Dynamic Free Trade Networks: Some Numerical Results

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Abstract

To investigate dynamic paths of network formation of free trade agreements (FTAs), we conduct simulations of Goyal and Joshi’s (2006) model of FTA network formation game with many countries. We compare the results between two protocols regarding the choice of the pair that decides if they form (or sever) an FTA link in each round in the network evolution process. The first protocol is the random protocol in which a pair of countries is randomly chosen in each period. The other protocol is the maximum protocol in which the country that has the largest incentive can propose to form or sever an FTA link. We find that with the random protocol, FTA evolution processes reach the complete FTA network in some cases and its likelihood becomes smaller if the number of countries grows. With the maximum protocol, the network evolution always ends with a unique final FTA network for each case of \( n \in 5, 6, \cdots, 100 \) countries. The final FTA network may or may not be the complete network.

Very preliminary and incomplete

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1 Introduction

In recent decades, most countries in the world appear to be eager to form free trade agreements (FTAs) with other countries. Whether this trend is a building block or a stumbling block for the movement towards global free trade partly and importantly depends on how far this FTA formation goes. Most of the existing works for FTAs are not appropriate to answer to this question in a meaningful way. Large part of the literature analyzes FTA formation in three-country model, which is obviously insufficient to derive practically meaningful predictions about dynamic paths of FTA formation in the world of many countries.

Furusawa and Konishi (2005, 2007) and Goyal and Joshi (2006) analyze this problem in the context of the network formation game (Jackson and Wolinsky, 1996). The network formation game is a powerful tool to analyze a complex web of free trade agreements in the world. However, their main goals are to find pairwise stable networks in static models, so they are silent about dynamic paths leading to the stable network that they have derived. Thus, there is no reason to believe that the worldwide FTA network formation will lead to the stable networks.

To derive a meaningful prediction of the worldwide FTA evolution, we need a dynamic analysis of the model that involves reasonably many countries. Obviously it is not an easy task. Indeed, it is almost impossible to theoretically derive rich insightful results that can be applied to the reality. Numerical simulations are quite useful in such situations. They show us dynamic paths leading to stable FTA networks, from which we can derive some insights on the characteristics of dynamic paths of FTA formation.

We conduct simulations of Goyal and Joshi’s (2006) model in which there are $n$ symmetric countries. We adopt the symmetric model because we want to isolate purely dynamic effects of FTA formation on dynamic paths and the final FTA network that will be reached after many rounds of bilateral FTA formation. We compare the results between two protocols regarding the choice of the pair that decides if they form (or sever) an FTA link in each round in the network evolution process. The first protocol is the random protocol in which a pair of countries is randomly chosen in each period. The other protocol is the maximum
protocol in which the country that has the largest incentive can propose to form or sever an FTA link.

First, we conduct simulations in the case of 10 countries. With the random protocol, about a half of simulations show that FTA evolution reaches the complete network, in which every pair of countries form a link, so that global free trade will result. With the maximum protocol, the network evolution always reaches the complete network. But if countries incur some costs to form or sever a link, the final network may consist of three components each of which is the complete sub-network.

We check the robustness of the results obtained in the case of 10 countries, by extending simulations to the cases of various numbers of countries. In the case of 30 countries, for example, the final network is less likely to be the complete network with the random protocol, and it consists of two complete components with different size with the maximum protocol. We further examine FTA evolution in the case of $n$ countries with $n$ ranges from 5 to 100. We find that with the maximum protocol, all network evolution processes reach a unique final FTA network in each case of a particular $n$. The final network may or may not be the complete network, but all of incomplete networks are almost complete in the sense that each of them consists of a large complete component and one or two small complete components.

