Aggregate Evidence of Nominal Price Rigidities and the Inflation-Output Trade-Off: The Relationship between the Income Share of Labor and the Rate of Inflation

Christian Jensen
Moore School of Business
University of South Carolina

January, 2011

JEL classification: E10; E30; E32
Keywords: Nominal price rigidities; inflation-output trade-off; Phillips curve

Abstract
In order for an inflation-output trade-off to arise due to nominal rigidities such as menu costs, negotiation costs, monetary misperceptions or imperfect information, the income share of labor must increase with inflation. However, we find it falls with inflation, in a statistically significant manner in the U.S. and the other OECD countries for which at least 25 years of data is available, after controlling for changes in competition, the only variable that affects the factor payments in the absence of nominal rigidities. Explaining the observed relationship between inflation and the income share of labor without nominal rigidities is difficult, since the income share is not likely to impact inflation or monetary policy, and is independent of most variables and shocks, including those to productivity. Hence, the relationship is evidence of the relevance of nominal rigidities at the aggregate level, but the fact that it is negative refutes that the rigidities give rise to a trade-off between inflation and output, since it implies that inflation has a negative impact on output. The reason is that contrary to what models of nominal rigidities propose, the average mark-up paid by households increases with inflation.

*Department of Economics; Moore School of Business; 1705 College Street; Columbia, SC 29208; cjensen@alumni.cmu.edu.
1 Introduction

The hypothesis that money affects aggregate real economic variables in the short run, in spite of being neutral in the long run, is one of the most controversial issues in economics, mainly due to a lack of convincing empirical evidence of short-run non-neutrality and its relevance at the aggregate level. Despite this, much effort has been exerted studying nominal rigidities through which money can affect the real economy, and using these frameworks, especially the sticky-price models proposed by Calvo (1983) and Rotemberg (1982), not only to analyze monetary policy, but also to guide it.\(^1\) Much of the work was sparked by the infamous Phillips (1958) curve, which did not stand the test of time, but has nevertheless had an enormous impact on the field. In fact, later contributions have tended to rationalize why the Phillips curve shifts over time, thus justifying the deterioration of the original empirical evidence. Consequently, many appear to have accepted that nominal rigidities are not easily detectable in aggregate data, and some have turned to look for these in disaggregate data, see for example Bils and Klenow (2004).\(^2\) However, such studies cannot demonstrate the relevance of nominal rigidities at the aggregate level. The present paper studies how money can affect real aggregate variables through nominal rigidities that distort price-setting, in order to assess the aggregate relevance of such rigidities empirically. We find evidence of nominal rigidities distorting price-setting in U.S. aggregate data from 1947 to 2009, in that inflation lowers the income share of labor. This evidence is statistically significant not only for the U.S., but for the majority of the seven OECD countries for which at least 25 years of data is available, and for the group of eight countries as a whole. While the finding is consistent with the aggregate importance of nominal rigidities, it refutes the existence of an inflation-output trade-off, since it implies that inflation has a negative impact on output.

\(^1\)Some prominent examples include Clardia, Gali and Gertler (1999) and Woodford (2003).

\(^2\)The evidence of the negative impact hyperinflation has on aggregate output is convincing. However, such situations are considered abnormalities in that the channels through which extremely high inflation affects real variables are considered different from those through which moderate inflation could have an impact.
The main difficulty with identifying the impact of nominal rigidities in aggregates is that economic theory predicts that most of these variables should be related even without such rigidities. For example, Freeman and Huffman (1991) argue that a positive co-movement between inflation and output can be explained without nominal rigidities by both variables reacting to productivity shocks, since these would tend to affect interest rates, and thus the money supply endogenously through bank deposits. The problem is attenuated by the fact that monetary policy responds to aggregates, especially output. For example, Wang and Wen (2005) argue that many of the features observed in U.S. inflation and output data that might look like evidence of nominal rigidities can be explained by a model without rigidities when policy follows the Taylor (1993) rule. This is why the present study focuses on the income share of labor instead of output. It is an aggregate variable that economic theory predicts should be independent of shocks to productivity, government spending, household preferences, and most other variables usually considered relevant for business cycles, but is affected by nominal rigidities through the way these distort price-setting. Furthermore, the income share of labor is not usually considered to have an impact on monetary policy, nor does it depend on variables typically thought to influence policymaking.

Many a hypothesis has been proposed describing nominal rigidities through which money could affect the real aggregate economy by distorting price-setting. The most prominent ones are menu costs (Calvo (1983) and Rotemberg (1982)), negotiation costs (Taylor (1980)), monetary misperceptions (Lucas (1972)) and costly information-gathering (Mankiw and Reis (2002)). Our framework encompasses all of these in a general model of the aggregate impact of price-setting distortions. There are two distinct effects, one working through relative prices, the other through the average mark-up. By making price-adjustment idiosyncratic, nominal rigidities distort relative prices, reducing total factor productivity as a result of the cost-minimizing mix of intermediate goods differing from the efficient mix, hence having a negative impact on output and welfare. At the same time, nominal rigidities affect the mark-ups households pay, which can reduce or raise output and welfare, depend-
ing on whether they increase or decrease the average mark-up paid and the dead-weight loss from imperfect competition. An inflation-output trade-off requires that the average mark-up paid fall with inflation, and that this effect dominate the one through relative prices and total factor productivity. While the relative strength of the two effects depends on the degree of competition, which can vary over time and across countries, the evidence against nominal price rigidities generating an inflation-output trade-off is persistent over time and across countries. In fact, contrary to what is assumed in models of nominal rigidities, the data suggests that the average mark-up paid increases with inflation.

