_0^\alpha u(c_{-0})^{1-\alpha}$, where $0 < \alpha < 1$ and $u(\cdot)$ is homogeneous.\(^{24}\)

Another simple case arises when preferences are quasi-linear, i.e., if $U(c_0, u(c_{-0}) = c_0 + u(c_{-0})$, a specification discussed in Section 3.1. In this case there are no income effects in the demand for non-numeraire goods, hence $\varepsilon_{ji} = \bar{\varepsilon}_{ji} = - (\sigma_{ij} + \bar{\varepsilon}_{i0})/n$ and

$$\varepsilon_{ii} = -\frac{n - 1}{n} \sigma_{ij} - \frac{\bar{\varepsilon}_{i0}}{n}. \quad (62)$$

Note, from (61) and (62), that $n$ directly affects both the income and the substitution effect, and that $\varepsilon_{ii} \approx -\sigma_{ij}$ for $n$ large.\(^{25}\)

Turning to the elasticity of substitution, a general result is that, if $u(\cdot)$ is additive, i.e.,

\(^{23}\)For expositional purposes, in this Appendix it prove convenient to denote the demand elasticity by $\varepsilon_{ii} = \partial \ln c_i / \partial \ln p_i$, rather than by $\varepsilon_i = \partial \ln c_i / \partial \ln p_i$ as in the main text.

\(^{24}\)The reason is that in this case the income and substitution effects partially cancel out (i.e., $\sigma_{0i} = \varepsilon_{i0Y} = 1$, see the footnote immediately below).

\(^{25}\)It is possible to show that, for a symmetric consumption pattern over $\mathbf{c}_{-0}$, (61) and (62) are special cases of the general expression in (60).
if \( u(c_{-0}) = \sum_{i=1}^{n} u(c_i) \), then \( c_i = c_j \ (i, j = 1, \ldots, n, i \neq j) \) implies:

\[
\sigma_{ij} = -\frac{u'(c_i)}{u''(c_i)c_i}. \tag{63}
\]

To see this, note that differentiating the first-order conditions

\[
p_i = \mu U_u(c_0, \sum_i u(c_i))u'(c_i),
\]

where \( \mu \) is the relevant Lagrangian multiplier and \( U_u = \partial U/\partial u \), yields:

\[
1 = \mu_i U_u(c_i) + \mu U_u u''(c_i)\tilde{c}_{ii} + \mu u'(c_i) \left[ U_{u0} \tilde{c}_{0i} + \sum_{h=1}^{n} u'(c_h)\tilde{c}_{hi} \right], \tag{64}
\]

\[
0 = \mu_j U_u(c_i) + \mu U_u u''(c_i)\tilde{c}_{ij} + \mu u'(c_i) \left[ U_{u0} \tilde{c}_{0j} + \sum_{h=1}^{n} u'(c_h)\tilde{c}_{hj} \right],
\]

where \( U_{u0} = \partial^2 U/\partial u\partial c_0 \), \( U_{uu} = \partial^2 U/\partial u^2 \) and \( \mu_i = \partial \mu_i/\partial p_i \). Subtracting the second expression from the first, exploiting the symmetry of price effects implied by the compensated demand functions (i.e., \( \tilde{c}_{ij} = \tilde{c}_{ji} \)) and manipulating yields:

\[
1 = p_i \left[ \frac{\mu_i - \mu_j}{\mu} + \frac{U_{u0}}{U_u} (\tilde{c}_{0i} - \tilde{c}_{0j}) + \frac{U_{uu}}{U_u} \sum_{h=1}^{n} u'(c_h)(\tilde{c}_{hi} - \tilde{c}_{hj}) \right] + \frac{u''(c_i)c_i}{u'(c_i)} (\tilde{c}_{ii} - \frac{c_j}{c_i} \tilde{c}_{jj}). \tag{64}
\]

Note that, for \( p_i = p_j \), then, due to symmetry of preferences over \( c_{-0}, c_i = c_j, \mu_i = \mu_j, \tilde{c}_{hi} = \tilde{c}_{hj} \ (h = 0, \ldots, n, i \neq h \neq j) \) and \( \tilde{c}_{ii} = \tilde{c}_{jj} \). Thus (64) boils down to (63).

