Adverse Selection, Uncertainty Shocks and Business Cycles*

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Abstract

I develop dynamic general equilibrium models in which private information about investment projects causes adverse selection in credit markets. I study the effect of uncertainty shocks which change the degree of asymmetric information. An increase in asymmetric information aggravates adverse selection, raises interest rate spreads and shrinks intermediation. Amplified by a counter-cyclical markup in wages and variable capital utilization rates, the uncertainty shock drives business cycles. I establish a close link between the uncertainty shock and other shocks including financial shocks.

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Key words: Adverse selection, uncertainty shocks.

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1. Introduction

Financial markets bring prosperity, but sometimes they become dysfunctional and cause disruption. The great recession starting in late 2007 reminds strongly of the latter. Financial markets feature fragility, which seems obvious, yet they feature complexity too. Specifically, financial markets differ in what is traded and in the source of financial frictions if any. In this paper I focus on the role of asymmetric information in credit (loan) markets and show that a change in the asymmetric information aggravates adverse selection and causes recession in a dynamic general equilibrium framework.

Credit markets play a dominant role in funding, yet they involve some degree of asymmetric information. Entrepreneurs raise funds by issuing bonds or by applying loans to a bank. While a bank has some information on the entrepreneurs, the bank faces asymmetric information about the likelihood of default. Some entrepreneurs may engage in risky projects, which make them less likely to repay. With this asymmetric information, adverse selection could occur, as shown by Stiglitz and Weiss (1981). Adverse selection makes some safe and productive entrepreneurs fail to raise funds.

I develop dynamic general equilibrium models with adverse selection in credit markets, building upon Stiglitz and Weiss (1981). I introduce three new elements to the original Stiglitz and Weiss model. First, I introduce a variable scale of projects while the scale is fixed at unity in Stiglitz and Weiss (1981). Second, I introduce an agency problem: a borrower can pledge at most a fraction of its return. Third, I introduce exogenous shocks, named uncertainty shocks, which change the degree of asymmetric information in credit markets. The first element is essential in a dynamic general equilibrium model. The second element not only adds another credit frictions but also helps to incorporate the variable scale of investment. The third element, the uncertainty shock, triggers business cycles. Specifically, the negative uncertainty shock aggravates adverse selection and shrinks intermediation, accompanied by a spike in interest rate spreads, and it eventually causes recession.

The uncertainty shock could have different effects on a real economy, depending on the role of entrepreneurs who have private information. To make clear the point I consider two dynamic general equilibrium models with adverse selection. In the first model (Model-I), entrepreneurs own, trade and rent out capital, whose activities constitute the aggregate demand for capital. This modeling parallels with Bernanke, Gertler and Gilchrist (1999, BGG in short hereafter) who consider Townsend (1983) costly state verification (CSV) problems with such entrepreneurs. In the second model (Model-II), entrepreneurs produce investment goods, whose activities constitute the aggregate supply of investment. This modeling parallels with Carlstrom and Fuerst (1997) who consider CSV problems with such entrepreneurs.

I show analytically that the uncertainty shock emerges as well-known shocks. In Model-
I, the uncertainty shock emerges as financial shocks which exogenously change an interest rate spread, a wedge between a return to capital and a risk free rate. Hall (2010), Gilchrist, Yankov and Zakrajsek (2009) and Gilchrist and Zakrajsek (2010) among others claim that the financial shock can explain the key features of the great recession. In Model-II, the uncertainty shock emerges as shocks to the marginal efficiency of investment (MEI), or an investment wedge. Justiniano, Primiceri and Tambalotti (2009a,b) show that the MEI shock constitutes the main driving force in the U.S. including the great recession. Justiniano, et al (2009b) make a conjecture that the MEI shock appears as a reduced form shock related with financial intermediation.

The adverse selection with uncertainty shocks provides micro-foundations for the financial shock and the MEI shock. While the financial shock and the MEI shock changes wedges exogenously, the uncertainty shock builds upon micro-foundations so that it answers why the wedges change. The negative uncertainty shock increases the degree of overall uncertainty about the riskiness of entrepreneurial projects. A bank faces more asymmetric information and responds by raising loan rates, resulting in a spike in interest rate spreads in Model-I or a drop in the MEI in Model-II. Thus, this paper contributes in understanding the source of the financial shock and the MEI shock.

I show quantitatively that the uncertainty shocks generate significant fluctuations consistent with business cycles in both the two models. The key mechanisms which amplify the uncertainty shock consist of a counter-cyclical markup in wages and variable capital utilization rates. I embed the markup exogenously to focus on the role of amplification mechanisms. With the markup the marginal product of labor is equated to the markup times the marginal rate of substitution between consumption and hours. The model overcomes a co-movement problem, first pointed by Barro and King (1984), if the counter-cyclicality of markup is great enough.

The uncertainty shock is related with a literature that studies time-varying volatility. Uncertainty, measured by various second moments, appears to increase after major economic and political incidents (Bloom, 2009) and during recessions in the U.S. including the great recession (Bloom, Floetotto and Jaimovich, 2010).1 This paper is closely related with Williamson (1987) and Christiano, Motto and Rostagno (2010, in short CMR hereafter) who consider so called risk shocks, shocks to the riskiness of the entrepreneurs return in CSV problems. I show that the uncertainty shock in Model-I brings similar but slightly different quantitative implications relative to the risk shock. Since the degree of frictions in CMR depends on the cost of state verification technologies, the risk shock affects the degree of frictions and so does the cost. As a result the risk shock generates different responses of output and hours from those of the uncertainty shock.

1Gilchrist, Sim and Zakrajsek (2010) and Kiley and Sim (2011), using firm-level data or macro data, provide empirical evidence on uncertainty shocks as a source of business cycles. Kehrig (2011), using plant-level data, shows that the dispersion of total factor productivity is procyclical.
This paper contributes to a growing literature on adverse selection in a dynamic general equilibrium model. Eisfeldt (2004), Kurlat (2012) and Bigio (2011) study lemons problem in a dynamic framework, where entrepreneurs raise funds only by selling assets subject to asymmetric information. While they focus on adverse selection in buyer-seller problems a la Akerlof (1970), I focus on adverse selection in borrower-lender problems a la Stiglitz and Weiss (1981), which enables me to study an interest rate spread and a leverage. There are only a few dynamic general equilibrium model adopting Stiglitz and Weiss (1981). House (2005) studies adverse selection in an overlapping generations framework. Christiano and Ikeda (2011) builds upon Mankiew (1986) and studies a two-period general equilibrium model with adverse selection. Both of them fix a scale of projects at unity. I allow a variable scale of projects and embed adverse selection into a dynamic general equilibrium model in a reasonable manner.

Broadly this paper contributes to a literature on financial frictions in a dynamic general equilibrium model. The literature has studied various financial frictions. A selective list includes CSV problems (BGG 1999, Carlstrom and Fuerst 1997, CMR 2010), collateral constraints (Kiyotaki and Moore 1997, Iacoviello 2005, Jerman and Quadrini 2012), asset resaleability constraints (Kiyotaki and Moore 2008, Del Negro, Eggertsson, Ferrero and Kiyotaki 2011), moral hazard problems (Gertler and Karadi 2011, Gertler and Kiyotaki 2010), and adverse selection in buyer-seller problems (Eisfeldt 2004, Kurlat 2012, Bigio 2011). This paper adds adverse selection in borrower-lender problems to the literature.

I organize the rest of the paper as follows. In Section 2, I present a partial equilibrium model with asymmetric information and an agency problem. I start from a symmetric information model with an agency problem as a baseline and proceed to an asymmetric information model. In Section 3, I embed the partial equilibrium model into dynamic general equilibrium models and present two models. In each model I make clear the role of uncertainty shocks analytically. In Section 4, I conduct simulations to explore the quantitative effects of uncertainty shocks. In Section 6, I conclude the paper.

2. Adverse Selection: A Partial Equilibrium Model

I present a partial equilibrium model with adverse selection in credit (loan) markets. Entrepreneurs have private information and borrow funds from a bank. I solve for the optimal contract between entrepreneurs and a bank and derive aggregate relationships. In this section I omit time subscript $t$ for notational simplicity. In the subsequent sections I embed this model into dynamic general equilibrium models and explore the impact of uncertainty shocks.
2.1. Environment

Overview: There exist many entrepreneurs and a bank. A financing problem is static in that it involves only one time borrowing and lending. An entrepreneur combines its own net worth and a loan made by a bank to invest in a project. The entrepreneur knows the riskiness of its project, which could differ among entrepreneurs, while the bank does not. Also, the entrepreneur can pledge to repay at most a fraction of its return. The bank provides a schedule of contracts without knowing an entrepreneur’s riskiness. The entrepreneur chooses the best contract among the schedule. After the outcome of project is realized, the entrepreneur and the bank receive returns according to the contract.

The financing problem features two sources of agency problems. An entrepreneur’s private information about the riskiness of its project causes adverse selection. An entrepreneur’s limited pledgeability constrains the amount of loan made by a bank.

Entrepreneurs: There exist many risk neutral entrepreneurs whose objective is to maximize their expected return. Initially an entrepreneur starts its business with net worth $N_n$, indexed by $n$. The entrepreneur has a risky project with default probability $1 - p$. Success probability $p$ is private information to the entrepreneur and is drawn from distribution function $F: [p, \bar{p}] \rightarrow [0, 1]$ with $0 < p < \bar{p} \leq 1$, independently and identically across entrepreneurs. I assume that distribution $F(p)$ has full support.

A set of index, $(n, p)$, characterizes each entrepreneur. The type-$(n, p)$ entrepreneur has net worth $N_n$ and has a project with success probability $p$. I assume that for given $n$ there exist many entrepreneurs so that distribution $F(\cdot)$ coincides with the distribution of the type-$n$ entrepreneurs.

The type-$p$ project either succeeds or defaults. In case of success the project yields gross return $\theta(p)R^e$ per unit of goods invested. In case of default the project yields zero return. I assume $\theta(p) = 1/p$ for analytical tractability so that the expected gross return, just after the realization of the project’s outcome (a success or a default), becomes identical for all projects. This simplification allows me to focus on the riskiness of project, which is private information to entrepreneurs.

A limited liability law protects entrepreneurs. Entrepreneurs do not have any liability after paying to a bank. The limited liability implies that a bank cannot force an entrepreneur to pay when the entrepreneur defaults with its project, because the entrepreneur has nothing to pay at hand. Therefore, a financing contract between the type-$(n, p)$ entrepreneur and a bank is characterized by two objects: the amount of loan, $B_n(p)$, and the amount of payment in case of success, $X_n(p)$. The sum of loan $B_n(p)$ and net worth $N_n$ constitutes the entrepreneur’s asset which is invested in a project.

In addition to asymmetric information, there exits another friction. An entrepreneur can pledge at most a fraction, $0 < 1 - \phi < 1$, of its expected return to repay to a bank. This limited pledgeability imposes a constraint on a pair of loan $B_n(p)$ and payment $X_n(p)$
in case of success:

\[ pX_n(p) \leq (1 - \phi)Re[N_n + B_n(p)]. \]  

(1)

The left-hand-side of (1) denotes the expected payment to a bank while the right-hand-side of (1) denotes the maximum amount that the type-\((n, p)\) entrepreneur can pledge to repay. Though I do not provide micro-foundations behind the limited pledgeability, constraint (1) can be derived from a moral hazard problem in which an entrepreneur can divert fraction \(\phi\) of its return.

**Bank**: There exists a single monopolistic bank.\(^2\) The bank takes in deposits from households with risk-free rate \(R^f\) and makes a loan to entrepreneurs. In the process of making a loan contract, the bank, without knowing an entrepreneur’s riskiness \(p\), provides a schedule of contracts, \(\{B_n(p), X_n(p)\}\), as a function of entrepreneur’s riskiness. Since the bank can observe an entrepreneur’s net worth, the schedule also depends on the net worth indexed by \(n\). I restrict my attention to a truth telling schedule without loss of generality, so that the type-\((n, p)\) entrepreneur chooses contract \(\{B_n(p), X_n(p)\}\) offered by the bank. After the outcome of project is realized, the bank receives payment from entrepreneurs according to the contract. The bank lends to many entrepreneurs so that it diversifies the individual entrepreneur’s risk and pay interests for sure to households.

I impose following conditions to the financing problem.

**Assumption 1**: (i) \(R^e > R^f\), (ii) \(R^f > (1 - \phi)R^e\).

Assumption 1(i) is necessary for a bank to lend to entrepreneurs. Assumption 1(ii) is necessary for a loan being bounded. If the assumption does not hold, the more a bank makes a loan the more the bank makes profits.

### 2.2. Symmetric Information

Before solving a model with asymmetric information I consider a model with symmetric information as a benchmark to make clear the role of asymmetric information. In this model a bank observes entrepreneurial riskiness \(p\) while it still faces pledgeability constraint (1).