Without nominal rigidities that distort price-setting, changes in the income share of labor must, according to our theoretical framework, be due to variations in the degree of competition. As mark-ups are reduced due to intensified competition, the share of income that goes to profits falls, and the shares of labor and capital increase. While inflation has no impact on the degree of competition in the absence of nominal rigidities, the causality could go in the opposite direction, since output increases with the degree of competition, which could affect the rate of inflation through money demand or monetary policy. Hence, controlling for changes in competition is crucial when studying the relationship between the income share of labor and inflation. However, doing so is a non-trivial matter, since no direct measures are available, and the ones that are at hand are very noisy measures of competition affected by other variables, such as productivity. To get around this, Solow-residual estimates of productivity are used to correct our measure of competition, which is real GDP, estimating a Taylor approximation of the solution to our dynamic stochastic general equilibrium model. In addition, we control for other variables that affect real GDP, and instrument for measurement error using lags and data from different countries. The null hypothesis of no nominal rigidities is rejected in aggregate data for Australia, Belgium, Canada, the U.K. and U.S. It is not rejected for France, Italy and Japan, but we have only between 115 and 127 observations for each of these three countries, while we have almost 200 for most of the others. Pooling our data, exploiting that one cannot reject that the
impact inflation has on the income share of labor is the same across the eight countries in our sample, the null hypothesis of no nominal rigidities is rejected.

Our dynamic general equilibrium model builds on that of Blanchard and Kiyotaki (1987). It consists of an infinite number of a priori identical monopolistically competing producers that rent capital and labor from households in competitive factor markets to produce differentiated intermediate goods that households purchase to compose final goods. The next three sections present the producers, households and equilibrium conditions, respectively. The following three sections study the impact nominal rigidities have on total factor productivity, factor markets and factor shares, respectively. Finding that evidence of nominal rigidities should be easiest to identify in the income share of labor, the subsequent section presents the data for the U.S. The following section studies how inflation affects the income share of labor, controlling for changes in the degree of competition. The issue is then studied with aggregate data for seven other OECD countries.

2 Producers

In any period \( t \), each of the continuum of measure one identical households produce \( y_t \) units of final good by combining a continuum of differentiated intermediate goods \( x_{it} \), indexed by \( i \in [0, 1] \), according to the Dixit-Stiglitz (1977) production function

\[
y_t = \gamma_t \left( \int_0^1 \frac{x_{it}^{\theta_t-1}}{\theta_t} di \right)^{\frac{\theta_t}{\theta_t-1}}
\]

where \( \theta_t \in (1, \infty) \) is the elasticity of substitution between any two intermediate goods, while \( \gamma_t > 0 \) is the productivity with which intermediate goods can be combined into final goods. Assuming intermediate goods are the only inputs required to produce final goods, each household chooses the optimal mix of these so as to minimize the cost of providing
final goods by solving

$$\min_{\{x_it\}_{i=0}^1} \int_0^1 P_it x_it \, di \quad (2)$$

subject to the production function (1), where $P_it$ is the period-$t$ price of intermediate good $i$. The resulting demand for intermediate good $i$ from each of the households is

$$x_it = \left( \frac{P_it}{P_t} \right)^{-\theta_t} \gamma^{-\theta_t - 1}_t \, y_t \quad (3)$$

for any $i \in [0, 1]$. Inserting into the production function (1) yields the final-good price,

$$P_t = \left( \int_0^1 P_it^{1-\theta_t} \, di \right)^{-1} \gamma^{-1}_t \quad (4)$$

which equals its marginal cost of production, since all households can compose identical final goods at identical cost. Aggregating intermediate-good demands (3) across households yields the aggregate demand for intermediate good $i$,

$$X_it = \left( \frac{P_it}{P_t} \right)^{-\theta_t} \gamma^{-\theta_t - 1}_t Y_t \quad (5)$$

where $Y_t$ is the aggregate demand for final goods at time $t$.

In each period $t$, intermediate-good producer $i$ finds the optimal mix of inputs, capital $k_it$, labor $n_it$ and land $l_it$, so as to minimize production costs by solving

$$\min_{k_it, n_it, l_it} R_t k_it + W_t n_it + F_t l_it \quad (6)$$

subject to its production technology

$$X_it = z_it k_it^{\alpha} n_it^{1-\alpha-\nu} l_it^\nu \quad (7)$$

where $W_t$ is the nominal wage, $R_t$ is the nominal rental rate of capital, while $F_t$ is the nominal
rental rate of land. The weight each of these factors carry in production is determined by the coefficients $\alpha \in (0, 1)$ and $\nu \in (0, 1)$. As usual, $z_t > 0$ is an exogenous productivity shock. The first-order conditions from the cost-minimization problem yield producer $i$’s factor demands

\[ k_{it} = \alpha \frac{\lambda_t X_{it}}{R_t} \]  
\[ n_{it} = (1 - \alpha - \nu) \frac{\lambda_t X_{it}}{W_t} \]  
\[ l_{it} = \nu \frac{\lambda_t X_{it}}{F_t} \]

where

\[ \lambda_t = \frac{1}{z_t} \left( \frac{R_t}{\alpha} \right)^{\alpha} \left( \frac{W_t}{1 - \alpha - \nu} \right)^{1 - \alpha - \nu} \left( \frac{F_t}{\nu} \right)^{\nu} \]

is the marginal cost of producing any kind of intermediate good at time $t$.

In the absence of rigidities, imperfect information and anything else that interferes with price-setting, intermediate-good producer $i$ chooses the price $P_{it}$ that maximizes its period-$t$ profits given the demand it faces (5), and thus solves

\[
\max_{P_{it}} (P_{it} - \lambda_t) \left( \frac{P_{it}}{P_t} \right)^{-\theta_t} \gamma_t \theta_t - 1 Y_t
\]

which yields

\[ P_{it} = \frac{\theta_t}{\theta_t - 1} \lambda_t \]

a gross mark-up $\frac{\theta_t}{\theta_t - 1} \in (1, \infty)$ of its marginal cost of production $\lambda_t$. With menu costs, imperfect information, or anything else that distorts price-setting, intermediate-good producer $i$ will apply a potentially different mark-up $\omega_{it}$ to its marginal cost $\lambda_t$, charging

\[ P_{it} = \omega_{it} \lambda_t \]

where the mark-up $\omega_{it}$ can differ over time and across producers. Depending on the source
of the distortions to price-setting, $\omega_{it}$ can be sensitive to present expectations of future inflation, past expectations of present inflation, expectations of other variables, and the degree of nominal rigidity or uncertainty.

