House Prices Booms and Current Account Deficits*

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Abstract

One of the most striking features of the period before the Great Recession of 2007-2009 is the strong positive correlation between house price appreciation and current account deficits in countries that have subsequently experienced the highest degree of financial turmoil. A progressive relaxation of credit constraints can rationalize this empirical observation. Lower collateral requirements facilitate access to external funding and drive up house prices. Households increase their leverage borrowing from the rest of the world so that the current account turns negative. Several pieces of evidence support this view. The paper further compares this mechanism with the role of monetary policy, the exchange rate regime and foreign saving shocks in accounting for the evidence.

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1 Introduction

“...[C]ountries in which current accounts worsened...had greater house price appreciation over this period [2001Q4-2006Q3]. ... This simple relationship requires more interpretation before any strong conclusions about causality can be drawn...”

Speech by Chairman Ben S. Bernanke
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During the five years before the eruption of the recent financial crisis, countries that have witnessed a notable increase in house prices have also experienced substantial external deficits.\(^1\) This fact is well-known for the U.S., where the acceleration of house prices and the emergence of a large negative current account were perhaps the two most discussed topics in the pre-crisis period. As figure 1 shows, this phenomenon goes beyond the U.S. being a robust feature among advanced economies.\(^2\)

\(^1\)Similar dynamics for capital inflows and real estate prices occurred before the Asian crisis in the late 1990s (see Obstfeld and Rogoff (2010) and the references therein).

\(^2\)Figure 1 replicates figure 10 in Bernanke (2010). The August 2007 ECB Monthly Bulletin presents a similar plot for the period 1997-2005.
This paper argues that a progressive relaxation of borrowing constraints can generate a strong negative correlation between house prices and current account. Lower collateral requirements facilitate access to external funding and drive up house prices. Households increase their leverage borrowing from the rest of the world so that the current account turns negative.

The analysis relies on a two-country model with tradable consumption goods and housing. The expected value of housing represents the collateral for households’ debt. This endogenous borrowing constraint is buffered by a time-varying parameter which constitutes the key shock in the model. This parameter controls the loan-to-value threshold. An increase in this threshold, for given value of the collateral, leads households to lever up and demand more housing, hence driving up house prices. To the extent that the relaxation of credit constraints affects the whole economy, the increase in domestic borrowing must be financed from abroad, thus generating a current account deficit.

Few empirical studies discuss the relation between house prices and external imbalances. Ahearne et al. (2005) document the co-movement between house price dynamics and current account balance since 1970. Aizenman and Jinjarak (2009) provide a precise estimate of the relation between the two variables: a one standard deviation increase in lagged current account deficits is associated with a 10% appreciation of real estate prices. Fratzscher, Juvenal and Sarno (2010) adopt the opposite perspective: together with equity market shocks, house price shocks account for up to 32% of the movements in the U.S. trade balance over a 20-quarter horizon.

Three recent papers investigate the connection between house prices and the current account balance in quantitative general equilibrium frameworks. The mechanism that generates the negative correlation between house prices and current account balance in this work is similar to one of the shocks in Punzi (2006), who considers a richer model with residential investment and preference heterogeneity also within countries. Shocks to the loan-to-value ratio provide a rationale for increased housing demand, which is the more basic type of shocks investigated in Gete (2010). These demand-driven explanations contrast with the idea that differentials in expected output growth lead to house price booms and current account deficits, as in Kole and Martin (2009).

In a closed economy setting, Favilukis, Ludvigson and Van Nieuwerburgh (2011) develop a rich two-sector model with heterogenous households and idiosyncratic risk. Together with the reduction in transaction costs for housing and business cycle factors, the relaxation of borrowing constraints accounts for the observed increase in the observed price-to-rent ratio. Iacoviello and Neri (2010) estimate a DSGE model with housing and find that slow technological progress in the housing sector explain the long run upward trend in U.S. house prices. Housing preference and technology shocks account for about 50% of the variance of

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3 The model essentially interprets the non-tradable sector in Ferrero, Gertler and Svensson (2010) as housing.
4 Kole and Martin (2009) find similar results.
5 Midrigan and Philippon (2010) use credit constraint shocks in an island economy to match the distribution of house prices across U.S. counties.
housing investment and prices at business cycle frequencies.

