The Interplay Between Student Loans and Credit Cards
and Amplification of Consumer Default∗

Felicia Ionescu† Marius Ionescu‡
Colgate University Colgate University
February 15, 2012

Abstract

We analyze, theoretically and quantitatively, the interactions between two different forms of unsecured credit and their implications for default behavior of young U.S. households. One type of credit mimics credit cards in the U.S. and the default option resembles a bankruptcy filing under Chapter 7 and the other type of credit mimics student loans in the U.S. and the default option resembles Chapter 13 of the U.S. Bankruptcy Code. In the credit card market financial intermediary offers a menu of interest rates based on individual default risk. In the student loan market the government sets the interest rate and chooses a wage garnishment to pay for the cost associated with default. We prove the existence of a steady-state equilibrium and characterize the circumstances under which a household defaults on each of these loans depending on household characteristics in both markets. Our model is consistent with the main facts regarding borrowing and default on both forms of unsecured credit for young U.S. households. We demonstrate that the institutional differences between the two markets make borrowers prefer default on student loans rather than on credit card debt.

We quantify the effects of increased college debt burdens versus more severe credit card terms on the increase in default rates for student loans in recent years and find that the increase in college debt burden explains only 4% of the increase in the default rate for student loans, whereas worse terms on credit card accounts help fully explain the increase in default rates for student loans. We plan to explore the policy implications of our theory.

JEL Codes: D91; I22; G19;

Keywords: Default, Bankruptcy, Student Loans, Credit Cards

*The authors thank Kartik Athreya, Jeff Baldani, Satyajit Chatterjee, Juan-Carlos Hatchondo, Takao Kato, Dirk Krueger, Wenli Li, Leo Martinez, Makoto Nakajima, Pierre-Daniel Sarte, and Nicole Simpson for helpful comments.
† Department of Economics, 13 Oak Drive, Hamilton NY, 13346; (315)228-7955, fionescu@colgate.edu.
‡ Department of Mathematics, 13 Oak Drive, Hamilton NY, 13346; (315)228-7955, mionescu@colgate.edu.
1 Introduction

College debt has steadily increased in the last two decades and has reached records high in the past several years (with a cumulative growth rate of 282% from 1990 to 2008). In fact, as of June 2010, total student loan debt passed total credit card debt for the first time (see Figure 1).\(^1\) At the same time, the two-year basis cohort default rate (CDR) for student loans has steadily declined from 22.4% in 1990 to 4.6% in 2005 and increased ever since reaching records high in the last decade (at 8.5% in 2009).\(^2\)

The increase in college debt alone cannot explain the recent increases in student loan default rates. A second market is needed to understand this behavior: currently the majority of individuals with college debt (76%) also have credit card debt, according to our findings from the Survey of Consumer Finance (SCF). While both of these loans represent important components of young households’ portfolios in the U.S., the financial arrangements in the two markets are very different, and in particular with respect to the roles played by bankruptcy arrangements and default pricing. In addition, credit terms on credit card accounts have worsen in the recent years adversely affecting households’s capability to diversify risk. For young borrowers this is particularly problematic, because even modest balances have more of an impact than the same balance belonging to a consumer who has a much older or robust credit history.\(^3\)

We propose a theory about the interactions between student loans and credit cards in the U.S. and their impact on default incentives of young U.S. households. As we argue in this paper, the interaction between different bankruptcy arrangements induces significant trade-offs in default incentives in the two markets. However, there is no existing work embedding this trade-off into a quantitative dynamic theory. Yet, understanding the interaction between these financial arrangements is important in the light of the recent trends in borrowing and default behavior. Data show that young U.S. households (of which a large percentage have both college and credit card debt) now have the second highest rate of bankruptcy (just after those aged 35 to 44) and the rate among 25- to 34-year-olds increased between 1991 and 2001, indicating that this generation is more likely to file for bankruptcy as young adults than were young boomers at the same age.\(^4\)

\(^1\) According to the Federal Reserve releases, U.S. households owe $826.5 billion in revolving credit (98% of revolving credit is credit card debt) and they owe $829.785 billion in student loans — both federal and private. Partially, this is due large increases in college cost by 40% in the past decade and partially due to paying down credit card debt. Currently 70% of individuals who enroll in college taking out student loans (College Board, 2008).

\(^2\) The 2-year CDR is computed as the percentage of borrowers who enter repayment in a fiscal year and default by the end of the next fiscal year.

\(^3\) This is because of the principle of revolving utilization as well as the weight given in credit scoring to the age of a consumer’s credit file. These borrowers have younger credit reports and fewer accounts, which implies that they are likely be scored in a “thin file” or “young file” score card.

\(^4\) Source: http://www.creditcards.com/
student loans have a higher default rate than credit cards or any other loan, including car loans and home loans.\footnote{According to a survey conducted by the FRB New York, the national student loan delinquency rate 60+ days in 2010 is 10.4 percent compared to only 5.6 percent for the mortgage delinquency rate 90+ days, 1.9 percent for bank card delinquency rate and 1.3 percent for auto loans delinquency rate. Based on an analysis of the Presidents FY2011 budget, in FY2009 the total defaulted loans outstanding are around $45 billion.}

These trends are alarming considering the large risks that young borrowers face: the college dropout rate has increased dramatically in the past decade (from 38% to 50% for the cohorts that enroll in college in 1995 and 2003 respectively).\footnote{We define the dropout rate as the fraction of students who enroll in college and do not obtain a degree 6 years after they enroll. Numbers are based on the BPS 1995 and 2003 data.} In addition, the unemployment rate among young workers with college education has jumped up significantly during the Great recession: 8% of young college graduates and 14.1% of young workers with some college are unemployed in 2010 (Bureau of Labor Statistics).

The combination of high income risks, high indebtedness and worse financial terms implies that borrowers are more likely to default on at least one of their loans. A couple of questions arise immediately: Which default option do young borrowers find more attractive and why? In particular, is the current environment conducive to higher default incentives in the student loan market? Secondly, how much of the increase in default on student loans is explained by trends in the student loan market and how much by the interaction between the two markets?

In order to address the proposed issues we develop a general equilibrium economy that mimics features of student and credit card loans. Infinitely lived agents differ in college debt and income levels as well as a \textit{default risk}, which is determined by the amount of debt that they owe in both markets and the default status on their student loans. These elements, in turn, determine the loan terms agents face on their credit card accounts, including loan prices. In contrast, the interest rate in the student loan market does not account for the risk that some borrowers may default. Agents face uncertainty in earnings and may save/borrow and as in practice, borrowing terms are individual specific. Central to the model is the decision of young households to repay or default on their credit card and student loans. Consequences to default for student and credit card loans differ in several important ways: for student loans they include a wage garnishment and for credit cards they induce exclusion from borrowing for several periods. More importantly, credit card loans can be discharged in bankruptcy (under Chapter 7), whereas student loans cannot be discharged (borrowers need to reorganize and repay under Chapter 13 in the Bankruptcy code).

In the theoretical part of the paper, we first characterize the default behavior for both markets. We determine the set of income levels for which borrowers default on student loans, on credit card debt and on both types of loans and show how these sets vary with households characteristics and
behavior in both markets. Our theory explains facts related to borrowing and default behavior of young U.S. households (presented in Section 2): 1) The incentive to default on student loans increases in college debt burden (debt-to-income ratio), i.e. default on student loans is more likely to occur for individuals with low levels of earnings and high levels of college debt. This result is in line with empirical evidence in Dynarski (1994), Ionescu (2008) and with our findings from the SCF. 2) Similarly, the incentive to default on credit card debt increases in credit card debt, result which is consistent with findings in Chatterjee et. al. (2007).

Our main theoretical contribution consists in demonstrating the existence of cross-market effects and their implications for default behavior. This contribution is two fold:

1) Our theory delivers that in equilibrium credit card loan prices differ across loan sizes, i.e. the interest rate increases in the size of the loan, result which is consistent with Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). In addition, we show that in equilibrium borrowers with high risk of default receive higher rates on their credit card loans (for the same loan size). In our model the default risk conditional on the size of the loan is delivered by the amount of debt and the default status in the student loan market. Our result arises from the fact that the probability of default on any credit card loan decreases in the amount of debt owed in the student loan market. Also this probability is higher for an individual with a default flag in the student loan market relative to an individual without a default flag. This set of results are new in the literature and provide a rationale for pricing credit card loans by using a default risk that takes into account participation in other credit markets. This result is related to findings in Chatterjee, Corbae, and Rios-Rull (2010) who show that credit scores (which in practice include borrowing and default in multiple markets) represent a proxy for the probability of individual default and thus provide a rationale behind using credit scores in pricing loans.

2) In any steady-state equilibrium, for any credit card debt there is a threshold of student loan such that if the amount owed on student loans is higher than this threshold then the agent defaults on at least one type of her loans. Moreover, we find a higher threshold of student loans above which default for student loans occurs. Both of these thresholds decrease in the amount owed on credit card. This result innovates by showing that while a high college debt is necessary to induce default on student loans, this effect is amplified by high indebtedness in the credit card market. This arises from the differences in bankruptcy arrangements in the two markets: the financially constrained borrower finds it optimally to default on student loans (even though she cannot discharge her debt) in order to be able to access the credit card market at relatively better terms. Since there is no effect on her credit card market participation from defaulting on student loans the borrower with high college debt prefers the default penalty in the student loan market over restricted credit card market participation.
In the quantitative part of our paper, we parametrize the model to match statistics regarding college debt, credit card debt, and income of young borrowers with student loans aged 20-26 as delivered by the SCF 2004 as well as default rates for these two types of loan. In our preferred parametrization, findings reveal large gaps in credit card rates across individuals with different levels of college debt and default status in the student loan market (rates almost double for individuals in the top half of student loan levels relative to individuals in the bottom half and for individuals with a default flag for student loans relative to individuals without a default flag). This set of findings strengthens our theory and emphasizes the quantitative importance of correctly pricing credit card debt based on behavior in other credit markets. Results also show that the thresholds that deliver default for any type of loans and default for student loans decline in the credit card debt level and this relationship is stronger for lower levels of credit card debt. In fact, for low levels of credit card debt the two thresholds coincide such that individuals with low levels of credit card debt default only on student loans and do not choose to default on credit card debt. For them the benefit of discharging their debt is small compared to the large cost associated with default (of being excluded from borrowing).

Lastly, our results deliver that the credit card default probability decreases in income. This is consistent with empirical evidence (see Sullivan, Warren, and Westbrook (2001)) and represents a new finding in the quantitative research on consumer credit, which for the most part delivers the opposite (counterfactual) result. Our research shows the importance of accounting for other types of loans when analyzing default behavior, feature that is absent in previous models of consumer default. In our model, both income and college debt levels matter for the default decision on credit card debt.

We use our theory to answer the proposed quantitative question of how much of the recent increase in default rates for student loans is due to an increase in college debt and how much is due to worse terms in the credit card market? To answer this question, we run two experiments: 1) we first quantify the effects of the changes in college debt burdens on default rates for student loans and find that the increase in college debt burden from SCF 2004 to SCF 2007 induces an increase in default rates from 4.6% to 5.8%, with most of this effect coming from the increase in student loan amounts. 2) we then quantify the effects of the change in credit card limits on default rates and find that a decline of 22% in credit card limits from SCF 2004 to SCF 2007 delivers an increase in default rates to 7.1%. We conclude that worse credit card terms amplify default on student loans. We plan to explore the policy implications of our theory, in particular we study loan repayment policies contingent on terms in the other market.
1.1 Related literature

Our paper is related to two strands of existing literature: default within credit card markets and within student loans. The first strand relates to research by Athreya, Tam, and Young (2009), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Chatterjee, Corbae, and Rios-Rull (2010), and Livshits, MacGee, and Tertilt (2007). The two first studies explicitly model a menu of credit levels and interest rates offered by credit suppliers with the focus on default under Chapter 7 within the credit card market. Livshits, MacGee, and Tertilt (2007) quantitatively compare liquidation in the U.S. to reorganization in Germany in a life-cycle model with incomplete markets, earnings and expense uncertainty.

In the student loan literature there are several papers closely related to the current study including research by Ionescu (2010), Ionescu and Simpson (2010) and Lochner and Monge (2010). These papers incorporate the option to default on student loans when analyzing various government policies. Out of these studies, the only one that accounts for the role of individual default risk in pricing loans is Ionescu and Simpson (2010) who recognize the importance of this risk in the context of the private market of student loans. The model, however, is silent with respect to the role of credit risk for credit cards or for the allocation of consumer credit, the study being restricted to the analysis of the student loan market. Ionescu (2010) is the only study that models both dischargeability and non-dischargeability of loans for young U.S. households in the U.S., but only in the context of the student loan market. Furthermore, as in Livshits, MacGee, and Tertilt (2007), Ionescu (2010) studies various bankruptcy rules in distinct environments that mimic different periods in the student loan program (in Livshits, MacGee, and Tertilt (2007) in different countries) rather than modeling them as alternative insurance mechanisms available to borrowers.\footnote{The modeling of alternative bankruptcy rules and induced trade-offs in default decisions poses obvious technical challenges which will be addressed in this paper.}

Our paper builds on this body of work and improves on the modeling of the outside insurance options available to borrowers with student loans and credit card debt. On a methodological level, our paper is more related to Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). As in their paper, we model a menu of prices for credit card loans based on the individual risk of default. In their paper the probability of default is linked to the size of the loan. We take a step further in this direction. First, we model the menu of prices based on individual default risk which consists of three elements: the size of the loan, the amount owed on student loans and the default status on student loans. These three components altogether represent a proxy for the default probability. In this direction our study is in the spirit of recent work by Chatterjee, Corbae, and Rios-Rull (2010) who provide a theory that explores the importance of credit scores (which incorporate borrowing
behavior and past repayment behavior in different types of markets) for consumer credit based on a limited information environment. Furthermore, in addition to endogenizing interest rates on credit card debt, we also allow them to respond to changes in default incentives created by alternative bankruptcy arrangements in the two markets.

