Corporate Credit Spreads and Business Cycles*

Seon Tae Kim†
Email: seon.kim@itam.mx
February 16, 2012

Abstract

I study the implications of fluctuations in corporate credit spreads for TFP and output. Motivated by that corporate credit spreads are countercyclical, I build a simple model in which the difference in default probabilities on corporate debts leads to the spread in interest rates paid by firms. In the model, firms finance their capital by issuing one-period bonds, and differ in the variance of the firm-level productivity, tightly linked to the default probability. The higher default probability for risky firms relative to safe firms results in the misallocation of capital: capital is allocated too little for risky firms and too much for safe firms. As the default probability for risky firms increases, the extent of misallocation becomes larger, and thereby TFP and output drop. I embed the basic mechanism into an otherwise standard growth model, and calibrate it to data for the post-war U.S. economy: the shock process for default probabilities for risky and safe firms are calibrated to match the fluctuations in the historical default rates of corporate bonds by credit ratings. In my benchmark simulation results, I find that shocks to the distribution of default probabilities across firms can account for about 60% of output fluctuations and 90% of TFP fluctuations in the U.S. economy. In addition, I find that the misallocation mechanism proposed in my paper is supported by the firm-level investment data.

I am greatly indebted to Richard Rogerson, my advisor, for his advice and encouragement. I also thank seminar participants at Arizona State University. All errors are mine.

†Contact: Department of Business Administration, School of Business, ITAM, D.F., Mexico.
1 Introduction

This paper studies a classical issue in business cycle research, to identify the shocks that drive the output fluctuations. The results in this literature indicate that the changes in TFP can account for large part of the output fluctuations, see, e.g., Chari et al. (2007), which motivated many researchers to explore the sources and mechanism of the TFP fluctuations. In particular, it has been paid much attention to how financial market imperfections interact with the real sector. For instance, it is well documented that corporate credit spreads fluctuate countercyclically: the yield spread between Baa and Aaa corporate bonds is negatively correlated with detrended output for the U.S. during the period 1964-2009. This paper studies the implications of fluctuations in corporate credit spreads for TFP and output in a DSGE framework.

For the link between the corporate credit spreads and TFP, I focus on the channel of resource misallocation: difference in the cost of capital across firms indicates the misallocation of capital. (See, e.g., Restuccia and Rogerson (2008), and Hsieh and Klenow (2009).) It is highly likely that capital is allocated too little for firms of low credit grade, labeled as risky firms, and too much for firms of high credit grade, labeled as safe firms, compared to the optimal allocation. As the corporate credit spreads increase, the extent of resource misallocation becomes larger, and thereby TFP drops. Put differently, an increase in corporate credit spreads leads to reallocation of capital away from risky firms toward safe firms.

I provide an evidence supporting the above hypothesis of capital-reallocation driven by corporate credit spreads by using the COMPUSTAT dataset on the firm-level investment during the period 1980-2007. I find that capital is indeed reallocated away from risky firms toward safe firms as the corporate credit spreads increase: the negative response of the firm-level ratio of investment to lagged capital to an increase in the Baa-Aaa corporate credit spread is stronger for a firm of higher risk, measured as the stock return volatility.

I build a simple model to study the impact of corporate credit spreads on TFP and output via the channel of resource misallocation. In my model, firms borrow their capital in the one-
period bond market. Importantly, firms differ in the risk (i.e., the variance) of their idiosyncratic productivity shocks, tightly linked to the firms’ default probability. That is, low risk firms are safe in the sense that their default probability is low while high risk firms are risky in the sense that their default probability is high. Default event is essentially exogenous, i.e., jump shock; a firm hit by the idiosyncratic jump shock goes out of business and loses a fraction of its capital, labeled as liquidation costs. The corporate bond market is imperfect in the sense that contingent bond is not available and that bondholders lose a positive fraction of their principal in the event of default, labeled as default costs. In this economy, the difference in the default probability between risky and safe firms results in the misallocation of capital: capital is allocated too little for risky firms and too much for safe firms compared to the optimal allocation.

In the model, as the risk, equivalent to the default probability, for risky firms increases, the credit spread increases, capital is reallocated away from risky firms toward safe firms, and output and TFP decrease. The induced decrease in TFP in turn leads to lower labor supply, consumption and investment. The key to the negative response of TFP to an increase in the default probability for risky firms is that the extent of misallocation of capital becomes larger: the ratio of capital allocated to a risky firm to a safe firm decreases further below the optimal level because of the higher spread in the cost of capital between risky and safe firms.

After I establish analytically the above basic mechanisms in a simple static model, I embed the basic mechanisms into an otherwise standard growth model, and calibrate it to data for the post-war U.S. economy. In particular, the shock process for risk distribution between risky and safe firms are calibrated to match the fluctuations in the historical default rates of corporate bonds by credit ratings. The properties of the calibrated dynamic model is analyzed numerically, and standard statistics studied in business cycle research are compared to the data.

In my benchmark simulation results, shocks to the risk distribution between risky and safe firms account for about 60% of output fluctuations and 90% of TFP fluctuations. As predicted by the static model, the default costs, measured as one minus recovery rate, plays an important role as an amplification device: the effects of those shocks on output and TFP are almost zero for the counterfactual case of the zero default costs, i.e., 100% recovery rate. In addition, I repeat
the same experiment for the case in which recovery rate is negatively correlated with default rates, i.e., low recovery rate for periods of high default rate, as in the data. In this case, effects on output and TFP are larger compared to the benchmark case of the constant recovery rate.

I turn to discuss the sensitivity of my results with respect to the capital structure. The key to the misallocation between a risky firm and a safe firm is the difference in the cost of capital between the two firms, for which the capital structure might be important; a risky firm can reduce, to some extent, its high cost of capital by substituting cheap internal cash flows for the expensive debt. Therefore, it is in order to discuss the implications for my results of the simplifying assumption of 100% debt financing imposed on my model.

For the impact of the alternative means of financing on my results, I focus on the internal financing because of the dominance of internal and debt financing for corporate investment in the data\(^2\). The capital structure indeed affects the quantitative, but not the qualitative, properties of my dynamic model. More specifically, I consider an extreme case in which firms are restricted to use internal financing and to roll over the sufficiently large (constant) debt so that every firm’s marginal capital should be always financed by internal cash flows and that risky firms are still faced by default risk. In this case, I find that fluctuations in the extent of misallocation, TFP and output are slightly larger than they are for the benchmark case of 100% debt financing\(^3\).

This paper mainly contributes to the literature studying the sources and mechanism of business cycles. This literature can be classified to two categories: one is the amplification mechanism of aggregate productivity shocks while the other considers impulses other than aggregate productivity shocks, for instance, shocks to the second moment of aggregate or idiosyncratic productivity as in Bloom (2009).

Compared to the first category of literature, i.e., the amplification mechanism, fluctuations in

\(^2\)See Myers (2001) for the discussion of corporate capital structure.

\(^3\)In the event of default, the capital held by the defaulting risky firm is taken over by the debt holders and therefore the equity holders lose 100% of the marginal capital. Meanwhile, for the case of 100% debt financing, the suppliers of capital, i.e., debt holders, lose less than 100% of their (marginal) capital in the event of default. Thus, the impact of an increase in the default risk on a risky firm’s cost of capital is larger for the case of internal financing combined with roll-over debt than it is for the benchmark case of 100% debt financing.
TFP and output are obtained in this paper without shocks to the aggregate productivity. In my model, even though the expected firm-level productivity is constant, endogenous variations in the extent of misallocation, driven by shocks to the risk distribution between risky and safe firms, lead to fluctuations in TFP and output. The default costs, one minus recovery rate, amplifies the response of TFP to those risk shocks. The shock to the cross-firm distribution of risk studied in this paper indeed generates business cycles including the TFP fluctuations in the sense that it has no impact on output and TFP for the counterfactual case of zero default costs.

For the second category of literature seeking sources of business cycles, this paper focuses on the dispersion, or the extent of heterogeneity, between risky and safe firms in the risk of the firm-level productivity. In contrast, many papers in this literature focuses on the common level of risk for every firm. (See, e.g., Bloom (2009), Bloom et al. (2009) and Arellano et al. (2010).)

This paper also contributes to another body of literature studying comovements in credit spreads and output. Gertler and Lown (1999) and Gilchrist and Zakrajsk (2011) studied the empirical relationship between credit-spread cycles and business cycles, which I study in a DSGE framework in this paper. Gomes and Schmid (2009) study the implications of aggregate productivity shocks for credit-spread cycles and business cycles in a general equilibrium framework; in their model, the firm-level capital is assumed to be fixed over time while endogenous fluctuations in the firm-level capital are the key mechanism studied in this paper.

**Related Literature**

This paper is also related with a recently growing body of literature studying cyclical fluctuations in resources-allocation and business cycles. For instance, Khan and Thomas (2010) and this paper share that TFP and output would fluctuate channeled by the reallocation of resources across firms. This paper differs from Khan and Thomas (2010) in the shocks and mechanism studied. This paper focuses on shocks to the distribution of default risk between risky and safe firms and the mechanism of capital reallocation between firms of different risk while Khan and Thomas (2010) consider shocks to the collateral value of capital and the mechanism of capital...
reallocation between firms of different size\textsuperscript{4}. Gourio(2011) studies the implications of shocks to the value of capital after installment for credit spreads and output; he focuses on disastrous shocks to the value of capital and the channel of intertemporal substitution of consumption and investment.

Lastly, this paper is related with Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) in a broad sense of studying the implications of credit market imperfections for business cycles. This paper differs from them in the results that output and TFP are highly positively correlated in my model but they are not in those two papers.

This paper is organized as follows. Section 2 provides an empirical evidence on the hypothesis of capital-reallocation driven by corporate credit spreads. Section 3 sets up the static model and presents the main analytic results. Section 4 embeds the previous static model to a standard growth model. Section 5 calibrates the dynamic model and discusses the simulation results. Section 6 extends the baseline dynamic model to the case of the internal-fiancing combined with roll-over debt, calibrates it and discusses the simulation results. Section 7 concludes.

\section{Corporate Credit Spreads and Cross-Section of Capital}

In this section, I empirically document how the firm-level investment is correlated with corporate credit spreads. I put my emphasis on the heterogeneity between risky and safe firms in the response of investment to changes in the corporate credit spreads. My empirical analysis of the firm-level investment will provide an evidence supporting my hypothesis of the credit-spread driven reallocation of capital between firms of different risk.

\textsuperscript{4}For corporate debt instruments, I focus on the bond market while Khan and Thomas (2010) focus on the loan market; for the corporate debts traded in the U.S., the bond market is larger than the loan market.
2.1 Augmented Two-Factor Model

I analyze the firm-level investment by using a two-factor model:

\[ E[i_t(j) \mid (F_{1,t}, F_{2,t})] = \alpha_0 + \lambda_1(j) \cdot F_{1,t} + \lambda_2(j) \cdot F_{2,t} \]

where the subscript \( t \) indexes time, \( i_t(j) \) denotes firm \( j \)'s investment, \( (F_{1,t}, F_{2,t}) \) denotes the two common factors, \( \alpha_0 \) is the constant term common to every firm and every period, and \( (\lambda_1(j), \lambda_2(j)) \) refers to the firm \( j \)'s two factor loadings. The above two-factor model of investment assumes that firms respond in their investment decision to the two common factors with differing sensitivity. For the two factors, I consider aggregate efficiency, proxied by real GDP per capita, and aggregate risk, proxied by the Moody’s Baa-Aaa corporate credit spreads.

The key in my empirical analysis is to identify the structure of the heterogeneity in factor loadings. That is, I address the questions of which firms are, in terms of investment, more procyclical and more sensitive to the economy-wide risk. I model the factor loadings as functions of two characteristics, firm size and firm-level risk. The firm size is likely to represent the firm’s efficiency while the firm-level risk is intended to measure the firm’s vulnerability, including default probability, to changes in the business environment.

For the functional form of factor loadings, as in Kim and Choi (2012), I assume that they are linear:

\[ E[\lambda_h(j) \mid x_t(j)] = \alpha_h + \beta_h x_t(j), \quad h \in \{1, 2\} \]

where \( x_t(j) \equiv (size_t(j), risk_t(j)) \) is the vector of firm \( j \)'s (time-varying) two characteristics, size and risk. In the above expression of the \( h \)-th factor loading for firm \( j \), \( \alpha_h \) refers to the component common to every firm, and \( \beta_h \) represents the impact of firm \( j \)'s size and risk on the firm’s own \( h \)-th factor loading.

In addition, I augment the basic two-factor model by adding additional independent variables out of several firm characteristics discussed in the literature studying the cross-section of the firm-level investment. Let the vector \( z_t(j) \) denote such characteristics variables, e.g., cash flows and leverage ratio, affecting firm \( j \)'s investment. The fully specified, augmented two-factor model
of the firm-level investment is given by:

\[ i_t(j) = \alpha_0 + \nu(j) + [\alpha_1 + \beta_1 x_t(j)] \cdot F_{1,t} + [\alpha_2 + \beta_2 x_t(j)] \cdot F_{2,t} + \gamma z_t(j) + \delta_1 \cdot t + \delta_2 \cdot t^2 + \epsilon_t(j) \]  

(1)

where \( \nu(j) \) refers to the fixed-effect term, \( \gamma \) refers to the sensitivity vector to the firm characteristic vector \( z_t(j) \), and \( \epsilon_t(j) \) is an i.i.d. Gaussian random variable. Lastly, as in Covas and Den Haan (2011), a linear and a quadratic time-trend terms are also included so that trend in a dependent variable, if any, should be controlled for.

2.2 Data Sources

One period is one calendar year. The first factor, per capita real GDP, is measured as the chain-weighted real GDP per capita provided by NIPA and then logged and detrended by the HP-Filter with the smoothing parameter equal to 100. For the second factor, the monthly Moody’s Baa-Aaa corporate credit spread is aggregated to the annual frequency and then logged and demeaned.

For the firm-level investment and characteristics, annual industrial COMPUSTAT files from 1980 to 2007 for publicly listed non-financial firms are used\(^5\). For the firm-level risk, CRSP daily stock returns for NYSE, AMEX and NASDAQ stocks between 1979 and 2007 are used.

Investment is measured as expenditures on ‘Property, Plant and Equipment Capital’ (COMPUSTAT data item #30) plus ‘Acquisiton’ (item #129). As in the literature, the ratio of the current investment to one-period lagged capital stock (item #8) is used for \( i_t(j) \) so that it is stationary:

\[ i_t(j) = \frac{INVESTMENT_t(j)}{CAPITAL_{t-1}(j)}. \]

For the firm size, the lagged book value of assets (item #6) is used as in the literature. To make the firm size stationary, I express it as the differential from the median firm size:

\[ size_t(j) = \frac{ASSETS_{t-1}(j) - ASSETS_{t-1}}{ASSETS_{t-1}} \]

\(^5\)Financial firms (SIC codes 6021,6022,6029,6035,6036) are excluded from the sample. In addition, I also exclude from the sample firms with a missing value for the book value of assets and firms most affected by the accounting change in 1988. See Covas and Den Haan (2011).
where $\overline{\text{ASSETS}}_{t-1}$ denotes the (cross-sectional) median book value of the lagged assets. The firm-level risk is measured as the volatility of the firm’s stock returns, lagged by 180 days, for one year as in Campbell and Taksler (2003). To control for the time-varying component of stock returns common to every firm, I express the firm-level risk as the difference of the firm’s stock return volatility from the (cross-sectional) median volatility of stock returns:

$$risk_t(j) = \text{Var}(\text{RETURN}_{t-\Delta}(j)) - \text{Var}(\text{RETURN}_{t-\Delta}(\cdot))$$

where $\text{Var}(\text{RETURN}_{t-\Delta}(\cdot))$ refers to the median volatility of stock returns, and subscript $t-\Delta$ represents that the volatility is calculated for the stock returns lagged by 180 days.

Next, for additional firm characteristics, as discussed in the literature, the following lagged variables are included: marginal product of capital $mpk_{t-1}(j)$, leverage ratio $lvg_{t-1}(j)$, cash flows $cf_{t-1}(j)$, and Tobin’s Q denoted by $q_{t-1}(j)$:

$$z_t(j) \equiv (mpk_{t-1}(j), lvg_{t-1}(j), cf_{t-1}(j), q_{t-1}(j))$$

where $mpk_{t-1}(j)$ is measured as the ratio of net sales (item #12) to capital stock, and $lvg_{t-1}(j)$ is measured as the ratio of the book value of total liabilities (item #181) to assets. Cash flow is measured as the income before extraordinary items (item #18) plus depreciation and amortization (item #14), and then expressed as the ratio to the lagged assets:

$$cf_t(j) = \text{CASH FLOW}_{t-1}(j)/\text{ASSET}_{t-2}(j).$$

Lastly, $q_{t-1}(j)$ is measured as the ratio of the market value of assets to the book value of assets.

As in the literature, I windsorize variables so that statistical results are not severely influenced by outliers. For each variable, observations higher than the 99th percentile is replaced by the the 99th percentile and observations lower than the first percentile is replaced by the first percentile.
Table 1: Panel Regression Results for Investment: Fixed Effect Model

<table>
<thead>
<tr>
<th>RGDP</th>
<th>Baa-Aaa Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><code>const</code></td>
<td><code>size</code></td>
</tr>
<tr>
<td>1.07</td>
<td>-0.007</td>
</tr>
<tr>
<td>(7.7)</td>
<td>(-1.7)</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.61</td>
</tr>
<tr>
<td>(46.7)</td>
<td>(-24.6)</td>
</tr>
</tbody>
</table>

Note: Numbers in the first line refer to estimates of coefficients and numbers in the bottom line refer to \( t \) statistics.

