The Behavioral Economics of Crime Rates and Punishment Levels

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Abstract

Empirical studies have shown that increasing the severity of punishment increases the crime rate. We develop a theory of "personal rules" based on the tradeoffs between self-image of one's own criminal productivity and the temptation-salience of the present -of taking the easy way out by committing a crime, which transforms lapses into precedents that undermine future self-restraint. The foundation for this mechanism is the imperfect recall of one’s own criminal productivity, leading people to draw inferences from their past actions. The rationalization may lead them to overestimate the expected utility of committing crimes when opportunities present themselves.


JEL Codes: D03, D81, K42.

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1 Introduction

If our objective is to minimize crime rates, this naturally raises two questions. First, what are the costs and benefits an agent takes into consideration before deciding whether to commit a crime? Second, if the authority in charge is resource constrained, should it use up all its resources to enforce the laws in order to achieve its objective?

According to Becker’s [1968] seminal paper on criminal behavior, if criminals are rational and behave according to the subjective expected utility framework, a higher probability of apprehension will lead to a fall in criminal activities. The probability of apprehension is costly whereas the fine is a costless transfer from the criminal to the state. Therefore, one should set the fine at its highest value. The probability of apprehension is used to complement the fine in deterring individuals from committing criminal activities.

Let us illustrate how this model works with an example. Suppose people live for two periods. In both periods they have the opportunity to commit a crime. Each individual has a best estimate about the utility of committing a crime. In both periods, there is the same probability of being apprehended if one commits a crime and the same utility cost—a monetary fine—if one is apprehended. If the agent decides to commit a crime today, will commit a crime tomorrow since the costs and benefits of a criminal activity remain the same in every period. As the probability of apprehension increases, he is less likely to commit a crime in every period. The prediction is monotonicity between the probability of apprehension and the crime rates.
On the other hand, this theory’s prediction is not always consistent with the empirical literature. Many studies indicate that increasing the likelihood of punishment deters criminal activity, but increased punishment is not associated with lower crime rates. This is Beyleveld’s [1980:306] conclusion after a survey of social science literature on crime and deterrence. Similarly, using a large sample of offenders released from federal prisons, Myers [1980, 1983] finds that increases in the certainty of punishment are positively related to participation in crime.

We will propose an alternative theory that can be reconciled with the empirical findings.

Our model works as follows. A decision maker (DM) has the opportunity to commit a crime in each of the two available periods. At the outset, he gets a noisy but informative signal about his criminal productivity. This signal represents soft information which does not leave a material record, hence, is very difficult to be verified later. For example, the signal could be the momentary impression DM obtains from a personal interaction with his acquaintance who is trying to recruit DM by telling DM how profitable crime is for you.

If he chooses not to become a criminal in period one, he leads a honest life as a regular worker in the period. He does not need to access to his signal, which is his impression about his criminal abilities, in period one since it is irrelevant to him. Then, at the start of the second period, DM will only be able to recall the signal—which represents soft and unverifiable information-imperfectly\(^1\). In other words, with positive

\(^1\)The idea that makes a distinction between hard information and soft information is used by Mullainathan[2002], Bernheim and Thomadsen[2005] and Tirole and Bénabou [2004, 2009]. We will discuss their works in further detail in section 2.
probability, at time period two he will remember the signal he saw at time period one. With the remaining probability, at time period two, he will forget the signal he saw in time period one completely. The signal is soft information because only the individual has access to it. Suppose you choose not to undertake criminal activities and take on a full time regular job. During this time, you will not access this information about your criminal signal received in period 1 because this is irrelevant in your life. Your memory about information that has not been assessed over a long period of time will degrade.

On the other hand, if DM commits a crime at period one, he immediately receives benefits from his criminal action. The benefits are increasing as he has higher criminal productivity. Hence, he will verify firsthand his criminal ability. This is because the amount of money he has obtained though crime is hard evidence of his criminal productivity. In other words, DM will discover his true criminal productivity after committing a crime. Then, in period two, he decides whether to commit a crime or not based on the hard information.

In each period, if he commits a crime, two things could happen. If the decision maker is not apprehended, he keeps payoffs from his criminal productivity. Else, his payoffs from his criminal action are forfeited and, in addition, he is charged a fine. The probability of apprehension is the same across periods.

We will assume that it is faster to earn money from criminal activities as compared to a regular white/blue collar job. For example, a prostitute can make a few hundred dollars an hour but if she chooses to work at McDonald’s, it will take her a week’s work before the getting
this amount of money. When a women faces two options, prostituting herself or working at McDonald’s, the first option is salient and tempting since it provides her immediate benefits. On the other hand, the second option requires her to incur costs of working efforts at the first moment before receiving benefits later. Because of temptation, when she decides whether to commit a crime or not, the costs she incurs at the working place is magnified. In other words, at the moment of making a criminal decision, DM becomes myopic and hence discounts the payoffs from non-criminal activities hyperbolically. Because of this form of imperfect willpower, the decision maker knows that every time he is faced with the choice about whether to commit a crime or not, he will be tempted to commit a crime even though it is not profitable due to his low criminal productivity from an ex-ante standpoint.

If the decision maker realizes that he is of low criminal productivity, he may want to avoid committing a crime in the first period so as to convince himself in period two that the costs outweigh the benefits of being a criminal. In other words, whenever people look back to their own past actions to infer what they are likely to do in the future because the motives that led to these choices at this later point in time are no longer accessible with complete accuracy or reliability. As the probability of apprehension increases, crime rates in the period one will fall. However, this also makes the decision maker more likely infer that he is of high criminal productivity in period two. This will result in an increase in crime rates during the second period. Thus, there is a

\footnote{Lee and McCrary [2005, 2009] use a large sample on felony arrests in Florida and find that hyperbolic discounting affect offenders’ decisions on criminal actions.}
non-monotone relationship between the probability of apprehension and aggregate crime rates.\textsuperscript{3}

Is there support for our model in the literature? Gottfredson and Hirschi [1990] propose that crime rates are linked to a stable individual level trait such as low self-control. They assert that "people who lack self-control will tend to be impulsive, insensitive, physical (as opposed to mental), risk taking, short-sighted, and nonverbal, and they will tend therefore to engage in crime and analogous acts" [Gottfredson & Hirschi 1990:90]. This hypothesis has gained substantial empirical support. A lot of attention has been devoted to testing the major components of this self-control theory. A variety of studies have found self-control to be important in predicting criminal behaviors.\textsuperscript{4}

The body of the paper is organized as follows. Section 2 presents the related literature. Section 3 displays some other applications of the model. Section 4 provides the model’s setup. Section 4.1 explains the parametric assumption we have imposed on the model. Section 4.2 details the solution concept that will be used to analyze the model. Section 5 analyzes the model. Section 6 provides a discussion about why imperfect recall and hyperbolic discounting are both indispensable in our setting if we want to obtain a non monotonic relationship between crime rates and the probability of apprehension. Section 7 considers what happens to the results of the model when we relax the assumptions made in the model section. Section 8 concludes. Proofs are gathered in

\textsuperscript{3}We thank Barton Lipman for suggesting this mechanism.

\textsuperscript{4}For example, low self-control is significantly related to drunken driving [Piquero and Tibbetts 1996], self-reported juvenile delinquency [Wood et al. 1993], official-delinquency [Brownfield et al. 1993], and adult criminal and imprudent behaviors [Burton et al. 1994; Grasmick et al. 1993].
the Appendix.