Inserting for the potentially distorted prices (14) into the price aggregator (4), after substituting for the marginal cost of production (11), yields the aggregate price level

$$P_t = \gamma_t^{-1} z_t^{-1} \left( \frac{R_t}{\alpha} \right)^\alpha \left( \frac{W_t}{1 - \alpha - \nu} \right)^{1-\alpha-\nu} \left( \frac{F_t}{\nu} \right)^\nu \left( \int_{0}^{1} \omega_{it}^{1-\theta_t} di \right)^{1-\theta_t}$$ (15)

and the relative price

$$\frac{P_{it}}{P_t} = \gamma_t \frac{\omega_{it}}{\left( \int_{0}^{1} \omega_{it}^{1-\theta_t} di \right)^{1-\theta_t}}$$ (16)

Substituting this relative price into the demand function for intermediate good $i$ (5), and inserting the resulting equation and the marginal production cost (11) into the factor demands (8), (9) and (10), and aggregating over all intermediate-good producers, we find the aggregate demands for capital, labor and land,

$$K_t = \gamma_t^{-1} z_t^{-1} \left( \frac{R_t}{\alpha} \right)^{\alpha-1} \left( \frac{W_t}{1 - \alpha - \nu} \right)^{1-\alpha-\nu} \left( \frac{F_t}{\nu} \right)^\nu Y_t \frac{\int_{0}^{1} \omega_{it}^{1-\theta_t} di}{\left( \int_{0}^{1} \omega_{it}^{1-\theta_t} di \right)^{1-\theta_t}}$$ (17)

$$N_t = \gamma_t^{-1} z_t^{-1} \left( \frac{R_t}{\alpha} \right)^{\alpha} \left( \frac{W_t}{1 - \alpha - \nu} \right)^{-\alpha-\nu} \left( \frac{F_t}{\nu} \right)^\nu Y_t \frac{\int_{0}^{1} \omega_{it}^{1-\theta_t} di}{\left( \int_{0}^{1} \omega_{it}^{1-\theta_t} di \right)^{1-\theta_t}}$$ (18)

$$L_t = \gamma_t^{-1} z_t^{-1} \left( \frac{R_t}{\alpha} \right)^{\alpha} \left( \frac{W_t}{1 - \alpha - \nu} \right)^{1-\alpha-\nu} \left( \frac{F_t}{\nu} \right)^{\nu-1} Y_t \frac{\int_{0}^{1} \omega_{it}^{1-\theta_t} di}{\left( \int_{0}^{1} \omega_{it}^{1-\theta_t} di \right)^{1-\theta_t}}$$ (19)

respectively.
3 Households

In addition to effortlessly composing final goods, households rent their labor $N_t$, capital $K_t$ and land $L_t$ to the collectively owned intermediate-good producers in order to provide for consumption $C_t$ and the accumulation of assets: physical capital $K_t$, money $M_t$ and bonds $B_t$. Since households are assumed to be identical, aggregation is trivial, so we focus on aggregates directly. Each of the continuum of measure one of identical households solves

$$\max_{C_t, N_t, K_{t+1}, B_{t+1}, M_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t u (C_t, 1 - N_t)$$

subject to the budget constraint

$$K_{t+1} + \frac{B_{t+1}}{P_t} + \frac{M_{t+1}}{P_t} + C_t = \frac{W_t}{P_t} N_t + \frac{R_t}{P_t} K_t + \frac{F_t}{P_t} (1 - \delta) K_t + \frac{(1 + \Re_t) B_t}{P_t} + \frac{M_t}{P_t} + \Pi_t + S_t$$

given a discount rate $\beta \in (0, 1)$, depreciation rate $\delta \in (0, 1)$, and initial conditions $K_t$, $B_t$, $M_t$, where $\frac{W_t}{P_t}$ is the real wage, $\frac{R_t}{P_t}$ is the real rental rate of capital, $\frac{F_t}{P_t}$ is the real rental cost of land, $\Re_t$ is the nominal interest rate on bonds, $\Pi_t$ are profits from the production of intermediate goods, $S_t$ are transfers to and from the government, and $P_t$ is the aggregate price level, equal to the price of the final good. In order to guarantee a strictly positive demand for money, one can introduce a cash-in-advance constraint, or include money in the utility function. To simplify, the supply of land is normalized to one.\(^3\)

\(^3\)Including land as an inelastically supplied input permits obtaining an explicit solution for aggregate output. The production side only determines the optimal factor mix, not the levels, but fixing the level of one of the inputs, all other levels are determined. Below, we let the importance of land converge to zero.
4 Equilibrium

The government budget constraint is

\[ P_t S_t + P_t G_t = M_{t+1} - M_t + B_{t+1} - (1 + \Re_t) B_t \]  \hspace{1cm} (22)