2 The Model

Let $N = \{1, 2, \ldots, n\}$ be the set of $n$ countries ($n \geq 3$), each of which is populated by identical consumers who consume a numeraire good and a manufacture good. There are $n$ firms that produce the homogeneous manufacture good, each of which is located in each of $n$ countries (firm $i$ is located in country $i$). The manufacture good market in each country is segmented from other markets in other countries, such that firms engage in the Cournot competition in each country. This setting of the market structure is common in the literature of international trade of a good in oligopoly; Goyal and Joshi (2006) adopt the setting in their analysis of FTA network formation game. The basic model of this paper, which we describe below, is the same as that of Goyal and Joshi (2006).
Every consumer’s preferences are given by a quasi-linear utility function with quadratic subutility for the manufacture good. Let $q_{ki}$ denote the quantity of the manufacture good that firm $k$ sells in country $i$ and $Q_i = \sum_{k \in N} q_{ki}$ denote the total quantity of the manufacture good that is sold in country $i$. Then, the representative consumer’s utility is given by

$$u = \alpha Q_i - \frac{1}{2} Q_i^2 + y,$$

where $y$ denotes the consumption level of the numeraire good. The derived demands for the manufacture good of the representative consumer in country $i$ is given by

$$p_i = \alpha - Q_i,$$

(1)

where $\alpha > 0$ is a parameter. We normalize the mass of consumers in each country to be one, so the demand function expressed in (1) also represents the aggregate demand function in country $i$.

Firms are assumed to have the same cost function, but have different costs of supply in each country. We assume that firms incur no fixed costs and that the unit cost of production equals $c > 0$. Firm $k$ is faced with the tariff $\tau_{ki}$ when it sells its products in country $k$. The tariff rate is zero for firm $k$ if country $k$ has an FTA with country $i$ or $k = i$ so that firm $k$ is located in country $i$. Firm $k$ is faced with a positive tariff $\tau_i$, on the other hand, if country $k$ does not have an FTA with country $i$. That is, letting $C_i$ denote the set of FTA partner countries for country $i$ plus country $i$ itself, we have

$$\tau_{ki} = \begin{cases} 0 & \text{if } k \in C_i \\ \tau_i & \text{if } k \notin C_i \end{cases}$$

Let $l_i$ denote the number of FTA partner countries for country $i$ (or the number of country $i$’s FTA links), i.e., $|C_i| - 1$. Then, we define the average external tariff of country $i$ can be written by

$$\bar{\tau}_i \equiv \frac{1}{n-1} \sum_{k \in N} \tau_{ki} = \frac{n-1-l_i}{n} \tau_i.$$  

(2)

In each country $i$, all individual firms compete with others in the Cournot fashion, so the Nash equilibrium supply by firm $j$, aggregate supply, and price are given by

$$q_{ji} = \frac{\alpha - c - [(n+1)\tau_{ji} - n\bar{\tau}_i]}{n+1},$$
\[ Q_i = \frac{n(\alpha - c - \bar{\tau}_i)}{n + 1}, \]
\[ p_i = \frac{\alpha + n(c + \bar{\tau}_i)}{n + 1}, \]
respectively. Then it follows that the consumer surplus \( CS_i \), country \( i \)'s tariff revenue \( TR_i \), and firm \( i \)'s profits earned in country \( k \) can be computed as

\[
CS_i = \frac{1}{2} \left[ \frac{n(\alpha - c - \bar{\tau}_i)}{n + 1} \right]^2,
\]
\[
TR_i = \sum_{k \in N \setminus C_i} \frac{\tau_i \{\alpha - c - [(n + 1)\tau_i - n\bar{\tau}_i]\}}{n + 1},
\]
\[
\pi_{ik} = \left\{ \frac{\alpha - c - [(n + 1)\tau_{ik} - n\bar{\tau}_k]}{n + 1} \right\}^2.
\]