Starting with Kiyotaki and Moore (1997), several authors have highlighted the role of endogenous borrowing constraint as source of amplification of other types of shocks. Monacelli (2009) shows that borrowing constraints can reconcile the empirical evidence on the response of durable and non-durable spending to monetary shocks with the dynamics of an otherwise standard New Keynesian model. In a small open economy, Iacoviello and Minetti (2003) relate the strength of the impact of monetary policy shocks on house prices to the degree of financial liberalization.

The view that reductions in collateral requirements have generated increases in house prices and consequently external imbalances is obviously not uncontroversial. Glaeser, Gottlieb and Gyourko (2010) find no evidence that changes in approval rates or loan-to-value levels explain a large part of the increase in house price in the U.S. between 1996 and 2006. By their own admission, however, their empirical estimates suffer from an endogeneity problem and the quality of the data in the empirical analysis may be questionable.

The paper presents several pieces of evidence supporting the role financial liberalizations in generating the increase in house prices. Not surprisingly, several alternative explanations exist. Taylor (2008) argues that loose monetary policy both in the U.S. and abroad is at the heart of the problem. Developments in mortgage markets and in the securitization process only contributed to worsen the problem. Extending the model to include nominal rigidities and a role for active monetary policy allows for a quantitative evaluation of the relative importance of this explanation vis-a-vis the role of shocks to the loan-to-value ratio. Importantly, this extension allows for studying the role of foreign exchange rate pegs to the U.S. dollar – a popular rationale for the recent U.S. current account deficits which has not been much discussed in relation to the house price boom.

The “saving glut” hypothesis (Bernanke, 2005) is another potential explanation of the negative correlation between house prices and current account balances. Building on their earlier work, Caballero, Fahri and Gourinchas (2008) argue that global demand for liquid assets generated capital flows from the rest of the world toward the U.S. where asset prices, and hence house prices too, took off. The paper evaluates the quantitative appeal of this competing story by including preference shocks that make the rest of the world more patient relative to the U.S.

The rest of the paper proceeds as follows. The next section provides some evidence on the relaxation of borrowing constraints induced by the process of financial innovation. The third section presents the general model and then focuses on the special case without nominal rigidities. The fourth section discusses the calibration and the basic quantitative experiment. The fifth section focus on nominal rigidities and addresses the quantitative importance of alternative explanations. Finally, the sixth section concludes.

Sterk (2010), however, shows that this result depends crucially on nominal rigidities in the durable sector.
2 Evidence

1. Mian and Sufi
2. NY Fed Equifax data and paper
3. Indicators in Glaeser et al. (2010)
4. Appendix of Favilukis, Ludvigson and Van Nieuwerburgh (2011)

3 Model

Time is discrete and indexed by \( t \). The world consists of two countries, Home and Foreign, of equal size. In each country, a continuum of measure one of firms produce a final tradable good using a labor aggregate as the only factor of production. The representative household in each country comprises a continuum of measure one of workers who supply differentiated labor inputs and consume a composite of the tradable goods produced in each country as well as housing services, which are assumed to be proportional to the fixed housing stock. The value of housing represents the collateral needed to obtain private credit. Goods and labor markets are imperfectly competitive. Goods prices and wages are set on a staggered basis.

Household’s Preferences and Constraints

This section presents the household objective and constraints from the perspective of the Home country. An asterisk * denotes foreign variables when relevant.

The representative household maximizes

\[
U_t \equiv E_t \left( \sum_{s=0}^{\infty} \delta_{t+s-1} u_{t+s} \right).
\]  

The endogenous discount factor \( \delta_t \) is defined by the recursion

\[
\delta_t = \beta_t \delta_{t-1}
\]

where

\[
\beta_t \equiv \frac{e^{\sigma_t}}{1 + \vartheta_1 (\ln C_t - \vartheta_0)}.
\]

The sensitivity of the discount factor to the level of current consumption \( \vartheta_1 \) is chosen to be small enough not to significantly affect the equilibrium dynamics. Given the choice for \( \vartheta_1 \), the constant \( \vartheta_0 \) pins down the steady state value of the discount factor to a desired target. The endogenous discount factor is a widely-used device to ensure the existence of a determinate steady state stock of net foreign assets in the presence of incomplete international financial
markets (Uzawa, 1968). The variable $\varsigma_t$ is a first order autoregressive preference shock with i.i.d. normal innovations.

