Bailouts and Bank Runs: Theory and Evidence from TARP

Chunyang Wang*

Peking University

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Abstract

During the recent Financial Crisis, there were bank runs right after government bailout announcements. This paper develops a global game model of information based bank runs to analyze how the announcement of bailouts affects investors’ bank run incentives. The equilibrium probability of bank runs is uniquely determined. I conclude that before the announcement, the existence of such bailout policy reduces investors’ bank run incentives, but after the announcement, investors may run on the bank, since such an announcement reflects the government’s information about the bad bank asset. The empirical evidence from TARP is consistent with my theory.

Keywords: Bailout, Bank Run

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1 Introduction

This paper attempts to answer the question, “how does the announcement of bailouts affect the probability of bank runs?” There are two reasons why I focus on this question. First, this question is important because bank runs and government bailouts became more common during the 2007-2009 financial crisis. After the Great Depression, bank runs rarely happened until the recent crisis. In the years of 2008 and 2009, 165 banks failed. However, only 11 banks failed in the five years before 2008. One notable cause was bank runs. There were runs on large banks such as Northern Rock, Bear Stearns, IndyMac Bank, Washington Mutual, and AIG. The government interventions during this crisis in the banking sector were also the largest in U.S. history. Government interventions are in the form of various bailouts. Second, answering the above question is not a trivial matter because even though the stated goal of these interventions is to “restore confidence to our financial system” several bank runs happened right after their bailout announcements. Below I will present some evidence that supports this statement.

In his description of bank runs during the 2007-2009 crisis, Markus K. Brunnermeier (Brunnermeier (2009)) wrote

“The...on March 11 [2008], ... the Federal Reserve announced its $200 billion Term Securities Lending Facility. This program allowed investment banks to swap agency and other mortgage-related bonds for Treasury bonds for up to 28 days.... Naturally, they (market participants) pointed to the smallest, most leveraged investment bank with large mortgage exposure: Bear Stearns.......Bear’s liquidity situation

1A bank run occurs when a massive number of depositors withdraw their deposits from the bank because they believe the bank might be insolvent. I will define bank runs more precisely later in the model.
2Bank runs are also called debt runs for non-bank financial institutions, which use lots of short term debt contracts such as commercial paper and repo transactions. Debt runs happen when a large number of debt holders stop rolling over their debts.
3The failed bank list can be access at the Federal Deposit Insurance Corporation (FDIC) website, http://www.fdic.gov/bank/individual/failed/banklist.html.
4For a detailed description of these runs, please refer to Brunnermeier (2009).
5In this paper, for simplicity, I define banks as all the financial institutions, including commercial banks, investment banks, hedge funds, and insurance companies.
6AIG faced “margin runs” as described by Gorton (2008).
worsened dramatically the next day as it was suddenly unable to secure funding on the repo market.

The bank run on Bear Stearns is different from the traditional bank runs where depositors run to the bank to withdraw their deposits. The run on Bear Stearns occurred when hedge funds, which place their liquid wealth with their prime brokers, retrieved those funds (Brunnermeier (2009)). In spite of the different style, the runs on these non-bank financial institutions are the same as the traditional bank runs, in the sense that financial institutions have to execute a costly liquidation of their investment to repay their debt. In this example the announcement of government interventions preceded a bank run. In this paper, bailouts take the form of capital injection or government loans before banks go bankruptcy. I view arrangements after bank runs are less important than capital injection ex ante, since costly inefficient liquidation has already happened after bank runs. Although the bailout of Bear Stearns often stands for its sale to JP Morgan arranged by NY FED, this was done after the bank run. The bailout after bank runs was intended for avoiding financial contagion as banks are interconnected.

Another example associating bank runs with bailout announcements is the case of Northern Rock. Before the bank run, Northern Rock was the fifth-largest bank in the United Kingdom by mortgage assets. The most recent U.K. bank run before Northern Rock was in 1866 at Overend Gurney. The run on Northern Rock is examined in detail by Shin (2009). He wrote “On September 13, 2007, the BBC’s evening television news broadcast first broke the news that Northern Rock had sought the Bank of England’s support. The next morning, the Bank of England announced that it would provide emergency liquidity support. It was only after that announcement—that is, after the central bank had announced its intervention to support the bank—that retail depositors started queuing outside the branch offices.” The run on Northern Rock was a classic bank run where depositors queued in a line in front of their banks to withdraw their deposits.

After the U.S. Treasury Secretary Henry Paulson proposed his $700 billion bailout plan...
on September 20, 2008, Washington Mutual experienced a 10-day online withdrawal, or a so-called “silent bank run”, which led to the largest bank failure in U.S. history. Depositors withdrew about $16.4 billion. Almost in the same period, Wachovia, which was the fourth-largest bank holding company in the United States, suffered a bank run as well, where large depositors withdrew their deposits, which led to its takeover by Wells Fargo. Some of the banks which were in the Troubled Asset Relief Program (TARP), chose to return the bailout money afterwards by claiming they were forced to take the money. For example, Jason Korstange, a spokesman for Minnesota-based TCF Financial Corp., who received 361 million dollars, announced that the bank wanted to pay it back. “The perception is that any bank that took this money is weak. Well, that isn’t our case. We were asked to take this money.”

All of the above facts and examples contradict the stated goal of government interventions to “restore confidence”. For these cases, bailouts did not prevent but triggered bank runs. In this paper, I will analyze the effect of bailout announcements on the probability of bank runs and provide evidence to support my arguments.

I consider the following environment. Investors put their funds in a bank. The bank invests these funds in an asset. The quality of a bank’s asset is random. Investors choose whether to withdraw their investments early or to wait until the asset is mature. The liquidation of the investment is costly. The government and investors receive private noisy signals of the quality of the bank’s asset after its realization. Government bails out a bank in the form of capital injection only if its signal is below some cutoff threshold, i.e., the government helps a bank that is in trouble.

The model builds on the work of Goldstein and Pauzner by adding the government sector. Building on the classic bank run model by Diamond and Dybvig, which is unsatisfactory for policy analysis because of its multiple equilibrium, “run” and “no run”, Goldstein and Pauzner utilize the global game technique to achieve a unique equilibrium by assuming information dispersion as stated in the above environment. Goldstein and Pauzner

\(^{10}\) http://articles.latimes.com/2009/mar/14/business/fi-tainted-money
\(^{11}\) I will call the return of the investment in long term technology, quality of the asset held by the bank, and the bank fundamental interchangeably below.
\(^{12}\) I also call these actions “run” or “no run” on the bank.
combine the coordination failure approach for bank runs (Diamond and Dybvig (1983)) with the fundamental approach (Chari and Jagannathan (1988), Allen and Gale (1998)). The former argues that investors run on a bank because investors believe others will run on the bank. The latter argues that investors run on a bank because they perceive the bank is bad. The unique equilibrium achieved by Goldstein and Pauzner (2005) is characterized as follows. Investors whose signals about the bank fundamental are below a cutoff choose to run on the bank, otherwise not. Banks are fragile in the sense that a small change in fundamentals can lead to a large change in bank run outcomes. During the Subprime Mortgage Crisis, the subprime mortgage only accounted for 12% of the outstanding US mortgage market, which was a small part of banks’ assets. But it still had a tremendous effect on the financial system. This fact provides a real world example to use the global game model.

Addressing my question at the beginning of this paper, “How does the announcement of bailouts affect the probability of bank runs?”, I answer that the announcement of bailouts may increase the probability of bank runs. I use the word “may” because there are two effects. The first one is the capital injection effect in the sense that money transfer to a bank reduces the probability of bank runs. The second is the signaling effect, i.e., the announcement signals the government’s information that the bank’s asset quality is low, which increases the probability of bank runs. The total effect depends on the magnitude of the two separate effects. The model implies that the probability of bank runs after bailout announcements depends on three factors. First, the probability of a bank run is higher if the bailout amount is smaller. The government providing any positive bailout amount has a constant information effect no matter

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13See Weaver (2008) for more details.

14The root of the banking crisis this time is different from the one during the Great Depression. Fundamentals played a crucial role in triggering banking panics this time. In his famous speech, Bernanke said to Milton and Anna, “Regarding the Great Depression. You’re right, we did it. We’re very sorry. But thanks to you, we won’t do it again.” For more details on their work, please refer to Friedman and Schwartz (1971). The Fed policy this time was thus based on the experience from the Great Depression, according to Bernanke, so that the Fed was flooding the banking system with cash. However, the Fed policy was criticized again by Anna Schwartz. In an interview with Anna Schwartz, it states “But ‘that’s not what’s going on in the market now,’ Ms. Schwartz says. Today, the banks have a problem on the asset side of their ledgers – ‘all these exotic securities that the market does not know how to value.’” Therefore, both the fundamental argument and the coordination argument play roles during this crisis. This is why I exploit the global game model in this paper.

15During the Subprime Mortgage Crisis, the Mortgage-Backed Security (MBS) was very hard to value. The government’s action may convey important information to the investors about the value of these assets. Therefore, government bailouts, which alleviate the liquidity problem, may cause the situation to worsen by reflecting the bad quality of banks’ assets.
what the bailout amount is. But the capital injection effect is stronger for a larger bailout. Second, the probability of a bank run is higher if the government signal is more precise. Once a bailout is announced, for the government with a more precise signal, investors will believe it is more likely that the bank fundamental is below the bailout cutoff. Third, the probability of a bank run is higher when the government uses a lower cutoff. Conditional on a bailout, investors deduce from a lower cutoff that the government believes the bank is worse.

The model also provides insights from an ex ante perspective. I consider two economies. The government in economy A commits never to bail out any banks. The government in economy B uses the above assumed exogenous strategy to bail out banks. The probability of bank runs may increase after the announcement of bailouts. However, ex ante, i.e., before the announcement of bailouts, the probability of bank runs in economy B is lower than the probability of bank runs in economy A, because the capital injection effect dominates in economy B. Signaling does not impact the investors’ bank run incentives since the government has not announced yet whether the bank will be bailed out or not. Another insight is about transparency, defined here as the precision of government signal. After the announcement of bailouts, more transparency increases the probability of bank runs as discussed above. Before the announcement of bailouts, more transparency reduces the probability of bank runs, because the policy mistakes, defined as bailing out banks with good fundamentals and not bailing out banks with bad fundamentals, are less frequent.

I provide evidence to support the theory by studying government bailouts and bank runs during the 2008-2009 Financial Crisis. The bailouts are from the $700 billion TARP, which enables the Treasury to purchase assets and equity from financial institutions. The program was proposed by the then U.S. Treasury Secretary Henry Paulson on September 20, 2008, passed by the Congress on October 3, 2008, and signed into law by President Bush on the same day. The original plan was revised on October 14th to include debt guarantee and other more specific details. The names of the banks that received bailouts were not announced until their own individual bailout announcements. I use two methods to measure the probability of bank runs. The two methods complement each other with their distinctive advantages. The first approach follows from the work of Veronesi and Zingales (2010), which constructs a bank
run index by using the Credit Default Swap (CDS) rates. A larger bank run index indicates a higher probability of bank runs. I find that after Paulson’s bailout proposal, bank run indices for some banks among the eight biggest recipients of bailouts increased dramatically. The indices stayed very high even after the Congress passed the bailout bill. The indices fell steeply after the announcement of the revised Paulson plan that included debt guarantee, which functions as deposit insurance.

The CDS rates are only available for a very limited number of banks. In order to measure the probability of bank runs more extensively, I adopt the second approach to include many more banks. There are 241 banks in the data. I conduct event studies of bank stock price reactions in response to the bailout announcements. Abnormal returns are used as a proxy for the probabilities of bank runs. An abnormal return, which is triggered by events, is the difference between the actual return and the expected return of a security. The theoretical reason that enables abnormal returns to be exploited as a proxy for the probability of bank runs works as follows. The classic bank runs or debt runs can be extended to the equity holders, such as investors in a hedge fund or mutual funds (Shleifer and Vishny (1997); Brunnermeier (2009)). There is an early-mover advantage for fund managers to sell liquid assets first. This first-mover advantage can make financial institutions, not only banks, subject to runs (Brunnermeier (2009)). See Cheng and Milbradt (2010) for an attempt to add the equity holders in the global game bank run model. A higher abnormal return indicates a lower probability of bank runs. A positive abnormal return indicates a reduction of the probability of bank runs, compared to the case when there is no bailout announcement. Consistent with the theory, I find that banks displayed significant negative abnormal returns after their own individual bailout announcements, and the abnormal return was positively correlated with the bailout ratio, defined as the bailout amount divided by its total asset. However, the abnormal returns after the recipient unspecified announcement of TARP were significantly positive, which indicates that ex ante, the existence of bailouts reduced the probability of bank runs.