Without loss of generality, we simplify the exposition by setting \( \alpha - c = 1 \). Then, substituting (2) into the above to obtain the social welfare, which is nothing but the total surplus derived from the market of the manufacture good, for country \( i \):

\[
W_i = \frac{1}{2} \left[ \frac{n - (n - 1 - l_i)n\tau_i}{n + 1} \right]^2 + \left( n - 1 - l_i \right) \left[ 1 - \left( l_i + 2 \right)n\bar{\tau}_i \right] \frac{n\tau_i}{n + 1} + \sum_{k \in C_i} \left[ \frac{1 + (n - 1 - l_k)n\tau_k}{n + 1} \right]^2 + \sum_{k \notin C_i} \left[ \frac{1 - (l_k + 2)n\tau_k}{n + 1} \right]^2.
\]

As shown in this equation, social welfare consists of the consumer surplus, tariff revenue, firm \( i \)'s profits derived from its domestic country and its FTA partner countries, and firm \( i \)'s profits derived from other outside countries. Note that due to the quasi-linearity of a consumer’s utility function, social welfare correctly measures every individual’s utility.

We suppose that each country \( i \) selects its optimum external tariff for any given FTA network. The optimum tariff that maximizes social welfare is given by

\[
\tau(l_i) = \frac{3}{n + 7 + (2n + 5)l_i}.
\]

Note that the optimum tariff decline as country \( i \) forms more FTA links with other countries, i.e., \( \tau'(l_i) < 0 \), which is known as the tariff complementarity effect.

The worldwide FTA network is described as a collection of FTA links that pairs of countries form. Let \( ij \) denote the link between countries \( i \) and \( j \). The network \( g \) is the
collection of all such links; \( ij \in g \) if country \( i \) and country \( j \) has an FTA and \( ij \not\in g \) otherwise. We represents the number of FTA links that country \( i \) has in the FTA network \( g \) by \( l_i(g) \).

For any given FTA network \( g \), social welfare of country \( i \) is given by

\[
W_i(g) = \frac{1}{2} \left[ \frac{n - (n - 1 - l_i(g))\tau(l_i(g))}{n + 1} \right]^2 + \frac{(n - 1 - l_i(g))[1 - (l_i(g) + 2)\tau(l_i(g))]\tau(l_i(g))}{n + 1}
+ \sum_{k \in C_i(g)} \left[ \frac{1 + (n - 1 - l_k(g))\tau(l_k(g))}{n + 1} \right]^2 + \sum_{k \not\in C_i} \left[ \frac{1 - (l_k(g) + 2)\tau(l_k(g))}{n + 1} \right]^2. \tag{3}
\]

Now, we are ready to derive some results, all of which (perhaps except for Proposition 1 below) have already been known.

The first one asserts that welfare of a country declines as an FTA partner country signs an FTA with another country. We see it readily from (3) recalling that an increase in \( l_j \), where \( j \in C_i \), will decrease \( \tau(l_j) \). We record this result as a lemma.

**Lemma 1** Social welfare of country \( i \) declines with the number of its FTA partner country \( j \)’s FTA links \( l_j \) (where \( j \in C_i \) and \( j \neq i \)).

The second lemma, which confirms Yi’s (2000) result, is what can be called the strong tariff complementarity effect. The lemma claims that an FTA benefits outside countries. We have seen that the external tariff decline as a country forms an FTA link. On the one hand, the markets of the countries that form the FTA become more competitive, so the FTA tends to reduce profits for a firm that supplies the good from outside of these countries. The outside firm will benefit from the FTA, on the other hand, because these countries lower their individual external tariffs as we have seen. The lemma (surprisingly) shows that the latter effect always outweighs the former.

**Lemma 2** Social welfare of country \( i \) increases with the number of an unlinked country’s FTA partners, i.e., \( W_i \) increases with \( l_j \) if \( j \not\in C_i \).

**Proof.** We see from (3) that we need only show that \((l_j + 2)\tau(l_j)\) decreases with \( l_j \). But it is easy to see that the derivative of \((l_j + 2)\tau(l_j)\) with respect to \( l_j \) (which is the expression
on the left-hand side of the inequality below) is negative:
\[-\frac{9(n + 1)}{(n + 7 + (2n + 5)l_j)^2} < 0.\]

Q.E.D.

