Excessive Perks to Restrain the Hidden Saving Problem*

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Abstract

We offer an explanation to why perks are over-provided to high profile CEOs. Hidden saving of an agent makes it difficult for a principal to control the agent’s moral hazard problem, e.g. Rogerson (1985). However, an agent typically cannot save perks, for example, a CEO who owns the right to use a private jet for personal use cannot bank the unused airplane hours. Thus, the principal may over-supply the agent perks to avoid the hidden saving problem. When the agent can both exert lower effort and save wage income, i.e., in the presence of double deviation problem, we show that the principal over-supplies perks more than the agent would have purchased on their own under two fairly standard environments: (i) when there are two states of outcome, or (ii) when the probability distribution satisfies the monotone likelihood ratio property, and the agent’s utility function exhibits constant absolute risk aversion. More generally, we show that excessive perks are a part of the second-best compensation if the optimal wage increases in the outcome, or if the condition of increasing wage is imposed.

Keywords: hidden saving, moral hazard problem, double deviation

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1 Introduction

Employers usually over-supply perks to their employees, especially to high profile CEOs. The question thus arises: why does an employer want her employees to consume more perks than the employees would purchase on their own? A possible clue lies in the fact that many such perks are products that the employee may find difficult to put on sale, to save, or to postpone consuming. In other words, the employer can easily monitor the employee’s consumption of perks, in contrast to his consumption of the other goods (which we henceforth call money\(\text{1}\)).

We consider a principal-agent framework where the agent has a hidden saving/borrowing technology for money but not for perk goods. With the hidden saving/borrowing technology, the agent can deviate from the contract by exerting less effort and can also save or borrow money. This phenomenon is called the double deviation problem. We demonstrate that the principal may over-supply perk goods (in the sense that at the amount supplied, the per-dollar marginal utility of the perk good is less than that of money) to mitigate the double deviation problem.

Before considering a compensation scheme including perks, we provide a benchmark model where compensation comes solely from monetary payments in the presence of the hidden saving. In solving the benchmark model, the first-order approach may be invalid because the agent’s decision problem may not be globally concave in effort and money consumption\(\text{2}\). Abraham and Pavoni (2009) characterizes a sufficient condition for global concavity in consumption and effort for the agent’s problem, under which the first-order

\(\text{1}\)We do not assume that an employee actually consumes money. This money can be a composite good (excluding the perk goods).

\(\text{2}\)The first-order approach considers the agent’s first-order condition (with respect to the agent’s effort) as an incentive compatibility constraint. This approach essentially assumes that the agent would (locally) deviate only from the implemented effort. In this sense, the constraint is (weakly) weaker than the canonical incentive compatibility constraint where the agent can deviate both from the implemented effort and from the implemented consumption. Kocherlakota (2004) shows that the first-order approach is indeed invalid in a context of dynamic unemployment insurance.
approach is valid. The sufficient condition for the global concavity requires that the gain from double deviation be sufficiently smaller than the loss of deviation from the optimal consumption or from effort level alone. Koehne (2010) further generalizes their results.

We take a different route in order to investigate the consequence of the double deviation problem. Instead of assuming the global concavity and using the first-order approach in terms of effort, we assume discrete effort levels and consider the agent’s optimal saving decision at each effort level.

Although abandoning the first-order approach means more complex calculation and more difficult extension to infinite horizon problems, our approach allows us to simply analyze the issue of double deviation. Firstly, our approach makes it possible to measure exactly how much the agent wants to change the choice of consumption when he deviates from the contracted effort. Secondly, our approach does not require the assumption of global concavity. To make an analogy, our approach is to Abraham and Pavoni (2009) and Koehne (2010) as Grossman and Hart (1983) is to the classical principal-agent models using the first-order approach.

In the benchmark model, we characterize the agent’s optimal consumption choices when he chooses high effort and when he chooses low effort. If the agent’s choice of consumption in the first period at his low effort is smaller than the choice at high effort, then the principal’s optimal contract is deterred by the agent’s simultaneous deviation of “saving more” and “working less”. In this case, we say that the principal faces the hidden saving problem.

For example, Harris and Raviv (1979), Holmstrom (1979), Mirrlees (1976), Shavell (1979a, b), Rogerson (1985b), and Jewitt (1988) employ and/or justify the first-order approach. On the other hand, Grossman and Hart do away with the first-order approach and assume discrete effort levels.

Our definition of the hidden saving problem is conceptually similar to the one noted by Rogerson (1985). However, there is a subtle difference. Rogerson’s terminology means the agent’s incentive to save even when the agent chooses implemented effort. To avoid this problem, the principal needs to satisfy the Euler equation balancing today’s marginal utility and tomorrow’s (expected) marginal utility at the implemented effort. Later literature considering the double deviation problem more explicitly (such as Kocherlakota [2004], Abraham and Pavoni [2008, 2009]) uses the term in a slightly different way: even if the agent’s consumption
If the agent’s choice of consumption in the first period at high effort is lower than his consumption at low effort, we say that the principal faces the hidden borrowing problem.

Firstly, we show that the double deviation problem always exists: whenever the agent deviates from the contracted effort, he always changes his consumption as well. Secondly, we show that the double deviation problem causes an additional intertemporal distortion, which makes it more costly for the principal to implement high effort. Thirdly, we show that the principal faces the hidden saving problem, as opposed to the hidden borrowing problem, under two fairly standard environments: (i) when there are two outcome states, or (ii) when the agent’s utility function exhibits constant absolute risk aversion and probability distributions satisfy the monotone likelihood ratio property. More generally, we show that if an optimal wage scheme is monotonic in the outcome, the principal faces the hidden saving problem. Moreover, if the principal faces the additional constraint that the wage has to be monotonic in the outcome, then the principal always faces the hidden saving problem.

After analyzing the benchmark model, we add a perk good into this framework and explain why the principal may over-supply the perk good. We consider perks as goods which the agent cannot save because perk consumptions can be monitored at a low cost by the principal, in contrast to money. For example, a CEO who owns the right to use a private jet for personal use cannot bank the unused airplane hours, yet the CEO could set up a saving account for wage income.