Given riskiness \(p\) and net worth \(N_n\), a bank chooses the amount of loan, \(B_n(p) \geq 0\), and the amount of payment in case of success, \(X_n(p) \geq 0\), to maximize its expected profits, \(\pi_n(p)\). The bank solves

\[
\max_{\{B_n(p), X_n(p)\}} \pi_n(p) = pX_n(p) - R^fB_n(p),
\]

subject to pledgeability constraint (1) and an entrepreneur’s participation constraint

\[
R^e[N_n + B_n(p)] - pX_n(p) \geq R^eN_n.
\]

\(^2\)I discuss competitive banks in Section 2.5.
The expected profits, $\pi_n(p)$, consist of the expected payment from the type-$(n,p)$ entrepreneur, $pX_n(p)$, and minus the cost of funds, $RF_n(p)$. Participation constraint (3) requires that the type-$(n,p)$ entrepreneur should earn at least $ReN_n$.

In solving problem (2) I rewrite participation constraint (3) as follows:

$$pX_n(p) = ReB_n(p) - \epsilon, \quad 0 \leq \epsilon \leq ReB_n(p).$$

Substituting the above constraint and the pledgeability constraint into $\pi_n(p)$, I express the profits as:

$$\pi_n(p) \leq (Re - Rf) \frac{1-\phi}{\phi} N_n - \frac{Rf - (1-\phi)Re}{\phi Re} \epsilon. \quad (4)$$

The inequality in (4) corresponds to that in the pledgeability constraint. From Assumption 1, maximizing the profits in (4) results in $\epsilon = 0$ and the inequality in (4) holding with equality. This implies that both pledgeability constraint (1) and participation constraint (3) are binding. As a result, the solution is given by

$$B_n(p) = \frac{1-\phi}{\phi} N_n, \quad (5)$$

and $X_n(p) = ReB/p$. The amount of loan, $B_n(p)$, is constrained by the amount of net worth, $N_n$. The leverage, $[B_n(p) + N_n(p)]/N_n(p)$, is given by $1/\phi$, which increases as $\phi$ becomes lower, i.e. as an entrepreneur can pledge more repayment.

The solution, (5), has a convenient feature for aggregation. Because an individual loan is linear in net worth and is independent of $p$, I can aggregate an individual loan over $n$ and $p$, and express the aggregate loan as

$$B = \frac{1-\phi}{\phi} N, \quad (6)$$

where $B$ and $N$ denote the aggregate loan and the aggregate net worth respectively. Importantly the aggregate loan is independent of the distribution of the riskiness of project, $F(p)$. A change in $F(p)$ affects neither the aggregate loan nor any other aggregate variables in a model with symmetric information.

2.3. Asymmetric Information

Now I consider a model with asymmetric information. Without loss of generality, I consider a truth telling contract in which entrepreneurs reveal their private information voluntarily. A monopolistic bank provides a schedule of contracts which specify the amount of loan, $B_n(p) \geq 0$, and the amount of payment in case of success, $X_n(p) \geq 0$. The bank’s problem is to maximize its expected profits subject to various constraints:

$$\max_{\{X_n(p), B_n(p)\}} \int_{p \geq 0} \left[ pX_n(p) - RfB_n(p) \right] dF(p), \quad (P1)$$
subject to, for all \( p \in [\underline{p}, \bar{p}] \), pledgeability constraint (1), participation constraint (3), and incentive constraint

\[
R^eB_n(p) - pX_n(p) \geq R^eB_n(\bar{p}) - pX_n(\bar{p}), \quad \text{for all } \bar{p},
\]

(7)

Incentive constraint (7) implies the type-(\( n, p \)) entrepreneur’s profits when it reveals its riskiness \( p \) are equal or greater than that when it deceives to be \( \bar{p} \neq p \).

I solve this problem in four steps.

**Step 1:** I start from a guess that there exists threshold \( p^* \) above which entrepreneurs do not get loans. This guess, combined with incentive constraint (7), implies

\[
\begin{align*}
&\text{for } p \geq p^* \quad R^eB_n(p) - pX_n(p) = 0, \\
&\text{for } p < p^* \quad R^eB_n(p) - pX_n(p) > 0.
\end{align*}
\]

(8)

That is, participation constraint (3) holds with equality for \( p \geq p^* \) and it holds with strict inequality for \( p < p^* \). From (7) and (8), I obtain for \( p \geq p^* \),

\[
0 \geq R^eB_n(p) - p^*X_n(p) = (p - p^*)X_n(p).
\]

(9)

I have derived the inequality in (9) using (7) for \( p = p^* \). I have derived the equality in (9) using (8). For \( p > p^* \) the payment, \( X_n(p) \), has to be zero since \( p - p^* > 0 \), otherwise condition (9) would not hold, violating either (7) or (8). Therefore, for \( p > p^* \) both a payment and a loan become zero: \( X_n(p) = B_n(p) = 0 \). For \( p < p^* \) it is straightforward to show that both a payment and a loan have to be positive. Given threshold \( p^* \), a participation constraint (3) is satisfied for all \( p \). Now I can rewrite problem (P1) as follows:

\[
\max_{\{B_n(p), X_n(p)\}_{p,p^*}} \int_{p^*}^{\bar{p}} \left[ pX_n(p) - R^eB_n(p) \right] dF(p),
\]

(P2)

subject to, for all \( p \in [\underline{p}, \bar{p}] \), pledgeability constraint (1) and incentive constraint (7).

**Step 2:** I replace incentive constraint (7) by a local incentive compatibility constraint and a monotonicity constraint as in a standard mechanism design problem: for all \( p \),

\[
\begin{align*}
&\quad R^e[dB_n(p)/dp] - p[dX_n(p)/dp] = 0, \\
&\quad dX_n(p)/dp \leq 0,
\end{align*}
\]

(10) \hspace{1cm} (11)

Intuitively, constraints (10) and (11) correspond to the first and second order conditions of an entrepreneur’s problem respectively. In the problem the type-(\( n, p \)) entrepreneur maximizes its expected profits, \( W_n(p) \equiv R^eB_n(p) - pX_n(p) \), by choosing a pair, \( \{B_n(p), X_n(p)\} \), among a schedule offered by a bank, \( \{B_n(\bar{p}), X_n(\bar{p})\}_{\bar{p}} \). I express the profits of the type-(\( n, p \)
entrepreneur with $p \leq p^*$ as a function of payment schedule $X_n(p)$ using local incentive compatibility constraint (10) and the envelope theorem as follows:

$$W_n(p) = \int_p^{p^*} X_n(x) dx.$$  \hspace{1cm} (12)

Expression (12) implies that the entrepreneur’s profits are increasing in payment $X_n(p')$ with $p \leq p' \leq p^*$. The intuition behind (12) has to do with the fact that risky entrepreneurs with lower $p$ repay less in expected values than safer entrepreneurs with higher $p$. For example, if the type-$p^*$ entrepreneur re-paid more, it would have to receive more loan to satisfy participation constraint (3). This new pair of loan and payment for $p = p^*$ is also available to other entrepreneurs. For $p < p^*$, an increase in loan raises the type-$p$ entrepreneur’s expected gross return by the same amount as an increase in the type-$p^*$ entrepreneur’s return, while an expected repayment is smaller for the type-$p$ entrepreneur than for the type-$p^*$ entrepreneur. Therefore, risky entrepreneurs with lower $p$ can enjoy rents accrued from a lower repayment in expected values.

Using (10) and (12) I express loan schedule $B_n(p)$ as a function of $X_n(p)$:

$$B_n(p) = \left[pX_n(p) + \int_p^{p^*} X_n(x) dx \right] / R.$$  \hspace{1cm} (13)

Local incentive compatibility constraint (10) is satisfied as long as a loan schedule is given by (13). Using loan schedule (13), I express the banker’s profits in (P2) as follows:

$$V_n(p^*) \equiv \int_p^{p^*} \omega(p) X_n(p) dp,$$  \hspace{1cm} (14)

where $\omega : [p, \bar{p}] \rightarrow \mathcal{R}$ is given by

$$\omega(p) \equiv pf(p) - \frac{Rf}{Re} pf(p) - \frac{Rf}{Re} F(p).$$  \hspace{1cm} (15)

Also, using loan schedule (13) I rewrite pledgeability constraint (1) as

$$X_n(p) \leq \frac{(1 - \phi)}{\phi p} R^e N_n + \frac{(1 - \phi)}{\phi p} \int_p^{p^*} X_n(x) dx.$$  \hspace{1cm} (16)

Now I simplify problem (P2) as follows.

$$\max_{\{\{X_n(p)\}_{p,p^*}\}} \int_p^{p^*} \omega(p) X_n(p) dp,$$  \hspace{1cm} (P3)

subject to monotonicity constraint (11) and pledgeability constraint (16).
Step 3: I conjecture that monotonicity constraint (11) holds and will confirm it later. Let \( \xi(p) \geq 0 \) denote the Lagrange multiplier associated with constraint (16). I express problem (P3) in a Lagrangean form:

\[
\max_{\{X_n(p)\}_{p,p^*}} \int_p^{p^*} \left\{ \omega(p)X_n(p) + \xi(p) \left[ \frac{1 - \phi}{\phi} R^c N_n + \frac{1 - \phi}{\phi} \int_p^{p^*} X_n(x) - pX_n(p) \right] \right\} dp.
\]

Using integral by part, I rewrite this problem as follows.

\[
\max_{\{X_n(p)\}_{p,p^*}} \int_p^{p^*} \left\{ \left[ \omega(p) - p\xi(p) + \frac{1 - \phi}{\phi} \Xi(p) \right] X_n(p) + \frac{1 - \phi}{\phi} \xi(p)R^c N_n \right\} dp, \tag{17}
\]

where \( \Xi(p) \equiv \int_p^{p^*} \xi(x)dx \) (I have assumed that there is no mass point in \( \Xi(p) \) and I have used condition \( \overline{\Xi}(p) = 0 \)). Because of linearity in \( X_n(p) \), payment schedule \( X_n(p) \) has a corner solution: either \( X_n(p) = 0 \) or \( X_n(p) \) is maximized so that constraint (16) holds with equality. Given \( X_n(p) > 0 \), an optimality condition with respect to \( X_n(p) \) is

\[
\omega(p) - p\xi(p) + \frac{1 - \phi}{\phi} \Xi(p) = 0. \tag{18}
\]

This condition constitutes an integral equation with terminal condition \( \xi(p) = \omega(p)/p > 0 \). The solution is:

\[
\xi(p) = \frac{\omega(p)}{p} + \frac{1 - \phi}{\phi} \frac{1}{p} \int_p^{p^*} \omega(x)(1/x)^{1/\phi} dx \tag{19}
\]

Here I impose a following condition to ensure a unique interior solution.

Assumption 2: Distribution \( F(p) \) and parameters \( R^f \) and \( R^c \) satisfy that there exists a unique \( p = p^o \in (p, \bar{p}) \) such that the right-hand-side of equation (19) is zero.

Assumption 2 implies that \( \xi(p) > 0 \) for \( p < p^o \) and \( \xi(p) = 0 \) for \( p = p^o \). Since \( \xi(p) \), a Lagrange multiplier, has to satisfy \( \xi(p) \geq 0 \), and since payment schedule \( X_n(p) \) has a corner solution, it must be that \( \xi(p) = 0 \) and \( X_n(p) = 0 \) for \( p > p^o \). This implies that threshold \( p^* \) has to satisfy \( p^* \leq p^o \). Suppose \( p^* < p^o \). Since \( \xi(p) > 0 \) for \( p \leq p^* \), Lagrangean (17) implies that an increase in \( p^* \) raises the banker’s profits. So, \( p^* < p^o \) cannot be a solution. Therefore, the threshold is given by \( p^* = p^o \). Specifically, threshold \( p^* \) is a solution to:

\[
0 = \frac{\omega(p^*)}{p^*} + \frac{1 - \phi}{\phi p^*} \frac{1}{p^*} \int_p^{p^*} \omega(x)(1/x)^{1/\phi} dx. \tag{20}
\]

Step 4: I have shown that constraint (16) holds with equality for \( p \leq p^* \), where \( p^* \) is given by (20). With inequality holding with equality constraint (16) constitutes an integral equation for unknown function \( X_n(p) \) with terminal condition \( X_n(p^*) = [(1-\phi)/(\phi p^*)]R^c N_n \).
I solve the integral equation and obtain a schedule of contract as follows: for \( p \leq p^* \)

\[
X_n(p) = \left[ \frac{(1 - \phi)R^e}{\phi} (p^*)^{\frac{1 - \phi}{\phi}} \right] \left( \frac{1}{p} \right)^{\frac{1}{\phi}} N_n, \tag{21}
\]

\[
B_n(p) = \frac{1 - \phi}{\phi} \left\{ 1 + \frac{1}{1 - \phi} \left[ \left( \frac{p^*}{p} \right)^{\frac{1 - \phi}{\phi}} - 1 \right] \right\} N_n, \tag{22}
\]

where I have derived equation (22) by substituting out \( X_n(p) \) in (13) using (21). Note that \( X_n(p) \) is strictly decreasing in \( p \) so that it satisfies monotonicity condition (11). I conclude that, the schedules, (21) and (22), with \( p^* \) given by (20) is a solution to the original banker’s problem, (P1), for \( p \leq p^* \). For \( p > p^* \), both loan and payment are zero: \( X_n(p) = B_n(p) = 0 \).