Per-period utility $u_t$ depends positively on consumption of both goods $C_t$ and housing services $H_t$ and negatively on hours worked by each member of the representative household $L_t(i)$

$$u_t \equiv \ln C_t + \eta \ln H_t - \frac{1}{1 + \nu} \int_0^1 L_t(i)^{1+\nu} \, di,$$

where $\eta > 0$ is the relative utility weight of housing and $\nu > 0$ is the inverse Frisch elasticity of supply of a specific labor input.

The tradable bundle $C_t$ combines consumption of goods produced in the Home ($C_{ht}$) and Foreign ($C_{ft}$) country with constant elasticity of substitution $\gamma > 0$

$$C_t \equiv \left[ \alpha \gamma C_{ht}^{\gamma} + (1 - \alpha) \gamma C_{ft}^{\gamma} \right]^{\frac{1}{\gamma}},$$

where $\alpha \in [0.5, 1)$ is the share of domestic tradable goods. If $\alpha > 0.5$, preferences for tradable goods exhibit home bias. The Foreign tradable bundle places a weight $\alpha$ on consumption of Foreign tradable goods.

Household’s members perfectly pool their consumption risk within each country. The representative household can smooth consumption intertemporally by borrowing and lending in international financial markets, subject to a collateral constraint in the form of housing wealth

$$(1 + i_t)B_t \leq \theta_t E_t(\eta + H_t),$$

where the borrowing constraint parameter $\theta_t$ is an exogenous shock with mean $\theta$ and support over the unit interval. The idea behind the borrowing constraint is that the Foreign household can only recover a fraction $\theta_t$ of the collateral in case of default, possibly due to various costs associated with the bankruptcy process. 

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7 See Bodenstein (2009) for a comparison with alternative methods to close two-country models with incomplete international financial markets.

8 If $\alpha > 0.5$, preferences for tradable goods exhibit home bias. The Foreign tradable bundle places a weight $\alpha$ on consumption of Foreign tradable goods.

9 See, for instance, Kiyotaki and Moore (1997) or Kocherlakota (2000).
Firms and Production

Final goods producing firms pack intermediate goods according to a constant return technology with elasticity of substitution $\phi_p > 1$

$$Y_{ht} = \left[ \int_0^1 Y_t(h) \frac{\varphi_{p-1}}{\varphi_p} dh \right]^\frac{\varphi_p}{\varphi_p - 1}. \quad (8)$$

All intermediate goods producing firms have access to the same constant return technology which uses a labor aggregate $L_t$ as the only factor of production

$$Y_t(h) = AL_t, \quad (9)$$

where $A$ is a constant productivity factor.

The labor aggregate combines differentiated labor inputs with an elasticity of substitution $\phi_w > 1$

$$L_t = \left[ \int_0^1 L_t(i) \frac{\varphi_{w-1}}{\varphi_w} di \right]^\frac{\varphi_w}{\varphi_w - 1}. \quad (10)$$

Finally, the stock of housing (land) is assumed to be fixed

$$H_t = H. \quad (11)$$

Monetary Policy

The central bank sets the short-term nominal interest rate in response to deviations of inflation and output from their targets

$$(1 + i_t) = (1 + i_t-1)^{\rho_i} \left[ \left( \frac{\Pi_t}{\Pi_{t-1}} \right)^{\varphi_\pi} \left( \frac{Y_{ht}}{\tilde{Y}_{ht}} \right)^{\varphi_y} \right]^{1-\rho_i} e^{\varepsilon_{it}}, \quad (12)$$

where $\rho_i$ is the degree of interest rate smoothing, $\Pi_t = P_t / P_{t-1}$ is the inflation rate, $\tilde{\Pi}_t$ and $\tilde{Y}_{ht}$ are the targets for inflation and output respectively and $\varepsilon_{it}$ is an i.i.d. normal innovation to the interest rate rule with mean zero and standard deviation $\sigma_i$.