Equalizing aggregate demand for land (19) to its inelastic unitary supply yields the aggregate production function

\[ Y_t = \gamma_t z_t K_t^\alpha N_t^{1-\alpha-\nu} \left( \int_0^1 \omega_t^{1-\theta_t} \, di \right)^{-\frac{\theta_t}{1-\theta_t}} \]  \hspace{1cm} (23)

after exploiting that the aggregate demands for factors of production (17)-(19) imply that

\[ \frac{R_t}{F_t} = \frac{\alpha}{\nu K_t} \]  \hspace{1cm} (24)

\[ \frac{W_t}{F_t} = \frac{1 - \alpha - \nu}{\nu N_t} \]  \hspace{1cm} (25)

which guarantees an optimal factor mix in the production of intermediate goods. Combining these two conditions with the one for the price level (15), yields

\[ \frac{R_t}{P_t} = \alpha \gamma_t z_t K_t^{\alpha-1} N_t^{1-\alpha-\nu} \left( \int_0^1 \omega_t^{1-\theta_t} \, di \right)^{-\frac{1}{1-\theta_t}} \]  \hspace{1cm} (26)

\[ \frac{W_t}{P_t} = (1 - \alpha - \nu) \gamma_t z_t K_t^{\alpha} N_t^{1-\alpha-\nu} \left( \int_0^1 \omega_t^{1-\theta_t} \, di \right)^{-\frac{1}{1-\theta_t}} \]  \hspace{1cm} (27)

\[ \frac{F_t}{P_t} = \nu \gamma_t z_t K_t^{\alpha} N_t^{1-\alpha-\nu} \left( \int_0^1 \omega_t^{1-\theta_t} \, di \right)^{-\frac{1}{1-\theta_t}} \]  \hspace{1cm} (28)

which are the real rental rates and real wage. Due to the lack of significance of land as a source of fluctuations, we let \( \nu \) converge toward zero, so that land is eliminated from the model henceforth.
5 Total factor productivity

Aggregate output is \((\nu \to 0)\)

\[ Y_t = A_t K_t^\alpha N_t^{1-\alpha} \]  

Equation (29)

where total factor productivity

\[ A_t = \gamma_t z_t \left( \frac{\int_0^1 \omega_{it}^{1-\theta_t} \, di}{\int_0^1 \omega_{it}^{-\theta_t} \, di} \right)^{\frac{\theta_t}{\theta_t}} \]  

Equation (30)

depends on the level of the productivity shocks \(\gamma_t\) and \(z_t\), but only on the dispersion in the mark-ups \(\omega_{it}\). To see this, note that when all intermediate-good producers \(i\) apply the same mark-up \(\omega_t\), total factor productivity (30) simplifies to \(A_t = \gamma_t z_t\) independently of the value of \(\omega_t\).

Intuitively, increased productivity \(z_t \gamma_t\) for all producers of intermediate goods raises total factor productivity \(A_t\), since more final goods \(Y_t\) can be produced with any given (strictly positive) quantities of capital and labor. A higher \(z_t\) makes capital and labor more efficient in the production of intermediate goods, while a higher \(\gamma_t\) raises the amount of final good that can be produced with a given quantity of intermediate goods. Because producers are a priori identical and face the same elasticity \(\theta_t\), they should all apply the same mark-up. When they do not, relative prices become distorted, which in turn makes the composition of the final good inefficient, resulting in less of it being produced with any given quantities of capital and labor. The level of the mark-ups has no impact on relative prices, or the composition of final goods, so it has no effect on total factor productivity. The negative impact dispersion in mark-ups has on total factor productivity is larger the smaller the value of \(\theta_t \in (1, \infty)\), since this reduces the substitutability between intermediate goods, making productivity drop more when distortions to relative prices drive the cost-minimizing mix of intermediate goods away from the socially efficient mix.
Observing the effects nominal rigidities have on total factor productivity empirically promises to be difficult. Productivity is affected by many variables, including the development and diffusion of new technologies. In addition, economic theory predicts that total factor productivity should impact most other variables, including those expected to be key in distorting price setting, such as inflation. Moreover, monetary policy responding to movements in output spurred by changes in productivity can hide the impact that variables such as inflation can have on total factor productivity in combination with nominal rigidities.

6 Factor markets

The real rental rate of capital and real wage are \((\nu \to 0)\)

\[
\frac{R_t}{P_t} = \alpha K_t^{\alpha - 1} N_t^{1-\alpha} Q_t
\]

\(31\)

\[
\frac{W_t}{P_t} = (1 - \alpha) K_t^{\alpha} N_t^{-\alpha} Q_t
\]

\(32\)

respectively, where

\[
Q_t = \gamma_t z_t \left( \int_0^1 \omega_{it}^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}}
\]

\(33\)

captures the direct impact of distortions to price-setting. Without dispersion in mark-ups, 

\[Q_t = \frac{\gamma_t z_t}{\omega_t} \]

so in general \(Q_t\) does not only depend on the heterogeneity of the mark-ups, but also on their level.

Increased productivity \(\gamma_t z_t\) for all producers raises total factor productivity \(A_t\), and therefore also tends to raise factor prices, just as a regular positive productivity shock would. By leading to an inefficient mix of intermediate goods, distorted mark-ups make capital and labor be used less efficiently in the production of final goods, which contributes to lowering the real wage and rental rate. In addition, some firms apply mark-ups that are higher, and others apply mark-ups that are lower, than they otherwise would, affecting the
size of the average mark-up. This impacts real factor prices because the higher the mark-up a producer applies, the lower its production will be, and the less factors it demands, which tends to reduce factor prices. Whether the effect on real factor prices is positive or negative depends on the skewness of the distribution of the mark-ups, and since firms that apply low mark-ups become larger than those that apply high ones, the distribution needs to be positively skewed for the impact on factor prices to be negative (so that the producers that charge too much shrink by more than the growth of those that charge too little). When the distribution is not positively skewed to a sufficient extent, dispersion in mark-ups can raise factor prices despite lowering total factor productivity. Combined with elastic factor supplies, this can make aggregate output increase as the dispersion in mark-ups grows, even if total factor productivity falls. An example of this is the inflation-output trade-off that arises in sticky-price models such as that proposed by Calvo (1983). Inflation distorts mark-ups, which reduces total factor productivity, but at the same time firms that charge mark-ups that are too low increase their sales so much that total output grows. Producing more at a time when total factor productivity falls is obviously highly costly in terms of household utility, providing a strong rationale for avoiding inflation. However, with price rigidities such a rationale exists even without an inflation-output trade-off, since inflation temporarily reduces total factor productivity.

