Price Discrimination and the Hold-Up Problem:
A Contribution to the Net-Neutrality Debate*

Dominik Grafenhofer †

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Abstract

This paper studies ex-ante investment incentives of a buyer (a content provider) facing a monopoly input seller (an internet service provider), who employs second-degree price discrimination. As a benchmark, we extend an adverse-selection model by a non-contractible investment stage, which allows the buyer to improve her type. The buyer underinvests due to partial rent extraction by the seller. When the buyer uses the seller’s product as an input in a downstream production process, and the seller also charges downstream customers of the buyer, the seller faces a two-sided market. We show that the underinvestment problem in the two-sided setting is less severe compared to the one-sided market, because the seller cross-subsidizes the content provider in order to extract more rent from downstream customers. When we introduce competition, this result is reversed: first-best investment can be achieved in the one-sided case, while even fierce competition can leave investment incentives unchanged in the two-sided setting. Finally, we provide conditions under which a net-neutrality regulation (no charges for the buyer) can alleviate the hold-up problem and demonstrate that net neutrality might even arise endogenously.

Keywords: hold-up problem, adverse-selection, price discrimination, two-sided markets

JEL Classification: D82, L22, L23

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†Max Planck Institute for Research on Collective Goods, Kurt-Schumacher-Str. 10, 53113 Bonn, Germany, e-mail: grafenhofer@coll.mpg.de. Financial support by the NSF (NSF Grant Patent pools and biomedical innovation, Award No. 0830288) is acknowledged.
1 Introduction

In many situations of economic exchange the value of trade is affected by investments before trade occurs. Ex-ante investment is indeed one of the concerns in the recent debate on net neutrality. Internet service providers (ISPs) argue¹ that they should be allowed to charge content providers for traffic on their network and access to their customers, so that they have incentives to invest in the quality of their network (e.g. connection speed). On the other hand, content providers also invest in cost reduction or the quality of their content (e.g. vis-à-vis final customers or advertisers), which increases their value of connecting to the ISP’s platform and its users. If this investment cannot be contracted on ex-ante, a hold-up problem may occur when there is some bargaining power on the ISP’s side. The aim of the paper is to study this hold-up, and to investigate ways to overcome it: competition between ISPs² and net-neutrality regulation (no charges for content providers).

ISPs are two-sided platforms. They sell Internet access to end-users, and allow content providers to reach these end-users. On the other hand, content providers (apart from very big ones) connect to these ISPs via intermediaries: Internet backbone providers, content delivery networks or other ISPs. Typically they do not charge end-users but only content providers for the uplink (a one-sided market). For tractability we discuss the one-sided and two-sided setting separately. Furthermore, two-sided markets exhibit additional features like cross subsidies which interfere with the hold-up.

In section 2 we study a one-sided market, which serves as a benchmark: a buyer (content provider) uses an input provided by a monopoly seller (content-delivery network) in order to serve end-users in a downstream market. Non-contractible ex-ante investment allows the buyer to improve her type (e.g. content quality or cost). After investment has been sunk, the seller (the non-investing party) has full bargaining power, and uses second-degree price discrimination to screen for the content provider’s type³. In equilibrium there is underin-

¹AT&T CEO Edward Whitacre in BusinessWeek, November 7th 2005: How do you think they’re going to get to customers? Through a broadband pipe. Cable companies have them. We have them. Now what they would like to do is use my pipes free, but I ain’t going to let them do that because we have spent this capital and we have to have a return on it. So there’s going to have to be some mechanism for these people who use these pipes to pay for the portion they’re using. Why should they be allowed to use my pipes?

The Internet can’t be free in that sense, because we and the cable companies have made an investment and for a Google or Yahoo or Vonage or anybody to expect to use these pipes [for] free is nuts!

²The New York Times, August 15th 2010: The Google-Verizon proposal acknowledges the power of competition, arguing that there is no need for neutrality on the wireless Internet because there is more rivalry among U.S. mobile network operators than among fixed-line broadband providers. (By contrast, Google and Verizon call for a neutrality mandate on the fixed-line Internet, with the exception of vaguely defined new broadband services.)

³Appendix E contains a short overview of the case when the seller is restricted to charge a linear price. However, to carry out comparative statics requires detailed information on demand together with the distri-
vestment, because returns from investment are partially extracted by the seller. We provide conditions ensuring existence and/or uniqueness of a pure-strategy equilibrium.

When the seller (ISP) also charges end-users, we get a two-sided market (section 3). We show that we can apply the mechanics of the one-sided in the two-sided setting. The underinvestment, existence, and uniqueness results carry over directly. We find that the seller sets marginal tariffs below marginal cost for some (at least the most efficient) types of buyers in order to extract more surplus from end-users connecting to the platform. Thus, the underinvestment problem is less severe in the two-sided case, because more surplus is left to the buyer compared to the one-sided setting. If the seller’s marginal cost is low, she will actually pay at least some types of buyers. When the buyer can freely dispose of unneeded units of the input provided by the seller, negative-marginal tariffs induce the buyer to purchase excessive amounts\(^4\). Thus, free disposal limits the seller’s ability to subsidize the buyer side of the market, because it constrains marginal tariffs to be non-negative. When this free-disposal constraint binds for all types of buyers, sellers will charge the buyer a fixed fee. This could explain why net neutrality prevails in practice, \(i.e.,\) that we do not observe ISPs (sellers) charging content providers (buyers) for traffic.

Studying competition between sellers in section 4 we find a dichotomy: fierce enough competition between sellers in the one-sided setting leads to a cost-plus-fixed-fee tariff\(^5\) in equilibrium, which solves the hold-up problem, \(i.e.,\) first-best equilibrium investment. In the two-sided market, however, the buyer faces a competitive bottleneck. Sellers act like monopolists vis-à-vis the buyer even under perfect competition. They use all their profits to lower end-user fees in order to attract more of them, \(i.e.,\) competition takes place only on the end-user side of the market. Therefore, contrary to the one-sided case, competition is ineffective or only partially effective in solving the hold-up.

When the seller can commit herself or write a contract with the buyer before investment takes place, the distortion is alleviated or even eliminated (section 5). In principle, this result holds for the one-sided and two-sided setting. However, it should be taken with a grain of salt when applied to a setting with a multitude of sellers as it is the case for ISPs, where there are thousands of ISPs in the US only. Due to the huge number of parties ex-ante contracting seems to be infeasible. Adding to that, ex-ante commitment is plagued by a free-riding problem when there are many sellers around.

Because competition does not solve the underinvestment problem in the two-sided case,

\(^4\)This relies on the assumption that the seller can only track the amount of the input used by the buyer, but not the actual downstream market transactions.

\(^5\)A result which has been shown in Rochet and Stole (2002).
the question arises whether net neutrality, \textit{i.e.} sellers (ISPs) not being allowed to charge buyers (content providers), should be mandated: In general net neutrality can lead to over- and underinvestment in equilibrium, and its welfare impact is ambiguous. However, if the marginal cost of traffic is low and if the buyer can freely dispose of the seller’s input, net neutrality increases equilibrium investment and welfare.

Departing from our standard assumption that the seller does not observe the buyer’s investment, we get that equilibrium investment decreases compared to the non-observable case (discussed in appendix C). This is the second alley for regulative intervention. A simple measure to bring back the equilibrium investment to the level from the unobservable investment case is to introduce anonymity requirements, \textit{i.e.} to restrict the seller to offer the same tariff to all buyers.

\textbf{Relationship to the literature}

The debate on net neutrality contains a whole range of more or less connected issues, which includes for example the discussion on product-line restrictions (\textit{i.e.} whether a \textit{discrimination} based on different quality levels should be allowed or prohibited). Recently, more and more of these questions get an analytical treatment: Schuett (2010) provides a survey of this growing literature.

One of the arguments in the political debate on net neutrality is that ISPs should be allowed to charge content providers for access to their customers in order to provide them with better incentives to invest in the quality of their platform. Economides and Tåg (2009) find in a two-sided market setting, that for \textit{reasonable} parameter ranges\footnote{Whether these parameters are realistic or applicable empirically is discussed by Njoroge et al. (2010).} a net-neutrality regulation (imposing on a platform not to charge content providers) improves total welfare under monopoly and competition. Cañon (2009) shows that ISPs invest more, when they are not allowed to charge content providers. This translates into higher welfare of net-neutrality regulation.

Hermalin and Katz (2007) discuss the question whether ISPs should be restricted to only provide a single quality to all users from a traditional and a two-sided markets perspective. They find that putting such restrictions on a platform \textit{often} reduces welfare: content providers which would otherwise have bought low-quality access will drop out of the market and those willing to go for the high-quality will have to settle with a mediocre quality level, which is, in many configurations, not compensated for by relatively higher-quality purchases in the segment of content providers heading for medium-quality access. Choi and Kim (2010)
consider the impact such product-line restrictions on the ISPs and on content-providers incentives to invest. Their results are mixed: The ISP’s and the content providers’ investment can be harmed or boosted by the product-line restriction. When the ISPs bargaining power is high enough the content providers will invest more under the single quality regulation.

The ineffectiveness of competition in a competitive bottleneck mirrors what has been found in Armstrong (2006) and Armstrong and Wright (2007) in a simpler setting. The multi-homing side of the market faces local monopolies for access to end-users connected to the respective platforms. Platforms maximize their joint surplus including end-users, which is (partially) distributed to end-users due to competition with other platforms. The interests of the multi-homing side are not taken into account by the platform.

On a broader scope this paper is related to many contributions in the vertical integration literature. Williamson (1975, 1979) and Klein et al. (1978) were among the first to discuss the hold-up problem. They describe that opportunistic behavior in ex-post bargaining over the distribution of joint-surplus may hurt incentives to carry out relationship specific investment ex-ante. They discuss long-term contracts and vertical integration as ways to solve the problem. Grossman and Hart (1986) formalize some of these ideas. In all scenarios (vertical integration versus non-integration) efficient ex-post bargaining takes place, but non-contractible ex-ante actions influence the status-quo of the bargaining process. Hence, actions are not chosen to maximize joint gains from trade ex-post, but individual surplus after negotiations (i.e. too little weight is put on joint surplus, too much on improving the status-quo). In an alternative model of vertical integration Riordan (1990) uses a hold-up model with adverse selection to study the non-integration case. Schmidt’s (1996) goal is to analyze the trade-off between allocative and productive efficiency in the context of privatization, for which he employs a model with non-contractible ex-ante investment and adverse selection. Laffont and Martimort (2002) present a similar model, where in certain parameter ranges the investing party mixes over investment levels in equilibrium.

The contracting stage between seller and buyer follows the spirit of Maskin and Riley (1984), who analyze the problem of a monopolist with incomplete information over customers’ types. To maximize profit the monopolist uses non-linear prices for screening. Guesnerie and Laffont (1984) study in detail assumptions ensuring the existence and uniqueness of solutions in these kinds of adverse-selection problems. While in the monopoly case the downward incentive constraint plays the key role (hindering high valuation types to mimic low valuation ones), Rochet and Stole (2002) find for the duopoly case that also the upward incentive constraint can be binding, which significantly complicates the analysis.

Tirole (1986) studies ex-ante investment in procurement. Inefficient bargaining between
two parties takes place after a non-contractible investment has been sunk. He finds under-
investment independent of whether investment is observed. In case of observable and at
the same time contractible investment (although keeping the assumption that bargaining
on trade takes place only after the investment stage) equilibrium will be higher than under
unobservable investment and might even exceed the first-best level.

Hermalin and Katz (2009) study a similar setup like in section 2: A seller proposes to sell
at a certain price to the buyer who has unit-demand. Non-contractible ex-ante investment
influences the buyer’s surplus from trade. Like in appendix C of this paper they find that
observability of investment by the seller decreases equilibrium investment under the monotone
hazard rate assumption. In Hermalin and Katz (2009) the distribution over types ties down
demand completely, because the buyer’s type equals the buyer’s valuation, while in our
setting each type has her own demand (or rather surplus) function. Hence, it is surprising
that this result carries over from the unit-demand case with the only additional assumption
being single-crossing\(^7\).

The discussion of the impact of observability of investment points at another interesting
connection to the literature on vertical integration: Crémer (1995). He finds that in some
settings the principal wants to commit not to acquire information about an agent in order not
to discover excuses for bad performance, and thus to boost the agent’s efforts. This serves
as a commitment device to be tough on a badly performing agent in the future. In this
paper the non-observability helps the seller to not react to deviations from the equilibrium
investment level, which leaves the buyer with a higher marginal return from investment.

2 A monopoly input provider - one-sided market

We start with a one-sided market analysis. This captures examples like Internet-service-
providing connecting content providers to the Internet, Internet-backbone-providers selling
traffic connecting a content provider to a certain geographical area, or Internet content de-
ivery networks (e.g. Akamai Technologies), which allow content owners to outsource file
download, video streaming,... to a (more efficient) third party. In these settings the final
customers (end-users) of the content provider are not charged for the Internet traffic con-
sumption originating from the content provider. Typically only the content provider has to
pay for the service.

In more abstract terms we study a situation in which a buyer\(^8\) (content provider) uses

\(^7\)This is, however, not the case when tariffs are restricted to be linear like studied in appendix E.

\(^8\)or multiple buyers
a seller’s input (e.g. a Internet content delivery network) in order to serve end-users in a downstream market. We assume that a linear price $p$ is used in the downstream market and that the buyer is a monopolist in this market. The monopoly input seller has the full bargaining power vis-à-vis the buyer, and does not engage in any kind in the downstream market, nor does she observe any downstream interaction taking place. The production function of the seller exhibits constant marginal cost $c$.