3.1 Equilibrium with Perfect Competition and Flexible Prices/Wages

Consider first a version of the model in which workers and firms have no market power ($\phi_k \to \infty$) and prices and wages are flexible ($\zeta_k \to 0$) for $k = \{p, w\}$. Further suppose that the monetary authority in each country stabilizes inflation at zero so that the nominal interest rate coincides with the real interest rate ($R_t \equiv 1 + i_t$ and $R^*_t \equiv 1 + i^*_t$).

In the absence of market power and nominal rigidities, the real wage equals the marginal
disutility of labor in units of marginal utility of consumption

\[
\frac{W_t}{P_t} = \frac{L_t}{1/C_t}. \quad (13)
\]

On the firm side, optimality implies

\[
P_{ht} = \frac{W_t}{A}. \quad (14)
\]

Therefore, equilibrium in the labor market requires

\[
\frac{P_{ht}}{P_t} = \frac{AL_t}{1/C_t}, \quad (15)
\]

or, using the production function

\[
\frac{P_{ht}}{P_t} = A^{1-\nu}Y_{ht}^{\nu}C_t, \quad (16)
\]

where the last equation makes use of the production function to eliminate labor. Similarly, in the Foreign country

\[
\frac{P_{ht}^*}{P_t^*} = A^{*1-\nu}Y_{ft}^{\nu}C_t^*, \quad (17)
\]

The relative prices of the Home and Foreign tradable goods depend on the terms of trade

\[
T_t \equiv \frac{P_{ft}}{P_{ht}} = \frac{P_{ft}^*}{P_{ht}^*} \text{ according to}
\]

\[
\frac{P_{ht}}{P_t} = \left[\alpha + (1-\alpha)T_t^{1-\gamma}\right]^{-\frac{1}{1-\gamma}}, \quad (18)
\]

and

\[
\frac{P_{ht}^*}{P_t^*} = \left[\alpha + (1-\alpha)T_t^{-(1-\gamma)}\right]^{-\frac{1}{1-\gamma}}, \quad (19)
\]

The labor market equilibrium pins down Home and Foreign output of tradable goods as a function of the terms of trade and Home and Foreign consumption respectively

\[
\left[\alpha + (1-\alpha)T_t^{1-\gamma}\right]^{-\frac{1}{1-\gamma}} = A^{1-\nu}Y_{ht}^{\nu}C_t, \quad (20)
\]

and

\[
\left[\alpha + (1-\alpha)T_t^{-(1-\gamma)}\right]^{-\frac{1}{1-\gamma}} = A^{*1-\nu}Y_{ft}^{\nu}C_t^*, \quad (21)
\]

Next, the goods market equilibrium pins down Home and Foreign consumption as a
function of the terms of trade and the real exchange rate

\[
Y_{ht} = \left[ \alpha + (1 - \alpha)T_t^{1-\gamma} \right] \gamma^{-\gamma} \left[ \alpha C_t + (1 - \alpha)S_t^\gamma C_t^\gamma \right]. \tag{22}
\]

The Foreign country counterpart of the last equation is

\[
Y_{ft} = \left[ \alpha + (1 - \alpha)T_t^{-(1-\gamma)} \right] \gamma^{\gamma} \left[ (1 - \alpha)S_t^{-\gamma} C_t + \alpha C_t^\gamma \right]. \tag{23}
\]

Real house prices are

\[
Q_t = \frac{\eta C_t}{H} + \beta_t E_t \left( \frac{C_t}{C_{t+1}} Q_{t+1} \right) + \mu_t \theta_t E_t \left( \frac{Q_{t+1}}{\Pi_{t+1}} \right). \tag{24}
\]

The Foreign counterpart of equation (24) is

\[
Q_t^* = \frac{\eta C_t^*}{H^*} + \beta_t^* E_t \left( \frac{C_t^*}{C_{t+1}^*} Q_{t+1}^* \right). \tag{25}
\]

Differently from the Home economy, the borrowing constraint never binds in the Foreign country, therefore \( \mu_t^* = 0 \) at all times.