Distortions to price-setting that have a uniform effect on mark-ups, not generating dispersion, have no effect on total factor productivity, and only affect aggregate output through the quantity of factors employed. Since they can lower or raise mark-ups, such distortions can, in theory, contribute to either raise or lower aggregate output. By reducing mark-ups and the dead-weight loss of imperfect competition, distortions to price-setting can boost welfare if they raise output. When distortions affect the dispersion of mark-ups, they lower total factor productivity, which always has a negative impact on welfare. Still, the total effect on welfare, aggregate output, and factor markets, can go either way.
The impact heterogeneous mark-ups have on total factor productivity $A_t$ and factor prices through $Q_t$, depend on the elasticity of substitution $\theta_t$, which can vary over time. As a result, one should not expect the effects of mark-up heterogeneity to be constant over time. In particular, when it is inflation that generates the dispersion in the mark-ups, one should not expect the inflation-output relationship to be a stable one, since in theory, even its sign could change, as the relative importance of the effects through $A_t$ and $Q_t$ vary with $\theta_t$. This implies that putting our model, and the relevance of nominal price rigidities, to the test empirically by studying output is futile. The same applies to factor demands and prices.

7 Income shares

The income share of labor is ($\nu \to 0$)

$$
\frac{W_t N_t}{P_t Y_t} = (1 - \alpha) \frac{\int_0^1 \omega_{it}^{-\theta_t} di}{\int_0^1 \omega_{it}^{1-\theta_t} di} = (1 - \alpha) \frac{Q_t}{A_t}
$$

which simplifies to

$$
\frac{W_t N_t}{P_t Y_t} = 1 - \frac{\alpha}{\omega_t}
$$

when mark-ups are identical across intermediate-good producers. Hence, the share depends on both the level and dispersion of mark-ups, but is independent of productivity. The income share of labor also depends on the value of $\alpha$, and with heterogeneous mark-ups, the elasticity of substitution $\theta_t$ (through $A_t$, $Q_t$ and $\omega_{it}$). The same applies to the income share of capital, which is

$$
\frac{R_t K_t}{P_t Y_t} = \alpha \frac{\int_0^1 \omega_{it}^{-\theta_t} di}{\int_0^1 \omega_{it}^{1-\theta_t} di} = \alpha \frac{Q_t}{A_t}
$$
so barred any changes in $\alpha$, it behaves exactly the same as the income share of labor (34). Whatever income is not used to pay capital and labor goes to profits, so their share is

$$\frac{\Pi_t}{P_t Y_t} = 1 - \frac{\int_{0}^{1} \omega_t^{-\theta_t} di}{\int_{0}^{1} \omega_t^{1-\theta_t} di} = 1 - \frac{Q_t}{A_t}$$

(37)

which moves in the opposite direction of the capital and labor shares.

Without distortions to price-setting, the income shares of labor, capital and profits simplify to $\frac{1-\alpha}{\omega_t}$, $\frac{\alpha}{\omega_t}$ and $1 - \frac{1}{\omega_t}$, respectively, where the mark-up $\omega_t = \frac{\theta_t}{\theta_t - 1}$, so the shares would only depend on the level of competition $\theta_t$ (apart from the constant $\alpha$). When the economy becomes more competitive, producers charge lower mark-ups, reducing the share of income that goes to profits and raising the shares that go to labor and capital. When the economy becomes less competitive, the process is reversed. Given the discussion on $A_t$ and $Q_t$ above, it should be clear that our model offers no unambiguous prediction as to how the income shares should be affected by distortions to mark-ups, since their impact on $Q_t$ is ambiguous.

8 Income share data

Table 1 shows U.S. gross domestic income by type, as reported quarterly by the BEA (seasonally adjusted). It lists the average percentage of gross domestic income for each category over the period stretching from the first quarter of 1947 to the fourth quarter of 2009. Our model has no taxes or subsidies on production, and only three categories of income: compensation of labor, payments to capital and profits. Hence, the mapping between the model and the data is not unambiguous. While it is fairly safe to assign compensation of employees to the labor share and consumption of fixed capital to the capital share, it is not clear how most of the other categories should be allocated. In particular, this applies to net operating

4See NIPA table 1.10 at www.bea.gov. We use the abbreviation IVA for inventory valuation adjustment and CCA for capital consumption adjustment. Our numbers are as reported by the BEA in April 2010, but are subject to retroactive revisions.
Table 1: Gross domestic income by type, relative to total, averages 1947-2009 (*1959-2009).

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic income</td>
<td>100.0%</td>
</tr>
<tr>
<td>Compensation of employees, paid</td>
<td>57.0%</td>
</tr>
<tr>
<td>Wage and salary accruals</td>
<td>49.3%</td>
</tr>
<tr>
<td>Supplements to wages and salaries</td>
<td>7.7%</td>
</tr>
<tr>
<td>Taxes on production and imports</td>
<td>7.8%</td>
</tr>
<tr>
<td>Less: Subsidies</td>
<td>0.4%</td>
</tr>
<tr>
<td>Net operating surplus</td>
<td>*23.5%</td>
</tr>
<tr>
<td>Private enterprises</td>
<td>24.2%</td>
</tr>
<tr>
<td>Net interest and misc payments</td>
<td>4.7%</td>
</tr>
<tr>
<td>Business current transfer payments</td>
<td>0.6%</td>
</tr>
<tr>
<td>Proprietors’ income with IVA &amp; CCA</td>
<td>8.5%</td>
</tr>
<tr>
<td>Rental income of persons with CCA</td>
<td>2.0%</td>
</tr>
<tr>
<td>Corporate profits with IVA &amp; CCA</td>
<td>8.5%</td>
</tr>
<tr>
<td>Taxes on corporate income</td>
<td>3.5%</td>
</tr>
<tr>
<td>Profits after tax with IVA &amp; CCA</td>
<td>5.0%</td>
</tr>
<tr>
<td>Current surplus of govn’t enterprises</td>
<td>*0.0%</td>
</tr>
<tr>
<td>Consumption of fixed capital</td>
<td>11.4%</td>
</tr>
<tr>
<td>Statistical discrepancy</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

The contribution of capital owned and used by the enterprise itself is not specified either. This is a problem because the income shares of labor and capital on one hand, and that of profits on the other, should, according to our model, be affected differently by nominal rigidities. As a result, the only income share that is measured adequately is that of labor, estimated by the compensation of employees.\(^5\) It is obviously biased downward, since some compensation of labor is probably categorized as net operating surplus, but averaging 57% of gross domestic income, compensation of employees contains the bulk of the income share of labor (at least 80% by traditional measures).