We assume that a buyer’s type $\theta$ is private information. The parameter $\theta$ has different interpretations: It can denote the quality of the buyer’s product as perceived by end-users. More general $\theta$ can be seen as a parameter of the end-users’ aggregate demand. End-users’ inverse demand depends positively on the buyer’s type $\theta$: $p(\theta, q)$. Hence, buyer $\theta$’s profit from trade in the downstream market(s) necessitating the use of $q$ units of the seller’s product is given by $V(\theta, q) = q p(\theta, q)$. Another interpretation is to view $\theta$ as a cost parameter in the buyer’s production. Inverse downstream demand (of a single downstream user) does not depend on $\theta$ in this case, and the buyer’s downstream profit is given by $V(\theta, q) = q p(q) - C(\theta, q)$. Last but not least $\theta$ can be the size of the downstream market: $V(\theta, q) = \theta q p(q)$.

The buyer’s surplus (i.e. profit) also depends on an ex-ante investment of the buyer (e.g. in the quality of the downstream product, in a cost reduction of the buyer’s production process).

2.1 A simple one seller one buyer model

In the outlined model we abstract away from the details of the downstream market and study a more stylized and simplified situation in which a potential downstream market can be summarized by the buyer’s profit function $V(\theta, q)$. In this baseline model there is a single buyer of a product or service, who faces a monopoly seller. Before trade occurs the buyer might sink an investment $I$ which increases her valuation of the seller’s output. The game evolves as follows:

1. The buyer chooses an investment level $I \geq 0$. The seller does not observe investment $^{10}$.

$^{9}$This extreme distribution of the bargaining power is necessary for tractability reasons. In practice buyers (content providers) may have different degrees of bargaining power. For example a local web television station has considerably less weight than Google dealing with France Télécom. Under a more realistic distribution of the bargaining power the effects identified in the upcoming discussion should still be present.

$^{10}$In appendix C we study the case of observable investment.
2. The seller offers a tariff \( T(q) \) to the buyer.

3. Nature assigns a type \( \theta \in [\underline{\theta}, \bar{\theta}] \) to the buyer, where \( \underline{\theta} < \bar{\theta} \), according to the probability distribution function \( f(\theta|I) \) (and cumulative distribution function \( F(\theta|I) \) respectively)\(^{11}\).

4. The buyer learns her type and chooses \( q \), the quantity of the good to buy from the monopoly seller. A buyer of type \( \theta \) maximizes her net surplus \( V(\theta, q) - T(q) \).

If the seller could observe the buyer’s type, she would extract the buyer’s joint surplus from trade, which would in turn remove any incentives for the buyer to invest in the first place. This is the well known hold-up problem. Due to the assumption that the buyer’s type cannot be observed directly, the seller cannot perfectly extract the buyer’s surplus, which leaves the possibility of positive investment in step 1. Whether the resulting investment level is distorted compared to an yet to be defined benchmark and in which direction, is analyzed in different scenarios in the following sections.

We focus on non-linear tariffs for different reasons: First, it is common practice in vertical relationships to use non-linear tariffs. Hence, a focus on linear tariffs would be restrictive. Second, one could interpret second-degree price discrimination as an approximation for third-degree price discrimination (which is often intractable). Third, appendix E provides a short discussion of the linear pricing case. There we find, that the crucial parameter in comparative statics is the sensitivity of the buyer’s demand elasticity on investment, on which we have little information. Under non-linear prices this issue does not arise.

To be able to apply standard techniques, we have to put some structure onto the model. The following assumption collects restrictions on the distribution of the buyer’s types \( \theta \) and the impact of investment on it:

**Assumption 1** (Monotone hazard rate in \( \theta \) & First-order stochastic dominance in \( I \)). For any \( I > 0 \) and \( \theta \in [\underline{\theta}, \bar{\theta}] \) the hazard rate

\[
h(\theta|I) := \frac{f(\theta|I)}{1 - F(\theta|I)}
\]

is non-decreasing in \( \theta \). Furthermore, the distribution function fulfills the first-order stochastic-dominance property in \( I \), i.e., \( \frac{\partial F}{\partial I} < 0 \). For every \( I \geq 0 \) we assume full support of \( f(\theta|I) \).

\(^{11}\)An extension to the case where \( \bar{\theta} = \infty \) is straightforward, but additional care has to be taken such that all involved integrals converge, and exchanging differentiation and integration needs more attention as well.
The assumption that the hazard rate is non-decreasing in $\theta$ is a standard one in the literature on price discrimination. The second part of the assumption says that a higher investment $I$ improves the distribution of $\theta$, i.e. that a higher investment level leads to higher returns. All occurring functions, distributions etc. are assumed to be smooth (i.e. differentiable as many times as needed).

The second assumption concerns the buyer’s surplus function:

**Assumption 2** (Normalization and single-crossing condition). For any type $\theta$ the buyer’s surplus from consuming nothing is zero: $V(\theta, 0)$. Profit and marginal profit increase in $\theta$:

$$\frac{\partial V}{\partial \theta}, \frac{\partial^2 V}{\partial q \partial \theta} > 0$$

To rule out corner solutions we impose the following:

**Assumption 3** (Boundary conditions). For all types $\theta$ marginal buyer’s surplus is bigger than $c$ for $q = 0$, and joint surplus of the buyer and seller is bounded from above.

The assumptions up to now are standard in the literature on adverse selection. Nevertheless we have to check whether the assumptions hold when $V(\theta, q)$ stands for the buyer’s downstream profit. For assumption 2 to be fulfilled the following conditions must hold when $\theta$ is a parameter of downstream demand:

$$\frac{\partial V}{\partial \theta} = q \frac{\partial p}{\partial \theta} > 0, \quad \text{and} \quad \frac{\partial^2 V}{\partial \theta \partial q} = \frac{\partial p}{\partial \theta} + q \frac{\partial^2 p}{\partial \theta \partial q} > 0$$

The last inequality does not hold for all demand functions, but is fulfilled for linear demand. Other examples of inverse demand functions for which it holds are $p(\theta, q) = \theta p(q)$ and $p(\theta, q) = p(q) + \theta$. Note that for many parametrizations of demand the single-crossing assumption holds in the relevant range, i.e. as long as marginal revenue is increasing. When $\theta$ is a cost parameter, cost has to decrease in $\theta$ and single-crossing has to hold for the cost function: $\frac{\partial^2 C}{\partial \theta \partial q} < 0$.

### 2.2 Optimizing joint surplus

First, we take a look at the optimal joint surplus, by maximizing the sum of the seller’s and buyer’s surplus when there is no information and contracting problem. Put differently we look at the buyer’s surplus if she had access to the same technology for providing the good/input as the seller. We will use the results from this discussion as benchmark. We
determine the optimal investment level $I^{fb}$ and consumption levels $q^{fb}(\theta)$ for all $\theta$ such that joint surplus is maximized:\[\]
\[
\max_{I,\{q(\theta)\}} \int_0^\theta \left[ V(\theta, q(\theta)) - cq(\theta) \right] f(\theta | I) \, d\theta - I
\]
The necessary condition for the optimal quantity for type $\theta$ is
\[
\frac{\partial V}{\partial q}(\theta, q^{fb}(\theta)) = c \quad \text{for (almost) all } \theta \in [\underline{\theta}, \overline{\theta}].
\]
When the buyer’s type is $\theta$, joint-surplus can be written as $W(\theta) := V(\theta, q^{fb}(\theta)) - cq^{fb}(\theta)$. It will be useful later on to remember that the envelope theorem implies $\frac{dW}{d\theta}(\theta) = \frac{\partial V}{\partial q}(\theta, q^{fb}(\theta)) > 0$. Taking the expectation over $\theta$ given the investment level $I$ yields the expected gross joint-surplus
\[
W(I) = \int_0^\theta W(\theta) f(\theta | I) \, d\theta.
\]
From the first-order stochastically dominance property it follows that expected gross joint-surplus $W(I)$ is increasing in $I$. Furthermore, there exists a maximum of net joint surplus\[\]
$W(I) - I$ for an investment level $I^{fb}$.

## 2.3 Equilibrium investment

Let us now consider the decentralized case. When the buyer’s investment level from stage 1 is not known to the seller in stage 2, the buyer’s investment choice and the seller’s tariff choice are in fact simultaneous moves. Only after these decisions are made, the buyer chooses on the quantity to buy in step 4 after having learned the realization of $\theta$ and the tariff proposed by the seller.

To solve this game, we employ backwards induction starting with the buyer’s quantity choice and using the result as an input in the simultaneous move game. We consider pure strategy equilibria in the simultaneous move game\[\]
14. Given the anticipated investment level

\[\]
12Including downstream end-user surplus would lead to even higher first-best consumption and investment levels, which can be seen by a simple revealed preference argument. This would have no qualitative impact on the results to come.

13$W(I)$ is bounded from above by $W(\overline{\theta})$. Furthermore, due to the smoothness assumptions $W(I)$ continuous in $I$. Therefore, $[W(I)] = \max\{W(I) - I, 0\}$ has a compact support and is continuous in $I$, which in turn guarantees that $[W(I)]$ (and hence $W(I)$) attains its supremum for some $I^{fb}$.

14This is not to say that mixed strategy equilibria might not exist. Laffont and Martimort (2002) outline a situation with ex-ante non-contractible, non-observable investment by an agent and a follow-up adverse-
I_{a}$ the seller’s optimization problem in step 2 has a well known solution (see e.g. Maskin and Riley, 1984), which is characterized\textsuperscript{15} by the following equations:

\begin{align}
\frac{\partial V}{\partial q}(\theta, q^*(\theta, I_{a})) &= c + \frac{1}{b(\theta|I_{a})} \frac{\partial^2 V}{\partial q \partial \theta}(\theta, q^*(\theta, I_{a})) \\
\frac{\partial T}{\partial q}(q^*(\theta, I^*), I_{a}) &= \frac{\partial V}{\partial q}(\theta, q^*(\theta, I_{a})) \tag{2}
\end{align}

The first equation determines quantities bought by each type $\theta$, while the second allows to determine the optimal tariff. There is one caveat however, because a (non-negative) solution to the first equation might not exist for certain types, which signifies that the seller does not want to serve them. We get a corner solution $q^*(\theta, I_{a}) = 0$ for these types. Therefore, incentive compatibility necessitates a non-decreasing quantity schedule in types, there exists a cut-off point for shutdown $\tilde{\theta}(I_{a})$ given the level of investment: Below this cut-off the seller does not serve the respective type, above the point quantities are determined by equation (1). It is well known, that quantities traded are below first-best levels, \textit{i.e.} $q^*(\theta, I_{a}) < q^{fb}(\theta)$ for all $\theta \in [\overline{\theta}, \overline{\theta}]$, because

\[ \frac{\partial V}{\partial q}(\theta, q^*(\theta, I_{a})) > c. \]

Only type $\overline{\theta}$ gets the efficient quantity (\textit{no distortion at the top}). Due to the fact that buyer $\tilde{\theta}(I_{a})$’s surplus is zero and the envelope theorem, we know that buyer $\theta$’s surplus from trade is given by

\begin{align}
U(\theta, I_{a}) := \int_{\tilde{\theta}(I_{a})}^{\theta} \frac{\partial V}{\partial q}(t, q^*(t, I_{a})) \, dt & \quad \text{for } \theta > \tilde{\theta}(I_{a}) \quad \text{and} \\
U(\theta, I_{a}) := 0 & \quad \text{for } \theta \leq \tilde{\theta}(I_{a}). \tag{3}
\end{align}

Trivially $U(\theta, I_{a})$ is increasing in $\theta$. Using these results it is possible to discuss the optimal investment behavior of the buyer in step 1. We know that in equilibrium the seller sets an optimal tariff based on her anticipation $I_{a}$ of the prevalent investment level, so we can neglect other tariffs without loss of generality. We know that given the seller’s anticipated selection stage. The analysis is carried out under the restriction to two types and only two admissible investment levels ($0$ or $l$). Depending on parameters three different equilibria exist:

1. The agent invests (and thus first-best effort) although the principal anticipates this and designs the contract under this premise.

2. The agent does not invest (socially inefficient), which is taken into account by the principal as well.

3. The agent randomizes between investing and not investing. The principal offers contracts based on the induced probability distribution over types, which leaves the agent indifferent whether to invest or not.

Schmidt (1996) shows that in this model under quite strong assumptions and a continuous investment choice, that there exists only a unique pure-strategy equilibrium, and this is exhibiting underinvestment.\textsuperscript{15} necessary, but not sufficient conditions

\textsuperscript{15}necessary, but not sufficient conditions
investment level is \( I_a \), the buyer’s gross expected surplus of a buyer investing \( I \) is given by

\[
\mathcal{U}(I, I_a) := \int_0^\theta U(\theta, I_a) f(\theta|I) d\theta.
\]

The buyer chooses the level of investment to maximize her net expected surplus:

\[
\max_{I \geq 0} \mathcal{U}(I, I_a) - I
\]

Like in the section before discussing the first-best solution for the maximization problem \( \mathcal{U}(I, I_a) \) is bounded from above by \( W(\theta) \), continuous in \( I \) and non-negative only on a compact set. Thus, a maximum exists and is obtained for some level of investment. This is already enough information to show that investment lies below first-best level:

**Proposition 1.** In any pure strategy equilibrium the buyer’s equilibrium investment, denoted by \( I^* \), is below first-best investment level\(^{16} \), i.e. \( I^* < I^{fb} \).