The rest of the paper proceeds as follows. Section 2 is related literature. The benchmark model is introduced in Section 3. A full model with an exogenous government bailout policy is presented in Section 4. Section 5 provides empirical evidence for the theoretical arguments.
2 Related Literature

The model in this paper is most closely related to the work of Goldstein and Pauzner (2005) who apply the global game technique to the classic bank run model, i.e., the seminal work of Diamond and Dybvig (1983), to select a unique equilibrium. I add a government sector to their model to examine how government bailout announcements affect a bank’s risk of experiencing runs. Strategic complementarity in the coordination literature was plagued with multiple equilibrium. Global games are games of incomplete information where each player observes a noisy signal of the underlying state. This information dispersion in the strategic complementarity setting is used to generate a unique equilibrium. Morris and Shin (1998) start the global game application to currency crisis by building on the pioneering theoretical work of Carlsson and Van Damme (1993). Other applications include bank runs (Goldstein and Pauzner (2005) and Rochet and Vives (2004)), debt rollover (He and Xiong (2009)), reputation (Chari, Shourideh, and Zetlin-Jones (2010)), liquidity crash (Morris and Shin (2004)) and political riots (Atkeson (2000) and Edmond (2007)). See Morris and Shin (2003) for a recent survey. Technically, the unique equilibrium selection by Goldstein and Pauzner (2005) is different from other literature, because the global complementarity property among agents is not satisfied in the bank run model. Global games with signaling in a currency crisis context have been analyzed by Angeletos, Hellwig, and Pavan (2006). Their work has an unappealing multiple equilibrium feature by considering the feedback effect from investors’ behavior to that of the policy maker, while I obtain a unique equilibrium by getting rid of the feedback effect, since in this paper I put more emphasis on the policy analysis, which needs a unique equilibrium.

Recently, motivated by the recent financial crisis, there is a small but growing body of literature on bank bailouts (Keister (2010); Chari and Kehoe (2010); Green (2010)). The work by Keister (2010) is closest in theme with mine in studying bailouts and bank runs. In an environment with limited government commitment, he argues that the anticipation of
a bailout can have positive ex ante effects. First, bailouts are part of an efficient insurance arrangement between government and private consumption. Second, the anticipation of a bailout decreases the potential loss an investor faces if she does not withdraw her funds. The second effect coincides with the argument in this paper about the ex ante benefits of bailouts in reducing the probabilities of bank runs. Chari and Kehoe (2010) show that in the absence of commitment to not bailing out banks ex post, ex ante regulation of private contracts will be more efficient. Green (2010) argues that bailing out an insolvent corporate sector in some states of the world is essential to implementing efficient investment in a limited-liability regime.

For the empirical part of this paper, I use stock price abnormal returns as a proxy for the probability of bank runs. According to the survey by de Bandt and Hartmann (2000), the most popular approach to test the contagion effect is event studies of bank stock price reactions in response to announcements. Calomiris and Mason (1997) conclude that stock price returns are a good predictor of bank failure after examining the banking crises data during the Great Depression. Veronesi and Zingales (2010) construct an index for bank runs, and conclude that the advantage from the revised Paulson plan is the cost effectiveness of the debt guarantee, after calculating the increased benefits to banks of the original and revised Paulson plan respectively. Andritzky, Jobst, Nowak, Ait-Sahalia, and Tamirisa (2009) study the effect of bailing out an individual firm on the financial market during the recent crisis, and find that bailing out individual banks in an ad hoc manner had adverse repercussions, both domestically and abroad. Using the same event study methodology as mine, Goldsmith-Pinkham and Yorulmazer (2010) find that both the run on Northern Rock and the subsequent bailout announcement had a significant effect on the rest of the U.K. banking system.

3 Benchmark Model without Information Dispersion

The benchmark model without information dispersion is a variation of the Diamond-Dybvig economy (Diamond and Dybvig (1983)). The environment I introduce will be used throughout the paper. In this section I will show that the model has two equilibria, which cause policy

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16 The contagion effect is defined as the effect of one bank failure on the failure risk of other banks.
analysis impossible.

3.1 The Economy

There are three periods, $t = 0, 1, 2$, one homogeneous good, and a continuum $[0, 1]$ of agents. Each agent has an endowment of one unit. They enter the economy in Period 0, and consumption happens in either Period 1 or 2, denoted by $c_1$ and $c_2$. The timing for consumption depends on their types. Agents do not know their types until period 1. There are two types of agents, patient and impatient. Patient agents consume in both periods with perfect substitution, while impatient agents consume only in Period 1. The fraction of impatient agents is $\lambda$. So the probability for an agent to become patient agents in Period 1 is $1 - \lambda$. The utility functions for patient and impatient agents are $u(c_1 + c_2)$ and $u(c_1)$, respectively. The utility function, which is twice continuously differentiable and increasing, satisfies $u(0) = 0$ and relative risk aversion coefficient, $-cu''(c)/u'(c) > 1$ for any $c \geq 1$.

Agents can invest their endowments in Period 0 in an asset which in Period 2 yields $R$ with probability $\theta$, or 0 with probability $1 - \theta$, where $R > 1$, and $\theta$, the underlying fundamental of the asset, which determines the expected asset return, is distributed uniformly on $[0, 1]$. I assume $E_\theta[u(R)] > u(1)$ so that the expected long term return is higher than the short term return. If the asset is liquidated in Period 1, it will yield just one unit for any one unit of input.

3.2 Banks and Contract

Banks are financial intermediations which invest agents’ endowments in the asset mentioned above. Banks play the role of risk sharing by issuing demand deposit contract as illustrated in [Diamond and Dybvig (1983)]. The contract is specified as follows. Each agent deposits her endowment in Period 0. If she demands withdrawal in Period 1, she is promised a fixed payment, $r > 1$.[17] Sequential service constraint is satisfied, i.e., a bank pays $r$ to agents until

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[17] If considering the optimal risk sharing between patient and impatient agents, $r$ is set by $r = c^{FB}_1$, where $c^{FB}_1$ is determined by the optimal contract as follows.
it runs out of resources. If she waits to withdraw in Period 2 after the asset matures, she is
paid with the leftovers divided by the number of agents who remain (withdraw in Period 2).\textsuperscript{18}
So the payment in Period 2 will be a stochastic number $\tilde{r}$.

$s$ is an indicator to denote the strategies of the agents in Period 1. Actions for patient
agents

$$s = \begin{cases} 
1 \text{ if "withdraw"} \\
0 \text{ if "wait"}
\end{cases}$$

For impatient agents, their action is always to withdraw, since they only care about the first
period consumption.

The payment to the agents in Period 2, $\tilde{r}$, is depicted in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Ex Post Payments to Agents</th>
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<tbody>
<tr>
<td>Period</td>
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</table>
| 1      | $\tilde{r} = r$    | $\tilde{r} = \begin{cases} 
    r \text{ w. prob } \frac{1}{nr} \\
    0 \text{ w. prob } 1 - \frac{1}{nr}
\end{cases}$ |
| 2      | $\tilde{r} = \begin{cases} 
    \frac{1 - nr}{1 - n} R \text{ w. prob } \theta \\
    0 \text{ w. prob } 1 - \theta
\end{cases}$ | $\tilde{r} = 0$ |

where $n$ is the number of investors who demand early withdrawal in Period 1.

As demonstrated by Diamond and Dybvig (1983), there are two equilibria in this model.

\[ \max_{c_1} \lambda u(c_1) + (1 - \lambda)u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right)E_\theta[\theta] \]

The first order condition is

\[ u'(c_1^{FB}) = Ru'(\frac{1 - \lambda c_1^{FB}}{1 - \lambda} - R)E_\theta[\theta] \]

\textsuperscript{18}If I add equity to the model, then part of the matured return will go to the equity holders. This is a
justification for the empirical methodology where abnormal returns are used as a proxy for the probability of
bank run. $\theta$ determines the equity returns and debt returns.
One is “no bank run” equilibrium where $n = \lambda; s = 0$. The other one is “bank run” equilibrium where $n = 1; s = 1$. Bank runs are driven by coordination motives. The optimal action for an agent is to run if others run, and the optimal action for an agent is to wait if others wait. The model with such multiple equilibria feature is unsuitable for policy analysis, since it is impossible to address the effect of bailouts on the probability of bank runs. To achieve a unique equilibrium, I utilize the global game technique by adding information dispersion to the model.

4 Model with Information Dispersion and Government Bailout

I now turn to the full model with information dispersion and government bailout where information dispersion indicates that every agent observes a noisy signal of the asset quality after the quality is realized. Government bailout is in the form of capital injection. I will show that assuming information dispersion is essential to attain a unique equilibrium. The unique equilibrium makes it feasible to undertake comparative statics to determine how various factors affect the probability of bank runs.

4.1 Timing

\begin{itemize}
  \item \textit{t} \\
  0 \hspace{0.5cm} \text{Deposit her endowment} \\
  1 \hspace{0.5cm} \text{Promised a fixed payment $r$ if withdrawing } \\
  \hspace{1cm} \theta \text{ is realized, not publicly revealed} \\
  1.1 \hspace{0.5cm} \text{The government and investors draw private signals of } \theta \\
  1.2 \hspace{0.5cm} \text{The government announces whether to bail out the bank or not} \\
  1.3 \hspace{0.5cm} \text{Investors update posterior of } \theta \text{ based on government action} \\
  1.4 \hspace{0.5cm} \text{Patient Investors choose to demand early withdrawal or not } \\
  \hspace{1cm} \text{proceeds of the non-liquidated investment divided} \\
  2 \hspace{0.5cm} \text{by the number of remaining investors}
\end{itemize}
As illustrated above, the government announcement is before bank runs and liquidations. The realization of $\theta$ can be viewed as a shock to the bank assets, such as the collapse of housing price.

### 4.1.1 Information

The state of the economy, $\theta$, is realized at the beginning of Period 1. Agents cannot observe $\theta$, but for every $i$, agent $i$ receives a noisy signal about $\theta$, $\theta_i = \theta + \varepsilon_i$, where $\varepsilon_i$ is distributed uniformly over $[-\bar{\varepsilon}, \bar{\varepsilon}]$. The distribution of the signal is public knowledge. This signal is her private signal, while whether she is patient or not is her private information. This information perturbation leads to coordination in the sense that an agent who receives a high signal believes that other agents receive high signals as well, which reduces the probability of bank runs, while an agent who receives a low signal believes that other agents receive low signals as well, which increases the probability of bank runs. Under this information structure, the financial system becomes fragile due to the strategy complementarity among agents.

### 4.1.2 Government

The government tries to help banks which are in trouble, i.e., banks with a low fundamental. Even though sometimes bank runs are efficient for banks with low fundamentals, as argued in Allen and Gale (1998), which causes the government interventions unnecessary, in this paper, there are several reasons to assume such government strategy, such as “Too Big To Fail”\textsuperscript{19}

The government’s objective is to stabilize the financial market by lowering the probability of bank runs. The only instrument for the government is either capital injection or loans to the bank in Period 1. Due to its limited auditing ability, the government does not have perfect information about the bank fundamental. It obtains a noisy signal about the fundamental, $\theta_G = \theta + \eta$, where $\eta$ is distributed uniformly over $[-\bar{\eta}, \bar{\eta}]$. In Period 1, once the government observes the bank’s fundamental is so low that the bank is vulnerable to runs, the government will inject $\bar{B}$ amount of liquidity to the bank. So the government’s strategy takes the following

\textsuperscript{19}Government believes that failure of a “too big” institution will produce massive negative externalities to the economy so that it is more efficient to save it from failure.
form.

\[
\tilde{B} = \begin{cases} 
\bar{B} & \text{if } \theta_G < \theta^*_G \\
0 & \text{if } \theta_G \geq \theta^*_G 
\end{cases}
\]  \hspace{1cm} (1)

where \( \tilde{B} \) is a random variable which takes value \( \bar{B} \) if government’s perception about the bank fundamental is sufficiently low, below some exogenous cutoff threshold \( \theta^*_G \). Otherwise, \( \tilde{B} = 0 \), i.e., the government just leaves the bank untouched. Agents know the above strategy including the value of \( \theta^*_G \), but cannot directly observe \( \theta_G \).

I use \( B \) to denote the event that \( \theta_G < \theta^*_G \), and \( \bar{B} \) to denote the event that \( \theta_G \geq \theta^*_G \).

Under this assumption, conditional on a government bailout announcement, Table 2, which is symmetric to Table 1, depicts the ex post payment to the patient agents. The case without bailout announcement is the same as the one depicted in Table 1.