2 Related Literature

The type of information. - The idea of imperfect recall is widely supported by empirical studies in biology and psychology. From the structure, our model is closely related to Mullainathan[2002], Bernheim and Thomadsen[2005] and Tirole and Bénabou [2004, 2009] that assume that our memorability of information depends on a type of the information. If it is "hard information" which leaves a material record, the information is perfectly recalled since it’s verified later. If it is "soft information" which does not leave verifiable records, the information may not be recalled perfectly. Hence, the past criminal actions and monetary income from criminal acts are hard information since they are verified later. If a gang member approaches you in an attempt to recruit you, this is soft information since it does not leave any solid and verifiable record. Mullainathan[2002] assumes that people recall soft information more perfectly if it is rehearsed or/and as the cue information. If the person chooses to spend a honest working life, he is away from events which reminds him of the past impression, hence, his past impression is not perfectly recalled. However, Mullainathan[2002] focuses on the naive model where people do not adjust themselves to fallibility of memory while we consider the sophisticated model. In Bernheim and Thomadsen[2005], people know limitation of their memory and then directly make costly investment in his memory to sustain perfect memory in the future. In our model, like in Tirole and Bénabou [2004, 2009], people choose actions which affect payoffs today and this action affect his memory in the
future. Hence, when agents choose payoff relevant actions, they need to consider their marginal effect on their immediate payoffs and their marginal effect on their future payoffs through effect on their future memory.

Hyperbolic discounting.-A major part of psychology has been devoted to understanding behaviors that are characterized by strong internal conflicts, harmful impulses which causes the individual to succumb against his better judgment.⁵ Experimental psychologists have documented this robust feature of time inconsistent preferences that commonly gives rise to these self-control problems, namely people’s tendency to discount payoffs much more steeply at long than at short horizons.⁶ Criminologists suggest that criminal acts are essentially associated with impulsivity. Criminal acts yield easy and simple gains instead of waiting for benefits from complex tasks or tedious works.[Gottfredson and Hirschi 1990]⁷ Hence, being confronted with easy and immediate gratification from criminal acts, in decision making, people magnified costs of non-criminal honest working. For example, an agent walking to a Burger King restraint finds an unlocked and unattended car on the street. He is tempted to steal the car even if the car theft gives the same or smaller amount of money.

⁵In Beabou and Tirole [2000, 2002], the individual is faced with a choice between a course of action which requires no self restraint and a challenging his own capacity to resist temptation and hold out for larger, long run payoffs, The ability parameter on which uncertainty now bears is the degree to which his preferences may be subject, in certain circumstances, to a bias towards instant gratification.

⁶Ainsley and Haslam [1992] argues that humans share a general preference for immediate gains to future gains, which may cause hyperbolic discounting. Pratt and Cullen [2000] conducted a meta-analysis on existing empirical studies and suggest impulsive behaviors are associated with criminal acts.

⁷Frank [2005] and Utset [2007] argue that impulsivity is essentially connected with criminality in any case and crime deterrence can be improved by accounting for the effects of hyperbolic discounting. Kahneman et al. [1997] argue that hyperbolic discounting are likely accountable for drug use.
which he can earn from works at a Burger King.

Self-signaling.- People often learn about themselves by observing their past actions. Conversely, they often make choices to preserve favorable self images. This is well documented in psychology.\(^8\) Through this key feature, our work is most closely related to Tirole and Bénabou [2004, 2009] and Akerlof and Dickens[1982] and Dickens [1986]. In Bénabou and Tirole [2004], they develop a theory of personal rules based on an individual’s own willpower, which transforms lapses into precedents that undermine future self-restraint. However, our results have two implications that are different from them. The second implication is about the value of self-confidence. In Bénabou and Tirole [2004], an agent who has higher confidence in his ability will exercise self-control more effectively and avoid future mistakes. In our model, higher confidence weakens the effectiveness of self-control since it increases the temptation he faces in the next period. The agent wants to lower his self-confidence to protect himself from future temptation. Lower self-confidence will reduce crime rates and will improve his own welfare. The third implication is about the value of malleable beliefs-forgetting the signal. In Bénabou and Tirole [2004, 2009], malleable beliefs causes the agent to have the wrong beliefs that he may be of higher productivity than he actually is, which will increase the crime rates, and will reduce his own welfare.\(^9\)

\(^8\)In a well-known experiment conducted by Quattrone and Tversky [1984], they found that subjects who were led to believe that their tolerance for a certain kind of pain (keeping one’s hand in very cold water) was diagnostic of either a good or a bad heart condition reacted by, respectively, extending or shortening the amount of time they withstood that pain.

\(^9\)Soman [2001] and Wertenbroch et al. [2009] provides empirical supports for the memory management by testing the consumers’ recall of their recent expenditures.
our model, malleable beliefs may allow the agent to exercise self-control to avoid ex-ante non-optimal criminal actions. The malleable beliefs means that the agent can control his future action more effectively by abstaining today. An agent who knows that he will be tempted to commit a crime today will be willing to abstain in order to protect himself from the temptation he faces tomorrow. Thus, malleable beliefs may reduce crime rates in every period and hence improve welfare. Akerlof and Dickens[1982] and Dickens [1986] show non-monotonicity between punishment rates and crime rates by using a non-Bayesian framework. In their models, an agent directly chooses his belief about punishment rates in order to reduce his psychic cost of fear. Our model shows non-monotonicity by using a Bayesian framework. Our Bayesian agent does not directly choose his future inference. Instead, his current action affects his information, as we argued above, which affects his future inference.

In addition, the paper studies how exogenous factors such as punishment rates affects agent’s behavior, which is not argued by Bénabou and Tirole [2004, 2009]. We also consider a two period model whereby the past action affects how likely an agent forgets his criminal productivity and his path dependent memory affects how his future action is subject to temptation or weak willpower. On the other hand, in Bénabou and Tirole[2004], the past action affects how likely an agent forgets the level of his willpower. Bénabou and Tirole[2009] do not argue the effect of the past action on the future action, instead, in their model, an agent’s past action affects his anticipatory feeling in the future.
3 Other Applications

One of the most important uses of policies like punishments is in the behavior modification of a child so that it is acceptable in the eyes of the law. In the presence of direct intervention, it is relatively easy to ensure that the child behaves properly. But this is far from enough. The ultimate goal is to ensure that the child behaves correctly in the absence of such pressures. Usually, the attempt to shape a child’s behavior occurs under some sort of forced compliance under a parent’s supervision. For example, the child will be told not to do something then will be given certain amounts of pressure to comply. The parent will attempt to enforce these pressures with the threat of punishment if the child disobeys. These punishments may vary in magnitude. Thus, is that threatening children with harsh punishments if they disobey instruction may not be the best way guide them into taking the correct action?

To find answer, Freedman [1963, 1965] conducted the following experiments. Children were instructed not to play with a very attractive toy. One group was threatened with severe punishment if they disobeyed and another with mild punishment for non-compliance. Both groups of children were then allowed to play in a room containing the toy. Several weeks later the same two groups of children were once again put in a room with the toy, only this time the threat of punishment was withdrawn. Those who had been threatened with the more severe punishment proved more likely to play with the toy than those threatened with mild punishment. Our model with the modification of decreasing probability of apprehension as suggested in the robustness section that
can be applied to capture the setting in which this experiment took place.

4 Model

This game consists of a single decision maker (DM). Before the start of the game, DM is endowed with criminal productivity $v$ that is drawn from an exponential distribution with parameter $\lambda = 1$.

At $t = 0$, DM receives an informative but noisy signal

$$\sigma = v + \varepsilon$$

about his criminal productivity $v$, where $\varepsilon$ and $v$ are independently and identically distributed. The signal is his impression about his criminal abilities which he obtains through transient interactions with other people. Hence, this is soft information, which is not verified later. For example, when you are in a bar, you have a chat with an unknown guy standing next to you. He is trying to recruit you to be a drug dealer. You have some impression about how profitable drug dealing is to you but this impression is not verifiable later on.