**Proof.** The proof involves a simple revealed preference argument:

\[
\begin{align*}
U(I^*, I^*) - I^* & \geq U(I^{fb}, I^*) - I^{fb} \\
W(I^{fb}) - I^{fb} & \geq W(I^*) - I^*
\end{align*}
\]

Adding up both inequalities, plugging in and rearranging yields:

\[
\int_\theta^\infty [(W(\theta) - U(\theta, I^*)) [f(\theta|I^{fb}) - f(\theta|I^*)]] d\theta \geq 0
\]

Now observe that \( W(\theta) - U(\theta, I^*) \) is strictly increasing in \( \theta \) (due to the below first-best quantities under price discrimination and single-crossing):

\[
\frac{\partial}{\partial \theta} [W(\theta) - U(\theta, I)] = \frac{\partial V}{\partial \theta}(\theta, q^{fb}(\theta)) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I)) > 0
\]

Thus, first-order stochastic dominance mandates that \( I^* \leq I^{fb} \). Knowing this we want to show that the inequality is strict by contradiction: Suppose that \( I^* = I^{fb} \). Being an optimum level of investment \( I^* \) must fulfill the first-order condition for the maximization

\(^{16}\text{This result can be easily extended to all investment levels in the support of the buyer’s mixed investment strategy, in the case in which the seller continues to play a pure strategy.}\)
problem \( \max_I U(I, I^*) - I: \)

\[
\frac{\partial U}{\partial I}(I^*, I^*) = \int_\theta^\theta U(\theta, I^*) \frac{\partial f}{\partial I}(\theta | I^*) \, d\theta = 1
\]

The same holds for \( I^* = I^{fb} \) and first-order condition characterizing the socially optimal level of investment:

\[
\frac{\partial W}{\partial I}(I^*) = \int_\theta^\theta W(\theta) \frac{\partial f}{\partial I}(\theta | I^*) \, d\theta = 1
\]

We can construct the following contradiction, due to the first-order stochastic dominance together with the fact, that \( W(\theta) - U(\theta, I^*) \) is strictly increasing in \( \theta \) as discussed above:

\[ 1 = \int_\theta^\theta W(\theta) \frac{\partial f}{\partial I}(\theta | I^*) > \int_\theta^\theta U(\theta, I^*) \frac{\partial f}{\partial I}(\theta | I^*) = 1 \]

Thus, \( I^* < I^{fb} \) must hold.

The question arises whether one can expect a pure-strategy equilibrium to exist. It turns out that this requires strong regularity assumptions on the adverse-selection problem and on the distribution over types:

**Lemma 1** (Regularity of the buyer’s reaction function \( I(I_a) \)). We assume that the first-order condition of the seller’s problem has a unique solution for each type \( \theta \) and that the respective sufficient second-order condition (i.e. the second-order condition corresponding to (1)) is fulfilled\(^{17}\). Furthermore, the distribution of types is assumed to be strictly convex in investment, i.e. \( F_{II} > 0 \). Then for all levels of anticipated investment \( I_a \) the investment maximization problem has a unique solution and \( I(I_a) \) is a continuous function.

The proofs boil down to continuity arguments. For the details see the appendix. It should be noted that the implications of the lemma might also hold, when some of the conditions in the lemma are not met\(^{18}\). Using these results we can show the following proposition:

**Proposition 2.** If the assumptions or even only the implication of lemma 1 hold, an equilibrium in pure strategies exists. The equilibrium investment level is denoted by \( I^* \).

\(^{17}\)Guesnerie and Laffont (1984) study a class of principal-agent problems, for which a solution can be shown to exist and is unique. They assume \textit{concavity and regularity of the surrogate social welfare function}, which boils down in our case to the stated properties. For further details I refer to their exposition.

\(^{18}\)Especially the assumption on the convexity of the distribution function together with the later imposed monotone hazard rate assumption 4 is restrictive. In appendix I cite two examples for distributions over types fulfilling all properties imposed at some point in the paper.
Proof. Finding an equilibrium is equivalent to showing that a solution to fixed point equation \( I^* = I(I^*) \) exists. We know that \( I(0) \geq 0 \). If \( I(0) = 0 \), we have already found an equilibrium. In the other case \( I(0) > 0 \) we certainly know that \( I(W(\bar{\theta})) \leq W(\bar{\theta}) \) (because you would never want to invest more than can be recuperated at maximum). With the help of Bolzano’s theorem we show that a fixed point exists.

In general multiple pure-strategy equilibria cannot be ruled out, but under additional assumptions one can show that the buyer’s reaction function \( I(I_a) \) is decreasing, which immediately implies uniqueness. One could interpret this as a situation where the buyer’s decision on investment and the seller’s reaction to different anticipated investment levels are strategic substitutes.

**Assumption 4** (Decreasing hazard rate in investment). For any \( I > 0 \) and \( \theta \in [\bar{\theta}, \tilde{\theta}] \) the hazard rate \( h(\theta|I) \) is decreasing in \( I \):

\[
\frac{\partial h}{\partial I}(\theta|I) < 0 \quad \text{for all } \theta \in [\bar{\theta}, \tilde{\theta}]
\]

It is well known that the decreasing hazard rate assumption implies first-order stochastic dominance (assumption 1). For a method to construct probability distributions with the properties appearing in this paper see appendix D.

To show the uniqueness of the equilibrium under this assumption, we have to derive some preparative results, which will be useful later on as well. First, using these assumptions allows to pin down the impact of changes in investment on the quantities traded in stage 4:

**Lemma 2.** When assumption 4 holds, the quantity chosen by any type \( \theta > \tilde{\theta}(I_a) \) is decreasing in the level of anticipated investment \( I_a \) for all \( \theta \in [\tilde{\theta}(I_a), \bar{\theta}] \), i.e.

\[
\frac{\partial q^*}{\partial I_a}(\theta, I_a) < 0.
\]

The intuition for the result is that because of the hazard rate assumption the higher anticipated investment, the lower the likelihood of a given type \( \theta \) compared to the mass of types higher than \( \theta \). Hence, the seller puts less weight on this type and further distorts its quantity in order to extract more information rent from higher types. The result is proven in the appendix.

**Lemma 3.** Under assumption 4 buyer \( \theta \)'s surplus from trade is decreasing in anticipated investment levels \( I_a \) for all \( \theta \in [\tilde{\theta}(I_a), \bar{\theta}] \). Furthermore, marginal surplus w.r.t. anticipated investment is decreasing in \( \theta \).
Both results are a direct consequence of the distortion of quantities increasing in anticipated investment. The seller wants to extract more surplus from higher type buyers, and therefore leaves less rent to lower type ones (see appendix for the proofs of this and the next lemma). This in turn decreases the buyer’s incentives to invest:

**Lemma 4.** Under assumption 4 the buyer’s reaction function $I(I_a)$ is decreasing in the seller’s anticipation of the level of investment.

Using these results allows to state and prove the uniqueness of the equilibrium:

**Proposition 3.** Under assumption 4 the equilibrium (should it exist) is unique.

**Proof.** Suppose by contradiction that there are multiple equilibria in pure strategies. Pick two and denote the equilibrium levels of investment with $I_1^*$ and $I_2^*$, w.l.o.g. $I_1^* < I_2^*$. The following inequality chain establishes the desired contradiction, and thus establishes the uniqueness of the equilibrium in pure-strategies:

$$I_1^* = I(I_1^*) > I(I_2^*) = I_2^*$$

The inequality holds because the best-response function is decreasing due to lemma 4.

Summing up, we have seen that price-discrimination by the seller leading to a partial rent extraction from the buyer distorts the investment decision of the buyer, who does not take into account the returns of the investment going to the seller and is harmed by distorted traded quantities. Therefore, the investment level is too low compared to the efficient level. Existence of a unique pure-strategy equilibrium hinges on the decreasing hazard rate assumption (stricter than the first-order stochastic-dominance property) and regularity assumptions on the buyer’s best-response function w.r.t. investment, which establishes strategic substitutability between investment and anticipated investment.

### 3 A monopoly platform - two-sided market

Suppose that the seller (an ISP) connects end-users to the Internet, and the buyer (a content provider) would like to provide a service to these end-users\(^{19}\). Then, the seller might not only charge the buyer, but also the end user, and two-sided market effects have to be taken into account. We consider the case, where the seller charges the buyer a non-linear tariff and the

\(^{19}\)We stick to the assumption that the end-user pays for the service, but results would be the same, when the buyer would finance the service through advertisements.
end-users an access fee $P$. We rule out other forms of price discrimination or mechanisms. Consider a unit mass of end-users, each has unit demand. An end-user derives surplus from consuming one unit of the buyer’s product (e.g. access to her content library) given by $v + \theta$ where $v$ is an iid draw from the distribution $G(v)$ on $[\underline{v}, \overline{v}]$ which is assumed to have a strictly increasing hazard rate. In this formulation $\theta$ represents the (inverse) value of the outside option - the higher $\theta$ the lower the value of the outside option, the higher the demand. Checking the conditions from section 2 shows that $p(\theta, q) = G^{-1}(1 - q) + \theta$ indeed fulfills the assumptions of the base model. To ensure interior solutions of all optimization problems we assume that $\underline{v} + \overline{\theta} < c$, so that it is never (socially) optimal to serve all end-users in the downstream market. Like previously the buyer’s downstream profit is denoted by

$$V(\theta, q) = qp(\theta, q) = qG^{-1}(1 - q) + q\theta.$$ 

These additional features change the timing of events:

1. The buyer chooses an investment level $I \geq 0$.
2. The seller offers a tariff $T(q)$ to the buyer and access fee $P$ to the end-user.
3. Each end-user decides whether to join the seller’s platform.
4. Nature assigns a type $\theta$ to the buyer.
5. The buyer learns her type $\theta$, and chooses quantity $q$ observing the number of connected end-users, which induces the downstream price $p(\theta, q)$.
6. Nature assigns to each end-users independently a type $v$.
7. Each end-user learns her surplus from consuming, i.e. the sum of the buyer’s type $\theta$ and her own type $v$. Observing the price $p(\theta, q)$ each end-user decides whether to buy.
Our setup is restrictive under the interpretation of $\theta$ as a quality parameter or (more generally) as parameter of end-user demand. Then, information on $\theta$ is available to the buyer and end-users. In the two-sided setting the seller is also in contact with end-users. This changes the nature of the mechanism design problem significantly, and opens the alley for a full surplus extracting mechanism à la Crémére and McLean (1988), in which the seller can use the end-users to learn about the buyer’s “private” information. While this is an interesting problem itself, we stick to the outlined mechanism, because it matches ISPs’s stated intentions to levy fees on content providers for generated traffic. Another justification for this restriction is that the regulator might impose equal treatment rules (one non-linear tariff available to all buyers) on the seller for competition reasons\textsuperscript{20}.

By contrast, only the buyer has information on $\theta$ when it is a cost parameter. Then, the described behavior resembles the optimal contract for the seller, and the analysis below applies without any restriction of generality. For expositional reasons we stick to the interpretation of $\theta$ as parameter of end-user demand. The analysis in the cost parameter case is simpler, and all derived results apply in both cases.

The first new question to consider is whether the end-users join the platform or not. The seller can extract the whole expected end-user surplus via the fixed fee, as there is no ex-ante heterogeneity between end-users\textsuperscript{21}. The end-users’ expected surplus given a buyer’s type $\theta$ is given by

$$u(\theta, q) = \int_{\theta}^{\pi} [v + \theta - p(\theta, q)] g(v) dv.$$ 

Hence, the seller maximizes

$$\int_{\theta}^{\pi} [T(q(\theta|I)) + u(\theta, q(\theta|I)) - cq(\theta|I)] f(\theta|I) d\theta$$

subject to the buyer’s self-selection constraints. Define a modified surplus function

$$\hat{V}(\theta, q) := V(\theta, q) + u(\theta, q)$$

and observe that\textsuperscript{22}

$$\frac{\partial \hat{V}}{\partial \theta}(\theta, q) = \frac{\partial V}{\partial \theta}(\theta, q) \quad \text{for all } (\theta, q).$$

\textsuperscript{20}There may be also non-economic reasons, e.g. privacy concerns, to limit the set of admissible mechanisms to be used by the seller.

\textsuperscript{21}Ex-ante heterogeneity of end-users will be studied at a later stage.

\textsuperscript{22}Due to the additive structure of demand $\frac{\partial u}{\partial \theta} = 0$. Trivially, this also holds in the case of $\theta$ being a cost parameter. For more general demand $\frac{\partial u}{\partial \theta} \neq 0$ may hold. A rigorous treatment of this case is left for future research.
It is straightforward to show that all assumptions required from the surplus function in the previous exposition are met by the modified surplus function $\hat{V}$. With this information and the usual trick of plugging the buyer’s incentive constraint into the seller’s profit function, we get that the seller optimizes the following expression:

$$\int_{\theta}^{\tilde{\theta}} \left[ (\hat{V}(\theta, q(\theta|I)) - cq(\theta|I)) f(\theta|I) - \frac{\partial \hat{V}}{\partial \theta}(\theta, q(\theta|I))(1 - F(\theta|I)) \right] d\theta$$

The seller is faced with a standard adverse-selection problem for an agent’s surplus function $\hat{V}$. Therefore, the results from section 2 carry over from the base-model: There is underinvestment (here joint surplus also includes end-user surplus, because it is part of the seller’s profit). Existence and uniqueness considerations apply as well.

On top of these results we can go a bit further and compare the situation where the seller may charge the end-users with one where she cannot.

**Lemma 5.** Take the anticipated level of investment $I_a$ as given. Then the seller offers the buyer a tariff which induces higher quantities and leaves each type of buyer better off compared to a situation in which the seller might not charge end-users.