<table>
<thead>
<tr>
<th>Table 2: Ex Post Payments to Agents</th>
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<tbody>
<tr>
<td>Period</td>
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<tr>
<td>1</td>
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| 2      | \( \tilde{r} = \begin{cases} 
\frac{1+\tilde{B}-nr}{1-n}R & \text{with prob } \theta \\
0 & \text{with prob } 1 - \theta 
\end{cases} \) | \( \tilde{r} = 0 \) |

where \( \tilde{B} \) is the government bailout in Period 1. When \( n < \frac{(1+\tilde{B})}{r} \), I assume that banks invest the bailout money in the asset even though it is not efficient to do so if \( \theta \) is sufficiently small. Such an assumption does not affect the results since the focus in this paper is on “run” or “not run” which happens in Period 1. Despite the fact that investing in the asset with small \( \theta \) increases the probability of a run, the precondition is that \( n < \frac{(1+\tilde{B})}{r} \), i.e., there is no run.

\footnote{The result will still hold if the government announces its observation of the bank fundamental. Investors will put certain weights on their own signal and government signal, where the weight depends on investors’ belief about the government, i.e., whether the government is trustworthy.}
For a given state $\theta$, the incentive to withdraw in Period 2 is

$$v(\theta, n, \tilde{B}) = \begin{cases} 
\theta u(1+\tilde{B}-nr) - u(r) & \text{if } 1+\tilde{B} \geq n \geq \lambda \\
-\frac{1+\tilde{B}}{nr} u(r) & \text{if } 1 \geq n \geq \frac{1+\tilde{B}}{r} 
\end{cases}$$

(2)

$v$ decreases with $n$, only when $n \leq \frac{1+\tilde{B}}{r}$, as drawn in Figure 1.

Figure 1: The net incentive to wait

4.2 Equilibrium

It is necessary for me to describe the information structure before I define and characterize the equilibrium. Here, I show the posterior distribution of agent $i$’s information about bank fundamental $\theta$ conditional on the event of bank bailout and agent $i$’s private information $\theta_i$. For agent $i$, conditional on obtaining the signal $\theta_i$, her posterior about $\theta$ is

$$\theta \sim U[\max(\theta_i - \bar{\epsilon}, 0), \min(\theta_i + \bar{\epsilon}, 1)]$$

Boundaries of the interval are a concern because they have to be in $[0, 1]$. That’s why I use the max and min sign.
Conditional on the event that $\theta_G < \theta^*_G$, investors’ posterior is

$$\theta \sim U[0, \min(\theta^*_G + \bar{\eta}, 1)]$$

**Lemma 1** For agent $i$ who observes a bailout and obtains signal $\theta_i$, the support of the conditional distribution from $\theta|\theta_i$ and $\theta|B$ overlap with each other. The posterior is

$$\theta|\theta_i, B \sim U[\max(0, \theta_i - \bar{\varepsilon}), \min(1, \theta^*_G + \bar{\eta}, \theta_i + \bar{\varepsilon})]$$ (3)

In the same way, the posterior for an agent who observes $\theta_i$ and conditional on no bailout announcement is

$$\theta|\theta_i, N \sim U[\max(0, \theta^*_G - \bar{\eta}, \theta_i - \bar{\varepsilon}), \min(1, \theta_i + \bar{\varepsilon})]$$ (4)

**Proof** Please refer to Appendix A.

Since the two posterior intervals are used to derive $\theta$, it’s straightforward to get that they overlap with each other.

Conditional on a bailout announcement, a (mixed) strategy for agent $i$ is a measurable function $s_i : ([0 - \bar{\varepsilon}, 1 + \bar{\varepsilon}], B) \rightarrow [0, 1]$ which indicates the probability that patient agent $i$ demands early withdrawal given her signal $\theta_i$, and a bailout announcement $B$, where $s_i(\theta_i, B) = 1$ indicates withdrawing and $s_i(\theta_i, B) = 0$ indicates waiting.

Conditional on no bailout announcement, a (mixed) strategy for agent $i$ is a measurable function $s_i : ([0 - \bar{\varepsilon}, 1 + \bar{\varepsilon}], N) \rightarrow [0, 1]$ which indicates the probability that patient agent $i$ demands early withdrawal given her signal $\theta_i$, and no bailout announcement $N$, where $s_i(\theta_i, N) = 1$ indicates withdrawing and $s_i(\theta_i, N) = 0$ indicates waiting.

**Definition 1.** A Bayesian equilibrium is a measurable strategy profile $\{s_i(\theta_i, B), s_i(\theta_i, N)\}_{i \in [0, 1]}$, such that each patient agent chooses the best action at each signal, given the strategies of the other agents. Specifically, in equilibrium
(1) \( s_i(\theta_i, B) = 1 \) if \( \Delta(\theta_i; \cdot, B) < 0 \); \( s_i(\theta_i, B) = 0 \) if \( \Delta(\theta_i; \cdot, B) > 0 \); \( 0 \leq s_i(\theta_i, B) \leq 1 \) if \( \Delta(\theta_i; \cdot, B) = 0 \)

(2) \( s_i(\theta_i, N) = 1 \) if \( \Delta(\theta_i; \cdot, N) < 0 \); \( s_i(\theta_i, N) = 0 \) if \( \Delta(\theta_i; \cdot, N) > 0 \); \( 0 \leq s_i(\theta_i, N) \leq 1 \) if \( \Delta(\theta_i; \cdot, N) = 0 \)

where, conditional on bailout announcement \( B \), \( \Delta(\theta_i; \cdot, B) \) denotes the expected value of the utility differential \( v(\theta; \cdot, n(\theta, B), B) \) for agent \( i \) between waiting and withdrawing in Period 1; \( n(\cdot, B) \) denotes agent’s belief regarding the proportion of agents who run at each state \( \theta \),

\[
n(\theta, B) = \lambda + \frac{(1 - \lambda)}{2\varepsilon} \int_{-\varepsilon}^{\varepsilon} s_i(\theta + \varepsilon_i, B) d\varepsilon_i
\]

\[
\Delta(\theta_i; n(\cdot, B), B) = \frac{1}{\min(1, \theta^*_G + \eta_i + \varepsilon) - \max(0, \theta_i - \varepsilon)} \int_{\max(0, \theta_i - \varepsilon)}^{\min(1, \theta^*_G + \eta_i + \varepsilon)} v(\theta; n(\theta, B), B) d\theta
\]

\( v(\theta; n(\theta, B), B) \) is shown in (2) when \( \tilde{B} = B \).

conditional on no bailout announcement \( N \), \( \Delta(\theta_i; \cdot, N) \) denotes the expected value of the utility differential \( v(\theta; \cdot, n(\theta, N), N) \) for agent \( i \) between waiting and withdrawing in Period 1; \( n(\cdot, N) \) denotes agent’s belief regarding the proportion of agents who run at each state \( \theta \),

\[
n(\theta, N) = \lambda + \frac{(1 - \lambda)}{2\varepsilon} \int_{-\varepsilon}^{\varepsilon} s_i(\theta + \varepsilon_i, N) d\varepsilon_i
\]

\[
\Delta(\theta_i; n(\cdot, N), N) = \frac{1}{\min(1, \theta_i + \varepsilon) - \max(0, \theta^*_G - \eta_i, \theta_i - \varepsilon)} \int_{\max(0, \theta^*_G - \eta_i - \varepsilon)}^{\min(1, \theta_i + \varepsilon)} v(\theta; n(\theta, N), N) d\theta
\]

\( v(\theta; n(\theta, N), N) \) is shown in (2) when \( \tilde{B} = 0 \).
If all patient agents have the same threshold strategy, I use \( n(\theta, \theta') \) to denote \( n(\theta) \), where \( \theta' \) is the common threshold.

**Proposition 1.** Conditional on the event \( B \), the model has a unique threshold equilibrium in which patient agents who observe signals below threshold \( \theta^*_B(r, B) \) choose to run, \( s_i(\theta_i, B) = 1 \), and do not run above, \( s_i(\theta_i, B) = 0 \), and conditional on the event \( N \), the model has a unique threshold equilibrium in which patient agents who observe signals below threshold \( \theta^*_N(r, \bar{B} = 0) \) choose to run, \( s_i(\theta_i, N) = 1 \), and do not run above, \( s_i(\theta_i, N) = 1 \).

**Proof.** In the Appendix.

Below I sketch the proof and provide intuition for the case when there is a bailout announcement, i.e., \( \bar{B} = \bar{B} \). The proof for the case when \( \bar{B} = 0 \) is omitted since it is just the symmetry of the former. Before advancing to the main part of the proof, I need to characterize the lower dominance region which is essential for the main proof. But first of all, I need to define bank runs.

**Definition 2.** *(Bank Run)* I define Bank Run as the number of investors who demand early withdrawals in Period 1 is larger than the maximum number of investors the bank can serve, i.e.,

\[
n = \lambda + (1 - \lambda)\psi \geq \frac{1 + \bar{B}}{r}
\]

where \( \psi \) denotes the number of patient agents who demand early withdrawals.

In this paper, I restrict parameter \( \lambda \) to satisfy

\[
\lambda + \frac{1 - \lambda}{2} = \frac{1 + \bar{B}}{r}
\]

so that, as long as the the fraction of patient investors who demand early withdrawal is larger than \( \frac{1}{2} \), then I call there is a Bank Run.
Since I restrict attention to threshold equilibrium, it means that the number of investors who will run on the bank is

\[
    n(\theta) = \lambda + (1 - \lambda) \frac{\theta^*_B - (\theta - \bar{\varepsilon})}{2\bar{\varepsilon}}
\]  

(7)

when the cutoff threshold is $\theta^*_B$.

When the fundamental is very bad, patient agents run not considering other agents’ action. I refer to this region as the lower dominance region. Even if all the other patient agents choose to wait, she still runs. I set $n = \lambda$, and use $\theta(r, \bar{B})$ to denote the bank fundamental which makes investors indifferent.

\[
    u(r) = \theta u\left(\frac{1 + \bar{B} - \lambda r}{1 - \lambda} R\right)
\]

\[
    \theta(r, \bar{B}) = \frac{u(r)}{u\left(\frac{1 + \bar{B} - \lambda r}{1 - \lambda} R\right)}
\]

Investor $i$ demands early withdrawal if $\theta_i < \theta(r, \bar{B}) - \bar{\varepsilon}$.

In this paper, I only focus on threshold equilibria. There are two steps for the rest of the proof. First, I derive the basic properties of the run function, $\Delta(\theta; n(\cdot, B), B)$, defined as the expected utility differential between waiting and withdrawing. Second, I establish that there exists a unique threshold equilibrium.

Step 1: For a given state $\theta$, the incentive to withdraw in Period 2 is

\[
v(\theta, n, \bar{B}) = \begin{cases} 
    \theta u\left(\frac{1 + \bar{B} - n R}{1 - n} R\right) - u(r) & \text{if } \frac{1 + \bar{B}}{r} \geq n \geq \lambda \\
    0 - \frac{1 + \bar{B}}{n r} u(r) & \text{if } 1 \geq n \geq \frac{1 + \bar{B}}{r}
\end{cases}
\]  

(8)

$v$ decreases with $n$, only when $n \leq \frac{1 + \bar{B}}{r}$, as drawn in Figure 1. The proof for the unique equilibrium in the standard global games (Morris and Shin (1998)) is based on the iterated dominance elimination method. However the proof does not work here since the model does not satisfy the global complementarity property, i.e., an agent’s incentive to take an action increases as more agents take that action. The incentive to run on the bank is an increasing function
of \( n \) only when the bank is not bankrupt, i.e., \( n \leq \frac{1+B}{r} \). Otherwise, the incentive decreases as \( n \) increases. Once the bank is bankrupt, the incentive decreases since the expected payoff for the agent decreases as more agents demand early withdrawals, even though it is still optimal for that agent to run on the bank. Morris and Shin (2003) show that if the payoffs satisfy a single crossing property and the information distribution satisfies the monotone likelihood ratio property (MLRP), then there exists a unique monotone equilibrium. With the specification of uniform distribution, Goldstein and Pauzner (2005) show that there is no other equilibrium.

The expected utility differential for agent \( i \) is

\[
\Delta(\theta_i; n(\cdot, B), B) = \frac{1}{\min(1, \theta^*_G + \eta_i, \theta_i + \varepsilon)} \int_{\max(0, \theta_i - \varepsilon)}^{\min(1, \theta^*_G + \eta_i, \theta_i + \varepsilon)} v(\theta; n(\cdot, B), B) d\theta
\]

In the appendix, I show that the function \( \Delta(\theta_i; n(\cdot, B), B) \) is continuously increasing in \( \theta_i \) and it’s strictly increasing if \( n(\theta) \leq \frac{1+B}{r} \).