Since the agent’s saving insures the agent from future risk, the more the agent saves, the schedule already satisfies the Euler equation at the recommended effort, it may not satisfy the Euler equation at the deviated effort. Our terminology means the latter. Thus, the agent might save/borrow, at the same time while making lower effort. Formally, let $\bar{c}$ be the consumption in period 1 in the model where the principal can monitor and deter the agent’s saving, $c$ be the agent’s consumption in period 1 in our model, and $c_L$ be the agent’s optimal consumption when he deviates to low effort in our model. Our hidden saving problem means $c > c_L$, but not $\bar{c} > c$. Thus, it is possible to have the hidden saving problem in the sense of Rogerson and, at the same time, to have the hidden borrowing problem in the sense of Kocherlakota, Abraham and Pavoni, and ours.
more difficult it becomes for the principal to implement high effort. However, the principal may be able to (partially) avoid this problem by providing perks.\(^5\) We show that if the principal faces the hidden saving problem, the second-best compensation provides more perks than the agent would have purchased on their own; that is to say, at the allocated level, the per-dollar marginal utility of money is larger than the per-dollar marginal utility of the perk good. Intuitively, the difference between them depends on the severity of the moral hazard problem and how much the agent wants to change the choice of consumption when he deviates from the contracted effort. The severity of the moral hazard problem is measured by the shadow value of the incentive compatibility constraint. The more severe the double deviation problem is, the more perks that the principal wants to provide. The different saving technologies for perks and for money are the driving force of our results.

Scholars have studied the compensation of high profile CEOs empirically, e.g., Yermack (1995), Kole (1997), Jensen and Murphy (1990), and Murphy (1999). Others have approached this research agenda in a dynamic framework, e.g., Wang (1997), Hopenhayn and Jarque (2007), and Lustig et al. (2010), or in matching framework, e.g., Gabaix and Landier (2008) and Edmans, Gabaix, and Landier (2007). The main focus was to answer why CEOs are paid so much. Despite the large amount of research on CEO compensation, studies on perks are very limited.

There have been two views on perks in the literature\(^6\). The first considers perks as non-productive goods. The principal cannot monitor whether the agent abuses them or not; thus, a moral hazard problem is present. The second view considers perks as productive goods. Specifically, Alchian and Demsetz (1972) consider perks as a remedy for a moral

\(^5\)However, this does not necessary mean that the principal will supply only perks. Providing only perks will be an extremely inefficient way to ensure the agent’s participation, i.e., the individual rationality condition.

hazard problem rather than a source of the problem. When the members of a profit-sharing firm have to purchase input factors personally, an under-investment problem emerges, or, equivalently, a free-rider problem occurs because each member does not fully appropriate the profit from their investments. If the problem is severe, it could be more efficient to give the input factors as perks even under the risk of possible abuse.

Despite the difference between these two views, they share the same assumption that an employer cannot monitor the use of perks. However, an employer is quite capable of monitoring the use of many expensive perks. For example, an employer would have very little difficulty in checking whether a private jet was used for business or for personal reasons. The monitoring cost will be insignificant compared to the cost of flying the jet. In fact, it is often a legal requirement to report such expensive perks to the public. If the use of perks is observable, it should be explicitly contractible.

One might argue that perks reduce the cost of production or the cost of employees’ efforts, and that an agent and a principal can save on taxes by including perks in a compensation scheme. However, many perks do not seem to help production or reduce the agent’s cost of effort. For instance, corporate retreats involving horse back riding in Santa Fe, volleyball in Bari, or sailing in Greece may be useful for “team building”, but more frugal destinations and activities might be equally useful. Thus, we do not assume that perks reduce the cost of effort: we assume no complementarities between perks and effort; nor do we assume that perks have a productive use. Also, a principal could hypothetically report perks as a cost of the production, receive a tax deduction, and therefore provide the perks at a lower cost than the agent would pay privately. However, many perks are now fully subject to tax. As

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7 For example, new rules by Securities and Exchange Commission (SEC [2006]) require public companies to list all perks over $10,000.
8 Other examples include fancy company cars, a “training program” on a Mediterranean island, a car service home in a Lincoln town car, and a lavish corporate holiday party.
9 Since many perks are listed to the public, they are taxed. For example, Meg Whitman (eBay) was invited to use corporate planes for up to 200 hours of personal travel annually. That added up to more than $773,000, plus nearly $231,000 more to cover her tax bills for the perk.
a result, we do not assume that a principal and an agent can exclude perks in their taxes.

Bennardo, Chiappori and Song (2010) assume that an agent’s utility depends on effort and consumptions in a non-separable way. They also assume that both the consumption of perks and money are monitored. When the cross derivatives between effort and monetary income and between effort and a perk good are different, they find that the principal over-supplies perks in the presence of a moral hazard problem. In the present paper, we show that excessive perks can also appear due to different hidden saving/borrowing technologies for different commodities, even when the cross derivatives are identical. The agent can save money for the purpose of consumption smoothing against uncertainty in the future, but he cannot maintain perks into the next period. Therefore, the over-supply of the perk good can glean the efficiency loss from the hidden saving problem.

In section 2 we provide a benchmark model without perks as well as an extended model with perks. In section 3 we offer a model that constrains wages to be non-decreasing in output, and we also provide an interpretation of our model in an infinitely repeated environment. We conclude in section 4. Proofs are in the appendix.

2 Model

We demonstrate the double deviation problem in our benchmark model. A principal minimizes the cost of implementing an agent’s high effort, given that the agent has a hidden saving/borrowing technology. We then extend the model to include a perk good.

2.1 Benchmark model

We consider a two period principal agent model. The agent’s consumption in period 1 and 2 are denoted by $c$ and $w$. The agent makes an effort, $e$, in the first period, and there is no effort choice in the second period. This effort is the agent’s private information and determines the distribution of the outcome in the second period. There are two effort levels,
$e_H$ and $e_L$, where $e_H > e_L$. Outcome $y \in Y \subset \mathbb{R}$ is realized in the beginning of the second period with probability $P_H(y)$ when the agent chooses effort $e_H$, and with $P_L(y)$ when the agent chooses $e_L$. The set of outcomes, $Y$, is finite. In the second period, the principal awards the agent contingent payment $w(y)$, and the agent consumes it. We assume that the principal’s desired effort level is $e_H$. A contract is a vector $(c, (w(y))_{y \in Y}, e_H)$. Given the contract, the agent’s utility is