Next, we consider the effect of an increase in its own link on social welfare. An increase in \(l_i\) increases the competitiveness of the market in country \(i\). So the consumer surplus increases while firm \(i\)'s domestic profits decrease. Tariff revenue also decreases as a result because the tariff against the new partner country will be eliminated and the external tariff rate decreases (further away from its tariff-revenue maximizing level). The next lemma shows that the overall impact of an increase in \(l_i\) on social welfare of country \(i\) is negative and that this negative impact becomes smaller as \(l_i\) increases.

**Lemma 3** Social welfare of country \(i\) decreases with the number of its own FTA links, i.e., \(l_i\). The size of this negative impact becomes smaller as \(l_i\) increases, i.e., \(|\partial W_i/\partial l_i|\) decreases with \(l_i\).

**Proof.** We obtain from (3) that
\[
\frac{\partial W_i}{\partial l_i} = -\frac{\tau(l_i)}{(n + 1)^2} A(l_i),
\]
where \(A(l_i) = 3 + [(n + 3)(n - 2) - (2n + 5)l_i]\tau(l_i)\). So all we need to show is \(A(l_i) > 0\) for any \(l_i = 0, 1, 2, \ldots, n - 2\). Now, we have
\[
A'(l_i) = -\frac{n + 7 + (n + 3)(n - 2)}{(n + 7 + (2n + 5)l_i)^2} < 0,
\]
\[
A(n - 2) = \frac{3[n + 7 + (n + 3)(n - 2)]}{n + 7 + (2n + 5)(n - 2)} > 0.
\]
That is, \(A\) is decreasing in \(l_i\) with \(A(n - 2) > 0\). Thus, we have shown that \(A(l_i) > 0\) for any \(l_i = 0, 1, 2, \ldots, n - 2\). Q.E.D.

Despite that Lemma 3 claims that an FTA decreases the sum of the consumer surplus, tariff revenue, and the domestic firm’s profits in the domestic market, it does not necessarily mean that an FTA reduces social welfare of a signatory. The next lemma shows that an FTA increases the domestic firm’s profits earned in the partner country’s market, as expected.
Lemma 4 Forming an FTA link with country \( j \) increases firm \( i \)'s profits earned in country \( j \). This increment in profits becomes smaller as \( l_j \) increases.

**Proof.** Consider the case in which a link \( ij \) is added to the FTA network \( g \). Firm \( i \)'s profits in country \( j \) increases from

\[
\pi_{ij}(g) = \left[ 1 - \frac{(l_j + 2)\tau(l_j)}{n + 1} \right]^2
\]

to

\[
\pi_{ij}(g + ij) = \left[ 1 + \frac{(n - 2 - l_j)\tau(l_j + 1)}{n + 1} \right]^2.
\]

It is apparent that \( \pi_{ij}(g + ij) > \pi_{ij}(g) \). Furthermore, as \( l_j \) increases (so that \( g \) also changes accordingly), \( \pi_{ij}(g) \) increases as the proof of Lemma 2 shows while \( \pi_{ij}(g + ij) \) decreases. Thus, the increase in firm \( i \)'s profits in country, i.e., \( \pi_{ij}(g + ij) - \pi_{ij}(g) \), decreases as \( l_i \) increases. Q.E.D.

Lemma 3 and Lemma 4 imply a following important proposition.

**Proposition 1** Incentive for country \( i \) to form an FTA link with country \( j \) increases with the number of country \( i \)'s FTA links and decreases with the number of country \( j \)'s FTA links.

This incentive may be positive or negative depending on the FTA network as we will see shortly in the next section. Thus, interim networks in the evolution of FTA networks affect future path of the network evolution, and the final FTA networks may be quite different from one another if an interim network happens to be different. We study the evolution of FTA network and characterize its features appealing to numerical simulations in the next two sections.