Step 2: To prove there exists a unique threshold equilibrium, I only need to prove that given that all other patient agents use threshold strategy \( \theta^*_B \), patient agent \( i \) runs if and only if \( \theta_i < \theta^*_B \), and waits if and only if \( \theta_i > \theta^*_B \), i.e.,

\[
\Delta(\theta_i, n(\cdot, \theta^*_B), \bar{B}) < 0 \quad \forall \theta_i < \theta^*_B
\]

(10)

\[
\Delta(\theta_i, n(\cdot, \theta^*_B), \bar{B}) > 0 \quad \forall \theta_i > \theta^*_B
\]

(11)

\[
\Delta(\theta_i, n(\cdot, \theta^*_B), \bar{B}) = 0 \quad \forall \theta_i = \theta^*_B
\]

The appendix A shows that there is exactly one value of \( \theta^*_B \) that satisfies \( \Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) = 0 \), which follows from the existence of the lower dominance region, and that \( \Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) \) is continuously increasing in \( \theta^*_B \) and it is strictly increasing if \( n(\theta) \leq \frac{1+B}{r} \), which can be
directly derived from Lemma 2 and 3 in the Appendix. Then, in the appendix, I show given \( \Delta(\theta_B^r, n(\cdot, \theta_B^r), \bar{B}) = 0 \) by using the single crossing property of \( \Delta(\theta, n(\cdot, \theta_B^r), \bar{B}) \).

As can be seen from Figure 2, \( \Delta(\theta, n(\theta, \theta'), \bar{B}) \) crosses its 0 value only once, i.e., \( \Delta(\theta, n(\theta, \theta'), \bar{B}) \) is positive for high value of \( \theta \), negative for low value of \( \theta \), and crosses zero only once.

\[
\Delta(\theta, n(\theta, \theta'), \bar{B}) = 0, \quad (10) \quad \text{and} \quad (11)
\]

Figure 2: The run function

Figure 2 presents patient agents’ incentive to run as a function of \( \theta \). Suppose \( \theta_B^r \) is the indifference signal which makes the expected utility differential equal to 0, i.e., \( A_1 = A_2 \), where \( A_1 \) is the integrated area where agents choose to wait, and \( A_2 \) is the integrated area where agents choose to run. Any agent who receives a lower signal, such as \( \theta_1^B \), has a larger \( A_2 \), and thus lowers the expected utility differential below 0, which induces that agent to run on the bank. Q.E.D.

**Proposition 2.** Conditional on the event \( B \), the probability of bank runs, \( \Pr(\theta \leq \theta_B^r|\bar{B} = \bar{B}) \), which is proportional to \( \theta_B^r(\bar{B}, \theta_G^*, \bar{\eta}) \), is a decreasing function of the bailout level \( \bar{B} \), the government strategy cutoff \( \theta_G^* \), and the precision of government signal \( \bar{\eta} \).

\[
\frac{\partial \theta_B^r(r, \bar{B}, \theta_G^*, \bar{\eta})}{\partial \bar{B}}, \quad \frac{\partial \theta_B^r(r, \bar{B}, \theta_G^*, \bar{\eta})}{\partial \theta_G^*}, \quad \frac{\partial \theta_B^r(r, \bar{B}, \theta_G^*, \bar{\eta})}{\partial \bar{\eta}} \leq 0
\]
Proof. Please refer to Appendix B.

Proposition 2 provides three insights for bank runs after a bailout announcement. For \( \theta^*_G \), it states that the probability of bank runs, defined as the proportion of agents who demand early withdrawal, i.e., those who obtain signals below \( \theta^*_B \), becomes higher as the government uses a smaller cutoff \( \theta^*_G \) to bail out the bank. There are two interpretations in the real world. First, the government announces its bailout policy \( \theta^*_G \) explicitly. \( \theta^*_G \) can be a restriction on credit ratings or bank capital. The government announces bailouts only if these various valuations fall below \( \theta^*_G \). I can also interpret this in a dynamic way. Investors cannot directly observe \( \theta^*_G \) this time. However, investors can learn \( \theta^*_G \) based on government’s bailout history. Suppose the government bails out the bank less frequently. Once a bailout is announced, agents will infer that the government must be using a lower \( \theta^*_G \). I thus conjecture that bank runs happened more likely if the government has a rare history of bailing out banks. This theoretical insight provides an explanation of the run on Bear Stearns after the Fed announced its liquidity provision policy as described in the Introduction. Bailouts rarely happened before. The last major bailout is for Long-Term Capital Management in 1998 and it was bailed out by other banks under the arrangement from the Fed. Of the Bear Stearns rescue, Bernanke said, “I hope this is a rare event, and I hope it’s not something that we ever have to do again.” Investors probably deduced that the Fed must use a very low \( \theta^*_G \) for its bailout policy. Bear Stearns’ situation was more likely to be so bad that the Fed chose to bail it out.

For \( \bar{B} \), if I increase the value of \( \bar{B} \) while keeping other variables on the right hand side constant, \( \theta^*_B \) is lower, i.e., the probability of a bank run decreases. While keeping the information effect \( \theta^*_G \) constant, injecting more capital to a bank will reduce the probability of a bank run. Therefore, there are two opposite effects of a bailout announcement on the probability of bank runs. For \( \bar{\eta} \), a higher government signal precision, i.e., a lower \( \bar{\eta} \), will lead to a higher probability of bank runs. The reason is that a higher signal precision will make the investors be more sure that the bank fundamental is below \( \theta^*_G \).

Suppose \( \theta^*_G \) is observed with a small noise. \( \theta^{**}_G = \theta^*_G + \varphi \), where \( \varphi \) is a noisy term. It is straightforward to show that adding such noise has the same effect as increases in \( \bar{\eta} \), i.e., the
information effect from bailouts decreases, since investors think that it is more likely that the bank fundamental is not lower than $\theta^*_G$.

Figure 3 and 4 present more intuitive explanations of Proposition 2. Figure 3 shows that as the signal precision $\eta$ or the government cutoff threshold $\theta^*_G$ increases, the area above the $\theta$ line increases. If I still want to get the expected utility differential to be equal to 0, $\theta^*_B$ needs to be reduced to $\theta^*_{B, New}$.

Figure 3: The information effect

Figure 4 shows that as $\bar{B}$ increases, $\theta^*_B$ needs to be reduced to $\theta^*_{B, New}$ to get the expected utility differential to be equal to 0. Both Figure 3 and 4 confirm the claims in Proposition 2.

4.3 Comparing Economies With and Without Bailouts (Ex Ante)

To examine the efficiency of government bailout commitment in ensuring financial stability by reducing the probability of bank runs, I need to compare the probability of bank runs in an economy where the government executes the cutoff strategy as specified in (1) and the probability in another economy where the government commits never to bail out the bank.
This paper abstracts away from any bailout costs\textsuperscript{22} Proposition 3 demonstrates that the existence of bailouts reduces the probability of bank runs.

**Proposition 3.** Assume $r = \frac{1}{2}$. For $\bar{B} > 0$, the probability of a bank run in the economy where the government commits never to bail out a bank, $\Pr(\theta < \theta^*)$, is higher than the probability of bank runs in the economy where the government uses the above bailout strategy, i.e.,

$$\Pr(\theta < \theta^*) > \Pr(\theta < \theta^*_B|\theta_G < \theta^*_G) \Pr(\theta_G < \theta^*_G) + \Pr(\theta < \theta^*_N|\theta_G \geq \theta^*_G) \Pr(\theta_G \geq \theta^*_G)$$

where $\theta^*$ is the unique equilibrium cutoff threshold for an economy where the government commits never to bail out a bank; both $\theta^*_B$ and $\theta^*_N$ are equilibrium threshold cutoffs in the economy where the government uses the above bailout strategy; $\theta^*_B$ is the unique equilibrium cutoff threshold for the economy where the government commits never to bail out a bank.

\textsuperscript{22}The reason why I do not consider the cost of bailout is that the bailout financing source is very hard to specify. The transfer of resources in \cite{diamond1983bank} comes from the taxation imposed by the government on the early withdrawals. However, it is not feasible in reality that governments tax the investors in that bank to bail out the bank.

\textsuperscript{23}I use this assumption for the simplicity of the calculation of $\theta^*_B$. This assumption is also employed in the bank run paper by Morris and Shin (1999). Under this assumption, all the runs are information based.
old after a bailout announcement; \( \theta^*_N \) is the unique equilibrium cutoff threshold after no bailout announcement.

**Proof.** Please refer to Appendix B.

### 4.4 The 2008-2009 Financial Crisis and Government Policy

The government policies tackling the Subprime Mortgage Crisis were highly disputed (Philippon and Schnabl (2009)). I map the results from the model analyzed above to the circumstances during this crisis. The main conclusion that bailout announcements might increase the probability of bank runs, has been discussed in the motivation part of the Introduction. Therefore, the focus here is on other aspects of the model results, including the comparative statics and ex ante arguments.

Regarding the precision of the government signal, the model implies that, ex ante, a more precise government signal reduces the probability of bank runs, while ex post, a more precise government signal increases the probability of bank runs. The government’s signal precision here is called transparency or government auditing. The latter denotes the effort the government puts to investigate the bank to improve its transparency. First, I address the ex post case. It is widely argued that the public has the right to know basic information about the “unprecedented and highly controversial use” of public money, i.e., TARP. But the Fed warns that bailed-out banks may be hurt if the documents are made public and would cause a run or a sell-off by investors. Secret Bailouts did work in the past. For example, during the UK secondary banking crisis of 1973, the Bank of England arranged a secret bailout to National Westminster Bank. The bank survived during the crisis, and became profitable again after the crisis. However, the bailout for Northern Rock was criticized for being too public. According to Mervyn King, he could not arrange a secret bailout because of falling foul of EU rules on State Aid. Even though more transparency ex post is sometimes detrimental to bank stability. Ex ante, transparency lowers the probability of bank runs. Part of the reason why the Subprime Mortgage Crisis was severe both extensively and intensively is that investors lost their confidence about their governments. Investors believe the government’s signal is noisy,
i.e., the government does not have much information about the bank. The Fear Index VIX, which is the volatility implied in the prices of options, reached the record level of 69.25 during the government intervention period. Deborah Lucas argues that a more constructive role for the government is to improve transparency. “The government could send auditors to these finance corporations and they could say, ‘You have to open your books,’ and we could figure out who’s OK and who isn’t,” she suggested. “To the extent that this calms down the markets, that could be a help.”

One of the implications from the model is that bailout amounts should be large enough to contain bank runs. According to Moessner and Allen (2010), central bank liquidity provision was larger in 2008-09 than in 1931, when it had been constrained in many countries by the gold standard. That’s why the banking crises this time were less severe than that during the Great Depression. In the empirical part of the paper, I will show that during the recent crisis, the probability of a bank run is negatively correlated with a bank’s bailout ratio.

5 Empirical Evidence from the 2008-2009 Financial Crisis

In this section, I provide suggestive evidence to support the arguments from the theoretical model by investigating the Troubled Asset Relief Program in the US. First, I present the timeline of the key events with a highlight on the government interventions during the recent financial crisis. Second, I use the bank run index, one measurement of the probability of bank runs, developed by Veronesi and Zingales (2010) to demonstrate the effect of several key events including the bailout announcements on the probability of bank runs. Third, I introduce the data which will be utilized to investigate the effect of government announcements on the abnormal returns of stock prices. Fourth, I describe how to measure abnormal returns. In the end, I present the results from event studies and discuss the correspondences between the empirical results and the theoretical model.

25 The crisis was global. However I focus on the US because most of other countries, such as Iceland and UK, nationalized the banks, which is inconsistent with the model in this paper. Investors do not have incentive to run on a nationalized bank.
Two main conclusions from the theoretical model are 1) announcements of bank bailouts may increase the probability of bank runs; 2) before banks’ specified bailout announcements, the existence of the above mentioned bailout policy reduces the probability of bank runs, compared to the government policy which rules out bailouts.