To grasp the intuition (the proof is relegated to the appendix) let us take a look at the allocation profile in the two-sided market setting given the anticipated level of investment:

$$\frac{\partial V}{\partial q}(\theta, \hat{q}(\theta, I_a)) = c + \frac{1}{h(\theta|I_a)} \frac{\partial^2 V}{\partial q \partial \theta}(\theta, \hat{q}(\theta, I_a)) - \frac{\partial u}{\partial q}(\theta, \hat{q}(\theta, I_a))$$

Compared to the first-order condition of the monopoly input provider case, equation (1), we have an additional negative term on the left-hand side which captures the increase in end-user surplus due to an increase in quantities. This partially offsets the downstream inefficiency introduced by the buyer. As a result optimal quantities are higher compared to the monopoly input provider case. Further note that the seller is charging below marginal cost for type $\theta$ (and for types close to $\tilde{\theta}$), *i.e.* the seller is willing to make a marginal loss on the buyer side in order to extract a higher surplus from the end-user side. For low types there is also the conventional rent-extraction motive with the net effect depending on parameter values.

**Proposition 4.** Suppose assumption 4 holds. Then the buyer invests more when the seller is able to charge end-users.

**Proof.** Denote by $\hat{I}^*$ an equilibrium investment level under non-observable investment when the seller is able to charge end-users. Again a simple revealed preference argument like in
the proof of lemma 4 leads to the following inequality:

$$\int_{\theta} \left[ U(\theta, I^*) - \hat{U}(\theta, \hat{I}^*) \right] \left[ f(\theta | I^*) - f(\theta | \hat{I}^*) \right] d\theta \geq 0 \quad (7)$$

The first derivative with respect to $\theta$ of the first term in square brackets can be written as follows:

$$\frac{\partial}{\partial \theta} \left[ U(\theta, I^*) - \hat{U}(\theta, \hat{I}^*) \right] = \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I^*)) - \frac{\partial V}{\partial \theta}(\theta, \hat{q}(\theta, \hat{I}^*))$$

$$= \left[ \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I^*)) - \frac{\partial V}{\partial \theta}(\theta, \hat{q}(\theta, I^*)) \right] + \left[ \frac{\partial V}{\partial \theta}(\theta, \hat{q}(\theta, I^*)) - \frac{\partial V}{\partial \theta}(\theta, \hat{q}(\theta, \hat{I}^*)) \right]$$

The first term in square brackets is negative due to lemma 5. Now suppose by contradiction that $I^* > \hat{I}^*$. Then, the second-term is negative as well. Therefore, inequality (7) has to hold strictly in the opposite direction due to the first-order stochastic dominance property. This is the desired contradiction. Thus, $I^* \leq \hat{I}^*$ has to hold. The strictness of inequality can be shown by the usual first-order condition argument.

We have seen that giving the seller the opportunity to also charge the end-users promotes investment. Underinvestment still prevails.

**Corollary 1.** Suppose assumption 4 holds. Then giving the seller the possibility to also charge the end-users improves the seller’s surplus.

**Proof.** Due to proposition 4 equilibrium investment is higher when the seller may also charge end-users. Anticipating equilibrium investment the seller is still free to charge only the buyer and stick to the equilibrium tariff from the one-sided case. Due to the higher (anticipated) investment the seller is better off than in the one-sided case. Hence, the seller cannot be worse off, when she plays the equilibrium strategy instead.

**Free disposal**

One of the features of the tariff charged by the seller is that for some types the marginal tariff is below marginal cost. The seller subsidizes the buyer in order to increase end-user surplus by compensating for the market power of the buyer vis-à-vis end-users in the downstream market. To see this, consider the first-order condition (6) for type $\theta$:

$$\frac{\partial V}{\partial q}(\theta, \hat{q}(\theta, I_a)) = c - \frac{\partial u}{\partial q}(\theta, \hat{q}(\theta, I_a)) < c$$
When marginal cost is small (or zero), marginal tariffs given by the right-hand side of the equation are negative. Up to now we have assumed implicitly that there is no free disposal, i.e. that each unit of the seller’s input purchased by the buyer translates into one more trade between the buyer and an end-user. In this case the seller can subsidize buyers via a negative marginal tariff. When the buyer can freely dispose of the seller’s input, the buyer might serve less end-users than the amount of input procured would suggest. Consider

$$q^*(\theta) := \arg \max_q V(\theta, q),$$ (8)

which is the number of end-users the buyer would like to serve in case she would not have to pay for the seller’s input. Inducing the buyer to serve more end-users necessitates a subsidy on the seller’s input. Given free disposal the buyer would still serve $q^*(\theta)$ end-users under a subsidy and dispose of additional units of the input. This would eliminate the seller’s incentive to subsidize in the first place, because it would not boost end-user surplus anyway. Hence, the seller has to take an additional constraint into account when constructing the optimal tariff.

**Proposition 5.** Under free disposal the seller picks the quantity schedule $\min\{q^*(\theta), \hat{q}(\theta, I_a)\}$ given the anticipated-level of investment. Furthermore, under assumption 4 equilibrium investment is (weakly) lower compared to a situation without free disposal.

**Proof.** Free disposal introduces another constraint on the seller’s optimization problem, i.e. that the quantity schedule is bounded from above by $q^*(\theta)$. In case this constraint does not bind for type $\theta$, $\hat{q}(\theta, I_a)$ is still a solution of the optimization problem. When it binds, the quantity is determined by $q^*(\theta)$. By revealed preference one can show that $q^*(\theta)$ is increasing in $\theta$, thus, $\min\{q^*(\theta), \hat{q}(\theta, I_a)\}$ is increasing in $\theta$ as well. Hence, this quantity schedule is implementable and is, thus, indeed the optimal solution to the seller’s maximization problem. The result on investment can be derived along the lines of the proof of proposition 4 (higher quantities lead to a cheaper tariff, which induces more investment).

It can happen that the free disposal constraint is binding for all types, i.e. that the seller charges the buyer only a fixed fee. Even if the constraint is slack for some types, it binds for

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23In terminology of the net-neutrality debate, we have assumed up to now, that traffic and traded quantities with the end-user are equivalent.

24Considering the real-world situation we have in mind, assuming free disposal is reasonable, as there is only an indirect link between traffic generated on the sellers network and end-users served by the buyer. The buyer can always generate fake traffic (e.g. by using an inefficient video compression technology or by not caching already transmitted data efficiently) as long as the seller does not observe actual downstream market interactions between the buyer and the end-user.
high types, from which most information rents can be extracted. Thus, the presence of free disposal considerably limits the gains of the seller from being able to charge buyers, which may be outweighed by transaction costs for setting up contracts and running a billing system for traffic. Hence, in terms of the net-neutrality debate, free disposal could be rationalization for the current behavior of ISPs to not charge content providers for access to their network.

4 Competition

Next we study how these insights are altered by seller competition in the one- and two-sided settings: In the one-sided setting competition is about attracting the buyer to choose a certain seller. In the two-sided setting sellers compete for end-users to connect to their platform. Hence, as soon as they are connected the buyer faces a monopoly w.r.t. access to end-users connected to a certain platform.

4.1 Competition in the one-sided market

We add a second competing seller to the model from section 2 and give the buyer the choice between buying from one of the sellers, e.g. a content provider who has the choice between different backbone providers in order to serve the end-users.

In the literature price discrimination under competition has already been studied for example by Rochet and Stole (2002). In the following we briefly adopt their approach to our setting:\footnote{Alternatively building on Armstrong and Vickers (2001) and Armstrong and Vickers (2010) leads to the same results, but allows for a general buyer’s surplus function at the expense of a more restrictive assumption on the differentiation between sellers (uniform distribution over the Hotelling line).} We extend the buyer’s type by one additional dimension $x \in [0,1]$, which can be interpreted as the location of the buyer relative to two sellers, called left and right seller respectively.
Assumption 5. The distributions of the two dimensions $\theta$ and $x$ of the type are assumed to be independent. The marginal distribution over $x$ is log-concave and is denoted by the cumulative distribution function $L$ (where $L' = l$).

Buying from the left seller necessitates a transportation cost of $\tau x$ irrespective of the quantity, while buying from right seller costs $\tau(1-x)$, where $\tau > 0$ is the fixed marginal transportation cost per unit of distance from the respective seller. Hence, the sequence of events looks like this:

1. The buyer chooses an investment level $I$ to improve the value of the seller’s output.
2. Nature assigns a type $(\theta, x) \in [\underline{\theta}, \bar{\theta}] \times [0,1]$ to the buyer according to the joint probability distribution function $f(\theta|I)l(x)$.
3. The left and the right seller choose simultaneously tariffs $T_L(q)$ and $T_R(q)$.
4. The buyer chooses which seller $S \in \{L, R\}$ she wants to buy from and $q$, the quantity of the good to buy from the chosen seller.

Proposition 6. Assume that the buyer’s surplus $V(\theta, q)$ is strictly concave in $q$ and assumption 5 holds. If $\tau$ is sufficiently small, so that all types of buyers $(\theta, x) \in [\underline{\theta}, \bar{\theta}] \times [0,1]$ choose to buy positive quantities, the sellers offer a cost-plus-fixed-fee tariff in the symmetric equilibrium, i.e. $T_S = c + F_S$ for $S \in \{L, R\}$. The fixed-fee depends on the marginal transportation cost $\tau$ and the distribution $L$, but not on the anticipation on the buyer’s investment. The marginal consumer, denoted by $\hat{x} \in [0,1]$, is indifferent between buying from the left and the right seller is also independent of the investment level.

For the proof see Rochet and Stole (2002)\textsuperscript{26}. Sellers charge a cost-plus-fixed-fee tariff, which is independent of the anticipated investment level. Using this, we can show that the hold-up problem is solved in this case (for the proof see appendix):

Proposition 7. If $\tau$ is sufficiently small as required in proposition 6 (i.e. competition is fierce enough), then the equilibrium exhibits socially optimal quantities traded in stage 4, and the socially optimal level of investment is chosen by the buyer in stage 1.

\textsuperscript{26}This proposition yields a strikingly simple result, which unfortunately does not generalize to cases where competition is weak. Then the equilibrium (conditional on the anticipated investment level) takes a more complex form, which is in combination with ex-ante investment out of the scope of this paper.
4.2 Competitive bottleneck

In the one-sided market case fierce competition allows to get rid of the (downward) distortion of the investment level. In the two-sided setting the buyer needs the seller in order to connect to the end-users of the respective seller. Suppose again that there are two sellers $i = 1, 2$ present (a generalization to $n$ sellers is straightforward), and end-users single-home, while the buyer can multi-home. Sellers compete in non-linear tariffs $T_i$ (charged to the buyer) and prices $p_i$ (charged to end-users).

Compared to the monopoly case the timing of events basically stays the same augmented only for the competition between sellers: In stage 2 both sellers set their tariffs and access fees simultaneously. In stage 3 the end-users decide on whether to connect and to which platform, and finally in stage 5 the buyer decides on the quantities on both sellers’ platforms. In order to simplify the notation we drop anticipated investment from the distribution function and other variables.

Let $x_i \in [0, 1]$ be the fraction of end-users joining platform $i$. First note that end-user inverse demand is - up to scaling for the market share - unchanged from the section before: $p(\theta, \frac{x_i}{x_i})$, where we use a shorthand notation: $\frac{x_i}{x_i} = \frac{q_i(\theta, x_i)}{x_i}$. Given the quantity schedule $q_i(\cdot)$ offered by the seller to the buyer, an end-user’s expected surplus from joining seller $i$ is denoted by $u_i^e(q_i(\cdot), x_i)$, which is the expectation over

$$u(\theta, \frac{q_i}{x_i}) = \int_{p(\theta, \frac{x_i}{x_i})-\theta}^{\sigma} [v + \theta - p(\theta, \frac{q_i}{x_i})] g(v) dv.$$
We further note that the buyer’s surplus from serving end-users on seller i’s platform is given by \( x_i V(q_i x_i, \theta) \) because \( V(q_i x_i, \theta) = \frac{q_i}{x_i} p(q_i x_i, \theta) \). \( V \) and \( p \) are the buyer’s profit and end-users downstream inverse demand from section 3 on the monopoly case.

We denote by \( P_i \) the access fee for end-users charged by the seller joining platform \( i \), and by \( \bar{u}_i \) the expected net surplus from joining seller i’s platform. Sellers compete to attract end-users. Based on expected net surplus from joining the sellers’ platforms end-users decide on which platform to join:

\[
x_i \in \phi_i(\bar{u}_i, \bar{u}_j)
\]

(9)

The function \( \phi \) pins down the substitutability of the two seller’s platforms. For the case of perfect competition\(^{27}\) \( \phi \) is given by

\[
\begin{align*}
\phi_i &= \{0\} & \text{if } \bar{u}_i < \bar{u}_j \\
\phi_i &\in [0,1] & \text{if } \bar{u}_i = \bar{u}_j \\
\phi_i &= \{1\} & \text{otherwise}
\end{align*}
\]

Given this we can state seller i’s problem. She maximizes

\[
x_i \left\{ \int_{\theta} \left[ \left( V(\theta, q_i x_i) - \frac{q_i}{x_i} q_i x_i \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, q_i x_i)(1 - F(\theta)) \right] d\theta + P_i \right\},
\]

subject to equation (9), and where expected net end-user surplus is given by

\[
\bar{u}_i = u^e_i(q_i(\cdot), x_i) - P_i.
\]

(10)

Choosing the optimal tariff, sellers take the number of end-users on their platform as given. We find that tariffs charged to the buyer are the same as in the monopoly case up to scaling for the number of end-users having joined their respective platform. Hence, we get a dichotomy: Monopoly behavior on the buyer side, and competition on the end-user side:

**Proposition 8.** Under perfect competition the sellers charge the same tariff (only scaled for relative platform size) as under monopoly, yielding allocations \( x_i \hat{q}(\theta) \). Furthermore, the sellers do not make any profit and use all profits from trade with the buyer to attract end-users to their respective platform.