For each of the two factors, the factor loadings are decomposed into the three components: constant, the slope for size and the slope for risk. \( R^2 = 0.1037 \). Total number of observations = 45540 and the number of firms is 4502.

### 2.3 Estimation Results

Table 1 provides estimates of coefficients entering the equation (1). For a firm of median size and median risk, the investment is highly positively correlated with real GDP, dubbed as *output*, and highly negatively correlated with the Baa-Aaa corporate credit spread, dubbed as *credit spread*. Moreover, all of the four characteristic variables \((mpk, lvg, cf, q)\) are also significantly correlated with the firm-level investment.

Now, I turn to discuss how the heterogeneity in the two factor loadings are correlated with the two firm-level characteristics, size and risk. On the one hand, the impact of the firm size on the factor loading neither for output nor for the credit spread is significant. On the other hand, the negative impact of the firm-level risk on the factor loading for the credit spread is significant; as the credit spread increases, a high-risk firm decreases its investment more than a low-risk firm does. Put differently, for an increase in the credit spread, capital is reallocated away from risky firms toward safe firms, of which implications for TFP and output will be studied analytically in a general equilibrium framework in the next section of the static model and then quantified in the later section of the dynamic model.

---

6The volatility of the firm’s stock returns are calculated with the 6-month lead relative to the investment to capture the investors’ ex-ante expectations.

7Market value of assets = (item #181) + (item #25) \( \times \) (item #199) + (item #10)+(item #19). See Covas and Den Haan (2011).
3 The Basic Mechanism: A Static Analysis

In this section, I study a simple static model in order to illustrate the key economic mechanism, reallocation of capital between risky and safe firms, which links an increase in the risk for a risky firm relative to a safe firm to decreases in TFP and output. I also highlight the role of financial frictions as an amplification device.

3.1 Environment

Technology

There is a single final good. Capital is used in the production of the final good and can be converted to the final good one-for-one.

There is a continuum of measure one of firms that produce the (homogeneous) final good. The stochastic production function of firm $i \in [0,1]$ is given by:

$$y(i) = z(i)[k(i)]^\alpha$$

where $\alpha \in (0,1)$ is the returns-to-scale parameter and $z(i)$ is the firm-level productivity.

A key feature of the model is that there is heterogeneity across firms in the extent of riskiness. For simplicity, I assume that a half of the firms has no risk, i.e., $z(i) = 1, \forall i \in [0,1/2]$, and label them as safe firms; the other half of the firms are assumed to have a positive risk, i.e., they draw $z(i)$ independently from an identical distribution of the mean equal to one, and they are labeled as risky firms. It is a public information if firm $i$ is safe or risky.

Note that two types of firms differ only in terms of the variance, but not the expected value, of the firm-level productivity. To facilitate exposition, I assume that risky firm $i$’s productivity, $z(i)$, is a random variable with two possible outcomes given by:

$$z(i) = \begin{cases} 0 & \text{w.p. } \nu \\ 1/[1 - \nu] & \text{w.p. } 1 - \nu \end{cases}, \quad \forall i \in (1/2,1]$$

where $\nu \in (0,1)$ is the probability of drawing zero productivity for a risky firm. The event of $z(i) = 0$ is essentially intended to capture the jump shock leading to default such that investors
lose a large portion of their principal invested to the defaulting firm. Risky firms are exposed to the higher probability of such a jump shock than safe firms are.

The expected productivity for a risky firm is equal to the productivity for a safe firm as mentioned earlier:

\[ E[z(i)] = 1, \quad \forall i \in (1/2, 1]. \]

Note that the variance of risky firm \( i \)'s productivity is increasing in \( \nu \):

\[ \text{Var}(z(i)) = \frac{\nu}{1-\nu}, \quad \forall i \in (1/2, 1]. \]

The parameter \( \nu \), again representing the probability of a risky firm's jump-default shock, will play a key role in this paper. I will study the consequences of an increase in the (default) risk for risky firms relative to safe firms, i.e., an increase in \( \nu \), for capital-allocation and prices in this economy.

**Preferences**

Preferences of the representative household is given by the expected utility function \( E[u(c)] \) where \( u(\cdot) \) is concave, increasing and \( C^2 \).

The household is endowed with \( \kappa > 0 \) units of capital and owns all of the firms.

**Timing**

The timing of events for this economy is as follows. There are two subperiods: initial and final. In the initial subperiod, the household supplies capital to firms via debt contracts. In the final subperiod, productivities for risky firms \( \{z(i)\} \) are realized, production takes place, capital is depreciated by the rate of \( \delta \in (0, 1) \), default/repayment decisions are made, profits are transferred to the household, confiscated output of the defaulting firms are distributed to the lender/household, and consumption takes place.

**Debt Contracts**

A firm finances its capital with 100% debt financing. By a debt contract, I mean a contractual
arrangement such that firm \( i \) borrows \( k(i) \) units of capital and promises to pay back to the lender(s) the undepreciated capital \([1 - \delta]k(i)\) plus \( r(i) \) units of the final good per unit of capital:

\[
 r(i) = \begin{cases} 
  r^S, & \forall i \in [0, 1/2] \\
  r^R, & \forall i \in (1/2, 1] 
\end{cases}
\]

where \( r^S \) is the interest rate for a safe firm and \( r^R \) is the interest rate for a risky firm.

If a firm chooses to default, then the lenders take back their undepreciated capital, and confiscate the defaulting firm’s output. A defaulting firm’s confiscated output is distributed to the lenders according to the share of an individual lender’s capital supply to the defaulting firm. A key assumption is that the lenders lose a fraction \( \tau \geq 0 \) of undepreciated capital where \( \tau \) represents costs of collecting back capital from a defaulting firm, e.g., court-related bankruptcy costs, losses of capital incurred by poor management during the confiscation procedures and so on.

I close this section by briefly describing the market structure of debt contracts. The debt-contract market is competitive in the following sense:

1. lenders, i.e., the household, take as given the market interest rates, expected confiscated output per unit of capital, the expected probability of default for an individual firm, and can supply as much capital as they want.

2. borrowers, i.e., firms, take as given the market interest rates, and can borrow as much capital as they want.

The above two conditions essentially say that neither individual lenders nor individual firms affect the market interest rates and that individual lenders cannot affect the decision rules of firms. An individual lender-household is assumed to be infinitesimally small such that its lending behavior cannot affect the default decision rule of an individual borrower-firm for this economy.

**A Firm’s Problem**
I write the ex-post profit of firm \( i \) in the final subperiod as:

\[
\pi(r(i), k(i), z(i)) = \max_{x \in \{0,1\}} \left\{ (1 - x) \cdot \left[ z(i)[k(i)]^{\alpha} - r(i)k(i) \right] \right\}
\]

where \( x \) denotes the firm’s choice whether to default \( x = 1 \) or not \( x = 0 \).

First, it is obvious that safe firm \( i \) never defaults given that the firm faces no uncertainty and that the firm would lose its output if it defaults. Therefore, safe firm \( i \)’s optimal decision rule of borrowing capital is given by:

\[
\alpha[k(i)]^{\alpha-1} = r^S.
\]

It follows that the default probability for safe firm \( i \) is zero in equilibrium, which the lenders rationally expect.

Second, it is obvious that risky firm \( i \) defaults and its profit is equal to zero in the event of \( z(i) = 0 \) for any given positive values of \( r^R \) and \( k(i) \). It is also obvious given that the marginal product of capital conditional on \( z(i) = 1/[1 - \nu] \) is infinite at \( k(i) = 0 \) that risky firm \( i \) does not default in the event of \( z(i) = 1/[1 - \nu] \) and, taking it into its consideration, chooses \( k(i) \) to maximize its expected profit given by:

\[
\max_{k(i)} \left\{ \nu \cdot 0 + [1 - \nu] \cdot \left[ \frac{1}{1 - \nu} \cdot [k(i)]^{\alpha} - r^R k(i) \right] \right\}
\]

which implies that risky firm \( i \)’s optimal \( k(i) \) is given such that its profit conditional on \( z(i) = 1/[1 - \nu] \) should be maximized:\(^8\):

\[
\alpha \cdot \frac{1}{1 - \nu} \cdot [k(i)]^{\alpha-1} = r^R.
\]

Lastly, as only the risky firms hit by the jump shocks \( z(i) = 0 \) default in equilibrium, it is straightforward that a defaulting risky firm’s output is zero in equilibrium and that the default probability for risky firm \( i \) is equal to \( \nu \). Furthermore, by the law of large numbers, the ex-post profit in the event of \( z(i) = 0 \) becomes zero by the option of default, which provides the firm with an insurance, to some extent, against the negative profit in the event of \( z(i) = 0 \) in the absence of such an option.

\(^8\)The key here is that the firm’s profit in the event of \( z(i) = 0 \) becomes zero by the option of default, which provides the firm with an insurance, to some extent, against the negative profit in the event of \( z(i) = 0 \) in the absence of such an option.
The Household’s Problem

As mentioned earlier, taking as given the market interest rates, expected confiscated output per unit of capital, default probabilities and total profit, the household maximizes its expected utility subject to its budget constraint. More specifically, the household makes its decision of how much capital to supply to firms in the initial subperiod prior to realization of idiosyncratic productivities for risky firms \{z(i)\}. In the final subperiod, when \{z(i)\} for risky firms are realized, the household receives interest payments, undepreciated capital and total profit, which it uses for consumption of the final good\(^9\).

I present formally the household’s problem as follows:

\[
\max_{c,\{k(i)\}} \left\{ E[u(c)] \right\} \\
\text{s.t. } c = \pi + \int_0^{1/2} \left[ 1 - \delta + r^S \right] k(i)di + \int_{1/2}^1 \left[ \left[ 1 - \chi_{z(i)=0} \right] [1 - \delta + r^R] + \chi_{z(i)=0} [1 - \delta] [1 - \tau] \right] k(i)di \\
+ \left[ 1 - \delta \right] \left[ \kappa - \int_0^1 k(i)di \right], \\
\int_0^1 k(i)di \leq \kappa, \quad k(i) \geq 0, \forall i \in [0, 1]
\]

where \(\pi\) refers to the total profit and \(\chi_{z(i)=0}\) is an indicator on whether \(z(i)\) is zero or not:

\[
\chi_{z(i)=0} = \begin{cases} 
1 & \text{if } z(i) = 0 \\
0 & \text{otherwise} 
\end{cases}
\]

Recall that risky firm \(i\) defaults if and only if in the event of drawing \(z(i) = 0\), which is incorporated into the above household’s budget constraint via the indicator function \(\chi_{z(i)=0}\). In the event of \(z(i) = 0\), risky firm \(i\) defaults and the household loses a \(\tau\) fraction of undepreciated capital as well as interests for the capital supplied to the defaulting firm \(i\).

\(^9\)The household receives zero confiscated output for a defaulting risky firm in equilibrium as discussed earlier.
In this economy, there is no aggregate uncertainty and hence consumption is independent of the realized gross returns to individual debts\textsuperscript{10}. It follows that the household purchases only the debts of which expected gross returns are the highest. In order for the supply of capital to be positive for both risky and safe firms, which is the equilibrium outcome as I will show later on, the expected gross returns should be equalized between the two types of debts:

\[ 1 - \delta + r^S = [1 - \nu \left( 1 - \delta + r^R \right) + \nu [1 - \delta] [1 - \tau] \]

which simplifies to: \[ r^S = [1 - \nu] r^R - \nu \tau [1 - \delta] \]. It is straightforward that two parameters \( \nu \) and \( \tau \) are important for the equilibrium interest rates \( r^S \) and \( r^R \), and hence they are also important for \( r^R - r^S \), the equilibrium corporate credit spread. I will focus on the effects of increases in \( \nu \) and/or \( \tau \) on the credit spread in the comparative statics section later.

**Resource Constraint**

Capital can be converted to the final good one-for-one. Taking into consideration depreciation and losses of capital in the event of default, I write the resource constraint for this economy as:

\[ c = y + [1 - \delta] \left[ \frac{1}{2} k^S + \frac{1}{2} k^R \left[ [1 - \nu] + \nu [1 - \tau] \right] \right] + [1 - \delta] \left[ \kappa - \frac{1}{2} k^S - \frac{1}{2} k^R \right] \]

where \( y \) is aggregate output given by:

\[ y = \frac{1}{2} [k^S]^\alpha + \frac{1}{2} \left[ \nu \cdot 0 + [1 - \nu] \cdot \frac{1}{1 - \nu} \right] [k^R]^\alpha \]

which simplifies to: \( y = \frac{1}{2} \left[ [k^S]^\alpha + [k^R]^\alpha \right] \). The resource constraint says that consumption is equal to aggregate output plus capital net depreciation and losses owing to default costs.

### 3.2 Equilibrium

Competitive equilibrium for this economy is a list of allocation \((c, y, \{k(i)\})\), interest rates\((r^S, r^R)\), expected default probability for risky firms equal to \( \nu \), the expected confiscated output per unit

\textsuperscript{10}More formally, the equilibrium capital allocation is identical across the same type of firms as discussed earlier, which implies that the equilibrium consumption is independent of realizations of \( z(i) \)'s for risky firms because any idiosyncratic risk is diversified away in equilibrium.
of capital for a risky firm equal to zero, and total profit $\pi$ that satisfy:

1. Each firm maximizes its expected profit.

2. The household maximizes its expected utility subject to its budget constraint.

3. The expected default probability and expected confiscated output per unit of capital for risky firms are consistent with the decision rules of risky firms.

4. Total profit $\pi$ is consistent with the decision rules of safe and risky firms.

5. Markets clear.

3.3 Results

In this section, I characterize the equilibrium allocation and prices of capital and then analyze the effects of an increase in the default-risk for risky firms $\nu$ on aggregate output and productivity.

**A Pseudo Planner’s Problem**

I show that the equilibrium allocation can be obtained as the solution to the following Pseudo Planner’s Problem:

$$
\max_{c,k_S \geq 0, k_R \geq 0} \{ E[u(c)] \}
$$

s.t. $c = \frac{1}{2} [k_S]^\alpha + \frac{1}{2} [k_R]^\alpha + \frac{1}{2} [1 - \delta] k_S + \frac{1}{2} [1 - \delta] k_R \left[1 - \nu + \nu [1 - \tau] \right] + [1 - \delta] \left[ \kappa - \frac{1}{2} k_S - \frac{1}{2} k_R \right],$

$$\frac{1}{2} k_S + \frac{1}{2} k_R \leq \kappa.$$

Note that losses of capital by $\tau$ for an occurrence of $z(i) = 0$ are incorporated to the above problem so that one of the key financial frictions for this economy, the default costs $\tau$, is maintained. Furthermore, I have already simplified the above pseudo planner’s problem by imposing the obvious condition for the optimal decision rule of capital allocation such that capital allocation
should be identical across the same type of firms\textsuperscript{11}:
\[ k(i) = k_S \geq 0, \forall i \in [0, 1/2], \quad k(i) = k_R \geq 0, \forall i \in (1/2, 1]. \]

**Proposition 1.** 1. A unique solution to the above pseudo planner’s problem exists and is identical to the equilibrium allocation

2. The solution is characterized by:
\[
\alpha[k^S]^\alpha-1 = \alpha[k^R]^\alpha-1 - \nu \tau [1 - \delta],
\]
\[
\frac{1}{2} [k^S + k^R] = \kappa.
\]

It follows that $k^S > k^R > 0$ for $\tau \in (0, 1]$ and that $k^S = k^R = \kappa$ for $\tau = 0$.

**Proof.** See the appendix. \qed

A key result is that capital is allocated such that the expected gross returns are equalized across firms. The positive probability of the jump-default shock for a risky firm $\nu > 0$ implies the positive expected losses of capital via the friction $\tau$ for a risky firm while there are no such losses for a safe firm. Given that expected output from a given amount of capital is the same between a risky firm and a safe firm, it follows that the expected gross return curve of a risky firm is below that of a safe firm. The optimality condition for capital allocation, i.e., equalization of the expected gross returns across firms, implies that capital should be allocated more to a safe firm than it is to a risky firm for the case of $\tau > 0$. It is also interesting that there is no misallocation of capital for the case of $\tau = 0$.

So far, I have focused on the equilibrium allocation. I next turn to discuss the equilibrium credit spread. Lemma 1 states that credit spread is tightly related to the equilibrium capital allocation.

\textsuperscript{11}It is obvious in the sense that the same type of firms are homogeneous at the initial subperiod, when the decision for capital allocation is made.
Lemma 1. The equilibrium corporate credit spread $r^R - r^S$ is given by:

$$r^R - r^S = \alpha \frac{1}{1 - \nu} [2\kappa - k^S]^{\alpha-1} - \alpha [k^S]^{\alpha-1}$$

which is strictly positive because $k^S \geq k^R = 2\kappa - k^S > 0$ and $\nu \in (0, 1)$.

Proof. It follows from the market clearing condition and the earlier results that $\alpha \cdot [k(i)]^{\alpha-1} = r^S$ for safe firm $i$ and $\alpha \cdot 1/[1 - \nu] \cdot [k(i')]^{\alpha-1} = r^R$ for risky firm $i'$. 

The equilibrium condition for gross returns for risky and safe debts is simplified to:

$$r^S - [1 - \nu]r^R = -\nu \tau [1 - \delta]$$

which is, combined with the condition for the equilibrium capital demand of firms, simplified to:

$$\alpha [k^S]^{\alpha-1} - \alpha [2\kappa - k^S]^{\alpha-1} = -\nu \tau [1 - \delta].$$

It follows that the two parameters $\nu$ and $\tau$ are important for the equilibrium capital allocation and thereby they are also important for the equilibrium interest rates and credit spread.