I examine the announcement of TARP in this paper, which is composed of two parts. The first part is in October 2008, when the government announced the banking sector would be bailed out, but did not specify the names of banks who would receive bailouts except the nine largest banks. The Department of Treasury released the following announcement on its website after the pass of the TARP bill by the Congress, “This bill contains a broad set of tools that can be deployed to strengthen financial institutions, large and small, that serve businesses and families. Our financial institutions are varied – from large banks headquartered in New York, to regional banks that serve multi-state areas, to community banks and credit unions that are vital to the lives of our citizens and their towns and communities.” The announcement did not indicate the names of the banks who would be bailed out. But from the statement above, the probability for a large bank to receive bailouts is higher than that for a small bank, given the number of banks in their different size group. The first part can be viewed as an announcement of the existence of an ex ante bailout policy. Investors expected the bailouts were mainly for the large banks on Wall Street. I thus use this date, October 3, 2008 as the exact date of bailout announcements for the Top 10 banks. There are other two reasons besides the “Too Big To Fail” policy. First, as the name “Troubled Asset Relief Program” indicates, the bailouts will go to buy toxic asset from bank’s balance sheet, while these big banks have more holdings of MBS. Second, on the day President Bush signed the bill into law, he remarked “By coming together on this legislation, we have acted boldly to prevent the crisis on Wall Street from becoming a crisis in communities across our country”, which explicitly indicates the big Wall Street banks.

The second part of bailout announcements is periodically specified announcements which

27 http://www.financialstability.gov/latest/hp1175.html
contain more details, including banks’ names and the bailout amount. There are 241 banks with their stock price returns available in my dataset that received bailouts. The first measure of bank runs, the bank run index constructed by Veronesi and Zingales (2010), is more precise, but is limited to nine banks due to the availability of CDS rates, which are used to construct the index. I find that, for some banks, their bank run indices increased dramatically after their individual bailout announcements. The second approach utilizes the stock price abnormal returns in response to bailout announcements to measure the probability of bank runs, which can be applied to many more banks, 241 banks. The higher the stock price abnormal return, the lower the probability of bank runs. In this paper, investors’ payoff resembles that of the equity holder, in the sense that investors’ payoff depends on the asset return. As I argued in the Introduction section, the classic bank runs or debt runs can be extended to the equity holders, such as investors in a hedge fund or mutual funds (Shleifer and Vishny (1997); Brunnermeier (2009)). If I add equity holders in my model, I would obtain the result that the probability of a bank run increases as the stock price abnormal return goes down. Since Abnormal Return = Actual Return - Normal Return, a positive abnormal return after a bailout announcement signifies a reduction of the probability of a bank run. For the original Paulson plan which was proposed on September 20, 2008 and passed on October 3, 2008, the average stock price abnormal return for banks in my dataset was positive using various cumulative intervals. Since the names of these banks which would be bailed out were not known until after their own specific bank bailouts later on, I find this confirms the ex ante argument, i.e., the existence of bailouts reduces the probability of bank runs. After banks’ own individual bailout announcements, they displayed significant negative average abnormal returns, and the abnormal return was positively correlated with a bank’s bailout ratio, defined as the bailout amount divided by its total asset.
5.1 Timeline

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>September 15</td>
<td>Lehman Brothers files for bankruptcy protection</td>
</tr>
<tr>
<td>September 19</td>
<td>Paulson financial rescue plan is unveiled.</td>
</tr>
<tr>
<td>September 25</td>
<td>Washington Mutual failed and is seized by the FDIC.</td>
</tr>
<tr>
<td>September 29</td>
<td>EESA is defeated in the United States House of Representatives</td>
</tr>
<tr>
<td>September 29</td>
<td>Citigroup Inc. announced that it would acquire Wachovia.</td>
</tr>
<tr>
<td>October 1</td>
<td>The U.S. Senate passes HR1424, their version of the $700 billion bailout bill.</td>
</tr>
<tr>
<td>October 3</td>
<td>President Bush signs the EESA, creating a $700 billion Troubled Assets Relief Program to purchase failing bank asset.</td>
</tr>
<tr>
<td>October 6–10</td>
<td>Worst week for the stock market in 75 years.</td>
</tr>
<tr>
<td>October 6</td>
<td>Fed announces that it will provide $900 billion short-term cash loans to banks.</td>
</tr>
<tr>
<td>October 11</td>
<td>The Dow Jones Industrial Average caps its worst week ever.</td>
</tr>
<tr>
<td>October 14</td>
<td>Revised Paulson plan with debt guarantee is announced.</td>
</tr>
</tbody>
</table>

The timeline for the government interventions during the Subprime Mortgage Crisis is presented in Table 3. The TARP was passed by the Congress and signed by the President on October 3, 2008. During the period around the bailout announcement, US stock market, which is a measure of market confidence, was very volatile and had its worst week ever, after the pass of TARP. During this period, there were also bank runs and bank failures, such as the run on Washington Mutual, which led to the largest bank failure in the history of the U.S., after depositors withdrew $16.4 billion. Wachovia experienced a similar bank run, but mainly by large depositors, whose deposits were not fully insured, which caused the takeover of Wachovia by Wells Fargo.

5.2 Bank Run Index during the Crisis

Bank run index is constructed by Veronesi and Zingales (2010) using the Credit Default Swap rates, which are available for only a limited number of banks. Bank Run Index measures the
difference between the (risk neutral) probability of default in the immediately following year and the (risk neutral) probability of default between year 1 and year 2, conditional on surviving at the end of year one.

Computation of Bank Run Index

\[ R = P(1) - P(2) \]  

where

\[ P(n) = \text{prob}(\text{Default in year } n | \text{No Default before year } n) \]

Risk neutral default probabilities are bootstrapped from CDS rates. Please refer to the appendix in their paper for the bootstrapping procedure. A higher bank run index stands for a higher probability of bank runs.

Figure 5 plots the bank run index during the crisis and labels several key events that are related to this paper.\(^{28}\) These eight banks are among the biggest bailout recipients from TARP. Morgan Stanley experienced a huge volatility of bank run index. Goldman Sachs, Merrill Lynch, Wachovia, Citi also experienced large volatilities of their indices. The volatilities for commercial banks Bank of America, JP Morgan and Wells Fargo were almost zero. Part of the reason for these commercial banks to perform well is the announcement by the FDIC on October 3 of an increase of the deposit insurance coverage from $100,000 to $250,000 per depositor. The FDIC also provided full guarantee for the FDIC-insured depository institutions. The failure of Lehman Brothers on September 15, 2008, increased the bank run index for some banks dramatically. The bankruptcy of Lehman Brothers is the largest bankruptcy filing in U.S. history with Lehman holding over $600 billion in assets. Both the counterparty concern and the non-commitment of “Too Big To Fail” policy spurred the bank run index. Nevertheless the average magnitude of these increases was still much smaller than the effect of $700 billion

\(^{28}\)There were other events going on during this turbulent period, including European bank rescue. But no bad events happened. For example, the ban on international short sales did not occur. So the results are very robust, i.e., the indices could only go up, not go down if there were no such events.
bailout proposal from the then Treasury Secretary, Henry Paulson. The theory in this paper behind these facts suggests that the $700 billion bailout signaled to the investors how severe the financial crisis could potentially be, or how toxic banks’ assets actually would be. Immediately after the proposal, there was a “silent” bank run on Washington Mutual, where depositors withdrew their deposits online. Wachovia and Washington Mutual were the top two mortgage lenders at that time. Wachovia experienced a bank run as well, which was almost at the same time. The failure of Washington Mutual is the largest bank failure in US history, which caused abrupt increases in bank run index for some other banks. The bank run indices kept very high, except for Bank of America, JP Morgan and Wells Fargo, which had less mortgage exposure. The indices did not go down until the announcement of the revised bailout plan which explicitly included the debt guarantee, and thus bank runs happened much less likely, i.e., the bank run indices for these banks dropped steeply.

5.3 Data

The data I use for Method 2 in this paper is from multiple sources. For the main bailout information, I assemble the bank level data from Grail research’s “Global Financial Crisis: Bailout Tracker” (see Grail (2009)) and Federal Reserve Bank (2009), which provide detailed information on bailout enactment dates and bailout amounts. Since some of the enactment dates are different from the announcement dates, I also manually searched the announcement date online including the press releases on the U.S. Treasury department web page. I then gather manually the announcement date of the bailout for each bank. Daily stock prices for all the banks are available at Center for Research in Security Prices at the University of Chicago (CRSP). The data on banks’ total assets is from Bankscope. As I stated at the beginning of this section, the subsequent event studies will be conducted in two parts. The first part deals with the recipient unspecified TARP announcement without banks’ name specifications. The second part turns to specified individual bank bailouts.
5.4 Measurement of Abnormal Returns

Following Fama (1976), the following model is specified.

\[ R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \]

where \( R_{it} \) and \( R_{mt} \) are the rates of return for bank \( i \) and the FTSE All-Share index at date \( t \), respectively. The error term \( \epsilon_{it} \) is assumed to have zero mean, to be independent of \( R_{mt} \), and to be uncorrelated across banks.\(^{29} \)

\(^{29}\)One concern is that these events could be endogenous, i.e., the government bails out a bank when its abnormal return is low. However, this bias would only drive the effect from bailout announcements downwards. See Viswanathan and Wei (2008) for more econometric methodologies on this.
Then I run an OLS regression to estimate $\hat{\alpha}_i$ and $\hat{\beta}_i$, with the Newey-West HAC adjustment to control for the presence of autocorrelation and heteroskedasticity. The abnormal returns are calculated as follows,

$$AR_{it} = R_{it} - \hat{\alpha}_i - \hat{\beta}_i R_{mt}$$

which is the difference between the realized return and the estimated return from the OLS regression.

I define day ‘0’ as the date of a hypothetical event for a given security. For the first part of bailout announcements, it is October 3, 2008, on which the recipient unspecified TARP was announced after being passed and signed into law. The parameters of the market model are estimated using a $T = 250$ trading day (1 year) window, starting at day -244 and ending at day +5 relative to the event. The first 235 days in this period (-244 through -9) is designated as the ‘estimation period’, and the following 15 days (-9 through +5) is designated the ‘event window’. For the individual bank bailout case, the only different place is that I use (-3 through +5) as ‘event window’, since there was no uncertain period as that for the original Paulson plan, which was proposed on September 20, 2008 but was passed on October 3.

I calculate the standardized abnormal returns as

$$SAR_{it} = \frac{AR_{it}}{\hat{\sigma}_{it}} \sim t(T - 2)$$

to determine whether abnormal returns are significantly different from zero, where $\hat{\sigma}_{it}$ is the sample standard deviation of the abnormal return.

I use cumulative abnormal returns (CAR) to measure the aggregate effect of an announcement, which is defined as

$$CAR_i(t_1, t_2) = \sum_{t=t_1}^{t_2} AR_{it}$$
for $t_1 \leq t \leq t_2$. The standard deviation of the CAR is calculated as

$$\hat{\sigma}_{it}(t_1, t_2) = \left( \sum_{t=t_1}^{t_2} \hat{\sigma}_{it} \right)^{1/2}$$

Then, the standardized cumulative abnormal returns is

$$SCAR_{it} = \frac{CAR_i(t_1, t_2)}{\hat{\sigma}_i(t_1, t_2)} \sim t(T - 2)$$

5.5 Results

Table 4 and 5 report results on the abnormal returns and cumulative abnormal returns around the day of the enactment of TARP, October 3, 2008. Table 6 and 7 present results on the abnormal returns and cumulative abnormal returns around the exact dates of individual banks’ bailout announcements. Table 8 shows the relationship between abnormal returns and bailout ratio.

Table 4 and 5 show that banks exhibited positive abnormal returns most of the time. On the day of announcement, i.e., October 3, 2008, the total sample, including the 241 banks, had insignificant abnormal returns which is close to 0. But largest banks, which were highly expected to be on the bailout list because of “Too-Big-To-Fail”, with their names explicitly announced on October 14, 2008, generated significant -3.16% abnormal returns. This supports the theory in this paper that ex ante, before the name of the bank who will receive a bailout is announced, the existence of a potential bailout decreases the probability of bank runs, as indicated by the positive cumulative abnormal returns for the banking sector. Ex post, when the government announced the names of the banks who would be bailed out, i.e., the ten largest banks, these banks displayed negative abnormal returns. The performance of the abnormal returns in the general banking sector corresponds to my first argument, where investors know the existence of a potential bailout, but cannot identify which one. The largest banks were explicitly revealed or anticipated to be bailed out.

These effects are illustrated more clearly by the CAR for the event window (0,5), where
investors highly expected that big banks would be bailed out, as pointed out by various media, reporting that the largest banks would be bailed out. The large banks generated very significant negative abnormal returns, which was about -22%, while the banking sector displayed significant 5.9% positive abnormal returns.
Table 4

Mean Abnormal Returns (%)

This table reports abnormal returns for a fifteen-day period surrounding the announcement of the recipient unspecified TARP.