Let $f_{v,\sigma}(\cdot, \cdot)$ be the probability distribution function (p.d.f.) of DM's criminal productivity $v$ and the signal $\sigma$ that he received. We will denote DM's information at the beginning of each period $t \in \{1, 2\}$ by $\Omega_t$.

At $t = 1$, the information DM has is expressed by $\Omega_1 := \sigma \in [0,1]$. Two options are available to DM, he can either commit a crime ($a_1 = 1$) or not ($a_1 = 0$). DM chooses the probability of taking each ac-
tion $a_1 \in \{1, 0\}$, which is given by $\mu_1(a_1)$ such that $\sum_{a_1 \in \{1, 0\}} \mu_1(a_1) = 1^{10}$. If DM does not commit a crime $a_t = 0$, he will receive a wage income $W > 0$ from a performing a honest day's work. If he commits a crime $a_1 = 1$, the known probability of being apprehended at $t = 1$ is $p \in (0, 1)$. If he does not get caught, DM will receives a monetary payoff $v > 0$, else he will be fined $F > 0$.

If DM commits a crime at $t = 1$, he will discover his true $v$ and be perfectly aware of it. On the other hand, if DM does not commit a crime, he will not discover his true $v$ and completely forget his signal $\sigma$. In either case, he will be perfectly aware of his criminal choice $a_1$.\footnote{We make a distinction between "hard information", which leaves a trace of hard evidence, and "soft information", which does not leave such an evidence. DM perfectly recalls hard information while he imperfectly recalls soft information. As we argued above, his signal $\sigma$ is his impression about his criminal ability he obtained through temporary interactions with others. This is unverifiable soft information. Hence, he will forget $\sigma$. On the other hand, his knowledge $v$ which he obtains after committing a criminal act is hard information. It's since he discovers $v$ through monetary gains from the criminal act, which is verifiable evidence. In addition, his criminal choice is also hard evidence. If he commits a crime such as car theft, he breaks a car and gains monetary by selling the car. If he chooses a honest working life, he goes to his office every day and}

\footnote{If DM does not commit a crime, he should not forget his signal with some positive probability. Since adding this complication leads to no qualitative changes to all our results, we will focus on the simplest case whereby DM forgets his signal for sure when he chooses not to commit a crime.}
receives salary. Either action leaves hard evidence.

DM’s action at \( t = 1 \) affects his information at \( t = 2 \). Let DM’s memory about \( v \) in period 2 be denoted by \( \hat{\sigma} \). Given \( a_1 = 1 \), DM will recall the truth, i.e. \( \hat{\sigma} = v \). When \( a_1 = 0 \), DM will forget the received signal, i.e. \( \hat{\sigma} = \emptyset \). He will infer it from his past action \( a_1 \) by using Bayes rule. DM’s information at \( t = 2 \) is given by \( \Omega_2 := (\hat{\sigma}, a_1) \in [0,1] \cup \{\emptyset\} \times \{1,0\} \). See Figure 1.

At \( t = 2 \), DM has the option to commit a crime (\( a_2 = 1 \)) or not (\( a_2 = 0 \)). DM chooses the probability of taking each action \( a_2 \in \{1,0\} \), which is given by \( \mu_2(a_2) \) such that \( \sum_{a_2 \in \{1,0\}} \mu_2(a_2) = 1 \). At this point in time, the costs and benefits of committing a crime are the same as at \( t = 1 \).

Suppose DM committed a crime at \( t = 2 \). Then, if he chooses not to commit a crime at \( t = 2 \), his decision means that DM is quitting a life of crime and turning over a new life. If DM chooses to commit a crime
at $t = 2$ as well, he is going to continue a life of crime.

On the other hand, suppose that DM chooses not to commit a crime at $t = 1$, which means that he is currently refusing to participate in criminal acts and chooses to lead a honest life by working at a normal job. Then, if he still chooses not to commit a crime at $t = 2$, he decides to continue leading a honest life. If he decides to commit a crime at $t = 2$, this means that after working at an honest job for sometime, he has become tempted by the payoffs from criminal activity as it is more attractive than working hard. Thus, he has decided to quit his current job and accept the offer presented to you at $t = 1$ and lead a life of crime.

In addition to a standard discount rate across periods, we will assume that DM’s preference exhibits time inconsistency. In each period $t \in \{1, 2\}$, when DM is deciding on $\mu_t$, he feels tempted or experiences “salience of the present” at the thought obtaining of "easy" money instantly through criminal activities instead of working hard for it. Criminal act yields immediate gain $v$ to DM while honest working life requires him to work for a week to receive payment. Hence, being confronted with temptation from criminal acts, DM discounts the delayed benefits and costs at a rate $\beta_t \in (0, 1)$; equivalently, he values the immediate gratification from a criminal action at $\frac{v}{\beta_t}$ instead of $v$ in each period.\footnote{Our form of hyperbolic discounting is closely related to Bénabou and Tirole\cite{benabou2004difference}. In their model, DM chooses between one action which yields immediate benefits and another action which yields higher benefits later. Since immediate gain is tempting, in decision making, DM discounts delayed benefits and costs hyperbolically.}

Due to this form of imperfect willpower, DM will commit a criminal act

\footnote{See Ainslie \cite{ainslie1992self, ainslie2001self} for the evidence, and Strotz \cite{strotz1956myopia}, Phelps and Pollack \cite{phelps1968theory}, Loewenstein and Prelec \cite{loewenstein1992time}, Laibson \cite{laibson1997golden, laibson2001golden}, and O'Donoghue and Rabin\cite{odonoghue1999hyperbolic} for formal models and economic implications.}
even though it is unprofitable from an ex-ante standpoint. For example, if DM sells drugs at a street corner, he gets paid as soon as a buyer approaches him. In others, you earn a lot of money by handing out a few items which takes you a few hours. However, if DM works at Burger King, he needs to complete a week's worth of hard work/manual like mopping the floor, throwing out the trash before earning the money.

DM will face increasing amounts of temptation to commit a crime. For example, the amount of temptation DM faces today measures how much someone new to the workforce is tempted to obtain "easy" money through a criminal act. The temptation DM faces tomorrow represents how much he craves it after working for sometime and realizing how difficult it is to make an honest living. For this reason, we will assume that temptation he faces tomorrow is stronger than it is today.\textsuperscript{14} Since DM has no way to resist his temptation at \( t = 1 \), we will assume that \( \beta_1 = 1 \). \( \beta_1 \) is common knowledge.\textsuperscript{15}

At \( t = 1 \), DM's expected utility can be expressed by the expected utility function

\[
U_1 = E_1[\sum_{a_1 \in \{1,0\}} \mu_1(a_1) \cdot u_1(a_1, v) + \sum_{a_2 \in \{1,0\}} \mu_2(a_2) \cdot u_1(a_2, v)] \tag{2}
\]

Likewise, the expected utility perceived by DM at \( t = 2 \) is given by

\textsuperscript{14}Hoch and Loewenstein [1991], Shiv and Fedorikin[1999], Hinson, et al.[2003] and Vohs and Faber [2007] study consumer behaviors and provide evidence that self-control (or willpower) becomes more difficult over time.