$$u(c) - e + \sum_y U(w(y))P_H(y),$$

where $u(\cdot)$ and $U(\cdot)$ are temporal utility functions for period 1 and 2 respectively. Note that there is no discount between period 1 and 2 – this simplification does not change our results qualitatively. Temporal utility functions $u(\cdot)$ and $U(\cdot)$ could be identical as in the most of traditional economic models. However, we stick to the different notations to distinguish the first period and the second period utilities more conveniently. The agent has a hidden saving/borrowing technology, i.e., the agent can transfer the first period’s consumption into the second period without the principal detecting, and vice versa. As a result, the agent might deviate not only to exert low effort but also to save or borrow. Let a deviation be $(\tilde{c}, \tilde{e})$ where $\tilde{c}$ is the deviated consumption in period 1 and $\tilde{e}$ is the deviated effort. We assume that the agent faces a zero interest rate both for saving and for borrowing – non-zero interest rate would not change our results qualitatively. The consumption in period 2 after deviation $(\tilde{c}, \tilde{e})$ would thus be $w(y) + c - \tilde{c}$. The agent’s consumption schedule and effort after the deviation is $(\tilde{c}, (w(y) + c - \tilde{c})_{y \in Y}, \tilde{e})$.

The principal wants to induce the agent’s participation and implement high effort $e_H$ at the lowest cost and prevent any possible deviation $(\tilde{c}, \tilde{e})$. Hence, an optimal contract $(c, (w(y))_{y \in Y}, e_H)$ has to satisfy constraints (1) - (4) below.

The first constraint is an individual rationality constraint. The principal must guarantee the agent some minimum expected utility $\bar{V}$ to get the agent to sign the contract. We assume that the agent cannot opt out of a contract once the contract starts; thus, there is
an individual rationality constraint only for period 1, that is,

\[ u(c) + \sum_y U(w(y))P_H(y) - e_H \geq \bar{V}. \] (1)

Secondly, the implemented consumption schedule is required to be consistent with the agent’s optimal choice, given the hidden saving/borrowing technology. This leads to a saving/borrowing constraint (or an Euler equation) when the agent chooses high effort \( e_H \),

\[ u'(c) = \sum_y U'(w(y))P_H(y), \] (2)

i.e., the marginal utility in the first period is equivalent to the (expected) marginal utility in the second period, given effort \( e_H \).

Finally, we state the incentive compatibility constraint. The incentive compatibility constraint is:

\[ u(c) + \sum_y U(w(y))P_H(y) - e_H \geq u(\bar{c}) + \sum_y U(w(y) + c - \bar{c})P_L(y) - e_L, \forall \bar{c}. \]

Consider deviation \((\bar{c}, e_L)\). The agent’s optimal consumption in period 1 when he chooses low effort \( e_L \), that is denoted by \( c_L \), satisfies:

\[ u'(c_L) = \sum_y U'(w(y) + c - c_L)P_L(y). \] (3)

With the above \( c_L \), we can write the incentive compatibility constraint as:

\[ u(c) + \sum_y U(w(y))P_H(y) - e_H \geq u(c_L) + \sum_y U(w(y) + c - c_L)P_L(y) - e_L \] (4)

Now the principal’s problem is reduced to:

\[
\max_{\{c_{\cdot}, c_L, w(y)\}} - \sum w(y)P_H(y) - c \quad \text{subject to} \quad (1), (2), (3), \text{and} \quad (4).
\]

Let \( \rho, \eta_H, \eta_L \) and \( \gamma \) be the Lagrangean multipliers for constraints (1), (2), (3), and (4).
respectively. The first order conditions with respect to $w(y)$, $c$ and $c_L$ are:

$$w(y) : -P_H(y) + \rho U'(w(y))P_H(y) + \gamma [U'(w(y))P_H(y) - U''(w(y) + c - c_L)P_L(y)]$$

$$+ \eta_H [-U''(w(y))P_H(y)] + \eta_L [-U''(w(y) + c - c_L)P_L(y)] = 0,$$

(5)

$$c : -1 + \rho u'(c) + \gamma [u''(w(y) + c - c_L)P_L(y)]$$

$$+ \eta_H u''(c) + \eta_L \left[ - \sum U''(w(y) + c - c_L)P_L(y) \right] = 0,$$

(6)

$$c_L : \gamma \left[ -u'(c_L) + \sum U'(w(y) + c - c_L)P_L(y) \right] + \eta_L \left[ u''(c_L) + \sum U''(w(y) + c - c_L)P_L(y) \right] = 0.$$  

(7)

**Lemma 1** $\eta_L = \eta_H = 0.$

**Remark:** The zero shadow value for equation (3), $\eta_L = 0$, does not mean that the saving/borrowing constraint at effort $e_L$ is meaningless. The principal would want to change $c_L$ if she could. This change would eventually reduce the agent’s utility when making low effort $e_L$ (the value of the incentive compatibility constraint’s right-hand side). However, the reduction is possible by either the increase or the decrease of $c_L$. The effects of the increase and the decrease have identical magnitude, but in the opposite directions. Thus, the shadow value becomes zero. See Appendix A.2 for details.

Finally, we derive the following equations from equation (5) and (6) by applying the results of Lemma 1.

$$\frac{1}{u'(c)} = \rho + \gamma \left[ 1 - \frac{u'(c_L)}{u'(c)} \right],$$

(8)

$$\frac{1}{U'(w(y))} = \rho + \gamma \left[ 1 - \frac{U''(w(y) + c - c_L)P_L(y)}{U'(w(y))P_H(y)} \right].$$

(9)

We next prove that the individual rationality and the incentive compatibility constraints are binding, then we explain equations (8) and (9).

**Lemma 2** $\gamma > 0$ and $\rho > 0.$
Equations (8) and (9) describe how the principal balances the marginal benefits and costs in the presence of the individual rationality constraint, the moral hazard problem, and the hidden saving/borrowing problem. To illustrate this balance, consider increasing the agent’s utility. There are two methods to do this: one is to increase the first period consumption by $1/u'(c)$ units, and the other is to increase the second period consumption at each output state $y$ by $\frac{1}{U'(w(y))}$.