Comparative Statics

The primary objective of this paper is to assess the consequences of an increase in the corporate credit spread on aggregate output. As noted above, two key parameters that influence the equilibrium corporate credit spread are $\nu$ and $\tau$. In this section, I present comparative statics results concerning how $\nu$ and $\tau$ affect equilibrium allocations and credit spread. As we will see in the next section, these effects will continue to be present in the dynamic analysis.

I show that, in the case of $\tau > 0$, in response to increases in both $\nu$ and $\tau$, the credit spread increases, capital is reallocated away from risky firms toward safe firms, and aggregate output decreases. Because inputs are constant in the equilibrium, it follows that TFP will also be decreasing in both $\nu$ and $\tau$.

This result is intuitive. In response to increases in either $\nu$ or $\tau$, the expected gross returns to risky debts relative to safe debts shift downward because the expected losses of capital increases.
To equalize the expected gross returns between safe and risky debts, capital should be reallocated from risky firms to safe firms. Aggregate output decreases because more capital is allocated to safe firms, which at the margin have lower (expected) productivity since \( k^S > k^R \).

Proposition 2 reports the effects of an increase in \( \nu \).

**Proposition 2.**

1. The credit spread is increasing in \( \nu \): 
\[
d[\nu R - \nu S]/d\nu > 0.
\]

2. \( k^S \) is increasing in \( \nu \) and \( k^R \) is decreasing in \( \nu \) for \( \tau \in (0, 1] \):
\[
dk^S/d\nu > 0, \quad dk^R/d\nu < 0, \quad \forall \tau \in (0, 1].
\]

3. Aggregate output is decreasing in \( \nu \) for \( \tau \in (0, 1] \):
\[
dy/d\nu = \frac{1}{2} \left[ -\nu \tau [1 - \delta] \right] dk^S/d\nu < 0, \quad \forall \tau \in (0, 1].
\]

It follows that TFP, defined as \( y/\kappa \), also decreases in \( \nu \) for \( \tau \in (0, 1] \).

**Proof.** Recall that I have already derived, in the earlier discussion of the lemma 1, the equilibrium condition for the household’s capital supply such that the expected gross returns are equalized between safe and risky debts:
\[
\alpha [k^S]^{\alpha - 1} - \alpha [2\kappa - k^S]^{\alpha - 1} = -\nu \tau [1 - \delta].
\]

From the above equation, for the case of \( \tau \in (0, 1] \), it is obvious that \( k^S \) is strictly increasing in \( \nu \) because of \( \alpha \in (0, 1) \), which in turn implies that \( k^R = [2\kappa - k^S] \) is strictly decreasing in \( \nu \). It follows that \( r^S \) is strictly decreasing in \( \nu \) and \( r^R \) is strictly increasing in \( \nu \), and hence the credit spread, \( r^R - r^S \), is strictly increasing in \( \nu \). Lastly, 
\[
dy/d\nu = \frac{1}{2} \left[ \alpha [k^S]^{\alpha - 1} - \alpha [2\kappa - k^S]^{\alpha - 1} \right] dk^S/d\nu
\]

is simplified to 
\[
dy/d\nu = \frac{1}{2} \left[ -\nu \tau [1 - \delta] \right] dk^S/d\nu,
\]

which is obviously negative for \( \tau > 0 \) and zero for \( \tau = 0 \). □

The third part of the above result shows that the response of \( y \) to an increase in \( \nu \) is determined by the endogenous reallocation of capital away from risky firms toward safe firms. This reallocation effect on output is always non-positive. But, it is important to note that it is
zero for the case of \( \tau = 0 \) and negative whenever \( \tau > 0 \). The reason for this is that equalization of the expected gross returns between risky and safe debts implies that the expected marginal product of capital is equalized between risky and safe firms for the case of \( \tau = 0 \). In this case, the marginal effect of misallocation is zero. To see this, recall the first equation in the above proof of the proposition 2:

\[
\alpha [k^S]^{\alpha-1} - \alpha [2\kappa - k^S]^{\alpha-1} = -\nu \tau [1 - \delta]
\]

which simplifies, in the case of \( \tau = 0 \), to:

\[
\alpha [k^S]^{\alpha-1} - [1 - \nu] \alpha \cdot 1/[1 - \nu] \cdot [k^R]^{\alpha-1} = 0,
\]

i.e., no difference in the expected marginal product of capital between the two types of firms\(^\text{12}\). This result shows the importance of \( \tau \) in determining the response of \( y \) to an increase in \( \nu \), which is quite intuitive given that the larger \( \tau \), the larger the current extent of misallocation; therefore, the impact of one unit increase in \( \nu \) on TFP becomes larger for higher \( \tau \).

Next, proposition 3 states the results concerning the effects of an increase in \( \tau \) on the equilibrium allocations and credit spread.

**Proposition 3.**

1. The credit spread is increasing in \( \tau \): \( d[r^R - r^S]/d\tau > 0 \).

2. \( k^S \) is increasing in \( \tau \in (0, 1] \) and \( k^R \) is decreasing in \( \tau \in (0, 1] \):

\[
dk^S/d\tau > 0, \quad dk^R/d\tau < 0, \quad \forall \tau \in (0, 1].
\]

3. Aggregate output is decreasing in \( \tau \in (0, 1] \): \( dy/d\tau < 0, \quad \forall \tau \in (0, 1] \).

It follows that TFP, defined as \( y/\kappa \), also decreases in \( \tau \in (0, 1] \).

**Proof.** It is essentially the same with the proof of the proposition 2. Note again the equilibrium condition for capital allocation:

\[
\alpha [k^S]^{\alpha-1} - \alpha [2\kappa - k^S]^{\alpha-1} = -\nu \tau [1 - \delta].
\]

From the above equation, it is obvious that \( k^S \) is strictly increasing in \( \tau \in (0, 1] \), from which all of the other parts of the proposition 3 follow, see the proof of the proposition 2.\(\square\)

\(^{12}\)Moreover, in the case of \( \tau = 0 \), an increase in \( \nu \) leads to no reallocation of capital as I have already shown, in proposition 1, that allocation of capital is independent of \( \nu \) in this case: \( k^S = k^R = \kappa, \forall \nu \in (0, 1) \) for \( \tau = 0 \).
The proposition 3 says that an increase in \( \tau \) leads to reallocation of capital away from risky firms toward safe firms and a decrease in aggregate output, which is quite similar to the effects of an increase in \( \nu \). The key is that an increase in \( \tau \) leads to an increase in the expected losses of capital for risky debts, which is the same as for the effect of an increase in \( \nu \), and thereby induces a downward shift of the expected gross returns to risky debts. It follows that, in response to an increase in \( \tau \), capital is reallocated from risky firms to safe firms, and aggregate output and productivity decrease.

Note that an increase in default costs \( \tau \) represents a decrease in the recovery rate on defaulted debt, which could be interpreted as a negative financial shock to the collateral value of a firm as studied by Khan and Thomas (2010). I do not explore further to compare my result to the literature because I do not focus on studying the effects of financial shocks. (See, e.g., Khan and Thomas (2010) and references therein for this issue.)

In this section, I illustrated the key economic mechanism which links an increase in the default-risk for a risky firm \( \nu \) to decreases in output and productivity. The key mechanism is that an increase in \( \nu \) makes the extent of misallocation higher in the presence of the financial friction, in particular, positive default costs \( \tau > 0 \). I obtained the equilibrium condition for capital allocation such that the expected gross returns should be equalized between safe and risky debts. I presented comparative statics results concerning how the default-risk for a risky firm \( \nu \) and the default costs \( \tau > 0 \) affect the equilibrium allocation and credit spread.

In short, an increase in \( \nu \), again the default-risk for a risky firm relative to a safe firm, leads to an increase in the credit spread and reallocation of capital away from risky firms toward safe firms, which in turn results in decreases in output and productivity. In addition, I also have shown the importance of financial frictions, in particular, the default costs \( \tau > 0 \). First, I have shown that the effect on aggregate output of the endogenous capital reallocation, if any, induced by an increase in \( \nu \) is negligible as long as there is no default costs \( \tau = 0 \). Second, I have also shown that an increase in \( \tau > 0 \) itself leads to the results similar to the effects of an increase in \( \nu \), which implies that the effects of an increase in \( \nu \) are larger as long as it also leads to an increase in \( \tau \) at the same time as in the data.
4 Dynamic Model

In this section, I embed the previous static model into an otherwise standard growth model and use the dynamic model to assess the quantitative implications of stochastic fluctuations in the extent of uncertainty for risky firms relative to safe firms.

4.1 Environment

Two features are added in this dynamic model. First, the household makes decisions on consumption/leisure and consumption/investment. Second, the probability of the jump-default shock for a risky firm evolves stochastically so that its default probability and the credit spread fluctuate stochastically.

Technology

There is a continuum of safe firms of measure $\lambda \in (0, 1)$ and a continuum of risky firms of measure $1 - \lambda$: firm $i$ is safe for $i \in [0, \lambda]$ and is risky for $i \in (\lambda, 1]$. As in the previous section, a firm’s type is a public information. For the simplicity of exposition, I abstract from the changes in the measure of firms and assume that the measure of firms is constant$^{13}$.

Firm $i$’s production function is the same as in the previous static analysis except that labor services are also used in the production of the final good. First, safe firm $i$ faces no risk as in the previous static model and its production function is given by:

$$y_t(i) = \left[k_t(i)^{\alpha} h_t(i)^{1-\alpha}\right]^{\theta}, \quad \forall i \in [0, \lambda], \quad \alpha \in (0, 1), \quad \theta \in (0, 1)$$

where $y_t(i)$ is firm $i$’s output, $k_t(i)$ is capital services employed by firm $i$ and $h_t(i)$ is labor services hired by firm $i$.

Next, risky firm $i$ faces positive risk as in the previous static model; its production function

---

$^{13}$This assumption seems reasonable for the purpose of assessing the quantitative implications for TFP and output of shocks to the risk-distribution between risky and safe firms because it is well known in the literature that entering and/or exiting firms are small and thereby their effects on output are small.
is stochastic and given by:

\[ y_t(i) = z_t(i) \left[ k_t(i)^\theta [h_t(i)]^{1-\theta} \right]^\alpha, \quad \forall i \in (\lambda, 1) \]

where \( z_t(i) \) is an idiosyncratic productivity shock and independent across \( i \in (\lambda, 1) \). The probability distribution of \( z_t(i) \) is given by:

\[
\begin{cases} 
0 & \text{w.p. } \nu_t \\
1/[1 - \nu_t] & \text{w.p. } (1 - \nu_t) 
\end{cases}, \quad \forall i \in (\lambda, 1)
\]

where \( \nu_t \) refers to the time-varying probability of the jump shock for a risky firm, intended to capture the default risk as in the corporate-bond-pricing literature, see, e.g., Leland (2006). If risky firm \( i \) draws the jump-shock \( z_t(i) = 0 \) in period \( t \), then the firm’s productivity thereafter is permanently equal to zero:

\[
\text{If } z_t(i) = 0, \text{ then } z_{t+s}(i) = 0, \forall s > 0,
\]

and this firm is immediately liquidated and replaced by a new risky firm, which is indexed by \( i \) from the period \( t + 1 \) and thereafter until it is liquidated. Importantly, liquidation is costly; \( \bar{\tau} \geq 0 \), labeled as liquidation costs, fraction of the remaining capital held by the liquidated firm is lost.

I turn to discuss how the jump-shock probability \( \nu_t \) evolves over time. \( \nu_t \) is stochastic and follows the first order Markov process; let \( f(\nu_{t+1}|\nu_t) \) denote the density of \( \nu_{t+1} \) conditional on \( \nu_t \). By definition, \( \nu_{t+1} \) should be restricted to belong to the unit interval: \( \nu_{t+1} \in [0, 1] \). That is, \( \nu_{t+1} \) has a finite support, a subset of the unit interval.

**Household**

Preferences of the infinitely-lived representative household is given by:

\[
E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) - \psi \frac{h_t^{1+\omega}}{1+\omega} \right] \right], \quad \beta \in (0, 1)
\]

where \( E_0[\cdot] \) is the expectation operator in \( t = 0 \), \( c_t \) is consumption in period \( t \), \( h_t \) is labor supply, and \( \beta \) is the discount factor.
The household is endowed with one unit of time every period and \( k_0 \) units of capital in the initial period \( t = 0 \). The household owns all of the firms.

**Timing**

The timing of events for this economy is as follows. In each period \( t \), there are two subperiods, labeled initial and final, as in the earlier analysis. In the initial subperiod, \( \nu_t \) is realized, which everyone observes, the household supplies to firms labor services via (defaultable) wage contracts and capital stock via (defaultable) debt contracts. In the final subperiod, idiosyncratic productivities for risky firms \( \{z_t(i)\} \) are realized, production takes place, firms make default decisions, profits are transferred to the household, confiscated outputs of the defaulting firms are distributed to the suppliers of labor and capital, and the household makes its consumption/investment decision.

**Debt and Wage Contracts**

In this section, I describe the structure of debt and wage contracts. I maintain the key features of the debt contracts in the previous static analysis. I also maintain to assume that a firm finances its capital with 100% debt financing as in the previous section. After finishing to describe the dynamic model, I will discuss the implications of my simplifying assumption of 100% debt financing for the properties of the dynamic model.

Regarding wage contracts, I assume that wage payments to the suppliers of labor services are prone to default risk similarly to the debt contracts. Upon default of a firm’s promises on wage contracts and debt contracts\(^\text{14}\), the defaulting firm’s output is confiscated and distributed to suppliers of labor services and capital. I will discuss debt and wage contracts in more detail in what follows.

A debt contract is, as standard in the literature, a one-period contract specifying promised repayment and punishment-to-default. The details of the one-period debt contracts in this

\(^{14}\text{As we will see later on, a firm either defaults on both of the two contracts or does not default on either in equilibrium.}\)
section are essentially the same as for the previous static analysis; only key features of debt contracts are discussed here. Firm \( i \) promises to pay back to the lenders undepreciated capital, \([1 - \delta]k_t(i)\), and \( r_t(i) \) units of the final good per unit of capital:

\[
\begin{cases}
  r_t^S, & \forall i \in [0, \lambda] \\
  r_t^R, & \forall i \in (\lambda, 1]
\end{cases}
\]

where \( r_t^S \) refers to the interest rate for a safe firm and \( r_t^R \) does the same for a risky firm.

Renegotiation is not allowed and hence the firm has two options: either fully repay or default. In the event that the firm defaults on its promises, the suppliers of capital take back the undepreciated capital and confiscate the firm’s output. The arrangement for default will be discussed in more detail later.

Next, I turn to describe wage contracts. A wage contract is a one-period contract specifying promised wages and punishment-to-default. Firm \( i \) hires \( h_t(i) \) units of labor in the initial sub-period of \( t \). In return for hiring \( h_t(i) \), the firm promises to pay \( w_t(i) \) units of the final good per unit of labor services in the final subperiod of \( t \):

\[
\begin{cases}
  w_t^S, & \forall i \in [0, \lambda] \\
  w_t^R, & \forall i \in (\lambda, 1]
\end{cases}
\]

where \( w_t^S \) is the wage rate for a safe firm and \( w_t^R \) is the (promised) wage rate for a risky firm. Note that wage rates differ between a safe firm and a risky firm because of the difference in the default risk, just as interest rates differ for the same reason.

Similarly to debt contracts, renegotiation is not allowed for the wage contract and hence the firm has two options: either fully repay or default. In the event that the firm defaults on its promises, the suppliers of labor confiscate the firm’s output.

I describe the arrangement for default as follows. For simplicity, I focus on the case in which a firm defaults on both its debt and wage contracts, which suffices for my analysis because in this model a firm either defaults on both of these contracts or does not default on either. If a firm defaults on both its debt and wage contracts, then the suppliers of labor lose wages and the suppliers of capital lose interest payments, \( r_t(i)k_t(i) \), and a fraction \( \tau \), labeled as default costs,
of undepreciated capital, \( \tau [1 - \delta] k_t(i) \). As mentioned earlier, the defaulting firm’s output is confiscated and distributed to the suppliers of labor services and suppliers of capital. Specifically, I assume that the suppliers of labor and suppliers of capital split the confiscated output in the proportions \((1 - \theta : \theta)\). Once the confiscated output is split between the suppliers of labor and suppliers of capital, individual suppliers of labor and/or capital receive the confiscated output based on their shares in labor and/or capital for the defaulting firm.

**A Firm’s Problem**

A firm’s per-period problem is essentially the same as for the previous static analysis except that a firm, in this section, makes a decision on labor as well as capital. I summarize the results about a firm’s problem and default decision rules rather than discuss them in detail.

First, firm \( i \) defaults if and only if \( z_t(i) = 0 \), from which it immediately follows that a safe firm never defaults and that, by the law of large numbers, \( \nu_t \) fraction of risky firms default in period \( t \). It follows that, in equilibrium, the (expected) confiscated output per unit of labor and per unit capital for a risky firm are equal to zero.

Second, because risky firm \( i \)’s profit in the event of drawing \( z_t(i) = 0 \) is zero, the firm’s optimal quantity of labor and capital services are given such that the firm maximizes its profit conditional on \( z_t(i) = 1 / [1 - \nu_t] \), and they are characterized in recursive form as: \( \forall i \in (\lambda, 1] \),

\[
\alpha \theta \frac{1}{1 - \nu} [k(i, \nu, K)]^{\alpha \theta - 1} [h(i, \nu, K)]^{\alpha [1 - \theta]} = r^R(\nu, K),
\]

\[
\alpha [1 - \theta] \frac{1}{1 - \nu} [k(i, \nu, K)]^{\alpha \theta} [h(i, \nu, K)]^{\alpha [1 - \theta] - 1} = w^R(\nu, K)
\]

where \( r^R(\nu, K) \) and \( w^R(\nu, K) \) are interest and wage rate functions for a risky firm, respectively, \( k(i, \nu, K) \) and \( h(i, \nu, K) \) are risky firm \( i \)’s policy functions for capital and labor, respectively, and \( K \) is aggregate capital stock. \( (\nu, K) \) is the aggregate state vector.