\(^5\)NIPA table 1.15, titled “Price, cost and profit per unit of real gross value added of nonfinancial domestic corporate business”, provides unit labor costs and profits. However, the unit profits are only from current production, and it is unclear exactly what the BEA includes as profits.
persistently wandered off from its average value of .57. The figure also includes the quarterly rate of inflation, as an annually compounded rate computed from the GDP deflator reported by the BEA. The two series appear to move together, however, according to our model, this co-movement could be due to both series responding to changes in the degree of competition \( \theta_t \), the only variable that affects the income share of labor in the absence of nominal rigidities (assuming \( \alpha \) is constant). When it increases and the economy becomes more competitive, a smaller share of income goes to profits and a larger share goes to labor and capital. While there is no reason to believe that the rate of inflation is affected by the income shares, increased competition raises real output, which can impact inflation. As a result, changes in the degree of competition could generate a co-movement between inflation and the income share of labor even without rigidities. Therefore, we need to control for such changes to study the relevance of price rigidities.

9 Estimation

Controlling for changes in competition \( \theta_t \) requires data on the overall degree of competition in the U.S. economy. This cannot be measured from mark-ups, since according to our model, nominal rigidities affect these. Elasticities of substitution and market shares are not available for the economy as a whole. Any aggregate measure of competition, such as the number of firms, hiring, bankruptcies, start-ups, or production, are very noisy measures of competition, since they are affected by all sorts of shocks, including those to productivity. Hence, our strategy is to correct the aggregate measure of competition for changes in productivity, and any other shocks and variables that might affect it.

According to the model above, in the absence of nominal rigidities, real aggregate output

---

6See NIPA table 1.1.4 at www.bea.gov.
7Real unit profits from NIPA table 1.15 are sometimes used as an aggregate measure of mark-ups and the degree of competition \( \theta_t \). However, according to our model, real unit profits are affected by nominal rigidities, and are therefore not a reliable measure of the degree of competition. In particular, they cannot distinguish between the effects of nominal rigidities and changes in competition.
is a function of the degree of competition, total factor productivity, capital and government purchases,

\[ Y_t = H(\theta_t, A_t, K_t, G_t) \]  

(38)

The function \( H \) implicitly defines the elasticity \( \theta_t \), and therefore the inverse of the mark-up, as a function of output, total factor productivity, capital and government purchases,

\[ \frac{\theta_t - 1}{\theta_t} = \tilde{H}(Y_t, A_t, K_t, G_t) \]  

(39)

so any change in output not caused by variations in productivity, capital or government purchases, must be due to changes in competition. Inserting \( \tilde{H} \) into the income share of labor in the absence of nominal rigidities (35), exploiting that \( \omega_t = \frac{\theta_t}{\theta_t - 1} \), yields

\[ \frac{W_tN_t}{P_tY_t} = (1 - \alpha) \times \tilde{H}(Y_t, A_t, K_t, G_t) \]  

(40)

Data for total factor productivity \( A_t \) is computed from Solow residuals,

\[ A_t = \frac{Y_t}{K_t^{\alpha}N_t^{1-\alpha}} \]  

(41)

Likewise, the capital stock \( K_t \) is computed with historical data for net investment,

\[ K_t = K_0 + \sum_{j=0}^{t-1} I_j - \delta K_j \]  

(42)

given an initial capital stock \( K_0 \). Inserting into the income share of labor (40) yields

\[ \frac{W_tN_t}{P_tY_t} = h \left( Y_t, N_t, G_t, \sum_{j=0}^{t-1} I_j - \delta K_j, K_0 \right) \]  

(43)
Applying a first-order Taylor expansion to $h$ yields

$$
\frac{W_t N_t}{P_t Y_t} = \phi_0 + \phi_1 Y_t + \phi_2 N_t + \phi_3 G_t + \phi_4 \left( \sum_{j=0}^{t-1} I_j - \delta K_j \right) + \varepsilon_t
$$

where $\varepsilon_t$ captures the approximation error. Higher-order approximations can be applied to achieve a better fit.

According to our model, in the absence of nominal rigidities the income share of labor only depends on the degree of competition $\theta_t$ and the constant $\alpha$, which should both be unaffected by inflation. Hence, controlling for changes in competition, measured as changes in GDP not caused by variations in productivity, capital or government purchases, the rate of inflation should be insignificant for the income share of labor if nominal rigidities are irrelevant. In other words, when including the rate of inflation in our approximated expression for the income share of labor (44) and estimating it with data, inflation should not be statistically significant.

Table 2 summarizes the results of estimating a first-order approximation of the income share of labor (44) using data from the BEA.\(^8\) In addition to including the rate of inflation, with coefficient $\phi_5$, the equation includes a first-order autoregressive error term, AR(1), to capture the serial correlation in the residuals (estimated with TSLS using four lags of both the dependent and independent variables as instruments). The numbers in parenthesis are the estimated standard deviations, which are robust to both heteroskedasticity and serial correlation (Newey-West 4\(^{th}\)-order HAC).\(^9\) The estimated autoregressive coefficient indicates the presence of a unit root, which could not be rejected with formal tests, so the equation was re-estimated in first differences (second row in table 2). With or without a

---

\(^8\)Employment is civilian employment measured in millions, while GDP, government expenditure and investment are in trillions of inflation-adjusted dollars. The rate of inflation is measured in decimals as a quarterly rate compounded annually.