### 6.2 Demand in Lancaster’s Ideal Variety Approach

We now derive the aggregate demand function, \( q(\omega) \), for a firm selling variety \( \omega \) at the price \( p(\omega) \), with contiguous competitors \( \omega_l \) and \( \omega_r \) charging prices \( p(\omega_l) \) and \( p(\omega_r) \). The clientele of firm \( \omega \) is a compact set ranging from \( \omega \) to \( \bar{\omega} \), where \( \omega \) and \( \bar{\omega} \) are the locations of consumers just indifferent between \( \omega \) and \( \omega_l \), and between \( \omega \) and \( \omega_r \). The values of \( \omega \)

\[
epsilon_{ii} = -\frac{n-1}{n} \sigma_{ij} - \frac{1}{n} \left[ 1 + \frac{c_0}{\bar{\omega}_0} (\sigma_{0i} - \varepsilon_{0Y}) \right].
\]

Thus, the result that \( \varepsilon_{ii} \approx -\sigma_{ij} \) for \( n \) large holds for a wide class of utility functions.

As is well known, if in addition preferences over varieties are CES, i.e., if \( u(c_i) = c_i^\rho/\rho \ (i = 1, \ldots, n, 0 \leq \rho \leq 1) \), then the elasticity of substitution does not even depend on the consumption level \( c_i \), as it is constant and equal to \( 1/\rho \).
and $\varpi$ are therefore implicitly defined by:

$$
\begin{align*}
\quad p(\omega_l)h(\delta(\omega_l, \omega)) &= p(\omega)h(\delta(\omega, \omega)), \\
\quad p(\omega_r)h(\delta(\omega_r, \varpi)) &= p(\omega)h(\delta(\omega, \varpi)).
\end{align*}
$$

(65)

Denote by $d^* = \delta(\omega_l, \omega_r)$ the distance between firm $\omega$’s competitors, by $d = \delta(\omega_l, \omega)$ the distance between $\omega_l$ and $\omega$, and by $\overline{d} = \delta(\omega_r, \omega)$ and $\overline{d} = \delta(\varpi, \omega)$ firm $\omega$’s distance from its marginal consumers. It follows that $\delta(\omega_l, \omega) = d - \overline{d}$, and $\delta(\omega_r, \varpi) = d^* - d - \overline{d}$.

Substituting into (65) yields:

$$
\begin{align*}
\quad p(\omega_l)h(d - \overline{d}) &= p(\omega)h(d), \\
\quad p(\omega_r)h(d^* - d - \overline{d}) &= p(\omega)h(\overline{d}).
\end{align*}
$$

(66)

A firm’s market width is obtained by inverting the above implicit conditions:

$$
\begin{align*}
\quad d &= \delta(p(\omega), p(\omega_l), p(\omega_r), d^*, d), \\
\quad \overline{d} &= \overline{\delta}(p(\omega), p(\omega_l), p(\omega_r), d^*, d).
\end{align*}
$$

(67)

The aggregate demand for firm $\omega$ is therefore given by:

$$
q(\omega) = \left[\delta(\cdot) + \overline{\delta}(\cdot)\right] c(\omega) L = \left[\delta(\cdot) + \overline{\delta}(\cdot)\right] \frac{L}{p(\omega)}.
$$

(68)

Implicit differentiation of the two-equation system in (66) yields:

$$
\begin{align*}
\frac{\partial \delta(\cdot)}{\partial p(\omega)} &= -\frac{h(d)}{p(\omega_l)h'(d - \overline{d}) + p(\omega)h'(d)} , \\
\frac{\partial \overline{\delta}(\cdot)}{\partial p(\omega)} &= -\frac{h(\overline{d})}{p(\omega_r)h'(d^* - d - \overline{d}) + p(\omega)h'(\overline{d})}.
\end{align*}
$$

(69)

Note that, in a symmetric equilibrium, $p(\omega_l) = p(\omega_r) = p(\omega)$, $d = \overline{d} = d - \overline{d} = d^* - d - \overline{d} = \frac{d}{4}$, and $n = 1/d$. Substituting into (68) and (69) yields:

$$
\begin{align*}
\frac{\partial \delta(\cdot)}{\partial p(\omega)} &= -\frac{h(\frac{d}{4})}{2p(\omega)h'(\frac{d}{4})} , \\
\frac{\partial q(\omega)}{\partial p(\omega)} &= \left[\frac{\partial \delta(\cdot)}{\partial p(\omega)} + \frac{\partial \overline{\delta}(\cdot)}{\partial p(\omega)}\right] \frac{L}{p(\omega)} - \left[\delta(\cdot) + \overline{\delta}(\cdot)\right] \frac{L}{p(\omega)^2}
\end{align*}
$$

$$
= -\frac{h(\frac{d}{4})}{p(\omega)h'(\frac{d}{4})} \frac{L}{p(\omega)} - \left[\delta(\cdot) + \overline{\delta}(\cdot)\right] \frac{L}{p(\omega)^2}.
$$

thus leading to the expression for the demand elasticity $\varepsilon(q(\omega))$ reported in the main text.
References