<table>
<thead>
<tr>
<th>Day Relative to the Announcement Day</th>
<th>Total Sample (Num = 241)</th>
<th>Big Banks (Num = 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t value</td>
<td>t value</td>
</tr>
<tr>
<td>-9</td>
<td>-2.02**</td>
<td>-0.45</td>
</tr>
<tr>
<td>-8</td>
<td>1.04</td>
<td>1.48</td>
</tr>
<tr>
<td>-7</td>
<td>-0.14</td>
<td>-5.68***</td>
</tr>
<tr>
<td>-6</td>
<td>-0.77</td>
<td>-0.09</td>
</tr>
<tr>
<td>-5</td>
<td>0.46</td>
<td>3.41**</td>
</tr>
<tr>
<td>-4</td>
<td>1.88**</td>
<td>-6.30****</td>
</tr>
<tr>
<td>-3</td>
<td>-1.28</td>
<td>10.06****</td>
</tr>
<tr>
<td>-2</td>
<td>2.60***</td>
<td>7.45****</td>
</tr>
<tr>
<td>-1</td>
<td>3.75****</td>
<td>2.06</td>
</tr>
<tr>
<td>0</td>
<td>0.05</td>
<td>-3.16*</td>
</tr>
<tr>
<td>1</td>
<td>1.22</td>
<td>0.64</td>
</tr>
<tr>
<td>2</td>
<td>0.17</td>
<td>-8.08****</td>
</tr>
<tr>
<td>3</td>
<td>-1.08</td>
<td>-2.85*</td>
</tr>
<tr>
<td>4</td>
<td>-1.41</td>
<td>-9.55****</td>
</tr>
<tr>
<td>5</td>
<td>6.95****</td>
<td>6.408</td>
</tr>
</tbody>
</table>

The symbols *, **, *** and **** denote statistical significance at the 0.10, 0.05, 0.01 and 0.001 levels, respectively, using a generic one-tail test.
Table 5

Mean Cumulative Abnormal Returns (%)

This table reports mean cumulative abnormal returns for the fifteen-day period surrounding the announcement of the recipient unspecified TARP.

<table>
<thead>
<tr>
<th>Day Relative to the Announcement Day</th>
<th>Total Sample (N = 241)</th>
<th>Big Banks (N = 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.05</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.16*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.054</td>
</tr>
<tr>
<td>(0,1)</td>
<td>1.27</td>
<td>0.831</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.875</td>
</tr>
<tr>
<td>(0,5)</td>
<td>5.90**</td>
<td>2.222</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-22.00****</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4.413</td>
</tr>
<tr>
<td>(-9,5)</td>
<td>11.43***</td>
<td>2.722</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-10.06*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.276</td>
</tr>
</tbody>
</table>

The symbols *, **, *** and **** denote statistical significance at the 0.10, 0.05, 0.01 and 0.001 levels, respectively, using a generic one-tail test.

Table 6 and 7 present the abnormal and cumulative abnormal returns for all the banks that received bailouts. Every bank in the sample was bailed out explicitly, i.e., investors knew which banks obtained bailouts and how much. These bailout information can be reached at the U.S. Treasury department[^30] and various media sources. The banks experienced negative average abnormal returns, -0.41% for the announcement day, -1.24% for the event window (0,1), and -2.33% for the event window (0,5). The CAR for the two event windows after a bank’s individual announcement are significantly negative. Since there is a time lag for investors to receive the information on a bailout, the last event window exhibits a lower abnormal return. The abnormal return before the day ‘0’ was insignificant because a bailout announcement is hard to be predicted. Almost two thirds of banks experienced negative abnormal returns. This supports the conclusion in my model that ex post, after bailout announcements, the probability of bank runs may increase.

### Table 6

**Mean Abnormal Returns for Individual Banks (%)**

This table reports mean abnormal returns for an eight-day period surrounding the announcements of individual bank bailouts.

<table>
<thead>
<tr>
<th>Day Relative to the Announcement Day</th>
<th>Total Sample</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Day 0 = Announcement Day)</td>
<td>(Num = 241)</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>-0.74∗</td>
<td>-1.491</td>
</tr>
<tr>
<td>-1</td>
<td>0.38</td>
<td>0.754</td>
</tr>
<tr>
<td>0</td>
<td>-0.41</td>
<td>-0.829</td>
</tr>
<tr>
<td>1</td>
<td>-0.82**</td>
<td>-1.657</td>
</tr>
<tr>
<td>2</td>
<td>-0.84**</td>
<td>-1.693</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
<td>0.243</td>
</tr>
<tr>
<td>4</td>
<td>-0.26</td>
<td>-0.519</td>
</tr>
<tr>
<td>5</td>
<td>-0.11</td>
<td>-0.221</td>
</tr>
</tbody>
</table>

The symbols ∗, ∗∗, ∗∗∗, and ∗∗∗∗ denote statistical significance at the 0.10, 0.05, 0.01 and 0.001 levels, respectively, using a generic one-tail test.

### Table 7

**Mean Abnormal Returns (%)**

This table reports mean cumulative abnormal returns for an eight-day period surrounding the announcements of individual bank bailouts.

<table>
<thead>
<tr>
<th>Day Relative to the Announcement Day</th>
<th>Total Sample</th>
<th>Positive:Negative</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Day 0 = Announcement Day)</td>
<td>(N = 241)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,0)</td>
<td>-0.41</td>
<td>(108:133)</td>
<td>-0.829</td>
</tr>
<tr>
<td>(0,1)</td>
<td>-1.24**</td>
<td>(98:143)</td>
<td>-1.758</td>
</tr>
<tr>
<td>(0,5)</td>
<td>-2.33**</td>
<td>(100:141)</td>
<td>-1.909</td>
</tr>
</tbody>
</table>

The symbols ∗, ∗∗, ∗∗∗, and ∗∗∗∗ denote statistical significance at the 0.10, 0.05, 0.01 and 0.001 levels, respectively, using a generic one-tail test.
Table 8

<table>
<thead>
<tr>
<th>Event Window</th>
<th>(0, 0)</th>
<th>(0, 1)</th>
<th>(0, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.017</td>
<td>-0.034***</td>
<td>-0.040***</td>
</tr>
<tr>
<td></td>
<td>(-1.23)</td>
<td>(-3.14)</td>
<td>(-2.84)</td>
</tr>
<tr>
<td>Bailout Ratio</td>
<td>0.037</td>
<td>0.051**</td>
<td>0.069**</td>
</tr>
<tr>
<td></td>
<td>(1.18)</td>
<td>(2.03)</td>
<td>(2.09)</td>
</tr>
</tbody>
</table>

The symbols *, **, *** , and **** denote statistical significance at the 0.10, 0.05, 0.01 and 0.001 levels, respectively, using a generic one-tail test.

Table 8 shows that the bailout ratio, defined as the bailout amount for a bank divided by its total asset, is positively correlated with the abnormal return and the coefficient is significant at the 5% level for the last two windows, even though not for the announcement day, due to information delays.

For the banks I have analyzed in the Bank Run Index section, Bank of America, JP Morgan and Wells Fargo had a much higher abnormal return than the other banks, which is consistent with the result from Bank Run Index. This gives another justification to use abnormal returns as a proxy for the probabilities of bank runs.

### 6 Conclusion

This paper is motivated by various facts listed in the introduction which contradict the conventional wisdom that government bailouts can restore investors’ confidence and thus reduce

\[31\text{The table is not presented here, but is available upon request.}\]
the probability of bank runs. I develop an information based bank run model to examine the effect of bailout announcements on the probability of bank runs. The model highlights that the signaling effect from government bailouts may increase the probability of bank runs, since bailouts signal the government’s information that the bank is in trouble. The model characterizes the probability of bank runs after bailout announcements as a function of bailout amount, government bailout policy and government signal precision. The model also examines whether the existence of such bailout policy is better than the circumstance when the government commits never to bail out banks. I explore the relation between bank runs and TARP bailouts during the recent financial crisis to provide evidence for the theoretical results. From the two measures of the probability of bank runs, I find that bailout announcements generated negative impacts on probabilities of bank runs during the recent crisis. This paper suggests that bailout policy should be carefully designed in order to mitigate the potential bank run probabilities. The empirical evidence implies that the force identified in the theory may be important in practice.

7 Appendix A

Lemma 1 For agent \(i\) who observes a bailout and obtains signal \(\theta_i\), the support of the two posterior intervals from \(\theta|\theta_i\) and \(\theta|B\) overlap with each other. The posterior is

\[
\theta|\theta_i, B \sim U[\max(0, \theta_i - \bar{\epsilon}), \min(1, \theta^\ast_G + \bar{\eta}, \theta_i + \bar{\epsilon})]
\]

In the same way, The posterior for an agent who observes \(\theta_i\) and conditional on no bailout announcement is

\[
\theta|\theta_i, N \sim U[\max(0, \theta^\ast_G - \bar{\eta}, \theta_i - \bar{\epsilon}), \min(1, \theta_i + \bar{\epsilon})]
\]

Proof \(\theta\) is realized at the beginning of period 1. The government will bail out the bank if \(\theta_G \leq \theta^\ast_G\), i.e., \(\theta \leq \theta^\ast_G - \eta \leq \theta^\ast_G + \bar{\eta}\).

\[
\theta|B \sim U[0, \min(1, \theta^\ast_G + \bar{\eta})]
\]
For agent \(i\) who can observe the bailout, her signal must satisfy, 
\[ \theta_i \leq \theta_G^* + \bar{\eta} + \bar{\varepsilon}, \]
where \(\theta_i \in [\max(0, \theta_i - \bar{\varepsilon}), \min(1, \theta_i + \bar{\varepsilon})]\), then
\[
0 \leq \max(0, \theta_i - \bar{\varepsilon}) \leq \min(1, \theta_G^* + \bar{\eta})
\]

I can then conclude that the posterior interval conditional on \(B\) and \(\theta_i\) is nonempty. The posterior interval can be obtained directly, i.e., \(\theta_i, B \in [\max(0, \theta_i - \bar{\varepsilon}), \min(1, \theta_G^* + \bar{\eta}, \theta_i + \bar{\varepsilon})]\). Now I only need to show that the distribution is uniform. Below I only show the case when \(\theta_i, B \in [\theta_i - \bar{\varepsilon}, \theta_G^* + \bar{\eta}]\). Other cases can be derived in the same way.

If the observations are made in more than one stage, then the posterior distribution can be computed in different stages by letting the posterior distribution after each stage serve as the prior distribution for the next stage. I use \(\xi, f, g, h\) to denote all the p.d.f.

\[
\xi(\theta | \theta_i) = \frac{1}{(\theta_i + \bar{\varepsilon}) - (\theta_i - \bar{\varepsilon})} = \frac{1}{2\bar{\varepsilon}}
\]

\[
h(\tilde{B}) = \tilde{B}(\theta, \theta_i) = \frac{f(\theta_i, \tilde{B} = \tilde{B}(\theta)}{g(\theta_i | \theta)} = \frac{1}{2\bar{\varepsilon}} \frac{2\bar{\varepsilon}}{\theta_G^* + \bar{\eta} - (\theta_i - \bar{\varepsilon})}
\]

\[
\xi(\theta | \theta_i, \tilde{B}) = \tilde{B} = \tilde{B}(\theta, \theta_i) = \frac{h(\tilde{B} = \tilde{B}(\theta, \theta_i))}{\int_0^1 h(\tilde{B} = \tilde{B}(\theta', \theta_i)) \xi(\theta | \theta_i) d\theta'} = \frac{1}{\theta_G^* + \bar{\eta} - (\theta_i - \bar{\varepsilon})}
\]

Since \(\xi(\theta | \theta_i, \tilde{B} = \tilde{B})\) is a constant, then \(\theta_i, B \sim \mathcal{U}[\theta_i - \bar{\varepsilon}, \theta_G^* + \bar{\eta}]\).
The posterior when there is no bailout announcement can be obtained in the same way.

Proof of Proposition 1 The proof of this proposition is based on [Goldstein and Pauzner (2005)]. I first derive properties of the function that relates the investors’ signals to the probability of runs, which is crucial for the proof thereafter. Second, I show that there exists a unique threshold equilibrium.

A1. Properties of the Run Function

From equation (5), the expected utility differential between waiting and withdrawing for agent $i$ who obtains signal $\theta_i$ and sees government bails out the bank is

$$
\Delta(\theta_i; n(\cdot, B), B) = \frac{1}{\min(1, \theta_G^* + \bar{\eta}, \theta_i + \bar{\varepsilon}) - \max(0, \theta_i - \bar{\varepsilon})} \int_{\max(0, \theta_i - \bar{\varepsilon})}^{\min(1, \theta_G^* + \bar{\eta}, \theta_i + \bar{\varepsilon})} v(\theta; n(\theta, B), B) d\theta
$$

Lemma 2 Function $\Delta(\theta_i; n(\cdot, B), B)$ is continuous in $\theta_i$.