To this end, the novelty of our work consists in providing a theory about interactions between credit markets with different financial arrangements and their role in amplifying consumer default for student loans. Previous research analyzed these two markets separately with the main focus being on credit card debt. Our paper attempts to bridge this gap. Our results are not specific to the interpretation for student loans and credit cards and speak to consumer default in any environments that feature differences in financial market arrangements and thus induce a trade-off in default incentives for consumers that participate in these markets.\footnote{In this respect our paper is related to Chatterjee, Corbae, and Rios-Rull (2008) who provide a theory of unsecured credit based on the interaction between unsecured credit and insurance markets.} In related empirical work, Edelberg (2006) studies the evolution of credit card and student loan markets and finds that there has been an increase in the cross-sectional variance of interest rates charged to consumers which is largely due to movements in credit card loans: the premium spread for credit card loans more than doubled, but education loan and other consumer loan premiums are statistically unchanged.

The paper is organized as follows. In Section 2, we describe several important facts about student loans and credit card terms and the legal and financial environment in the two markets. We develop the model and present the theoretical results in Section 3. We calibrate the economy to match important features of the markets for student and credit card loans and present quantitative results in Section 4. Section 5 concludes.

2 Background

2.1 Legal environment

The financial and legal environment surrounding the U.S. credit cards and student loan markets is characterized by the following features.

1) Student loans are not secured by any tangible asset, so there might be some similarities with credit card markets, but unlike credit card loans, guaranteed student loans are uniquely risky,\footnote{For instance, another application of this theory may be international consumer credit markets (credit card loans in the U.S. and euro area that feature differences in eligibility conditions and bankruptcy rules).}

\footnote{In Chatterjee, Corbae, and Rios-Rull (2008) there is private information about a person’s type. The type signaling incentive in insurance markets may change households’ incentives to default in credit markets. This approach is motivated by the role of credit scores in unsecured credit and auto insurance markets.}
since the eligibility conditions are very different. Loans are based on financial need, not on credit ratings, and are subsidized by the government. Agents are eligible to borrow up to the full college cost minus expected family contributions.

2) In contrast, for credit card loans, lenders use credit scores (FICO) as a proxy for the risk of default. These scores include information about repayment and borrowing behavior on all types of loans that borrowers have, in addition to credit card debt.

3) For credit cards, lenders impose interest rates that vary significantly across individuals of different risks of default. In general, in unsecured credit markets the feedback of any bankruptcy law into the interest rate is exactly how the default is paid for.

4) However, the interest rate on student loans does not incorporate the risk that some borrowers might exercise the option to default. The interest rate is fixed by the government. Several default penalties implemented in the student loan program such as wage garnishments upon default might bear part of the default risk.

5) Default on credit card debt triggers limited market participation. At the same time, default in the student loan market has no effect on credit card market participation.

6) Finally, individuals can file for bankruptcy for credit cards under Chapter 7 of the U.S. Bankruptcy Code, which implies permanent discharge of net debt (liabilities minus assets above exemption levels). In contrast, individuals can file for bankruptcy for student loans only under Chapter 13, which does not allow for discharge and implies a fixed term repayment schedule.

2.2 Data facts

Findings documented in this section are based on the SCF data for young borrowers aged 20-26 years old who have some college education (with or without having a college degree) and who took out student loans to finance their college education. They are no longer enrolled in college and they need to repay their student loans. We construct these samples using the SCF 2004 and the SCF 2007. Data details for the facts documented below are provided in the Appendix.

1) According to the statistics from the Department of Education, the national two-year basis cohort default rates on student loans increased from 4.5% in 2005 to 7% in 2008 (see Figure 1 in the Appendix).

2) College debt borrowed increased by almost 35% during this period of time.

3) Income of young individuals has not kept pace: it only increased by 28.7% from 2004 to 2007.

4) Consequently, college debt burdens (debt-to-income ratios) increased from 0.98 (in 2004) to 1.15 (in 2007).
5) Young borrowers with student loans use credit cards at very high rates: around 72% of young U.S. households in both years in the SCF have at least a credit card and 76% of those who are credit card users have positive balances.

6) Terms on credit card accounts of young borrowers worsen: credit limits decline from $14,014 in 2004 to $10,949 in 2007 and the average interest rate increased from 9.9% to 13% during this period. Additionally, the credit card utilization ratio increased from 0.48 to 0.74.

7) Credit card debt burden increased from 2004 (at 0.11) to 2007 (at 0.13) and a higher fraction of young borrowers hardly repay on their credit card debt in 2007 relative to 2004.

8) High college debt burdens increase the likelihood of default for student loans (see Dynarsky (1994) and Ionescu (2008) for details).

9) High credit card burdens increase the likelihood of default on credit card debt (see Athreya et. al. (2008), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007)).

10) Trends in default behavior, college debt-to-income ratios and credit terms differ across 1990-2005, 2005-2008 such that default rates for student loans decline before 2005 and increase afterwards. At the same time college debt-to-income ratios increase before 2005, however at a slower pace than after 2005.

11) Finally, terms on credit cards improve before 2005 and deteriorate after 2005. In the past three years, credit card providers have levied some of the largest increases in interest rates, fees and minimum payments.\textsuperscript{10} Also, retailers have become stingier with credit.\textsuperscript{11} But the terms on credit cards greatly affect the young household’s capability to smooth consumption and pay their education loans.

\textsuperscript{10}For instance, JPMorgan Chase, the biggest credit card provider raised the minimum payment on outstanding balances from 2% to 5% for some customers, raised its balance-transfer fee from 3% to 5% – the highest rate among the large consumer banks and also changed its United Mileage Plus Visa Signature card from a single 13.24% rate to a range of 13.24% to 19.24%, meaning most cardholders are likely to qualify for those costlier rates (June 30 Bloomberg article). Citigroup has reportedly raised rates on outstanding balances nearly 3 percentage points to an average of 24% for 13 million to 15 million cardholders (July 1 2009 Financial Times article).

\textsuperscript{11}For instance, American Express has taken the most heat over slashing credit limits. Nearly half of its portfolio underwent a major overhaul that included cutting limits by a half or more. Chase decreased credit lines or closed accounts in 2008 totaling $129 billion. Most credit card issuers enforced fees, and reduced credit lines on their in-store cards (examples include Home Depot, Target).
3 Model

3.1 Legal environment

Consumers who participate in the student loan and credit cards markets, namely young U.S. households with student loans, are small, risk-averse, price takers. They differ in levels of college debt, \( d \) and income, \( y \). They are endowed with a line of credit, which they may use for transactions and consumption smoothing. They choose to repay or default on their student loans as well as on their credit cards. Default in each of these two markets has very different consequences, which we explain below.

3.1.1 Credit cards

Bankruptcy for credit cards in the model resembles Chapter 7 “total liquidation” bankruptcy. Additionally, the model captures the fact that credit card issuers use consumer characteristics to assess the likelihood that any single borrower will default. Loan prices and credit limits imposed by credit card issuers are set to account for the individual default risk and are tailored to each credit account.

Consider a household that starts the period with some credit card debt, \( b_t \). Depending on the household decision to declare bankruptcy as well as on the household borrowing behavior, the following things happen:

1. If a household files for bankruptcy, \( \lambda_b = 1 \) (and she can do so irrespective of current income or past consumption), then the household unsecured debt is discharged and liabilities are set to 0.

2. The household cannot save during the period when default occurs, which is a simple way of modeling that the U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.

3. The household begins next period with a record of default on credit cards. Let \( f_t \in F = \{0, 1\} \) denote the default flag for a household in period \( t \), where \( f_t = 1 \) indicates in period \( t \) a record of default in the previous period and \( f_t = 0 \) denotes the absence of such record. Thus a household who defaults on credit in period \( t \) starts period \( t+1 \) with \( f_{t+1} = 1 \).

4. A household who starts the period with a default flag cannot borrow and the default flag can be erased with a probability \( p_f \).
5. In contrast, a household who starts the period with $f_t = 0$ is allowed to borrow and save according to individual credit terms: credit rates assigned to household by credit lenders vary with individual characteristics. This latter feature is important to allow for capturing default risk pricing in equilibrium.

This formulation captures the idea that there is restricted market participation for borrowers who have defaulted in the credit card market relative to borrowers who have not. It also implies more stringent credit terms for consumers who take on more credit card debt, i.e. precisely the type of borrowers who are more constrained in their capability to repay their loans. In addition, creditors also take into account borrowing behavior in the other type of market, i.e. the student loan amount owed, $d$ as well as the default status for student loans, $h$. These features are consistent with the fact that credit card issuers reward good repayment behavior and penalize bad repayment behavior taking into account this behavior in all markets that borrowers participate in. Finally, we assume that defaulters on credit cards are not completely in autarky, which is consistent with evidence. In U.S. consumer credit markets, households retain a storage technology after bankruptcy, namely, the ability to save. We assume without loss of generality that defaulters cannot borrow. In practice, borrowers who have defaulted in the past several years are still able to obtain credit at worse terms. In our model, allowing them a small negative amount or 0 does not have an effect on the results.

3.1.2 Student loans

Bankruptcy for student loans in the model resembles Chapter 13 “reorganization” bankruptcy, which requires reorganization and repayment of defaulted loans. Under the current Federal Loan Program students who participate cannot discharge on their student loans. Consequently default on student loans in the model at period $t$ (denoted by $\lambda_d = 1$) simply means a delay in repayment that triggers the following consequences:

1. There is no debt repayment in period $t$. However, the college debt is not discharged. The defaulter must repay the amount owed for payment in period $t + 1$.

2. The defaulter is not allowed to borrow or save in period $t$, which is line with the fact that credit bureaus are notified when default occurs and thus access to the credit card market is

---

\[^{12}\text{Borrowers are considered in default if they do not make any payments within 270 days in the case of a loan repayable in monthly installments or 330 days in the case of a loan repayable in less frequent installments. Loan forgiveness is very limited. It is granted only in the case constant payments are made for 25 years or in the case where repayment causes undue hardship, but as a practical matter, it is very difficult to demonstrate undue hardship unless the defaulter is physically unable to work.}\]
restricted. Also, as before, this feature captures the fact that the U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.

3. A fraction $\gamma$ of the defaulter’s wages is garnished in period $t$. Once the defaulter rehabilitates her student loan, the wage garnishment is interrupted. This penalty captures the default risk for student loans in the model.\textsuperscript{13}

4. The household begins next period with a record of default on student loans. Let $h_t \in H = \{0, 1\}$ denote the default flag for a household in period $t$, where $h_t = 1$ indicates a record of default in period $t$ and $h_t = 0$ denotes the absence of such record. Thus a household who defaults in period $t$ starts period $t + 1$ with $h_{t+1} = 1$.

5. A household that begins period $t$ with a record of default must pay the debt owed in period $t$, $d_t$. The default flag is erased with probability $p_h$. In period $t + 1$ the household starts with $h_{t+1} = 0$ and a new round of loan payment due, $d_t$.\textsuperscript{14}

6. There are no consequences on market participation during the periods after default on student loan occurs. However, there are consequences on the pricing of credit card loans from defaulting on student loans, as mentioned above. This assumption is justified by the fact that in practice student loan default is reported to credit bureaus and so creditors can observe the default status immediate after default occurs and adjust terms on loans. However, immediate repayment and rehabilitation of the defaulted loan will result in deleting the default status reported by the loan holder to the national credit bureaus. In practice, most of defaulters rehabilitate their loans. Therefore they are still able to access the credit card market (on worse terms as explained above).\textsuperscript{15}

\section*{3.2 Preferences and endowments}

At any point in time the economy is composed of a continuum of infinitely lived households with unit mass.\textsuperscript{16} Agents differ in student loan payment levels, $d \in D = \{d_{\text{min}}, \ldots, d_{\text{max}}\}$ and

\begin{itemize}
\item \textsuperscript{13}This penalty can be as high as 15\% of defaulter’s wages. In addition, consequences include seizure of federal tax refunds, possible hold on transcripts and ineligibility for future student loans.
\item \textsuperscript{14}The household cannot default the following period after default occurs. In practice, less than 1\% of borrowers repeat default given that the U.S. government seizes tax refunds in the case when the defaulter does not rehabilitate her loan soon after default occurs. This penalty is severe enough to induce immediate repayment after default (and at a larger amount).
\item \textsuperscript{15}In the quantitative part of the paper, we estimate the probability of erasing the default flag on student loans such that the period in which such a flag appears is short, consistent with the data.
\item \textsuperscript{16}The use of infinitely lived households is justified by the fact that we focus on the cohort default rate for young borrowers, which means that age distributions are not crucial for analyzing default rates in the current study. The
income levels, \( y \in Y = [y_{\min}, y_{\max}] \). There is a constant probability \((1 - \rho)\) that households will die at the end of each period. Households that do not survive are replaced by newborns who have not defaulted on student loans \((h = 0)\), or on their credit cards \((f = 0)\), have zero assets \((b = 0)\) and with labor income and college debt drawn independently from the probability measure space \((Y \times D, B(Y \times D), \psi)\) where \(B(\cdot)\) denotes the Borel sigma algebra and \(\psi\) denotes the joint probability measure. Surviving households independently draw their labor income at time \(t\) from a stochastic process. The amount that the household needs to pay on her student loan is the same.\(^{17}\) Household characteristics are then defined on the measurable space \((Y \times D, B(Y \times D))\).

The transition function is given by \(\Phi(y_{t+1})\delta_{d_t}(d_{t+1})\), where \(\Phi(y_t)\) is an i.i.d. process and \(\delta_d\) is the probability measure supported at \(d\).

**Assumption 1.**

The preferences of the households are given by the expected value of a discounted lifetime utility consists of:

\[
E_0 \sum_{t=0}^{\infty} (\rho \beta)^t U(c_t)
\]

where \(c_t\) represents the consumption of the agent during period \(t\) and \(\beta \in (0, 1)\) is the discount factor and \(\rho \in (0, 1)\) the survival probability.

**Assumption 2.** The utility function \(U(\cdot)\) is increasing, concave and twice differentiable. It also satisfies Inada condition: \(\lim_{c \to 0^+} U(c) = -\infty\).

### 3.3 Markets Arrangements

There are several similarities as well as important differences between the credit card market and the market for student loans.