\(^{27}\)We stick to the perfect competition case for simplicity, but the following analysis also applies to e.g. competition à la Hotelling.
Proof. First we solve equation (10) for $P_i$, which allows to rewrite seller $i$’s profit as

$$x_i \left\{ \int_{\theta}^{\bar{\theta}} \left[ \left( V(\theta, \frac{q_i}{x_i}) + u_i(\theta, \frac{q_i}{x_i}) - c \frac{q_i}{x_i} \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta - \bar{\theta} \right\}.$$

Recalling $\hat{V} = V + u$ and $\hat{V}_\theta = V_\theta$ we can write seller $i$’s profit as

$$x_i \left\{ \int_{\theta}^{\bar{\theta}} \left[ \left( \hat{V}(\theta, \frac{q_i}{x_i}) - c \frac{q_i}{x_i} \right) f(\theta) - \frac{\partial \hat{V}}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta - \bar{\theta} \right\}.$$

From this expression we see that the choice of the allocation profile or tariff scaled for market share $x_i$ is independent of the choice of the optimal net surplus left to end-users. It is also independent of what the other seller is doing, i.e. the surplus $\bar{u}_j$ left to end-users by seller $j$, and therefore also independent of the actual market share $x_i$. The seller’s profit from trade with the buyer looks the same up to scaling as the seller’s profit in the monopoly case. Hence, the optimal quantity schedule has to be a scaled version of the optimal quantity schedule from the monopoly case: $x_i \hat{q}(\theta)$. Summing over both sellers it is already clear that the buyer is faced with the same situation as under the monopoly: $\hat{q}(\theta) = x_i \hat{q}(\theta) + x_j \hat{q}(\theta)$.

Given this behavior of the sellers vis-à-vis the buyer we can study the competition in net end-user surplus. Denote by $x_i \pi_B$ the profit each seller makes from trade with the buyer, and by $x_i u_i$ the gross surplus of all end-users joining platform $i$. With this notation and by using equation (9) seller $i$’s profit is given by

$$\phi_i(\bar{u}_i, \bar{u}_j) \left\{ \pi_B + u_i - \bar{u}_i \right\}.$$

Remember that $\pi_B + u_i$ is constant with respect to the surplus $\bar{u}_i$ promised to end-users. Thus, the only equilibrium of this game has both sellers offering all the surplus generated to the end-users: $\bar{u}_i = \pi_B + u_i$. Suppose not, then $\pi_B + u_i > \bar{u}_i$ for some $i$. Promising $\bar{u}_i$ can only be optimal if any end-users are attracted, hence $\pi_B + u_j > \bar{u}_j$ must hold too. This cannot be optimal for seller $j$, because she could attract all end-users by just undercutting seller $i$, which is a contradiction to assertion.

Finally, using the definition of $\bar{u}_i$ we can recuperate the access fee

$$P_i = u_i - \bar{u}_i = -\pi_B.$$
The intuition is simple: Due to competition each seller wants to offer maximum net surplus to end-users. Thus, she maximizes her profit plus the end-users’ surplus and redistributes her profit to the end-user via the access fee. The access fee can be positive or negative. In situations where the sellers are willing to make a loss from trade with the buyer in order to boost end-user surplus, access fees are positive (in order to recuperate the loss on the buyer side). When sellers make a profit out of dealing with the buyer, they redistribute their profits to the end-users via a negative access fee (see appendix B for the treatment of the case when negative access fees are infeasible). For other forms of competition, e.g. for competition à la Hotelling, the only change is that sellers do not transfer all surplus to the end-users, and thus end up with positive profits.

For the discussion on the hold-up problem this result is disappointing. Contrary to the monopoly input provider case, even fierce competition between sellers (no differentiation between sellers, same cost structure) does not increase investment towards first-best investment. The only effect of competition is that the sellers are not capable anymore to extract end-user surplus, the seller’s surplus goes to the end-users instead. Finally, all of these results also hold under free disposal.

Ex-ante heterogeneity of end-users

As a robustness check for the inefficacy result we now allow for ex-ante heterogeneity of end-users before joining the platform. Let us consider the monopoly case first. Suppose that every end-user has a different valuation $w$ for being connected to the platform, which is known to the end-user at the time she decides whether to join the platform. We assume that this valuation is independent of the expected returns from future interaction with the content provider. We denote the distribution over these valuations $w \in [w_l, w_h]$ by $H(w)$ where $0 \leq w_l < w_h \leq \infty$. Hence, the number of end-users joining the platform is given by (by slight abuse of notation we denote the anticipated and equilibrium investment level by $\hat{I}^*$ and respective quantities by $\hat{q}^*(\theta, \hat{I}^*)$ again):

$$H \left( \int_{\frac{\hat{q}^*(\theta, \hat{I}^*)}{\hat{I}^*}} \hat{I}^* u(\theta, \hat{q}^*(\theta, \hat{I}^*), f(\theta|\hat{I}^*) d\theta - P \right)$$

---

28This means that the draw of an end-user’s valuation of the buyer’s good $v$ is independent of the previously drawn $w$. This may not be true in some applications. Eg. Young people typically have a higher valuation for being connected not only because they anticipate to have a higher demand for content in the future, but also because they typically have lower psychological or cognitive cost of connecting. Allowing for correlation between the ex-ante and ex-post type of end-users is feasible, but does not add additional insights apart from complicating the analysis.
Like in the competitive bottleneck section one can show that the monopolist’s optimal quantity schedule controlled for the number of end-users connecting is independent of the number of end-users connected. Given this schedule the access fee is chosen to maximize the platform’s profit. This problem can be reformulated in the cutoff ex-ante valuation $\overline{w}$ of joining the platform (using the same shortcut notation as before):

$$H(\overline{w})[\pi_B + u - \overline{w}]$$

We assume that the monopolist finds it optimal to exclude some end-users from joining the platform. Otherwise the analysis is unchanged from the case without end-user heterogeneity. If we go back to the competitive bottleneck, we get similar result as in the previous section: The same quantity schedule controlled for the number of users connected to platform $i$ arises as in the monopoly case. Hence, profit is given by

$$\phi(\overline{w}_i, \overline{w}_j) H(\overline{w}_i)[\pi_B + u_i - \overline{w}_i].$$

Under perfect competition profits are zero again and the equilibrium cutoff valuation is $\overline{w}_i = \max\{\pi_B + u_i, w_h\}$. Again platforms use all their profits to attract end-users to join their platform. This time more end-users join one of the two platforms under competition compared to the monopoly participation level\(^{29}\). Under the hazard rate assumption \(^4\) this translates into higher equilibrium investment under competition\(^{30}\).

**Proposition 9.** With ex-ante heterogeneity of end-users the sellers charge the same tariff (scaled for respective platform size) under perfect competition as under monopoly. Furthermore, the sellers do not make any profit and use all profits from trade with the buyer to attract end-users to their respective platform. This induces more end-users to join the two platforms and under assumption \(^4\) a higher equilibrium investment compared to the monopoly. However, investment remains below the first-best level.

The underinvestment part of the proposition is mainly driven by the fact, that (scaled) tariffs are the same under competition and in the monopoly, such that even when all end-users join the platform under competition, i.e. $\overline{w}_i = w_h$, investment incentives are downward distorted. On top of that it may happen\(^{31}\) that even under competition some end-users decide

\(^{29}\) A monopolist would never give away all profits in order to attract end-users.

\(^{30}\) One has to show first that the best investment response to a tariff based on a fixed anticipated investment is higher under the competitive bottleneck via a revealed preference argument. In the second step we exploit that this best response function is decreasing under assumption \(^4\) to show by contradiction that the equilibrium investment is higher in the competitive bottleneck.

\(^{31}\) $\pi_B + u$, does not depend on the distribution of $w$, and thus neither on $w_h$. 

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not to join a platform, i.e. $\overline{w}_i > \overline{w}_b$ causing investment incentives to be even lower. This means that also under ex-ante heterogeneity of end-users perfect competition does not solve the hold-up problem, but it improves innovation incentives and hence equilibrium investment.

5 Ex-ante contracting/commitment

So far we have seen that only sub-optimal amounts are invested by the buyer, because the seller’s decision on the tariff $T(q)$ takes place after the buyer’s investment decision, which is why the seller does not take into account her impact on the ex-ante investment decision. Let us depart from the non-contractibility of investment assumption in this section, and consider two ways to solve or alleviate the hold-up: ex-ante contracting and ex-ante commitment by the seller. We consider first the one-sided setting, before we discuss their applicability to the two-sided market case.

5.1 Ex-ante contracting

In this section we allow both parties to sign a contract before investment takes place. This change in the admissible contracts modifies the sequence of events of the game (stages 3 and 4 are unchanged):

1. The seller commits to offer a tariff $T(q)$ to the buyer in the future, and additionally might charge a fixed fee $A$ upfront.

2. The buyer observes the seller’s tariff $T(q)$ and fixed fee $A$, chooses whether to accept the seller’s offer. Then she decides on the amount $I \geq 0$ to invest in order to improve the value of the seller’s output.

Proposition 10. The equilibrium contract proposed by the seller is composed of a fixed fee $A = W(I^b) - I^b$ and a tariff $T(q) = cq$. Thus, first-best quantities are traded and investment attains the socially optimal level. (Proof: see appendix)

Unsurprisingly, the seller is able to fully extract the buyer’s follow-up profit in the first stage, which allows her to sell at cost at stage 4. This in turn induces the buyer to choose first-best quantities and investment. The buyer breaks even only in expected terms, i.e. there are states of nature in which the buyer makes loss (when taking the upfront fixed fee $A$ into account).
5.2 Ex-ante commitment with ex-post participation constraints

Assume now, that the seller can commit to a tariff in stage 1, but that the buyer can only sign the contract after the investment has been sunk and the draw of nature has materialized. The buyer consumes a positive amount only if she is able to at least break even. This renders the seller’s problem more complicated, because she has to take into account an additional incentive compatibility constraint for investment (i.e. has to bear in mind the buyer’s best response investment).

Proposition 11. Equilibrium investment under ex-ante commitment with ex-post participation constraints is higher than the equilibrium investment level $I^*$ under non-commitment. (Proof: see appendix)

    Under commitment the seller takes into account the impact of the chosen tariff on investment behavior. Committing to the tariff chosen in equilibrium under non-commitment is a feasible strategy of the seller, because it induces investment $I^*$. Therefore, inducing a lower investment level cannot be profitable for the seller. One can show that there is a profitable deviation from the tariff charged under non-commitment, which makes the seller strictly better off. Hence, investment has to be higher than under non-commitment.

5.3 Discussion and two-sided market case

The results derived in the one-sided setting (propositions 10 and 11) hold, in principle, in the two-sided monopoly platform case too. The problem applying these results to the ISP market arises from the sheer number of ISPs\(^{32}\). In order to serve all end-users ex-ante contracts have to be written with thousands of content providers and content distribution networks. In practice this seems to be infeasible e.g. because of high transaction cost due to the huge number of negotiations. Furthermore, content providers typically only launch in a few geographically constraint markets. When they extend their reach to other areas or countries later on, a considerable amount of their investment has already been sunk before they get into negotiations with local ISPs.

This curse of large numbers also applies to the ex-ante commitment case, because common agency problems arise. Studying commitment in this situation with a finite number of competing sellers is tricky, but for the limit case of an unit mass of sellers competing for the buyer the analysis is trivial: Commitment by a seller to a tariff trying to induce more

\(^{32}\)There are thousands of ISPs worldwide, most of them are not even competitors because they serve different geographical markets.
investment is in vain, because she has zero mass in the buyer’s investment considerations. Hence, ex-ante commitment does not change the equilibrium investment and tariff.

6 Regulation

As we have seen before competition is ineffective (or of limited effectiveness) in solving the investment hold-up in a two-sided market because sellers compete on the end-user side. This brings up the scope for regulation. In this section we study the impact of net neutrality on investment and provide a short discussion of observability of investment.

6.1 Net-neutrality regulation

Currently ISPs do not charge content providers for traffic and access to the respective end-users. This pricing scheme (or absence of traffic charges) is very likely to be in place for historic reasons, because the Internet has been conceived originally as platform for fail-safe military communication and later opened up to research institutions. Only over time the Internet became more and more a platform for commercial exchange. We call this status-quo pricing scheme net-neutrality regime (zero-prices for buyers: \( T(q) = 0 \), access fees for end-users \( \mathcal{P} \) in the terminology of the paper). The question arises, whether a net-neutrality regime should be imposed on ISPs, who push in many statements for a deviation from it. The answer to the question is not clear cut:

First note that the quantities chosen by a type \( \theta \) buyer, who does not have to pay for the input, is \( q^n(\theta) \) like defined in the discussion of free disposal in equation (8) in section 3. For high marginal cost \( c \) excessive use of the input occurs under net-neutrality regulation, i.e. \( q^n(\theta) > \hat{q}^{fb}(\theta) \) for all \( \theta \). Overinvestment occurs in equilibrium (proof analogous to the one of proposition 1). When marginal cost is too high, the seller cannot recuperate the loss on the buyer side, and quits the market. Even if this does not happen, excessive use of the input and the overinvestment problem under net neutrality can be more damaging than the downward distortions in quantities and investment under the unregulated regime. It is, however, difficult to find meaningful conditions which imply welfare decreasing effects of net neutrality in this case.

Even if marginal cost \( c \) is low, the case is not clear cut. Suppose that \( c = 0 \) then

\[
\frac{\partial V}{\partial q} (\theta, \hat{q}(\theta, I_a)) = \frac{1}{h(\theta|I_a)} \frac{\partial^2 V}{\partial q \partial \theta} (\theta, \hat{q}(\theta, I_a)) \quad \text{for all } \theta, \hat{q}(\theta, I_a) \quad \text{and } \theta \in \{ \theta \}.
\]

Depending on the parameters the marginal tariff can be negative for all \( \theta \), i.e.
\[
\frac{\partial V}{\partial q}(\theta, \hat{q}(\theta, I_a)) < 0.
\]
Then, in the absence of free disposal imposing net neutrality decreases traded quantities for all types (further away from first-best), which in turn leads to lower equilibrium investment. Thus, we need an even stricter requirement to unambiguously show welfare improving effects of net-neutrality regulation:

**Proposition 12.** Under free disposal and when the marginal cost \( c \) is zero (or very small) net neutrality is (weakly) welfare improving.