\(^9\)Applying a first-order Taylor approximation to the income share of labor (43) in terms of the logarithm of the variables could help remedy heteroskedasticity, but then the estimates become sensitive to the unknown initial capital stock $K_0$. Estimating a second-order approximation without logarithms yields similar results to those reported above, but almost none of the second-order terms are statistically significant.
unit root, our first-order Taylor expansion (44) should apply both in levels and differences. In fact, not only are the estimates not significantly different, they are very similar. In both cases, inflation is significant at 5%, but not at 1%.

Controlling for changes in the degree of competition and serial correlation, the co-movement between inflation and the income share of labor is negative ($\phi_5 < 0$).\textsuperscript{10} Accepting that nominal rigidities make total factor productivity $A_t$ deteriorate with inflation, or at least that it does not improve with inflation, the impact on $Q_t$ must, according to our model (34), be negative. This requires the average mark-up paid by households to increase with inflation, implying that inflation reduces output. Hence, the data suggests that nominal rigidities have a significant effect on aggregate variables, but that contrary to what the Phillips curve suggests, inflation reduces output, due to both a negative impact on total factor productivity and a higher average mark-up, so that there is no trade-off between inflation and output.

The errors with which economic aggregates are measured can produce endogeneity and lead to biased and inconsistent estimates. Assuming such measurement errors are independent over time, they can be corrected for by estimating our equation (in differences), using two-stage least squares with (four) lags of both the dependent and independent variables as instruments.\textsuperscript{11} The results, reported in the third row of table 2, show that the obtained

\begin{table}
\centering
\begin{tabular}{l|ccccccc}
 & $\phi_0$ & $\phi_1$ & $\phi_2$ & $\phi_3$ & $\phi_4$ & AR(1) & $\phi_5$ & $R^2$
\hline
$W_{t+1}$ & .441 & -28.564 & .003 & 46.275 & .227 & .970 & -0.057 & .97
\hline
$W_tN_t$ & .000 & -29.512 & .002 & 31.813 & 2.065 & - & -0.025 & .21
\hline
$\Delta W_tN_t$ & -0.001 & -26.450 & .004 & 83.356 & .041 & - & -0.073 & .02
\hline
\end{tabular}
\caption{Regressions of the income share of labor on real GDP, employment, government expenditure, cumulative net investment and the GDP deflator, in levels and differences.}
\end{table}

\textsuperscript{10} Including lags of inflation in the estimated equation (in levels or differences) we rule out that the negative coefficient on inflation is due to a lagged positive effect.

\textsuperscript{11} Since our measure of the income share of labor is computed with estimates of gross domestic income,
estimates are not significantly different from our earlier ones, so either measurement error has a negligible impact on the estimates, or it is correlated over time and therefore not corrected for by instrumenting with lags. A Hausman test cannot reject the null hypothesis of no measurement error.

10 International data

The OECD provides quarterly national income account data for more than thirty countries. Unfortunately, for most of these it has less than twenty years of observations. For the U.S it starts in 1947, the U.K. in 1955, Australia in 1960, Canada in 1961, France in 1978, Japan in 1980, Belgium and Italy in 1981. For the remaining countries, the series start anywhere between 1988 and 2000, so these were discarded for having too few observations. The data, which is seasonally adjusted, is plotted in figure 2, which includes both the income share of labor and the rate of inflation computed from the GDP deflator. Table 3 provides summary statistics.

Table 4 reports the results from running the same regression estimated above for the U.S. with the OECD data. This is done for each country individually, in terms of differences, without instrumenting for measurement error. Controlling for changes in the degree of competition, inflation is statistically significant for explaining movements in the income whose measurement error could easily be correlated with that of GDP, which is an independent variable, we must also instrument for the income share of labor.\footnote{See www.sourceoecd.org/database/OECDStat. The data is subject to retroactive revisions, the numbers used are those reported in April 2010. In order to facilitate the comparison across countries, this section uses the data for the U.S. reported by the OECD. The Belgian data is from Eurostat, see epp.eurostat.ec.europa.eu. The investment data for the UK is also from Eurostat. The Japanese data is from the Japanese cabinet office, see www.esri.cao.go.jp. Employment is measured in millions, while GDP, government expenditure and investment are in trillions of inflation-adjusted local currency. The quarterly rate of inflation is measured in decimals at an annually compounded rate. All series have been seasonally adjusted.}

\footnote{Employment is measured as civilian employment, except for France, for which total employment is used due to it being available further back in time. For many countries, the unavailability of employment data far enough back in time limits the number of observations in the regressions.}

\footnote{One cannot reject that the income share has a unit root for any of the countries in our sample.}
Table 3: Summary statistics for GDP deflator and income share of labor, seasonally adjusted, for Australia, Belgium, Canada, France, Italy, Japan, the U.K. and U.S.

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP deflator</th>
<th>Income share labor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>AU</td>
<td>5.5</td>
<td>5.1</td>
</tr>
<tr>
<td>BE</td>
<td>2.7</td>
<td>2.1</td>
</tr>
<tr>
<td>CA</td>
<td>4.4</td>
<td>4.0</td>
</tr>
<tr>
<td>FR</td>
<td>3.8</td>
<td>3.6</td>
</tr>
<tr>
<td>IT</td>
<td>5.5</td>
<td>4.8</td>
</tr>
<tr>
<td>JP</td>
<td>.26</td>
<td>2.3</td>
</tr>
<tr>
<td>UK</td>
<td>6.0</td>
<td>6.4</td>
</tr>
<tr>
<td>US</td>
<td>3.4</td>
<td>2.7</td>
</tr>
</tbody>
</table>

It is not significant for France, Italy and Japan, but we have fewer observations for these countries than for most of the others. In addition, Japan has experienced considerably less inflation than any of the other countries in our sample. As above, we find that inflation has a negative impact on the income share of labor. Assuming that inflation does not raise total factor productivity $A_t$, this implies that $Q_t$ must fall with inflation, meaning inflation reduces output, so that there is no trade-off between the two.