Using the first method, the cost of the increase is $1/u'(c)$ since the price of $c$ is assumed to be one. This cost is represented by the left-hand side of equation (8). The first benefit of the increase comes from the relaxation of the individual rationality constraint, the benefit of which is its shadow value $\rho$ in the right-hand side of equation (8). The second benefit of the increase is the relaxation of the incentive compatibility constraint, the benefit of which is denoted by $\gamma$. However, the second benefit comes with a cost: the agent will save some of the increased first period consumption that will tighten the incentive compatibility constraint, the cost of which is denoted by $\gamma u'(c_L)u'(c)$. Thus, the second benefit net of the cost is $\gamma \left[1 - \frac{1}{U'(w(y))}\right]$. This net effect is negative if there is a hidden saving problem, $c > c_L$. Thus, equation (8) illustrates the balance of marginal cost and marginal benefit.

To illustrate the net effect of the second method of increasing the agent’s utility, we multiply the probability $P_H(y)$ and equation (9) and sum over all $y \in Y$. Then we derive

$$\sum_y \frac{1}{U'(w(y))}P_H(y) = \rho + \gamma \left[1 - \sum_y \frac{U'(w(y) + c - c_L)}{U'(w(y))}P_L(y)\right].$$

The left-hand side of the equation represents the marginal cost of the second method. The first term on the right-hand side, $\rho$, is the marginal benefit due to the relaxation of the individual rationality constraint. The second term on the right-hand side, $\gamma \left[1 - \sum_y \frac{U'(w(y) + c - c_L)}{U'(w(y))}P_L(y)\right]$, represents the net benefit from the relaxation of the incentive compatibility constraint. If the principal faces a hidden saving problem, the increase of the second period consumption helps to mitigate the hidden saving problem; thus, the net effect is positive. Mathematically, $c > c_L$ implies the positive value of the second term.
In summary, these two different methods have different net effects on the principal’s profit, due to the double deviation problem. For example, suppose that the principal faces the hidden saving problem. We derive:

\[
\frac{1}{u'(c)} < \rho < \sum_y \frac{1}{U'(w(y))} P_H(y).
\]

Firstly, these inequalities show that there is a difference between the net effects of providing an extra unit of utility through the first period consumption or the second period. Put simply, there is an intertemporal distortion when the double deviation problem exists. Secondly, these inequalities show that the inverse Euler equation does not hold; more specifically, the first period consumption is decreased and the second period average consumption is increased. Lastly, the net marginal benefit of providing one extra unit of utility through the first period consumption is less than the shadow value of the individual rationality constraint, \(\rho\), while the marginal benefit through the second period consumption is greater than the shadow value.

In contrast, when the principal faces the hidden borrowing problem, these inequalities are reversed. If there were no double deviation problem, the two methods would have the same net marginal effect, and the inverse Euler equation would hold. However, we show in Corollary 6 (below) that the double deviation problem always exists.

From the above illustration, we can see that the hidden saving or borrowing problem has an important effect on the principal’s maximization problem. We now characterize a few conditions that imply the hidden saving problem.

**Lemma 3** \(c_L < c\) if and only if \(\sum_y U'(w(y))P_H(y) < \sum_y U''(w(y))P_L(y)\).

Lemma 3 shows that the principal faces the hidden saving or borrowing problem depending on the relative size of the expected marginal utilities for high versus low effort. Lemma 3 may seem unhelpful, since the wage schedule \(w(y)\) is an endogenous variable determined by the principal. However, Lemma 3 helps identify many important cases that imply \(c_L < c\), as shown below.
Let $Y = \{1, \ldots, n\}$ be the set of outcomes. We assume the monotone likelihood ratio property (MLRP) holds: $\frac{P_H(y)}{P_L(y)}$ decreases in $y$, i.e., $\frac{P_L(y+1)}{P_H(y+1)} < \frac{P_L(y)}{P_H(y)}$ for all $y \in \{1, \ldots, n-1\}$. Then we prove that a monotonic wage scheme guarantees $\sum U'(w(y))P_H(y) < \sum U'(w(y))P_L(y)$. Next we prove the following.

**Lemma 4** Assume the MLRP. $\sum U'(w(y))P_H(y) < \sum U'(w(y))P_L(y)$ if the optimal wage $w(y)$ increases in $y$, i.e., $w(y+1) \geq w(y)$ for all $y \in \{1, \ldots, n-1\}$.

We trivially derive the following by Lemmas 3 and 4.

**Proposition 5** Assume the MLRP. If the optimal wage $w(y)$ increases in $y$, then $c > c_L$.

Proposition 5 shows the existence of the hidden saving problem when the optimal wage is monotonically increasing in output. But, it leaves the possibility that the optimal wage is not monotonically increasing; thus, hidden saving problem may not exist. However, we first show that the hidden saving or borrowing problem always exists.

**Corollary 6** Assume the MLRP. The double deviation problem exists, i.e. $c \neq c_L$.

The following two propositions characterize two conditions for monotonically increasing $w(y)$; hence, they imply the existence of the hidden saving problem (instead of the hidden borrowing problem) by Lemma 4.

**Proposition 7** If there are two states, $Y = \{G, B\}$, with $P_H(G) > P_L(G)$, then $w(G) > w(B)$ and the agent wants to save when the he chooses low effort, i.e. $c_L < c$.

In the case of two output states, the optimal contract has to satisfy $w(G) > w(B)$ to induce high effort. Given that, putting in low effort decreases the second period consumption in average. Thus, if the agent puts in low effort, he will also want to save wage.

**Proposition 8** Assume the MLRP and that $U(\cdot)$ exhibits constant absolute risk aversion (CARA). Then the optimal wage $w(y)$ increases in the outcome. Furthermore, $c_L < c$ will hold.
As shown in equation (9), the optimal wage contract is determined not only by the probability ratio, \( \frac{P_L(y)}{P_H(y)} \), but also by the ratio of the marginal utilities, \( \frac{U'(w(y) + c - c_L)}{U'(w(y))} \). The two marginal utilities in the ratio are evaluated at two points that are apart by \( c - c_L \); thus, the ratio of the marginal utilities essentially measures the absolute risk aversion. If \( U(\cdot) \) exhibits CARA, the ratio of the marginal utilities is constant in \( w(y) \). Therefore, in the optimal wage schedule \( w(y) \), difference between the high and low wage mainly depends on the probability ratio \( \frac{P_L(y)}{P_H(y)} \). This implies that the optimal wage increases in the outcome as in a typical principal agent model under the MLRP. This, in turn, implies that the agent making low effort faces higher expected marginal utility in the second period; hence, the agent wants to save (Proposition 5). We can show the same statement as Proposition 8 when \( U(\cdot) \) exhibits increasing absolute risk aversion (the statement and the proof are in Appendix A.9). However, the results do not hold when the agent’s utility exhibits decreasing absolute risk aversion (DARA). Although the optimal wage is still determined by the ratio of the marginal utilities and the ratio of the probabilities, the ratio of the marginal utilities could either increase or decrease in \( w(y) \), depending on the sign of \( c - c_L \). As a result, the two forces (given by the two ratios) might not work in the same direction.