**Proof.** First notice that in this case net neutrality increases the quantity traded for all types, for which the free disposal constraint is not binding. For types bounded by the free disposal constraint quantity stays the same under net neutrality. Furthermore, quantities are below the first-best, which can be shown by a revealed preference argument and the fact that marginal buyer surplus has to be negative in social optimum for all types (because \( c = 0 \)):

\[
\frac{\partial V}{\partial q}(\theta, \hat{q}^{fb}(\theta)) = -\frac{\partial u}{\partial q}(\theta, \hat{q}^{fb}(\theta)) < 0
\]

Marginal social surplus given a buyer’s type \( \theta \) is increasing in quantities below first-best quantities, i.e. for \( q < \hat{q}^{fb}(\theta) \):

\[
\frac{\partial}{\partial q}[V(\theta, q) + u(\theta, q)] = \frac{\partial}{\partial q}\left[\int_{\hat{p}(\theta, q)}^{\infty} (v + \theta)g(v)\,dv\right] = p(\theta, q) > 0
\]

Hence, any increase in quantity towards first-best increases social surplus. Furthermore, using a revealed preference argument like in proposition 1 we can show that (equilibrium) investment under net neutrality is (weakly) higher. We get strict inequalities when net-neutrality regulation has a bite, i.e. that in the absence of regulation there are some types for which the free-disposal constraint is not binding. Together, this implies that a net-neutrality regulation is welfare increasing, as welfare improves for each realized type and types improve due to higher investment. The proof can be extended for small deviations from zero marginal costs.

This result sheds light on the impact of net-neutrality regulation on the fixed vs. mobile Internet market. Marginal traffic cost is very small for fixed line ISPs, which makes mandated net neutrality a viable option to curb the hold-up problem. For mobile Internet carriers, however, marginal traffic cost can be high due to congestion, as the bandwidth of a local loop is shared by all end-users in the respective geographical area\(^{33}\). Hence, congestion

\(^{33}\)This may also apply to other technologies like Internet over power lines.
may be the more severe problem than potential underinvestment by content providers, and thus a net-neutrality regulation could aggravate congestion via overinvestment by content providers.\footnote{Via a revealed preference argument it is possible to lift this result to situations when the content provider’s investment is not transmission technology specific: e.g., a content provider active in the fixed-line and mobile market, where the net-neutrality regulation only applies to the fixed-line market.}

6.2 Anonymity rule

In appendix C we consider the case of the seller being able to observe investment in the one-sided monopoly input provider case, but the results are also valid for the two-sided monopoly platform. This discussion can be seen as a generalization of Hermalin and Katz (2009), who study only the unit-demand case. Here we provide a rough overview of the results, for details see the appendix. We show that under the decreasing hazard rate assumption, equilibrium investment is lower when investment becomes observable. The seller benefits from non-observability of investment due to the higher equilibrium investment level. Thus, the seller prefers not to be informed about the level of investment chosen by the buyer, which resembles results in Crémé (1995) pointed out in the introduction. The buyer, on the other hand, prefers the situation where investment is observable, because she benefits from her role as the Stackelberg leader of the game. The impact on joint surplus cannot be determined in general, but when joint-surplus is concave in investment, non-observability is welfare improving. Suppose that the seller would be able to commit ex-ante to not observe investment, she would choose commit, and thus bring investment back to the same level as under non-observability. If such a commitment is not feasible, one can achieve the same goal by regulation: Differing from our model sellers (ISPs) typically deal with a large number of buyers. Consider the limit case of a continuum of buyers. Then, under a regulation which allows for non-linear prices, but prohibits the seller from discriminating between buyers (i.e., to offer the same tariff to all buyers), there exists an equilibrium in which all buyers invest $I_a$ and the deviation of a single buyer does not induce the seller to change the tariff.

7 Implications for the net-neutrality debate and alleys for future research

What are the implications of the paper for the net-neutrality debate? Content-delivery networks and backbone providers already charge content providers for their services today. In a one-sided market competition amongst these input providers protects investment by
content providers from a hold-up. This result does not apply in the two-sided market case: The hold-up problem is immune to competition in this setting. This means that even fierce competition between ISPs in order to attract end-users does not influence the severity of the hold-up. Ex-ante commitment by ISPs to a tariff before investment takes place still works in the monopoly case, but as there are many ISPs from whom the content provider must secure an agreement, the free-riding problem between ISPs significantly limits the effectiveness of commitment. The punchline is that in a two-sided market of the type outlined in the paper the hold-up problem seems to be pervasive. Net-neutrality regulation can alleviate the hold-up when the ISPs’ marginal traffic costs are small (and there is free disposal of traffic), because traded quantities are closer to first-best, which improves content providers’ incentives to innovate. In general, though, this result does not hold. For high marginal cost net neutrality leads to over-consumption of traffic and overinvestment - the resulting welfare effects can be negative.

Of course this paper studies only a small subset of the issues around net neutrality, and thus does not provide final insights into this debate. However, the framework of the model is general enough to be useful in future research, e.g. to identify other regulatory measures to curb the negative impact on investment of ISPs charging content providers. For example, similar effects should be present when different service qualities are introduced into the model. Furthermore, investment incentives for ISPs to invest in the quality of their networks should be brought into the picture as well. Another question which could be studied with only minor modifications to the present model is how the two-sided tariff structure might influence content providers’ efforts to serve their content more efficiently, i.e. in a way which uses less traffic, and hence lowers the ISP’s costs. This question might be very important in the context of mobile ISPs, who are faced by a more or less fixed total bandwidth for all end-users in a certain area given the transmission technology used. In fact, traffic cost is exogenous to the present model. However, most of the social traffic cost comes from congestion, which is usually borne by the end-users due to degraded quality or incapability to consume some services (like video telephony) at all. So, another alley for future research could be to include these congestion costs into the model.
References


Appendices

A Proofs

Proof of lemma 1. First we show that $U(I, I_a)$ is also continuous in $I_a$. We know that $q^*(\theta, I_a)$ is continuous in $I_a$ due to the regularity assumption on the seller’s problem and the smoothness of all involved functions, i.e. $\frac{\partial V}{\partial q}$ and $\frac{\partial^2 V}{\partial q \partial \theta}$ are continuous in $q$ and $h(\theta|I)$ is continuous in $I$. Now we have to show that $U(\theta, I_a)$ is continuous in $I_a$, i.e. that if $I_n \to I_a$ then

$$U(\theta, I_a) = \lim_{I_n \to I_a} U(\theta, I_n) = \lim_{I_n \to I_a} \int_\theta^\bar{\theta} \frac{\partial V}{\partial \theta}(t, q^*(t, I_n)) dt.$$ 

This is a direct consequence of the Dominated Convergence Theorem, because $\frac{\partial V}{\partial q}(t, q^*(t, I_n)) < \frac{\partial V}{\partial q}(t, q(t))$ for all $n$ (the inequality holds because of the single-crossing condition and a downward distortion of the quantities traded due to second-degree price discrimination).

Now we need to prove that $U(I, I_a)$ is continuous in $I_a$, which follows again from the Dominated Convergence Theorem, because $U(\theta, I_a) < W(\theta)$ for all $I_a$. Therefore, the correspondence $\mathcal{I}(I_a)$ mapping the seller’s anticipation to optimal buyer’s investment level is upper-hemicontinuous due to Berge’s maximum theorem. So, there might be jumps in the buyer’s best-response, which may lead to non-existence of a pure-strategy equilibrium.

The convexity of the distribution function renders the buyer’s problem in stage 1 concave given $I_a$:

$$\int_\theta^\bar{\theta} U(\theta, I_a) \frac{\partial^2 f}{\partial I^2}(\theta|I) > 0$$

Therefore, the investment choice problem has a unique solution, and thus $\mathcal{I}(I_a)$ is also continuous (upper-hemicontinuous and single-valued).

Proof of lemma 2. It is well known that the IC constraint of the seller’s problem can be backed out and plugged into the seller’s objective function:

$$\max_{\theta(I)} \int_\theta^\bar{\theta} \left[ (V(\theta, q(\theta|I)) - cq(\theta|I)) f(\theta|I) - \frac{\partial V}{\partial \theta}(\theta, q(\theta|I))(1 - F(\theta|I)) \right] d\theta$$

This reformulation allows to optimize surplus type by type. Hence, it is possible to carry out
a revealed preference argument. Suppose $I'_a > I_a$:

$$V(\theta, q^*(\theta|I_a)) - cq^*(\theta|I_a) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I_a)) \frac{1}{h(\theta|I_a)} \geq V(\theta, q^*(\theta|I'_a)) - cq^*(\theta|I'_a) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I'_a)) \frac{1}{h(\theta|I'_a)}$$

Subtracting the second inequality from the first yields:

$$\left[ \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I'_a)) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I_a)) \right] \left[ \frac{1}{h(\theta|I_a)} - \frac{1}{h(\theta|I'_a)} \right] \geq 0$$

Due to the monotone hazard rate assumption the right term in brackets is negative, such that the first term in brackets has to be non-positive. This can only be true if $q^*(\theta|I_a) \geq q^*(\theta|I'_a)$.

The inequality is strict when there is no shutdown for $I_a$, because if $q^*(\theta|I_a) = q^*(\theta|I'_a)$ held, at least one first-order condition would be violated.

**Proof of lemma 3.** Straight-forward differentiating buyer $\theta$’s surplus with respect to the anticipated level investment and application of lemma 2 yields the first result:

$$\frac{\partial U}{\partial I_a}(\theta, I_a) = \int_0^\theta \left[ \frac{\partial^2 V}{\partial \theta \partial q}(t, q^*(t, I_a)) \frac{\partial q^*}{\partial I_a}(t, I_a) \right] dt < 0$$

Differentiating once more by $\theta$ shows the second part of the result:

$$\frac{\partial^2 U}{\partial I_a \partial \theta}(\theta, I_a) = \frac{\partial^2 V}{\partial \theta \partial q}(\theta, q^*(\theta, I_a)) \frac{\partial q^*}{\partial I_a}(\theta, I_a) < 0$$

**Proof of lemma 4.** By revealed preference we get for $I' > I$

$$\mathcal{U}(\mathcal{I}(I), I) - I \geq \mathcal{U}(\mathcal{I}(I'), I) - I'$$
$$\mathcal{U}(\mathcal{I}(I'), I') - I' \geq \mathcal{U}(\mathcal{I}(I), I') - I.$$

Subtracting the second from the first inequality yields

$$\int_0^\theta \left[ U(\theta, I) - U(\theta, I') \right] \left[ f(\theta|I) - f(\theta|I') \right] d\theta \geq 0.$$

We know that the first difference inside the integral is increasing in $\theta$:

$$\frac{\partial}{\partial \theta} [U(\theta, I) - U(\theta, I')] = \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I)) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I')) > 0$$
This together with the FOSD property shows that $\mathcal{I}(I') > \mathcal{I}(I)$. 

**Proof of lemma 5.** We denote by $\hat{q}(\theta, I_a)$ the quantity schedule prevailing in the current setting given the anticipated level of investment. We carry out the usual revealed preference argument (see the proof of lemma 2 for a reference) and use $V_\theta = \hat{V}_\theta$ to get:

$$V(\theta, \hat{q}(\theta, I_a)) - V(\theta, \hat{q}(\theta, I_a)) \geq \hat{V}(\theta, q^*(\theta, I_a)) - V(\theta, q^*(\theta, I_a))$$

This is equivalent to

$$u(\theta, \hat{q}(\theta, I_a)) \geq u(\theta, q^*(\theta, I_a)),$$

but $u(\theta, q)$ is increasing in $q$ because the end-users' inverse demand is falling in $q$:

$$\frac{\partial u}{\partial q} (\theta, q) = -\frac{\partial p}{\partial q} (\theta, q) \int_0^\pi g(v) dv = -q \frac{\partial p}{\partial q} (\theta, q) > 0$$

Hence, $\hat{q}(\theta, I_a) \geq q^*(\theta, I_a)$. The strict inequality can be shown to hold by plugging into both first-order conditions, which leads to a contradiction in case equality would hold.

Let’s call $\hat{U}(\theta, I_a)$ a buyer $\theta$’s surplus given anticipated investment. It can be computed analogously to the base-model setting with $\hat{q}$ playing the role of $q^*$ in equation (3). From this equation it is straightforward to see that a higher quantity schedule increases a buyer $\theta$’s surplus given the anticipated level of investment. 

**Proof of proposition 7.** Given $\tau$ is sufficiently small proposition 6 applies, the buyer chooses to consume irrespective of her type $(\theta, x)$. Irrespective of which seller she buys from, she consumes socially optimal units $q_{fb}^b(\theta)$, which maximizes $V(\theta, q) - cq$.

The choice which seller to buy from is, however, in general not efficient, i.e. $\hat{x}$ lies somewhere inside the unit interval. However, for symmetric distributions on $x$, efficient choice of the seller arises (i.e. $\hat{x} = \frac{1}{2}$) (it might also happen for asymmetric distributions by chance).