The borrowing constraint (7) pins down the stock of internationally-traded real debt \( B_t \equiv B_t/P_t \)

\[
R_t B_t = \theta_t E_t (Q_{t+1} H). \tag{26}
\]

The shadow price of the borrowing constraint is

\[
R_t \mu_t = 1 - \beta_t R_t E_t \left( \frac{C_t}{C_{t+1} \Pi_{t+1}} \right). \tag{27}
\]

The Euler equation for the Foreign country pins down the real return in international financial markets

\[
1 = \beta_t^* R_t E_t \left( \frac{S_t C_t^*}{S_{t+1} C_{t+1}^*} \right). \tag{28}
\]

No arbitrage (residually) pins down the real return in the Foreign country

\[
R_t^* E_t \left( \frac{C_t^*}{C_{t+1}^*} \right) = R_t E_t \left( \frac{S_t C_t^*}{S_{t+1} C_{t+1}^*} \right). \tag{29}
\]

The law of motion of foreign debt (from the resource constraint) pins down the terms of trade

\[
-B_t = -R_{t-1} B_{t-1} + \left[ \alpha + (1 - \alpha)T_t^{1-\gamma} \right]^{-\gamma} Y_{ht} - C_t. \tag{30}
\]
The world resource constraint pins down the real exchange rate

\[
\left[ \alpha + (1 - \alpha)T_t^{1-\gamma} \right]^{-\frac{1}{1-\gamma}} Y_{ht} + \left[ \alpha + (1 - \alpha)T_t^{-(1-\gamma)} \right]^{-\frac{1}{1-\gamma}} Y_{ft} = C_t + S_tC_t^*.
\] (31)

Finally, the endogenous discount factors are

\[
\beta_t = \frac{e^{\varsigma_t}}{1 + \vartheta_t (\ln C_t - \vartheta_0)} \quad \text{and} \quad \beta_t^* = \frac{e^{\varsigma_t^*}}{1 + \vartheta_t^* (\ln C_t^* - \vartheta_0^*)}.
\] (32)

3.1.1 Steady State

If the borrowing constraint is not binding in either country, the model admits a symmetric steady state in which all relative prices (those of tradable goods, the terms of trade and the real exchange rate) are equal to one and foreign debt is zero. In this steady state, each country is in autarky and the level of productivity pins down output (and hence consumption). House prices are equal to the present discounted value of the marginal utility of housing services while the real return is equal to the inverse of the discount factor. The unattractive feature of such a perfectly symmetric steady state is that, up to a linear approximation, borrowing constraints are irrelevant for house prices dynamics, as it can be seen from equation (24).

Considering an asymmetric steady state is one way to resuscitate a role for borrowing constraints in affecting house prices dynamics while retaining the tractability of a linear approximation.\(^{10}\) Even with identical preferences and technologies, simply imposing that one country’s borrowing constraint is binding is enough to generate an asymmetric steady state. However, assuming a different degree of patience across countries provides a more fundamental reason why one country’s steady state may be binding. In particular, a calibration such that \(\vartheta_1 > \vartheta_1^*\) (but arbitrarily close) generates a scenario in which the Foreign country ends up being a net saver in international financial markets. Differences in the steady state level of productivity can still be used to obtain a steady state in which relative prices are equal to one.

4 Quantitative Results

This section discusses the calibration of the parameters and presents the central quantitative experiment of the paper – a relaxation of the borrowing constraint parameter \(\theta\).

4.1 Calibration

Two parameters are fairly standard in the international macroeconomics literature. The domestic share of tradable consumption \(\alpha\) is set to 0.7. The elasticity of substitution between

\(^{10}\) An alternative would be to use non-linear methods for solving models with occasionally binding constraints (e.g. Christiano and Fisher, 1999)
Home and Foreign tradable goods $\gamma$ equals 2.

The parameter $\eta$ is chosen so that in steady state the housing share of total expenditure corresponds to 20%. A loan-to-value ratio ratio of 70% pins down the steady state value of $\theta$.

The inverse Frisch elasticity of labor supply is assumed to be equal to 2. The level of Home productivity $A$ is fixed to one while the Foreign level $A^*$ is adjusted so that the steady state terms of trade and real exchange rate are equal to one.

The sensitivities of the discount factors to domestic consumption $\vartheta_1$ and $\vartheta_1^*$ and the constant $\vartheta_0$ are chosen so that in steady state $\beta = 0.98$ and $\beta^* = 0.99$.