In spite of this indeterminacy, we can at least show that if the principal faces the hidden saving problem with \( U(\cdot) \) exhibiting DARA, the wage increases in \( y \) monotonically. The intuition is the following. If there is a hidden saving problem, we have \( U'(w(y)) > U'(w(y) + c - c_L) \). Suppose \( w(y) = w(y + 1) \). Then increasing \( w(y + 1) \) is a cheaper way to relax the incentive compatibility constraint than increasing \( w(y) \), since the ratio of the benefit to the cost is larger for \( y + 1 \), i.e., \( \frac{U'(w(y+1))P_H(y+1)}{U'(w(y)+c-c_L)P_L(y+1)} > \frac{U'(w(y))P_H(y)}{U'(w(y)+c-c_L)P_L(y)} \), from the MLRP and the DARA. Thus, we will have an increasing wage \( w(y) \) in \( y \). Thus, we derive the following proposition.

**Proposition 9** Assume that the probability distributions \( P_H(y) \) and \( P_L(y) \) satisfy the MLRP.

\(^{10}\)This does not mean that we can make \( w(y+1) \) arbitrarily large since the ratio \( \frac{U'(w(y)+c-c_L)}{U'(w(y))} \) increases to approach 1 under the assumption of DARA.
and that $U(\cdot)$ exhibits decreasing absolute risk aversion (DARA). Then, if $c_L < c$, $w(y)$ increases in $y$.

Propositions 7 and 8 cover two important cases: a two state outcome space is often used in the literature, and many models assume constant absolute risk aversion and the MLRP. Proposition 9 characterizes a condition for an increasing wage. Although this finding is a somewhat weaker characterization (as $c$ and $c_L$ are endogenous variables), we believe it is worth noting, since the hidden saving problem is the main concern of most of the literature (e.g., Rogerson [1985a], Kocherlakota [2004], Abraham and Pavoni [2008, 2009]).

2.2 Extended model with perks

In this section, we show that the hidden saving problem implies the over-supply of perks. We introduce a perk good, which is denoted by $d$. The key difference between the monetary compensation $c$ and the perk good $d$ is that the agent can save $c$ but not $d$. For semantical convenience, we call $c$ “money” and $w(y)$ “wage” (or “wage scheme”). We consider this money as a composite good excluding the perk good. We normalize the price of $c$ to be unity and define $p$ as the price of $d$. A contract is $(c, d, (w(y))_{y \in Y}, e_H)$, and a deviation is $(\tilde{c}, \tilde{e})$.

We assume additive separability of the agent’s utility function between the consumption of money and the perk good. This allows us to ignore the issue of substitutability/complementarity between the perk good and money, and focus on our main insight. We denote the agent’s temporal utility function by $u(c) + v(d)$. At the end of the contract, the principal pays only money, but not the perk good, which the agent can purchase on the market. Thus, the second period temporal utility is:

$$U(w(y)) := \max_{c,d} u(c) + v(d) \text{ subject to } w(y) = c + pd.$$  

Similarly to the previous section, the principal faces the following individual rationality
constraint and the Euler equation when the agent chooses high effort $e_H$.

$$u(c) + v(d) + \sum_y U(w(y))P_H(y) - e_H \geq \bar{V}$$  (10)

$$u'(c) - \sum_y U'(w(y))P_H(y) = 0$$  (11)

The principal also faces the following incentive compatibility constraint

$$u(c) + v(d) + \sum_y U(w(y))P_H(y) - e_H \geq u(c_L) + v(d) + \sum_y U(w(y) + c - c_L)P_L(y) - e_L$$  (12)

where $c_L$ is defined by the following:

$$u'(c_L) - \sum_y U'(w(y) + c - c_L)P_L(y) = 0.$$  (13)

Thus, the principal’s problem is

$$\max_{\{c, d, w(y), c_L\}} \sum w(y)P_H(y) - c - pd \quad \text{subject to (10) - (13).}$$  (14)

First order conditions are:

$w(y) : \quad -P_H(y) + \rho U'(w(y))P_H(y) + \gamma [U'(w(y))P_H(y) - U'(w(y) + c - c_L)P_L(y)] + \eta_H [U''(w(y))P_H(y)] + \eta_L [U''(w(y) + c - c_L)P_L(y)] = 0,$

$c : \quad -1 + \rho u'(c) + \gamma [u''(w(y) + c - c_L)P_L(y)] + \eta_H u''(c) + \eta_L [U''(w(y) + c - c_L)P_L(y)] = 0,$

$d : \quad -p + \rho v'(d) = 0,$

$c_L : \quad \gamma [-u'(c_L) + \sum U'(w(y) + c - c_L)P_L(y)] + \eta_L [U''(c_L) + \sum U''(w(y) + c - c_L)P_L(y)] = 0.$

Similarly to the results from the benchmark model, we derive the following lemma.

**Lemma 10** (i) $\eta_L = \eta_H = 0$, (ii) $\rho > 0$, $\gamma > 0$, (iii) $c_L < c$ if and only if $\sum_y U'(w(y))P_H(y) < \sum_y U'(w(y))P_L(y).$
By applying the results of Lemma 10 to the first order conditions, we derive the following two key equations:

\[ \frac{p}{v'(d)} = \rho \quad \text{and} \quad \frac{1}{u'(c)} + \gamma \left[ \frac{u'(c_L)}{u'(c)} - 1 \right] = \rho. \]  

Equations (15) show that the principal balances the marginal cost and the marginal benefit of giving one extra unit of utility to the agent. The principal has two methods of increasing the agent’s utility in period 1: by adding \( \frac{1}{u'(d)} \) units of the perk good or by adding \( \frac{1}{u'(c)} \) units of money. On one hand, the marginal benefit of either of these is the relaxed individual rationality condition, which is represented by the shadow value of the individual rationality condition, \( \rho \), on the right hand sides of the two equations. On the other hand, there is a difference in the marginal costs of the two different methods. Firstly, giving one unit of utility to the agent through the perk good costs \( p \times \frac{1}{u'(d)} \) units of money to the principal. Secondly, giving one unit of utility to the agent by giving him money not only costs \( \frac{1}{u'(c)} \) units of money, but also costs \( \gamma \left[ \frac{u'(c_L)}{u'(c)} - 1 \right] \) due to the hidden saving problem. The agent can save the increased monetary payment against the uncertainty in the future, and this reduced uncertainty makes it more costly for the principal to implement higher effort. This marginal cost is higher when the moral hazard problem is more severe (larger \( \gamma > 0 \)) and/or when the hidden saving problem is more severe (larger \( \frac{u'(c_L)}{u'(c)} > 1 \), i.e., larger \( (c - c_L) > 0 \)).