#### 3.3.1 Credit cards

The market for privately issued unsecured credit in the U.S. is characterized by a large, competitive market place where price-taking lenders issue credit through the purchase of securities backed by repayments from those who borrow. These transactions are intermediated principally by credit card issuers. Given a default option and consequences on the credit record from default behavior as well as borrowing behavior, the market arrangement departs from the conventional modeling

---

\(^{17}\)Government student loan payments are fixed and computed based on a fixed interest rate and duration of the loan.
of borrowing and lending. As in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) our model handles the competitive pricing of default risk, a risk that varies with household characteristics. In this dimension, our model departs from Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) in several important ways: the default risk is based on the borrowing behavior in both markets, i.e. it depends on the size of the loan on credit cards, $b$ as well as on the amount of student loans owed, $d$. In addition, it depends on the default status on student loans, $h$. Obviously in the case of a default flag on credit cards, no loan is provided. Competitive default pricing is achieved through letting prices vary with all these three elements. This modeling feature is novel in the literature and is meant to capture the fact that in practice the price of the loan depends on various elements such as past repayment and borrowing behavior in all the markets that borrowers participate in. Unsecured credit card lenders use this behavior (which in practice is captured in a credit score) as a signal for household credit risks and thus their probability of default. They tailor loan prices to individual default risk, not only to individual loan sizes.

A household can borrow or save by purchasing a single one-period pure discount bond with a face value in a finite set $B \subset \mathbb{R}$. The set $B = \{b_{\min}, \ldots, b_{\max}\}$ contains 0 and positive and negative elements. Let $N_B$ be the cardinality of this set. Individuals with $f = 1$ (which is a result of defaulting on credit cards in one of the previous periods) is limited in her market participation, $b \geq 0$.

A purchase of a discount bond in period $t$ with a nonnegative face value $b_{t+1}$ means that the household has entered into a contract where it will receive $b_{t+1} \geq 0$ units of the consumption good in period $t+1$. The purchase of a discount bond with a negative face value $b_{t+1}$ means that the household receives $q_{b_{t+1}, d_t, h_t}(-b_{t+1})$ units of the period-$t$ consumption good and promises to deliver, conditional on not declaring bankruptcy, $-b_{t+1} > 0$ units of the consumption good in period $t+1$; if it declares bankruptcy, the household delivers nothing. The total number of credit indexes is $N_B \times N_D \times N_H$. Let the entire set of $N_B \times N_D \times N_H$ prices in period $t$ be denoted by the vector $q_t \in \mathbb{R}^{N_B \times N_D \times N_H}$. We restrict $q_t$ to lie in a compact set $Q \equiv [0, q_{\max}]^{N_B \times N_D \times N_H}$ where $0 < q_{\max} < 1$.

---

18 Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) handles the competitive pricing of default risk by expanding the “asset space” and treating unsecured loans of different sizes for different types of households (of different characteristics) as distinct financial assets.

19 A similar approach is taken in Chatterjee, Corbae, and Rios-Rull (2010) where credit limits and loan pricing arise from the optimal response of private lenders to limited information about the agent’s credit risk.

20 Note that households are liquidity constrained in the model. The existence of such constraints in credit card markets has been documented by Gross and Souleles (2000). Overall credit availability has not decreased along with bankruptcy rates over the past several years before the crisis and so aggregate response of credit supply to changing default has not been that large (see Athreya 2002).
3.3.2 Student loans

Student loans represent a different form of unsecured credit. First, loans are primarily provided by the government (either direct or indirect and guaranteed through the FSLP), and do not share the features of a competitive market.\textsuperscript{21} Unlike for credit cards, the interest rate on student loans, \( r_g \), is fixed by the government and does not reflect the risk of default in the student loan market.\textsuperscript{22} However, the penalties for default capture some of this risk. In particular, the wage garnishment is adjusted to cover default. More generally, loan terms are based on financial need, not on default risk. Secondly, taking out student loans is a decision made during college years. Once they are out of college, households need to repay their loans in equal rounds over a determined period of time subject to the fixed interest rate. Thus the amount of student loan owed each period, \( d \), is fixed given the total college debt, loan duration, and the interest rate. We model college loan bound households that are out of school and need to repay \( d \) per period; there is no borrowing decision for student loans.\textsuperscript{23}

Agents decide to repay on their student loans (\( \lambda_d \in \{0, 1\} \)). Recall the defaulter starts the next period after default occurs with student loan default flag, \( h = 1 \) and the household starts the next period with \( h = 0 \) in the case default has not occurred. In the case the household has a default flag, a wage garnishment is imposed and they keep repaying the amount owed during the following periods after default occurs.

We define the state space of credit characteristics of the households by \( S = B \times F \times H \) to represent the asset position, the credit card, and student loan default flags. Let \( N_S = N_B \times 2 \times 2 \) be the cardinality of this set.

To this end, an important note is that the assumption that all debt that young borrowers access is unsecured is made for a specific purpose and is not restrictive. The model is designed to represent the section of households who have student loans and credit card debt. As argued, these borrowers rely on credit cards to smooth consumption and have little or no collateral debt.

\textsuperscript{21}Recently, students have started to use pure private student loans, not guaranteed by the government. This new market is a hybrid between government loans and credit cards featuring characteristics of both markets. However, this new market is still small and concerns about the national default rates are specific to student loans in the government program, default rates for pure private loans being of much lower magnitudes (for details see Ionescu and Simpson (2010)). Therefore we focus on government student loans in the current study.

\textsuperscript{22}After the Higher Education Reconciliation Act of 2005 was passed, the interest rate has been set to 6.8%. Before 2005 the rate was based on the 91-day Treasury-bill rate.

\textsuperscript{23}While returning to school and borrowing another round of loans is a possibility, this decision is beyond the scope of the paper.
3.4 Decision problems

The timing of events in any period is: (i) idiosyncratic shocks, \( y_t \) are drawn for survivors and newborns and college debt is drawn for newborns; (ii) households default/repay on both credit card and student loans and borrowing/savings decisions are made; also consumption takes place and default flags for the next period are determined. We focus on steady state equilibria where \( q_t = q \).

3.4.1 Households

We will present the households’ decision problem in a recursive formulation where any period \( t \) variable \( x_t \) is denoted by \( x \) and its period \( t + 1 \) value by \( x' \).

Each period, given their college debt, \( d \), current income, \( y \), and beginning-of-period assets, \( b \), households must choose consumption, \( c \) and asset holdings to carry forward into the next period, \( b' \).

In addition, agents may decide to repay/default on their student loans, \( \lambda_d \in \{0, 1\} \) and credit card loans, \( \lambda_b \in \{0, 1\} \). As described before, these decisions have different consequences: while default on student loans implies a wage garnishment \( \gamma \) and no effect on market participation (however it may deteriorate terms on credit card accounts), default on credit card payments triggers exclusion from borrowing for several periods and has no effect on income.

The household’s current budget correspondence, \( B_{b,f,h}(d, y; q) \) depends on the exogenously given income, \( y \), college debt, \( d \), beginning of period asset position, \( b \), credit card default record, \( f \), student loan default record, \( h \), and the prices in the credit card market, \( q \). It consists of elements of the form \((c, b', h', \lambda_d, \lambda_b) \in (0, \infty) \times B \times H \times F \times \{0, 1\} \times \{0, 1\}\) such that

\[
c + q_b, d, h b' \leq y(1 - g) + b(1 - \lambda_b) - d(1 - \lambda_d),
\]

and such that the following cases hold:

1. If a household with income \( y \) has a good student loan record, \( h = 0 \) and a good credit card record, \( f = 0 \), then \( \lambda_b \in \{0, 1\} \) if \( b < 0 \) and \( \lambda_b = 0 \) if \( b \geq 0 \), \( \lambda_d \in \{0, 1\} \), \( b' = 0 \) if \( \lambda_d = 1 \) or \( \lambda_b = 1 \), \( b' \in B \) if \( \lambda_d = \lambda_b = 0 \), \( g = 0 \), \( h' = \lambda_d, f' = \lambda_d \). The household can chose to pay off both loans \((\lambda_b = \lambda_d = 0)\), in which case the household can borrow freely on the credit card market.

If the household choses to exercise its default market on either one of the markets \((\lambda_d = 1 \) or \( \lambda_b = 1))\), then the household cannot borrow or accumulate assets. Since \( h = 0 \) there is no income garnishment \((g = 0)\).

2. If a household with income \( y \) has a good student loan record, \( h = 0 \) and a bad credit card record, \( f = 1 \), then \( \lambda_b = 0, \lambda_d \in \{0, 1\} \), \( b' \geq 0 \), \( g = 0 \), \( h' = \lambda_d, f' = 1 \). In this case, there is no
repayment on credit card debt; the household chooses to pay or default on the student loan debt. The household cannot borrow or accumulate assets and the credit record will will stay 1.

3. If a household with income $y$ has a bad student loan record, $h = 1$ and a good credit card record, $f = 0$, then
   
   \[
   \lambda_b \in \{0, 1\} \text{ if } b < 0 \text{ and } \lambda_b = 0 \text{ if } b \geq 0, \lambda_d = 0, b' \in B \text{ if } \lambda_b = 0 \text{ and } b' = 0 \text{ if } \lambda_b = 1, g = \gamma, f' = \lambda_b, \text{ and } h' = 1. \]

   The household pays back the credit card debt (if net liabilities, $b < 0$) or defaults, pays the student loan and its income is garnished by a factor of $\gamma$. The student record will stay 1.

4. If a household with income $y$ has a bad student loan record, $h = 1$ and a bad credit card record, $f = 1$, then
   
   \[
   \lambda_d = \lambda_b = 0, b' \geq 0, g = \gamma, f' = 1, h' = 1. \]

   The household cannot borrow in the credit card market, pays the student loan, and her income is garnished.

There are several important observations: 1) we account for the fact that the budget constraint may be empty; in particular if the household is deep in debt, earnings are low, new loans are expensive, then the household may not be able to afford non-negative consumption. The implication of this is that involuntary default may occur. and 2) Repeated default on student loans occurs on a limited basis (i.e. when $B_{b,f,1}(d, y; q) = \emptyset$) and it is followed by partial dischargeability, which is in line with the data.

**Assumption 3.** We assume that consuming $y_{\min}$ today and starting with zero assets, $b = 0$ and a bad credit card record, $f = 1$ and student loan default record, $h = 1$ with garnished wage (i.e. the worst utility with a feasible action) gives a better utility than consuming zero today and starting next period with maximum savings, $b_{\max}$ and a good credit card record, $f = 0$ and student loan default record, $h = 0$ (i.e. the best utility with an unfeasible action).

Let $v(d, y; q)(b, f, h)$ or $v_{b,f,h}(d, y; q)$ denote the expected lifetime utility of a household that starts with asset $b$, student loan debt $d$, credit card default record $f$, and default record $h$, has earnings $y$ and faces prices $q$. Then $v$ is in the set $\mathcal{V}$ of all continuous functions $v : D \times Y \times Q \rightarrow \mathbb{R}^{Ns}$. The household’s optimization problem can be described in terms of an operator $(Tv)(d, y; q)(b, f, h)$ which yields the maximum lifetime utility achievable if the household’s future lifetime utility is assessed according to a given function $v(d, y; q)(b, f, h)$.

**Definition 1.** For $v \in \mathcal{V}$, let $(Tv)(d, y; q)(b, f, h)$ be defined as follows:

1. For $h = 0$ and $f = 0$

   \[
   (Tv)(d, y; q)(b, f, h) = \max \left\{ \max_{(c, b', f', \lambda_d, \lambda_b) \in B_{b,f,h}(d, y; q)} \left( U(c) - \tau_d \lambda_d - \tau_b \lambda_b + \beta \rho \int v_{b', f', h'}(d, y'; q) \Phi(dy') \right), \right. \]

   \[
   U(y) + \beta \rho \int v_{0,1,1}(d, y'; q) \Phi(dy') \right\}, \]

   where $\Phi$ is the cumulative distribution function of the income distribution.
where $\tau_d$ and $\tau_b$ are utility costs that the household incurs in case of default on the student loan market ($\tau_d$) or the credit card market ($\tau_b$).

2. For $h = 0$ and $f = 1$ (in which case $\lambda_b = 0$ and $f' = 1$ with probability $1 - p_f$ and $f' = 0$ with probability $p_f$)

$$(Tv)(d, y; q)(b, f, h) = \max\{\max_{B_{b,f,h}(d,y;q)} \left\{ U(c) - \tau_d \lambda_d + (1 - p_f) \beta \rho \int v_{b',f',h'}(d, y'; q) \Phi(dy') \\
+ p_f \beta \rho \int v_{b',0,h'}(d, y'; q) \Phi(dy'), U(y) + \beta \rho \int v_{0,1,1}(d, y'; q) \Phi(dy') \right\} \}.$$ 

3. For $h = 1$ and $f = 0$ (in which case $\lambda_d = 0$ $h' = 1$ with probability $1 - p_h$ and $h' = 0$ with probability $p_h$)

$$(Tv)(d, y; q)(b, f, h) = \max\{\max_{B_{b,f,h}(d,y;q)} \left\{ U(c) - \tau_b \lambda_b + (1 - p_h) \beta \rho \int v_{b',f',h'}(d, y'; q) \Phi(dy') \\
+ p_h \beta \rho \int v_{b',0,0}(d, y'; q) \Phi(dy') \right\} \}.$$ 

4. For $h = 1$ and $f = 1$

$$(Tv)(d, y; q)(b, f, h) = \max\{\max_{B_{b,f,h}(d,y;q)} \left\{ U(c) + (1 - p_f)(1 - p_h) \beta \rho \int v_{b',f',h'}(d, y'; q) \Phi(dy') \\
+ (1 - p_f)p_h \beta \rho \int v_{b',f',0}(d, y'; q) \Phi(dy') \\
+ p_f(1 - p_h) \beta \rho \int v_{b',0,h'}(d, y'; q) \Phi(dy') \\
+ p_f \beta \rho \int v_{0,0,0}(d, y'; q) \Phi(dy') \right\} \}.$$ 

The first part of this definition says that a household with good student loan and credit card default records may choose to default on either type of loan, on both or on none of them. For all these cases to be feasible we need to have that the budget sets conditional on not defaulting on both loans and conditional on defaulting on student loans or on credit card debt are non-empty. In the case at least one of these sets is empty, then automatically the attached option may not be available. In the case all of these sets are empty then involuntarily default occurs. We assume that when involuntarily default happens it will occur on both markets. In the case where both default and no default options deliver the same utility the household may choose either. $^{24}$ Finally, recall

$^{24}$These two assumptions are made such that default is not biased towards one of the two markets.
that in the case the household chooses to repay on her student loans or on her credit card debt she may also choose borrowing and savings and in the case she decides to default on either of these loans there is no choice on assets position.