2.4. Loan Contract Properties

I study the loan contract under asymmetric information derived above. First, I study its micro properties. Second, I study its macro implications.

**Micro properties:** Under asymmetric information the payment and loan schedules, (21) and (22), have the following properties:

(i) A pair of payment (21) and loan (22) features a separating contract.

(ii) Both the payment and the loan are decreasing in riskiness \( p \) and increasing in threshold \( p^* \).

(iii) The borrowing interest rate, \( R^b(p) \equiv X_n(p)/B_n(p) \), is decreasing in riskiness \( p \).

(iv) Threshold \( p^* \) is increasing in the premium, \( R^e/R^f \).

Property (i) distinguishes this model from Stiglitz and Weiss (1981) model which features a pooling equilibrium. Interestingly this model features heterogeneous borrowing interest rates, as stated in (iii), which implies heterogeneous interest rate spreads. Though a bank does not observe riskiness \( p \), a riskier borrower faces higher interest rate spreads.

Properties (ii) and (iii) imply that an entrepreneur with a higher risk project gets more loans and pays back more in case of success. Behind the properties lies in pledgeability constraint (1). The pledgeability constraint is less tightly binding for riskier entrepreneurs, given the amount of loan and repayment. A riskier entrepreneur borrows more and pays more than a safer entrepreneur does, resulting in (ii) and (iii).

Property (iv) implies that entrepreneurs get more loans as their return gets higher. In deciding threshold \( p^* \), a bank faces a following trade-off. On the one hand, the bank makes profits from making loans to new entrepreneurs by increasing \( p^* \). On the other hand, the bank has to bear the cost of entrepreneurs’ default, if the bank increases loan \( B_n(p) \) by
raising $p^*$ according to contract (22). A rise in premium $R^e/R^f$ increases the profitability of entrepreneurs, so the former effect outweighs the latter.

**Macro implications:** The loan contract, summarized by (20), (21) and (22), has two nice properties for aggregation. First, threshold $p^*$ does not depend on the amount of net worth as is clear from equation (20). This property implies that the same threshold applies to all entrepreneurs. Even an entrepreneur with great amount of net worth faces the same threshold. An entrepreneur cannot get a loan if the probability of success exceeds the threshold: $p > p^*$. Second, both loan schedule (22) and payment schedule (21) are linear in net worth. This implies that one can aggregate loan $B_n(p)$ and payment $X_n(p)$ without paying attention to the distribution of net worth.

As a consequence of the above two properties, the aggregate loan has a simple expression. From loan schedule (22) I can express the aggregate loan, $B$, as

$$B = \int_n \int_{p^*} B_n(p)dF(p)dH(n),$$

$$= \left[ \frac{1}{\phi} \int_{p^*}^{p} \left( \frac{p^*}{p} \right)^{\frac{1-\phi}{\phi}} dF(p) - F(p^*) \right] N,$$

where $F(\cdot)$ denotes the distribution of $p$ and $H(\cdot)$ denotes the distribution of net worth indexed by $n$. Let $W$ denotes a bank’s profits. Using the expression in (P3) I express the profits as

$$W = \int_n \int_{p^*} \omega(p)X_n(p)dpdH(n),$$

$$= \left[ \frac{1 - \phi}{\phi} \left( \frac{R^e}{\phi} \right) \right] \int_{p^*}^{p} \omega(p) \left( \frac{1}{p} \right)^{\frac{1}{\phi}} dp N,$$

$$= - R^e \omega(p^*) N,$$

where I have substituted out $X_n(p)$ using (21) in the second equality, and I have used (20) in the third equality. Since $\omega(p^*) < 0$ the profits are positive\(^3\). Also, using the expression in (P1) I express the profits in another way as

$$W = \int_n \int_{p^*} [pX_n(p) - R^f B_n(p)]dF(p)dH(n),$$

$$= \frac{1 - \phi}{\phi} R^e \left[ \int_{p^*}^{p} \left( \frac{p^*}{p} \right)^{\frac{1-\phi}{\phi}} dF(p) \right] N - R^f B,$$

\(^3\)Equation (18) and $\xi(p^*) = 0$ implies that $\omega(p^*) < 0$. 

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Using (23)-(25) I substitute out $W$ and the integral term and obtain a simple expression for the aggregate loan:

$$B = \frac{(1 - \phi)(R^e/R^f)}{1 - (1 - \phi)(R^e/R^f)} \left[ F(p^*) + \frac{1}{1 - \phi} \omega(p^*) \right] N. \quad (26)$$

Equation (26) implies that two factors distort the aggregate loan. First, the monopoly of lending allows a bank to earn positive profits and decrease the aggregate loan, which is captured by $\omega(p^*) < 0$ in equation (26). If the bank earned zero profits, i.e., $\omega(p^*) = 0$, the aggregate loan would become greater. Second, due to adverse selection the bank makes loans to smaller number of entrepreneurs, which is captured by $F(p^*) < 1$ in equation (26). Although all entrepreneurs have the same expected return, $R^e$, entrepreneurs with $p > p^*$ do not get funded. In order to cover the default cost of high risk (low $p$) entrepreneurs, the bank offers relatively high interest rates to entrepreneurs. Low risk (high $p$) entrepreneurs are more likely to repay in expected values, which makes a loan arrangement unattractive. As a result, low risk entrepreneurs with $p > p^*$ do not borrow and the aggregate loan decreases.

Without a monopoly and adverse selection, the aggregate loan would be greater, given by (26) with $\omega(p^*) = 0$ and $F(p^*) = 1$. Without adverse selection but with monopoly, the aggregate loan is given by equation (6). Given that $R^e/R^f > 1$ is close to unity and $p^* < 1$ is away from unity, the aggregate loan with the two distortions, given by (26), is lower than the aggregate loan with monopoly only, given by (6).

### 2.5. Competitive Banks

I have assumed a monopoly bank in deriving the optimal contract. If banks are competitive, does an equilibrium exist? If any, how does the equilibrium contract look like? Here I briefly argue that there is no symmetric equilibrium under the current contracting procedure. I discuss how to resolve the non-existence problem by changing a contracting procedure.

I start from defining an equilibrium. Suppose that there exist some competitive banks. I assume that entrepreneurs choose a bank randomly if contracts provided by banks are indifferent. A triplet, $\{X_n(p), B_n(p), p^*\}$, constitutes a symmetric equilibrium if and only if it satisfies plegability constraint (1), participation constraint (3), incentive constraint (7), a bank’s zero profit condition and a bank’s no profitable deviation.

A natural candidate for an equilibrium is $\{X_n^{cm}(p), B_n^{cm}(p), p^{cm}\}$ where $X_n^{cm}(p)$ and $B_n^{cm}(p)$ are given by (21) and (22) respectively, with $p^*$ replaced by $p^{cm}$, which is given by a bank’s zero profit condition:

$$\int_{\underline{p}}^{p^{cm}} \omega(p)X_n^{cm}(p)dp = 0.$$ 

Here I simply assume that such $p^{cm}$ uniquely exists. By construction, plegability constraint (1) holds for all $p \in [\underline{p}, p^{cm}]$, so that payment $X_n^{cm}(p)$ attains its maximum for each $p \in$
This, in turn, implies that the type-\((n,p)\) entrepreneur’s profits, given by (12), attains their maximum for each \(p \in [p, p^{cm}]\). Therefore, this contract is the most attractive one to entrepreneurs with \(p \leq p^{cm}\) among those satisfying equilibrium conditions.

Now I shall show that there exists no symmetric equilibrium. Suppose contrarily that such an equilibrium exists, denoted by \(\{X_n^c(p), B_n^c(p), p^*\}\). First, consider a case that the equilibrium differs from \(\{X_n^{cm}(p), B_n^{cm}(p), p^{cm}\}\). Then, there exists \(\epsilon > 0\) such that a triplet, \(\{X_n^{cm}(p), B_n^{cm}(p), p^{cm} - \epsilon\}\), attracts all entrepreneurs with \(p \leq p^{cm} - \epsilon\) and the profits of a deviated bank are positive. Because such a deviated schedule of contract is close enough to the one which maximizes the profits of entrepreneurs with \(p \leq p^{cm}\), entrepreneurs with \(p \leq p^{cm} - \epsilon\) prefer the deviation over the incumbent. The positive profits result from the assumption of the existence of unique \(p^{cm}\). Next, consider a case that the equilibrium coincides with \(\{X_n^{cm}(p), B_n^{cm}(p), p^{cm}\}\). There are many profitable deviations but I propose a simple one, \(\{X_n^d(p), B_n^d(p), p^{cm} + \epsilon\}\), where for small \(\epsilon > 0\),

\[
X_n^d(p) = X_n^d = \frac{(1 - \phi)R_e}{\phi} (p^{cm} + \epsilon)^{\frac{1-\phi}{\phi}} N_n,
\]

and \(B_n^d(p)\) is given by (22) with \(p^*\) replaced by \(p^{cm} + \epsilon\). Given that other banks offer \(\{X_n^{cm}(p), B_n^{cm}(p), p^{cm}\}\), this deviation attracts only safe entrepreneurs with \(p'(\epsilon) < p \leq p^{cm} + \epsilon\), where \(p'(\epsilon)\) is given by

\[
\int_{p'(\epsilon)}^{p^{cm}} X_n^{cm}(x)dx = [p^* + \epsilon - p'(\epsilon)]X_n^d.
\]

The left-hand side denotes the profits of the type-\((n, p'(\epsilon))\) entrepreneur when it chooses the incumbent, while the right-hand side denotes the profits when it chooses the deviated bank. From each entrepreneur with \(p'(\epsilon) < p \leq p^{cm} + \epsilon\), a deviated bank can make profits, given by

\[
pX_n^d(p) - RfB_n^d(p) = X_n^d\left[p\left(1 - \frac{Rf}{R_e}\right) - (p^{cm} + \epsilon - p'(\epsilon))\right].
\]

The profits are positive for small \(\epsilon\). Therefore, the deviation attracts only safe entrepreneurs and makes positive profits, taking advantage of a situation where other banks try to attract not only safe entrepreneurs but also risky entrepreneurs.

The non-existence problem in this model shares a similar feature with Rothchild and Stiglitz (1976) who show that there exists no equilibrium in competitive insurance markets with adverse selection. Wislon (1977) and Hellwig (1987) propose a solution how to resolve the non-existence problem. The idea is to prevent a bank from attracting only safe entrepreneurs by adding an additional contracting procedure in which a bank, after observing the other banks’ schedule of contracts, chooses to stay in or leave the market. With the additional procedure, a triplet, \(\{X_n^{cm}(p), B_n^{cm}(p), p^{cm}\}\), constitutes an equilibrium with competitive banks. I leave the detail in the appendix.
3. General Equilibrium Models

Now I embed adverse selection in credit markets, analyzed in the previous section, into a
dynamic general equilibrium model. In doing so I restrict my attention to a one-period
financing contract so that I can apply the results in the previous section directly. Set aside
the adverse selection, the basic framework is the standard real business cycle model with
exogenous countercyclical markups and endogenous capital utilization. As I show later
those additional features serve as critical mechanisms amplifying uncertainty shocks.

I study analytically the effect of uncertainty shocks which change the distribution of
entrepreneurs’ riskiness, in dynamic general equilibrium models. In the first model, I embed
the adverse selection into the demand side of capital as in BGG (1999). In the second model,
I embed the adverse selection into the supply side of investment as in Carlstrom and Fuerst
(1997). In the following I first describe technologies, preferences and shocks which are
common between the two models. Then, I describe the two models separately.

3.1. Common Building Blocks

Preferences: There is a continuum of household, with preferences given by
\[
E_t \sum_{s=0}^{\infty} \beta^s U(C_t, L_t), \quad 0 < \beta < 1,
\]
where \( C_t \) denotes consumption, \( L_t \) denotes labor supply and \( U(\cdot) \) satisfies \( U_1 > 0, U_2 < 0, U_{11}, U_{22} < 0 \) and \( U_{11} U_{22} - U_{12}^2 > 0 \).