From equation (15), we derive:

\[ \frac{1}{u'(c)} - \frac{p}{v'(d)} = \gamma \left[ 1 - \frac{u'(c_L)}{u'(c)} \right] \Rightarrow MRS := \frac{v'(d)}{u'(c)} = \frac{p}{1 + \gamma (u'(c_L) - u'(c))}. \]

If \( u'(c_L) > u'(c) \), the marginal rate of substitution is smaller than the price ratio \( p \). This means that, if the agent were allowed to sell the perk good, \( d \), at the market price \( p \), the agent would choose to sell some of his perks.

The following main proposition summarizes the conditions implying the existence of the hidden saving problem; thus, the proposition characterizes conditions for the over-supply of the perk good.
**Proposition 11**  When one of the following conditions holds, the principal provides the agent more perks than the agent would have chosen if the agent were given only money.

1. There are two states, $Y = \{G, B\}$ with $P_H(G) > P_L(G)$.

2. The probability distributions $P_H(y)$ and $P_L(y)$ satisfy MLRP, and $U(\cdot)$ exhibits CARA.

3. The optimal $w(y)$ increases in $y$.

### 3 Extensions

#### 3.1 Additional constraints: increasing wage in outcome

In the case of a utility function $U(\cdot)$ exhibiting CARA and probability distributions satisfying MLRP, we can show that wages increase in the outcome, i.e., $w(y + 1) > w(y)$ for $y \in \{1, \ldots, n - 1\}$. This result is particularly useful since it implies $c_L < c$, a necessary and sufficient condition for the over-supply of perks. However, without constant absolute risk aversion, we may not always obtain this result.

We rarely observe non-monotone contracts in the real world. There could be many reasons. For example, the principal might be afraid of the agent destroying output *ex-post*.11 Whatever the reason is, we can think an exogenous increasing wage constraint:

$$w(y + 1) \geq w(y) \text{ for } y \in \{1, \ldots, n - 1\}. \quad (16)$$

This additional constraint alters only the first FOC with respect to $w(y)$, and we still derive equation (15). Lemma 3 and 4 also do not change, and we can prove $c_L < c$ from the new constraint with the MLRP. Thus, we can show the over-supply of perks.

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11Note that this is different from putting low effort. The effort choice is *ex-ante* while destroying output is *ex-post*. 
3.2 Infinite horizon model

We may interpret our model as a reduced form of a model in an infinitely repeated environment. Suppose that an agent owns wealth \( s \) before signing the contract. We can develop an analogous model. Let \((c^*, (w^*(y))_{y \in Y}, e_H)\) be the optimal contract. Suppose that function \( U(\cdot) \) satisfies the following:

\[
U(s) = u(c^* + s) - e_H + \beta \sum_y U(w^*(y)) \Pr(y; e_H)
\]

where the discount factor is now \( \beta < 1 \).

Then we can interpret function \( U(\cdot) \) as a continuation utility, and the optimal contract in our model is the repetition of the short-term optimal contract. In fact, Fudenberg, Holmstrom, and Milgrom (1990) suggest that short-term contracts are sufficient under fairly general environments.\(^{12}\) A functional fixed-point theorem may be able to derive such function \( U(\cdot) \).

4 Conclusion

We develop a principal-agent model with the hidden saving/borrowing problem. A principal faces the hidden saving problem when the agent deviates from the optimal contract by simultaneously saving more and working less. Instead of using the first-order approach in terms of effort, we assume discrete effort levels and calculate an agent’s saving/borrowing choice at each effort level. This approach allows us to measure exactly how much the agent wants to change the choice of consumption when the agent deviates from the contracted effort. We find that the hidden saving problem exists under fairly standard environments.

\(^{12}\)To be precise, short-term contracts are sufficient if (1) all public information can be used in contracting, (2) the agent can access a bank on equal terms with the principal, (3) re-contracting takes place with common knowledge about technology and preferences and (4) the frontier of expected utility payoffs generated by the set of incentive-compatible contracts is downward sloping at all times.
To avoid the (shadow) cost caused by the hidden saving problem, the principal may want to increase the agent’s consumption of goods that the agent cannot save. We extend the developed model to explain why perks may exist. We find that the principal calculates the amount of perks for the agent by measuring how much the agent wants to save/borrow when the agent deviates from the contracted effort.

We also discuss two possible extensions. The first extension imposes the constraint that wages must be non-decreasing in output. For example, a principal might be afraid of an agent destroying output ex-post, so that she imposes the non-decreasing wage constraint. We show that the hidden saving problem always exists with this constraint. The second extension embeds our two-period contract in an infinitely repeated environment.