Because measurement error is more likely to be independent across countries than inter-temporally within a country, we now use the data for the other countries in our sample as instruments when estimating for each country using two-stage least squares. The results are given in table 5, and in terms of the impact of inflation, they are similar to those obtained with ordinary least squares, except that inflation is now not significant for the U.S. However, using Hausman’s test for measurement error, we cannot reject the null hypothesis of no measurement error for any of the countries in our sample, meaning that either measurement error is not important, or it is correlated across countries and therefore not instrumented for correctly. Exploiting this, and the fact that one cannot reject that the coefficient on inflation is the same for all the countries in our sample (estimated jointly as a panel with

---

15The estimates for the U.S. in table 3 differ somewhat from those in table 2 because the data reported for the U.S. by the OECD and the BEA are not identical.
Table 4: OLS regressions of the income share of labor on real GDP, employment, government expenditure, cumulative net investment and the GDP deflator, all in differences, for various countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>$\phi_0$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
<th>$\phi_5$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU</td>
<td>.002</td>
<td>-1.756</td>
<td>.009</td>
<td>-1.061</td>
<td>.041</td>
<td>-.032</td>
<td>.16</td>
</tr>
<tr>
<td>BE</td>
<td>-.002</td>
<td>-7.352</td>
<td>.074</td>
<td>5.240</td>
<td>.821</td>
<td>-.067</td>
<td>.40</td>
</tr>
<tr>
<td>CA</td>
<td>.001</td>
<td>-2.031</td>
<td>.017</td>
<td>.458</td>
<td>.073</td>
<td>-.031</td>
<td>.35</td>
</tr>
<tr>
<td>FR</td>
<td>-.000</td>
<td>-1.506</td>
<td>.023</td>
<td>1.193</td>
<td>.038</td>
<td>-.011</td>
<td>.36</td>
</tr>
<tr>
<td>IT</td>
<td>-.004</td>
<td>-1.030</td>
<td>.002</td>
<td>1.284</td>
<td>.291</td>
<td>-.021</td>
<td>.32</td>
</tr>
<tr>
<td>JP</td>
<td>.000</td>
<td>-3.228</td>
<td>.007</td>
<td>-.769</td>
<td>.124</td>
<td>-.006</td>
<td>.41</td>
</tr>
<tr>
<td>UK</td>
<td>-.000</td>
<td>-2.238</td>
<td>.009</td>
<td>1.832</td>
<td>.198</td>
<td>-.049</td>
<td>.37</td>
</tr>
<tr>
<td>US</td>
<td>.000</td>
<td>-48.398</td>
<td>.004</td>
<td>-1.540</td>
<td>3.598</td>
<td>-.028</td>
<td>.42</td>
</tr>
</tbody>
</table>

Table 5: TSLS regressions of the income share of labor on real GDP, employment, government expenditure, cumulative net investment and the GDP deflator, all in differences, with 109 observations, using data for all other countries as instruments.

<table>
<thead>
<tr>
<th>Country</th>
<th>$\phi_0$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
<th>$\phi_5$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU</td>
<td>-.000</td>
<td>-.736</td>
<td>.018</td>
<td>-3.883</td>
<td>.047</td>
<td>-.067</td>
<td>.28</td>
</tr>
<tr>
<td>BE</td>
<td>-.001</td>
<td>-8.743</td>
<td>.134</td>
<td>7.440</td>
<td>.555</td>
<td>-.060</td>
<td>.38</td>
</tr>
<tr>
<td>CA</td>
<td>-.000</td>
<td>-1.205</td>
<td>.001</td>
<td>2.130</td>
<td>.083</td>
<td>-.067</td>
<td>.42</td>
</tr>
<tr>
<td>FR</td>
<td>.001</td>
<td>-1.650</td>
<td>.029</td>
<td>.801</td>
<td>.008</td>
<td>-.014</td>
<td>.36</td>
</tr>
<tr>
<td>IT</td>
<td>-.005</td>
<td>-.907</td>
<td>.004</td>
<td>.294</td>
<td>.338</td>
<td>-.020</td>
<td>.30</td>
</tr>
<tr>
<td>JP</td>
<td>.001</td>
<td>-3.064</td>
<td>.008</td>
<td>-6.749</td>
<td>.108</td>
<td>-.020</td>
<td>.40</td>
</tr>
<tr>
<td>UK</td>
<td>-.001</td>
<td>-.873</td>
<td>.006</td>
<td>-.631</td>
<td>.170</td>
<td>-.052</td>
<td>.45</td>
</tr>
<tr>
<td>US</td>
<td>-.001</td>
<td>-41.031</td>
<td>.004</td>
<td>-38.941</td>
<td>5.778</td>
<td>-.028</td>
<td>.42</td>
</tr>
</tbody>
</table>

OLS), this coefficient is re-estimated assuming it is identical across countries, using ordinary least squares. Doing so, we find that inflation is statistically significant for the group as a whole, with a coefficient estimate on inflation of -.038 and an estimated standard error of .005**, based on a total of 1,309 observations.
11 Conclusions

In order for an inflation-output trade-off to arise as a result of nominal rigidities making inflation distort price-setting due to menu costs, sticky prices, sticky information or monetary misperceptions, the income share of labor must increase with inflation. However, after controlling for changes in the degree of competition and serial correlation, we find that the income share of labor falls with inflation, in the U.S. and the seven other OECD countries for which we have at least 25 years of data. This negative co-movement is statistically significant for the group of countries as a whole, and for each country individually, except for the three countries for which we have the least amount of data. These results are evidence of the relevance of nominal rigidities at the aggregate level, but refute that such rigidities give rise to an inflation-output trade-off, since they imply that inflation has a negative impact on output.
References


Figure 1: Income share of labor and rate of inflation in U.S.
Figure 2: Income shares of labor and rates of inflation for select OECD countries

AU BE CA FR IT JP UK US