Proof Since functions $\frac{1}{\min(1, \theta_G^* + \bar{\eta}, \theta_i + \bar{\varepsilon}) - \max(0, \theta_i - \bar{\varepsilon})}$ and $\int_{\max(0, \theta_i - \bar{\varepsilon})}^{\min(1, \theta_G^* + \bar{\eta}, \theta_i + \bar{\varepsilon})} v(\theta; n(\theta, B), B) d\theta$ are bounded and continuous in $\theta_i$, their products are continuous in $\theta_i$.

Lemma 3 Function $\Delta(\theta_i + a; (n + a)(\cdot, B), B)$ is continuous and nonincreasing in $a$, where $(n + a)(\theta, B)$ denotes $n(\theta + a, B)$. It is strictly increasing in $a$ if $n(\theta) < \frac{1 + \bar{B}}{r}$.

Proof Function $\Delta(\theta_i + a; (n + a)(\cdot, B), B)$ is continuous in $a$ because $v$ is bounded, and $\Delta$ is an integral over a segment of $\theta$’s. As $a$ increases, agent $i$ sees the same distribution of $n$, but the expected $\theta$ is higher. Since $v(\theta; n, B)$ in equation (5) is non-decreasing in $\theta$, $\Delta(\theta_i + a; (n + a)(\cdot, B), B)$ is nondecreasing in $a$. If over the limits of integration there is a positive probability that $n(\theta) < \frac{1 + \bar{B}}{r}$, from (5), $v(\theta, n, B)$ strictly increases in $\theta$, $\Delta(\theta_i + a; (n + a)(\cdot, B), B)$ strictly increases in $a$. Q.E.D.

A2. There Exists a Unique Threshold Equilibrium

To prove there exists a unique threshold equilibrium, the only condition I need to prove is that given that all other patient agents use threshold strategy $\theta_B^*$, patient agent $i$ runs if and
only if $\theta_i < \theta^*_B$, and waits if and only if $\theta_i > \theta^*_B$, i.e.,

$$\Delta(\theta_i, n(\cdot, \theta^*_B), \bar{B}) < 0 \quad \forall \theta_i < \theta^*_B \quad (A.1)$$

$$\Delta(\theta_i, n(\cdot, \theta^*_B), \bar{B}) > 0 \quad \forall \theta_i > \theta^*_B \quad (A.2)$$

Since I restrict attention to threshold equilibrium, it means there exists at least one value of $\theta^*_B$ such that the following equation holds.

$$\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) = 0$$

There is exactly one value of $\theta^*_B$ that satisfies $\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) = 0$, which follows from the existence of the lower dominance region, and that $\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B})$ is continuous in $\theta^*_B$ and it is strictly increasing if $n(\theta) \leq \frac{1+\bar{B}}{\bar{r}}$, which can be directly derived from Lemma 2 and 3 in the Appendix.

**Lemma 4** $\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B})$ is increasing in $\theta^*_B$.

**Proof** From equation (5),

$$\Delta(\theta_i, n(\cdot, \theta^*_B), \bar{B}) = \frac{1}{\min(1, \theta^*_G + \bar{\eta}, \theta_i + \bar{\varepsilon}) - \max(0, \theta_i - \bar{\varepsilon})} \times \int_{\min(1, \theta^*_G + \bar{\eta}, \theta_i + \bar{\varepsilon})}^{\max(0, \theta_i - \bar{\varepsilon})} v(\theta, n(\theta, \theta^*_B), \bar{B}) d\theta$$

Suppose $\theta_i = \theta^*_B$. Then,

$$\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) = \frac{1}{\min(1, \theta^*_G + \bar{\eta}, \theta^*_B + \bar{\varepsilon}) - \max(0, \theta^*_B - \bar{\varepsilon})} \times \int_{\max(0, \theta^*_B - \bar{\varepsilon})}^{\min(1, \theta^*_G + \bar{\eta}, \theta^*_B + \bar{\varepsilon})} v(\theta, n(\theta, \theta^*_B), \bar{B}) d\theta \quad (A.3)$$
Define $\theta_T$ as the one satisfying $n(\theta_T) = \frac{1 + \bar{B}}{r}$. Then, from Equation (8),

$$\theta_T = \theta^*_B + \bar{\varepsilon} + (\lambda - \frac{1 + \bar{B}}{r}) \frac{2\bar{\varepsilon}}{1 - \lambda}$$

From (8) and (7), (A.3) can be rewritten as

$$\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) = q(\theta^*_B)(\int_{\theta_T(\theta^*_B)}^{\theta_T(\theta^*_B)} (-\frac{1 + \bar{B}}{nr} u(r)) d\theta$$

$$+ \int_{\theta_T(\theta^*_B)}^{\theta_h(\theta^*_B)} (\theta u(\frac{1 + \bar{B} - nr}{1 - n} R - u(r))d\theta)$$

where $\theta_l(\theta^*_B) = \max(0, \theta^*_B - \bar{\varepsilon})$, and $\theta_h(\theta^*_B) = \min(1, \theta^*_G + \bar{\eta}, \theta^*_B + \bar{\varepsilon})$. Define

$$W_1(\theta) = \int_{-\infty}^{\theta} -\frac{1 + \bar{B}}{nr} u(r) d\theta$$

$$W_2(\theta) = \int_{-\infty}^{\theta} \theta u(\frac{1 + \bar{B} - nr}{1 - n} R - u(r))d\theta$$

Then,

$$\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) = q(\theta^*_B)(W_1(\theta_T(\theta^*_B)) - W_1(\theta_l(\theta^*_B))) + W_2(\theta_h(\theta^*_B)) - W_2(\theta_T(\theta^*_B))$$

After differentiation with respect to $\theta^*_B$, it can be obtained that

$$\frac{\partial \Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B})}{\partial \theta^*_B} \geq 0$$

Q.E.D.

Thus, there is only one value of $\theta^*_B$ which is a candidate for a threshold equilibrium. To establish that it is indeed an equilibrium, it needs to show that given that $\Delta(\theta^*_B, n(\cdot, \theta^*_B), \bar{B}) = 0$ holds, (A.1) and (A.2) must also hold. The only thing I need to prove now is for $\forall \theta_i < \theta^*_B$, $\Delta'(\theta_i, n(\cdot, \theta^*_B), \bar{B}) < 0$. The case when $\theta_i > \theta^*_B$ is symmetry. Below I only show the case when
\( \theta_i, B \sim U[\theta_i - \bar{\varepsilon}, \theta_G + \bar{\eta}] \). Other cases can be derived in the same way.

\[
\Delta(\theta_B', n(\cdot, \theta_B'), \bar{B}) = \frac{1}{\theta_G + \bar{\eta} - (\theta_B' - \bar{\varepsilon})} \int_{\theta = \theta_B' - \bar{\varepsilon}}^{\theta_G + \bar{\eta}} v(\theta, n(\theta), \bar{B}) d\theta = 0
\]

\[
\Delta(\theta_i, n(\cdot, \theta_B'), \bar{B}) = \frac{1}{\theta_G + \bar{\eta} - (\theta_i - \bar{\varepsilon})} \int_{\theta = \theta_B' - \bar{\varepsilon}}^{\theta_G + \bar{\eta}} v(\theta, n(\theta), \bar{B}) d\theta
\]

\[
= \frac{1}{\theta_G + \bar{\eta} - (\theta_i^* - \bar{\varepsilon})} \int_{\theta = \theta_i^* - \bar{\varepsilon}}^{\theta_B' - \bar{\varepsilon}} v(\theta, n(\theta), \bar{B}) d\theta
\]

\[
+ \frac{1}{\theta_G + \bar{\eta} - (\theta_B' - \bar{\varepsilon})} \int_{\theta = \theta_B' - \bar{\varepsilon}}^{\theta_B' - \bar{\varepsilon}} v(\theta, n(\theta), \bar{B}) d\theta
\]

\[
= \frac{1}{\theta_G + \bar{\eta} - (\theta_B' - \bar{\varepsilon})} \int_{\theta = \theta_i - \bar{\varepsilon}}^{\theta_B' - \bar{\varepsilon}} v(\theta, n(\theta), \bar{B}) d\theta
\]

where the last step is from \( \frac{1}{\theta_G + \bar{\eta} - (\theta_B' - \bar{\varepsilon})} \int_{\theta = \theta_B' - \bar{\varepsilon}}^{\theta_G + \bar{\eta}} v(\theta_i, n(\cdot, \theta_B'), \bar{B}) d\theta = 0 \). \( v(\theta, n(\theta, \theta_B'), \bar{B}) \) have the single crossing property. The single crossing property can be obtained easily from (8). For \( \frac{1+\bar{B}}{r} \geq n \geq \lambda, \frac{\partial v(\theta, n(\theta, \theta_B'), \bar{B})}{\partial \theta} > 0 \), and \( v(1, \lambda, \bar{B}) > 0 \). For \( 1 \geq n \geq \frac{1+\bar{B}}{r}, \frac{\partial v(\theta, n(\theta, \theta_B'), \bar{B})}{\partial \theta} = 0 \), and \( v(\theta, n, \bar{B}) < 0 \). According to the single crossing property of \( v(\theta, n(\theta, \theta_B'), \bar{B}) \), i.e., \( v(\theta, n(\theta, \theta_B'), \bar{B}) \) crosses 0 only once, \( v(\theta, n(\theta, \theta_B'), \bar{B}) \) is positive for high values of \( \theta \), negative for low values of \( \theta \). Since the integration of \( v \) in the interval \([\theta_B' - \bar{\varepsilon}, \theta_G + \bar{\eta}]\) is 0, then the integration of \( v \) in the lower value interval \([\theta_i - \bar{\varepsilon}, \theta_B' - \bar{\varepsilon}]\) is negative. This proves that equation \( \Delta \) holds.

### 8 Appendix B

**Proposition 2** Conditional on the event \( B \), the probability of bank runs, \( \Pr(\theta \leq \theta_B^*|\bar{B} = \bar{B}) \), which is proportional to \( \theta_B^*(\bar{B}, \theta_G, \bar{\eta}) \), is a decreasing function of the bailout level \( \bar{B} \), the
government strategy cutoff $\theta_*^G$, and the precision of government signal $\bar{\eta}$.

$$\frac{\partial \theta^*_B(r, \bar{B}, \theta^*_G, \bar{\eta})}{\partial B}, \frac{\partial \theta^*_B(r, \bar{B}, \theta^*_G, \bar{\eta})}{\partial \theta^*_G}, \frac{\partial \theta^*_B(r, \bar{B}, \theta^*_G, \bar{\eta})}{\partial \bar{\eta}} \leq 0$$

**Proof.** Since the run threshold is $\theta^*_B$, i.e., investors who observe signals below $\theta^*_B$ choose to withdraw from the bank, the number of investors who will run is

$$n(\theta) = \lambda + (1 - \lambda) \frac{\theta^*_B - \theta - \bar{\varepsilon}}{2\bar{\varepsilon}} \tag{B.1}$$

where $\lambda$ is the number of impatient agents, $1 - \lambda$ is the total number of patient agents, and $\frac{\theta^*_B - \theta - \bar{\varepsilon}}{2\bar{\varepsilon}}$ is the fraction of patient agents who get signals below $\theta^*_B$.