Similarly, safe firm \( i \)’s optimal quantity of labor and capital services are characterized as:

\[
\alpha \theta [k(i, \nu, K)]^{\alpha \theta - 1} [h(i, \nu, K)]^{\alpha [1 - \theta]} = r^S(\nu, K), \quad \forall i \in [0, \lambda],
\]

\[
\alpha [1 - \theta] [k(i, \nu, K)]^{\alpha \theta} [h(i, \nu, K)]^{\alpha [1 - \theta] - 1} = w^S(\nu, K) \quad \forall i \in [0, \lambda]
\]
where \( r^S(\nu, K) \) and \( w^S(\nu, K) \) are interest and wage rate functions for a safe firm.

Lastly, allocations of capital and labor are identical to the same type of firms:

\[
k(i, \nu, K) = \begin{cases} 
  k^S(\nu, K) & \text{for } i \in [0, \lambda] \\
  k^R(\nu, K) & \text{for } i \in (\lambda, 1]
\end{cases}, \quad h(i, \nu, K) = \begin{cases} 
  h^S(\nu, K) & \text{for } i \in [0, \lambda] \\
  h^R(\nu, K) & \text{for } i \in (\lambda, 1]
\end{cases}.
\]

The Household’s Problem

For simplicity, I focus on the case in which the household’s supply of capital and labor are symmetric across firms of the same type. In this case, the household’s problem is given in recursive form as:

\[
v(k, \nu, K) = \max_{c, k', h^S, h^R, k^S, k^R} \left\{ \left[ \log(c) - \psi \frac{h^{1+\omega}}{1 + \omega} \right] + \beta \int v(k', \nu', K'(\nu, K)) f(\nu'|\nu) d\nu' \right\}
\]

\[
s.t. \quad c + k' = \pi(\nu, K) + \lambda \left[ w^S(\nu, K)h^S + [1 - \delta + r^S(\nu, K)]k^S \right] \\
+ [1 - \lambda] \left[ w^R(\nu, K)h^R + [1 - \delta + r^R(\nu, K)]k^R \right] + \nu[1 - \delta][1 - \tau]k^R,
\]

\[
\lambda h^S + [1 - \lambda]h^R = h \in [0, 1], \quad \lambda k^S + [1 - \lambda]k^R = k,
\]

\[
h^S \geq 0, h^R \geq 0, \quad k^S \geq 0, k^R \geq 0
\]

where \( k \) denotes the household’s current capital, \( v(k, \nu, K) \) is the value function for the recursive problem of the household, \( K'(\nu, K) \) is the law of motion for aggregate capital, and \( \pi(\nu, K) \) is the aggregate profit function. As discussed earlier, the household takes as given the expected default probability for a risky firm equal to \( \nu \) and the expected confiscated output for a defaulting risky firm equal to zero.

The above household’s problem says that, taking as given price functions, default probability for risky firms, and the expected confiscated output per unit of labor and per unit of capital\(^{15}\) for a risky firm in the event of default, the household makes decisions on its next-period capital \( k' \) and its supply of labor and capital to firms \( (h^S, h^R, k^S, k^R) \).

\(^{15}\)They are zero as discussed earlier.
The equilibrium condition for the household’s optimal capital-supply to safe and risky firms is that the expected gross returns should be the same between the two types of debts for the same reason as discussed in the previous section:

\[ 1 - \delta + r^S(\nu, K) = [1 - \nu] \left[ 1 - \delta + r^R(\nu, K) \right] + \nu [1 - \delta][1 - \tau]. \]

Similarly, in equilibrium, the expected wage payments per unit of labor should be the same between the two types of wage contracts:

\[ w^S(\nu, K) = [1 - \nu] w^R(\nu, K). \]

**Resource Constraint**

The resource constraint for this economy is given by:

\[ c_t + k_{t+1} = y_t + [1 - \delta] \left[ \lambda k^S_t + [1 - \lambda] \left[ 1 - \nu_t \tau \right] k^R_t \right], \]

where \( y_t \) is aggregate output in period \( t \) given by:

\[ y_t = \lambda \left[ \left( k^S_t \right)^{\theta} \left( h^S_t \right)^{1-\theta} \right]^{\alpha} + [1 - \lambda] \left[ \left( k^R_t \right)^{\theta} \left( h^R_t \right)^{1-\theta} \right]^{\alpha}. \]

The resource constraint simply says that consumption plus the next-period capital equals to output plus undepreciated capital net default-losses.

### 4.2 Equilibrium

I study a recursive competitive equilibrium, which is a list consisting of the value function \( v(k, \nu, K) \), policy functions \( c(k, \nu, K) \), \( k'(k, \nu, K) \), \( h^S(k, \nu, K) \), \( h^R(k, \nu, K) \), \( k^S(k, \nu, K) \), \( k^R(k, \nu, K) \), interest rate functions \( r^S(\nu, K) \) and \( r^R(\nu, K) \), wage rate functions \( w^S(\nu, K) \) and \( w^R(\nu, K) \), the expected default probability for a risky firm equal to \( \nu \), the expected confiscated output in the event of default for a risky firm equal to zero, total profit function \( \pi(\nu, K) \) and aggregate capital transition function \( K'(\nu, K) \) that satisfy:

1. \( v(k, \nu, K) \) solves the Bellman equation for the household’s problem, and policy functions \( c(k, \nu, K) \), \( k'(k, \nu, K) \), \( h^S(k, \nu, K) \), \( h^R(k, \nu, K) \), \( k^S(k, \nu, K) \) and \( k^R(k, \nu, K) \) are optimal decision rules of such a problem.
2. \( \forall (\nu, K) \in (0, 1) \times [0, \infty) \), \( h^S(K, \nu, K) \) and \( k^S(K, \nu, K) \) are the optimal quantity of labor and capital for safe firm \( i \in [0, \lambda] \), and \( h^R(K, \nu, K) \) and \( k^R(K, \nu, K) \) are the optimal quantity of labor and capital for risky firm \( i \in (\lambda, 1] \).

3. Markets clear: \( \forall (\nu, K) \in (0, 1) \times [0, \infty) \),

\[
c(K, \nu, K) + k'(K, \nu, K) = y(\nu, K) + [1 - \delta] \left[ \lambda k^S(K, \nu, K) + [1 - \lambda][1 - \nu\tau] k^R(K, \nu, K) \right]
\]

where output function \( y(\nu, K) \) is given by:

\[
y(\nu, K) = \lambda \left[ [k^S(K, \nu, K)]^\alpha [h^S(K, \nu, K)]^{1-\theta} \right] + [1 - \lambda] \left[ [k^R(K, \nu, K)]^\theta [h^R(K, \nu, K)]^{1-\theta} \right]^\alpha.
\]

4. \( \pi(\nu, K) \) is consistent with decision rules of individual firms:

\[
\pi(\nu, K) = [1 - \alpha] y(\nu, K), \quad \forall (\nu, K) \in (0, 1) \times [0, \infty).
\]

5. \( K'(\nu, K) \) is consistent with \( k'(k, \nu, K) \):

\[
K'(\nu, K) = k'(K, \nu, K), \quad \forall (\nu, K) \in (0, 1) \times [0, \infty).
\]

### 4.3 Discussion: Capital Structure and Reallocation of Capital

In this subsection, I discuss how the capital structure other than 100% debt financing, assumed for my benchmark case, would affect the properties of the dynamic model. In particular, I focus on how the capital structure would alter the response of the capital allocation between a risky and a safe firm to an increase in the risk for a risky firm, the key mechanism for TFP and output.

On the one hand, as will be shown below, the capital structure does not affect the qualitative properties of the dynamic model. On the other hand, for the quantitative aspects of the model, the capital structure may be important because the availability of financial instruments other than debt, e.g., internal cash flows, would affect the relative cost of capital for a risky firm to that of a safe firm, the crux in determining the extent of misallocation of resources.

For the impact of the alternative means of financing on the properties of the dynamic model, I focus on the internal financing because of the dominance of internal and debt financing for
corporate investment in the data. (See Myers (2001).) The capital structure indeed affects
the quantitative properties of my dynamic model as will be discussed in the later section of
simulation results. More specifically, I consider an extreme case in which firms are restricted to
use internal financing and to roll over the sufficiently large (constant) debt so that every firm’s
marginal capital should be always financed by internal cash flows and that risky firms hit by
the jump-shock should still default. It turns out that fluctuations in key variables are larger for
this case than they are in the benchmark case of 100% debt financing, which will be discussed
in detail in the later section of the extended model of internal financing.

Lastly, there is another issue of debt financing: the availability of long-term debts. It is
well known that long-term debt provides the advantage of hedging against the interest-rate risk
compared to the one-period debt studied in this paper. If all of the risky firms use long-term
debts instead of one-period debts, then not all of but only a fraction of risky firms, whose debts
are matured, would be exposed to the risk of the stochastic cost of capital. Put differently, in
this case, a smaller extensive margin would be operative for the capital reallocation. At the same
time, as the longer-maturity induces the larger response in the cumulative default probability,
and hence, the required interest rate, to one unit increase in the per-period default probability, a
larger intensive margin would be operative for the capital reallocation. It is interesting, although
beyond the scope of this paper, to study which of the two opposing effects of a longer-maturity
debt on capital reallocation would be dominant, which I leave for future work.

5 Quantitative Analysis

In this section, I assess the quantitative consequences of shocks to $\nu$ to the U.S. economy during
the period 1964-2009. For this purpose, I calibrate the dynamic model presented in the previous
section by using NIPA data on the U.S. economy and Moody’s data on corporate bond market,
and I calibrate the shock process for $\nu$ by targeting fluctuations in historical default rates of
corporate bonds by credit ratings, in particular, for risky bonds. I find that these shocks are an
importance source of fluctuations in the U.S. economy.
5.1 U.S. Economy And Corporate Bond Market

In this subsection, I document facts about fluctuations in real economic aggregates and fluctuations in the yields and default rates of corporate bonds over the period 1964-2009 as well as recovery rates for the period 1982-2009.

Data

For the U.S. economy, I use NIPA to measure output, consumption and investment, and the BLS to measure labor. Based on the NIPA investment, capital stock is constructed by the perpetual inventory method as in the literature\textsuperscript{16}. TFP is measured as the Solow residual. Both NIPA and BLS series are for the period 1964-2009 at quarterly frequency. As in the literature, all series are detrended; they are logged and then HP-Filtered with the smoothing parameter of 1600.

I describe in more detail how I measure NIPA and BLS variables in the data. Output is measured as real GDP. For consumption, real personal consumption expenditures on non-durables and services are used. For investment, real private investment is used\textsuperscript{17}. All of the above NIPA variables are measured by the chain-weighted method and seasonally adjusted\textsuperscript{18}. For labor, I use the BLS aggregate hours index.

For the corporate bond market, I use Moody’s global corporate bond dataset\textsuperscript{19}. Moody’s uses letter grades to classify corporate bonds based on creditworthiness. This dataset provides annualized nominal yields for bonds of only two different letter grades, Aaa and Baa, at monthly frequency. I convert monthly series of annual yields for the two graded bonds to quarterly series by computing averages over the quarter, and calculate real yields by subtracting the concurrent quarterly growth rates of the BLS CPI index. The spread in real yields between Baa and Aaa grade corporate bonds is taken as my credit spread index. Moody’s dataset also provides annual default rates for all letter grades\textsuperscript{20} where the default rate is measured by the ratio of the num-

\textsuperscript{16}Detailed procedure is given in the appendix.
\textsuperscript{17}NIPA item #8, headed ‘Gross private domestic investment: Fixed Investment’.
\textsuperscript{18}See the appendix, Data Sources, for more detail.
\textsuperscript{19}See Moody’s (2010).
\textsuperscript{20}Specifically, seven letter grades as follows: Aaa, Aa, A, Baa, Ba, B and Caa-C.
ber of issuers defaulted for a year relative to the total number of issuers of the bond-cohorts\textsuperscript{21}. Recovery rates are measured as the ratio of a bond price in the aftermath of the event of default relative to the price prior to the event of default, which is interpretable as the loss rate of a bond in the event of default\textsuperscript{22}.

**U.S. Business Cycles And Credit Spread Cycles**

Figure 1 shows fluctuations in real GDP and the Baa-Aaa corporate credit spread for the U.S. economy.

![Figure 1: Output And an Index of Corporate Credit Spreads](image)

*Note*: Both of the two series are quarterly, logged and HP-filtered. The solid line refers to output while the dashed line refers to the Baa-Aaa corporate credit spread.

From Figure 1, we can see that detrended output is negatively correlated with the detrended Baa-Aaa corporate credit spread over the period 1964-2009. Table 2 presents descriptive statistics for key aggregates and the credit spread over the period 1964-2009.

The main message of table 2 is that the corporate credit spread is substantially negatively

\textsuperscript{21}Note that it is not a volume-weighted default rate as used by Giesecke et al. (2010) and Altman and Karlin (2010). Neither of these two papers provides default rates conditional on creditworthiness.

\textsuperscript{22}By construction, recovery rates are measured by the issued years prior to default. For a given letter grade (unsecured) bond, Moody’s provides 5 measures for the cases of 1-5 issued years prior to default: I use the sample mean of those 5 measures as my recovery rate.
Table 2: Statistics of Key Aggregates and the Corporate Credit Spread

<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th>h</th>
<th>k</th>
<th>TFP</th>
<th>c</th>
<th>i</th>
<th>Baa-Aaa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(x)/\sigma(y)$</td>
<td>1.00</td>
<td>1.27</td>
<td>0.34</td>
<td>0.53</td>
<td>0.54</td>
<td>3.35</td>
<td>13.77</td>
</tr>
<tr>
<td>$\text{corr}(x, y)$</td>
<td>1.00</td>
<td>0.88</td>
<td>0.15</td>
<td>0.45</td>
<td>0.84</td>
<td>0.91</td>
<td>-0.58</td>
</tr>
<tr>
<td>$\text{corr}(x, \text{Baa-Aaa})$</td>
<td>-0.58</td>
<td>-0.49</td>
<td>0.25</td>
<td>-0.37</td>
<td>-0.52</td>
<td>-0.65</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*Note:* The percentage standard deviation of output $\sigma(y)$ is 1.56. $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable $x$ to that of output $\sigma(y)$. $\text{corr}(x, y)$ refers to the correlation coefficient of a variable $x$ and output while $\text{corr}(x, \text{Baa-Aaa})$ does the same for the Baa-Aaa credit spread. The column headed ‘Baa-Aaa’ refers the Baa-Aaa corporate credit spread. All variables are log deviations from trend correlated with output, TFP and labor over the period 1964-2009\(^{23}\).

I next turn to discuss fluctuations in the default rates of corporate bonds. These fluctuations are of particular importance because I will use them to measure shocks to the default probability for a risky firm $\nu_t$ in the model. Below I summarize fluctuations in the default rates for safe and risky corporate bonds. I begin by discussing how to classify corporate bonds of different letter grades into safe and risky bonds, and then present statistics of the default rates for safe and risky corporate bonds.

As mentioned earlier, Moody’s provides yields for only two letter grade bonds, Aaa and Baa, and the Baa-Aaa yield spread is used as my index of the corporate credit spread. Note that Aaa bonds are the safest grade according to the Moody’s grade system. I use Baa letter grade as the threshold in classifying bonds into the safe and risky categories. There are three letter grades higher than Baa grade, i.e., Aaa, Aa and A, and I classify them as safe bonds. And I classify Baa, Ba and B grade bonds as risky bonds\(^{24}\). Then default rates of safe and risky bonds are

\(^{23}\text{See Gertler and Lown (1999) for more detailed discussion of empirical relationships between the ‘High-Yield/Aaa’ corporate credit spread and output over the U.S. business cycles since 1980s.}\)

\(^{24}\text{Note that I exclude the Moody’s Caa-C grade bonds from my consideration here because default rates of Caa-C grade bonds are substantially higher than those of Baa, Ba and B grade bonds. For instance, the default rate of B grade bonds, the riskiest out of the three risky bonds, is 4.7% on average for the period 1964-2009 while the default rate of Caa grade bonds is 20.2% on average for the same period, which is larger by a factor}\)
calculated as the cross-sectional averages of corporate bonds within the categories of safe and risky bonds, see the table 3²⁵.

Table 3: Annual Default Rates of Corporate Bonds During the Period 1964-2009

<table>
<thead>
<tr>
<th>Grade</th>
<th>Safe</th>
<th>Risky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.02%</td>
<td>1.93%</td>
</tr>
<tr>
<td>Std.</td>
<td>0.07%</td>
<td>1.76%</td>
</tr>
</tbody>
</table>

*Note:* The line headed ‘Mean’ presents sample means of historical default rates of safe and risky bonds at annual frequency and the line headed ‘Std.’ does the same for sample standard deviations.

The main result of table 3 is that default rates for risky bonds are high and volatile while default rates for safe bonds are almost constant quite close to zero. For simplicity, from now on, I assume that default rates of safe bonds are zero.

### 5.2 Calibration

In this subsection, I discuss how to calibrate parameter values of the dynamic model. As in the literature, I calibrate parameters of the dynamic model by targeting long-run averages of key aggregates of the U.S. economy during the period 1964-2009; parameter values are chosen so that the model economy in the steady state, essentially νₜ being constant, is consistent with the data for key statistics.