The second part of the definition says that if the household with a good student loan default record and with a default flag on credit cards will only have the choice to default/repay on student loans since she does not have any credit card debt. Recall that as long as the household carries the default flag in the credit card market she cannot borrow. As before, this problem assumes that the budget set conditional on not defaulting is non-empty, otherwise involuntarily default on student loans occurs.

The last two parts represent cases for a household with a bad student loan default record. In these last cases, default on student loans is not an option. In part three the household has the choice to default on credit card. As before, this is an option only if the budget set associated with no default on credit card is non-empty (otherwise involuntarily default on credit card debt occurs). In part four, however, there is no choice to default. Thus, the household simply solves a consumption/savings decision if the budget set conditional on not defaulting on either loan is non-empty. Otherwise, we assume as in the first part that involuntarily default occurs for both loans. Involuntarily default happens when borrowers with very low income realizations and high indebtedness have no choice but defaulting. In the case this occurs repeatedly in the student loan market (i.e. for a household with default flag, \( h = 1 \), we assume that the household may discharge her student loan and there is no wage garnishment. This features capture the fact that a very small proportion of households partially discharge their student loan debt.\(^{25}\) Note that in all the cases where default occurs, the household incurs a utility cost, which is denoted by \( \tau_d \) in the case default on student loans occurs and by \( \tau_b \) in the case default on credit card debt occurs. Consistent with modeling of consumer default in the literature, these utility costs are meant to capture stigma following default as well as attorney and collection fees associated with default (see Athreya, Tam, and Young (2009), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), and Livshits, MacGee, and Tertilt (2007)).

We next proceed as follows: we provide a first set of results which contains the existence and uniqueness of the household’s problem and the existence of the invariant distribution. The second set of results contains the characterization of both default decisions in terms of households characteristics and market arrangements. The last set of results contains the existence of the equilibrium and the characterization of prices. We prove the existence of cross-market effects and characterize how financial arrangements in one market affect default behavior in the other market.

\(^{25}\)In practice dischargeability is limited to undue hardships and occurs in less than 1% of the default cases.
Existence and uniqueness of a recursive solution to the household’s problem

**Theorem 1.** There exists a unique $v^* \in V$ such that $v^* = T v^*$ and

1. $v^*$ is increasing in $y$ and $b$.
2. Default decreases $v^*$.
3. The optimal policy correspondence implied by $T v^*$ is compact-valued, upper hemi-continuous.
4. Default is strictly preferable to zero consumption and optimal consumption is always positive.

Since $T v^*$ is a compact-valued upper-hemicontinuous correspondence by Theorem 7.6 in Stockey, Lucas, and Prescott (Measurable Selection Theorem) there are measurable policy functions, $c^*(d, y; q)(b, f, h)$, $b^*(d, y; q)(b, f, h)$, $\lambda_b^*(d, y; q)(b, f, h)$ and $\lambda_d^*(d, y; q)(b, f, h)$. These measurable functions determine a transition matrix for $f$ and $f'$, namely $F_{y,d,b,h,q}^* : F \times F \to [0, 1]$

<table>
<thead>
<tr>
<th>$F_{y,d,b,h,q}^*(f' = 1)$</th>
<th>$F_{y,d,b,h,q}^*(f' = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$1 - p_f$</td>
<td>$p_f$</td>
</tr>
<tr>
<td>$0$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Also the policy functions imply a transition matrix for the student loan default record, $H_{y,d,b,f,q}^* : H \times H \to [0, 1]$ which gives the student loan record for the next period, $h'$

<table>
<thead>
<tr>
<th>$H_{y,d,b,f,q}^*(h' = 1)$</th>
<th>$H_{y,d,b,f,q}^*(h' = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$1 - p_h$</td>
<td>$p_h$</td>
</tr>
<tr>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$1 - p_h$</td>
<td>$p_h$</td>
</tr>
<tr>
<td>$0$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

All the proves are provided in the Appendix.
Existence of invariant distribution

Let \( X = Y \times D \times B \times F \times H \) be the space of household characteristics. In the following we will write \( F_q^*(y, d, b, f, h, f') := F_{y, d, b, h, q}^*(f, f') \) and \( H_q^*(y, d, b, f, h, h') := H_{y, d, b, f, h, q}^*(h, h') \). Then the transition function for the surviving households’ state variable \( TS_q^*: X \times \mathcal{B}(X) \rightarrow [0, 1] \) is given by

\[
TS_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d \times Z_f \times Z_h} 1_{\{y' \in Z_y\}} F_q^*(y, d, b, f, df') H_q^*(y, d, b, f, h, dh') \Phi(dy') \delta_d(d')
\]

where \( Z = Z_y \times Z_d \times Z_b \times Z_f \times Z_h \) and \( 1 \) is the indicator function. The households that die are replaced with newborns. The transition function for the newborn’s initial conditions, \( TN_q^*: X \times \mathcal{B}(X) \rightarrow [0, 1] \) is given by

\[
TN_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d \times Z_f} 1_{\{(y', h') = (0, 0)\}} \Psi(dy', dd', df')
\]

Combining the two transitions we can define the transition function for the economy, \( T_q^*: X \times \mathcal{B}(X) \rightarrow [0, 1] \) by

\[
T_q^*(y, d, b, f, h, Z) = \rho TS_q(y, d, b, f, h, Z) + (1 - \rho) TN_q(y, d, b, f, h, Z)
\]

Given the transition function \( T_q^* \), we can describe the evolution of the distribution of households \( \mu \) across their state variables \( (y, d, b, f, h) \) for any given prices \( q \). Specifically, let \( \mathcal{M}(x) \) be the space of probability measures on \( X \). Define the operator \( \Gamma_q: \mathcal{M}(x) \rightarrow \mathcal{M}(x) \):

\[
(\Gamma_q \mu)(Z) = \int T_q^*((y, d, b, f, h), Z) d\mu(y, d, b, f, h).
\]

**Theorem 2.** For any \( q \in Q \) and any measurable selection from the optimal policy correspondence there exists a unique \( \mu_q \in \mathcal{M}(x) \) such that \( \Gamma_q \mu_q = \mu_q \).

**3.4.2 Characterization of the default decisions**

We establish results on how the default decision in each market is determined by college debt burdens (college debt-to-income ratios). We first determine the set for which default occurs for student loans (including the form of default with partial dischargeability), the set for which default occurs for credit card debt, as well as the set for which default occurs for both of these two loans. Let \( D_{b, f, i}^S(q) \) be the set for which involuntarily default on student loans and partial dischargeability
occurs. This set is defined as combinations of earnings, \( y \), and student loan amount, \( d \), for which \( B_{b,f,1}(d,y; q) = \emptyset \) in the case \( h = 1 \). For \( h = 0 \) let \( D^{SL}_{b,f,0}(d; q) \) be the set of earnings for which the value from defaulting on student loans exceeds the value of not defaulting on student loans. Similarly, let \( D^{CC}_{b,0,h}(d; q) \) be the set of earnings for which the value from defaulting on credit card debt exceeds the value of not defaulting on credit card debt in the case \( f = 0 \). Finally, let \( D^{Both}_{b,0,0}(d; q) \) be the set of earnings for which default on both types of loans occurs with \( h = 0 \) and \( f = 0 \). Note that the last two sets are defined only in the case \( f = 0 \) since for \( f = 1 \) there is no credit card debt to default on.

Theorem 3 characterizes the sets when default on student loans occurs (voluntarily or nonvoluntarily). Theorem 4 characterizes the sets when default occurs on credit card debt and Theorem 5 presents the set for which default occurs for both types of loans.

Theorem 3. Let \( q \in Q, b \in B \). If \( h = 1 \) and the set \( D^{SL}_{b,f,1}(q) \) is nonempty, then \( D^{SL}_{b,f,1}(q) \) is closed and convex. In particular the sets \( D^{SL}_{b,f,1}(d; q) \) are closed intervals for all \( d \). If \( h = 0 \) and the set \( D^{SL}_{b,f,0}(d; q) \) is nonempty, then \( D^{SL}_{b,f,0}(d; q) \) is a closed interval for all \( d \).

Theorem 4. Let \( q \in Q, (b, 0, h) \in S \). If \( D^{CC}_{b,0,h}(d; q) \) is nonempty then it is a closed interval for all \( d \).

Theorem 5. Let \( q \in Q, (b, 0, 0) \in S \). If the set \( D^{Both}_{b,0,0}(d; q) \) is nonempty then it is a closed interval for all \( d \).

Next we determine how the set of default on credit card debt varies with the credit card debt, the student loan debt and the default status on student loans of the individual. Specifically, Theorem 6 shows that the set of default on credit card debt expands with the amount of debt for credit cards. This result was first demonstrated in Chatterjee et. a. (2007).

Theorem 6. For any price \( q \in Q, d \in D, f \in F, \) and \( h \in H \), the sets \( D^{CC}_{b,f,h}(d; q) \) expand when \( b \) decreases.

In addition, we show two new results in the literature: 1) that this default set expands when the the student loan amount declines, which imply that individuals with lower levels of student loans are more likely to default on credit card debt (Theorem 7); and 2) that this default set is larger when \( h = 1 \) relative to the case where \( h = 1 \). This result implies that individuals with a default record on student loans have a higher risk of defaulting on their credit card debt (Theorem 8).

Theorem 7. For any price \( q \in Q, b \in B, f \in F, \) and \( h \in H \), the sets \( D^{CC}_{b,f,h}(d; q) \) shrink and \( D^{Both}_{b,f,h}(d; q) \) expand when \( d \) increases.
**Theorem 8.** For any price \( q \in Q, b \in B, d \in D, \) and \( f \in F, \) the set \( D^{CC}_{b,f,0}(d;q) \subset D^{CC}_{b,f,1}(d;q). \)

3.4.3 Financial intermediaries

The (representative) financial intermediary has access to an international credit market where it can borrow or lend at the risk-free interest rate \( r \geq 0. \) The intermediary operates in a competitive market and takes prices as given and chooses the number of loans \( \xi_{d_t,h_t,b_{t+1}} \) for all type \((d_t,h_t,b_{t+1})\) contracts for each \( t \) to maximize the present discounted value of current and future cash flows 

\[
\sum_{t=0}^{\infty} (1 + r)^{-t} \pi_t, \text{ given that } \xi_{d-1,h-1,b} = 0. \]

The period \( t \) cash flow is given by

\[
\pi_t = \rho \sum_{d_{t-1},h_{t-1}} \sum_{b_t \in B} (1 - p^b_{d_{t-1},h_{t-1},b_t}) \xi_{d_{t-1},h_{t-1},b_t} (-b_t) - \sum_{d_t,h_t} \sum_{b_{t+1} \in B} \xi_{d_t,h_t,b_{t+1}} (-b_{t+1}) q_{d_t,h_t,b_{t+1}}
\]

(2)

where \( p^b_{d_t,h_t,b_{t+1}} \) is the probability that a contract of type \((d_t,h_t,b_{t+1})\) where \( b_{t+1} < 0 \) experiences default; if \( b_{t+1} > 0, \) automatically \( p^b_{d_t,h_t,b_{t+1}} = 0. \) These calculations take into account the survival probability \( \rho. \)

If a solution to the financial intermediary’s problem exists, then optimization implies \( q_{d_t,h_t,b_{t+1}} \leq \frac{\rho}{(1+r)} (1 - p^b_{d_t,h_t,b_{t+1}}) \) if \( b_{t+1} < 0 \) and \( q_{d_t,h_t,b_{t+1}} \geq \frac{\rho}{(1+r)} \) if \( b_{t+1} \geq 0. \) If any optimal \( \xi_{d_t,h_t,b_{t+1}} \) are nonzero the associate conditions hold with equality. Our problem is a natural extension of the one described in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) in that we consider that the number of loans depend on the loan size (as in their paper). As mentioned above, the number of loans in our model also depends on the loan amount and default status in the student loan market.

3.4.4 Government

The only purpose for the government in this model is to operate the student loan program. The government needs to collect all student loans. The cost to the government is the total amount of college loans plus the interest rate subsidized in college. Denote by \( L \) this loan price. We compute the per period payment on student loans, \( d \) as the coupon payment of a student loan with its face value equals to its price (a debt instrument priced at par) and infinite maturity (console). Thus the coupon rate equals its yield rate, \( r_g. \) In practice, this represents the government interest rate on student loans. When no default occurs the present value of coupon payments from all borrowers (revenue) is equal to the price of all the loans made (cost), i.e. the government balances its budget.

However, since default is a possibility, government’s budget constraint may not hold. In this case the government revenue from a household in state \( b \) with credit card default status \( f, \) income \( y \) and college debt \( d \) is given by \( (1 - p^d_d)d + p^d_d \gamma y \) where \( p^d_d \) is the probability that a contract of type \( d \) experiences default for student loans. The government will choose the default penalty, \( \gamma \)
to recover the losses incurred when default for student loans arises. The budget constraint is then given by

$$\int dd\mu = \int [(1 - p_d^d) d + \sum_{t=1}^{\infty} (1 + r_g)^{-t} p_d^d \gamma y' p_t] \Phi(dy') d\mu$$

The penalty $\gamma$ is chosen such that the budget constraint balances. We turn now to the definition of equilibrium and characterize the equilibrium in the economy.

### 3.5 Steady-state equilibrium

In this section we define a steady state equilibrium, prove its existence and characterize the properties of the price schedule for individuals with different default records.