From equation (3)

$$\Delta^r(\theta, n(\cdot), \bar{B}) = \frac{1}{\min(1, \theta^*_G + \bar{\eta}, \theta + \bar{\varepsilon}) - \max(0, \theta - \bar{\varepsilon})} \int_{\max(0, \theta - \bar{\varepsilon})}^{\min(1, \theta^*_G + \bar{\eta}, \theta + \bar{\varepsilon})} v(\theta, n(\theta), \bar{B}) d\theta$$

Since the investor who observes $\theta^*_B$ is indifferent with withdrawing and waiting, then,

$$\Delta^r(\theta^*_B, n(\cdot), \bar{B}) = 0$$

i.e.,

$$\int_{\max(0, \theta - \bar{\varepsilon})}^{\min(1, \theta^*_G + \bar{\eta}, \theta + \bar{\varepsilon})} v(\theta, n(\theta), \bar{B}) d\theta = 0$$

When $\theta^*_G + \bar{\eta} > \theta^*_B + \bar{\varepsilon}$, $\min(1, \theta^*_G + \bar{\eta}, \theta^*_B + \bar{\varepsilon}) = \min(1, \theta^*_B + \bar{\varepsilon})$. So (3) will become

$$\theta | \theta_B, B \sim U[\max(0, \theta^*_B - \bar{\varepsilon}), \min(1, \theta^*_B + \bar{\varepsilon})]$$

i.e., there is no information effect from government bailout announcement. Then,

$$\frac{\partial \theta^*_B(r, \bar{B}, \theta^*_G, \bar{\eta})}{\partial \theta^*_G}, \frac{\partial \theta^*_B(r, \bar{B}, \theta^*_G, \bar{\eta})}{\partial \bar{\eta}} = 0$$

When $\theta^*_G + \bar{\eta} \geq 1$, there is no information effect from government bailout announcement as
well.

\[ \int_{\theta_B^{*+\bar{\eta}}}^{\theta_G^{*+\bar{\eta}}} v(\theta, n(\theta), \bar{B}) d\theta = 0 \] (B.2)

From the expression of \( v \) in (B.1), when \( n \) exceed the maximum number of agents the bank can serve, \( \frac{1+B}{r} \), it takes the value of \( -\frac{1}{n(\theta)r}u(\theta) \), otherwise, it takes the value of \( \theta u(\frac{1+B-n(\theta)r}{1-n(\theta)r}) - u(\theta^*) \). I restrict parameter \( \lambda \) to satisfy

\[ \lambda + \frac{1-\lambda}{2} = \frac{1+B}{r} \]

so that, as long as the the fraction of patient investors who demand early withdrawal is larger than \( \frac{1}{2} \). Therefore, as long as \( \theta < \theta_B^* \), it takes the value of \( -\frac{1}{n(\theta)r}u(\theta) \), otherwise, it takes the value of \( \theta u(\frac{1+B-n(\theta)r}{1-n(\theta)r}) - u(\theta^*) \).

Based on the above analysis, (B.2) can be transformed to

\[ -\int_{\theta_B^{*+\bar{\eta}}}^{\theta_B} 1+\bar{B} u(\theta) d\theta + \int_{\theta_B^{*+\bar{\eta}}}^{\theta_G^{*+\bar{\eta}}} [\theta u(\frac{1+B-n(\theta)r}{1-n(\theta)r}) - u(\theta^*)] d\theta = 0 \]

\[ \int_{\theta_B^{*+\bar{\eta}}}^{\theta_B} 1+\bar{B} u(\theta) d\theta = \int_{\theta_B^{*+\bar{\eta}}}^{\theta_G^{*+\bar{\eta}}} [\theta u(\frac{1+B-n(\theta)r}{1-n(\theta)r}) - u(\theta^*)] d\theta \] (B.3)

According to (B.1), there is a one to one correspondence between \( n \) and \( \theta \). So (B.3) implies that

\[ \int_{1}^{(1+\lambda)/2} \frac{1+B}{nr} u(\theta^*) d\theta = \int_{(1+\lambda)/2}^{(1+\lambda)/(1-\lambda)(\bar{\theta}_B^{*+\bar{\eta}}+\bar{\eta}-\bar{\epsilon})} (-\frac{2\bar{\epsilon}(n-\lambda)}{1-\lambda} + \theta_B^*) u(\frac{1+B-nr}{1-n} - u(\theta^*)) d\theta \]

\[ \int_{1}^{(1+\lambda)/2} \frac{1+B}{nr} u(\theta^*) d\theta = \int_{(1+\lambda)/2}^{(1+\lambda)/(1-\lambda)(\bar{\theta}_B^{*+\bar{\eta}}+\bar{\eta}-\bar{\epsilon})} (-\frac{2\bar{\epsilon}(n-\lambda)}{1-\lambda} + \theta_B^*) u(\frac{1+B-nr}{1-n} - u(\theta^*)) d\theta \] (B.4)

because

\[ n(\theta_B^*+\bar{\eta}) = \lambda + (1-\lambda) \frac{\theta_B^* - (\theta_B^* + \bar{\eta} - \bar{\epsilon})}{2\bar{\epsilon}} \]
and
\[ \theta = -\frac{2\bar{\varepsilon}(n-\lambda)}{1-\lambda} + \theta^*_B + \bar{\varepsilon} \]

Define
\[ l(n, \theta^*_B) = (-\frac{2\bar{\varepsilon}(n-\lambda)}{1-\lambda} + \theta^*_B + \bar{\varepsilon})u(\frac{1 + \bar{B} - nr}{1 - n}R) - u(r) \]
\[ D(\theta^*_B, \theta^*_G) = \lambda + (1-\lambda)\frac{\theta^*_B - (\theta^*_G + \bar{\eta} - \bar{\varepsilon})}{2\bar{\varepsilon}} \]

Some algebra after differentiating [B.4] with respect to \( \theta^*_G \) gives
\[ (\int_{\lambda+(1-\lambda)\frac{\theta^*_G - \theta^*_B}{2\bar{\varepsilon}}}^{(1+\lambda)/2} u(\frac{1 + \bar{B} - nr}{1 - n}R)dn) \frac{\partial \theta^*_B}{\partial \theta^*_G} = 2l(D(\theta^*_B, \theta^*_G)) \frac{\partial D(\theta^*_B, \theta^*_G)}{\partial \theta^*_G} \]

Since
\[ \int_{\lambda+(1-\lambda)\frac{\theta^*_G - \theta^*_B}{2\bar{\varepsilon}}}^{(1+\lambda)/2} u(\frac{1 + \bar{B} - nr}{1 - n}R)dn > 0 \]
\[ 2l(D(\theta^*_B, \theta^*_G)) \frac{\partial D(\theta^*_B, \theta^*_G)}{\partial \theta^*_G} \leq 0 \]

then,
\[ \frac{\partial \theta^*_B(r, \bar{B}, \theta^*_G, \bar{\eta})}{\partial \theta^*_G} \leq 0 \]

The other two results follow in the same way.

**Proposition B3** Assume \( r = 1 \) and \( \bar{B} = 0 \). I consider the case when \( \theta \mid \theta_i, B \sim U[\theta_i - \bar{\varepsilon}, \theta^*_G + \bar{\eta}] \).

\( \theta^*_B \) can be obtained analytically as follows,
\[ \theta^*_B = \bar{\varepsilon} + 2\frac{u(1)}{u(R)} - (\theta^*_G + \bar{\eta}) \]

**Proof.** Since the investor who observes \( \theta^*_B \) is indifferent with withdrawing and waiting, then,
from \textit{(B.2)},

\[
\int_{\theta_B^*-\bar{\varepsilon}}^{\theta_G^*+\bar{\eta}} v(\theta, n(\theta), r) d\theta = 0
\]

When \( r = 1 \), it can be obtained as

\[
\int_{\theta_B^*-\bar{\varepsilon}}^{\theta_G^*+\bar{\eta}} \left[ \theta - \frac{u(1)}{u(R)} \right] d\theta = 0
\]

\[
\left[ \frac{\theta^2}{2} - \theta \frac{u(1)}{u(R)} \right]_{\theta_B^*-\bar{\varepsilon}}^{\theta_G^*+\bar{\eta}} = 0
\]

Therefore,

\[
\theta_B^* = \bar{\varepsilon} + \frac{2u(1)}{u(R)} - (\theta_G^* + \bar{\eta})
\]

**Proposition 3** Assume \( r = 1 \). For \( \bar{B} > 0 \), the probability of a bank run in the economy where the government commits never to bail out a bank, \( \Pr(\theta < \theta^*) \), is higher than the probability of bank runs in the economy where the government uses the above bailout strategy, i.e.,

\[
\Pr(\theta < \theta^*) > \Pr(\theta < \theta_B^* | \theta > \theta_G^*) \Pr(\theta < \theta_G^*) + \Pr(\theta < \theta_N^* | \theta \geq \theta_G^*) \Pr(\theta \geq \theta_N^*)
\]

where \( \theta^* \) is the unique equilibrium cutoff threshold for an economy where the government commits never to bail out a bank; both \( \theta_B^* \) and \( \theta_N^* \) are equilibrium threshold cutoffs in the economy where the government uses the above bailout strategy; \( \theta_B^* \) is the unique equilibrium cutoff threshold after a bailout announcement; \( \theta_N^* \) is the unique equilibrium cutoff threshold after no bailout announcement.

**Proof.** I first consider the case when \( \theta | \theta_i, B \sim U[\theta_i - \bar{\varepsilon}, \theta_G^* + \bar{\eta}] \). The other posterior cases can be obtained in the same way. The probabilities for the government to bail out the bank or
\[
\Pr(\theta_G < \theta^*_G) = \int_0^1 \int_{\theta_G - \theta}^{\bar{\eta}} \frac{1}{2\eta} d\eta \, d\theta
= \frac{\theta^*_G + \frac{1}{2} + \bar{\eta}}{2\eta}
\]

\[
\Pr(\theta_G \geq \theta^*_G) = \int_0^1 \int_{\theta}^{\theta^*_G - \theta} \frac{1}{2\eta} d\eta \, d\theta
= \frac{\bar{\eta} - \theta^*_G + \frac{1}{2}}{2\eta}
\]

Since the investor who observes \( \theta^*_B \) is indifferent with withdrawing and waiting, then, from (B.2)

\[
\int_{\theta^*_B - \bar{\varepsilon}}^{\theta^*_G + \bar{\eta}} v(\theta, n(\theta), r, B) d\theta = 0
\]

When \( r = 1 \), and \( B = 0 \), from Proposition B3, the above equation can be obtained as

\[
\int_{\theta^*_B - \bar{\varepsilon}}^{\theta^*_G + \bar{\eta}} (\theta - \frac{u(1)}{u(R)}) d\theta = 0
\]

\[
\frac{\theta^2}{2} - \theta \frac{u(1)}{u(R)} \bigg|_{\theta^*_B - \bar{\varepsilon}}^{\theta^*_G + \bar{\eta}} = 0
\]

Therefore,

\[
\theta^*_B = \bar{\varepsilon} + \frac{2u(1)}{u(R)} - (\theta^*_G + \bar{\eta})
\]

\( \theta^*_N \) can be obtained in the same way,

\[
\int_{\theta^*_G - \bar{\eta}}^{\theta^*_N + \bar{\varepsilon}} (\theta - \frac{u(1)}{u(R)}) d\theta = 0
\]

\[
\frac{\theta^2}{2} - \theta \frac{u(1)}{u(R)} \bigg|_{\theta^*_G - \bar{\eta}}^{\theta^*_N + \bar{\varepsilon}} = 0
\]
Then,

\[ \theta_N^* = -\bar{\varepsilon} + \frac{2u(1)}{u(R)} - (\theta_G^* - \bar{\eta}) \]

Hence,

\[
\text{Pr}(\theta < \theta^*) = \text{Pr}(\theta < \theta_B^* | \theta_G < \theta_G^*) \Pr(\theta_G < \theta_G^*) + \text{Pr}(\theta < \theta_N^* | \theta_G \geq \theta_G^*) \Pr(\theta_G \geq \theta_G^*) - \text{Pr}(\theta < \theta^*)
\]

\[
= [\bar{\varepsilon} + \frac{2u(1)}{u(R)} - (\theta_G^* + \bar{\eta})] \frac{\theta_G^* - \frac{1}{2} + \bar{\eta}}{2\bar{\eta}}
\]

\[
+ [-\bar{\varepsilon} + \frac{2u(1)}{u(R)} - (\theta_G^* - \bar{\eta})] \frac{\bar{\eta} - \theta_G^* + \frac{1}{2}}{2\bar{\eta}}
\]

\[
- \frac{u(1)}{u(R)}
\]

\[
= \frac{(\bar{\varepsilon} - \bar{\eta})(2\theta_G^* - 1)}{2\bar{\eta}} + \frac{2\bar{\eta}(2\theta^* - \theta_G^*)}{2\bar{\eta}} - \theta^*
\]

\[
= -\bar{\varepsilon} + 2\bar{\varepsilon}\theta_G^* - 4\bar{\eta}\theta_G^* + \bar{\eta}
\]

\[
< \bar{\eta} - \bar{\varepsilon} + (2\bar{\varepsilon} - 4\bar{\eta})(\frac{1}{2} + \bar{\eta})
\]

\[
= \bar{\eta}(-1 + 2(\bar{\varepsilon} - 2\bar{\eta}))
\]

\[
< 0
\]

Therefore\textsuperscript{32},

\[
\text{Pr}(\theta < \theta^*) > \text{Pr}(\theta < \theta_B^* | \theta_G < \theta_G^*) \Pr(\theta_G < \theta_G^*) + \text{Pr}(\theta < \theta_N^* | \theta_G \geq \theta_G^*) \Pr(\theta_G \geq \theta_G^*)
\]

References


\textsuperscript{32}Here I proved the case when \( B = 0 \). But it applies to the general case, because the right hand side is a decreasing function of \( B \).