A household, with measure unity, consists of the large number of family members who
are either workers or entrepreneurs with their population \( f \) and \( 1 - f \) respectively where \( 0 < f < 1 \). Family members switch their job occupation randomly. Specifically, entrepreneurs
become workers randomly with probability \( 1 - \gamma \) where \( 0 < \gamma < 1 \). I call \( \gamma \) as the surviving
probability of entrepreneurs. The same number of workers become entrepreneurs randomly
so that the proportion of workers or entrepreneurs stays constant over time.

The household, as a representative agent of the family members, consumes and saves.
The household provides perfect consumption insurance among its family members. Work-
ers within the household supply labor and earn wage income. Entrepreneurs within the
household specialize in investing in a project and accumulate their net worth.

An entrepreneur can transfer its net worth to the household to which it belongs at the
beginning of period. The entrepreneur chooses to accumulate the net worth over time and
transfers the net worth only when it changes to a worker, because the average return from a
unit of goods invested in its project is strictly greater than the risk-free return earned by the
household who makes deposits in a bank. While the individual entrepreneur is subject to a
risk associated with its project, the household cares only about the average return because
there are many entrepreneurs as family members within the household. This modeling
device makes entrepreneurs behave as if they were risk-neutral as analyzed in Section 2. This setting, adopted from Gertler and Karadi (2010), allows me to embed entrepreneurs into the standard dynamic general equilibrium model, keeping the representative agent framework in a reasonable manner.

A household can save only through deposits in a bank with risk-free rate $R_t$. The household owns firms and a bank. Then, the flow budget constraint is given by

$$C_t + B_t = R_{t-1}B_{t-1} + w_tL_t + \Theta_t,$$

where $B_t$ denotes the amount of deposit at the end of time $t$, $w_t$ denotes wages, and $\Theta_t$ includes the sum of the net transfer from entrepreneurs who belong to the household, the profits of firms and of a bank, and the net payment to the state-contingent securities on the return to capital. The net payment presents only in Model-II as explained later.

I introduce an exogenous countercyclical markup in wages to make clear its role as important mechanisms amplifying uncertainty shocks. I model the markup in the simplest but an ad-hoc manner for the sake of exposition. Specifically, I assume that workers have a monopolistic power over labor supply and set real wages equal to the markup over the marginal rate of substitution between consumption and labor. I assume the markups are exogenous and countercyclical, given by

$$\lambda_{w,t} = \lambda_w \left( \frac{Y_t}{Y} \right)^{-\omega}, \quad \lambda_w > 1, \quad \omega > 0,$$

where $Y_t$ denotes output and $Y$ denotes output in steady state. The time-varying markup, $\lambda_{w,t}$, is decreasing in output and countercyclical by assumption.

Maximizing utility (27) subject to budget constraint (28) yields the first order conditions:

$$\frac{1}{R_t} = E_t \beta \left[ \frac{U_1(C_{t+1}, L_{t+1})}{U_1(C_t, L_t)} \right],$$

$$w_t = -\lambda_{w,t}U_2(C_t, L_t)/U_1(C_t, L_t),$$

where $\lambda_{w,t}$ denotes an exogenous countercyclical markup in wages. Equation (30) implies that the gross interest rate on deposit, $R_t$, is risk-free and determined by the intertemporal marginal rate of substitution. Equation (31) implies that wage is equated to the markup over the marginal rate of substitution between consumption and labor.

---

4One can introduce a countercyclical markup in wages in a rigorous manner by introducing nominal rigidities in wages as in Erceg, Henderson and Levin (2000). I find that introducing nominal rigidities in wages does not change quantitative results in this paper very much (Ikeda, 2012). While nominal rigidities in wages serve as a solid micro-foundation for the countercyclical markup, it complicates the model. Because I focus on the counter-cyclical markup as amplification mechanisms, I introduce the countercyclical markup in the simplest manner for the sake of exposition.
Technologies: There exist competitive producers, with production technologies given by

\[ Y_t = (u_t K_t)^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \]  

where \( K_t \) denotes the aggregate capital, \( L_t \) denotes the aggregate labor and \( u_t \) denotes capital utilization rates. Factor prices are given by marginal pricing:

\[ r^k_t = \alpha (u_t K_t)^{\alpha-1} L_t^{-\alpha}, \]  
\[ w_t = (1-\alpha) (u_t K_t)^\alpha L_t^{-\alpha}, \]

where \( r^k_t \) denotes the rental rate of capital services, \( u_t K_t \).

Capital providers set the capital utilization rate, \( u_t \). As I explain later, the capital providers are entrepreneurs in Model-I while they are entrepreneurs or a bank in Model-II. The capital providers earn rental rates, \( r^k_t u_t \), per unit of capital, with the associated cost of capital utilization, \( a(u_t) \), in goods unit. As in CEE (2005), the cost satisfies \( a'(u_t) > 0 \) and \( a(1) = 0 \). Maximizing the rental rate minus the cost with respect to \( u_t \) results in

\[ r^k_t = a'(u_t). \]  

As is clear from equation (35), the capital utilization rate depends only on the net return on capital, \( r^k_t \), and is increasing in \( r^k_t \).

The goods market clearing requires

\[ Y_t = C_t + I_t + a(u_t) K_t, \]

where \( I_t \) denotes the aggregate investment. Term \( a(u_t) K_t \) denotes the total cost associated with capital utilization rate \( u_t \). The law of motion for capital satisfies:

\[ K_{t+1} = (1-\delta) K_t + \bar{I}_t, \quad 0 < \delta < 1, \]

where \( \bar{I}_t \) denotes newly produced capital goods, which is different from investment \( I_t \) in unit of consumption goods, and \( \delta \) denotes capital depreciation rates.

Uncertainty Shocks: As in the partial equilibrium model in Section 2, an entrepreneur has a project with success probability \( p \), which follows distribution \( F_t(p) \). Here I introduce an exogenous disturbance to the distribution of the riskiness of project. In order to get a simple analytical expression and conduct simulations later I assume that the distribution of the riskiness of project is uniform over interval \([p^*, 1] \):

\[ F_t(p) = \frac{p - p^*}{1 - p^*}, \quad p^* = p e^{\nu_t}, \quad 0 < p < 1, \]

with

\[ \nu_t = \rho_v \nu_{t-1} + \epsilon_{v,t}, \quad 0 < \rho_v < 1, \]
where $\epsilon_{\nu, t}$ denotes a disturbance i.i.d. with mean zero. When the negative uncertainty shock, $\epsilon_{\nu, t} < 0$, hits the economy, the distribution becomes more dispersed and entrepreneurs’ projects become more risky on average. A bank, as a lender, faces higher degree of asymmetric information about the riskiness of project, $p$, than before, which will cause more severe adverse selection in credit markets.

3.2. Model-I: Adverse Selection in the Demand Side of Capital

I embed adverse selection in credit markets, analyzed in Section 2, into the demand side of capital as in BGG (1999). In this model, entrepreneurs own, trade and rent out capital. In trading capital entrepreneurs purchase capital so that the entrepreneurs’ activities constitute the aggregate demand for capital. In the supply side of capital there are competitive capital goods producers subject to investment adjustment costs. If the demand for capital increases, the price of capital increases and so does investment. In this model the uncertainty shock emerges as financial shocks which change a wedge between a return to capital and a risk-free rate.

**Entrepreneurs and a Bank:** There exist many entrepreneurs within a household. At time $t$ an entrepreneur starts its business with some amount of net worth in unit of consumption goods. The entrepreneur makes a one-period contract with a bank and receives a loan from the bank. Combining the net worth with the loan the entrepreneur purchases capital goods from capital goods producers at price $q_t$. In aggregate the balance sheet of entrepreneurs is expressed as:

$$q_t K_{t+1} = N_t + B_t. \quad (39)$$

The left hand side of equation (39) denotes the value of capital purchased by entrepreneurs and the right hand side of equation (39) denotes the liability side of balance sheet, consisting of the aggregate net worth, $N_t$, and the aggregate loan, $B_t$.

At the end of time $t$ an entrepreneur invests capital goods in its project with success probability $p$ and transforms the capital goods into specialized capital goods readily use for production. The riskiness of project, $p$, is private information to the entrepreneur. I assume that on average one unit of capital goods generates one unit of specialized capital goods for all projects. If the project fails the entrepreneur receives zero return. If the project succeeds the entrepreneur rents out the specialized capital goods to firms and earns rental rate $r_{t+1} u_{t+1}$ per unit of effective capital goods (specialized capital goods times capital utilization rates) at the beginning of time $t+1$. The entrepreneur incurs the cost of capital utilization rates, $a(u_{t+1})$, per unit of specialized capital goods. After renting out capital goods, the entrepreneur sells the depreciated capital goods to capital goods producers at price $q_{t+1}$. Consequently, on average the return from investing one unit of consumption
goods, $R_k^{t+1}$, is given by,

$$R_k^{t+1} = r_k^{t+1}u_{t+1} + q_{t+1}(1 - \delta) - a(u_{t+1}).$$  \hspace{1cm} (40)

The return consists of the net return from renting out capital goods, $r_k^{t+1}u_{t+1}$, plus the capital gain from the depreciated capital goods, $q_{t+1}(1 - \delta)$, minus the cost associated with capital utilization rate, $a(u_{t+1})$, per unit of capital purchased at price $q_t$.

After earning returns, the entrepreneur makes interest payments to the bank. The remaining amount of consumption goods constitutes the entrepreneur’s net worth at time $t+1$. At the beginning of time $t+1$, an idiosyncratic occupation shock hits the entrepreneur and it switches its job to a worker randomly with probability $1 - \gamma$, and it continues the current job with probability $\gamma$, where $0 < \gamma < 1$. If the entrepreneur becomes a worker, it brings the net worth to the household to which it belongs. If the entrepreneur continues its job, it starts its business with its net worth at time $t+1$ again. Those who just have become entrepreneurs from workers and those who do not have net worth receive a small amount of goods from the household to which they belong, so that they can run their businesses.

A financing problem between entrepreneurs and a bank proceeds as in the partial equilibrium model in Section 2. The returns, $R^e$ and $R^f$ in the model in Section 2, corresponds to $E_r R_{t+1}^k$ and $R_t$ in this general equilibrium model respectively. In applying the solution in Section 2, I assume that payment $X_n(p)$ does not depend on states at time $t+1$. This assumption is innocuous because entrepreneurs behave as if they were risk neutral.

Now I derive equilibrium conditions by applying the solution in Section 2. First, I derive an equation for the demand for capital. Substituting (23) into (39) and using the expression for $F_t(\cdot)$, (38), I can express the value of purchased capital as

$$q_t K_{t+1} = \left[ 1 - p_t^* \left( 1 - \frac{p_t^*}{p_t} - \phi \right) \right] N_t,$$  \hspace{1cm} (41)

where $s_t = E_t R_{t+1}^k / R_{t+1}$ denotes the discounted return to capital and $p_t^* = p e^{z_t}$ denotes the lower bound of the support of the distribution of riskiness. Equation (41) defines the demand for capital. If the right-hand-side of equation (41) increases, say, due to a change in the net worth, the demand curve shifts outward. The price of capital, $q_t$, increases, which, in turn, increases investment and output.

Next, I consider an equation that determines threshold $p_t^*$ above which entrepreneurs do not get loans. In Section 2, I showed that condition (20) determines $p_t^*$. Given uniform distribution $F_t(\cdot)$ I can express condition (20) as

$$0 = \phi \left( 1 - \frac{2}{s_t} \right) \left( p_t^* \right)^{2\phi - 1} + \frac{1 - (1 - \phi)s_t}{s_t} \left( p_t^* \right)^{2\phi - 1},$$  \hspace{1cm} (42)

where I have assumed $\phi \neq 1/2$. Equation (42) determines $p_t^*$ given $s_t$ and $p_t$.  

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Next, I derive an equation for an external finance premium, which constitutes an important financial variable in the model. I define an external finance premium, EFPₜ, as a ratio of the interest rate of external finance to the opportunity cost of internal funds (or risk-free interest rates), Rᵣ. In this model the interest rate of external finance corresponds to a loan interest rate. Because a loan interest rate differs among entrepreneurs with different level of riskiness, I use the average loan interest rate as the interest rate of external finance.

I derive an expression for average loan interest rates first and derive an expression for an external finance premium. Conditional on the success of project, an entrepreneur pays return Rᵣ to a bank following the payment schedule, (21). Reminding that because of the non-negative profits of a bank. I obtain the aggregate payment conditional on the success of project,

\[ X_{t+1} = \int_n \int_{\mathcal{P}_t} X_n(p) dF_t(p) dH_t(n) = \frac{E_t R_{t+1}^k}{1 - R_t} \left[ \left( \frac{p_t^*}{p_t} \right)^{\frac{\omega(t)}{\gamma}} - 1 \right] N_t. \]

I define average loan interest rates as a ratio of the aggregate payment to the aggregate loan: \( R_{t+1}^k = X_{t+1}/B_t \), where \( B_t = q_t K_{t+1} - N_t \) is given by (41). Then I can express the external finance premium, EFPₜ, as follows:

\[ \text{EFP}ₜ = \frac{R_{t+1}^k}{Rₜ} \].