Due to the fact that investment does not change the equilibrium tariffs of the sellers, it also does not influence the expected equilibrium transportation plus access cost:

$$C = \int_0^1 \min(F_L + \tau x, F_R + \tau(1 - x)) h(x) dx$$

Using this we see that the expected revenue from investing $I$ in stage 1 is given by $W(I) - C$. Then the socially optimal level $I_{fb}$ also maximizes the buyer’s profit, because the buyer’s profit is given by joint surplus minus a constant - the expected transportation.
plus access cost only, i.e. \( I^{fb} = \arg \max_I W(I) - C - I \). A corner solution can be ruled out, because it would imply that \( W(0) - C \leq W(I^{fb}) - C - I^{fb} < 0 \) which would be a contradiction to full-market coverage.

**Proof of proposition 10.** The seller’s profit equals the maximum of joint surplus outlined in section 2.2. What remains to be checked is, whether the buyer accepts the contract in stage 2. As a first step we observe that the buyer’s quantity choice in step 4 is socially optimal, i.e. a type \( \theta \) chooses to buy \( q^{fb}(\theta) \) units from the seller. Thus, the expected buyer’s profit is \( W(I) \) when investing \( I \) units in stage 2. Then the objective function of the buyer is identical to the one in the joint surplus maximization problem, so that the buyer chooses the socially efficient investment level \( I^{fb} \).

This leaves the buyer with a surplus of \( W(I^{fb}) - I^{fb} \) after having accepted the offer of the seller. This surplus equals the upfront fee \( A \) asked for by the seller, which leaves the buyer with zero profit when accepting the seller’s offer, i.e. she is indifferent whether to accept the contract proposed by the seller or not.

**Proof of proposition 11.** First note, that the seller is at least as well-off as under non-observable investment. She could just pick the equilibrium tariff from that situation, the buyer would invest the same amount as in the non-observable investment case. Furthermore, equilibrium investment under commitment can never be below the level under non-observability, because as pointed out before in the non-commitment case the seller’s profit is increasing in equilibrium investment level, which means that the seller even ignoring the incentive compatibility constraint for investment would be worse off inducing less investment compared to the case where she charges the equilibrium tariff from the non-observable investment case.

To show that the inequality is strict, we construct a deviation from the allocation schedule under non-observable investment towards the first-best schedule, which increases investment and the seller’s profit. Consider the following allocation schedules for \( \delta \in [0, 1] \):

\[
\bar{q}(\theta, \delta) := (1 - \delta)q^*(\theta, I^*) + \delta q^{fb}(\theta) \quad \text{for all } \theta > \tilde{\theta}(I^*)
\]

First we need to show that investment increases (strictly) in \( \delta \) (which can be proved via the usual revealed preference argument together with checking the first-order conditions). Now note that the seller’s profit depends on \( \delta \) via two channels: via a change in the allocation schedule per type and via a change in investment. Hence, profit can be denoted by
π(\(q(\cdot, \delta), I(\delta)\)). By differentiating with respect to \(\delta\) we get

\[
\frac{d\pi}{d\delta}(\delta = 0) = \int_{\theta}^{\bar{\theta}} \frac{\partial \tilde{q}}{\partial \delta}(\theta, 0) [f.o.c. at \delta = 0] f(\theta|I) d\theta + \frac{\partial \pi}{\partial I} dI \frac{\partial \pi}{\partial I} d\delta > 0.
\]

The first term of the expression is zero because of the envelope theorem, while the second part is strictly positive due to investment being strictly increasing in \(\delta\) and the first-order stochastic dominance property. Hence, there exists a small \(\delta\) such that the corresponding allocation schedule strictly improves the seller’s profit under commitment. As discussed before this can only be the case, when equilibrium investment has strictly increased. □

B Competitive bottleneck with non-negative access fees

We have seen in section 4.2 that in equilibrium competition might lead sellers to attract end-users via a negative access fee. There may be cases, however, where negative access fees are not feasible, e.g. because of the presence of end-users who are not interested in trading with the buyer anyway, but just want to cash in the negative access fee. Consider that such a non-negativity constraint on access fees is in place and binding. Using the notation from section 4.2 we can write seller \(i\)’s profit as

\[
\phi_i(u_i^e(q_i(\cdot), x_i), u_j^e(q_j(\cdot), x_j)) \left\{ \int_{\theta}^{\bar{\theta}} \left[ \left( V(\theta, \frac{q_i}{x_i}) - c \frac{q_i}{x_i} \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta \right\}.
\]

We first observe that under perfect competition both sellers make zero profit (from dealing with the buyer). Suppose both make a positive profit, then a seller with a market share below 1 could marginally increase the quantity schedule in order to increase the end-users’ expected surplus, and therefore attract all end-users which increases profit. Now suppose that only one seller makes a positive profit, then the other seller could mimic the profitable seller but increase the quantity schedule a little bit, hence attract all end-users, and thus make a positive profit.

Clearly a seller would just exit the market, if she made a negative profit. Hence, in equilibrium both sellers maximize end-user surplus under a zero-profit condition:

\[
\max \int_{\theta}^{\bar{\theta}} u(\theta, \frac{q_i}{x_i}) f(\theta) d\theta \\
\text{s.t.} \int_{\theta}^{\bar{\theta}} \left[ \left( V(\theta, \frac{q_i}{x_i}) - c \frac{q_i}{x_i} \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta = 0
\]
Denoting the Lagrange multiplier of the constraint by $\lambda$ we get the following necessary first-order condition:

$$\frac{\partial V}{\partial q}(\theta, \frac{q}{x_i}) = c + \frac{1}{h(\theta)} \frac{\partial V}{\partial q}(\theta, \frac{q}{x_i}) - \frac{1}{\lambda} \frac{\partial u_i}{\partial q}(\theta, \frac{q}{x_i})$$

Notice that for $\lambda = 1$ this is the first-order condition of the monopoly seller’s problem. By assumption we are in the case where the monopoly seller’s profit from trade with the buyer is positive. Hence, $\lambda < 1$ has to hold. Suppose not, i.e. $\lambda \geq 1$, then by a revealed preference argument quantities would drop for all $\theta$ compared to the monopoly case, and therefore end-user surplus would decrease as well. This cannot be true in equilibrium, because charging the (scaled) optimal tariff from the monopoly case would clearly be a profitable deviation.

Given that $\lambda < 1$ in equilibrium quantities can be shown to be larger than in the monopoly case by revealed preference. Hence, we get the following result:

**Lemma 6.** For a given anticipated investment level $I_a$ the seller offers the buyer a tariff inducing higher quantities compared to a situation in which access-fees are not restricted to be positive.

The proof uses the same technique as in lemma 5. Using this lemma in another revealed preference argument like in the proof of proposition 4 we get:

**Proposition 13.** Suppose assumption 4 holds and that the non-negativity constraint on the access fee binds, then under unobservable investment the buyer invests more under competition between sellers than in a monopoly.

When the non-negativity constraints are binding, sellers compete to attract end-users by offering better expected trades with the buyer by raising the quantity schedules to a point, where the sellers’ profits go to zero. This higher quantity schedule in turn induces the buyer to invest more in equilibrium than under the monopoly case (under non-observable investment). This is in contrast to the main result that competition does not change the equilibrium investment behavior of the buyer when there are no restrictions on the access fee.

**C Observable investment**

The analysis of this section is carried out for the one-sided markets case of a monopoly input provider. However, the results also apply directly to the two-sided monopoly platform case.

Again, as in analysis in section 2.3, the seller is allowed to propose any tariff $T(q)$ to the buyer in step 2. The difference is that this time we assume that the seller is able to observe
the investment in stage 1. As usual the problem is solved backwards by assuming that the
buyer has already sunk her investment in step 1. The reasoning of the seller is identical
to what was discussed before, with the only difference, that the seller observes, instead of
anticipates, the level of investment which has been sunk before. So, we still get that the
buyer’s surplus after $\theta$ has realized when the seller has observed an investment level $I$ is
given by $U(\theta, I)$.

The crucial difference from the setting before is, that in step 1 the buyer does not take
the tariff charged by the seller but the seller’s reaction function as given when deciding on
the level of investment, because the seller observes the level of investment after the buyer
has already sunk the investment and sets an optimal tariff based on this information. This
removes the simultaneous move from the game, and allows to solve the game using backwards
induction. Thus, the expected surplus of a buyer sinking an investment $I$ includes the seller’s
reaction and is, therefore, given by

$$U(I, I) = \int_{\theta} U(\theta, I) f(\theta | I) d\theta.$$  

By the same argument used before a maximum of $U(I, I) - I$ is obtained for some $I^{**}$. Hence, an equilibrium of the game exists, and while it is not necessarily the case that the
solution is unique, a multiplicity of equilibria might arise only in exceptional cases (as the
optimization problem for choosing $I$ is not necessarily strictly concave, multiple solutions
cannot be ruled out). As before there is an inefficiently low level of trade\(^{35}\).

With the very same revealed preference argument as used in the proof of proposition 1,
it is possible to rule out overinvestment:

**Proposition 14.** Under observable investment the buyer’s equilibrium investment does not
exceed the first-best investment level, i.e. $I^{**} \leq I^{fb}$.

One at the first sight surprising result follows without any further assumptions:

**Proposition 15.** The buyer is better off, when the seller observes the level of investment
compared to the situation where the investment level is not observed.

*Proof.* The proof is simple and short: $I^{**}$ maximizes $U(I, I) - I$. Thus, for other levels of
investment the net-returns are lower - this includes the equilibrium value of investment $I^*$
when investment is not observable by the seller: $U(I^{**}, I^{**}) - I^{**} \geq U(I^*, I^*) - I^*$.  \(\square\)

\(^{35}\)again with the exception of type $\bar{\theta}$, who consumes the first-best quantity.
When framed slightly differently the result becomes quite obvious: $I^*$ is in the set of admissible investment levels under observed investment, so the buyer’s net surplus must be at least as high as under non-observable investment. Put into other words again: Even under unobservable investment the buyer does not want to fool the seller by picking a non-anticipated investment level in equilibrium. This incentive not to deviate from the equilibrium level of investment is controlled by an equilibrium relationship. When the buyer’s investment is observable by the seller, the equilibrium level of investment is controlled directly by the buyer herself, i.e. the anticipated level of investment enters directly the objective function instead of being fixed at an arbitrary (from the point of view of the buyer) level.

We would expect that, because of the partial extraction of surplus by the seller, the buyer’s investment would be inefficiently low as well. However, this result does not always hold, and can be shown only after invoking additional assumptions on the impact of investment on the capability of the seller to extract rent from the buyer. To be able to characterize the distortion of investment, we need to assume again, that the hazard rate is decreasing in investment (i.e. assumption 4). Then with the help of lemma 2 and the fact, that $W(\theta) - U(\theta, I^*)$ is strictly increasing in $\theta$, it is straightforward to compare the equilibrium investment level with the socially optimal one.

**Proposition 16.** When the seller engages in second-degree price discrimination and assumption 4 holds, the equilibrium investment is strictly lower than the socially optimal level: $I^{**} < I^{fb}$.

**Proof (by contradiction).** Suppose that $I^{**} = I^{fb}$. Being an optimum level of investment $I^{**}$ must fulfill the first-order condition for the maximization problem $\max_I U(I, I) - I$:

$$
\int_0^\theta U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta|I^{**}) d\theta + \int_0^\theta \frac{\partial U}{\partial I}(\theta|I^{**}) f(\theta|I^{**}) d\theta = 1
$$

(11)

Note that the second term on the left-hand side is negative, because of lemma 3. On the other hand, the first-order condition characterizing the socially optimal level of investment is given by:

$$
\int_0^\theta W(\theta) \frac{\partial f}{\partial I}(\theta|I^{fb}) d\theta = 1
$$
By using lemma 3 we can establish the following contradiction:

\[
1 = \int_{\theta}^{\bar{\theta}} W(\theta) \frac{\partial f}{\partial I}(\theta | I^{**}) > \int_{\theta}^{\bar{\theta}} U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta | I^{**}) > \int_{\theta}^{\bar{\theta}} U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta | I^{**}) + \int_{\theta}^{\bar{\theta}} \frac{\partial U}{\partial I_a}(\theta | I^{**}) f(\theta | I^{**}) = 1
\]

Thus, \( I^{**} < I^{lb} \) must hold.

Contrary to the section before, existence and (in almost all cases) uniqueness follow directly from the sequential structure of the setup. In general, the equilibrium investment level is also distorted, but this time the strict downward distortion follows from the decreasing (in investment) hazard rate assumption. To better understand this, it helps to take a closer look at the respective first-order condition - equation (11). Marginal returns to investment can be split in two components: First on the left there is the rent effect, which comes from the improvement of the distribution of the types due to investment. This part is smaller than under maximization of the first-best, because parts of the rents go to the seller. The second term captures a strategic effect, which comes about by the seller’s reaction to the change in investment. While intuitively it might seem compelling at the first sight, that this term should be negative, it is not under all circumstances: The seller’s willingness and need to distort the low \( \theta \) quantities is tightly connected to the shape of the hazard rate over \( \theta \). Depending on how investment affects this shape determines whether the strategic effect is positive or negative. Under the decreasing hazard rate assumption we have seen that it turns out to be negative, and thus the total effect is negative and investment is strictly downward distorted.

**Proposition 17.** If assumption 4 holds, the equilibrium investment is smaller compared to the case when the seller can observe it before deciding on her choice of the tariff, i.e. \( I^{**} < I^* \).