Finally, the preference and borrowing constraint shocks follow AR(1) processes with persistence equal to 0.99 and i.i.d. innovations $\sim N(0, 1)$.

4.2 Financial Innovation, House Price Booms and External Imbalances

The model is approximated up to the first order about the asymmetric steady state described above. The appendix lists the log-linear system of equations that characterize the flexible price equilibrium.

The main experiment consists of shocking the collateral constraint parameter $\theta$. In particular, financial innovation corresponds to a higher loan-to-value ratio. Households can borrow a higher fraction of the expected value of their house. To illustrate the mechanism and its limitations, this section focuses on the response of the endogenous variables to a 1% innovation in $\theta$.

Figure 2 plots the response of house prices (top-left), the current account balance (top-right) and consumption (bottom-left) for the Home country, as well as the world real interest rate (bottom-right), defined as the weighted sum of the Home and Foreign interest rate.

At a very basic level, the model captures the negative correlation between house prices and current account balance. In response to the shock, house prices persistently increase while the current account worsens, at least for a couple of periods, before slightly overshooting its long run average of zero. A more realistic sequence of shocks approximating a progressive relaxation of collateral constraints (the process of financial liberalization) is likely to generate a run-up of house prices and a deterioration of the external balance more in line with what observed in the data.

The intuition for the result is simple. For given house prices, a relaxation of borrowing constraints induces households to lever up and demand more housing. Home households finance their higher leverage borrowing from abroad. As foreign debt increases, the current account turns negative, hence generating the negative correlation consistent with the empirical evidence.

\[11\] A very common assumption in macroeconomics is to set the inverse Frisch elasticity equal to 1. This choice, however, would prevent steady state productivity to play any role in determining relative prices, which is instead useful here to fix the steady state terms of trade and real exchange rate to one.
Figure 2: Impulse response function (selected variables) to a 1\% increase in the loan-to-value parameter $\theta$. 
Figure 2, however, also highlights some limitations of the basic model with flexible prices. First, in response to the shock, Home consumption drops, although the movement is almost negligible. In the data, consumption grew strongly during the house prices boom of the first half of the 2000s. The counterfactual movement of consumption partly depends on the preference assumption. If tradable consumption and housing were true complements, the shock would likely also generate an increase in consumption. Flexible prices, however, exacerbate this adjustment. In response to the shock, the terms of trade instantaneously adjust as to make Home goods more expensive. Because of home bias, domestic households disproportionately suffer from the adverse movement in relative prices.

A second issue with the basic model with flexible prices is that the drop in the world real interest rate is driven by the drop in the Foreign real rate. The Home real interest rate, however, is the relevant variable to compare with the data, which show a persistent decline of real rates on a global scale.

The next section reactivates price and wage rigidities with the objective of addressing these issues and evaluating the quantitative importance of some popular competing explanations.

5 Competing Explanations

1. Loose monetary policy
2. Foreign pegs
3. Global saving glut hypothesis

6 Conclusions

References


A Optimality Conditions for Households and Firms

This section presents the optimality conditions for households and firms of the Home country. Given the assumption of a representative household in each country, borrowing and lending occurs in equilibrium only at the international level. In what follows, the borrowing constraint is always assumed to bind for the Home economy and never for the Foreign economy.

Cost Minimization

Expenditure minimization determines the allocation of total consumption between Home and Foreign tradable goods as a function of their relative prices and total demand

\[ C_{ht} = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t \quad \text{and} \quad C_{ft} = (1 - \alpha) \left( \frac{P_{ft}}{P_t} \right)^{-\gamma} C_t, \]

(33)

where the price of the aggregate consumption bundle \( P_t \) is

\[ P_t = \left[ \alpha P_{ht}^{1-\gamma} + (1 - \alpha) P_{ft}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}. \]

Expenditure minimization also implies

\[ P_{ht}C_{ht} + P_{ft}C_{ft} = P_tC_t. \]

Final goods producers are perfectly competitive. Their cost minimization problem generates the demand for intermediate goods according to

\[ Y_t(h) = \left[ \frac{P_t(h)}{P_{ht}} \right]^{-\phi_p} Y_{ht}, \]