\textsuperscript{15}All our results remains unchanged even if we allow \( \beta_1 < 1 \).
where we define the utility function $u_i(a_t, v)$ at period $t$ during period $i \in \{1, 2\}$ as

$$u_i(a_t, v) = a_t \cdot \left( (1 - p_t) \cdot \frac{v}{\omega_{i,t}} - p_t \cdot F \right) + (1 - a_t) \cdot W$$

$$\omega_{i,t} = \begin{cases} \beta_t & \text{if } t = i = 2 \\ 1 & \text{otherwise} \end{cases}$$

The sequence of events occur according to the time line shown in Figure 2.
4.1 Parametric Assumptions

To keep the analysis interesting, we will assume there exists $p$ such that a DM who did not commit a crime at $t = 1$ will find it profitable to commit a crime at $t = 2$.

4.2 Solution Concept

The solution concept we will use is mixed strategy Perfect Bayesian Equilibrium (PBE). A PBE of this game is a pair $(\mu_1^*, \mu_2^*) \in [0,1]^4$ which are the arguments that maximize equations (1) and (2), meaning that

1. The criminal behavior of DM at $t = 1$ is optimal, given DM assessment of his own criminal productivity at $t = 2$.

2. At $t = 2$, DM assesses his criminal productivity using Bayes’ rule that takes into account his criminal behavior at $t = 1$.\(^\text{16}\)

5 Analysis

We will start by showing the existence of a PBE. Let us begin with the equilibrium analysis from the last period. Consider the DM’s decision at $t = 2$. At this moment, the DM simply considers the direct costs and benefits from committing a criminal act. To build some intuition, let us first consider the case whereby DM is free of the temptation $\beta$. Then the marginal payoff given his available information at this moment is

\[
(1 - p) \cdot E [v|\Omega_2, \mu_1^*, p] - pF - W
\]

\(^\text{16}\)See Definition 1 in the appendix for a formal mathematical definition of PBE.
The first term represents the immediate gain from criminal activity without being subjected to temptation. The second term represents the risk of punishment. The third term represents the wage income he will be paid if he abstains from committing a criminal act. The profitability of criminal activity $a_2 = 1$ is increasing in DM’s criminal productivity $v$, decreasing in punishment levels—either $p$ or $F$—and decreasing in the value of the outside option $W$. Thus, the ex-ante optimal rule DM uses to assess whether criminal activity is profitable or not at $t = 2$ can be expressed by the cutoff value

$$Y^* (p) := \frac{p \cdot F + W - C}{1 - p}$$

such that $a_2 = 1$ is ex-ante optimal if $E[v|\Omega_2, \mu^*_1, p] > Y^* (p)$ and $a_2 = 0$ is ex-ante optimal otherwise.

Now let us consider the case whereby at $t = 2$, DM’s action is affected by his temptation to commit a criminal act; i.e.

$$(1 - p) \cdot E\left[\frac{v}{\pi_2} | \Omega_2, \mu^*_1, p\right] - p \cdot F - W$$

Thus at this moment of choosing whether to commit a criminal act or not, he uses the ex-post rule $Y (p)$—which is affected by temptation $\beta_2$—which is now given by

$$Y (p) := \frac{p \cdot F + W}{1 - p} \cdot \beta_2 < Y^* (p).$$

DM’s temptation $\beta_2$ leads him to make a wrong decision—DM will commit a crime even though it is not profitable for him to do so—when
if his ex-post inference falls between the ex-ante cutoff rule $Y^*(p)$ and ex-post cutoff rule $Y(p)$; i.e.

$$E[v|\Omega_2, \mu_1^*, p] \in (Y(p), Y^*(p)).$$  \hspace{1cm} (9)$$

There are two possible outcomes at $t = 2$. First, if DM chose to commit a crime at $t = 1$ and discovered his true criminal productivity $v$ through his past criminal action ($a_1 = 1$ and $\hat{\sigma} = v$), he simply compares his true criminal productivity $v$ and his ex-post cutoff rule $Y(p)$ given by equation (4). Temptation leads DM to choose an unprofitable criminal decision-from a $t = 1$ standpoint-if his inference falls between the ex-ante optimal rule and the ex-post rule, i.e. $v \in (Y(p), Y^*(p))$.

Second, if DM did not commit a crime at $t = 1$ and loses his awareness ($a_1 = 0$ and $\hat{\sigma} = \emptyset$), he infers his criminal productivity from both his rule at $t = 1$ and his past action. He commits a crime if his inference about $v$ is large enough $E[v|\Omega_2 = (\emptyset, 0), \mu_1^*, p] > Y(p)$-and he abstains otherwise. For DM to successfully choose to not commit a crime at $t = 2$, his inference about his criminal productivity should be low. If it is high, he will commit a crime.

Now, let us consider DM’s decision at $t = 1$. Let $V_1(a_1, \sigma, \mu_2^*, p)$ denote a value function at period 1.

The net benefit of DM’s criminal decision $a_1 = 1$ instead of $a_0 = 0$ is decomposed into two parts. The immediate criminal benefit $M$ and the self-discipline effect $D$:  

$\text{20}$
\[ V_1 (a_1 = 1, \sigma, \mu^*_2, p) - V_1 (a_1 = 0, \sigma, \mu^*_2, p) =: M (\sigma, p) + D (\sigma, \mu^*_2, p) \] (10)

\( M \) is monotonically increasing with respect to the signal \( \sigma \) and monotonically decreasing with respect to the probability of apprehension \( p \): \( \frac{\partial}{\partial \sigma} M (\sigma, p) > 0 \).

The second term \( D \) is the forgone payoff he would have obtained at \( t = 2 \) if he did not commit a crime at \( t = 1 \) and forgot his signal (i.e. \( a_1 = 0 \) and \( \tilde{\sigma} = \emptyset \)). Whether he gains or loses by choosing \( a_1 = 1 \) today depends on how effectively he is able to stop himself from committing a crime at \( t = 2 \). For example, DM will choose \( a_2 = 0 \) if he chooses \( a_1 = 0 \). So by choosing \( a_1 = 1 \), this means that DM has lost an opportunity to stop himself from committing a crime tomorrow. This forgone payoff is monotonically increasing with respect to the signal \( \sigma \): \( \frac{\partial}{\partial \sigma} D (\sigma, \mu^*_2, p) > 0 \).

Thus, at \( t = 1 \), there is a unique cutoff \( X (p) \) whereby

\[ X (p) : \in \{ \sigma \in \mathbb{R}_+: M (\sigma, p) + D (\sigma, \mu^*_2, p) = 0 \} \] (11)

such that DM chooses \( a_1 = 1 \) if \( \sigma > X (p) \) and \( a_1 = 0 \) otherwise.

**Proposition 1**  For any \( p \), there exist a unique PBE.

**Proof.** See Appendix

We will explain why aggregate crime rates are not monotone in the probability of apprehension \( p \). Before that, let us remind ourselves about how the DM chooses whether or not to commit a crime at \( t = 2 \). DM compares his signal with the unique cutoff \( X (p) \) when he decides whether
to commit a crime or not at $t = 1$. When DM forgot his signal due to a lack of criminal experience ($a_1 = 0$ and $\tilde{\sigma} = \emptyset$), he will think that he abstained because his signal was below $X(p)$. DM’s inference about the signal he had received will be given by a monotone function of $X(p)$ such that

$$\psi(X(p)) := E[v|\Omega_2 = (\emptyset, 0), a_1^*, p]$$

$$\psi(t) := 1 - \left(\frac{t^2}{2} + t + 1\right) \cdot \exp(-t),$$

where $\psi(\cdot)$ function exhibits strict monotonicity and concavity given $X(p)$. On this path, DM’s inference at $t = 2$ is independent of $v$ and $\sigma$. Hence, DM will not commit a crime at $t = 2$ only if he infers that he is of sufficiently low criminal productivity; i.e. when

$$\psi(X(p)) < Y(p).$$

he will not commit a crime at $t = 2$, else he does.