One period in the model is one quarter in the data. The benchmark parameter values are listed in table 4.

First, I set the steady state ν to the average quarterly default rates for risky corporate bonds, of about 4 than the B grade bond default rate and by a factor of 10 than the average default rate of the three risky bonds combined. Lastly, it seems that the Caa-C grade bonds are not important for my purpose studying the implications of credit spread fluctuations on the output fluctuations because the share of the Caa-C grade bonds issuers in the corporate bond market is small, about 4% on average in 1997-2000.

²⁵See the appendix, Data Sources, for more detailed discussion on how default rates are calculated by the Moody’s.
### Table 4: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>TARGET</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu ): Prob. of ( z(i) = 0 ) for Risky Firm ( i )</td>
<td>.0048</td>
<td>Default Rate of Risky Bonds</td>
</tr>
<tr>
<td>( \delta ): Depreciation Rate</td>
<td>.015</td>
<td>Benchmark</td>
</tr>
<tr>
<td>( \tau ): Default Losses of Undepreciated Capital</td>
<td>.599</td>
<td>Recovery Rate of Risky Bonds</td>
</tr>
<tr>
<td>( \lambda ): Measure of safe firms</td>
<td>.289</td>
<td>Volume-Share of Safe Bonds</td>
</tr>
<tr>
<td>( \beta ): Discount Factor</td>
<td>.9909</td>
<td>Real Return to Aaa Bonds</td>
</tr>
<tr>
<td>( \alpha ): Returns to Scale</td>
<td>.87</td>
<td>Khan and Thomas (2010)</td>
</tr>
<tr>
<td>( \theta ): Capital Share</td>
<td>.33</td>
<td>Investment/GDP</td>
</tr>
<tr>
<td>( \omega ): Curvature of Labor-Disutility</td>
<td>.30</td>
<td>Benchmark</td>
</tr>
<tr>
<td>( \psi ): Level of Labor-Disutility</td>
<td>3.32</td>
<td>( h_{ss} = .33 )</td>
</tr>
</tbody>
</table>

1.93% at annual frequency. Setting the depreciation rate of capital \( \delta \) to .015, i.e., 6% at annual frequency, as in the literature, I choose default costs \( \tau = .599 \) by targeting the recovery rate for defaulted risky corporate bonds, about 40% on average during the period 1982-2009\(^{26}\):

Second, I choose the measure of safe firms \( \lambda = 0.389 \) by targeting the relative size of safe bonds to risky bonds:

\[
\frac{\lambda k^S}{\lambda k^S + [1 - \lambda]k^R} = \frac{\text{size of newly issued safe bonds}}{\text{total size of newly issued bonds}}
\]

where \( \lambda k^S \) refers to the size of newly issued safe bonds and \( [1 - \lambda]k^R \) does the same for risky bonds. For bond size, because of the data availability, the flow measure, i.e., volume of new issues, rather than the stock measure, i.e., the outstanding volume, is used. According to this measure, the relative size of safe bonds to the total is about 47.7% during the period 1997-2000\(^{27}\).

Third, discount factor \( \beta \) is set to .9909 to match the quarterly real-returns to the Moody’s Aaa

---

\(^{26}\)In the model, the recovery rate is measured as \([1 - \tau]\) if \( \delta = 0 \). For more detailed discussion, see the appendix, Calibration.

\(^{27}\)Given interest rates and the values of \( \alpha \) and \( \theta \), I can back out \( k^S \) and \( k^R \) and then solve for \( \lambda \) the above equation. Data on the size of corporate bonds by credit ratings is available to me for short periods, 1997-2000. See the appendix, Calibration, for more detailed discussion.
grade corporate bonds, about .92% on average during the period 1964-2009\textsuperscript{28}. For the returns-to-scale parameter, as in Khan and Thomas (2010), I choose \( \alpha = 0.87 \) as my benchmark. In turn, as is common in the literature, I choose \( \theta = .33 \) as my benchmark. This implies, in the model, about 29% non-proprietary capital income share in GDP and 58% non-proprietary labor income share in GDP, investment to GDP ratio equal to about 18% and capital to annual GDP ratio equal to 2.8, which is broadly consistent with the business cycle literature.

Lastly, I calibrate two preferences parameters: \( \omega \), the curvature parameter of disutility from labor, and \( \psi \), the level parameter of disutility from labor. I choose \( \omega = 0.30 \) as the benchmark case, which results in the volatility of labor about 70% relative to the output volatility in my simulation results where the counterpart in the data is about 127% as shown earlier. Then I choose \( \psi = 3.32 \) by targeting steady state labor supply of .33.

**Shock Process for \( \nu \)**

Next, I discuss how to calibrate the shock process for \( \nu \), which is important because the shock process for \( \nu \) is critical for the quantitative properties of my dynamic model. The key here is to match the distribution of the historical defaults rates for risky corporate bonds, featured by the clustering around zero; to incorporate into the model the risky-bond’s default rates clustered around zero, I posit that the probability distribution of \( \nu' \) conditional on \( \nu \) is a mixture of uniform and truncated normal distribution:

\[
f(\nu'|\nu) = \phi(\nu) \cdot f^U(\nu'|\nu) + (1 - \phi(\nu)) \cdot f^{TN}(\nu'|\nu)
\]

where \( f(\nu'|\nu) \) denotes the pdf of \( \nu' \) conditional on \( \nu \), \( f^U(\nu'|\nu) \) refers to the pdf for a uniform distribution over \([0, \nu]\), with \( \nu \in (0, 1) \), and \( f^{TN}(\nu'|\nu) \) refers to the pdf of a normal distribution truncated at 0 and 1. That is, the conditional density function \( f(\nu'|\nu) \) is the weighted average of the two (conditional) distributions \( f^U(\nu'|\nu) \) and \( f^{TN}(\nu'|\nu) \) where \( \phi(\nu) \) is the weight on \( f^U(\nu'|\nu) \).

For \( \nu \) very close to zero, the uniform distribution \( f^U(\nu'|\nu) \) over \([0, \nu]\) is essentially intended to capture that \( \nu' \) is clustered around zero for the invariant distribution of \( \nu' \) as in the data.

\textsuperscript{28}The Euler equation is given by: \( 1/\beta = 1 + \text{real return to a safe bond.} \)
Meanwhile, the truncated normal distribution \( f^{TN}(\nu' | \nu) \) mainly governs, together with the state-dependent weight \( \phi(\nu) \), the usual AR(1) mean-reverting process and is given by:

\[
\nu' = (1 - \rho)\nu + \rho \nu + \sigma \cdot \epsilon'
\]

where \( \epsilon' \) is a random variable truncated so that \( \nu' \in (0, 1) \), which would be an i.i.d. standard normal random variable if it were not truncated\(^2\).\(^9\).

I discretize the space of \( \nu \) into 21 points \( \nu \in \{\nu_1, \nu_2, \cdots, \nu_{21}\} \) and approximate the continuous process of \( \nu \) by using Tauchen (1986)’s method\(^3\).\(^0\). In particular, the uniform distribution \( f^U(\nu' | \nu) \) is discretized to the degenerate probability distribution for a single point, the minimum level \( \nu_1 = 0.025\% \), which is enough for my purpose to capture the default rate clustered around zero as in the data. The truncated normal distribution \( f^{TN}(\nu' | \nu) \) is discretized to the transition matrix \( \pi^{TN}(\nu', \nu) \). Lastly, the state-dependent weight on the uniform distribution \( \phi(\nu) \) is simplified such that \( \phi(\nu_1) = \phi_1 \) for \( \nu = \nu_1 \) and \( \phi(\nu) = \bar{\phi} \) for \( \nu > \nu_1 \) so that there are essentially two-regimes, which eventually leads to that in the model, the risky-bond’s default rate is clustered around zero and also mean-reverting. Then the transition matrix of discretized \( \nu' \) is given by:

\[
\pi(\nu', \nu) \equiv \text{Prob}(\nu' | \nu) = \begin{cases} 
\phi_1 + (1 - \phi_1) \cdot \pi^{TN}(\nu', \nu) & \text{if } \nu = \nu_1 \\
\bar{\phi} + (1 - \bar{\phi}) \cdot \pi^{TN}(\nu', \nu) & \text{if } \nu > \nu_1 
\end{cases}
\]

I estimate five parameters \( (\phi_1, \bar{\phi}, \rho, \nu, \sigma) \) and then calculate the transition matrix \( \pi(\nu', \nu) \). Moreover, I impose the restriction \( \phi_1 = \rho \) because \( \phi_1 \) and \( \rho \) are critical in determining the AR(1) persistence depending on if \( \nu = \nu_1 \) or \( \nu > \nu_1 \). Thus, I estimate jointly four parameters \( (\bar{\phi}, \rho, \nu, \sigma) \) by targeting four key statistics of the historical annual default rates for risky bonds: their sample mean, sample standard deviation, serial correlation, and, in addition, the frequency of annual

\[^{29}\] \( \epsilon' \) is truncated at \( -[(1 - \rho)\nu + \rho \nu]/\sigma \) and \( \int_{\nu_1}^{1} \exp\left( -\frac{1}{2} \left[ \frac{(\nu' - [(1 - \rho)\nu + \rho \nu]/\sigma)^2}{\sigma} \right] \right) d\nu' \), \( \forall \nu' \in (0, 1) \).

\[^{30}\]See the appendix, Calibration, for more detailed discussion.
default rates between zero and 5 basis point\textsuperscript{31}, see table 5 for the estimation results and table 6 for the default-rate statistics.

Table 5: Estimation Results for the Shock Process for $\nu$

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\rho = \phi_1$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3886</td>
<td>0.7501</td>
<td>0.0228</td>
</tr>
<tr>
<td></td>
<td>0.0010</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Statistics of Default Rates for Risky Bonds at Annual Frequency

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>Std.</th>
<th>$Corr(\nu', \nu)$</th>
<th>Prob($\nu &lt; .05%$)</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.9%</td>
<td>1.8%</td>
<td>0.32</td>
<td>15.2%</td>
<td>1.35</td>
</tr>
<tr>
<td>Model</td>
<td>1.6%</td>
<td>1.8%</td>
<td>0.23</td>
<td>25.9%</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Note: ‘Std.’ refers to the sample standard deviation of the risky bond’s default rate $\nu$, $Corr(\nu', \nu)$ refers to the OLS estimate of the slope parameter for the AR(1) auto-regression of $\nu$, and Prob($\nu < .05\%$) refers to the relative frequency of $\nu$ between zero and 5 basis point. All statistics are for $\nu$ aggregated at annual frequency.

As expected, the mean and standard deviation of the model-generated default rates for risky bonds are close to the data. The model-generated default rates are slightly less persistent and more clustered around zero relative to the data. Lastly, the skewness is, even though not targeted, almost the same between the model and data. In short, the calibrated model economy mimicks well the reality in terms of the distribution of default rates for risky bonds: the first three centered moments, serial correlation and clustering around zero.

5.3 Results

In this section, I present and discuss my results of simulating the calibrated economy. The results of the benchmark case is presented, and then results of repeating the same experiment

\textsuperscript{31}I use the method of weighted minimum distance between the model and the data. I use weights of (20:10:1:1) for the above four statistics in my estimation. Larger weights are put two the first two statistics, the sample mean and standard deviation, because of their importance. I calculate the model-generated statistics by simulating quarterly series and aggregating them at annual frequency.
for the two alternative cases for default costs $\tau$ are discussed: the counterfactual case of $\tau = 0$ and the case of $\tau$ positively correlated with $\nu$. The results of the first alternative case of $\tau = 0$ will show, as predicted by the previous static model, that shocks to $\nu$ have almost no impacts on allocation of capital, output and TFP. The second alternative case of $\tau$ positively correlated with $\nu$ is intended to capture that default costs are indeed positively correlated with default rates in the data. Below I discuss my simulation method and then proceed to discussing the simulation results in the order of the benchmark case and the two alternative cases of $\tau$.

**Simulation Method**

I feed a series of shocks to $\nu$ for 183 periods, which corresponds to the period 1964Q2-2009Q4, to the calibrated dynamic model. The model economy is always assumed to be in the deterministic steady state in the initial period. I do this simulation 1,000 times and report key statistics on average over the 1,000 times-simulations.

The equilibrium policy functions are numerically solved for, for which the first order finite element method is used as illustrated by McGrattan (1996). Once the policy functions are solved for, the price and other allocation functions are immediately calculated. More specifically, I numerically solve for four policy functions: consumption, interest and wage rates for safe firms, and aggregate labor supply. The aggregate state vector is the pair $(\nu, K)$. Even though the space of $\nu$ is already discretized, the space of $K$ is a continuum; for approximation of policy functions w.r.t. $K$, the finite element method is applied\(^{32}\).

**Results for the Benchmark Case**

I present the model-generated, for the benchmark parameter setting, statistics standard in the business cycle literature, see table 7.

\(^{32}\)I limit the space of $K$ to be sufficiently large, upper bound of 120% and lower bound of 80% relative to the steady state value of $K$. It turns out that the solved transition function of aggregate capital, $K'(\nu, K)$, never binds at those two boundaries. Numerical error is about $10^{-6}$ percentage point, between the guessed and updated policy functions, uniformly over the domain.
Table 7: Statistics of Key Aggregates: Benchmark

<table>
<thead>
<tr>
<th></th>
<th>σ(y)</th>
<th>σ(x)/σ(y)</th>
<th>corr(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>h k TFP c i</td>
<td>h k TFP c i</td>
</tr>
<tr>
<td>Data</td>
<td>1.56</td>
<td>1.27 0.34 0.53 0.54 3.35</td>
<td>0.88 0.15 0.45 0.84 0.91</td>
</tr>
<tr>
<td>Model</td>
<td>0.91</td>
<td>0.73 1.33 0.86 0.65 5.17</td>
<td>0.77 0.04 0.72 0.40 0.85</td>
</tr>
</tbody>
</table>

Note: σ(x)/σ(y) refers to the ratio of the percentage standard deviation of a variable x to that of output σ(y) and corr(x, y) refers to the correlation coefficient of x and output. All variables refer to their log deviations.

The main results of table 7 are that shocks to ν account for a substantial part of the fluctuations in output and TFP, about 60% for output and 90% for TFP\(^{33}\), which supports my claim that shocks to default-risk, more skewed for risky firms relative to safe firms, are an important source of fluctuations in output and productivity in the U.S. economy. The model-generated correlations between key aggregates with output are also consistent with the data.

As clarified in the previous static analysis, the key mechanism of the impact of an increase in the default risk for risky firms ν is the reallocation of capital away from risky firms toward safe firms, which has been also documented in the earlier section of the empirical analysis of the firm-level investment. I present the simulation results for capital allocation between risky and safe firms together with the credit spread, see table 8.

The main message of table 8 is that as expected, the corporate credit spread is countercyclical and that resource-allocation between risky and safe firms are highly correlated with the corporate

\[^{33}\text{I compute TFP as:}\]

\[
TFP_t = \frac{y_t}{[K_t^{PI}]^{a h_t^{1-g}}}
\]

where \(K_t^{PI}\) is the index of capital stock constructed by the perpetual inventory method consistent with the data and given by:

\[
K_{t+1}^{PI} = [1 - \tilde{\delta}]K_t^{PI} + I_t, \quad K_0^{PI} = K_0.
\]

\(\tilde{\delta}\) is the constant depreciation rate, equal to 6.60% per year, the same as what is used in constructing capital series in the data: this implies that .60% point of the annual depreciation rate is due to default-losses. See the appendix for more detail.
credit spread. As predicted by the static analysis, both capital and labor allocated for a risky firm relative to a safe firm decrease when the credit spread increases, i.e., the credit spread leads to the reallocation of resources between risky and safe firms as consistent with my earlier empirical analysis.

I discuss how some of important ingredients omitted in my model would affect my quantitative results and then proceed to discuss the results of the two alternative cases for $\tau$. My model predicts sizable effects on output and TFP of one unit increase in the default probability for risky firms via a substantial extent of reallocation of capital between risky and safe firms, for which the following three ingredients are important: availability of the third investment opportunity, sophisticated financing and adjustment costs of resource-reallocation.

First, if the third investment opportunity, i.e., government bond and foreign investment, other than investment to capital used for the domestic production is available, then the impact of risk-shocks for risky firms on resource-reallocation between risky and safe firms would be mitigated to the extent that capital is diverted to the third investment opportunity. On the contrary, the impact on aggregate-level investment would become larger and thereby the impact on output will be similar to my benchmark case of no availability of the third investment opportunity.

Second, if firms are allowed to use financing vehicles more sophisticated than the 100% debt financing, then my results would be affected because the response of a risky firm’s relative cost of capital to a safe firm would be also sensitive to the availability of financial instruments. For

---

Table 8: Statistics of Resources-Allocation and Credit Spread: Benchmark

<table>
<thead>
<tr>
<th></th>
<th>$x$</th>
<th>$r^R - r^S$</th>
<th>$k^S$</th>
<th>$k^R$</th>
<th>$h^S$</th>
<th>$h^R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(x)/\sigma(y)$</td>
<td>0.40</td>
<td>22.53</td>
<td>24.65</td>
<td>16.26</td>
<td>16.11</td>
<td></td>
</tr>
<tr>
<td>$corr(x,y)$</td>
<td>-0.95</td>
<td>-0.90</td>
<td>0.97</td>
<td>-0.92</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>$corr(x, r^R - r^S)$</td>
<td>1.00</td>
<td>0.99</td>
<td>-1.00</td>
<td>1.00</td>
<td>-1.00</td>
<td></td>
</tr>
</tbody>
</table>

Note: $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable $x$ to that of output, $corr(x,y)$ refers to the correlation coefficient of $x$ and output, and $corr(x, r^R - r^S)$ does the same for the credit spread $r^R - r^S$. All variables refer to their log deviations except for that $r^R - r^S$ is non-logged.
instance, a risky firm can substitute its internal cash flows for debts in financing its capital or insure against the interest rate shocks by issuing long-term bonds rather than one-period bonds. To check the robustness of my results to the availability of internal financing, a simple scenario of internal financing combined with rolling-over constant size of one-period bond will be examined in the later section of internal-financing model. The case of long-term bond is, although interesting, left for future work.