A Appendix

A.1 Proof for Lemma 1

From (7), we derive \( \eta_L = 0 \) since \( u'(c_L) = \sum_y U'(w(y) + c - c_L)P_L(y) \) and \( u''(c_L) + \sum_y U''(w(y) + c - c_L)P_L(y) < 0 \). Then (5) and (6) become

\[
-\rho u'(c) + \gamma [u'(c) - u'(c_L)] - \eta_H u''(c) = 0. \tag{18}
\]

Summing up (17) with respect to \( y \) and applying equation (2) and (3), we derive:

\[
-1 + \rho u'(c) + \gamma [u'(c) - u'(c_L)] - \eta_H \sum_y U''(w(y))P_H(y) = 0.
\]

Subtracting the above from (18), we derive \( \eta_H = 0 \) as well.
A.2 Meaning of $\eta_H = 0$ and $\eta_L = 0$

We separate the each saving/borrowing constraint into two inequality constraints. Thus we re-write the principal’s problem as:

\[
\max \left[ -\sum w(y)P_H(y) - c \right]
\]

IR : $u(c) + \sum_y U(w(y))P_H(y) - e_H \geq \bar{V} \tag*{\text{IR}}$

IC : $u(c) + \sum_y U(w(y))P_H(y) - e_H \geq u(c_L) + \sum_y U(w(y) + c - c_L)P_L(y) - c_L \tag*{\text{IC}}$

$S_H^+ : u'(c) - \sum_y U'(w(y))P_H(y) \geq 0, \quad S_H^- : \sum_y U'(w(y))P_H(y) - u'(c) \geq 0$

$S_L^+ : u'(c_L) - \sum_y U'(w(y) + c - c_L)P_L(y) \geq 0, \quad S_L^- : \sum_y U'(w(y) + c - c_L)P_L(y) - u'(c_L) \geq 0$

FOCs are:

$w(y) : \quad - P_H(y) + \rho U'(w(y))P_H(y) + \gamma \left[ U'(w(y))P_H(y) - U'(w(y) + c - c_L)P_L(y) \right]$

$+ (\eta_H^+ - \eta_H^-) \left[ -U''(w(y))P_H(y) \right] + (\eta_L^+ - \eta_L^-) \left[ -U''(w(y) + c - c_L)P_L(y) \right] = 0$

$c : \quad -1 + \rho u'(c) + \gamma \left[ u''(w(y) + c - c_L)P_L(y) \right]$

$+ (\eta_H^+ - \eta_H^-) \left[ u''(c) \right] + (\eta_L^+ - \eta_L^-) \left[ -\sum U''(w(y) + c - c_L)P_L(y) \right] = 0$

$c_L : \quad \gamma \left[ -u'(c_L) + \sum U'(w(y) + c - c_L)P_L(y) \right] + (\eta_L^+ - \eta_L^-) \left[ u''(c_L) + \sum U''(w(y) + c - c_L)P_L(y) \right] = 0$

where $\eta_H^+$ is a multiplier for $S_H^+$, $\eta_H^-$ for $S_H^-$, $\eta_L^+$ for $S_L^+$, and $\eta_L^-$ for $S_L^-$. By the same logic in the analysis of the previous model, we derive $(\eta_H^+ - \eta_H^-) = 0$ and $(\eta_L^+ - \eta_L^-) = 0$. However, this does not mean that any of $\eta_H^+$, $\eta_H^-$, $\eta_L^+$, and $\eta_L^-$ is zero.

A.3 Proof for Lemma 2

First, suppose $\gamma = 0$. Then $w(y)$ is a constant function. Consider a simple deviation $(c, e_L)$, i.e., the agent does not optimize consumption with the effort deviation. Given that $w(y)$ is constant, $e_H$ is clearly not enforceable with the deviation. Thus we conclude $\gamma > 0$. 

21
Second, suppose $\rho = 0$. If $c_L \leq c$, we derive $u'(c_L) \geq u'(c)$, which means that $\frac{1}{u'(c)} \leq 0$ from equation (8). However, this is impossible. If $c_L > c$, $\frac{U'(w(y) + c - c_L)}{U'(w(y))} > 1$ for all $y \in Y$. Also $\frac{P_L(y)}{P_H(y)} > 1$ for at least one $y$. These two facts imply $\frac{1}{U'(w(y))} \leq 0$ for at least one $y$ from equation (9). This is again impossible.

A.4 Proof for Lemma 3

(If.) Note $u'(c) = \sum_y U'(w(y))P_H(y) < \sum_y U'(w(y))P_L(y)$. Thus, given that the agent has deviated to exert $e_L$, the agent wants to save in period 1 to increase the marginal utility in period 1 and to decrease the marginal utility in period 2.

(Only if.) Since $c - c_L > 0$, we derive the result from the following inequalities:

$$\sum_y U'(w(y))P_H(y) = u'(c) < u'(c_L) = \sum_y U'(w(y) + c - c_L)P_L(y) < \sum_y U'(w(y))P_L(y).$$

A.5 Proof for Lemma 4

We prove the statement by a mathematical induction.

1. If the outcome space is of size two, i.e., $Y = \{1, 2\}$, the result holds trivially.

2. Assume that the statement is correct for the outcome space of size $n$, i.e., $Y = \{1, 2, \ldots, n\}$.

3. Consider the case of $Y = \{1, 2, \ldots, n, n + 1\}$.

Consider $Y = \{2, 3, \ldots, n, n + 1\}$ with the following re-defined probabilities:

For each $e \in \{H, L\}$, $\bar{P}_e(y) := \begin{cases} P_e(y) & \text{if } y \in \{3, \ldots, n, n + 1\} \\ P_e(1) + P_e(2) & \text{if } y = 2 \end{cases}$

Note that these redefined probabilities, $\bar{P}_H(y)$ and $\bar{P}_L(y)$, satisfy MLRP since

$$\frac{P_L(1)}{P_H(1)} > \frac{P_L(1) + P_L(2)}{P_H(1) + P_H(2)} > \frac{P_L(2)}{P_H(2)} > \frac{P_L(3)}{P_H(3)} > \ldots.$$
Then from the assumption, we derive
\[
U'(w(2))(P_H(1) + P_H(2)) + \sum_{y=3}^{n+1} U'(w(y))P_H(y) < U'(w(2))(P_L(1) + P_L(2)) + \sum_{y=3}^{n+1} U'(w(y))P_L(y)
\]

\[\Leftrightarrow U'(w(2))P_H(1) - U'(w(1))P_H(1) + \sum_{y=1}^{n+1} U'(w(y))P_H(y)
< U'(w(2))P_L(1) - U'(w(1))P_L(1) + \sum_{y=1}^{n+1} U'(w(y))P_L(y).\]

To complete the proof, it is enough to show
\[
U'(w(2))P_H(1) - U'(w(1))P_H(1) \geq U'(w(2))P_L(1) - U'(w(1))P_L(1)
\]

\[\Leftrightarrow U'(w(2))(P_H(1) - P_L(1)) - U'(w(1))(P_H(1) - P_L(1)) \geq 0\]

\[\Leftrightarrow (U'(w(2)) - U'(w(1))(P_H(1) - P_L(1)) \geq 0.\]

Note that \(P_L(1) > P_H(1)\) (otherwise, MRLP cannot be satisfied) and \(U'(w(2)) - U'(w(1)) \leq 0\). Thus, the proof is completed.