Now, we will explain how different levels of $p$ affects DM’s aggregate criminal behavior. We will explain why DM is able to resist the temptation of a criminal act and not commit a crime at $t = 2$ for small $p$ and large $p$, but not for mid ranged values of $p$. 

22
Recall, we have shown that DM can resist temptation and stop himself from committing a crime at \( t = 2 \) by choosing \( a_1 = 0 \) only if \( \psi (X (p)) < Y (p) \). We can interpret \( Y (p) \) as the ex-post marginal cost of a criminal act at \( t = 2 \). Then, \( \psi (X (p)) \) is DM’s inferred criminal productivity on the path \( a_1 = 0 \) and \( \sigma = \emptyset \) or ex-post marginal benefit of a criminal act at \( t = 2 \) on this path. The ex-post marginal cost is increasing and convex in \( p \); i.e. \( \frac{\partial}{\partial p} Y (p) > 0 \) and \( \frac{\partial^2}{\partial p^2} Y (p) > 0 \).

On the other hand, DM’s inferred criminal productivity on the path \( a_1 = 0 \) and \( \sigma = \emptyset \) is increasing but diminishing in \( p \); i.e. \( \frac{\partial}{\partial p} \psi (X (p)) > 0 \) and \( \frac{\partial^2}{\partial p^2} \psi (X (p)) < 0 \).

See Figure 3.

For small values of \( p \), undertaking a criminal action in every period is very profitable. DM will set the cutoff value \( X (p) \) at a very low level because not committing a crime is profitable only when he observes very low signal \( \sigma \). So when DM receives low signal \( \sigma < X (p) \), he will not
commit a crime at $t = 1$. DM forgets his signal and will infer that he is of low criminal productivity in period $t = 2$ which will cause him to choose $a_2 = 0$.

Now consider mid ranged values of $p$. As $p$ increases, DM will increase $X(p)$. This causes DM to infer that he is of high criminal productivity and this effect dominates the ex-post marginal cost; i.e. $\psi(X(p)) > Y(p)$.

If DM chose not to commit a crime at $t = 1$ and forgot his signal, he will infer that he is of high criminal productivity and commit a crime tomorrow.

For large values of $p$, DM wants to prevent himself from committing a crime if he is of low criminal productivity because the punishment is very severe. Thus, DM will set $X(p)$ at high value. This causes DM to infer that he is of high criminal productivity. However, the high marginal cost dominates the benefits of committing a crime at $t = 2$ from DM inferred criminal productivity. Thus, DM abstains from committing a crime at $t = 2$.

Hence, when $\Omega_2 = (0, \varnothing)$, crime rates at $t = 2$ is not monotonically decreasing in $p$. See Figure 4.

Let us consider the average aggregate criminal activities which is given by

$$E_0[a_1 + a_2] := \int_{\psi} \int_{\sigma} \int_{\sigma} E[a_1^*(\sigma) + a_2^*(\Omega_2)|\sigma, v] \cdot f_{\sigma, v}(\sigma, v).$$

As explained earlier, an increase in $p$ may cause an increase in crim-
inal activity at $t = 2$. This increase in criminal behavior at $t = 2$ will be greater than the decrease in criminal behavioral at $t = 1$. Thus, aggregate crime rates are not monotonically decreasing in $p$. See Figure 5.

**Proposition 2** (Non-Monotonicity between Crime Rates and Probability of Apprehension) The ex-ante equilibrium average $E_0 [a_1 + a_2]$ is not monotonically decreasing in $p$.

**Proof.** See Appendix 

6 Discussion

Let us discuss the two key ingredients of our model-temptation and imperfect recall. What happens if we remove one component but no the other? Will our results that aggregate crime rates are not monotone in the probability of apprehension still hold?
Consider a model in which there is no temptation and no imperfect recall. The agent is not affected by temptation and perfectly remembers what he observed in the past. If the agent finds it profitable to commit a crime today, he will also find it profitable to commit a crime tomorrow. As the apprehension rate increases, the agent is less likely to commit a crime at every period. Thus, it will produce a monotonic relationship between crime rates and the probability of apprehension.

Suppose that the agent is affected by temptation but he has perfect recall. He will be tempted to commit a crime tomorrow even if it’s not a profitable decision from the ex-ante standpoint. Hence, even if he does not commit a crime today, he cannot stop himself from committing a crime tomorrow. As the probability of apprehension increases, the benefit from committing a crime act is dominated by the cost of committing a crime act in every period. Thus, crime rates are monotonically decreasing in the probability of apprehension.
Suppose that the agent has imperfect recall but he is not affected by temptation; i.e., he forgets the signal he receives about his own criminal productivity if he does not commit a crime but he discovers his true ability if when he chooses to commit a crime. If the agent did not commit a crime in an earlier period, he will have to infer his criminal productivity from his past action and surroundings like $p$, $F$ and $W$. Without temptation, the agent knows that if he abstained from committing a crime in the previous period, it was not because he was trying to impose some self control on himself. He choose not to commit a crime because it was not profitable. Thus, if criminal activity is not profitable today, the agent will be able to infer that it will not be profitable tomorrow. As the probability of apprehension increases, the agent is less likely to commit a crime in every period.

Thus, in order to obtain a non monotonic relationship between crimes rates and the probability of apprehension in our setting, we have shown that both imperfect recall and temptation are indispensable.

7 Robustness

Now let us consider what would happen to our results if we relaxed some of the assumptions made in the model section. First, we have assumed that the probability of apprehension is constant across both periods. What happens if we allow the probability of apprehension to change across periods? For example, let the probability of apprehension in the first period is twice that of the second period. Compared to our current model, the criminal action here is more profitable at $t = 2$ compared to $t = 1$. So the DM would be more likely to commit a crime at $t = 2$
compared to our current model. Thus, the non monotonicity of crime rates and probability of apprehension remains unchanged.

8 Conclusion

Is understanding the relationship between the probability of apprehension and crime rates important? Suppose the objective is the attainment of the lowest possible crime rates. At the same time, the government is constrained by the fact that it cannot increase the probability of apprehension substantially due to resource constraints. Then, no increase in the probability of apprehension may be better than a small increase.

We will single out an interesting avenue for further research. The main idea presented here is that people have imperfect access to their own abilities and motives, and must therefore infer them from their past actions. This framework can provide the foundation for a theory of personal, professional, or sociocultural identity\textsuperscript{17} as a cognitive investment. Many interesting questions remain, such as the optimal design of contracts or the political economy of reforms when agents have motivated beliefs.

\textsuperscript{17}Beabou and Tirole [2010] develop a theory of moral behavior, individual and collective, based on a general model of identity in which people care about "who they are" and infer their own values from past choices.
Appendix

Definition

A Perfect Bayesian Equilibrium (PBE) is a pair \((\mu_1^*, \mu_2^*)\) such that

1. \(\mu_1^*(\Omega_1) = \arg\max_{\mu_1: \sum_{a_1 \in \{1,0\}} \mu_1(a_1) = 1} \{ E[\sum_{a_1 \in \{1,0\}} \mu_1(a_1) \cdot u(a_1, v) + \sum_{a_2 \in \{1,0\}} \mu_2(a_2) \cdot u(a_2, v) | \mu_2^*, \Omega_1] \} \) for \(\forall \Omega_1\)

2. \(\mu_2^*(\Omega_2) = \arg\max_{\mu_2: \sum_{a_2 \in \{1,0\}} \mu_2(a_2) = 1} \{ E[\sum_{a_2 \in \{1,0\}} \mu_2(a_2) \cdot u(a_2, v) | \mu_1^*, \Omega_2] \} \) for \(\forall \Omega_2\)

3. DM’s inference in each period \(t \in \{1, 2\}\) is consistent; i.e. the following are satisfied:

   (a) At \(t = 1\), the inference DM makes about his own criminal productivity \(v\) given \(\Omega_1 = \sigma\) is \(E[f_v(v') | \Omega_1 = \sigma] = \frac{f_{v, \sigma}(v', \sigma)}{\int f_{v, \sigma}(v'', \sigma) \, dv''} \).