The external finance premium must be positive or \( \text{EFP}ₜ > 1 \).²

Finally I derive the law of motion for aggregate net worth. In aggregate entrepreneurs earn return \( R_{t+1}^k(N_t + B_t) \) and pay \( \int_n \int_{\mathcal{P}_t} p X_n(p) dF_t(p) dH_t(n) \) to a bank at the beginning of period \( t + 1 \). The aggregate payment amounts to \( R_{t+1}^k B_t - E_t R_{t+1}^k \omega(p_t^*) N_t \) from conditions (24) and (25), where \( \omega(p_t^*) \) is given by (15).² A fraction, \( 1 - \gamma \), of entrepreneurs become workers and bring their net worth to households to which they belong. The same number of workers become new entrepreneurs. The new entrepreneurs and those who do not have any net worth receive small amount of start up funds from households. I assume that transfers from households at time \( t \) are proportional to output, given by \( \xi Y_t \), where \( 0 < \xi < 1 \). Then I can write the law of motion for aggregate net worth as

\[ N_{t+1} = \gamma[(R_{t+1}^k - R_{t+1}) B_t + (R_{t+1}^k + E_t R_{t+1}^k \omega(p_t^*)) N_t] + \xi Y_{t+1} \]  

²To see this note that the conditional repayment, given by (??), is strictly greater than the average repayment, \( \int_n \int_{\mathcal{P}_t} p X_n(p) dF_t(p) dH_t(n) \), and the ratio of the average repayment to \( B_t \) is greater than \( R_t \) because of the non-negative profits of a bank.

²The bank takes in deposits \( B_t \) from households with risk-free interest rate \( R_t \) and lends to entrepreneurs and earns profits. Reminding that \( R_c \) and \( R^k \) in the partial equilibrium model in Section 2 correspond to \( E_t R_{t+1}^k \) and \( R_t \) respectively in Model-I, conditions (24) and (25) implies that the aggregate payment, \( \int_n \int_{\mathcal{P}_t} p X_n(p) dF_t(p) dH_t(n) \), is given by \( R_{t+1}^k B_t - E_t R_{t+1}^k \omega(p_t^*) N_t \).
where \( B_t = q_t K_{t+1} - N_t \) is given by (41)

In sum, four equations, (41), (42), (43) and (44), describe the aggregate relationship of the financing problem between entrepreneurs and a bank.

**Capital goods Producers:** Competitive capital goods producers run two types of business. First, they produce new capital goods. Second, they purchase depreciated capital goods from entrepreneurs and sell new capital goods to entrepreneurs.

A capital goods producer purchases consumption goods, \( I_t \), and transforms it into new capital goods, \( \bar{I}_t \), using following technologies,

\[
\bar{I}_t = \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t, \tag{45}
\]

where function \( S(\cdot) \) denotes investment adjustment costs satisfying \( S(1) = S'(1) = 0 \) and \( S''(1) > 0 \) in steady state. The adjustment costs make the investment depend on the price of capital, playing the important role in a financial accelerator.\(^7\) The negative uncertainty shock decreases the demand for capital and then dampens the price of capital and so does investment. A decrease in the price of capital also decreases the net worth and the demand for capital, and this cycle (amplification) continues.

A capital goods producer purchases depreciate capital goods from entrepreneurs at price \( q_t \), combines them with newly produced capital goods and produce capital goods, using linear capital accumulation technologies, (37). Then the capital goods producer sells the capital goods at price \( q_t \) to entrepreneurs. Given the price of capital, \( q_t \), the capital goods producer chooses the amount of investment to maximize its expected profit:

\[
\max_{\{I_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U_1(C_{t+s}, L_{t+s}) \frac{\beta^s K_{t+s+1} - [I_{t+s} + q_{t+s}(1 - \delta)K_{t+s}]}{U_1(C_t, L_t)}, \tag{46}
\]

subject to production technologies, (37) and (45). Because a household owns capital goods producers, the capital goods producer discounts future profits using the household’s discount factor.

**Equilibrium of Model-I:** In defining a competitive equilibrium I suppose that the economy starts from period 0. Given a set of initial conditions, \( \{K_0, N_0, L_{-1}, A_{-1}\} \) and the processes of shocks, a *recursive competitive equilibrium* consists of decision rules for the allocation, \( \{Y_t, C_t, I_t, K_{t+1}, L_t, N_{t+1}, u_t, p_t^i\} \), and pricing rules for a set of prices \( \{r_t^p, w_t, q_t, R_{t+1}\} \), where both the rules are the functions of the states of the economy, satisfying:

- Given the pricing rules, the decision rules satisfy the consumption goods producers’ first order conditions, (33) and (34), the household’s first order conditions, (30) and

\(^7\)The adjustment costs, introduced by CEE (2005), allow the model to generate a hump-shaped response of investment and output to various shocks, consistent with VAR-based evidence.
(31), the condition for capital utilization rate, (35), and solve capital goods producer’s problem (46). Also, the decision rules satisfy the optimality conditions of entrepreneurs and a bank, (41) and (42), and the law of motion for net worth, (44).

- All markets clear. That is, (36) and (37) hold, where \( Y_t \) in (36) is given by (32) and \( \bar{I}_t \) in (37) is given by (45).

Uncertainty Shocks as Financial Shocks: Now I study the role of uncertainty shocks. Let \( \hat{x}_t \) denote the deviation of variable \( x_t \) from its steady state at time \( t \). Log-linearizing equations (41) and (42) and substituting out \( p_t^* \) I obtain:

\[
\hat{s}_t = -\chi_1 \left( \hat{N}_t - \hat{q}_t - \hat{K}_{t+1} \right) - \chi_2 \nu_t, \quad \chi_1, \chi_2 > 0.
\]

For the derivation of equation (47), see Appendix.

Equation (47) summarizes both the effect of the uncertainty shock and the role of credit frictions. Without the uncertainty shock, \( \nu_t = 0 \), equation (47) coincides with the equation of the demand for capital in BGG (1999) after log-linearization. The discounted return to capital, \( s_t \), rises when the net worth decreases. The negative relationship between the discounted return to capital and the net worth serves as a financial accelerator or a balance sheet channel: a decrease in net worth increases the discounted return, which, in turn, decreases the net worth, and the cycle continues.

Equation (47) includes the uncertainty shock, \( \nu_t \), which does not appear in BGG (1999). In equation (47) the negative uncertainty shock, \( \nu_t < 0 \), raises the discounted return to capital, \( s_t \), or a wedge between a return to capital and a risk free rate. The negative shock decreases the net worth and the demand for capital from equation (41). It results in a fall in the price of capital, investment and output, generating a co-movement consistent with business cycle facts as I explore quantitatively later.

The uncertainty shock plays exactly the same role as financial shocks analyzed by Hall (2010), Gilchrist, Ortiz and Zakrajsek (2009) and Gilchrist and Zakrajsek (2010). They introduce financial shocks which change a wedge between a return to capital and a risk free rate in a reduced form manner. In this model a residual term in equation (47) has a solid micro-foundation and a clear interpretation. The uncertainty shock affects the distribution of the riskiness of entrepreneurial project. The uncertainty shock changes the degree of asymmetric information and affects the severity of adverse selection endogenously, generating business fluctuations.

3.3. Model-II: Adverse Selection in the Supply Side of Investment

I embed adverse selection in credit markets, analyzed in Section 2, into the supply side of investment as in Carlstrom and Fuerst (1997). In this model, entrepreneurs own net
worth and produce new capital goods. Entrepreneurs combine their own net worth and loans from a bank, purchase consumption goods and transform them into newly produced capital goods. In contrast to Model-I, the entrepreneurs’ activities determine the supply of investment. The bank not only provides loans to entrepreneurs but also purchases depreciated capital goods from entrepreneurs. In this model the uncertainty shock emerges nearly as shocks to the marginal efficiency of investment.

**Entrepreneurs and a Bank:** A bank takes in deposits from households with risk free rate $R_t$. The bank can earn return either by purchasing capital, or by supplying funds to entrepreneurs and receiving newly produced capital from entrepreneurs. If the bank purchases capital with one unit of consumption goods, the bank earns the rental rate of capital and sells the depreciated capital in the next period, so that return $R^k_{t+1}$ is given by (40). Because the bank promises to pay $R_t$ to households, which may exceed risky return $R^k_{t+1}$, the bank sells contingent claims with return $R^k_{t+1} - R_t$ to households. Consequently, the following arbitrage condition holds:

$$1 = E_t \beta \left[ \frac{U_1(C_{t+1}, L_{t+1})}{U_1(C_t, L_t)} R^k_{t+1} \right].$$

(48)

The bank pays the risk free rate to households by combining a return to capital, $R^k_{t+1}$, and a net return from contingent claims, $R_t - R^k_{t+1}$.

An entrepreneur specializes in producing new capital goods from consumption goods. The entrepreneur has linear technologies (projects) transforming one unit of consumption goods into $\mu_t$ unit of new capital goods on average, where $\mu_t$ denotes shocks to the marginal efficiency of investment (MEI), following a stochastic process:

$$\log(\mu_t) = \rho_\mu \log(\mu_{t-1}) + \epsilon_{\mu,t}, \quad 0 \leq \rho_\mu < 1.$$

(49)

As in the partial equilibrium model in Section 2, if the entrepreneur invests one unit of consumption goods in its project, the entrepreneur successfully produces $\theta(p)\mu_t$ units of new capital goods with probability $p$, where $\theta(p) \equiv 1/p$. Riskiness $p$ is private information to the entrepreneur. In this model, an expected return on project is $\mu_t$, which corresponds to $R^e$ in the model in Section 2. A risk-free rate, the amount of new capital goods obtained by giving up one unit of consumption, is the inverse of the price of capital, $1/q_t$, which corresponds to $R^f$ in the model in Section 2.

A financing problem between entrepreneurs and a bank proceeds as in the model in Section 2. At the beginning of time $t$, an entrepreneur combines its own net worth and a loan from the bank to finance the purchase of consumption goods. In aggregate, the entrepreneurs’ balance sheet is given by

$$I_t = N_t + B^e_t,$$

(50)

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where \( I_t \) denotes the investment in terms of consumption goods and \( B_t^e \) denotes the loan from the bank. Applying the result in Section 2, in equilibrium I can express the aggregate loan, \( B_t^e \), using equation (26) with \( R^e/R^f \) in (26) replaced by \( \mu_t/(q_t) \). Then, I can rewrite (50) as

\[
I_t = \left[ \frac{1 - p_t^*}{1 - p_t} + \frac{q_t \mu_t - \frac{1}{1 - (1 - \phi)q_t \mu_t}}{1 - \frac{p_t^*}{1 - p_t}} \right] N_t, \tag{51}
\]

where I have used the distribution of the riskiness of project, given by (38). Threshold, \( p_t^* \), is determined by the zero profit condition, (42) with \( s_t \) replaced by \( q_t \mu_t \), which is essentially the same as in Model-I. The aggregate newly produced capital satisfies:

\[
\bar{I}_t = \mu_t I_t. \tag{52}
\]

In contrast to Model-I, there is no investment adjustment cost in this model. Actually, credit frictions embedded in the supply side of investment play a role similar to investment adjustment costs, as pointed out by Carlstrom and Fuerst (1997).

In aggregate entrepreneurs produce new capital goods \( \bar{I}_t \), given by (52), and pay \( \int_n \int_p pX_n(p) dF_t(p) dH_t(n) \) units of capital goods to the bank, which amounts to \( (1/q_t)B_t^e - \mu_t \omega(p_t^*)N_t \). Entrepreneurs rent remaining capital goods to consumption goods firms and sell the depreciated capital to the bank in the beginning of next period. Then, the law of motion for aggregate net worth is given by

\[
N_{t+1} = \gamma [r_t^k + (1 - \delta)q_{t+1}][\bar{I}_t - (1/q_t)B_t^e + \mu_t \omega(p_t^*)N_t] + \xi Y_{t+1}, \tag{53}
\]

where \( \gamma \) denotes the survival probability of entrepreneurs and \( \xi Y_{t+1} \) denote lump-sum transfers at time \( t + 1 \) from households to entrepreneurs who have failed in their projects or to newly born entrepreneurs.

