Entry, Exit and Endogenous Firm Dispersion over the Business Cycles*

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Abstract

Various empirical works have shown that dispersion of firm-level profitability is significantly countercyclical. I incorporate firms’ technology adoption decision into firm dynamics model with business cycle features to explain these empirical findings both qualitatively and quantitatively. The option of endogenous exiting and credit constraint jointly play an important role in motivating firms’ risk taking behavior. The model predicts that relatively small sized firms are more likely to take risk, and that the dispersion measured as the variance/standard deviation of firm-level profitability is larger in recessions, which are consistent to the data.

Keywords: Firm Dynamics, Business Cycles, Countercyclical Dispersion.

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1 Introduction

A growing branch of literature has focused on the cyclical changes in firm-level uncertainty. Recently, many works show that firm level productivity dispersion is negatively correlated with

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Standard models on industry dynamics that incorporate firm heterogeneity and entry/exit behavior build on the seminal work of Hopenhayn (1992). While being successful in explaining cyclicality of entry/exit rate, these models give counterfactual prediction on cyclical behavior of dispersion of cross-sectional distribution of firm-level productivity: in good times, entry rate is high and exit rate is low, firms with relatively low productivity stay in the industry, therefore the dispersion is high; when bad time comes, only high productivity firms can survive, and the dispersion is low.

The real option literature that aims at explaining such countercyclical dispersion suggests that the causal relationship goes from the second moment variation of economic activities to the first moment. An influential paper dedicated in this direction is Bloom (2009) focusing on partial equilibrium, in which time varying dispersion is modeled as second-moment shocks to firm specific productivity. Bloom shows that increased volatility, together with various adjustment costs to capital and labor, leads to larger option value of waiting and a pause in investment and employment. A sizable drop in aggregate economic activity occurs because of this "wait-and-see" effect. This idea is generalized into general equilibrium model in Bloom et al. (2009). Combined first- and second-moment shocks are then adopted in many consequent papers, for example Bachmann and Bayer (2009a, 2009b), Chugh (2010). However, Bachmann and Bayer (2009a, 2009b) show that once a joint first- and second-moment shock process is calibrated to match the German data in a neoclassical model, shocks to variance alone in firm-level TFP can hardly be the driving force of the economy. Bachmann, Elstner and Sims (2010) estimates SVAR using different identification schemes based on U.S. and German survey data and finds no evidence of "wait-and-see" effect. In addition, all firms of different sizes bear the same amount of risk in the second-moment driven model and the process of entry and exit is neglected.

An alternative way of explaining the countercyclical dispersion on the firm level is to take the opposite direction on the causality. It is possible that it is the first-moment shock that actually generates the second-moment variation. I take this view and explore a potential mechanism as a complement to the prevailing opinion on uncertainty shocks. In this paper, I incorporate firms’ technology adoption decision into an otherwise standard firm dynamics model with business cycle features. The main feature of this model is that, conditional on staying, an operating firm can choose from two different types of technology: a safe one, and a risky one with no risky premium. The option of exiting naturally forms a lower bound on the value of a firm, and therefore provides
a certain degree of incentive on risk taking behavior on the margin resulting from the convexity of the value function.

The model at work is built on the one developed in Vereshchagina and Hopenhayn (2009). Empirical works show that although the risk premium to risky entrepreneurial activity is very low, some entrepreneurs do not diversify the risk, especially the relatively poor ones. Motivated by this fact, Hopenhayn and Vereshchagina develop a theory of entrepreneurs’ endogenous risk taking based on the assumption of discrete occupational choice. The indivisibility occupational choice between being an entrepreneur and becoming a worker can result in convexity in an agent’s value function even when the agent is strictly risk averse, and the local convexity motivates risk taking behavior of a current entrepreneur with relatively low wealth level around the cutoff point. This paper concentrates on the individual decision problem, and aggregate consequence is not the main focus.

I take this model into the setting of industry dynamics where an incumbent firm has the option of exiting the industry and the option value of exiting can depend on the size of the firm. The convexity of value function of a firm owner occurs around the cutoff level of firm size, below which exiting is a better choice and above which adopting safe technology leads to higher value. Firms in the neighborhood of this cutoff may find risky technology appealing because the randomness in production partly eliminates the convexity. It is precisely the fraction of firms adopting the risky technology that determines the overall dispersion or uncertainty of all firms in profitability. The average profitability of safe technology also works as the cyclical indicator of economic condition, which is positively correlated to aggregate output. In a recession, the fraction of firms choosing the risky technology is larger hence the dispersion in firm profitability is larger, and the exiting rate is higher. The opposite happens in a boom.