Third, adjustment costs of resource-reallocation would reduce the fluctuations in key aggregates in my model. The key here is how large, if any, adjustment costs of resource-reallocation would be, which calls for an empirical investigatoin. I leave it for future work, too.

**Importance of τ in Amplifying the Effects of Shocks to ν**

As mentioned earlier, I examine the importance of τ, which captures a key friction in the corporate bond market, for the previous benchmark results for output and TFP. I consider the case of τ = 0 keeping other parameter values the same as for the benchmark case and repeat the same experiment as done for the benchmark case, see table 9 for the results in this case.

<table>
<thead>
<tr>
<th></th>
<th>σ(y)</th>
<th>σ(x)/σ(y)</th>
<th>corr(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>h k TFP c</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>1.56</td>
<td>0.34 0.53</td>
<td>3.35</td>
</tr>
<tr>
<td>Bench</td>
<td>0.91</td>
<td>1.33 0.86</td>
<td>5.17</td>
</tr>
<tr>
<td>τ = 0</td>
<td>0.00</td>
<td>3.64 0.16</td>
<td>12.09</td>
</tr>
</tbody>
</table>

*Note: The line headed τ = 0 refers to the case of τ = 0 and the line headed ‘Bench’ does the same for the benchmark case of τ = 0.999. σ(x)/σ(y) refers to the ratio of the percentage standard deviation of a variable x to that of output σ(y) and corr(x, y) refers to the correlation coefficient of x and output. All variables refer to their log deviations.*

The main results of table 9 are that there are almost no effects of shocks to ν for the alternative case of τ = 0, consistent with what the previous static model predicted. I conclude
that the extent of financial frictions, represented by $\tau$, is an important amplification device for this economy. The key here is that, in the case of $\tau = 0$, there is no misallocation of resources as clarified in the previous static analysis. And therefore, reallocation of capital, if any, would have no first order impact on TFP and output. It also follows that the shocks to $\nu$ studied in this paper differs from the aggregate productivity shocks, which directly affects output even if resource-allocation is kept constant.

**Case of Positive Correlation Between $\tau$ and $\nu$**

I turn to examine the second alternative case in which $\tau$ is time-varying and positively correlated with $\tau$ as is in the data. For simplicity, I consider the case in which $\tau$ is perfectly positively correlated with $\nu$ and given by:

$$\tau(\nu) = 1 - \tau_0 \nu^{-\tau_1}, \quad \tau_0 > 0, \tau_1 > 0$$

where the above functional form is taken from Altman and Karlin (2010). I calibrate $(\tau_0, \tau_1)$ based on the data on recovery and default rates for risky bonds$^{34}$, which results in $(\tau_0 = 0.1931, \tau_1 = 0.1288)$. For the above case, I repeat the earlier analysis done for the benchmark case of the constant $\tau$, see table 10.

From table 10, we can see that as predicted by the previous static analysis, the effects of shocks to $\nu$ are slightly larger in the alternative case of $\tau$ perfectly positively correlated with $\nu$ than they are for the benchmark case of constant $\tau$.

6 **Extension: Internal-Financing Dynamic Model**

In this section, I examine the sensitivity of my previous results of my baseline model of 100% debt financing to the availability of alternative methods of financing. As discussed earlier, I focus on discussing how the availability of internal financing would alter my results. More specifically, I will examine the resource-allocation for a risky firm under the time-varying default risk for

$^{34}$See Altman and Karlin (2010) for more detailed discussion.
Table 10: Statistics of Key Aggregates: Case of $\text{corr}(\tau, \nu) = 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\sigma(y)$</th>
<th>$\sigma(x)/\sigma(y)$</th>
<th>$\text{corr}(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.56</td>
<td>1.27 0.34 0.53 0.54 3.35</td>
<td>0.88 0.15 0.45 0.84 0.91</td>
</tr>
<tr>
<td>Bench</td>
<td>0.91</td>
<td>0.73 1.33 0.86 0.65 5.17</td>
<td>0.77 0.04 0.72 0.40 0.85</td>
</tr>
<tr>
<td>$\text{corr}(\tau, \nu) = 1$</td>
<td>1.09</td>
<td>0.71 1.14 0.77 0.57 5.15</td>
<td>0.82 0.06 0.77 0.41 0.89</td>
</tr>
</tbody>
</table>

Note: The line headed $\text{corr}(\tau, \nu) = 1$ refers to the alternative case of $\tau$ perfectly positively correlated with $\nu$ and the line headed ‘Bench’ does the same for the benchmark case of constant $\tau$. $\sigma(x)/\sigma(y)$ refers to the ratio of the percentage standard deviation of a variable $x$ to that of output $\sigma(y)$ and $\text{corr}(x, y)$ refers to the correlation coefficient of $x$ and output. All variables refer to their log deviations.

the case in which the internal cash-flow is operative in financing the marginal capital. For the purpose of exposition, I consider a simple scenario in which firms use internal financing and roll over constant and sufficiently large size of one-period bonds so that the same default-rate dynamics are still obtained and internal financing is always operative for marginal capital. The simulation results for this case will be provided and discussed.

6.1 Environment

Two features are added in this model. First, non-defaulting firms roll over their debt and finance their investments via their internal cash flows. Second, constant mass of risky firms are created and enter the market immediately every period.

Technology

The technology for this economy is essentially the same as for the previous baseline economy except that (i) measure of risky firms changes over time by the simplifying assumption of the constant entry and (ii) every firm receives a small amount of endowment income.

There is a continuum of safe firms of measure $\lambda \in (0, 1)$ and a continuum of risky firms of
measure $N_t > 0$: firm $i$ is safe for $i \in [0, \lambda]$ and is risky for $i \in (\lambda, N_t + \lambda]$ at period $t$. Note that $N_t$ can change over time $t$ whereas $N_t$ is constant equal to $1 - \lambda$ in the baseline model.

For simplicity, I assume that constant mass $M$ of risky firms are created and enter the market immediately every period incurring expenses $\chi$ for such a new entrant firm. As $\nu_t$ mass of risky firms will be liquidated, the mass of operative risky firms, again denoted by $N_t$, evolves as:

$$N_{t+1} = (1 - \nu_t)N_t + M, \quad \forall t = 0, 1, 2, \ldots$$

If the mass of newly created risky-firms were assumed to equal $(1 - \nu_t)$, then $N_{t+1}$ would be constant equal to $1 - \lambda$ as in the baseline model. In this case, aggregate entry-costs $\chi \cdot [1 - \nu_t]$ would be cyclical and affect a firm’s investment decision largely. Therefore, I purposefully assume that constant mass of new risky-firms enter for every period.

Next, firm $i$’s production function is the same as for the baseline model except for that the firm receives additonaly an endowment-income. Safe firm $i$’s production function is given by:

$$y_t(i) = \left\{ \left[ k_t(i) \right]^\theta [h_t(i)]^{1-\theta} \right\}^\alpha + \kappa, \quad \forall i \in [0, \lambda], \quad \alpha \in (0, 1), \quad \theta \in (0, 1), \quad \kappa > 0$$

where the constant term $\kappa > 0$ represents firm $i$’s endowment-income, which makes firm $i$’s production function as non-convex\(^{35}\). Essentially $\kappa > 0$ does the role of making the firm value high enough to match the observed leverage ratio in the data: $\kappa > 0$ is interpretable as the growth option in the valuation of a firm or the locally increasing-returns-to-scale for initial employment of resources.

Risky firm $i$’s production function is stochastic and given by:

$$y_t(i) = z_t(i) \left\{ \left[ k_t(i) \right]^\theta [h_t(i)]^{1-\theta} \right\}^\alpha + \kappa, \quad \forall i \in (\lambda, N_t]$$

where $z_t(i)$ is the stochastic productivity, captures the jump-default risk as in the baseline dynamic model and its probability distribution is the same as in the baseline dynamic model. Note that the expected endowment-income for a risky firm equals $\kappa$, the same as for a safe firm.

\(^{35}\)Even in the present of $\kappa > 0$, the production function still satisfy decreasing returns to scale globally.
I keep the notation $\nu_t$, the probability of a jump-default shock for a risky firm in period $t$, and $f(\nu_{t+1}|\nu_t)$, the density of $\nu_{t+1}$ conditional on $\nu_t$, as in the baseline dynamic model.

**Timing**

I discuss only the key point of timing of events here and skip the others same as for the baseline model. The time interval between the current final subperiod and the next initial subperiod is equal to zero as in the baseline model. It follows that, at the final subperiod of $t$, firm $i$ makes its decision on whether to revolve/default their current debt after observing its current period productivity $z_t(i)$ as well as $\nu_{t+1}$, which implies that the next period allocation of resources, $(k_{t+1}(i), h_{t+1}(i))$, is a function of $\nu_{t+1}$ as was the case in the baseline model.

**Capital Structure**

First, survived firms roll over, if not defaulted, their debt and finance their investments via their internal cash flows. Letting $B_t(i)$ denote firm $i$’s debt at $t$, I write:

$$B_t(i) = B > 0$$

where I assumed that every non-defaulted/operative firm carries costant size of debt, which captures the debts used in financing the initial entry costs and etc. Moreover, for simplicity, I assume the size of the rolled-over debt is the same between risky and safe firms; the size of debt does not matter for a safe firm because of no default risk for a safe firm. Letting $q_t(i)$ denote the price of one-period (non-contingent) bond issued by firm $i$ at the initial subperiod of $t$, I write the firm $i$’s interest expenses at the final subperiod of $t$ as $[1 - q_t(i)]B$.

Letting $x_t(i) \equiv k_{t+1}(i) - [1 - \delta]k_t(i)$ denote firm $i$’s investment at the final subperiod of $t$, I write firm $i$’s dividend $d_t(i)$, restricted to be non-negative, as:

$$d_t(i) = \max \left\{ 0, \ y_t(i) - x_t(i) - w_t(i)h_t(i) - [1 - q_t(i)]B \right\}$$

which states that firm $i$’s dividend is equal to its output net investment, labor expenses and interest expenses and that it can not be negative. By the non-negativity restriction imposed on
the dividend policy, I already ruled out the option of external financing via issuing equity, which was again motivated by the empirical fact reported by Myers(2001).

Second, new risky firm \( i \) entering at the initial subperiod of \( t + 1 \) also issues one-period (non-contingent) bond, of the size equal to \( B \) same as for the survived firms, at the given price of \( q_{t+1}(i) \). New risky firm \( i \) uses the proceeds of debt \( q_{t+1}(i) \cdot B \) for its capital in use for the production and immediate, at the initial subperiod of \( t+1 \), dividend-payouts to its equity holders.

**Costs of Default**

As in the baseline model, the bondholders of defaulting firm \( i \) lose \( \tilde{\tau} \) fraction of capital held by the defaulting firm in the liquidation process. Letting \((1 - \delta)k_t(i)\) denote the undepreciated capital owned by the defaulting firm \( i \) at the final subperiod of \( t \), I write the ex-post return, again at the final subperiod of \( t \), to the defaulted bond issued by the firm \( i \) as:

\[
\frac{(1 - \tilde{\tau})(1 - \delta)k_t(i)}{B}
\]

which is decreasing in both \( \tilde{\tau} \) and debt-to-capital ratio, \( B/k_t(i) \). The two factors, \( \tilde{\tau} \) and \( B/k_t(i) \), will play a key role in determining the recovery rate for defaulted risky debts, which is important in determining corporate credit spread. That is, the default costs \( \tau \) is given as:

\[
1 - \tau = (1 - \tilde{\tau}) \cdot \frac{k_t(i)}{B}
\]

which implies that default costs \( \tau \) is larger than the liquidation costs \( \tilde{\tau} \) if debt is larger than the capital held by firm \( i \).

**A Firm’s Problem**

I analyze a firm’s problem in recursive form. Safe firm \( i \)’s problem is represented by the following value functions. At the final subperiod, safe firm \( i \) chooses whether to default or not:

\[
V^S_+(k, h, K^S, K^R, N, \nu, \nu') = \max \left\{ 0, V^S_{i,n}(k, h, K^S, K^R, N, \nu, \nu') \right\}
\]

where \((k, h)\) is the vector of firm \( i \)'s individual state variables, chosen at the initial subperiod, \((K^S, K^R, N, \nu, \nu')\) is the vector of aggregate state variables and \(V^S_{i,n}(k, h, K^S, K^R, N, \nu, \nu')\) refers
to the firm’s value function conditional on non-default. Letting $V^S_S(k, K^S, K^R, N, \nu)$ denote the firm’s value function at the initial subperiod and $k'$ denote the firm’s choice of next period capital, I write $V^{S,n}_+(k, h, K^S, K^R, N, \nu, \nu')$ as:

$$V^{S,n}_+(k, h, K^S, K^R, N, \nu, \nu') = \max_{k' \geq 0} \left\{ \left[ [k]^\theta [h]^{1-\theta} \right]^\alpha + \kappa + (1 - \delta) k - w^S h - [1 - q^S] B - k' + V^S_S(k', K^S, K^R, N', \nu') \right\}$$

where $(K^S, K^R, N')$ denotes, aggregate (endogenous) state variables at the next period, capital owned by an individual safe firm and an individual risky firm and the number of (operative) risky firms, respectively. Next, I write $V^-_S(k, K^S, K^R, N, \nu)$ as:

$$V^-_S(k, K^S, K^R, N, \nu) = \max_{h \geq 0} \left\{ q^S \cdot \int_0^1 V^S_S(k, h, K^S, K^R, N, \nu, \nu') \cdot f(\nu', \nu) d\nu' \right\}$$

which means that the firm optimally chooses, at the initial subperiod, its current period labor to maximize its present value at the final subperiod where I already assumed that every firm discounts its future cash flows by the risk-free rate.

Similarly, risky firm $i$’s problem is also represented by the following value functions. The final-subperiod value function for the risky firm of productivity equal to $z$ is given by:

$$V^R_+(k, h, K^S, K^R, N, \nu, \nu', z) = \max \left\{ 0, V^{S,n}_+(k, h, K^S, K^R, N, \nu, \nu', z) \right\}$$

where the non-default value function $V^{R,n}_+(k, h, K^S, K^R, N, \nu, \nu', z)$ is given by:

$$V^{R,n}_+(k, h, K^S, K^R, N, \nu, \nu', z) = \max_{k' \geq 0} \left\{ z \left( \left[ [k]^\theta [h]^{1-\theta} \right]^\alpha + \kappa \right) + (1 - \delta) k - w^R h - [1 - q^R] B - k' + V^-_R(k', K^S, K^R, N', \nu') \right\}$$

and the risky firm’s initial-subperiod value function $V^R_-(k, K^S, K^R, N, \nu)$ is given by:

$$V^-_R(k, K^S, K^R, N, \nu) = \max_{h \geq 0} \left\{ q^R \cdot \int_0^1 \nu \cdot V^R_+(k, h, K^S, K^R, N, \nu, \nu', 0) \right\}$$
\[ + (1 - \nu) \cdot V^R_+ \left( k, h, K^S, K^R, N, \nu, \nu', \frac{1}{1 - \nu} \right) \cdot f(\nu', \nu) d\nu' \right\}. \]

In analyzing a firm’s problem below, I focus on the case that (i) firm i’s current capital \( k(i) \) is smaller than debt \( B \) so that the firm should default if hit by the jump-default shock \( z(i) = 0 \) and (ii) \( k(i) \) is not too small relative to \( B \) such that the firm should not default in the event of drawing positive productivity \( z(i) > 0 \). That is, I focus on the firm-level debt-to-capital ratio which might results in the same (equilibrium) default-rate dynamics for risky firms as in the previous baseline economy. In this case, therefore, safe firms never default and risky firms default if and only if in the event of drawing the jump-default shock \( z(i) = 0 \) as was the case in the baseline model.

First, I analyze a safe firm’s optimal decision rule for hiring labor and also a risky firm’s decision rule, too. Safe firm i’s optimal choice for labor denoted by \( h^S \), given its current capital \( k^S \), is given by: \[ \alpha (1 - \theta) [k^S]^\alpha [h^S]^\alpha (1 - \theta) - 1 = w^S. \]

Similarly, risky firm i’s optimal choice for labor denoted by \( h^R \), given its current capital \( k^R \), is also given by: \[ \frac{1}{1 - \nu} \alpha (1 - \theta) [k^R]^\alpha [h^R]^\alpha (1 - \theta) - 1 = w^R \]

which I simplify, by using the equilibrium condition \( w^S = [1 - \nu] \cdot w^R \), as: \[ \alpha (1 - \theta) [k^R]^\alpha [h^R]^\alpha (1 - \theta) - 1 = w^S. \]

It immediately follows that \( J \)-type firm i’s optimal choice for labor \( h^J \) is a function of its current capital \( k^J \) and current wage rate for a safe firm \( w^S \):

\[ h^J = \left[ \frac{\alpha (1 - \theta)}{w^S} \right]^{\frac{1}{1 - \alpha (1 - \theta)}} [k^J]^{\frac{\alpha (1 - \theta)}{1 - \alpha (1 - \theta)}}, \quad \forall J \in \{S, R\}. \]

Next, I turn to analyzing optimal investment decision. Safe firm i’s optimal decision for its next-period capital \( k^S_+ \) is given by:

\[ 1 = q^S \cdot \left[ 1 - \delta + \alpha \theta [\alpha (1 - \theta)]^{\frac{\alpha (1 - \theta)}{1 - \alpha (1 - \theta)}} [k^S_+]^{\frac{- (1 - \alpha)}{1 - \alpha (1 - \theta)}} [w^S_+]^{\frac{- \alpha (1 - \theta)}{1 - \alpha (1 - \theta)}} \right] \]
where “+” subindex of the (safe firm’s) wage rate indicates that it is the wage rate\textsuperscript{36} in the next period. The above optimality condition for the safe firm’s next period capital is the usual Euler equation for a firm’s problem where the firm discounts future cash flows by the risk-free rate.