**Equilibrium of Model-II**: Given a set of initial condition, \( \{K_0, N_0, A_{-1}\} \) and the processes of shocks, a recursive competitive equilibrium consists of decision rules for the allocation, \( \{Y_t, C_t, I_t, K_{t+1}, L_t, N_{t+1}, u_t, p_t^*\} \), and pricing rules for a set of prices \( \{r_t^k, w_t, q_t, R_{t+1}\} \), where the both rules are the functions of the states of the economy, satisfying:

- Given the pricing rules, the decision rules satisfy the consumption goods producer’s first order conditions, (33) and (34), the household’s first order conditions, (30) and (31), the condition for capital utilization rate, (35), and the arbitrage condition, (48). Also, the decision rules satisfy the optimality conditions of entrepreneurs and a bank, (51) and (42), and the law of motion for aggregate net worth, (53).

- All markets clear. That is, (36) and (37) hold, where \( Y_t \) in (36) is given by (32) and \( \bar{I}_t \) in (37) is given by (52).
Uncertainty Shocks as MEI Shocks: Now I study the role of uncertainty shocks in the model with adverse selection in the supply side of investment. I show that the uncertainty shock emerges nearly as the MEI shock which changes the marginal efficiency of investment.

Two equations, (42) and (51) with $s_t$ replaced by $q_t\mu_t$, determine the investment, $I_t$. Log-linearizing those two equations I obtain:

$$
\hat{I}_t = \left(\frac{1}{\chi_3}\right) (\hat{q}_t + \hat{\mu}_t) + \hat{N}_t + \left(\frac{\chi_4}{\chi_3}\right) \nu_t, \quad \chi_3, \chi_4 > 0, \quad (54)
$$

where exact expressions for coefficients $\chi_3$ and $\chi_4$ are given in Appendix. Then, the newly produced capital, $\bar{I}_t = \mu_t I_t$, after log-linearization, is given by

$$
\hat{\bar{I}}_t = \left(\frac{1}{\chi_3}\right) \hat{q}_t + \hat{\bar{I}}_t + \left[\left(1 + \frac{\chi_4}{\chi_3}\right) \hat{\mu}_t + \left(\frac{\chi_4}{\chi_3}\right) \nu_t\right]. \quad (55)
$$

Equation (55) makes clear that the uncertainty shock, $\nu_t$, emerges nearly as the MEI shock, $\hat{\mu}_t$. Given $\hat{q}_t$ and $\hat{\bar{I}}_t$ both the uncertainty shocks and the MEI shocks change the newly produced capital. On the one hand, the uncertainty shock changes the severeness of adverse selection and affect the amount of loan and investment. On the other hand, the MEI shock affects not only the amount of loan but also the efficiency of investment. After log-linearization the two shocks affect the newly produced capital in the same manner.

In this model both the uncertainty shock and the MEI shock appear in the two equilibrium conditions: the condition for $\bar{I}_t$, (52) with $I_t$ substituted out using (51), and the law of motion for aggregate net worth, (53). Adjusted the magnitude of the two shocks, the two shocks have the same effect on the newly produced capital in equation (52). A slight difference appears in the law of motion for aggregate net worth, (53). In (53), the effect on $\bar{I}_t$ of the two shocks is the same while the effect on $B^e_t$ of the two shocks is different. I will explore the difference between the uncertainty shock and the MEI shock quantitatively in the next section.

4. Simulation

I log-linearize the two general equilibrium models, presented in Section 3, around steady state and conduct simulations to explore the quantitative effect of uncertainty shocks. In the following I first parameterize the two models. Next, I study impulse responses to uncertainty shocks for the two models. Then, I study mechanisms which amplify uncertainty shocks and help the models generate business cycles. Next, I conduct stochastic simulations and examine how much volatility of key macroeconomic variables can be explained by uncertainty shocks. Finally, I compare uncertainty shocks with risk shocks analyzed by CMR (2010).
4.1. Model Parameterization

Before setting parameters I specify the utility function as follows:

\[ U(C, L) = \log(C) - \psi \frac{L^{1+1/\nu}}{1 + 1/\nu}, \quad \psi, \nu > 0. \]

I list the choice of parameters values in Table 1. Out of thirteen parameters, five \( \{\phi, p, \gamma, \xi, \rho_u\} \) are specific to the models with credit frictions and the others are conventional except the elasticity of markups.

I begin with conventional parameters. The period of time is quarterly. I set a preference discount factor as \( \beta = 0.993 \) so that the net real risk-free rate becomes 3 percent annual rate in steady state. I set the coefficient of the disutility of labor, \( \psi \), in a way that the average hours worked becomes unity in steady state. I choose conventional values for a labor supply elasticity (\( \nu = 1 \)), a capital income share (\( \alpha = 0.36 \)) and a capital depreciation rate (\( \delta = 0.025 \)). I set the curvature of investment adjustment costs to \( S''(1) = 1 \), which locates at the lower range of estimated values in the DSGE literature, though there is little guidance in the empirical literature about appropriate values. I choose this value because the implied curvature is enough to generate the reasonable hump-shaped responses of output and investment in Model-I. I set the parameter of capital utilization costs as \( \chi = 5 \), which is consistent with estimated values in the DSGE literature including Justiniano, et al (2009a,b). I set a markup in wages in steady state to \( \lambda_w = 1.2 \) and the elasticity of markup in wages to \( \omega = 2 \). The elasticity is consistent with Gali, Gertler and Lopez-Salido (2007) who report that a wage markup is slightly more than twice as volatile as output. Also, they report that a contemporaneous correlation between a wage markup and output is \( -0.83 \), consistent with a countercyclical markup in the models.

Next I set parameters specific to the models with credit frictions, \( \{\phi, p, \gamma, \xi, \rho_u\} \). I set \( \xi = 0.001 \) which determines the amount of lump-sum transfers from households to entrepreneurs. As in Carlstrom and Fuerst (1997) I make the transfers small so that the
transfers do not add additional dynamics. I set the AR(1) coefficient of uncertainty shocks to $\rho_u = 0.75$, following Gilchrist and Zakrajsek (2010) who use VARs and estimate the auto-correlation of financial shocks equal to 0.75. As shown in Section 3 the uncertainty shock in Model-I turns out to be equivalent to financial shocks after log-linearization.

I set remaining three parameters to hit following three targets in steady state: the leverage ratio of 1.5, the external finance premium of 2 percent annual rate, and the expected equity premium of 1 percent annual rate. The target value of the leverage ratio is lower than the literature\(^8\), but it is consistent with U.S. Flow of Funds Accounts according to which the debt net worth ratio of non-farm non-financial corporate business is around 0.5, implying a leverage ratio of 1.5.\(^9\) I set the target value of the external finance premium to the average spread of corporate bonds in the US.\(^10\) The target value of the expected equity premium does not necessarily build upon empirical evidence but suggests that there exists a discrepancy between a return of capital and a risk-free rate due to credit frictions.

4.2. Responses to Uncertainty Shocks

I study responses to the uncertainty shocks in the two models presented in Section 3. I show that the uncertainty shocks generate the co-movement of variables consistent with business cycles for key variables both in Model-I and in Model-II.

Before proceeding to simulations I would like to emphasize the unique nature of the uncertainty shock. The shock has real effects because of asymmetric information and its by-product of adverse selection. With symmetric information the shock would not have any real effects as I showed in Section 2.2.

Model-I: Figure 1 plots impulse responses to the negative uncertainty shock. In period $t = 0$, an economy is in steady state. In period $t = 1$, the negative uncertainty shock hits the economy. I set the magnitude of the shock to $\nu_t = -0.25\%$ so that an external finance premium rises 1 percent annual rate from its steady state level at the impact of the shock. Except the external financial premium the vertical axis of figures shows the percent deviation of variable from its steady state.

Figure 1 shows that the uncertainty shock in Model-I generates the co-movement of variables consistent with business cycles. In response to the negative uncertainty shock which increases the external finance premium by 1 percent annual rate, all variables decrease. The output decreases about 0.4 percent with its bottom reached in four periods.

\(^8\)For example, the leverage ratio in steady state is 2 in BGG (1999) and 4 in Gertler and Karadi (2010).

\(^9\)Ajello (2010) analyzes the U.S. public non-financial companies included in Compustat and reports that 33 percent of the capital expenditures of those firms is funded using financial markets. While a leverage ratio in general heavily depends on the type of borrowers and markets, Ajello (2010)’s finding implies that leverage ratio of 1.5 is not low.

\(^10\)According to Gilchrist, Yankov and Zakrajsek (2009), the average spread of corporate bonds with various credit quality relative to comparable maturity Treasury yield from 1990 to 2008 is 1.92 percent.
after the shock. The output shows a hump-shaped response thanks to the CEE adjustment costs. While the auto-correlation of shocks is relatively low, \( \rho = 0.75 \), the output shows a persistent response: the output stays below a half of its bottom even after 20 periods.

The investment decreases about 1.2 percent with a hump-shaped response. The magnitude of the change is about three times as great as output. The consumption decreases slightly in the initial periods and continue decreasing until eighteen periods after the shocks. The hours show a response similar to the output. The capital utilization rate also decreases with a hump-shaped response.

The notable feature of Model-I appears in the responses of the price of capital and the net worth. The price of capital decreases about 0.3 percent at the impact of the shocks and quickly moves back to the steady state in ten periods. The net worth shows a response similar to the price of capital, because the current net worth is mainly determined by the gross return on capital, which, in turn, is mainly determined by the current price of capital, as is clear from equations (40) and (44). With the right co-movement of the price of capital and the net worth, the uncertainty shock in Model-I generates fluctuations consistent with

Figure 1: Impulse Responses to Uncertainty Shocks (Model I)
Figure 2: Impulse Responses to Uncertainty Shocks (Model II)

business cycles.

Model-II: Next I proceed to responses to the uncertainty shock in Model-II. Figure 2 plots two impulse responses: one to the negative uncertainty shock (solid line) and the other to the negative MEI shock (dashed line). I set the magnitude of the uncertainty shock as same as before. I set the magnitude of the MEI shock to \( \mu_t = 0.14\% \) so that the responses of output coincide with those to the uncertainty shock at an impact.

Figure 2 makes three important observations. First, the uncertainty shock succeeds in generating the co-movement of variables for key business cycle variables. In response to the negative uncertainty shock, an external finance premium increases about 1 percent annual rate. The output decrease about 0.8 percent at the impact of the shock and moves gradually back to the steady state. The investment decreases about 2.1 percent whose magnitude is slightly more than three times as great as the output. The consumption decreases slightly in the initial periods and continue decreasing, showing a very persistent response. The hours show responses similar to the output and the capital utilization rate decreases too.

Second, the uncertainty shock fails to generate the co-movement of the price of capital and the net worth in Model-II. In this model the negative uncertainty shock shifts the supply curve of investment inward. As a result, the price of capital increases, while the
investment decreases. As shown in the law of motion for net worth, (53), the current net worth is increasing in the price of capital. An increase in the price of capital causes an increase in the net worth.

Third, the uncertainty shock generates observationally equivalent responses to those to the MEI shock for all variables except the net worth and the external finance premium. As argued in Section 3.3, the two shocks appear in two equations in Model II. On the one hand, the uncertainty shock, after adjusted the magnitude of the shocks, has the same effect on newly produced capital, $\bar{I}_t$, as does the MEI shock. On the other hand, the two shocks have the different effect on the aggregate loan, $B^e_t$, and so do on the net worth. Because the two shocks affect the newly produced capital in the same manner, the responses of all variables except the net worth and the external finance premium almost coincide with minor differences.

The third observation of the equivalence between the uncertainty shock and the MEI shock provides a rationale for the finding by Justiniano, et al (2009b). They estimate a DSGE model and find that the MEI shock serves as the most important shocks driving the U.S. business cycles. They interpret the MEI shock as something related with financial factors. They show empirically that their estimated MEI shocks are negatively correlated with an external finance premium, though their estimated model abstracts from financial factors and does not have an external finance premium. In Model-II, an increase in asymmetric information in credit markets raises the external finance premium and drives business cycles, consistent with the finding by Justiniano, et al (2009b). The result obtained here suggests that the uncertainty shock combined with imperfect financial markets can be a candidate for the source of the MEI shock. The result also confirms that the uncertainty shock in Model-II or the MEI shock may not constitute the driving force of business cycles once we take financial variables such as net worth into account, as pointed out by CMR (2010).

4.3. Amplification Mechanisms

The previous analysis on responses to the uncertainty shocks shows that both Model-I and Model-II generate the co-movement of key variables. Here I study mechanisms behind the success in generating the co-movement. Set aside the adverse selection in financial markets, the two models have additional features relative to the standard real business cycle model: a countercyclical markup in wages and variable capital utilization rates. The two features amplify the uncertainty shocks and help generate the co-movement of key variables.