### 3 Numerical Simulation: Case of 10 countries

Do countries keep forming FTA links until the complete FTA network, which realizes the worldwide free trade, is reached? If not, how do final FTA networks look like? Is the final network unique? Or are there multiple possible final FTA networks? If there are multiple final networks, what is the chance for each particular final FTA network to be realized? How
does the FTA network evolve until it reaches the final network? To answer these questions, we conduct numerical simulations of the model described in the previous section. In this section, we consider the case where \( n = 10 \). We will observe some important features of the FTA dynamics in these simulations, and will examine their robustness in the next section by repeating the simulations with different numbers of countries.

The simulation starts with the empty network, in which no pair of countries has an FTA link. In each round, a pair of countries is chosen and the decision as to whether they form a link (if they are not linked) or whether they sever the link (if they are linked). The selected pair of countries form a link (or keep a link) if and only if both of them have incentive to do so. Whereas the pair of countries sever a link if and only if at least one of the countries benefits by doing so. Rounds are repeated sufficiently long until the network evolution ceases.

We consider two protocols for the choice of a pair of countries in each round. The first one is the random protocol in which a pair of countries is chosen randomly in each round. If the pair has been linked, the choice is made on whether they keep the link. If the pair has not been linked, on the other hand, the choice is made on whether they form a link. The second one is the maximum protocol in which the country that has the largest incentive in making a decision is chosen in each period as a proposer. First, for each country \( i \), an increment of its payoff by forming a new link or severing the exiting link is calculated for every possible pair \( ij \) with another country \( j \). Then the maximum welfare increment for country \( i \) is found with the corresponding country \( j \). The country that has the largest welfare increment is chosen as a proposer in each round, and the proposer chooses the corresponding partner country to form a new link or sever the existing link.

We also examine the effect of negotiation costs (and/or adjustment costs) of forming or severing an FTA. We conduct simulations varying these costs. The benchmark case assumes zero negotiation costs. When we introduce negotiation costs, we assume for simplicity that the costs of forming a link are the same as the costs of severing a link. In reality, they may well be different. But it is not obvious which one is greater.

As Proposition 1 indicates, country \( i \)'s incentive to form a link with country \( j \) depends
positively with $l_i$ and negatively with $l_j$. Table 3 shows increments of country $i$’s social welfare for all possible $l_i$ and $l_j$ when $n = 10$. As the table indicates, country $i$ has incentive to form a link with $j$ if $l_i$ is large and $l_j$ is small. But country $i$ does not have incentive to form a link with $j$ otherwise. The table also gives us an important information about country $i$’s choice of its partner in any FTA network.

### 3.1 Random Protocol

First, we report simulation results when the proposer is chosen according to the random protocol. We conduct 1,000 simulations, each of which consists of 1,000 rounds.

When the negotiation costs are zero, each FTA network formation process lead to one of three final FTA networks. The first one is the complete network in which the number of total links is 45. This arises 527 times out of 1,000 simulations. In the final FTA network, social welfare of any country equals .495868. The second one is an incomplete FTA network in which 9 countries form complete network within themselves while the remaining country has a link with one of these 9 countries, as Figure 1 shows. This arises 176 times out of 1,000 simulations. In the final network, the hub country has the highest social welfare of .509852, which is also higher than the social welfare for a country in the complete network. The remaining 8 countries in the group have the second highest social welfare of .494487.

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1Each entry of Table 3 is inflated 1,000 times and rounded off for clarity.
This social welfare is smaller than the one in the complete network. The spoke country has the lowest social welfare of .485934.