**Definition 2.** A steady-state competitive equilibrium is a set of non-negative price vector $q^* = (q^*_{d,h,b'})$, non-negative credit card loan default frequency vector $p^{b*} = (p^{b*}_{d,h,b'})$, a non-negative student loan default frequency $p_d^d$, non-negative default penalty, $\gamma^*$, a vector of non-trivial credit card loan measures, $\xi^* = (\xi_{d,h,b'})$, decision rules $b^*(y, d, f, b, h, q^*)$, $\lambda_d^d(y, d, f, b, h, q^*)$, $\lambda_d^d(y, d, f, b, h, q^*)$, $c^*(y, d, f, b, h, q^*)$, and a probability measure $\mu^*$ such that:

1. $b^*(y, d, f, b, h, q^*)$, $\lambda_d^d(y, d, f, b, h, q^*)$, $\lambda_d^d(y, d, f, b, h, q^*)$, $c^*(y, d, f, b, h, q^*)$ solve the household’s optimization problem;
2. $\gamma^*$ solves the government’s budget constraint;
3. $p_d^d = \int \lambda_d^d(y, d, f, b, h) d\mu^*(dy, d, df, db, dh)$ (government consistency);
4. $\xi^*$ solves the intermediary’s optimization problem;
5. $p^{b*}_{d,h,b'} = \int \lambda_b^b(y', d, 0, b', 0) \Phi(dy') \mu^*(dy, d, df, db, dh)$ for $b' < 0$ and $p^{b*}_{d,h,b'} = 0$ for $b' \geq 0$ (intermediary consistency);
6. $\xi^*_{d,h,b'} = \int 1_{(b^*(y, d, f, b, h, q^*) = b')} \mu^*(dy, d, df, db, h)$ (market clearing conditions (for each type $(d, h, b')$);
7. $\mu^* = \mu_{q^*}$ where $\mu_{q^*} = \Gamma_{q^*} \mu_{q^*}$ ($\mu^*$ is an invariant probability measure).

The computation of equilibrium in incomplete markets models has been made standard by a series of papers including (Aiyagari, 1994) and (Huggett, 1993) and have been extensively used in recent papers with (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007) the most related to the current study. The dimensionality of the state vector, the non-trivial market clearing conditions which include a menu of loan prices, the condition for the government balancing budget as well as the interaction between the two types of credit make computation more involved than previous work.
### 3.5.1 Existence of equilibrium and characterization

**Theorem 9. Existence** A steady-state competitive equilibrium exists.

In equilibrium the credit card loan price vector has the property that all possible face-value loans (household deposits) bear the risk-free rate and negative face-value loans (household borrowings) bear a rate that reflects the risk-free rate and a premium that accounts for the default probability, results which are in line with those in (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007). In our setup the default probability depends on the characteristics in the student loan markets, such as loan amount and default status (in addition to on the size of the loan) and thus loan prices will depend on these elements as well. This result is delivered by the free entry condition of the financial intermediary which implies that cross-subsidization across loans made to individuals of different characteristics in the student loan market is not possible. Each \((d, h)\) market clears in equilibrium and it is not possible for intermediary to charge more than the cost of funds for individuals with very low risk in order to offset losses on loans made to high risk individuals. Positive profits in some contracts would offset the losses in others, and so intermediaries could enter the market for those profitable loans. We turn now to characterizing the equilibrium price schedule.

**Theorem 10. Characterization of equilibrium prices** In any steady-state equilibrium the following is true:

1. For any \(b' \geq 0\), \(q_{d,h}^* = \rho/(1 + r)\) for all \((d, h)\).
2. If the grid of \(D\) is sufficiently fine, and \(h = 0\) there is \(d\) such that \(q_{d}^* = \rho/(1 + r)\) for any \(b' \in B\).
3. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then \(d_1 < d_2\) implies \(q_{d_1}^* > q_{d_2}^*\) for any \(h\).
4. There is \(d\) such that \(q_{d}^* = 0\) for any \(h\).
5. If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then \(q_{h=1}^* > q_{h=0}^*\) for any \(d\).

Theorem 10 demonstrates that firms charge the risk-free interest rate on deposits (property 1) and to any loan size made to individuals with no default record on student loans and with student loan \(d\), the lowest possible value (property 2). Property 3 shows that individuals with lower levels of student loans are assigned higher loan prices, which implies that interest rates are lower for
individuals with lower levels of student loans. Property 4 says that individuals with the highest level of student loan debt face zero prices on their loans. At zero price, financial intermediaries are indifferent as to how many loans to offer since they expect no payoff in the next period at no cost (zero price). At the same time, individuals taking out these loans buy nothing in the current period but start the following period with liabilities. Consequently, households are better off by choosing \( b' = 0 \) so there is no demand for zero priced loans. The last property shows that individuals with a default record on student loans pay higher prices than individuals with no default record for any loan size, \( b' \) and amount of student loans they owe, \( d \).

3.5.2 The interplay between the two markets

Since the novel feature in this paper is the interaction between different types of markets and its effects on default decisions, we show how the default decision varies not only with the loan amount in the respective market, but also with the loan amount in the other market. We already established that the default probability on credit card loans increases in the amount of student loans. In this section we demonstrate that a borrower with high enough loans to repay will prefer defaulting on her student loans rather than on her credit card debt. Theorem 11 shows that for any level of credit card debt, a high enough college debt induces a borrower to default. Furthermore, if the amount owed to student loans is very high, the borrower will choose to default on student loans rather than on credit card debt. These two thresholds that trigger default on any type of loan and default on student loans decrease in the credit card debt, i.e. for higher credit card debt, the likelihood of defaulting in general, and defaulting on student loans, in particular is higher.

Assumption 4. We assume that there is \( b < 0 \) such that if \( y \in Y \) and \( b < \bar{b} \) then there is \( \bar{d} \leq d_{\text{max}} \) such that \( y + b - \bar{d} \leq 0 \). Note that if such a \( \bar{d} \) exists, then this property holds for all \( d \geq \bar{d} \).

Theorem 11. In any steady-state equilibrium for any credit card debt there is a threshold \( d_1(b) \) of student loans such that if \( d \geq d_1(b) \) then the agent defaults. Moreover there is a threshold \( d_2(b) \geq d_1(b) \) such that if \( d \geq d_2(b) \) then the agent defaults on student loans. These thresholds decrease in \( b \).

The intuition behind this result is that with high college debt levels, consumption is very small in the case the agent does not default at all. Consequently she finds it optimal to default. In the case where the student loan amount is very high, defaulting on student loans is optimal since the option of defaulting only on credit card debt delivers almost 0 consumption. In addition, if the borrower’s credit card debt is also very high then the borrower’s incentive to default increases. This result clearly illustrates the fact that when borrowers find themselves in financial hardship.
and have to default they will always choose to default on student loans. The financial arrangements in the two markets, and in particular the difference in bankruptcy rules and default consequences between the two types of credit certainly play an important role in shifting these default incentives for these types of borrowers.

To conclude, our theory produces several facts consistent with the reality (presented in Section 2): First, the incentive to default on student loans increases in college debt burden (debt-to-income ratio), i.e. default on student loans is more likely to occur for individuals with low levels of earnings and high levels of college debt. Second, the incentive to default on credit card debt increases in credit card debt, which is consistent with findings in Chatterjee et. al. (2007).

Our theory innovates by showing that a household with a high amount of student loans or with a record of default on student loans is more likely to default on credit card debt. This result is new in the literature and emphasizes the importance of accounting for other markets in which the individual participates in when studying default on credit card debt. Finally, we show that while a high college debt burden is necessary to induce default on student loans, this effect is amplified by high indebtedness in the credit card market.

4 Quantitative analysis

4.1 Mapping the model to the data

There are four sets of parameters that we calibrate: 1) standard parameters such as the discount factor, the coefficient of risk aversion; 2) parameters for the initial distribution of characteristics: college debt and income; 3) parameters specific to student loan markets such as default consequences, interest rates on student loans; and 4) parameters specific to credit card markets. Our approach includes a combination of setting some parameters to values that are standard in the literature, calibrating some parameters directly to data, and jointly estimating the parameters that we do not observe in the data by matching moments for several observable implications of the model.

Our model is representative for college educated individuals who are out of college and have student loans. The model period is one year and the coefficient of risk aversion chosen ($\sigma = 2$) are standard in the literature. We set the interest rate on student loans $r_g = 0.068$ as in the data. Table presents 1 presents the basic parameters of the model. We estimate the survival probability $\rho = 0.975$ to match average years of life to 40. The probabilities to keep default flags in the two markets are set to $p_f = 0.9$ and $p_h = 0.5$ to match average years of punishments in the credit card

\footnote{Since our agents are 26 years old, this matches a lifetime expectancy of 68 years old.}
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Targets</td>
<td>determined independently</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Coef of risk aversion</td>
<td>2</td>
<td>standard</td>
</tr>
<tr>
<td>( r_g )</td>
<td>Interest on student loans</td>
<td>0.068</td>
<td>Dept. of Education</td>
</tr>
<tr>
<td>( p_f )</td>
<td>Probability to keep CC default flag</td>
<td>0.9</td>
<td>Avg years of punishment=10</td>
</tr>
<tr>
<td>( p_h )</td>
<td>Probability to keep SL default flag</td>
<td>0.5</td>
<td>Avg years of punishment=2</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Survival probability</td>
<td>0.975</td>
<td></td>
</tr>
<tr>
<td>Targets</td>
<td>determined jointly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau_g )</td>
<td>Utility loss from SL default</td>
<td>0.1</td>
<td>Default rate on SL</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>Utility loss from CC default</td>
<td>3</td>
<td>Default rate on CC</td>
</tr>
<tr>
<td>( r_f )</td>
<td>Risk-free rate</td>
<td>0.04</td>
<td>Wealth-to-income ratio</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.96</td>
<td>Perc in debt</td>
</tr>
</tbody>
</table>

and in the student loan markets of ten and two years, respectively. The first choice is consistent with estimates in the literature (see Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007)) and the second choice is consistent with regulations from the Department of Education.

We use the joint distribution of debt and income for young households as delivered by the SCF 2004. The sample consists of households aged 20-26 years old with college education and college debt who are out of college. We assume a log normal distribution with parameters \((\mu_y, \sigma_y, \mu_d, \sigma_d, \rho_{yd}) = (-1.3689; 0.5674; -5.9053; 1.4833; -0.1063)\) on \([0, 1] \times [0, 0.234]\).

We pick the grid for assets such that to match the average credit card limits in the SCF 2004, which is $14,014 in 2004 dollars. We deliver a wage garnishment \(\gamma = 0.048\) to balance the government’s budget.\(^{27}\) Finally, we jointly estimate the following parameters: the utility loss from defaulting on student loans \((\tau_g)\), the utility loss from defaulting on credit card loans \((\tau_p)\), the discount factor \((\beta)\) and the risk-free rate \((r_f)\) to match the default rate for student loans of 4.6% (according to the Department of Education annual releases), the default rate for credit card debt of 0.9% for young adults (see Athreya, Tam, and Young (2009)), the wealth-to-income ratio and percentage with negative worth as delivered by our sample in the SCF 2004.

---

\(^{27}\)As in practice wage garnishments do not apply if income levels are below a minimum threshold below which the borrower experiences financial hardship.
4.2 Results: benchmark economy

4.2.1 Model versus data

The model does a good job at reproducing behavior regarding asset holdings: participation and amount as well as debt burdens in the two markets for borrowers in the SCF 2004 as evident from Table 2. The model predicts that 31% of individuals have negative assets and the interest rate that they pay on their loans is 5.1%. Also, a negligible fraction defaults on both types of loans. All these model predictions cannot necessarily be tested in the data. For instance, the interest rate in the model is lower compared to the credit card rate in the data. However the interest rate in the model represents the effective rate at which borrowers pay whereas in the data borrowers pay the high rate only in the case they roll over their debt.

<table>
<thead>
<tr>
<th>Table 2: Data versus model</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit card balance</td>
<td>3,319</td>
<td>2,568</td>
</tr>
<tr>
<td>Credit card debt-to-income ratio</td>
<td>0.1069</td>
<td>0.085</td>
</tr>
<tr>
<td>Per period college debt-to-income ratio</td>
<td>0.07</td>
<td>0.068</td>
</tr>
<tr>
<td>Default rate - both</td>
<td>-</td>
<td>0.11%</td>
</tr>
</tbody>
</table>

4.2.2 Default behavior

We quantify the probability of default in the two markets across individual characteristics (student loan amount, $d$, credit card debt, $b$, and income, $y$). Table 3 shows these findings across individuals with high levels of $d$, $b$, $y$ respectively (defined as the top 50 percentile) versus individuals with low levels of these variables (defined as the bottom 50 percentile).

Default rates for student loans are higher for individuals with high amounts owed to the student loan program and with low income levels. Overall, the default probability for student loans is higher for individuals with relatively low debt burdens in the student loan market, fact consistent with the data. In addition, the effects of the student loan debt on the default probability in the student loan market is larger than the effect of income. At the same time, individuals with high levels of credit card debt have slightly lower default rates on student loans relative to individuals with low levels of credit card debt.

Default rates on credit card debt are higher for individuals with low levels of income and with high levels of both types of loans. These results reveal the trade-off in default behavior captured in the model: individuals with low levels of credit card debt do not default on their credit cards but rather default on their student loans (if they must default) since the benefit of discharging
their credit card debt upon default is too small compared to the large cost of being excluded from borrowing. At the same time, the penalties associated with default in the student loan market are not contingent on their credit card debt. Individuals with high levels of credit card debt have higher rates of default overall, but since for them the benefit if discharging is relatively high, some of these individuals optimally choose to default on credit card debt and therefore fewer individuals default on student loans. Recall that in our model the fraction of individuals who default on both types of loans is negligible.

Note that for both types of loans, the likelihood of default is higher for individuals with low income and it is lower for individuals with high income. This finding is consistent with empirical evidence (see Musto and Souleles (2006)). However, research so far has generally generated the opposite result. The intuition behind this previous result in the literature is that agents with relatively low income levels stand to lose more from defaulting on credit cards relative to individuals with high income levels, for whom the penalties associated with default are less costly in relative terms. In our model, however, individuals also possess other types of loans, fact which pushes them into default. As we have seen, default rates on credit card increase in college debt levels. In addition, college debt is negatively correlated with income. These facts deliver the default probability to decrease in income in our model. This is a new finding in the literature and shows the importance of accounting for other types of loans when analyzing default behavior, feature that is absent in previous models of consumer default.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>“Low”</th>
<th>“High”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default SL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>1.5%</td>
<td>12.5%</td>
</tr>
<tr>
<td>(b)</td>
<td>6.4%</td>
<td>5.2%</td>
</tr>
<tr>
<td>(y)</td>
<td>5.28%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Default CC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>0.12%</td>
<td>0.7%</td>
</tr>
<tr>
<td>(b)</td>
<td>0</td>
<td>1.62%</td>
</tr>
<tr>
<td>(y)</td>
<td>0.33%</td>
<td>0.14%</td>
</tr>
</tbody>
</table>

### 4.2.3 Loan pricing

Consistent with our results on the individual probability of default for credit cards, the model delivers the pricing scheme of credit card loans based on individual default risk as proxied by the size of the loan, the amount owed in the student loan market and the default status in the student loan market. Recall that our theoretical results show that the interest rate on credit card debt increases in both amounts of loans and also it is higher for individuals with a default flag on student
loans. In our quantitative analysis we find that:

First, agents with high levels of credit card debt (top 50 percentile) have a credit card rate of 5.8% and agents with low credit card debt (bottom 50 percentile) have the risk-free rate 4%. Individuals with low levels of credit card debt have 0 probability of default, as shown in Table 3 and therefore receive the risk-free rate, whereas individuals with high levels of credit card debt pay a premium of 1.8% to account for the likelihood of defaulting (of 1.65%) on their credit card debt.