The steady states of this model also have the potential of explaining the empirical finding that less developed regions and countries tend to have larger dispersion in firm profitability, as documented in, for example, Hsieh and Klenow (2009).

The rest of the paper is organized as follows. Section 2 contains a simple three-period model that illustrates the mechanism and shows preliminary results. Section 3 takes the simple model into infinite horizon. Section 4 concludes.
2 A Simple Model

2.1 Setup

There are 3 periods, $t = 0, 1, 2$. There are a continuum of risk neutral entrepreneurs, each of whom owns a firm with different level of initial wealth $w_0 \in [0, \bar{w}]$. The c.d.f. of entrepreneurs is given as $G(w_0)$. At period 0, initial wealth $w_0$ can be divided into investment $k_0$ for future wealth and immediate consumption $w_0 - k_0$. If an entrepreneur decides to invest $k_0$, then she will get $w_1 = F(Z, k)$ as period 1 wealth, where

$$F(Z, k) = Zk^\alpha, 0 < \alpha < 1,$$

and $Z$ represents the realized profitability of technology the entrepreneur chooses after investment decision. An entrepreneur can choose either one out of two available technologies: a safe one and a risky one. For the safe technology, $Z = A$ for sure, while for a risky one, with probability $p \in (0, 1)$, $Z = y > A$, and with probability $1 - p$, $Z = 0$. Both technologies give the same expected value of $Z$, that is, $py + (1 - p)0 = A$. As a result of linearity of $F(Z, k)$ in $Z$, the expected return of risky project is the same as the safe one, i.e., there is no ex ante risk premium. At period 1, after the uncertainty in $Z$ realizes, the agent can decide whether to close her firm, exit the industry and get outside option value $V^0$, or stay. Conditional on staying, she makes the investment choice $k_1$ and technology adoption choice again based on period 1 wealth $w_1$. In the last period, she simply consumes her final wealth $w_2$. The objective of an agent with initial wealth $w_0$ is to maximize her discounted consumption, with discount factor $\beta$:

$$V_0(w_0) = \max_{\{c_t\}_{t=0,1,2}} \sum_{t=0}^{2} \beta^t c_t$$

$$= \max_{0 \leq k_0 \leq w_0} \left\{ (w_0 - k_0) + \beta \max \{V_1(Ak_0^\alpha), (1 - p)V_1(0) + pV_1(yk_0^\alpha) \} \right\}$$

where $V_t(w_t)$ is the time $t$ value for an agent with wealth $w_t$.

It is convenient to work backwards. At time $t = 2$,

$$V_2(w_2) = w_2.$$

At time $t = 1$, an agent with $k_1 > 0$ will be indifferent between operating a safe project and a risky one. Assume that all agents will perform safely in this case, which is consistent with their choice if they were risk averse. The period 1 value for a staying firm will be:

$$V_1(w_1) = \max_{0 \leq k_1 \leq w_1} \left\{ (w_1 - k_1) + \beta Ak_1^\alpha \right\}.$$
Let $k^* = (\alpha \beta A)^{1/(1-\alpha)}$, then the optimal choice for $k_1$ will be $w_1$ if $w_1 < k^*$ or $k^*$ otherwise. And

$$V_1^1(w_1) = \begin{cases} 
\beta Aw_1^\alpha & \text{if } w_1 < k^* \\
 w_1 - k^* + \beta Ak^\alpha & \text{o/w}
\end{cases}$$

Therefore, the value of a firm with wealth level $w_1$ at the beginning of period 1 will be given by

$$V_1(w_1) = \max \{ V_0, V_1^1(w_1) \}.$$ 

Let $w_1^*$ be such that $V_0^0 = V_1^1(w_1^*)$. Note that there is a kink at $w_1^*$ and $V_1(w_1)$ is convex in a neighborhood of $w_1^*$. This gives a firm with relatively low wealth level an incentive to take a risky project before it enters period 1. At $t = 0$, a firm makes the investment decision and chooses a technology:

$$V_0(w_0) = \max_{0 \leq k_0 \leq w_0} \{(w_0 - k_0) + \beta \max \{ V_1(Ak_0^\alpha), (1 - p) V_1(0) + pV_1(yk_0^\alpha) \}\}$$

$$= \max_{0 \leq k_0 \leq w_0} \{(w_0 - k_0) + \beta \max \{ V_0^0, V_1^1(Ak_0^\alpha), pV_1^1(yk_0^\alpha) + (1 - p)V_0^0 \}\}.$$ 

To characterize a firm’s technology choice, it is useful to introduce the following condition on parameters.