I show analytically that both a countercyclical markup in wages and variable capital utilization rates play an essential role in generating the co-movement of key variables. The equation governing a co-movement between hours and consumption is the intra-temporal optimality condition of households, (31), which equates wages to a markup over the marginal rate of substitution between consumption and hours. After substitut-
Figure 3: Impulse Responses to Uncertainty Shocks with Various Values of $\omega$

ing out wages $w_t$, capital utilization rates $u_t$, markup $\lambda_{w,t}$ and output $Y_t$ using (34), (35), (29) and (32) respectively, I log-linearize equation (31) and obtain a relationship between consumption and hours:

$$\dot{C}_t = \left[ (\omega + 1)(1 - \alpha) \left( 1 + \frac{\alpha}{\chi + 1 - \alpha} \right) - (1 + 1/\nu) \right] \dot{L}_t + \frac{(\omega + 1)\alpha\chi}{\chi + 1 - \alpha} \dot{K}_t,$$

(56)

where $\omega \geq 0$ denotes the elasticity of markup and $\chi > 0$ denotes the elasticity of capital utilization costs. If $\omega = 0$ a counter-cyclical markup in wages vanishes. If $\chi = \infty$ variable capital utilization rates vanish.

In the standard business cycle model, we have no counter-cyclical markup in wages and no capital utilization rates: $\omega = 0$ and $\chi = \infty$. In this case, a coefficient on hours becomes negative, $-(\alpha + 1/\nu) < 0$, because the two parameters, $\alpha$ and $\nu$, have to satisfy $0 < \alpha < 1$ and $\nu > 0$. A co-movement between consumption and hours depends mainly on a coefficient on hours, because the capital moves very slowly. Therefore, without a counter-cyclical markup in wages and variable capital utilization rates, both Model-I and Model-II would fail to generate a co-movement between the two variables. Intuitively, in recessions hours decrease and wages increase without neutral technological shocks and variable capital utilization rates. As a result of substitution from hours to consumption, consumption increases without a counter-cyclical markup in wages. This result reflects a famous co-movement problem: only neutral technology shock can easily generate the co-movement among key variables in a real business cycle framework, first pointed out by

Equation (56) shows that a coefficient on hours in equation (56) is increasing in the degree of a counter-cyclical markup in wages ($\omega$) and is increasing in the degree of variable capital utilization rates ($1/\chi$). If the effect of a counter-cyclical markup supported by variable capital utilization rates dominates the substitution effect, consumption decreases. Under the baseline parameters values, a coefficient on hours is positive and that’s why the models succeed in generating a co-movement between hours and consumption.

The mechanisms generating a co-movement between hours and consumption also serve as amplification mechanisms. In response to the negative uncertainty shocks, an decrease in the aggregate demand is enhanced by a decrease in consumption as well as a decrease in investment, resulting in an amplified response of output.

To see the effect of the amplification mechanisms, Figure 3 plots impulse responses to the negative uncertainty shocks with various degree of a counter-cyclical markup: from baseline $\omega = 2$ to no variable markup $\omega = 0$. The first row of Figure 3 plots the responses of output and consumption in Model-I and the second row of Figure 3 plots those in Model-II. The responses in the two models exhibit high sensitivity to the value of $\omega$ around baseline $\omega = 2$. When $\omega$ drops to $\omega = 1.8$, the output decreases less by about one-fourth and the consumption increases initially in Model-I, while in Model II the output decreases less by about a half and consumption increases initially too.

4.4. Business Fluctuations

I have shown that the uncertainty shocks drive business cycles for key variables in both Model-I and Model-II. Still, the result is qualitative rather than quantitative, focusing on a co-movement among key variables. Now I take the uncertainty shocks slightly more seriously and ask the following question: how much volatility can the uncertainty shocks explain for key variables in the U.S.?

In order to answer the question I conduct a typical business cycle stochastic simulation. I assume that the uncertainty shocks, $\nu_{u,t}$, follow an AR(1) process with its disturbance following a normal distribution with mean zero. I simulate the time series of the disturbances and generate the time series of key variables from a log-linearized model. Using the artificial data with the same size as sample data, I calculate the key statistics in the same manner as I do for the sample data. I repeat this process for one-thousand times and report the average values of the statistics.\(^\text{11}\)

A central issue in conducting the simulation lies in the value of the standard deviation of the uncertainty shocks. I set the value so that the simulated data hits the standard

\(^{11}\)For some series of the disturbances, threshold $p_t^\ast$ exceeds unity, which violates condition $p_t^\ast < 1$. I exclude those series and repeat the process until I obtain one-thousand series of artificial data satisfying $p_t^\ast < 1$ for all $t$. 

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Table 2: Cyclical Behavior of the U.S. and the Model Economy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data (1987-2010)</th>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SD (%)</td>
<td>Cross-Correlation of Output with x(-1) x x(+1)</td>
<td>SD (%) [ratio to Data]</td>
</tr>
<tr>
<td>Output</td>
<td>1.12</td>
<td>0.87 1.00 0.87</td>
<td>0.24 [0.22 ] 0.88 1.00 0.88</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.80</td>
<td>0.68 0.87 0.85</td>
<td>0.08 [0.10 ] 0.11 0.38 0.60</td>
</tr>
<tr>
<td>Investment</td>
<td>4.67</td>
<td>0.86 0.93 0.80</td>
<td>0.82 [0.18 ] 0.91 0.97 0.79</td>
</tr>
<tr>
<td>Hours</td>
<td>1.79</td>
<td>0.74 0.88 0.90</td>
<td>0.35 [0.19 ] 0.90 0.99 0.85</td>
</tr>
<tr>
<td>Premium, BAA-Treasury</td>
<td>0.77</td>
<td>-0.50 -0.39 -0.20</td>
<td>0.77 [1.00 ] -0.64 -0.48 -0.09</td>
</tr>
<tr>
<td>Premium, High-AAA</td>
<td>2.17</td>
<td>-0.48 -0.29 -0.03</td>
<td></td>
</tr>
</tbody>
</table>

Note: The data of output, consumption, investment, hours are taken logs and are detrended by Hodrick-Prescott filter with smoothing parameter value 1600. The simulated numbers are the average of the statistics calculated in the same manner as the data for the artificial data generated from the model for 1000 times.

deviation of the premium of sample data (0.77 percent), defined by a difference between Moody’s BAA corporate bond yield versus the U.S. 10-year Treasury yield. Uncertainty shocks in my model are likely to be a main driving force for the variation of the premium. The choice of the standard deviation is conservative. The standard deviation of the average corporate bond premium, analyzed by Gilchrist and Zakrajsek (2011), is about 3 percent.

Table 2 shows the result of the stochastic business cycle simulations. The upper left in Table 2 shows standard deviations and cross correlations for sample data from 1987 to 2010, detrended by Hodrick-Prescott filter with smoothing parameter 1600 except for premiums. For comparison I use another premium defined by a difference between high-yield B-rated corporate bonds from the Merrill Lynch’s High Yield Master file versus AAA corporate bond yields of comparable maturity, in addition to a premium defined by a difference between BAA corporate bonds yield versus the U.S. 10-year Treasury yield. The upper right and the lower left in Table 2 show the corresponding average statistics calculated from one-thousand artificial data series with the same sample size in Model-I and Model-II respectively.

The simulation result, reported in Table 2, reveals three findings. First, both in Model-I and in Model-II the uncertainty shocks generate the significant fluctuations of output, consumption, investment and hours. In Model-I and in Model-II the uncertainty shocks...
explain about 25 percent and 40 percent of the volatility of the data respectively for output, investment and hours, while the shocks explain about 10 percent of the volatility for consumption, respectively.

Second, both in Model-I and in Model-II the uncertainty shocks generate a co-movement among output, consumption, investment and hours. For investment and hours both Model-I and Model-II succeeds in reproducing high contemporaneous correlation of output. Model-I also succeeds in reproducing high cross correlation of output with investment and hours, while Model-II reproduces smaller cross correlation relative to sample data. The success of Model-I reflects its persistence mechanisms. The CEE adjustment costs generate a hump-shaped response of various variables and make responses persistent. For consumption both Model-I and Model-II reproduce mild contemporaneous correlation of output relative to sample data. While Model-I reproduces positive cross correlations, Model-II reproduces smaller cross-correlation of output with lagged consumption. Relatively poor performance on consumption reflects a co-movement problem in a real business framework, though the problem is mitigated by amplification mechanisms embedded in the two models.

Third, the uncertainty shocks succeed in reproducing the predictive power of an external finance premium in Model-I, while it is not the case for Model-II. According to the two data series of premiums, the lagged premium shows a higher correlation with the current output (−0.50 and −0.48 respectively) more than do the current premiums (−0.39 and −0.29 respectively) and the lead premiums (−0.20 and −0.03 respectively). On the one hand, Model-I shows a similar pattern; −0.65, −0.48 and −0.10 for the correlation of output with the lagged, the current and the lead premium respectively, while the degree of correlations is slightly higher relative to sample data. On the other hand, Model-II does not show such a pattern: a premium is highly contemporaneously correlated with output. The difference between Model-I and Model-II has to do with a difference in persistence mechanisms. In Model-I the response of output is hump-shaped thanks to the CEE investment adjustment costs, while in Model-II the response is monotone. This observation implies that a hump-shaped response is crucial for the model to reproduce the predictive power of premiums.

4.5. Comparison with Risk Shocks

The uncertainty shocks in Model-I share similar quantitative implications with risk shocks considered by CMR (2010). However, the two shocks are conceptually different. In the following I explain the risk shocks briefly and provide two examples to shed light on conceptual differences between the two shocks.

CMR (2010) builds a DSGE model incorporating BGG (1999) and introduce shocks to the standard deviation of idiosyncratic productivity of entrepreneurs, which they call risk shocks. In their model entrepreneurs own, trade and rent out capital as in Model-I. All entrepreneurs are identical ex-ante. The entrepreneurs make a debt contract with in-
As presented in Appendix, three equations summarize credit frictions in CMR (2010): (i) a balance sheet equation, (39), (ii) an optimality condition relating excess return \( s_t \equiv E_t R_{t+1}^k/R_{t+1} \) and the threshold of realized productivity under which entrepreneurs go bankrupt, and (iii) an intermediary’s zero profit condition. Let \( u_{r,t} \) denote the risk shocks to the standard deviation of idiosyncratic productivity. Log-linearizing the three equations results in the following relationship:

\[
\hat{s}_t = -\chi_{1,r} (\hat{N}_t - \hat{q}_t - \hat{K}_{t+1}) - \chi_{2,r} E_t u_{r,t+1}.
\]

Under conventional parameters values, coefficients satisfy \( \chi_{1,r}, \chi_{2,r} > 0 \). Given that the risk shocks are persistent, for example \( u_{r,t} = \rho_r u_{r,t-1} + \epsilon_{r,t} \) with \( 0 < \rho_r < 1 \), the risk shocks emerge as financial shocks as do the uncertainty shocks. Equation (57) share the same structure with its counterpart of the uncertainty shocks, (47). This observation makes clear that the two shocks have similar quantitative implications.

Now I discuss conceptual distinctions between the uncertainty shocks and the risk shocks. The uncertainty shocks concern the degree of uncertainty about the riskiness of project, while the risk shocks concern the riskiness of project. In other words, the riskiness of project itself does not matter to the uncertainty shocks, while it does matter to the risk shocks. What matters to the uncertainty shocks is the degree of uncertainty (asymmetric information) about the riskiness, not the riskiness itself.

In order to make clear distinctions between the two shocks I provide two examples. As shown in CMR (2010) an increase in risk, caused by the risk shocks, decreases output and vice-versa. Contrary to the risk shocks, the first example shows that an increase in risk, caused by the uncertainty shocks, increases output. The second example shows that a decrease in risk, caused by the uncertainty shocks, decreases output. While those results sound counter-intuitive, the results clarify distinctions between the two shocks.

Instead of the distribution of riskiness, given by (38), suppose that the distribution stays uniform but the uncertainty shock appears in the upper bound of the support:

\[
F_t(p) = \frac{p - \bar{p}}{\tilde{p}_t - \bar{p}}, \quad \tilde{p}_t \equiv \bar{p} e^{-v_t}, \quad 0 < \bar{p} < \tilde{p} < 1.
\]

As before the negative uncertainty shock, \( v_t < 0 \), increases the degree of uncertainty of riskiness. The difference from the baseline distribution, (38), appears in the riskiness. The negative uncertainty shocks decrease the overall riskiness of project, because an increase in
\( \bar{p}_t \) with \( p \) fixed implies that there are more projects with less riskiness. (Remember that the riskiness of project is measured by \( 1/p \)).

Using new distribution (58), I derive a log-linearized equation summarizing the effect of uncertainty shocks, similar to equation (47), as follows:

\[
\hat{s}_t = -\chi_1 (\hat{N}_t - \hat{q}_t - \hat{K}_{t+1}) - \chi_3 \nu_t, \quad \chi_1, \chi_3 > 0,
\]

where coefficient \( \chi_1 \) is the same as in equation (47). For the derivation of equation (59) see Appendix.