Second, agents with high levels of student loans receive a credit card rate of 7.3% on average and agents with low levels of student loans receive a credit card rate of 4.4%. The wedge in the interest rates accounts for the gap in the likelihood of default between this two groups (0.7% versus 0.12%) presented in Table 3.

Finally, defaulters on student loans ($h = 1$) have a credit card rate of 8.32% and nondefaulters on student loans ($h = 0$) have a credit card rate of 4.2%. This result shows that the default status on student loans is an important component of the pricing of credit card loans - the interest rate almost doubles for individuals who have defaulted on their student loans. This is a consequence of the result that individuals with a default flag on student loans are more likely to default on their credit card debt.

These three finding represent the quantitative counterpart of our theoretical results in Theorem ??.

In addition, our quantitative analysis predicts that agents with low income receive higher rates on average than agents with high income. This is a direct implication of the differences in default rates across income groups presented in Table 3.

4.2.4 The interplay between the two markets

We turn now to the interaction between the two markets and its effect on default behavior, the main theoretical result of the paper. Recall from Theorem 10 that in any steady-state equilibrium we can find a threshold of student loans which depends on the amount of credit card debt such that individuals will default for any level of student loans higher than this threshold. In addition, there exists an even higher threshold of the student loan amount above which individuals default on student loans. Table 4 shows the quantitative counterpart of this result on the interplay between the two markets. We divide individuals in two groups based on the amount owed to the student loan program, $d$ (low and high defined as before) and in three groups based on the $b$ variable: one group with positive assets and two groups (low and high defined as before) with negative assets.
Table 4: Default rates across debt levels in the two markets

<table>
<thead>
<tr>
<th></th>
<th>&quot;b &lt; 0 High&quot;</th>
<th>&quot;b &lt; 0 Low&quot;</th>
<th>b ≥ 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>d &quot;Low&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default SL</td>
<td>5.72%</td>
<td>5.64%</td>
<td>4.38%</td>
</tr>
<tr>
<td>Default CC</td>
<td>2.78%</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>d &quot;High&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default SL</td>
<td>15.9%</td>
<td>19.2%</td>
<td>9.8%</td>
</tr>
<tr>
<td>Default CC</td>
<td>4.7%</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Results show that conditional on the asset position, b the probability of defaulting on student loans increases in the amount of student loans owed. Similarly, conditional on the amount of student loans, the default probability on credit card debt increases in the amount of credit card debt. Overall, the default rate is larger (default on both types of loans) for high levels of credit card debt versus low levels of credit card debt. This result implies that the threshold \( d_1(b) \) above which default occurs in the model decreases in credit card debt. Furthermore, as discussed above, for low levels of credit card debt levels no one chooses to default. This result implies that the thresholds, \( d_1(b) \) and \( d_2(b) \) coincide for low levels of b. Finally, note that the gap in default rates on student loans across individuals with different student loan levels conditional on credit card debt is larger for the group with low b relative to the group with high b. This result implies that the rate at which threshold \( d_2(b) \) declines in b is larger for individuals with low levels of b than for individuals with high levels of b. This difference in the rates is accounted for by the fraction of people who default on their credit card debt, but not on their student loan debt. This quantitative analysis complements our theoretical result on the interaction between the two types of credit and its implication for default behavior.

4.3 Experiments

Recall that the national default rate for student loans increases from 4.5% in 2004 to 7% in 2007. At the same time, the college debt burden increased on average by 48% during this period. We conjecture that college debt burdens alone cannot explain the increase in student loan defaults. This claim is based on the following two observations: 1) Default rates for student loans decline from 1990 to 2004 and then increased after 2005 and 2) During this period student loan amounts increase steadily fueled by increases in college costs. In this section we study how much of the increase in default rates is due to an increase in college debt burdens and how much of this increase is due to worse terms in the credit card market. To answer these questions, we run two experiments: 1) we first quantify the effects of the changes in college debt burdens on default rates for student
loans and 2) we quantify the effects of the change in credit card limits on default rates.

4.3.1 Effects of college debt burdens on default

In this experiment we change the joint distribution of college debt and income from the baseline economy to the distribution of college debt and income from SCF 2007. This change implies that college debt burden increases on average from 0.07 to 0.081. Everything else is kept the same as in the baseline economy, in particular we keep the credit terms (limits that individuals receive on their credit card accounts) as in 2004. This experiment delivers an increase in the default rate on student loans from 4.6% in the baseline economy to 5.8%. The fact that changes in college debt burdens do not contribute much to the increase in default rates confirms our conjecture that the increase in college debt burdens alone cannot explain the increase in default rates for student loans.

We next disentangle the effects of the change in the student loan amount borrowed and of the change in income during this period. We change only the distribution of college debt and keep the income distribution the same as in the baseline economy, i.e. we only change the first two moments in the joint distribution. This counterfactual experiment delivers that the default rate on student loans increases to 6.4%. We therefore find that most of the increase in student loan default is accounted for by the increase in the student loan amount. In fact, the change in income from 2004-2007 works toward lowering the incentive to default. Note that this last experiment still does not deliver the full increase in default rates on student loans.

We also find that the change in the joint distribution of college debt and income to match the 2007 data delivers that the default rate on credit cards increases from 0.95% in the baseline economy to 1.4%. Also, the average interest rate on credit card loans in the economy increases from 5.1% to 5.6%. This result emphasizes the importance of college debt burden in default behavior not only for the student loan market but also for the other credit markets that individuals participate in. Taking on more student loan debt will increase the likelihood of default for all types of loans in the household’s portfolio. We next turn to analyzing the effect of the credit card market for default on student loans.

4.3.2 Effects of credit card terms on default on student loans

We run an experiment to study the effect of worse credit terms on credit card accounts on the default behavior in the student loan market. As mentioned in Section 2, credit card issuers raised interest rates and fees in the recent years. They cite the recent economic turmoil to explain these changes and argue that they are responding to record defaults and new regulations (see the Credit
Card Act of 2009) by raising fees and interest rates to help the banks absorb losses and maintain profit margins. They have also become stingier with credit. We document using the SCF 2004 and SCF 2007 that young borrowers with college loans receive higher rates (from 9.9% to 13%) and lower credit limits (by 22%) on average on their credit card accounts in 2007 relative to 2004 (fact 6 in Section 2). These facts motivate the experiment in this section.

To assess the importance of the effect of worse credit card terms on default on student loans, we expand our previous experiment and take into account the changes in the credit card market. To do so, in addition to using the joint distribution of college debt and income from SCF 2007 as in the previous experiment, we now re-parametrize the economy to match credit card terms that young U.S. households receive in 2007. This procedure implies accounting for and estimating a transaction cost of the financial intermediary such that to match credit card limits as delivered by SCF 2007. In this case the solution to the financial intermediary’s problem implies

\[
q_{dd,ht,b_t+1} \leq \frac{\rho}{(1+r+\phi)} (1 - p_{dd,ht,b_t+1}) \quad \text{if } b_{t+1} < 0 \quad \text{and} \quad q_{dd,ht,b_t+1} \geq \frac{\rho}{(1+r)} \quad \text{if } b_{t+1} \geq 0
\]

where \( r \) is the risk-free savings rate and \( \phi \) represents a transaction cost for lending, so that \( r + \phi \) is the risk-free borrowing rate.

We find that this experiment delivers an increase in the default rate on student loans from 4.6% to 7.1%. We conclude that worse terms on credit cards amplify default on student loans. Financially constrained borrowers who need to repay on their student loans use the credit card market in order to smooth out consumption and repay their college debt. When the credit market tightens this channel is restricted and capability of repayment of student loan borrowers is impeded. Given this set of results, we conjecture that a student loan repayment scheme which is contingent on terms in the credit card market may be welfare improving to the current fixed payment scheme for student loans. Next we use the model to verify this claim.

### 4.3.3 Student loans policy contingent on credit card terms

We conduct the following policy on student loans: we assume that the government takes into account the market tightness when setting default penalties on student loans. In our model this implies that the transaction cost to the financial intermediary enters the government’s problem: when the transaction cost is low, the government sets the wage garnishment to balance the budget constraint (as in the baseline economy) but when the transaction cost is high (as in the experiment where credit terms worsen), the government sets the wage garnishment for defaulters to \( \gamma = 0 \). In this case since there is no wage garnishment after default occurs, the government collects taxes to balance the budget. Taxes are equally distributed in the economy such that the cost of default for student loans is borne by all agents with student loans, not only by defaulters. We next analyze the welfare implications and redistributional effects of such a policy.
5 Conclusion

We developed a quantitative theory of unsecured credit and default behavior of young U.S. households based on the interplay between two forms of unsecured credit and we analyzed the implications of this interaction for default incentives. Our theory is motivated by facts related to borrowing and repayment behavior of young U.S. households with college and credit card debt, and in particular by recent (alarming) trends in the default rates for student loans. Specifically, different financial market arrangements and in particular, different bankruptcy rules in these two markets alter incentives to default conducing to, in some circumstances, amplification of the default behavior (as we show in this paper).

We built a general equilibrium economy that mimics features of student and credit card loans. In particular, our model accounts for 1) the bankruptcy arrangement differences between the two types of loans and 2) differences in pricing default risk in the two markets. Our theory explains borrowing and default behavior of young U.S. households: the incentive to default on student loans increases in college debt burdens and the incentive to default on credit card debt increases in credit card debt.

Our model predicts that the likelihood to default on credit card debt increases in the amount of student loans. Also individuals with a default flag in the student loan market have higher default probabilities of default in the credit card market than individuals who have not defaulted on their student loans. In the quantitative part of our paper we show that individuals with high levels of income are less likely to default in the credit card market relative to individuals with low levels of income. This result is consistent with empirical evidence, however it was difficult to be established in previous models of unsecured credit. The fact that individuals in our model also have other types of loans produces this result. These last three results are new in the literature and reveal the importance of accounting for interactions between different financial markets in which individuals participate when one analyses default behavior.

Our theory reveals that borrowers prefer defaulting on their student loans rather than on credit cards. Our main theoretical result shows that a borrower with high enough college debt burden defaults for sure and will choose to default in the student loan market. Moreover, our paper innovates in demonstrating how differences in market arrangements can lead to amplification of default in the student loan market. In the quantitative part of the paper we show that while a high college debt burden is necessary to increase default on student loans (with most of the effect coming from the amount of student loans owed), our results imply that this effect is amplified by worse terms for credit card accounts. This set of results are particularly important in the current market conditions when due to a significant increase in college costs, students borrow more than
ever and, at the same time, they face extremely tight conditions on their credit card accounts as a result of the adjustments creditors have made following the financial crisis.

References


A Appendix

Data details

Figure 1 presents trends in student loan and credit card debt in the U.S. and Figure 2 presents trends in the two year cohort default rate for student loans according to press releases from the Department of Education.

Figure 1:

![Figure 1](image)

Source: Federal Reserve’s G19 Consumer Credit
Facts are based on the SCF data sets for 2004 and 2007. Table 5 provides details on facts related to college debt burdens documented in Section 2 (facts 2-4) and Table 6 provides details for the facts related to credit cards documented in Section 2 (facts 5-8). We also discuss details on the trends in the past 20 years which are documented at point 11 in Section 2.

Table 5: College debt burdens for borrowers with student loans aged 20-26 years old

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amt. college debt borrowed</td>
<td>16467.74 (16968.41)</td>
<td>22204.85 (19283.8)</td>
</tr>
<tr>
<td>Amt. college debt outstanding</td>
<td>15607.82 (19857.65)</td>
<td>19920.9 (20712.3)</td>
</tr>
<tr>
<td>Income</td>
<td>31029.32 (19128.63)</td>
<td>39944.24 (26287.2)</td>
</tr>
<tr>
<td>Debt-to-income ratio</td>
<td>0.9677 (2.2591)</td>
<td>1.151 (3.404)</td>
</tr>
</tbody>
</table>

Note: the first number represents the mean and the number in parenthesis the standard deviation.

Table 6: Credit cards usage and terms for borrowers with student loans aged 20-26 years old

<table>
<thead>
<tr>
<th>Variable</th>
<th>2004</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage with credit card</td>
<td>69.9%</td>
<td>72.7%</td>
</tr>
<tr>
<td>Percentage with credit card balance</td>
<td>78.5%</td>
<td>75%</td>
</tr>
<tr>
<td>Credit card rate</td>
<td>10.6 (5.547)</td>
<td>12.56 (4.86)</td>
</tr>
<tr>
<td>Credit card limit</td>
<td>14014.36 (22499.5)</td>
<td>10949 (9546.97)</td>
</tr>
<tr>
<td>Credit card balance</td>
<td>3318.76 (4837.8)</td>
<td>4155.58 (6258.1)</td>
</tr>
<tr>
<td>Credit card utilization ratio</td>
<td>0.479 (0.3296)</td>
<td>0.735 (0.664)</td>
</tr>
<tr>
<td>Credit card debt burden</td>
<td>0.1069 (0.1543)</td>
<td>0.1269 (0.1758)</td>
</tr>
<tr>
<td>Perc. hardly repay credit card</td>
<td>40.3%</td>
<td>43.48%</td>
</tr>
</tbody>
</table>

Note: the first number represents the mean and the number in parenthesis the standard deviation.
Proofs of theorems

A.1 Proofs of Theorems 1 and 2

Let \( c_{\min} = y_{\min}(1 - \gamma) \) and \( c_{\max} = y_{\max} + b_{\max} - b_{\min} \). Then, if \( c \) is the consumption in any of the cases in the definition of \( T \), we have that \( U(c_{\min}) \leq U(c) \leq U(c_{\max}) \) and that \( c_{\min} \) is a feasible consumption. Recall that \( S = B \times F \times H \) is a finite set and let \( N_S \) be the cardinality of \( S \).