(34)

where the price index of the tradable bundle \( P_{ht} \) is

\[ P_{ht} = \left[ \int_0^1 P_t(h)^{1-\phi_p} dh \right]^{\frac{1}{1-\phi_p}}. \]

Intermediate goods producing firms minimize labor costs and demand the generic labor
input \( L_t(i) \) according to

\[
L_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{-\phi_w} L_t, \tag{35}
\]

where \( W_t \) is the aggregate wage index

\[
W_t = \left[ \int_0^1 W_t(i)^{1-\phi_w} di \right]^{\frac{1}{1-\phi_w}}.
\]

**Utility Maximization**

The representative household maximizes utility (1) subject to the budget constraint (6) and the borrowing constraint (7).\(^{12}\) Let \( \lambda_t \) and \( \lambda_t \mu_t \) be the lagrange multipliers on the two constraints. Workers operate in monopolistic competition taking the demand for their generic labor input as given. Therefore, equation (35) becomes an additional constraint for the household problem.

The first order condition for consumption is

\[
\frac{1}{C_t} - \lambda_t P_t = 0. \tag{36}
\]

The first order condition for housing services is

\[
\frac{\eta}{H_t} - \lambda_t Q_t + \beta_t E_t(\lambda_{t+1} Q_{t+1}) + \lambda_t \mu_t E_t(Q_{t+1}) = 0. \tag{37}
\]

The first order condition for debt is

\[
\lambda_t - \beta_t (1 + i_t) E_t(\lambda_{t+1}) - \lambda_t \mu_t (1 + i_t) = 0. \tag{38}
\]

Wages are set on a staggered basis (Calvo, 1983). The probability of not being able to adjust the wage is \( \zeta_w \). The optimality condition for a worker who is able to adjust the wage at time \( t \) is

\[
E_t \left\{ \sum_{s=0}^\infty \zeta_w^{s} \beta^{s} \lambda_{t+s} \left[ L_{t+s}(i) \left( \frac{\phi_w}{\phi_w - 1} \right) L_{t+s}(i)^{\nu} \right] \right\} = 0. \tag{39}
\]

Using the first order condition for consumption (36), the first order conditions for housing services (37) becomes

\[
\frac{\eta}{H_t} - \frac{Q_t}{P_t C_t} + \beta_t E_t \left( \frac{Q_{t+1}}{P_{t+1} C_{t+1}} \right) + \frac{\mu_t \theta_t}{P_t C_t} E_t(Q_{t+1}) = 0 \tag{40}
\]

\(^{12}\)The representative household treats the discount factor (2) as exogenous.
or
\[ Q_t = \frac{\eta C_t}{H_t} + \beta_t \mathbb{E}_t \left( \frac{C_t}{C_{t+1}} Q_{t+1} \right) + \mu_t \theta_t \mathbb{E}_t \left( \frac{1}{\Pi_{t+1}} Q_{t+1} \right), \] (41)

where \( Q_t \equiv Q_t/P_t \) defines real house prices. Equation (41) consists of a standard part, according to which real house prices are equal to the marginal utility of housing services in units of marginal utility of consumption plus expected discounted future house prices, and a second part which measures the contribution of the borrowing constraint via the shadow price \( \mu_t \).

Similarly, using again the first order condition for consumption (36), the first order condition for debt (38) becomes
\[
\frac{1}{P_t C_t} - \beta_t (1 + i_t) \mathbb{E}_t \left( \frac{1}{P_{t+1} C_{t+1}} \right) - (1 + i_t) \frac{\mu_t}{P_t C_t} = 0
\]
or
\[
(1 + i_t) \mu_t = 1 - \beta_t (1 + i_t) \mathbb{E}_t \left( \frac{C_t}{C_{t+1}} \frac{1}{\Pi_{t+1}} \right). \quad (42)
\]

Equation (42) shows that the shadow price \( \mu_t \) represents a wedge in the standard consumption Euler equation due to the borrowing constraint.