   (b) At \(t = 2\), the inference DM makes about his own criminal productivity \(v\) given

      i. \(\Omega_2 = (v, 1)\) is \(E[\Pr(v = v^*) | \Omega_2 = (v', 1)] = \begin{cases} 1 & \text{if } v^* = v' \\ 0 & \text{otherwise} \end{cases} \).

      ii. \(\Omega_2 = (\emptyset, 0)\), is \(E[f_{v}(v') | \Omega_2 = (\emptyset, 0)] = \frac{\int_{\emptyset} f_{v, \sigma}(v'', \sigma') \, dv'' \, d\sigma'}{\int_{\Omega_2} f_{v, \sigma}(v'', \sigma') \, d\sigma'} \).

Assumption

There exist non-measure set of \(p\) given which DM commits a crime at \(t = 2\) given \(\Omega_2 = (0, \emptyset)\) such that

\[ \sup_{p \in (0, 1)} \psi(X(p)) - Y(p) > 0 \]
Proof of Proposition 1

In the proof, let $\mu_1^* (a_t | \Omega_t, p)$ denote the optimal mapping at period $t=1, 2$ in equilibrium strategy such that

$$
\mu_1^* : [0, 1] \times (0, 1) \to [0, 1]^2
$$

$$
\mu_2^* : [0, 1] \cup \{\emptyset\} \times \{1, 0\} \times (0, 1) \to [0, 1]^2.
$$

Let $f_\sigma$ denote the unconditional pdf of $\sigma$ and $f_{v|\sigma}$ denote the pdf of $v$ conditional on $\sigma$. Then, from assumption, each pdf is given by

$$
f_\sigma (\sigma) = \begin{cases}
\sigma \exp (-\sigma) & \text{for } \forall \sigma > 0 \\
0 & \text{otherwise}
\end{cases}
$$

and

$$
f_{v|\sigma} (v|\sigma) = \begin{cases}
\frac{1}{\sigma} & \text{for } v > 0 \& \sigma > v \\
0 & \text{otherwise}
\end{cases}
$$

(16)

We claim that there exist functions $X (\cdot), Y (\cdot), \phi (\cdot)$ and $\psi (\cdot)$ which characterize a unique Pareto dominant PBE in pure strategies as follows.

Fix $\forall p \in (0, 1)$. Then, the optimization at $t = 1$ is

$$
\mu_1^* (a_1=1|\sigma, p) = \begin{cases}
1 & \text{for } \sigma > X (p) \\
0 & \text{for } \sigma < X (p)
\end{cases}
$$

(17)

and the optimization at $t = 2$ is

(i) for $\Omega_2=(1, v)$

$$
\mu_2^* (a_2=1|\Omega_2, p) = \begin{cases}
1 & \text{if } v > Y (p) \\
0 & \text{otherwise}
\end{cases}
$$

(18)
(ii) for $\Omega_2=(0, \varnothing)$

$$
\mu_2^2 (a_2=1|\Omega_2, p) = \phi (p) : \in \begin{cases} 
\{1\} & \text{if } \psi (X (p)) > Y (p) \\
(0, 1) & \text{if } \psi (X (p)) = Y (p) , \\
\{0\} & \text{if } \psi (X (p)) < Y (p) 
\end{cases}
$$

where

$$(X (p), \phi (p)) := \{(x, \lambda) \in [0, 1]^2 \text{ satisfying the three conditions below } \}$$

(X (p), \phi (p)) := \{(x, \lambda) \in [0, 1]^2 \text{ satisfying the three conditions below } \} \quad (20)

$$
x = \left(\frac{W + F}{1-p} - F\right) \cdot \left(1 + \sqrt{1 - \frac{(2 - \beta_2) \cdot \beta_2}{2 - \lambda}} \right), \quad (21)
$$

$$
\lambda \in \begin{cases} 
\{1\} & \text{for } H (p, 1) > 0 \\
\{\lambda \in (0, 1) : H (p, \lambda) = 0\} & , \\
\{0\} & \text{for } H (p, 0) < 0 
\end{cases}
$$

\(H (p, \lambda) := \psi (x) - Y (p) . \quad (22)\)

\(H (p, \lambda) := \psi (x) - Y (p) . \quad (23)\)

\(X (\cdot)\) is a period 1 cutoff. \(Y (\cdot)\) is a period 2 cutoff. \(\phi (\cdot)\) is period 2 commitment on the path \(a_1 = 0\). \(\psi (\cdot)\) is period 2 inference on the path \(a_1 = 0\).
We will start analysis.

First, we will show $\mu_2^* (\cdot)$. The problem at $t = 2$ is

$$\mu_2^*(a_2|\Omega_2, p) \in \max_{\mu_2} V_2 (\mu_2, \Omega_2, \mu_1^*, p)$$  \hspace{1cm} (24)

where

$$V_2 (\mu_2, \Omega_2, \mu_1^*, p) := \sum_{a_2 \in \{1,0\}} \mu_2 (a_2) \cdot a_2 \cdot \{ (1-p) \cdot E \left[ \frac{v}{\beta_2} | \Omega_2, \mu_1^* \right] -pF-W \} + W.$$  \hspace{1cm} (25)

Given any $\mu_1^*$, DM has a dominant pure strategy at $t = 2$ unless $E [v|\Omega_2, \mu_1^*] = Y (p)$ as follows.

$$\mu_2^*(a_2=1|\Omega_2, p) \in \begin{cases} 
1 & \text{if } E [v|\Omega_2, \mu_1^*] > Y (p) \\
(0, 1) & \text{if } E [v|\Omega_2, \mu_1^*] = Y (p) \cdot \text{ or }
0 & \text{if } E [v|\Omega_2, \mu_1^*] < Y (p) \end{cases}$$  \hspace{1cm} (26)

Suppose unique period 1 cutoff $X (p)$, which we will verify later.

Then, the equilibrium inference at $t = 2$ is

$$E [v|\Omega_2, \mu_1^*] = \begin{cases} 
v & \text{for } \Omega_2 = (v, 1) \\ 
\psi (X (p)) & \text{for } \Omega_2 = (\varnothing, 0) \end{cases}$$  \hspace{1cm} (27)

where
\[
E \left[ v | \Omega_2 = (\emptyset, 0), \mu_1^* \right] = \int_{\sigma = 0}^{X(p)} E \left[ v | \sigma \right] \cdot f (\sigma) \, d\sigma = \int_{\sigma = 0}^{X(p)} \frac{\sigma^2}{2} \exp (-\sigma) \, d\sigma \\
= 1 - \left( \frac{X(p)^2}{2} + X(p) + 1 \right) \cdot \exp (-X(p)) =: \psi (X(p)).
\]

Period 2 inference \( \psi (\cdot) \) on the path \( a_1 = 0 \) is monotone and has a finite codomain \((0, 1)\) over its domain \((0, \infty)\).