**Definition A1.** Define \( V \) to be the set of continuous functions \( v : D \times Y \times Q \to \mathbb{R}^{N_S} \) such that

1. For all \( (b, f, h) \in S \) and \( (d, y, q) \in D \times Y \times Q \)
   \[
   \frac{U(c_{\min})}{1 - \beta \rho} \leq v(d, y, q)(b, f, h) \leq \frac{U(c_{\max})}{1 - \beta \rho}.
   \] (3)

2. \( v \) is increasing in \( b \) and \( y \).

3. \( v \) is decreasing in \( f \): \( v(d, y, q)(b, 0, h) \geq v(d, y, q)(b, 1, h) \) for all \( d, y, q, b, h \).

Let \( (C(D \times Y \times Q; \mathbb{R}^{N_S}), \| \cdot \|) \) denote the space of continuous functions \( v : D \times Y \times Q \to \mathbb{R}^{N_S} \) endowed with the supremum norm

\[
\|v\| = \max_{(d,y,q)} \|v(d,y,q)\|,
\]

where the norm of a vector \( w = (w(b, f, h)) \in \mathbb{R}^{N_S} \) is

\[
\|w\| = \max_{(b,f,h) \in S} |w(b, f, h)|.
\]

Then \( V \) is a subset of \( C(D \times Y \times Q; \mathbb{R}^{N_S}) \). Define also \( C(D \times Y \times Q \times S) \) to be the set of continuous real valued functions \( v : D \times Y \times Q \times S \to \mathbb{R} \) with the norm

\[
\|v\| = \max_{(d,y,q,b,f,h)} |v(d, y, q, b, f, h)|.
\]

In the first lemma we show that the two spaces of functions that we defined above are interchangeable.

**Lemma A1.** The map \( V : C(D \times Y \times Q; \mathbb{R}^{N_S}) \to C(D \times Y \times Q \times S) \) defined by

\[
V(v)(d, y, q, b, f, h) = v(d, y, q)(b, f, h)
\]
is a surjective isomorphism.

Proof. We prove first that if \( v \in C(D \times Y \times Q; \mathbb{R}^{Ns}) \) then \( V(v) \) is continuous. Let \( (d_n, y_n, q_n, b_n, f_n, h_n)_{n \in \mathbb{N}} \) be a sequence that converges to \( (d, y, q, b, f, h) \) and let \( \varepsilon > 0 \). Since \( S \) is a finite set it follows that there is some \( N_1 \geq 1 \) such that \( b_n = b, f_n = f, \) and \( h_n = h \) for all \( n \geq N_1 \). Since \( v \) is continuous then there is \( N_2 \geq 1 \) such that if \( n \geq N_2 \) then
\[
\|v(d_n, y_n, q_n) - v(d, y, q)\| < \varepsilon.
\]
Thus \( |v(d_n, y_n, q_n)(b, f, h) - v(d, y, q)(b, f, h)| < \varepsilon \) for all \( n \geq N := \max\{N_1, N_2\} \). Therefore
\[
|V(v)(d_n, y_n, q_n, b_n, f_n, h_n) - V(v)(d, y, q, b, f, h)| < \varepsilon \text{ for all } n \geq N
\]
and \( V(v) \) is continuous. It is clear from the definition of the norms that \( \|V(v)\| = \|v\| \) for all \( v \in C(D \times Y \times Q; \mathbb{R}^{Ns}) \). Thus \( V \) is an isomorphism. Finally, if \( w \in C(D \times Y \times Q \times S) \) then one can define \( v \in C(D \times Y \times Q; \mathbb{R}^{Ns}) \) by
\[
v(d, y, q)(b, f, h) = w(d, y, q, b, f, h).
\]
Then \( T(v) = w \) and \( T \) is surjective.

In the following we are going to tacitly view \( V \) either as a subset of \( C(D \times Y \times Q; \mathbb{R}^{Ns}) \) or as a subset of \( C(D \times Y \times Q \times S) \) via \( V(V) \). For example, we are going to prove in the following lemma that \( (V, \| \cdot \|) \) is a complete metric space by showing that it’s image under \( V \) is a closed subspace of \( C(D \times Y \times Q \times S) \), which is a complete metric space.

**Lemma A2.** \( (V, \| \cdot \|) \) is a complete metric space.

**Proof.** We are going to show that \( V \) is a closed subspace of \( C(D \times Y \times Q \times S) \). Notice first that \( V \) is nonempty because any constant function that satisfies (3) is in \( V \). Let now \( \{v_n\}_{n \in \mathbb{N}} \) be a sequence of functions in \( V \) that converge to a function \( v \). Then, since \( C(D \times Y \times Q \times S) \) is complete, it follows that \( v \) is continuous. Since inequalities are preserved by taking limits it follows immediately that \( v \) satisfies the conditions of Definition A1, because each \( v_n \) satisfies those conditions. Therefore \( v \in V \) and, thus, \( (V, \| \cdot \|) \) is a closed subspace of \( C(D \times Y \times Q \times S) \) and, hence, a complete metric space.

**Lemma A3.** The operator \( T \) defined on \( C(D \times Y \times Q; \mathbb{R}^{Ns}) \) maps \( V \) into \( V \) and its restriction to \( V \) is a contraction with factor \( \beta \rho \).
Proof. We will show first that if \( v \in \mathcal{V} \) then \( Tv \in \mathcal{V} \). Since \( v \in \mathcal{V} \) we have that
\[
\frac{U(c_{\text{min}})}{1 - \beta \gamma} \leq v(d, y', q)(b', f', h') \leq \frac{U(c_{\text{max}})}{1 - \beta \gamma}
\]
for all \( (d, y', q) \in D \times Y \times Q \) and \( (b', f', h') \in \mathcal{S} \). Integrating with respect to \( y' \) we obtain that
\[
\frac{U(c_{\text{min}})}{1 - \beta \gamma} \leq \int v_{(\cdot, f', h')}(d, y'; q) \Phi(dy') \leq \frac{U(c_{\text{max}})}{1 - \beta \gamma}.
\]
because \( \int \Phi(dy') = 1 \). Since \( U(c_{\text{min}}) \leq U(c) \leq U(c_{\text{max}}) \) for all \( c \) appearing in the definition of \( T \), it follows that
\[
U(c) + \beta \rho \int v_{(\cdot, f', h')}(d, y'; q) \Phi(dy') \leq U(c_{\text{max}}) + \frac{\beta \rho U(c_{\text{max}})}{1 - \beta \rho} = \frac{U(c_{\text{max}})}{1 - \beta \rho},
\]
and, similarly
\[
\frac{U(c_{\text{min}})}{1 - \beta \rho} \leq U(c) + \beta \rho \int v_{(\cdot, f', h')}(d, y'; q) \Phi(dy').
\]
Thus the condition (3) of Definition A1 is satisfied. To prove that \( Tv \) is increasing in \( b \) and \( y \) and decreasing in \( f \), note that the sets \( B_{b,f,h}(d, y'; q) \) are increasing with respect to \( b \) and \( y \), and decreasing with respect to \( f \). These facts coupled with the same properties for \( v \) (which are preserved by the integration with respect to \( y' \)) imply that \( Tv \) satisfies the remaining conditions from Definition A1, with the exception of the continuity, which we prove next.

Since \( B, F, H \) and \( D \) are finite spaces, it suffices to show that \( Tv \) is continuous with respect to \( y \) and \( q \). Since \( Q \) is compact and \( v \) is uniformly continuous with respect to \( q \), it follows by a simple \( \varepsilon - \delta \) argument that the integral is continuous with respect to \( q \). Since \( U(\cdot) \) is continuous with respect to \( c \) and \( c \) is continuous with respect to \( d \) and \( y \), it follows that \( T(v) \) is continuous.

Finally we prove that \( T \) is a contraction with factor \( \beta \rho \) by showing that \( T \) satisfies Blackwell’s conditions. For simplicity, we are going to view \( \mathcal{V} \) one more time as a subset of \( C(D \times Y \times Q \times \mathcal{S}) \). Let \( v, w \in \mathcal{V} \) such that \( v(d, y, q, b, f, h) \leq w(d, y, q, b, f, h) \) for all \( (d, y, q, b, f, h) \in D \times Y \times Q \times \mathcal{S} \). Then
\[
\beta \rho \int v_{(\cdot, f', h')}(d, y'; q) \Phi(dy') \leq \beta \rho \int w_{(\cdot, f', h')}(d, y'; q) \Phi(dy')
\]
for all \( (d, y, q, b', f', h') \). This implies that \( Tv \leq Tw \). Next, if \( v \in \mathcal{V} \) and \( a \) is a constant it follows that
\[
\beta \rho \int (v_{(\cdot, f', h')}(d, y'; q) + a) \Phi(dy') = \beta \rho \int v_{(\cdot, f', h')}(d, y'; q) \Phi(dy') + \beta \rho a.
\]
Thus \( T(v + a) = Tv + \beta \rho a \). Therefore \( T \) is a contraction with factor \( \beta \rho \).
Theorem 1. There exists a unique $v^* \in V$ such that $v^* = Tv^*$ and

1. $v^*$ is increasing in $y$ and $b$.
2. Default decreases $v^*$.
3. The optimal policy correspondence implied by $Tv^*$ is compact-valued, upper hemi-continuous.
4. Default is strictly preferable to zero consumption and optimal consumption is always positive.

Proof. The first two parts follows from Definition A1 and Lemmas A2 and A3. The last part follows from our assumptions on $U$. So we need only to prove the third part of the theorem. The optimal policy correspondence is

$$\Xi_{(d, y, q, b, f, h)} = \{(c, b', h', f', \lambda_d, \lambda_b) \in B_{b, f, h}(d, y; q) \text{ that attain } v^*_{b, f, h}(d, y, q)\}.$$

For simplicity of our notation we will write $x = (d, y, q, b, f, h)$. For a fixed $x$ we need to show that if $\Xi_x$ is nonempty then it is compact. First notice that

$$\Xi_x \subset [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$$

and, thus, it is a bounded set. We need to prove that it is closed. Let $\{(c_n, b'_n, h'_n, f'_n, \lambda^n_d, \lambda^n_b)\}_{n \in \mathbb{N}}$ be a sequence in $\Xi_x$ that converges to some $$(c, b', h', f', \lambda_d, \lambda_b) \in [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}.$$ 

Since $B, F$, and $\{0, 1\}$ are finite sets it follows that there is some $N \geq 1$ such that $b'_n = b'$, $h'_n = h'$, $f'_n = f'$, $\lambda^n_d = \lambda_d$, and $\lambda^n_b = \lambda_b$ for all $n \geq N$. Define

$$\phi(c) = U(c) + \beta \rho \int v_{(y', f', h')}(d, y; q) \Phi(dy').$$

Then $\phi$ is continuous and, since $\phi(c_n) = v^*_{b, f, h}(d, y; q)$ for all $n \geq 1$, we have that

$$\phi(c) = \lim_{n \to \infty} \phi(c_n) = v^*_{b, f, h}(d, y; q).$$

Thus $(c, b', h', f', \lambda_d, \lambda_b) \in \Xi_x$ and $\Xi_x$ is a closed and, hence, compact set.

To prove that $\Xi$ is upper hemi-continuous consider $x = (d, y, q, b, f, h) \in D \times Y \times Q \times S$ and let $\{x_n\} \in D \times Y \times Q \times S$, $x_n = (d_n, y_n, q_n, b_n, f_n, h_n)$ be a sequence that converges to $x$. Since $D, B, F$, and $H$ are finite sets it follows that there is $N \geq 1$ such that if $n \geq N$ then $d_n = d$, $b_n = b$, $f_n = f$, $h_n = h$. Then for all $n \geq N$ we have $\phi(c_n) = \phi(c)$, and $\Xi_{x_n}$ converges to $\Xi_x$ in the Hausdorff metric $d_H$.

Therefore $\Xi_x$ is upper hemi-continuous.
\( b_n = b, \ f_n = f, \) and \( h_n = h. \) Let \( y_n = (c_n, b'_n, h'_n, f'_n, \lambda'_d, \lambda'_b) \in \Xi \) for all \( n \geq N. \) We need to find a convergent subsequence of \( \{y_n\} \) whose limit point is in \( \Xi. \) Since \( B, \ H, \ F, \) and \( \{0, 1\} \) are finite sets we can find a subsequence \( \{y_{n_k}\} \) such that \( b'_n = b', \ h'_n = h', \ f'_n = f', \lambda'_d = \lambda_d, \lambda'_b = \lambda_b \) for some \( b' \in B, \ h' \in H, \ f' \in F, \lambda_d, \lambda_b \in \{0, 1\}. \) Since \( \{c_{n_k}\} \subset [c_{\min}, c_{\max}] \) which is a compact interval, there must be a convergent subsequence, which we still label \( c_{n_k} \) for simplicity. Let \( c = \lim_{k \to \infty} c_{n_k} \) and let \( y_{n_k} = (c_{n_k}, b', h', f', \lambda_d, \lambda_b) \) for all \( k. \) Then \( \{y_{n_k}\} \) is a subsequence of \( \{y_n\} \) such that

\[
\lim_{k \to \infty} y_{n_k} = y := (c, b', h', f', \lambda_d, \lambda_b).
\]

Moreover, since
\[
\phi(c_k) = v^*_b f h(d_{n_k}, y_{n_k}; q_{n_k}) \quad \text{for all} \quad k
\]
and since \( \phi \) and \( v^* \) are continuous functions it follows that
\[
\phi(c) = \lim_{k \to \infty} \phi(c_k) = \lim_{k \to \infty} v^*_b f h(d_{n_k}, y_{n_k}; q_{n_k}) = v^*_b f h(d, y; q).
\]

Thus \( y \in \Xi \) and \( \Xi \) is an upper hemi-continuous correspondence. \( \square \)

**Theorem 2.** For any \( q \in Q \) and any measurable selection from the optimal policy correspondence there exists a unique \( \mu_q \in \mathcal{M}(X) \) such that \( \Gamma_q \mu_q = \mu_q. \)

**Proof.** The Measurable Selection Theorem implies that there exists an optimal policy rule that is measurable in \( X \times \mathcal{B}(X) \) and, thus, \( T_q^* \) is well defined. We show first that \( T_q^* \) satisfies Doeblin’s condition. It suffices to prove that \( T_q^* \) satisfies Doeblin’s condition (see Exercise 11.4g of Stockey, Lucas, Prescott (1989)). If we let \( \phi(Z) = TN_q^*(y, d, b, f, h, Z) \) for any \( (y, d, b, f, h) \in X \) it follows that if \( \varepsilon < 1/2 \) and \( \phi(Z) < \varepsilon \) then \( 1 - \varepsilon > 1/2 \) and

\[
TN_q^*(y, d, b, f, h, Z) < \varepsilon < \frac{1}{2} < 1 - \varepsilon
\]

for all \( (y, d, b, f, h) \in X. \) Thus Doeblin’s condition is satisfied.