**Condition 1.** $V_0^0 < \alpha^{\frac{\alpha}{1-\alpha}} \beta \frac{1}{1-\alpha} y^{\frac{1}{1-\alpha}} p^{1+\frac{\alpha}{1-\alpha}} (1-p^{1-\alpha}) / (1 - p) = (\alpha \beta A)^{\frac{1}{1-\alpha}} (p^\alpha - p) / (1 - p)$.

**Proposition 1.** At $t = 0$, if Condition 1 holds, then there exist $0 < k_0^I < k_0^{III}$ such that a firm that chooses to invest $k_0$ will

1. Exit the industry and get $V_0^0$ for sure in period 1 if $0 \leq k_0 \leq k_0^I$;
2. Invest $k_0$ in a risky project if $k_0^I < k_0 < k_0^{III}$;
3. Invest $k_0$ in a safe project if $k_0 > k_0^{III}$.

**Proof.** Let $k_0^{A}$ be such that $A(k_0^{A})^\alpha = k^*$, that is,

$$k_0^{A} = (\alpha \beta) \frac{1}{\alpha(1-\alpha)} A^{\frac{1}{1-\alpha}}.$$ 

Similarly, let $k_0^{y}$ be such that $y(k_0^{y})^\alpha = k^*$, that is

$$k_0^{y} = (\alpha \beta p)^{\frac{1}{\alpha(1-\alpha)}} y^{\frac{1}{1-\alpha}} < k_0^{A}.$$
When $k_0 < k_0^{III}$, the firm that stays in period 1 will further invest all $w_1$ in a safe project since $w_1 < k^*$. Let $k_0^{III}$ be the investment level at which a firm is indifferent between investing in a safe project and a risky one, meaning $V_1^i(Ak_0^{III}) = pV_1^i(yk_0^{III}) + (1 - p)V^0$. When Condition 1 holds, it is straightforward to show that

$$k_0^{III} = \left[ \frac{(1 - p)V^0}{\beta y^{1+\alpha}(p^{1+\alpha} - p^2)} \right]^{\frac{1}{\alpha^2}} < k_0^\beta.$$

Let $k_0^{II}$ be the investment level at which a firm is indifferent between exiting and investing in a risky project, that is, $V^0 = V_1^i(Ak_0^{II})$, then

$$k_0^{II} = \left[ \frac{V^0}{\beta A^{1+\alpha}} \right]^{\frac{1}{\alpha^2}} < k_0^{III}.$$

Let $k_0^I$ be the investment level at which a firm is indifferent between exiting and investing in a risky project, that is, $V^0 = pV_1^i(yk_0^I) + (1 - p)V^0$, then

$$k_0^I = \left[ \frac{V^0}{\beta Ay^{\alpha}} \right]^{\frac{1}{\alpha^2}} < k_0^{II}.$$

\[\blacksquare\]

FIGURE 1

![Graph showing continuation value at t=0 with k0](graph.png)

With this proposition established, we are ready to analyze a firm’s investment choice in period 0.
**Condition 2.**

\[ V^0 < \alpha \frac{2\alpha^2}{\beta^{1+\alpha^2}} y^{1-\alpha} p \frac{1+\alpha^2-\alpha^3}{1-\alpha^2} (p^{\alpha(1-\alpha)} - p) / (1 - p). \]

\[ V^0 < \alpha \frac{2\alpha^2}{\beta^{1+\alpha^2}} y^{1-\alpha} p^{2 \alpha^2} (p^{1+\alpha} - p^2) / (1 - p). \] [If this is true, then Cdt2 \Rightarrow Cdt1]

**Proposition 2.** At \( t = 0 \), if Condition 1 and 2 hold, then the decision rule of an entrepreneur with initial wealth \( w_0 \) will be one of the following:

1. If \( 0 < w_0 \leq k_0^f \), she consumes all \( w_0 \) and exits in period 1 for sure;

2. If \( k_0^f < w_0 < k_0^{III} \), she invests all \( w_0 \) in a risky project, then with probability \( p \), \( w_1 = yk_0^\alpha \), she in turn invests all \( w_1 \) in period 1; with probability \( 1 - p \), \( w_1 = 0 \), she exits in period 1;

3. If \( k_0^{III} \leq w_0 \leq k_0^A \), she invests all \( w_0 \) in a safe project, then invests all \( w_1 = Ak_0^\alpha \) in period 1;

4. If \( k_0^A < w_0 \leq k^* \), she invests all \( w_0 \) in a safe project, then invests \( k^* \) and consumes the rest in period 1;

5. If \( w_0 > k^* \), she invests \( k^* \) and consumes the rest in both periods.

**Proof.** This proposition can be proved by similar logic as in proof of Proposition 1. Condition 2 assures that when \( w_0 < k_0^{III} \), an entrepreneur always wants to invest all \( w_0 \). If Condition 2 is violated, the only consequence is that some entrepreneur with \( w_0 < k_0^{III} \) will consume a positive fraction of \( w_0 \) and invest the rest.

Assume a form of "Law of Large Numbers" holds, meaning that there are many firms at each \( w_0 \), and the fraction of risky project with \( y \) realized is \( p \). The average variation in TFP that firms choose to take in period 0 can be calculated as

\[
\int_{W} var (Z) dG (w_0|k_0 > 0) \\
= \int_{k_0^f}^{k_0^{III}} p (1 - p) y^2 dG (w_0|k_0 > 0) + 0 \\
= p (1 - p) y^2 \frac{G (k_0^{III}) - G (k_0^f)}{1 - G (k_0^f)}.
\]

At the same time, the aggregate or average output is:

\[
\int_{W} E (F (Z, k)) dG (w_0|k_0 > 0) \\
= py \int_{k_0^f}^{k^*} w_0^dG (w_0|k_0 > 0) + py (k^*)^\alpha \frac{1 - G (k^*)}{1 - G (k_0^f)}
\]
2.2 Comparative Statics

Consider a change in available technologies such that $p$ decreases while $y$ (the range of risky technology) remains the same. This means the safe technology $A$ also decreases.

**Proposition 3.** Assume $V^0$ and $y$ remain the same and Condition 1 and 2 always hold. Then $k_0^I$ and $k_0^{III}$ are decreasing in $p$, and $k^*$ is increasing in $p$. In addition, $k_0^{III} - k_0^I$ and $k_0^{III}/k_0^I$ is decreasing in $p$.

This implies that, for example, when $G(\cdot)$ is Pareto, the fraction of entrepreneurs who are willing to operate a risky project is larger when $p$ is lower. In particular, let $p^H > p^L > 0$ and $p^H + p^L = 1$, then the variance in return from a risky project remains unchanged when $p^H$ decreases to $p^L$. However, the average variation in TFP becomes larger.

**Corollary 1.** Let $V^0$ and $y$ remain unchanged and Condition 1 and 2 always hold. Suppose the distribution of initial wealth $G(\cdot)$ has a Pareto tail to the right of $k_0^I$ when risky technology is $p^H$. Then by last proposition, the average variation in TFP that firms choose to take increases when $p^H$ changes to $p^L$. Meanwhile, the aggregate output decreases.

**FIGURE 2.** $p^H = 0.55, p^L = 0.45$. 

![Graph showing continuation value at t=0 with k0 for safe, risky, and max(Safe, Risky) options.](image-url)
2.3 Extension

2.3.1 Employment Decision

In the simple 3 period model described above, labor is not explicitly modeled. In fact, including employment decision does not alter the result. Consider a decreasing returns to scale production function:

\[ F(Z, k, l) = Z^{\alpha} l^{\nu}, \alpha > 0, \nu > 0, \alpha + \nu < 1. \]

The timing is the same as before, and employment decision is made simultaneously with investment decision. Suppose the cost per unit of labor is \( c \), then the budget constraint becomes:

\[ k_t + c l_t \leq w_t \quad \text{for} \quad t = 0, 1. \]

It can be shown that the optimal choice of labor is always proportional to the optimal choice of investment, that is, \( l = \frac{\nu}{\alpha} k \), and maximum investment \( k^* = \left( \frac{\alpha \beta A}{1/(1-a)} \right)^{1/(1-a)} \). Taking this into account, the production function with optimal labor-investment ratio becomes:

\[ \tilde{F} \left( \tilde{Z}, k \right) = \tilde{Z}^{\alpha} k^{\nu} \text{ with } \tilde{Z} = Z \left( \frac{\nu}{\alpha c} \right)^{\nu}, a = \alpha + \nu. \]

And the budget constraint can be re-written as \( k_t \leq \frac{2}{a} w_t \) for \( t = 0, 1 \). Consequently, similar conditions and propositions can be established.