Similarly, risky firm \(i\)'s optimal choice for its next-period capital \(k_{R+}^i\) is given by:

\[
1 = q^S \cdot \left[ (1 - \nu') [1 - \delta] + (1 - \nu') \frac{1}{1 - \nu} \alpha \theta (1 - \theta) \right]^{\frac{(1 - \alpha)(1 - \theta)}{1 - \alpha(1 - \theta)}} [k_{R+}^i]^{\frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta)}} [w^S_+]^{\frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta)}}.
\]

The above equation implies that the risky firm’s next-period capital \(k_{R+}^i\) is independent of its current-period capital \(k_{R-}^i\), from which it follows that the above optimality condition holds for a survived risky firm as well as for an entrant risky firm. Therefore, I write the value of an entrant risky firm, at the initial period, as:

\[
V^E(k_{R^*}, k_{S^*}^i, k_{R^*}, N, \nu) \equiv q^RB - k_{R^*} + V^R(k_{R^*}, k_{S^*}^i, k_{R^*}, N, \nu)
\]

which I will use, in the later section of quantitative analysis, to calibrate \(\chi\), the costs of entry for a risky firm, so that the entry cost equals the value of an entrant risky firm in the steady state: \(\chi = V^E(k_{R^*}, k_{S^*}^i, k_{R^*}, N^*, \nu^*)\) where ‘\(^*\)' in the superscript denotes the steady state.

Combining the above Euler equations for safe and risky firms, I write the condition for capital allocation, one of the key factors in determining aggregate productivity in this economy, as:

\[
\frac{\nu'[1 - \delta]}{[1/q^S] - [1 - \delta]} + 1 = \left[ \frac{k_{R+}^i}{k_{S+}^i} \right]^{\frac{- (1 - \alpha)}{1 - \alpha(1 - \theta)}}
\]

from which it is obvious, given \(0 < q^S < 1\), that \(k_{R+}^i/k_{S+}^i\) is less than one and decreasing in \(\nu'\) holding \(q^S\) to constant. That is, capital is reallocated away from the risky firms toward safe firms as the risky-firm’s default probability \(\nu'\) increases holding the price of safe bonds to constant.

\textbf{The Household’s Problem}

The household’s problem is essentially the same as for the baseline model and I focus on the obvious equilibrium condition for prices of bonds that the expected returns should be equalized

\textsuperscript{36}I have already expressed the firm’s labor policy in terms of its capital and wage rate.
between safe and risky bonds. As discussed earlier, the ex-post (gross) return to risky bond is equal to $(1 - \tau)(1 - \delta)K^R/B$ in the event of default and $1/q^R$ in the event of non-default, respectively. The equalization of expected returns for risky and safe bonds leads to the following equilibrium risky-bond-pricing formula given the the safe-bond price:

$$\frac{1}{q^S} = [1 - \nu] \frac{1}{q^R} + \nu \frac{(1 - \tilde{\tau})(1 - \delta)K^R}{B}.$$ 

Meanwhile, the safe-bond price $q^S$ will be determined by the Household’s usual Euler equation.

### Resource Constraint

The resource constraint is given by:

$$c_t + \lambda k^S_{t+1} + N_{t+1}k^R_{t+1} + \chi M = y_t + (1 - \delta) \left[ \lambda k^S_t + N_t[1 - \nu_t \tilde{\tau}]k^R_t \right]$$

which states that output plus undepreciated capital net liquidation costs is used to consumption, next-period capital and entry-costs of new risky firms of measure $M$ where aggregate output $y_t$ is given by:

$$y_t = \lambda \left[ [k^S_t]^\theta [h^S_t]^{1-\theta}\right]^\alpha + N_t \left[ [k^R_t]^\theta [h^R_t]^{1-\theta}\right]^\alpha + \kappa \left[ \lambda + N_t \right].$$

Note that aggregate output is independent of $\nu_t$ holding allocation of resources and the mass of risky firms $N_t$ to constant.

### Capital Structure And Reallocation of Capital

In this section, I analyze the impact of capital structure on the magnitude of reallocation of capital, and thereby the change in TFP, in response to an increase in $\nu'$. More specifically, I compare the magnitude of the response of $k^R_+/k^S_+$ to an increase in $\nu'$ in this economy, the case of internal financing, to that in the baseline economy, the case of debt financing. For this purpose, I derive the derivative of $k^R_+/k^S_+$ w.r.t. $\nu'$ and take its absolute value, which is for this economy, given by:

$$\left| \frac{d}{d\nu'} \frac{k^R_+}{k^S_+} \right| = \left[ \frac{1 - \alpha (1 - \theta)}{(1 - \alpha)} \right] \frac{1 - \delta}{1/q^S - [1 - \delta]}$$
and the counterpart in the baseline economy of 100% debt financing is given by:

\[ \left| d\left(\frac{k^R}{k^S} \right)^{\text{BE}} / d\nu \right| = \left| \frac{1 - \alpha(1 - \theta)}{(1 - \alpha)} \right| \frac{\tau}{[1/q^S]^{\text{BE}} - 1} \]

where I use the superscript ‘BE’ to denote the variables in the baseline economy\(^{37}\). For the purpose of exposition, I consider the case in which \( \delta \) is sufficiently small and the default costs for the baseline economy is less than one \( \tau \in [0, 1) \); in this case, the magnitude of the derivative of \( k^R/k^S \) w.r.t. \( \nu \) is larger in this economy of internal financing combined with rolled-over debt than in the baseline economy of 100% debt financing if risk-free rates are the same between the two economy:

If \( \tau \in [0, 1) \) and \( 1/q^S = [1/q^S]^{\text{BE}} \), then \( \exists \delta > 0 \) s.t.

\[ \forall \delta \in [0, \delta) , \left| d\left(\frac{k^R}{k^S} \right)^{\text{BE}} / d\nu \right| > \left| d\left(\frac{k^R}{k^S} \right)^{\text{BE}} / d\nu \right| . \]

**Proof.** Assume that \( \tau \in [0, 1) \) and that \( 1/q^S = [1/q^S]^{\text{BE}} \). Consider the case of \( \delta = 0 \). It immediately follows that:

\[ \left| d\left(\frac{k^R}{k^S} \right)^{\text{BE}} / d\nu \right| = \left| \frac{1 - \alpha(1 - \theta)}{(1 - \alpha)} \right| \frac{1}{[1/q^S] - 1} > \left| \frac{1 - \alpha(1 - \theta)}{(1 - \alpha)} \right| \frac{\tau}{[1/q^S]^{\text{BE}} - 1} = \left| d\left(\frac{k^R}{k^S} \right)^{\text{BE}} / d\nu \right| . \]

The absolute value of the derivative of \( k^R/k^S \) w.r.t. \( \nu \) is continuous in \( \delta \) both for this economy and for the baseline economy. \( \square \)

The above result implies that the magnitude in the response of TFP to an increase in \( \nu \) will be larger in this economy relative to the baseline because aggregate output, given total inputs, is strictly increasing in \( [k^R/k^S] \in (0, 1) \) for both of the two economies holding the number of risky firms, \( N_t \), to constant. The key point giving rise to the above result is that the marginal cost of capital for a risky firm conditional on drawing zero productivity is higher in this economy than in the baseline. Both in this economy and in the baseline economy, a risky firm hit by the jump-default shock defaults. In this economy, the risky firm self-finances its investment and therefore loses, upon default, 100% of its marginal (undepreciated) capital; therefore, roughly

\(^{37}\)I use \( [1/q^S]^{\text{BE}} \equiv 1 + r^S \) for the baseline economy.
speaking, the marginal cost of capital for the risky firm increases by one unit in response to one unit increase in $\nu$. Meanwhile, in the baseline economy, the bondholder provides capital to a risky firm and loses $\tau < 1$ fraction of (undepreciated) capital in the event of default, which is less than 100% loss as is for the equity holders discussed above; thus, roughly speaking, the marginal cost of capital for the risky firm increases by $\tau < 1$ units in response to one unit increase in $\nu$.

In short, an increase in the marginal cost of capital for a risky firm in response to an increase in its default probability $\nu$ is larger in case of internal financing relative to the debt financing. As a result, the magnitude of capital reallocation in response to an increase in $\nu$ is larger in this economy relative to the baseline, and so is the change in TFP and output, which I will discuss in more detail in the later section of simulation results.

6.2 Equilibrium

I study a recursive equilibrium for this economy, which is a list of value functions

\[
\begin{align*}
&\left( V^S(k, h, K^S, K^R, N, \nu, \nu'), V^{S,n}(k, h, K^S, K^R, N, \nu, \nu'), V^S(k, K^S, K^R, N, \nu), V^R(k, h, K^S, K^R, N, \nu, \nu', z), V^{R,n}(k, h, K^S, K^R, N, \nu, \nu', z), V^R(k, K^S, K^R, N, \nu) \right); \\
&\text{policy functions } \left( h^S(k, K^S, K^R, N, \nu), h^R(k, K^S, K^R, N, \nu), k^S(k, h, K^S, K^R, N, \nu, \nu'), \right. \\
&\left. k^R(k, h, K^S, K^R, N, \nu, \nu'), c(b, h, K^S, K^R, N, \nu, \nu') \right); \\
&\text{price functions } \left( q^S(K^S, K^R, N, \nu, \nu'), q^R(K^S, K^R, N, \nu, \nu'), w^S(K^S, K^R, N, \nu), w^R(K^S, K^R, N, \nu) \right); \\
&\text{law-of-motion functions } \left( K^S_k(K^S, K^R, N, \nu, \nu'), K^R_k(K^S, K^R, N, \nu, \nu') \right); \\
&\text{total transfer function } \Pi(K^S, K^R, N, \nu, \nu') \text{ that satisfy the following conditions:}
\end{align*}
\]

1. Optimality:

   (a) Safe firm: the value functions $\left( V^S(k, h, K^S, K^R, N, \nu, \nu'), V^{S,n}(k, h, K^S, K^R, N, \nu, \nu'), V^S(k, K^S, K^R, N, \nu), V^R(k, h, K^S, K^R, N, \nu, \nu', z), V^{R,n}(k, h, K^S, K^R, N, \nu, \nu', z), V^R(k, K^S, K^R, N, \nu) \right)$ solve the Bellman equations for a safe firm’s problem, and policy functions $\left( h^S(k, K^S, K^R, N, \nu), k^S(k, h, K^S, K^R, N, \nu, \nu') \right)$ are the optimal decision rules for the safe firm’s problem where the safe firm takes as given the price functions $\left( q^S(K^S, K^R, N, \nu, \nu'), w^S(K^S, K^R, N, \nu) \right)$.

---

38Note that the above result in this paper is obtained even in the absence of tax advantage of debts.
(b) Risky firm: the value functions \( V^R_{+}(k, h, K^S, K^R, N, \nu, \nu', z), V^R_{-}(k, h, K^S, K^R, N, \nu, \nu', z) \) solve the Bellman equations for a risky firm’s problem, and policy functions \( h^R(k, K^S, K^R, N, \nu, \nu', z), k^R_{+}(k, h, K^S, K^R, N, \nu, \nu', z) \) are the optimal decision rules for the risky firm’s problem where the risky firm takes as given the price functions \( q^S(K^S, K^R, N, \nu, \nu'), q^R(K^S, K^R, N, \nu, \nu'), w^R(K^S, K^R, N, \nu) \).

(c) Household: \( h = \lambda \cdot h^S(K^S, K^S, K^R, N, \nu) + N \cdot h^R(K^R, K^S, K^R, N, \nu) \) is the household’s optimal decision rule, at the initial supberiod, on labor supply and \( b' = [\lambda + N']B > 0 \) is the household’s optimal decision rule, at the final supberiod, on consumption and savings, respectively, where the household takes as given the total transfer function \( \Pi(K^S, K^R, N, \nu, \nu') \) and price functions \( q^S(K^S, K^R, N, \nu, \nu'), q^R(K^S, K^R, N, \nu, \nu'), w^S(K^S, K^R, N, \nu), w^R(K^S, K^R, N, \nu) \).

2. Markets clear: for all \((K^S, K^R, N, \nu, \nu')\), the following condition holds:

\[
y(K^S, K^R, N, \nu, \nu') + (1 - \delta)[\lambda K^S + N[1 - \nu \tau]K^R] = c(b, h, K^S, K^S, K^R, N, \nu, \nu') + \lambda K^S + N' K^R + N' K^R + \chi M
\]

where \( b = [\lambda + N]B \) and \( h = \lambda \cdot h^S(K^S, K^S, K^R, N, \nu) + N \cdot h^R(K^R, K^S, K^R, N, \nu) \).

3. Consistency:

(a) Law-of-motion:

\[
k^S_{+}(K^S, K^S, K^R, N, \nu, \nu') = K^S_{+}(K^S, K^R, N, \nu, \nu'), \quad \forall(K^S, K^R, N, \nu, \nu')
\]

\[
k^R_{+}(K^R, K^S, K^R, N, \nu, \nu', z = \frac{1}{1 - \nu'}) = K^R_{+}(K^S, K^R, N, \nu, \nu'), \quad \forall(K^S, K^R, N, \nu, \nu').
\]

(b) Total transfer function: \( \Pi(K^S, K^R, N, \nu, \nu') \) is consistent with decision rules of safe and risky firms for all \((K^S, K^R, N, \nu, \nu')\).
6.3 Calibration

In this section, I discuss how to calibrate parameter values for the internal-financing dynamic model, which I label as the Internal Financing model, or simply IF model, by targeting long-run averages of key aggregates for the U.S. economy over the period 1964-2009 for the steady-state of the IF model economy such that \( \nu_t \) is constant and equal to its long-run average.

One period in the model corresponds to one quarter in the data. The benchmark parameter values are listed in table 11.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>TARGET</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu ): Prob. of ( z(i) = 0 ) for Risky Firm ( i )</td>
<td>.0048</td>
<td>Default Rate of Risky Bonds</td>
</tr>
<tr>
<td>( \beta ): Discount Factor</td>
<td>.9909</td>
<td>Real Return to Aaa Bonds</td>
</tr>
<tr>
<td>( \alpha ): Returns to Scale</td>
<td>.87</td>
<td>Khan and Thomas (2010)</td>
</tr>
<tr>
<td>( \delta ): Depreciation Rate</td>
<td>.015</td>
<td>Benchmark</td>
</tr>
<tr>
<td>( \theta ): Capital Share</td>
<td>.33</td>
<td>Investment/GDP</td>
</tr>
<tr>
<td>( \omega ): Curvature of Labor-Disutility</td>
<td>.30</td>
<td>Benchmark</td>
</tr>
<tr>
<td>( \psi ): Level of Labor-Disutility</td>
<td>3.32</td>
<td>( h_{ss} = .3333 )</td>
</tr>
<tr>
<td>( \lambda ): Measure of Safe Firms</td>
<td>.477</td>
<td>Volume-Share of Safe Bonds</td>
</tr>
<tr>
<td>( M ): Constant Measure of Risky-Entrant Firms</td>
<td>.00251</td>
<td>( N = 1 - \lambda )</td>
</tr>
<tr>
<td>( \tilde{\tau} ): Liquidation Costs</td>
<td>.30</td>
<td>Ramey and Shapiro (2001)</td>
</tr>
<tr>
<td>( B ): Size of Debt</td>
<td>14.79</td>
<td>Recovery Rate of Risky Bonds</td>
</tr>
<tr>
<td>( \kappa ): A Firm's Expected Endowment</td>
<td>.354</td>
<td>Leverage Ratio</td>
</tr>
<tr>
<td>( \chi ): Creation &amp; Entry Costs of a Risky Firm</td>
<td>35.20</td>
<td>Value of an Entrant Risky-Firm</td>
</tr>
</tbody>
</table>

I keep a set of parameters in the top half of the table 10, \( (\nu, \beta, \alpha, \delta, \theta, \omega, \psi) \), to the benchmark values used for the baseline model. And I proceed to choose values for the other parameters \( (\lambda, M, \tau, B, \kappa, \chi) \).

First, I choose the mass of safe firms \( \lambda = 0.477 \) by targeting the size of newly issued safe
bonds relative to the total bonds as in the baseline model. As the bond size is the same between risky and safe firms in this economy, \( \lambda \) is simply equal to the relative size of safe bonds to the total, which is 0.477, when I normalize the steady state mass of risky firms \( N \) to \((1 - \lambda)\). Then I choose \( M = .00251 \) because \( M = \nu \cdot N \), where \( N = 1 - \lambda \).