**Proof.** Again a revealed preference argument:

\[
\mathcal{U}(I^*, I^*) - I^* \geq \mathcal{U}(I^{**}, I^*) - I^{**} \\
\mathcal{U}(I^{**}, I^{**}) - I^{**} \geq \mathcal{U}(I^*, I^*) - I^*
\]

Adding up both inequalities, plugging in and rearranging yields:

\[
\int_{\theta}^{\bar{\theta}} [U(\theta, I^{**}) - U(\theta, I^*)] f(\theta | I^{**}) d\theta \geq 0
\]
Due to lemma 3 we get that $I^{**} \leq I^*$. Again, to get the strict inequality we suppose by contradiction that $I^{**} = I^*$. Then the following inequality chain holds and yields the desired contradiction:

$$1 = \int_\theta^\theta U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta|I^{**}) \, d\theta > \int_\theta^\theta U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta|I^{**}) \, d\theta + \int_\theta^\theta \frac{\partial U}{\partial I_a}(\theta|I^{**}) f(\theta|I^{**}) \, d\theta = 1$$

The first equality is the first-order condition for optimal investment under non-observable investment. The following inequality holds because of lemma 3. The last equation is the first-order condition for optimal investment under observable investment.

Comparing the results from propositions 15 and 17 is a bit striking: While, on the one hand, equilibrium investment drops compared to a situation when the seller might not observe investment, the buyer, on the other hand, benefits from the seller being able to observe investment. As pointed out before this is explained by the fact that in the game where the investment is not observed investment and tariff choice are de-facto simultaneous moves. While the sequential nature of the game with observable investment establishes the buyer (i.e. the investor) as the Stackelberg leader and the seller as the Stackelberg follower of the game.

**Proposition 18.** If assumption 4 holds, the seller is worse off in equilibrium when investment is observable, because she is always worse off if the prevailing value of investment is smaller.

It is easy to prove the last result via a simple revealed preference argument again, which is, therefore, left to the reader. The immediate follow-up question is how the joint buyer and seller surplus behave in the two different cases. Unfortunately this is not a clear cut case, it rather depends on the relationship between the inefficiency introduced by price discrimination and the level of investment. Joint surplus under price discrimination for a prevailing level of investment $I$ is given by

$$\int_\theta^\theta [V(\theta, q^*(\theta, I)) - cq^*(\theta, I)] f(\theta|I) \, d\theta - I.$$  

The marginal (gross) returns to investment are then

$$\int_\theta^\theta [V(\theta, q^*(\theta, I)) - cq^*(\theta, I)] \frac{\partial f}{\partial I}(\theta|I) \, d\theta + \int_\theta^\theta \left[ \frac{\partial V}{\partial q}(\theta, q^*(\theta, I)) - c \right] \frac{\partial q^*}{\partial I_a}(\theta) f(\theta|I) \, d\theta.$$  

The first term gives the improvement of the joint surplus to the increase of investment keeping the charged tariff fixed. It is clearly positive as $V(\theta, q^*(\theta, I)) - cq^*(\theta, I)$ is increasing in $\theta$. The second term results from the dead-weight loss due to price-discrimination, which is - given the decreasing hazard rate assumption - negative because $\frac{\partial q^*}{\partial I_a} < 0$ and $\frac{\partial V}{\partial q}(\theta, q^*(\theta, I)) - c > 0$.  

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Unfortunately it is not possible to trace out the joint effect, and thus we cannot conclude in general that joint surplus is higher under unobservable investment. Without stating the result formally it is, however, possible to show with a revealed preference argument that the investment level, which maximizes joint-surplus given the inefficiency introduced by second-degree price discrimination, is larger than the equilibrium investment level under observable and non-observable investment. Thus, if joint-surplus is concave in the prevailing investment level - as it is the case in the example below - we indeed get that joint surplus decreases when investment becomes observable.

D Discussion of the assumptions on the distribution over types

Throughout the paper we impose restrictions on the distribution function over the types and the impact of the investment level on it:

1. Monotone hazard rate (w.r.t. types): assumption 1
2. First-order stochastic dominance (w.r.t. investment): assumption 1
3. Decreasing hazard rate in investment: assumption 4

These assumptions are commonly used and have economic justifications, but the question arises how restrictive they are when invoked at the same time. It is well known that assumption 4 about the decreasing hazard rate in investment implies first-order stochastic dominance with respect to investment (i.e. assumption 1). One way to proceed is to use the relationship between the hazard rate and distribution function,

\[ F(\theta, I) = 1 - e^{-\int_0^\theta h(t|I) \, dt} , \]

choose a hazard rate function in line with the desired properties and compute the distribution function.

An alternative - probably more intuitive - construction procedure for a rich class of distributions in line with all mentioned assumptions starts from a one dimensional distribution over a finite or infinite interval \( J \) with a non-decreasing hazard rate and call the respective distribution function \( \tilde{F}(t) \) and the corresponding hazard rate by \( \tilde{h}(t) \) for \( t \in I \). Furthermore, we have to choose an (smooth) function \( m(\theta, I) \), which is increasing in \( \theta \) and maps onto \( J \).
for any $I \geq 0$. Now consider the following distribution function:

$$F(\theta, I) := \tilde{F}(m(\theta, I))$$

Denote by $f$ the respective density function and by $h$ its hazard rate. To find a set of sufficient restrictions on $m$ such that this distribution adheres to the properties above, we compute the relevant terms:

<table>
<thead>
<tr>
<th>Property</th>
<th>Needed sign</th>
<th>Sufficient condition(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_I = \tilde{F}' m_I$</td>
<td>$&lt; 0$</td>
<td>$m_I &lt; 0$ (also necessary!)</td>
</tr>
<tr>
<td>$h_{\theta} = \tilde{h}'(m_{\theta})^2 + \tilde{h} m_{\theta\theta}$</td>
<td>$\geq 0$</td>
<td>$m_{\theta\theta} \geq 0$</td>
</tr>
<tr>
<td>$h_I = \tilde{h}' m_{\theta} m_I + \tilde{h} m_{I\theta}$</td>
<td>$&lt; 0$</td>
<td>$m_{I\theta} &lt; 0$</td>
</tr>
</tbody>
</table>

**Example.** Take any distribution on $\mathbb{R}^+$ with a non-decreasing hazard rate, like the exponential or for certain parameter values the Weibull distribution, and consider this function:

$$m(\theta, I) = -\ln\left(\frac{\theta - \theta_0}{\theta - \theta_0 + I}\right)$$

It maps onto $\mathbb{R}^+$ for any non-negative value of $I$. Furthermore, all derivatives fulfill the conditions outlined above:

$$m_I, m_{I\theta} < 0 < m_{\theta}, m_{\theta\theta}$$

Thus, $F(\theta, I) := \tilde{F}(m(\theta, I))$ is a distribution function with the desired properties. Plugging in for the exponential distribution (for any parameter $\beta > 0$) yields the following result:

$$F(\theta, I) = 1 - \left(\frac{\theta - \theta}{\theta - \theta_0}\right)^\frac{1}{\beta(1+I)}$$

$$h(\theta, I) = \frac{1}{\beta(1+I)(\theta - \theta_0)}$$

It is obvious, that the two monotone hazard rate assumptions and the first-order stochastic-dominance property are fulfilled.

When we leave the case of a finite interval for $\theta$ there are even simpler examples. Start with a random variable $t$ on $[0, \infty]$ with a hazard rate $h(t)$ strictly increasing in $t$. Then consider $\theta = t + I$. It’s hazard rate is $h(\theta - I)$ which is clearly increasing in $\theta$ and decreasing in $I$. 

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Up to now I neglected the convex distribution function assumption which is used in proving that the buyer’s reaction function is single-valued. Distributions on finite intervals fulfilling this assumption and the two monotone hazard rate assumptions seem to be quite rare\(^{36}\). Two examples for such distributions are given in Spaeter (1998):

\[
\begin{align*}
F(\theta, I) &= \left[ \frac{\theta - \bar{\theta}}{\beta (1 + I \bar{\theta})} + 1 \right] \frac{\theta}{\bar{\theta}} \\
G(\theta, I) &= (I + k)^{\frac{\theta - \bar{\theta}}{\bar{\theta} - \theta}} \quad \text{for } k > 1
\end{align*}
\]

E Linear pricing

We have seen in the first part of the paper that the optimal tariff for the seller is non-linear, which is driven by the seller’s wish to extract as much surplus from the buyer as possible under the incentive compatibility constraint. Now we sketch what happens if the seller is restricted to (e.g. forced by regulation) charge a linear tariff \(T(q) = pq\) in stage 2. It turns out that the problem, while technically simpler at the first sight, is harder to analyze and it is more difficult to draw conclusive results. To demonstrate the difficulties in carrying out comparative statics the impact of observability on investment is analyzed.

E.1 Equilibrium investment

Let us start again with the case that the seller does not observe the level of investment sunk by the buyer before choosing the optimal price. Again the problem is solved by backwards induction: Given a per-unit price \(p\) in step 4 a buyer of type \(\theta\) maximizes her surplus:

\[
\max_q V(\theta, q) - pq
\]

A sufficient condition \((V(\theta, q)\) is strictly concave in \(q\)) for the optimal \(q(\theta, p)\) is given by the first-order condition (to simplify this section, assume that there are no corner solutions)

\[
\frac{\partial V}{\partial q}(\theta, q^*(\theta, p)) = p. 
\]

Using the single-crossing condition we get that chosen quantities are decreasing in price and increasing in type:

\[
\frac{\partial q^*}{\partial p}(\theta, p) < 0 < \frac{\partial q^*}{\partial \theta}(\theta, p)
\]

\(^{36}\)For an infinite domain it is easy to find examples: By shifting a one-dimensional distribution over \(\theta\) by an increasing and concave transformation of \(I\).
We can compute buyer $\theta$’s surplus with the help of the envelope theorem

$$U(\theta, p) = \int_{\bar{\theta}}^{\theta} \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, p)) \, dt + U(\bar{\theta}, p)$$ (13)

where buyer $\bar{\theta}$’s surplus is given by $U(\bar{\theta}, p) = V(\bar{\theta}, q^*(\bar{\theta}, p)) - \bar{p}q^*(\bar{\theta}, p)$. With the help of these expressions we can write the buyer’s expected surplus as follows:

$$\overline{U}(I, p) = \int_{\theta}^{\bar{\theta}} U(\theta, p) f(\theta | I) \, d\theta$$

Given the quantity choice of all buyer’s types (or all buyer’s potential types) the seller maximizes profits in stage 2 given anticipated investment $I$:

$$\max_p \pi(p | I) = \max_p (p - c) \int_{\theta}^{\bar{\theta}} q^*(\theta, p) f(\theta | I) \, d\theta$$ (14)

The optimal price $p(I)$ has to fulfill the first-order condition:

$$\frac{\partial \pi}{\partial p}(p(I) | I) = \int_{\theta}^{\bar{\theta}} \left[ q^*(\theta, p(I)) + (p(I) - c) \frac{\partial q^*}{\partial p}(\theta, p(I)) \right] f(\theta | I) \, d\theta = 0$$ (15)

or equivalently

$$\frac{p(I) - c}{p(I)} = - \frac{\int_{\theta}^{\bar{\theta}} q^*(\theta, p(I)) f(\theta | I) \, d\theta}{p(I) \int_{\theta}^{\bar{\theta}} \frac{\partial q^*}{\partial p}(\theta, p(I)) f(\theta | I) \, d\theta}.$$ (16)

In general it is not possible to tell how the seller anticipation of the buyer’s investment in step 1 influences the price. This depends on how investment affects the elasticity of (expected) demand. This relationship can be very complex, because it depends on the interaction between the demand of single buyer types, the distribution of types and the impact of investment on this distribution. The pure-strategy equilibrium investment level and price (assuming that such an equilibrium indeed exists) are called $I^*$ and $p^*$.

### E.2 Comparative statics: Observability

Now suppose that the seller observes the buyer’s investment when choosing the price level. We first note that the quantity choice of the buyer in step 4 stays the same as in the previous section: The quantity bought is still $q(\theta, p)$ - see equation (12). The seller’s reaction function $p(I)$ also stays the same, with $I$ being the observed level of investment (instead of the
Thus, the objective function of the buyer in stage 1 looks like this:

$$\int_0^\infty U(\theta, p(I)) f(\theta|I) d\theta - I$$

The existence of a pure strategy equilibrium is trivial, like in the case of non-linear tariffs and observable investment. We denote an equilibrium investment level by $I^{**}$ and the corresponding price level as $p^{**} = p(I^{**})$.

If we employ the usual revealed preference argument

$$U(I^*, p^*) - I^* \geq U(I^{**}, p^*) - I^{**}$$

$$U(I^{**}, p^{**}) - I^{**} \geq U(I^*, p^*) - I^* ,$$

add up both inequalities, plug in and rearrange, we get:

$$\int_0^\infty [U(\theta, p^{**}) - U(\theta, p^*)] f(\theta|I^{**}) d\theta \geq 0$$

This inequality can only hold if $p^{**} \leq p^*$, i.e. the equilibrium price charged by the seller drops when investment is observable. Whether investment drops or rises depends on the seller’s reaction function. If it increases for all (anticipated) investment levels, i.e. $p'(I) > 0$, we get that observability leads to a drop in investment like in the case where second-degree price discrimination was admissible. If not, we may even see investment rise (e.g. when the seller’s reaction function is decreasing in relevant levels of (anticipated) investment).

### E.2.1 Summary

This is only a very brief sketch of the linear-price case. Irrespective of the observability of investment one can easily show that underinvestment compared to first-best prevails, because the price chosen by the seller in equilibrium is trivially larger than marginal cost. Thus, an inefficiently low quantity is traded, which in turn leads to inefficiently low investment. However, the effect of observability on investment depends on the elasticity of aggregate (or expected) demand, which cannot be traced back to simple assumptions on buyer’s surplus and the distribution of types. This is in stark contrast to the case of second-degree price discrimination, where the influence of observability hinges only on an assumption on the impact of investment on the distribution of the buyer’s type (assumption 4). Thus, to assess the impact of observability requires much more detailed information on the demand structure under linear-prices, than under second-degree price discrimination.