Next, notice that if \( \phi(Z) > 0 \) then \( TN_q^*(y, d, b, f, h, Z) > 0 \) and, thus,

\[
T_q^*(y, d, b, f, h, Z) = TS_q^*(y, d, b, f, h, Z) + TN_q^*(y, d, b, f, h, Z) > 0.
\]

Then Theorem 11.10 of Stockey, Lucas, Prescott (1989) implies the conclusion of the theorem. \( \square \)
A.2 Proofs of Theorems 3-8

Let \((b, f, h) \in \mathcal{S}\) and \(q \in Q\) be fixed. Before proving the theorem we will introduce some notation which will ease the writing of our proofs. For \(y \in Y, d \in D\) we define the following maps:

\[
\psi_{\text{node}}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 0) := U(c) + \beta \rho \int \psi_{b',f',h'}(d, y'; q) \Phi(dy')
\]

for all \((c, b', f', h', 0, 0) \in B_{b,f,h}(d, y; q)\);

\[
\psi_{sL}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 0) = U(c) + \beta \rho \int \psi_{b',f',h'}(d, y'; q) \Phi(dy')
\]

for all \((c, b', f', h', 1, 0) \in B_{b,f,h}(d, y; q)\);

\[
\psi_{cc}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 1) = U(c) + \beta \rho \int \psi_{b',f',h'}(d, y'; q) \Phi(dy')
\]

for all \((c, b', f', h', 0, 1) \in B_{b,f,h}(d, y; q)\); and

\[
\psi_{\text{both}}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 1) = U(c) + \beta \rho \int \psi_{0,1,1}(d, y'; q) \Phi(dy')
\]

for all \((c, b', f', h', 0, 1) \in B_{b,f,h}(d, y; q)\). Note that these functions are continuous in \(y\) and \(d\). Also, these functions depend on \(b, f\), and \(q\).

**Theorem 3.** Let \(q \in Q, f \in F, b \in B(f)\). If \(h = 1\) and the set \(D_{b,f,1}^{SL}(q)\) is nonempty, then \(D_{b,f,1}^{SL}(q)\) is closed and convex. In particular the sets \(D_{b,f,1}^{SL}(d; q)\) are closed intervals for all \(d\). If \(h = 0\) and the set \(D_{b,f,0}^{SL}(d; q)\) is nonempty, then \(D_{b,f,0}^{SL}(d; q)\) is a closed interval for all \(d\).

**Proof.** If \(h = 1\) then \(D_{b,f,1}^{SL}(q)\) is the combinations of earnings \(y\) and student loan amount \(d\) for which \(B_{b,f,1}(d, y; q) = \emptyset\). Then they satisfy the inequality \(y(1 - \gamma) + b(1 - \lambda_b) - d - q_{y,d,h} b' \leq 0\) for all \(\lambda_b \in \{0, 1\}\) and \(b' \in B\). Thus \(D_{b,f,1}^{SL}(q)\) is closed. Moreover, if \((y_1, d_1)\) and \((y_2, d_2)\) are elements in \(D_{b,f,1}^{SL}(q)\) then if \((y, d) = t(y_1, d_1) + (1 - t)(y_2, d_2)\) with \(t \in (0, 1)\) it follows easily that

\[
y(1 - \gamma) + b(1 - \lambda_b) - d - q_{y,d,h} b' \leq 0
\]

and, thus, \((y, d) \in D_{b,f,1}^{SL}(q)\). So \(D_{b,f,1}^{SL}(q)\) is convex.

Assume now that \(h = 0\) and let \(d \in D\) be fixed. Let \(y_1\) and \(y_2\) with \(y_1 < y_2\) be in \(D_{b,f,0}^{SL}(d; q)\).
Therefore
\[
\psi_{sl}(y_i, d)(c_i^*, b_i^*, f_i^*, h_i^*, 1, 0) \geq \max \left\{ \psi_{node}(y_i, d)(c, b', f', h', 0, 0), \\
\psi_{ce}(y_i, d)(c, b', h', 0, 1), \\
\psi_{both}(y, d)(c, b', h', 1, 1) \right\}
\]
for all \((c, b', f', h', 0, 0), (c, b', f', h', 0, 1), (c, b', f', h', 1, 1) \in B_{b,f,0}(d, y_i; q), i = 1, 2\). Let \(y \in (y_1, y_2)\) and assume, by contradiction, that \(y \notin D_{b,f,0}(d; q)\). Assume, without loss of generality, that the agent chooses not to default on either market, i.e.
\[
\psi_{sl}(y, d)(c, b', f', h', 1, 0) < \psi_{node}(y, d)(c^*, b^*, f^*, h^*, 0, 0),
\]
for all \((c, b', f', h', 1, 0) \in B_{b,f,0}(d, y; q), \) where \((c^*, b^*, f^*, h^*, 0, 0) \in B_{b,f,0}(d, y; q)\) is the optimal choice for the maximization problem. Let \(\overline{c}_1 = c^* - (y - y_1)\). If \(\overline{c}_1 \leq 0\) then \(\overline{c}_1 < y_1 + b\) and thus
\[
c^* = \overline{c}_1 + (y - y_1) < y_1 + b + (y - y_1) = y + b.
\]
If \(\overline{c}_1 > 0\) we have that \((\overline{c}_1, b^*, f^*, h^*, 0, 0) \in B_{b,f,0}(d, y_1; q)\) and, thus,
\[
\psi_{sl}(y_1, d)(c_i^*, b_i^*, f_i^*, h_i^*, 1, 0) \geq \psi_{node}(y_1, d)(\overline{c}, b^*, f^*, h^*, 0, 0).
\]
Therefore
\[
U(y_1 + b) + \beta \rho \int v_{y_i, f_i, h_i, 1}(d, y'; q) \Phi(dy') \geq U(\overline{c}_1) + \beta \rho \int v_{y^*, f^*, h^*, 0}(d, y'; q) \Phi(dy'),
\]
Subtracting (7) from (5) we have that
\[
U(y + b) - U(y_1 + b) < U(c^*) - U(\overline{c}_1).
\]
Since \((y + b) - (y_1 + b) = y - y_1 = c^* - \overline{c}_1\) and \(U\) is strictly concave it follows that \(c^* < y + b\). Consider now \(\overline{c}_2 = c^* + (y_2 - y)\). Then \((\overline{c}_2, b^*, f^*, h^*, 0, 0) \in B_{b,f,0}(d, y_2; q)\) and thus
\[
U(y_2 + b) + \beta \rho \int v_{y_2, f_2, h_2, 1}(d, y'; q) \Phi(dy') \geq U(\overline{c}_2) + \beta \rho \int v_{y^*, f^*, h^*, 0}(d, y'; q) \Phi(dy').
\]
Using inequalities (5), and (8) we obtain that

\[ U(y_2 + b) - U(y + b) > U(\tau_2) - U(c^*). \]

Thus \( c^* > y + b \), and we obtain a contradiction with \( c^* < y + b \). Therefore \( y \in D_{b,f,0}^{SL}(d;q) \) and, thus, \( D_{b,f,0}^{SL}(d;q) \) is an interval. It is also a closed set because the maps \( \psi_{sl}, \psi_{both}, \psi_{cc}, \) and \( \psi_{node,f} \) are continuous with respect to \( y \). Thus, \( D_{b,f,0}^{SL}(d;q) \) is a closed interval. \( \square \)

**Theorem 4.** Let \( q \in Q, (b, f, 0) \in S \). If \( D_{b,f,0}^{CC}(d;q) \) is nonempty then it is a closed interval for all \( d \).

*Proof.* If \( b \geq 0 \) then \( D_{b,f,0}^{CC}(d;q) \) is empty. If \( b < 0 \) the proof of the theorem is very similar with the proof of Theorem 3 and we will omit it. \( \square \)

**Theorem 5.** Let \( q \in Q, (b, f, 0) \in S \). If the set \( D_{b,f,0}^{Both}(d;q) \) is nonempty then it is a closed interval for all \( d \).

*Proof.* If \( b \geq 0 \) then the set \( D_{b,f,0}^{Both}(d;q) \) is empty. For \( b < 0 \) the proof is similar with the proof of Theorem 3. \( \square \)

**Theorem 6.** For any price \( q \in Q, d \in D, f \in F, \) and \( h \in H \), the sets \( D_{b,f,h}^{CC}(d;q) \) expand when \( b \) decreases.

*Proof.* Let \( b_1 > b_2 \). Then

\[
\begin{align*}
\{(c, b', f', h', 0, 1) \in B_{b_1,f,h}(d, y; q)\} &= \{(c, b', f', h', 0, 1) \in B_{b_2,f,h}(d, y; q)\}, \\
\{(c, b', f', h', 1, 1) \in B_{b_1,f,h}(d, y; q)\} &= \{(c, b', f', h', 1, 1) \in B_{b_2,f,h}(d, y; q)\}, \\
\{(c, b', f', h', 0, 0) \in B_{b_2,f,h}(d, y; q)\} &\supseteq \{(c, b', f', h', 0, 0) \in B_{b_2,f,h}(d, y; q)\}, \\
\{(c, b', f', h', 1, 0) \in B_{b_2,f,h}(d, y; q)\} &\supseteq \{(c, b', f', h', 1, 0) \in B_{b_2,f,h}(d, y; q)\}.
\end{align*}
\]

Thus, if for \( b_1 \),

\[
\psi_{cc}(y, d)(c^*, b'^*, f'^*, h'^*, 0, 1) \geq \max \left\{ \psi_{node,f}(y, d)(c, b', f', h', 0, 0), \psi_{sl}(y, d)(c, b', h', 1, 0), \psi_{both}(y, d)(c, b', h', 1, 1) \right\},
\]

it follows that the same inequality will hold for \( b_2 \) as well. Therefore, \( D_{b_1,f,h}^{CC}(d;q) \subseteq D_{b_2,f,h}^{CC}(d;q) \). \( \square \)
Theorem 7. For any price \( q \in Q \), \( b \in B \), \( f \in F \), and \( h \in H \), the sets \( D^{CC}_{b,f,h}(d;q) \) shrink and \( D^{Both}_{b,f,h}(d;q) \) expand when \( d \) increases.

Proof. Let \( d_1 < d_2 \). Then

\[
\{ (c', f', h', 0, 1) \in B_{b,f,h}(d_1, y; q) \} \supseteq \{ (c, b', f', h', 0, 1) \in B_{b,f,h}(d_2, y; q) \},
\]

\[
\{ (c, b', f', h', 1, 1) \in B_{b,f,h}(d_1, y; q) \} = \{ (c, b', f', h', 1, 1) \in B_{b,f,h}(d_2, y; q) \},
\]

\[
\{ (c, b', f', h', 0, 0) \in B_{b,f,h}(d_1, y; q) \} \supseteq \{ (c, b', f', h', 0, 0) \in B_{b,f,h}(d_2, y; q) \},
\]

\[
\{ (c, b', f', h', 1, 0) \in B_{b,f,h}(d_1, y; q) \} = \{ (c, b', f', h', 1, 0) \in B_{b,f,h}(d_2, y; q) \}.
\]

Thus, if

\[
\psi_{both}(y, d_1)(c^*, b'^*, f'^*, h'^*, 1, 1) \geq \max \{ \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \psi_{sl}(y, d_1)(c, b', h', 1, 0), \psi_{cc}(y, d_1)(c, b', h', 0, 1) \},
\]

it follows that the same inequality holds for \( d_2 \). Therefore, \( D^{Both}_{b,f,h}(d_1; q) \subseteq D^{Both}_{b,f,h}(d_2; q) \). On the other hand, if

\[
\psi_{cc}(y, d_1)(c^*, b'^*, f'^*, h'^*, 0, 1) \geq \max \{ \psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \psi_{sl}(y, d_1)(c, b', h', 1, 0), \psi_{both}(y, d_1)(c, b', h', 1, 1) \},
\]

the inequalities can reverse for \( d_2 \). Therefore \( D^{CC}_{b,f,h}(d_1; q) \supseteq D^{CC}_{b,f,h}(d_2; q) \). \( \square \)

Theorem 8. For any price \( q \in Q \), \( b \in B \), \( d \in D \), and \( f \in F \), the set \( D^{CC}_{b,f,0}(d;q) \subset D^{CC}_{b,f,1}(d;q) \).

Proof. Let \( y \in Y \). For \( h = 1 \) we have that

\[
\{ (c, b', f', h', 1, 1) \in B_{b,f,1}(d; y; q) \} = \emptyset
\]

and

\[
\{ (c, b', f', h', 1, 0) \in B_{b,f,1}(d; y; q) \} = \emptyset.
\]
Therefore, if for $f = 0$ we have that

$$\psi_{cc}(y, d_1)(c^*, b'^*, f'^*, h'^*, 0, 1) \geq \max \{ \psi_{node, f}(y, d_1)(c, b', f', h', 0, 0),$$

$$\psi_{st}(y, d_1)(c, b', h', 1, 0),$$

$$\psi_{both}(y, d_1)(c, b', h', 1, 1) \},$$

then the same inequalities hold for $f = 1$. 

\qed