Second, I choose \( \tilde{\tau} = .30 \) as my benchmark where \( \tilde{\tau} \) represents the liquidation costs, which basically captures the high discount rates applied to the market price of a liquidated asset. I calibrate \( \tilde{\tau} \) based on estimates of the discount rate for a liquidated asset. There is a wide range of estimates of such discount rates. For instance, Ramey and Shapiro (2001) estimate that the “ratio of sales price to replacement cost” is about 10-40\%, which implies that the discount rate \( \tilde{\tau} \) is about 60-90\%. Taking into consideration that Ramey and Shapiro (2001)’s sample is limited to a number industries and that liquidation does not necessarily involve reployment of capital, e.g., selling off the equity of the defaulting firm to other investors, I choose \( \tilde{\tau} = .30 \) as my benchmark.

Third, I choose the constant size of rolled-over debt \( B \) by targeting the recovery rate in the event of default. Recall that the bondholders receives \( [1 - \tilde{\tau}][1 - \delta]k_R/B \), which is the recovery rate, per unit of defaulted risky bonds via liquidation process. The recovery rate for risky corporate bonds is again about 40\% on average during the period 1982-2009. Given \((\tau, \delta)\) and the model-implied capital held by a risky firm, \( k_R \), I choose \( B = 14.79 \) to equate the above expression to the average recovery rate for risky corporate bonds of 40\%.

Fourth, I choose the expected firm-level endowment-income \( \kappa \) to be as twice large as the interest expenses for a survived risky firm, which results in that the leverage ratio, i.e., the ratio of debt to the market value of equity plus debt, is 33\% for a survived risky firm, 20\% for a safe firm and 26-27\% in aggregate or cross-sectional average, which is broadly in line with the data.

6.4 Results

In this section, I present and discuss my simulation results for the internal financing model, see table 12.
Table 12: Statistics of Key Aggregates: Internal Financing Model

<table>
<thead>
<tr>
<th></th>
<th>( \sigma(y) )</th>
<th>( \sigma(x)/\sigma(y) )</th>
<th>( corr(x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x )</td>
<td>( y )</td>
<td>( h )</td>
</tr>
<tr>
<td>Data</td>
<td>1.56</td>
<td>1.27</td>
<td>0.34</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.91</td>
<td>0.73</td>
<td>1.33</td>
</tr>
<tr>
<td>IF</td>
<td>1.51</td>
<td>0.99</td>
<td>0.42</td>
</tr>
</tbody>
</table>

*Note:* The line headed ‘F’ refers to the internal financing model and the line headed ‘Baseline’ does the same for the baseline economy of 100% debt financing. \( \sigma(x)/\sigma(y) \) refers to the ratio of the percentage standard deviation of a variable \( x \) to that of output \( \sigma(y) \) and \( corr(x, y) \) refers to the correlation coefficient between \( x \) and output. All variables refer to their log deviations.

The main message of table 12 is that shocks to \( \nu \) account for quite large part of fluctuations in output and TFP. This result shows that even the availability of the internal-fianncing does not alter my earlier claim that shocks to default-risk, more skewed for risky firms relative to safe firms, are still an important source of fluctuations in the U.S. economy.

One caveat is that shocks to current \( \nu \) affects directly the next-period aggregate output via changes in the measure of risky firms in the next-period holding all else constant:

\[
y_{t+1} = \lambda \left[ k_{t+1}^S \right]^{\theta} \left[ h_{t+1}^S \right]^{1-\theta} + N_{t+1} \left[ k_{t+1}^R \right]^{\theta} \left[ h_{t+1}^R \right]^{1-\theta} + \lambda \kappa + N_{t+1} \kappa
\]

where the last term \( N_{t+1} \kappa \) refers to the endowment-incomes earned by the risky firms, which are subject to the changes in \( \nu_t \) through the changes in \( N_{t+1} \) because of \( N_{t+1} = (1 - \nu_t) N_t + M \).

I decompose changes in output and TFP to isolate the effect of shocks to \( \nu \) on fluctuations in output and TFP via *endogenous* changes in resource-allocation while controlling for the effects of *exogenous* changes in endowment \( N_{t+1} \kappa \). For this purpose, I measure output and TFP in an alternative way as follows:

\[
\tilde{y}_t = y_t - \left[ N_t - N_{ss} \right] \kappa, \quad \tilde{TFP}_t = TFP_t \cdot \frac{\tilde{y}_t}{y_t}.
\]

where \( \tilde{y}_t \) measures changes in output induced by the endogenous changes in resource-allocation and \( \tilde{TFP}_t \) does the same for TFP.
I present the percentage standard deviations of (detrended) \( \bar{y}_t \) and \( \bar{TFP}_t \), see table 13.

Table 13: Percentage Standard Deviations of Output and TFP

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>Baseline IF: ( \bar{y} )</td>
<td>Data Baseline IF: ( \bar{TFP} )</td>
</tr>
<tr>
<td>1.56</td>
<td>0.91</td>
<td>0.83</td>
</tr>
<tr>
<td>1.10</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>0.89</td>
<td></td>
</tr>
</tbody>
</table>

*Note:* \( \bar{y} \) measures changes in output induced by the endogenous changes in resource-allocation in the IF model and \( \bar{TFP} \) does the same for TFP.

From table 13, we can see that the effects of shocks to \( \nu \) on output and TFP in the internal financing model are still sizable compared to the data even after controlling for changes in the endowment term, \( N_t \cdot \kappa \); moreover, they are larger compared to the baseline economy of 100% debt financing. The key in generating the difference in the results for output and TFP between the internal financing model and the baseline model is that the marginal cost of capital for a risky firm responds to an increase in \( \nu \) more sensitively in the internal financing model relative to the baseline model. As a result, capital allocation \( k^R/k^S \) is more sensitive to an increase in \( \nu \) in the internal financing model than in the baseline model, see table 14.

Table 14: Percentage Standard Deviation of Capital Allocation \( k^R/k^S \)

<table>
<thead>
<tr>
<th></th>
<th>Baseline Model</th>
<th>IF Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>42.21</td>
<td>58.95</td>
</tr>
</tbody>
</table>

The results of table 14 show that capital allocation \( k^R/k^S \) indeed responds to \( \nu \)-shocks more sensitively in the internal financing model relative to the baseline model as was predicted in the earlier analysis.

I close this section by briefly discussing the volatility of the (equity) value of risky and safe firms. Note that risky firms are exposed to time-varying default-risk, which leads to the volatile equity value for a risky firm. On the contrary, a safe firm has no such risk. Therefore, the equity value should be more volatile for a risky firm relative to a safe firm, see table 15.
Table 15: Percentage Standard Deviation of Equity Value

<table>
<thead>
<tr>
<th></th>
<th>Risky Firm</th>
<th>Safe Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.59</td>
<td>5.89</td>
</tr>
</tbody>
</table>

Table 15 presents the percentage standard deviations of equity values at the final superiod, i.e., $V^R_+(\cdot)$ and $V^S_+(\cdot)$, for a survived risky and safe firms, respectively. We can see that the equity value for a risky firm is as about twice volatile as that for a safe firm.

7 Conclusion

In this paper, I studied the implications of fluctuations in corporate credit spreads for business cycle fluctuations. I built a simple model in which the difference in default probabilities on corporate debts leads to the spread in interest rates paid by firms. In the model, as risky firms become riskier, credit spread increases and capital is reallocated away from risky firms, which in turn leads to contractions in aggregate output, TFP, labor, consumption and investment. To quantify such effects, I calibrated my dynamic model to the U.S. economy and simulated it. In my benchmark simulation results, I found that shocks to the default-risk, more skewed to risky firms relative to safe firms, are an important source of fluctuations in the U.S. economy; those shocks account for about 60% of output fluctuations and 90% of TFP fluctuations during the period 1964-2009. I also found that the financial frictions, in particular the default costs, are an important amplification device.

As in the literature, I imposed the restriction that one-period bond is the only available instrument as an external debt financing. My quantitative results are quite similar even for the case in which firms use internal financing for their investments and roll over their constant size of one-period bond. It is interesting to extend my analysis presented in this paper to the environment in which more sophisticated financing vehicles are optimally chosen by firms, in particular, for the case of long-term bonds available. Furthermore, for simplicity, I abstracted from risk premium, which is known to be sizable and volatile for corporate bonds, see, e.g.,
8 References


Kim, S. and B. Choi (2012), “Cross-Section of Corporate Investment and Business Cycles”, manuscript, ITAM.


9 Appendix

9.1 Data Sources

NIPA Variables
I basically use the ‘Table 1.1.3. Real Gross Domestic Product, Quantity Indexes’ of NIPA, which provides quantity indexes for output, consumption and investment where the index for each item is seasonally adjusted and relative to the quarterly average over the year 2005. Then I convert the quantity index to the variables in terms of billions of chained 2005 dollars by using the quarterly average over the year 2005 based on ‘Table 1.1.6. Real Gross Domestic Product, Chained Dollars’.

Moody’s Dataset on Corporate Bond Market
Moody’s definition of default includes three types of events: missed or delayed repayment, bankruptcy and a distressed exchange. See Moody’s (2010) p.73-74 for more detail. The default rate of risky bonds is measured as the weighted average of default rates for the Baa, Ba and B grade bonds where the volume-shares by letter grades are used as the weights. Similarly, the default rate of safe bonds is measured as the weighted average of default rates for the Aaa, Aa and A grade bonds.

Table 11 present statistics, sample mean and standard deviation, of annual percentage default rates of corporate bonds according to the Moody’s grade system.

Table 16: Annual Percentage Default Rates of Corporate Bonds During the Period 1964-2009

<table>
<thead>
<tr>
<th>Grade</th>
<th>Safe</th>
<th>Risky</th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.02</td>
<td>1.93</td>
<td>0.00</td>
<td>0.03</td>
<td>0.03</td>
<td>0.16</td>
<td>1.00</td>
<td>4.72</td>
</tr>
<tr>
<td>Std.</td>
<td>0.07</td>
<td>1.76</td>
<td>0.00</td>
<td>0.12</td>
<td>0.08</td>
<td>0.28</td>
<td>1.18</td>
<td>4.34</td>
</tr>
</tbody>
</table>

Note: Default rates are measured by number of issuers rather than by the bond-volumes.

I next present the number-of-issuers shares of corporate bonds by their credit grades. Lastly, I present the recovery rates of corporate bonds.
Table 17: Percentage Shares of Corporate Bonds During the Period 1997-2000

<table>
<thead>
<tr>
<th></th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>3.3</td>
<td>13.2</td>
<td>25.4</td>
<td>21.9</td>
<td>13.7</td>
<td>18.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Note: Default rates are measured by number of issuers rather than by the bond-volumes.

Table 18: Average Percentage Recovery Rates of Corporate Bonds During the Period 1982-2009

<table>
<thead>
<tr>
<th></th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>61.96</td>
<td>44.37</td>
<td>41.44</td>
<td>43.794</td>
<td>42.36</td>
<td>37.53</td>
<td>34.85</td>
</tr>
</tbody>
</table>

Note: Recovery rates are measured by the year prior to defaults.

Recovery rate of a corporate bond is measured by the post-default price of that corporate bond relative to the price of the corporate bond in some years prior to default. The recovery rates reported in the above table are averages of them measured in 1, 2, 3, 4, and 5 years prior to default. See Moody’s (2010) for discussion in more detail.

9.2 Calibration

Recovery Rate

Recovery rate is defined as the price of a defaulted bond relative to the price of the bond prior to default. I begin by describing the procedure for calculating the recovery rate for risky corporate bonds in the data. I calculate recovery rates for the risky corporate bonds by a weighted average of recovery rates for Baa, Ba and B grade bonds, which results in about 41.3% on average over the period 1982-2009. Taking into consideration the Caa-C grade bonds omitted, of which recovery rates are about 34% on average for the same period, I adjust the above 41.3% recovery rate slightly downward to 40% and take it as my estimate of the long-run recovery rates of the risky corporate bonds.

In turn, I describe how to measure the recovery rate in the model economy consistently with the data. First, in the model, one unit of defaulted risky debt returns \([1 - \delta][1 - \tau]\) units of undepreciated capital where the price of capital in the final subperiod in terms of the final good
is equal to one. Next, I measure the price of a risky debt prior to default, i.e., in the initial subperiod, in terms of the final good in the final subperiod as the inverse of gross returns to the debt in the event of non-default as in the literature: $1/[1 - \delta + r^R]$. Therefore, the price of a defaulted risky debt relative to the price of the debt prior to default is given by the ratio of $[1 - \delta][1 - \tau]$ relative to $1/[1 - \delta + r^R]$:

$$\frac{[1 - \delta][1 - \tau]}{1 - \delta + r^R} = \text{recovery rate.}$$

**Bond Size by Credit Ratings**

Next, I describe the procedure for calculating the volume share of safe bonds which consist of Aaa, Aa and A grade bonds, or the investment-grade excluding Baa grade bonds. Note that the volume share of the speculative-grade bonds in the U.S. newly-issued, per year, corporate bond market is about 27.5% on average in 1993-2009 according to Altman and Karlin (2010). Therefore, the volume share of the investment-grade bonds is about 72.5% on average in 1993-2009 because corporate bonds are categorized as either investment-grade or speculative-grade. The Baa grade corporate bonds account for about 34.25%, on average, of the number of outstanding investment-grade bond-issuers in 1997-2000 according to Hamilton (2001). I assume that volume shares are proportional to the number-of-issuers shares for the investment-grade bonds and I calculate the volume-share of safe bonds in corporate bond market, which is given by:

$$0.725 \times [1 - .3425] = .4767$$

**Shock Process for $\nu$**

I discretize the space of $\nu$ by partitioning the interval $(0, 0.02)$ by an equal distance of 0.001 and then set the lower bound to 0.000025. Note that I set the maximum level of quarterly default rate to 2 percent, which amounts to 8 percent annual default rate as is the maximum level of the historical annual default rates.

**9.3 Simulation Result**

**Measuring Capital Stock with Perpetual Inventory Method**

I construct capital stock by using the perpetual inventory method as in the literature. I denote
by $K^{PI}_t$ such a constructed capital stock in period $t$, which is given by:

$$K^{PI}_{t+1} = [1 - \tilde{\delta}]K^{PI}_t + I_t, \quad K^{PI}_0 = K_0.$$ 

$\tilde{\delta}$ is the constant depreciation rate the same as what is used in constructing capital series in the data. I restrict $\tilde{\delta}$ such that $K^{PI}_t$ is the same with the correctly measured capital stock, $K_t$, in the deterministic steady state in which $\nu$ is constant equal to its average in the long-run. This results in $\tilde{\delta} = .0165$, which implies that capital is depreciated by 6.60% per year on average: 0.60% depreciation is due to losses for occurrences of defaults and the remaining 6.0% depreciation is due to non-defaults, e.g., physical and economical depreciation.

### 9.4 Mathematical Appendix

In this section, I provide the proof of proposition 1.

**Proof.** 1. I prove the existence and uniqueness of the solution to the pseudo planner’s problem.

The first order condition of the pseudo planner’s problem is given by:

$$1 - \delta + \alpha [k^S]^{\alpha - 1} = [1 - \nu] \left[ 1 - \delta + \alpha \frac{1}{1 - \nu} [k^R]^{\alpha - 1} \right] + \nu \left[ [1 - \delta][1 - \tau] \right],$$

$$\frac{1}{2} [k^S + k^R] = \kappa.$$

The first equation is the optimality condition of capital allocation, which says that the expected gross returns to capital should be equalized across firms, and the second equation corresponds to the resource constraint to capital. These two equations simplify to:

$$\alpha [k^S]^{\alpha - 1} = [1 - \nu] \alpha \frac{1}{1 - \nu} [k^R]^{\alpha - 1} - \nu \tau [1 - \delta],$$

$$k^R = 2\kappa - k^S.$$

I plug $k^R = 2\kappa - k^S$ into the optimality condition of capital allocation, i.e., equalization of the expected gross returns, and derive one equation with one unknown as follows:

$$\alpha [k^S]^{\alpha - 1} - \alpha [2\kappa - k^S]^{\alpha - 1} = -\nu \tau [1 - \delta]$$
where the LSH of the above equation is strictly decreasing in $k^S$ and the RHS of the above equation is constant in $k^S$.

Note that $\alpha \in (0,1)$, and it follows that the LHS explodes to negative infinity as $k^S \to 2\kappa$ while it explodes to positive infinity as $k^S \to 0$. The LHS is continuous with respect to $k^S$, and thereby existence of the solution $k^S \in (0,2\kappa)$ to the above equation follows from the intermediate value theorem. And the uniqueness of such a solution immediately follows from the strict monotonicity mentioned earlier. Finally, it is obvious that $k^S > k^R$ for the case of $\tau \in (0,1]$ because the RHS is negative in this case, and $k^S = k^R = \kappa/2$ for the case of $\tau = 0$.

2. I prove the equivalence between the planner’s solution and the equilibrium allocation.

Using the decision rules of firms discussed earlier, I have derived, see lemma 1, a firm’s optimal quantity of capital as:

$$\alpha [k^S]^{\alpha-1} = r^S \quad \text{and} \quad \alpha \frac{1}{1 - \nu} [k^R]^\alpha = r^R$$

which shows that allocation of capital and interest rates are in a tight relationship.

The optimality condition of the household’s problem implies that:

$$1 - \delta + r^S = [1 - \nu] (1 - \delta + r^R) + \nu [1 - \delta] [1 - \tau].$$

Plugging the earlier condition $\alpha [k^S]^{\alpha-1} = r^S, \alpha \cdot 1/[1 - \nu] \cdot [k^R]^{\alpha-1} = r^R$ and market clearing condition $[1/2][k^S+k^R] = \kappa$ into the above optimality condition of the household’s problem, I simplify the characterization of the equilibrium allocation as:

$$\alpha [k^S]^{\alpha-1} = \alpha [2\kappa - k^S]^{\alpha-1} - \nu \tau [1 - \delta]$$

which is identical to the earlier optimality condition of the pseudo planner’s problem. 

\[\square\]