The third one is another incomplete network that consists of two groups of countries: one group is the complete component with 8 countries and the other is the complete component with 2 countries (i.e., two countries with a link). Figure 2 shows an example of this network. This network arises as the final network 297 times out of 1,000 simulations. Social welfare of each country in the larger group is .493603, which is lower than the one in the complete network, while that of a country in the smaller group is .497808, which is greater than the one for a country in the larger group. It is even greater than the one in the complete network. These countries benefit from low tariff barriers by countries in the larger group, in addition to the benefit derived from their own FTA.
The first one of the following observations is the summary of the results reported above. The other two are the ones that we observe by closely examining the dynamic process of FTA network formation.

**Observation 1** With the random protocol without link costs, the final FTA networks are not unique. The complete network is the final network about a half of times. In the case of incomplete networks, countries are weakly partitioned into two groups and they form individual complete sub-networks.

**Observation 2** The key to predict the final FTA network is the timing of the link formation between the last isolated countries. If the last isolated pair forms a link at an early stage, the network will evolve to the complete network. If the last pair forms a link relatively late, the network will be the balloon type in which one country has only one link with a hub country in the large group. If the last pair forms a link very late, there will be two separate groups of countries in the final FTA network.

**Observation 3** Countries sometimes sever a link at an early stage of network evolution. It is because in some cases, benefits of a bilateral FTA will be lost for a country with only one link, for example, if the partner country has signed FTAs with other countries after they formed a link.

Next, we turn to the case in which countries incur the costs $c = .004$ to form or sever a link. As expected, the number of links each country forms will decrease. In this specific case, the maximum number of links over all countries is three. The following table summarizes the results of 1,000 simulations.

**Observation 4** Even when countries incur some link costs, characteristics of the final network are similar to the ones without a link cost. Naturally countries have smaller numbers of links in the final stage of network evolution.
<table>
<thead>
<tr>
<th>Number of total links</th>
<th>Likelihood</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>168/1000</td>
<td>2 groups (2 countries + 8 countries)</td>
</tr>
<tr>
<td>13</td>
<td>259/1000</td>
<td>2 groups (2 countries + 8 countries)</td>
</tr>
<tr>
<td>14</td>
<td>262/1000</td>
<td>1 group (Asymmetry in numbers of links)</td>
</tr>
<tr>
<td>15</td>
<td>311/1000</td>
<td>1 group (Every country has 3 links)</td>
</tr>
</tbody>
</table>

Table 2: Final Network with the Random Protocol with a Link Cost

### 3.2 Maximum Protocol

It is easy to characterize the final FTA networks with the maximum protocol without link costs. It is always (1,000 times out of 1,000 simulations) the complete network. The rounds end with the complete network with 45 links in 45 rounds. But we emphasize that evolution processes are not unique.

The evolution processes of early stages have a common pattern. After a pair of countries forms a link, another pair of countries forms another link. Then either one of a pair will form a link with either one of the other pair. Then the complete network is formed within these four countries, followed by another link with another pair of countries. Then this new pair of countries will start forming links with countries with the original group. The network will evolve until it reaches the complete network in 45 rounds, which means that no country severs a link in the process.

**Observation 5** With the maximum protocol without link costs, the final FTA network is always the complete network when \( n = 10 \), although evolution processes may be different across simulations.

Next we report the results of 1,000 simulations with the link cost \( c = .004 \). Final network is always the same. It consists of three components: two complete components each with four countries and one pair of linked countries. Social welfare of each of the two groups is .492718, while that of a country in the pair is .495293. Countries in the smaller group enjoy higher social welfare than those in the larger group.

**Observation 6** If there is a link cost, the maximum protocol will induce countries to form groups.
It can be considered that this model environment of the maximum protocol with a link cost captures the reality better than any other environments that we consider there. Then, this observation can be interpreted as a caveat of FTA formation.

4 Numerical Simulation: Case of \( n \) Countries

In the case of 10 countries, we have obtained a rather strong result that with the maximum protocol with a link cost, FTA network evolution always reaches the complete network. Is this result robust? The main purpose of this section is to check the robustness of this observation and others obtained in the case of 10 countries.