On the path \( a_1 = 1 \), his payoff is strictly increasing in \( v \) and hence there is a dominant action almost everywhere at \( t = 2 \) almost everywhere. On the other hand, on the path \( a_1 = 0 \), the decision is affected by inference \( \psi (X(p)) \) which is a function of period 1 cutoff \( X(p) \). There may exist mixed strategy.

Next, we consider DM’s problem at \( t=1 \) which is expressed by

\[
\mu_1^* (\sigma) \in \max_{\mu_1} \left\{ \max_{\mu_2} (\mu_1, \sigma, \mu_2^*, p) \right\} \\
= a \left\{ (1-p) \cdot E \left[ v | \sigma \right] - pF - W \right\} + a \left\{ \int_{v=Y} \left\{ (1-p) \cdot v - pF - W \right\} \cdot f_{v|\sigma} (v|\sigma) \cdot dv \right\} \\
+ (1-a) \cdot \phi (p) \cdot \left\{ (1-p) \cdot E \left[ v | \sigma \right] - pF - W \right\} + 2W
\]

We compare value functions between two actions:
\[ V_1 (a_1 = 1, \sigma, \mu_2^*, p) - V_1 (a_1 = 0, \sigma, \mu_2^*, p) =: (1 - p) E [v|\sigma] - pF - W \]

\[ + \int_{v=Y(p)}^{\infty} \left\{ (1 - p) v - pF - W \right\} \cdot f_{v|\sigma} (v|\sigma) dv - \phi (p) \cdot ((1 - p) E [v|\sigma] - pF - W) \]

\[ = \left( \frac{(1-p)\sigma}{2} - pF - W \right) + \left\{ \left( \frac{(1-p)\sigma}{2} - (pF + W) \right) - \frac{(1-p)Y(p)}{\sigma} \frac{Y(p)}{2} \right\} \text{if } \sigma > Y (p) \]

\[ - \phi (p) \cdot \left( \frac{(1-p)\sigma}{2} - pF - W \right). \]

Then,

\[ M (\sigma, p) + D (\sigma, p, \phi (p)) \]

\[ = \left\{ \begin{array}{ll}
(2 - \phi (p)) \left( \frac{(1-p)\sigma}{2} - (pF + W) \right) - \frac{(1-p)Y(p)}{\sigma} \frac{Y(p)}{2} \text{if } \sigma > Y (p) \\
(1 - \phi (p)) \left( \frac{(1-p)\sigma}{2} - pF - W \right) \text{otherwise}
\end{array} \right. \]

where

\[ M (\sigma, p) + D (\sigma, p, \phi (p)) < 0 \text{ for any } \sigma < Y (p) \]

and

\[ \text{34} \]
\[
\frac{\partial}{\partial \sigma} (M(\sigma, p) + D(\sigma, p, \phi(p)))
\]

\[
= \begin{cases} 
\frac{2(1-p)(1-p)}{2} + \left(\frac{(1-p)Y(\phi(p))}{2} - pFW\right) & \text{if } \sigma > k_\phi(p) \\
(1-p)(1-p) & \text{otherwise}
\end{cases}
\]  

and

\[
k_\phi(p) := \sqrt{\left[-(1-p) \frac{Y(p)}{2} + pF + W\right] \frac{2Y(p)}{(2-\phi(p)) (1-p)}}
\]

The results imply that \( M(\sigma, p) + D(\sigma, p, \phi(p)) < 0 \) for \( \sigma \in (0, k_\phi(p)) \), \( M(\sigma, p) + D(\sigma, p, \phi(p)) > 0 \) for \( \sigma = 1 \) and \( M(\sigma, p) + D(\sigma, p, \phi(p)) \) is strictly monotone for \( \sigma \in (k_\phi(p), 1) \). Hence, by intermediate value theorem, there defined a unique cutoff \( x(p, \phi(p)) \) given any \( \phi(p) \in [0, 1] \):

\[
\exists! x(p, \phi(p)) \in \{ \sigma \in (Y(p), \infty) : M(\sigma, p) + D(\sigma, p, \phi(p)) = 0 \}. 
\]  

Then, the unique cutoff is given by

\[
x(p, \phi(p)) := \left(\frac{W + F}{1-p}\right) \cdot \left(1 + \sqrt{\frac{1}{2-\phi(p)} (2-\beta) \beta}\right)
\]

Next, define Hyperplane \( H \) as follows.
\[ H(p, \lambda) := \psi(x(p, \lambda)) - Y(p). \] (36)

Then, \( \phi(p) \) and \( X(p) = x(p, \phi(p)) \) in equilibrium need to satisfy the following condition:

\[ H(p, \phi(p)) \in \begin{cases} (0, \infty] & \text{if and only if } \phi(p) = 1 \\ \{0\} & \text{if and only if } \phi(p) \in (0, 1) \\ (-\infty, 0) & \text{if and only if } \phi(p) = 0 \end{cases} \] (37)

Fixing any \( p \in (0, 1) \), \( x(p, \lambda) \) is continuous and strictly monotone with respect to \( \lambda \):

\[ \frac{\partial}{\partial \lambda} x(p, \lambda) < 0 \text{ for } \forall \lambda \in [0, 1]. \] (38)

Then, since \( \psi \) is continuous and monotone and \( Y(p) \) is independent of \( \lambda \), then, fixing any \( p \in (0, 1) \), \( x(p, \lambda) \) is continuous and strictly monotone with respect to \( \lambda \):

\[ \frac{\partial}{\partial \lambda} H(p, \lambda) < 0 \text{ for } \forall \lambda \in [0, 1]. \] (39)

Because of the strict monotonicity of \( H \), \( \phi(p) \) and \( X(p) \) are well defined for any \( p \in (0, 1). \) QED
Proof of Proposition 2

Proposition 2 is supported by two claims:

(1) Partial Non-Monotonicity: \(a_1\) and \(a_2\) are not monotonically decreasing in \(p\).

(2) Aggregate Non-Monotonicity: The ex-ante equilibrium average 
\(E_0[a_1 + a_2]\) is not monotonically increasing in \(p\).

We will take steps and show partial non-monotonicity in step 1 and then aggregate non-monotonicity in step 2.

(Step 1: Partial Non-Monotonicity)

In proof of proposition 1, we already showed period 2 cutoff \(Y(p)\) is strict monotone in \(p\) and is independent of period 1 cutoff \(X(p)\) and of period 2 commitment \(\phi(p)\) on the path \(a_1 = 0\).

Hence, we focus on finding non-monotonicity \(\phi(p)\), which is paired with its corresponding \(X(p) = x(p, \phi(p))\).

We claim that for any \(\lambda \in [0, 1]\) hyperplane \(H(p, \lambda)\) has two distinct cutoffs \(0 < \rho_L(\lambda) < \rho_H(\lambda) < 1\) such that

\[
H(p, \lambda) < 0 \text{ for } \forall p \in (0, \rho_L(\lambda)) \cup (\rho_H(\lambda), 1) \quad (40)
\]

\[
H(p, \lambda) > 0 \text{ for } \forall p \in (\rho_L(\lambda), \rho_H(\lambda)).
\]

Suppose existence of two distinct cutoffs \(\rho_L(\lambda)\) and \(\rho_H(\lambda)\) for each \(H(p, \lambda)\). Since \(H(p, \lambda)\) is strictly decreasing in \(\lambda\) fixing \(p\), there defined a unique \(\phi(p)\) for each \(p\), which exhibits non-monotonicity for \(\phi(p)\), as follows.
\[ \phi(p) = 1 \text{ for } \forall p \in (\rho_L(1), \rho_H(1)) \]

\[ \phi(p) = 0 \text{ for } \forall p \in (0, \rho_L(0)) \cup (\rho_H(0), 1) \]

\[ \phi(p) \in \{ \lambda \in (0, 1) : H(p, \lambda) = 0 \} \text{ otherwise.} \]

where \( \rho_L(0) < \rho_L(1) < \rho_H(1) < \rho_H(0) \) \hspace{1cm} (41)

Now, we will show existence of two distinct cutoffs for each \( H \).