**No Arbitrage**

The representative household in the Foreign country solves the same maximization problem with one substantial difference. While the Foreign representative household can purchase Home debt, Foreign debt only circulates domestically. No arbitrage then implies the consumption-based uncovered interest parity condition
\[
(1 + i^*_t) \mathbb{E}_t \left( \frac{C^*_t}{C^*_{t+1}} \frac{1}{\Pi^*_{t+1}} \right) = (1 + i_t) \mathbb{E}_t \left( \frac{E_t C^*_t}{E_{t+1} C^*_{t+1}} \frac{1}{\Pi^*_{t+1}} \right).
\]

Because of the representative household assumption, Foreign debt is in zero net supply in equilibrium. Additionally, the Foreign country is assumed to be a net saver in international financial markets so that the Foreign borrowing constraint to never bind (\( \mu^*_t = 0 \forall t \)).

**Profit Maximization**

Intermediate goods producers set prices on a staggered basis taking the demand for their product (34) as given and subject to the technology constraint (9). The probability of not being able to adjust the price is \( \zeta_p \). The optimality condition for a firm able to adjust its
price at time $t$ is\(^{13}\)

$$E_t \left\{ \sum_{s=0}^{\infty} \zeta_p^s \beta_t y_{t+s} Y_{t+s}(h) \left[ P_t(h) - \left( \frac{\phi_p}{\phi_p - 1} \right) W_{t+s} \right] \right\} = 0. \quad (43)$$

**Market Clearing**

The law of one price holds for tradable goods

$$P_{ht} = E_t P_{ht}^*, \quad (44)$$

where $E_t$ is nominal exchange rate (the price of the Foreign currency in terms of the Home currency). Home bias, however, implies that purchasing power parity does not hold (i.e. $P_t \neq E_t P_t^*$).

Final goods producing firms sell their products in the Home and Foreign market. Market clearing requires

$$Y_{ht} = C_{ht} + C_{ht}^* = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t + \left( \frac{P_{ht}}{P_t^*} \right)^{-\gamma} C_t^*, \quad (45)$$

where the second part of (45) uses (33) and its Foreign country counterpart. The Home tradable goods market equilibrium (45) can alternatively be expressed in terms of the Home relative price and the real exchange rate $S_t \equiv E_t P_t^*/P_t$

$$Y_{ht} = \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} \left[ \alpha C_t + (1 - \alpha) S_t^\gamma C_t^* \right]. \quad (46)$$

As mentioned, the housing stock is fixed in both countries

$$H_t = H \quad \text{and} \quad H_t^* = H^*.$$  

Market clearing for financial assets requires

$$B_t + B_t^* = 0, \quad (47)$$

where $B_t^*$ is the Foreign country stock of international debt.

**B Symmetric Steady State**

If the relative prices of tradable goods are equal to one, so are the terms of trade ($T_t = 1$) from equations (18) and (19). Given these results, the definition of the real exchange rate

\(^{13}\)The representative household in each country owns the domestic firms. Therefore, the lagrange multiplier on the budget constraint is the appropriate measure to convert the value of future profits in units of current consumption.
requires

\[ S = \frac{\xi P^*}{P} = \frac{\xi P^*_f P_h}{P} P^*_f = 1 \]  

Combining the labor market equilibrium conditions and the goods market equilibrium conditions yields

\[ Y_h = A^{1-\frac{1}{\phi}} C^{-\frac{1}{\phi}} = \alpha C + (1 - \alpha)C^* \quad \text{and} \quad Y_f = A^{1-\frac{1}{\phi}} C^*^{-\frac{1}{\phi}} = (1 - \alpha)C + \alpha C^*. \]  

If \( A = A^* = 1 \), the last two expressions imply that each country is always in autarchy (\( Y_h = C = C^* = Y_f = 1 \)). From the resource constraint, external debt is zero (\( B = 0 \)).

The endogenous discount factor is equal across countries

\[ \beta = \beta^* = \frac{1}{1 - \varphi_1 \varphi_0}. \]  

Furthermore, the borrowing constraint is slack (\( \mu = \mu^* = 0 \)) in both countries. House prices only depend on the present discounted value of the marginal utility of housing

\[ Q = Q^* = \frac{\eta/H}{1 - \beta}, \]  

under the assumption \( H = H^* \).

Finally, the real interest rate is equal to the inverse of the discount factor

\[ R = R^* = \frac{1}{\beta}. \]  

### C Log-Liner Approximation of the Model