We first report some observations when \( n = 30 \). Similarly to the case of 10 countries, we conduct 1,000 simulations in each case. In the case of 30 countries, there are 435 possible links, which are about 10 times more than those in the case of 10 countries. Thus, we allow 10,000 rounds of evolution, rather than 1,000 rounds, in the case of the random protocol.

With the random protocol without link costs, there are 15 final networks that FTA evolution has reached. The minimum number of total links is 331 while the maximum number of links is 435. The complete network results 48 times out of 1,000 simulations, which is significantly less than in the case of 10 countries. It is important but not surprising because chances that uneven interim networks emerge in the evolution process increases dramatically as the number of countries increases.

**Observation 7** With the maximum protocol without link costs, the likelihood that the final FTA network is the complete network is significantly lower when \( n = 30 \) than in the case where \( n = 10 \).

Introducing a link cost does not add much to our understanding in this case. We introduce the link cost \( c = .0003 \). The cost is much smaller than the cost we consider in the case of 10 countries. As the number of countries increases, the size of benefits or loss from a bilateral FTA link decreases. Thus, FTA evolution would stop at earlier stages if the link costs are the same. With this link cost, the final FTA network that we observe is either one-component
network of two-component network with asymmetric size (28 country component and 2 country component). The numbers of links are between 104 and 118.

Turning to the case of the maximum protocol, we find that the previous observation that the final FTA network is always the complete network is not robust to a change in the number of countries. When \( n = 30 \), the final network is always incomplete and always the one that consists of two components: one complete sub-network of 28 countries and the other complete sub-network of two countries. What remains the same when we increase the number of countries from \( n = 10 \) to \( n = 30 \) is that all 1,000 simulations lead to the same final network (of course beside the names of countries in particular positions).

**Observation 8** When there are 30 countries, FTA evolution always reach the same final FTA network that consists of two components of asymmetric size. The final FTA network with the maximum protocol without link costs is not necessarily the complete network.

With the link cost \( c = .0003 \), FTA evolution stops around 130 rounds with total links of 103 to 107. The final network consists of either 3 components (24 country component, 5 country component, and 1 isolated country) or two components (29 country component and 1 isolated country).

Finally, let us examine the case of the maximum protocol without link costs more closely. We have found that the final network is the complete network if \( n = 10 \), but it consists of two complete components of asymmetric size if \( n = 30 \). The task here is to see if which pattern is more likely in other cases with different numbers of countries. We conduct 1,000 simulations in each case of \( n \) countries with \( n \) varying from 5 to 100.

First, we find that in every case, the final FTA network is unique. The final FTA network may not be the complete network. Figure 3 depicts the number of total links in each case of \( n \) countries. The solid curve shows the number of total links in the complete network. As the figure shows, the final network is the complete network for some \( n \): this is indeed the case when \( n = 6, 8, 10, 12, 14, 50, 53, 58, 63, 68, 73, 78, 83, 88, 93, 98 \). The figure also suggests that the final network is not very different from the complete networks. Indeed, as expected, such a network consists of a few asymmetric complete components. For example, when \( n = 97 \), it
Observation 9 \textit{FTA evolution will lead to the complete FTA network in 16 cases out of 96 cases with different numbers of countries. In other cases, the final network consists of a few complete components of asymmetric size.}

5 Conclusion

We have conducted various simulations to reveal properties of dynamic FTA network formation. We have found among others that FTA network evolution reaches the complete FTA network in some cases if a pair of countries that make an FTA decision is chosen randomly in each round, but this likelihood decreases as the total number of countries increases. The complete FTA network is more likely to be reached if the country that has the highest incentive to form or sever a link makes a proposal in each round. The final network tends to have a few complete components with asymmetric size, if it is not the complete network.
We have built a model with symmetric countries to single out the dynamic effect of network evolution. But to predict a possible path of FTA formation in reality, it is important to conduct simulations with a model of asymmetric countries. This important task is left for future research.