From definition, DM does not commit a crime at period 2 at limiting \( p \to 1 \), i.e.

\[ \lim_{p \to 1} H(p, \lambda) = \infty \text{ for } \forall \lambda \in [0, 1] \] \hspace{1cm} (42)

From assumption, there exists \( p \) such that DM commits a crime at period 2 on the path \( a_1 = 0 \).

Hence, it’s suffice to show two results:

(Result 1) DM does not commits a crime at limit \( p \to 0 \)

\[ \lim_{p \to 0} H(p, \lambda) < 0 \text{ for } \forall \lambda \in [0, 1] \] \hspace{1cm} (43)

(Result 2) Fixing \( \forall \phi(p) = \lambda \in [0, 1] \), then, curvature of \( H(p, \lambda) \) is diminishing in \( p \).

\[ \frac{\partial^2 H(p, \lambda)}{\partial p^2} < 0. \] \hspace{1cm} (44)

Result 1 is achieved since
\[
\lim_{p \to 0} \max_{\lambda} H(p, \lambda) \leq \lim_{p \to 0} H(p, 0) = H(0, 0) \quad (45)
\]
\[
= 1 - \left( 1 + \sqrt{\frac{1 + (1 - \beta)^2}{2}} \right)^2 W^2 + \left( 1 + \sqrt{\frac{1 + (1 - \beta)^2}{2}} \right) W + 1
\]
\[
\cdot \exp \left( - \left( 1 + \sqrt{\frac{1 + (1 - \beta)^2}{2}} \right) W \right) - W \beta
\]
\[
< 1 - 1 - 0 = 0
\]

where the first inequality is obtained by setting \( \beta = 0 \) and \( W = 0 \) since \( \frac{\partial H(0, 0)}{\partial \beta} < 0 \) and \( \frac{\partial H(0, 0)}{\partial W} < 0 \).

For result 2, we first obtained a condition to decide curvature of \( H \) with respect to \( p \) conditional on \( \phi \): \( H \) is strictly concave if

\[
2 < x(p, \lambda) \cdot \left( 1 - \frac{\partial^2 x(p, \lambda)}{\partial p^2} \left( \frac{\partial x(p, \lambda)}{\partial p} \right)^2 \right) =: RHS(p, \lambda) \quad (47)
\]

and strictly convex if \( 2 > RHS(p, \lambda) \) since

\[
\frac{\partial^2 \psi(x(p, \lambda))}{\partial p^2} = \frac{x(p, \lambda) \cdot \exp(-x(p, \lambda))}{2 \cdot \left( \frac{\partial x(p, \lambda)}{\partial p} \right)^2} \left( 2 - x(p, \lambda) \cdot \left( 1 - \frac{\partial^2 x(p, \lambda)}{\partial p^2} \left( \frac{\partial x(p, \lambda)}{\partial p} \right)^2 \right) \right) \quad (49)
\]
Hence, it’s sufficient to show the following monotonicity of \( RHS(p, \lambda) \) fixing \( \lambda \):

\[
\frac{\partial RHS(p, \lambda)}{\partial p} = x(p, \lambda) \cdot \left( 1 - \frac{\partial^2 x(p, \lambda)}{\partial p^2} \right) > 0.
\] (50)

The monotonicity holds since

\[
\frac{\partial}{\partial p} \left( 1 - \frac{\partial^2 x(p, \lambda)}{\partial p^2} \right) = -\frac{\partial}{\partial p} \left( \frac{2(1-p)}{W+F} \left( 1 + \sqrt{1 + \frac{1}{2} \lambda (2-\beta) \beta} \right) \right) > 0
\]

and we already showed \( \frac{\partial x(p, \lambda)}{\partial p} > 0 \) and \( x(p, \lambda) > 0 \). We showed non-monotonicity with respect to period 2 commitment \( \phi(p) \) on the path \( a_1 = 0 \).

(Step 2: Aggregate Non-Monotonicity)

Then, estimate the across abilities and across periods aggregate average crime ratio.

It holds that \( X(p) > Y(p) \) since

\[
X(p) > x(p, 1) \bigg|_{\beta=1} = \frac{pF+W}{1-p} > Y(p).
\] (51)

Hence, if DM chooses \( a_1 = 1 \), then, he will choose \( a_2 = 1 \) on the path. On the other hand, if DM chooses \( a_1 = 0 \), period 2 strategy depends on whether the inferred marginal gain \( \Psi(X(p)) \) dominates the ex-post marginal cost \( Y(p) \).
Further, for any $p$

\[
\int_{0}^{\infty} E \left[ a_1^* + a_2^* | \sigma \right] f_{\sigma} (\sigma) \, d\sigma \quad (52)
\]

\[
= \int_{\sigma = X(p) \phi = \lambda}^{\infty} 2f_{\sigma} (\sigma) \, d\sigma + \phi (p) \int_{\sigma = 0}^{X(p)} f_{\sigma} (\sigma) \, d\sigma
\]

\[
= \phi (p) + (2 - \phi (p)) (X (p) + 1) \exp (-X (p))
\]

First consider $p \in (\rho_L (1) , \rho_H (1))$, i.e. $\phi (p) = 1$

\[
\int_{0}^{\infty} E \left[ a_1^* + a_2^* | \sigma \right] f_{\sigma} (\sigma) \, d\sigma = 1 + (X (p) + 1) \exp (-X (p)) \quad (53)
\]

which is increasing in $X (p)$ and $X (p)$ is increasing in $p$. Hence, the aggregate crimes are decreasing in $p$ over the space.

Further, for $p \in (0 , \rho_L (0)) \cup (\rho_H (0) , 1)$, i.e. $\phi (p) = 0$

\[
\int_{0}^{\infty} E \left[ a_1^* + a_2^* | \sigma \right] f_{\sigma} (\sigma) \, d\sigma = 2 (X (p) + 1) \exp (-X (p)) \quad (54)
\]

which is decreasing in $X (p)$ and $X (p)$ is increasing in $p$. Hence, the aggregate crimes are decreasing in $p$.

Further, for $p \in (\rho_L (0) , \rho_L (1))$
\[
\int_0^\infty E[a_1^* + a_2^*|\sigma] f_\sigma(\sigma) d\sigma
\]
\[
= \phi(p) + (2 - \phi(p)) (X(p) + 1) \exp(-X(p))
\]

which is increasing in \(\phi(p)\) while is decreasing in \(X(p)\).

For \(p \in (\rho_L(0), \rho_L(1))\), \(X(p)\) is decreasing in \(p\) and \(\phi(p)\) is increasing in \(p\), hence, the aggregate crimes are increasing in \(p\).

For \(p \in (\rho_H(1), \rho_H(0))\), \(X(p)\) is increasing in \(p\) and \(\phi(p)\) is decreasing in \(p\), hence, the aggregate crimes are decreasing in \(p\). QED.
Acknowledgement

We thank Ching-To Albert Ma, Michael Manove, Dilip Mookherjee, Andrew F. Newman, Hsueh-Ling Huynh, Laurent Bouton, Andrew Ellis, Chun Wing Tse and Tak Yuen Wong for comments that led to the improvement of this paper. We also thank seminar participants at Boston University for helpful suggestions. We are particularly grateful to Faruk Gul and Bart Lipman for their guidance and patience towards us during the entire project. All remaining errors are our own.

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