As the first example, consider the negative uncertainty shocks, \( \nu_t < 0 \). The negative uncertainty shocks in this case increase \( \bar{p}_t \) and decrease the overall riskiness of project. However, equation (59) implies that the negative uncertainty shocks increase an excess return to capital and decrease output as we saw in the previous section. Next as the second example, consider the positive uncertainty shocks, \( \nu_t > 0 \). The positive uncertainty shocks decrease \( \bar{p}_t \) and increase the overall riskiness of project. However, equation (59) implies that the positive uncertainty shocks decrease an excess return to capital and increase output. Unlike the risk shocks in CMR (2010), a decrease (increase) in riskiness does not necessarily result in an increase (decrease) in output.

The above two examples make a stark contrast between the uncertainty shocks and the risk shocks. Still, the negative uncertainty shocks have to be accompanied with an increase in riskiness in order to generate counter-cyclical external finance premiums as observed in the sample data.\(^{13}\) In that sense, the uncertainty shocks and the risk shocks share a similar empirical implication: both the two shocks are in some extent captured by an increase in riskiness. Bloom (2009) and Bloom, et al (2010) document that uncertainty, measured by various second moments, is counter-cyclical and propose a model in which a change in uncertainty drives business cycles. In my context, uncertainty referred by them can be interpreted as riskiness. The models proposed in this paper provide another mechanisms through which a change in riskiness (accompanied by a change in uncertainty about riskiness) drives business cycles.

6. Conclusion

In this paper I build a dynamic model in which imperfect financial markets materialize uncertainty shocks. I model imperfect financial markets by introducing asymmetric information on the riskiness of project and an agency problem. Asymmetric information causes adverse selection in financial markets, while an agency problem limits the amount

\(^{13}\)In practice, an increase in riskiness, caused by the uncertainty shocks, has to accompany with a decrease in output as in Model-I, because an increase in riskiness is reflected to an increase in an external finance premium. As shown in the data in Table 2, an external finance premium is counter-cyclical. Pro-cyclical external finance premium in the above two examples is at odd with the data.
of borrowing. I solve a static optimal contracting problem between intermediaries and entrepreneurs and embed it into dynamic general equilibrium models.

In a dynamic general equilibrium framework I consider the effect of uncertainty shocks which change the degree of uncertainty about the riskiness of project. On the one hand, the uncertainty shocks emerge as financial shocks if I embed credit frictions in the demand side of capital (Model-I). On the other hand, the uncertainty shocks emerge as shocks to the marginal efficiency of investment (MEI shocks) if I embed credit frictions in the supply side of investment (Model-II). The result suggests that the uncertainty shocks and the mechanisms studied here serve as micro-foundations for financial shocks and investment shocks, which have received a lot of attention as a source of business cycles and the causes of a financial crisis (Hall, 2010, Gilchrist, et al, 2009, and Gilchrist and Zakrajsek, 2010 for financial shocks, and Justiniano, et al, 2009a, 2009b for investment shocks).

In a quantitative analysis, I show that the uncertainty shocks generate significant fluctuations consistent with business cycles for standard variables both in Model-I and Model-II. Amplification mechanisms embedded in a counter-cyclical markup in wages and variable capital utilization rates play a crucial role in generating business cycles. A difference between the two models appears in the response of the price of capital and net worth. For those two variables, while the uncertainty shocks generate right co-movement in Model-I, the shocks generate wrong co-movement in Model-II. The result suggests either that the uncertainty shocks in the supply side of investment is not significant or that Model-II needs other mechanisms which solve the co-movement problem of the price of capital and net worth.

In this paper I focused on mechanisms through which shocks to uncertainty in financial markets generate business cycles, and did not discuss policy issues. In order to derive policy implications it is essential to model a counter-cyclical markup in wages in a micro-founded manner. One candidate is to introduce nominal wage rigidities. Introducing nominal rigidities opens a door to discussing both conventional and unconventional monetary policy. My another paper, Ikeda (2011), introduces nominal rigidities and discusses the policy issues.
Appendix

Equilibrium with Competitive Banks: I describe the environment of financing problem with competitive banks, define an equilibrium and solve for an equilibrium. There exist entrepreneurs and banks. Entrepreneurs are the same, as described in the main text. Unlike in the text, the banks are competitive, each of which is indexed by $i \in \{1, 2, \ldots, I\}$. I restrict my attention to a truth-telling contract.

A loan contract proceeds in three steps. First, for all $i$, the $i$-th bank offers a schedule of contract, $\sigma_{1,i} \equiv \{X_i^n(p), B_i^n(p)\}$, which specifies the amount of lending, $B_i^n(p)$, and the amount of repayment in case of success, $X_i^n(p)$. Second, for all $i$, after observing other banks’ offered schedules of contracts, the $i$-th bank decides whether to stay or leave: $\sigma_{2,i}(\sigma_1) \in \{\text{stay}, \text{leave}\}$, where $\sigma_1 \equiv \{\sigma_{1,i}\}_i$. If the bank leaves, its profits become zero. Otherwise, the bank proceeds to the next step. Third, entrepreneurs choose a bank and a contract is made. If some banks offer the same contract to an entrepreneur, the entrepreneur chooses one among the banks randomly.

A subgame-perfect equilibrium consists of the schedule of contracts offered by banks, $\sigma_i^* \equiv \{\sigma_{1,i}^*\}_i$, the banks’ strategies whether to stay or leave in the second stage, $\sigma_2^* \equiv \{\sigma_{2,i}^*\}_i$, satisfying (i) given $\sigma_1$ and $\sigma_2^*$, the $i$-th bank chooses to stay if profits from doing so are non-negative, for all $i$, and (ii) given $\sigma_1^*-i$ and $\sigma_2^*$, the $i$-th bank chooses $\sigma_{1,i}^*$ to maximize its profits, and (iii) pledgeability constraint (1), participation constraint (3), incentive constraint (7) and a bank’s zero profit condition holds. An equilibrium is symmetric if and only if a strategy in the first stage is the same for all banks: $\sigma_{1,i}^* = \sigma_{1,j}^*$ for all $i,j$.

I focus on a symmetric equilibrium and show that $\sigma_{1,i}^m \equiv \{X_{cm}^n(p), B_{cm}^n(p), p_{cm}\}$, defined in Section 2.5, constitutes an equilibrium contract. First, note that $\sigma_{1,i}^m$, by construction, satisfies all conditions stated in (iii). Second, note that $\sigma_{1,i}^m$, among those satisfying conditions in (iii), maximizes the profits of the type-$(n,p)$ entrepreneur for all $p \leq p_{cm}$. Third, assume that $\omega(p)$ is decreasing for $p \geq p^*$, where $p^*$, defined by (20), denotes a threshold under a monopolistic bank. Then, $\sigma_{1,i}^m$ maximizes a bank’s profits subject to constraints such that the profits of the type-$(n,p)$ entrepreneur is equal or greater than those attained under $\sigma_{1,i}^m$ for all $p$.

Now I show that there is no profitable deviation so that $\sigma_{1,i}^m$ constitutes an equilibrium. Suppose contrarily that there exists the $i$-th bank’s profitable deviation, $\sigma_{1,i}'$. Because it is profitable, the $i$-th bank stays in a market and earns positive profits. There could be two cases. In the first case, all the other banks stay in a market. In this case, those banks still earn zero profits. However, as I noted above, those banks’ offered schedule of contracts is not only the most attractive to entrepreneurs with $p \leq p_{cm}$ but also is the most profitable to a bank while keeping entrepreneurs as profitable as in choosing $\sigma_{1,i}^m$. Therefore, there is no way for the $i$-th bank to earn positive profits. In the second case, all the other banks leave a market. In this case, the $i$-th bank’s deviation, $\sigma_{1,i}'$, is, at least, as profitable as $\sigma_{1,i}^m$. 

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for some entrepreneurs with $p \leq p^{cm}$. However, given such an entrepreneur’s profits under $\sigma_{1,t}^{cm}$, the $i$-th bank’s profit maximization has to result in $\sigma_{1,t}^{cm}$, as I noted above. Hence, there is no profitable deviation. ■

**Derivation of Equation (56):** Taking into account the functional form of utility function I combine equation (31) and equation (34) and obtain:

$$ (1 - \alpha)(u_tK_t)^\alpha L_t^{-\alpha} = \lambda w, \psi L_t^{1/\nu} C_t. $$

Substituting out for $\lambda w, \psi$ using equation (29) I obtain:

$$ (1 - \alpha)(u_tK_t)^\alpha L_t^{-\alpha} = \lambda w(Y_t/Y)^{-\omega} \psi L_t^{1/\nu} C_t. $$

Substituting out for $Y_t$ using equation (32) I obtain:

$$ (1 - \alpha)Y^\omega/\lambda w, \psi = [(u_tK_t)^\alpha L_t^{1-\alpha}]^{-\omega} - \lambda w, \psi L_t^{1+1/\nu} C_t. $$

Log-linearizing this equation results in:

$$ -(\omega + 1)\alpha(\hat{u}_t + \tilde{K}_t) + [- \omega + 1 - \alpha + 1 + 1/\nu] \tilde{L}_t + \hat{C}_t = 0. $$

Log-linearizing equation (35) gives a relationship between the return to capital and capital utilization rates: $\hat{r}_t^k = \chi \hat{u}_t$. Substituting out for $\hat{r}_t^k$ using the log-linearized equation of equation (33), I express capital utilization rates as:

$$ \hat{u}_t = \frac{1 - \alpha}{\chi + 1 - \alpha} (-\tilde{K}_t + \tilde{L}_t). $$

From the above two equations I obtain:

$$ \hat{C}_t = [(\omega + 1)(1 - \alpha)]^{-\omega} \frac{\alpha}{\chi + 1 - \alpha} - (1 + 1/\nu) \tilde{L}_t + \frac{(\omega + 1)\alpha \chi}{\chi + 1 - \alpha} \hat{K}_t. $$

This completes the derivation of equation (56). ■

**Equations Relating to Risk Shocks:** Let $\omega_t$ denote an idiosyncratic productivity shock, following $\omega_t \sim F_t$, where $F_t$ denotes a c.d.f. with mean unity and standard deviation $\sigma_t = \sigma e^{u_{t-1}}$. Let $\overline{\omega}_t$ denote a threshold under which entrepreneurs go bankrupt. Define $\Gamma_t(\overline{\omega}_t)$ and $G_t(\overline{\omega}_t)$ as $\Gamma_t(\overline{\omega}_t) \equiv G_t(\overline{\omega}_t) + \overline{\omega} \int_0^{\overline{\omega}_t} dF_t(\overline{\omega}_t)$ and $G_t(\overline{\omega}_t) \equiv \int_0^{\overline{\omega}_t} \hat{\omega}_t dF_t(\overline{\omega}_t)$ respectively. Except the introduction of the risk shock, the setup and the notation is exactly the same as BGG (1999) and Christiano and Ikeda (2010, Section 6) who provide a simple analysis on BGG (1999). The equations characterizing the demand for capital consists of three equations:

$$ q_t K_{t+1} = N_t + B_t, $$

$$ 0 = E_t \left[ \left( 1 - \Gamma_{t+1}(\overline{\omega}_{t+1}) \right) s_{t+1} + \frac{\Gamma_{t+1}(\overline{\omega}_{t+1}) \left( [\Gamma_{t+1}(\overline{\omega}_{t+1}) - \mu G_{t+1}(\overline{\omega}_{t+1})] s_{t+1} - 1 \right)}{\Gamma_{t+1}(\overline{\omega}_{t+1}) - \mu G_{t+1}(\overline{\omega}_{t+1})} \right], $$

$$ 0 = [\Gamma_{t+1}(\overline{\omega}_{t+1}) - \mu G_{t+1}(\overline{\omega}_{t+1})] s_{t+1}(N_t + B_t) - B_t, $$

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where \( q_t \) denotes the price of capital, \( K_{t+1} \) denotes the capital stock, \( N_t \) denotes the net worth, \( B_t \) denotes the loan, and \( s_t \equiv E_t R_{t+1}^k / R_{t+1} \) as defined in Model-I, and parameter \( 0 < \mu < 1 \) determines monitoring costs. Readers who are interested in the model’s set up and in deriving those three equations may refer BGG (1999, Appendix A) or Christiano and Ikeda (2010, Section 6). I log-linearizing those three equations and substitute out for \( \hat{B}_t \) from the first equation using the third equation, and substitute out for \( \hat{\omega}_{t+1} \) using the second equation. As a result, I obtain equation (57). In practice, log-linearization is done numerically because of the complexity of the equations.
References