**A.6 Proof for Corollary 6**

Suppose \(c = c_L\) is possible, then equation [9] implies that \(w(y)\) increases in \(y\) because of MLRP. This again implies that \(c > c_L\) by Proposition 5, which is contradictory to the supposition of \(c = c_L\). Thus we have shown \(c \neq c_L\).

**A.7 Proof for Proposition 7**

Two states are “good” (\(G\)) and “bad” (\(B\)), \(Y = \{G, B\}\) with \(P_H(G) > P_L(G)\). First observation is that, as long as wage schedule \(w(\cdot)\) satisfies \(w(G) > w(B)\), \(\sum_y U'(w(y))P_H(y) < \sum_y U'(w(y))P_L(y)\) is satisfied. The second observation is that \(w(G) \leq w(B)\) cannot enforce high effort \(e_H^{13}\). Given that \(w(G) > w(B)\), it is trivial that \(\sum_y U'(w(y))P_H(y) < \)

\[13\text{This is trivial. Consider a simple deviation } (c, e_L), \text{ i.e., the agent deviates only to exert low effort. It is trivial to show that the incentive compatibility constraint is not satisfied.}\]
\[ \sum_y U'(w(y))P_L(y). \] Thus \( c_L < c \) is satisfied by Lemma 3

A.8 Proof for Proposition 8

First, we consider the case that function \( U(\cdot) \) exhibits CARA. The following equation shows that \( \frac{U'(w(y) + c - c_L)}{U'(w(y))} \) in equation (9) is constant in \( w(y) \) because of the property of CARA:

\[
\frac{d}{dw(y)} \left( \frac{U'(w(y) + c - c_L)}{U'(w(y))} \right) = \frac{U''(w(y) + c - c_L) - U'(w(y) + c - c_L)U''(w(y))}{U'(w(y))} \frac{U''(w(y))}{(U'^2)} = 0. \tag{19}
\]

Thus equation (9) implies that \( \frac{1}{U'(w(y))} \) increases in \( y \) since \( \frac{P_L(y)}{P_H(y)} \) decreases in \( y \). Thus we show decreasing \( U'(w(y)) \) and increasing \( w(y) \). Thus we show \( \sum_y U'(w(y))P_H(y) \leq \sum_y U'(w(y))P_L(y) \) by Lemma 4. This implies \( c_L < c \) by Lemma 3.

A.9 Statement and proof for Proposition 8 with IARA \( U(\cdot) \)

Proposition 12 Assume that the probability distributions \( P_H(y) \) and \( P_L(y) \) satisfy MLRP and that \( U(\cdot) \) exhibits increasing absolute risk aversion (IARA). Then the agent wants to save at low effort, i.e., \( c_L < c \).

Proof. Note the following.

\[
\frac{d}{dw(y)} \left[ \frac{U'(w(y) + c - c_L)}{U'(w(y))} \right] = \frac{U'(w(y) + c - c_L)}{U'(w(y))} \left[ \left( -\frac{U''(w(y))}{U'(w(y))} \right) - \left( -\frac{U''(w(y) + c - c_L)}{U'(w(y) + c - c_L)} \right) \right] \tag{20}
\]

Suppose \( c \leq c_L \). Then \( \frac{d}{dw(y)} \left[ \frac{U'(w(y) + c - c_L)}{U'(w(y))} \right] \geq 0 \). On one hand, if \( w(y) > w(y + 1) \), then

\[
\frac{U'(w(y+1)+c-c_L)}{U'(w(y+1))} \leq \frac{U'(w(y)+c-c_L)}{U'(w(y))}. \]Since \( \frac{P_L(y+1)}{P_H(y+1)} < \frac{P_L(y)}{P_H(y)} \) for \( y < w(y+1) \) and \( \frac{U'(w(y+1)+c-c_L)}{U'(w(y+1))} P_L(y+1) \frac{P_L(y+1)}{P_H(y+1)} < \frac{U'(w(y)+c-c_L)}{U'(w(y))} P_L(y) \frac{P_L(y)}{P_H(y)} \),

Then, we derive \( w(y) < w(y + 1) \) from equation (9). This is a contradiction. On the other hand, if \( w(y) < w(y + 1) \), then \( c > c_L \) by Lemma 3. This is a contradiction again. Thus we have shown \( c > c_L \).
A.10 Proof for Proposition 9

Consider equation (9) for $y$ and $y+1$. By subtracting the one for $y$ from the one for $y+1$, we derive:

$$
\frac{1}{U'(w(y+1))} - \frac{1}{U'(w(y))} = \gamma \left[ \frac{U'(w(y) + c - c_L) P_L(y)}{U'(w(y))} - \frac{U'(w(y+1) + c - c_L) P_L(y+1)}{U'(w(y+1))} \right]
$$

(21)

Note the following:

$$
\frac{d}{dw(y)} \left[ \frac{U'(w(y) + c - c_L)}{U'(w(y))} \right] = \frac{U'(w(y) + c - c_L)}{U'(w(y))} \left[ \left( - \frac{U''(w(y))}{U'(w(y))} \right) - \left( - \frac{U''(w(y) + c - c_L)}{U'(w(y) + c - c_L)} \right) \right]
$$

Thus, $\frac{d}{dw(y)} \left[ \frac{U'(w(y)+c-c_L)}{U'(w(y))} \right] > 0$ since $c > c_L$. Suppose $w(y) \geq w(y+1)$, then $\frac{U'(w(y)+c-c_L)}{U'(w(y))} \geq \frac{U'(w(y+1)+c-c_L)}{U'(w(y+1))}$. Also $\frac{P_L(y)}{P_H(y)} > \frac{P_L(y+1)}{P_H(y+1)}$ by MLRP. These two facts imply that the right-hand side of equation (21) is positive, which implies that $w(y+1) > w(y)$. This is a contradiction to the supposition. Thus, we conclude that $w(y+1) > w(y)$.

A.11 Proof for Proposition 11

1. Proposition 7 proves $c_L < c$, which proves the statement.
2. Proposition 9 proves $c_L < c$, which proves the statement.
3. Lemma 3 and 4 proves that $c_L < c$, which completes the proof.

References