**Condition 3 (1').** \( V^0 < \alpha^a \beta^{1-a} \tilde{y}^{1-a} p^{1+a-a^2} (1 - p^{1-a}) / (1 - p) = \left( \frac{\tilde{\alpha}^a \beta}{1/(1-a)} \right)^{1/\alpha} (p^a - p) / (1 - p). \)

Under the modified condition, the counterpart of Proposition 1 holds with modified cutoffs:

\[
\begin{align*}
  k_0^I &= \left( \frac{a}{\alpha} \right)^a \frac{V^0}{\beta \tilde{y}^a} \left( \frac{1}{\alpha} \right)^{1/\alpha} \\
  k_0^{II} &= \left( \frac{a}{\alpha} \right)^a \frac{V^0}{\beta \tilde{A}^{1+a}} \left( \frac{1}{\alpha} \right)^{1/\alpha} \\
  k_0^{III} &= \left( \frac{a}{\alpha} \right)^a \frac{1 - p}{\beta \tilde{y}^{1+a} (p^{1+a} - p^2)} \left( \frac{1}{\alpha} \right)^{1/\alpha} \\
  k_0^y &= \left( \frac{a}{\alpha} \right)^{\frac{1}{\alpha}} \left( \frac{1}{\alpha} \right)^{\frac{1}{\alpha}} \left( \frac{1}{\alpha} \right)^{\frac{1}{\alpha}} \tilde{y}^{1/\alpha} \\
  k_0^A &= \left( \frac{a}{\alpha} \right)^{\frac{1}{\alpha}} \left( \frac{1}{\alpha} \right)^{\frac{1}{\alpha}} \left( \frac{1}{\alpha} \right)^{\frac{1}{\alpha}} \tilde{A}^{1/\alpha} \\
\end{align*}
\]

**Condition 4 (2').** \( V^0 < \left( \frac{a}{\alpha} \right)^{1/a} \beta^{1-a} \tilde{y}^{1-a} p^{1+a-a^2} (1 - p^{1-a}) / (1 - p). \)

**Proposition 4.** At \( t = 0 \), if Condition 1' and 2' hold, then the decision rule of an entrepreneur with initial wealth \( w_0 \) will be:

1. If \( 0 < w_0 \leq \frac{a}{\alpha} k_0^I \), she consumes all \( w_0 \) and exits in period 1 for sure;
2. If $\frac{a_{k_{I}}^{I}}{\alpha} < w_{0} < \frac{a_{k_{I}}^{I I}}{\alpha}$, she inputs all $w_{0}$ for production and adopts the risky technology; then with probability $p$, $w_{1} = \tilde{y}k_{0}^{\alpha}$, she in turn inputs all $w_{1}$ in period 1; with probability $1 - p$, $w_{1} = 0$, she exits in period 1;

3. If $\frac{a_{k_{I}}^{I I}}{\alpha} \leq w_{0} \leq \frac{a_{k_{I}}^{A}}{\alpha}$, she inputs all $w_{0}$ for production and adopts the safe technology, then uses all $w_{1} = Ak_{0}^{\alpha}$ in period 1;

4. If $\frac{a_{k_{I}}^{A}}{\alpha} < w_{0} \leq \frac{a_{k_{I}}^{*}}{\alpha}$, she inputs all $w_{0}$ for production and adopts the safe technology, then uses $\frac{a_{k_{I}}^{*}}{\alpha}$ and consumes the rest in period 1;

5. If $w_{0} > \frac{a_{k_{I}}^{*}}{\alpha}$, she uses only $\frac{a_{k_{I}}^{*}}{\alpha}$ for safe production and consumes the rest in both periods.

The only difference now is that conditional on staying, an entrepreneur divides her total input into two parts: a fraction $\frac{a}{\alpha}$ for investment, and the rest for hiring labor.

The analysis of comparative statics remains unchanged.

### 2.3.2 Linear Outside Option and Operational Cost

For the sake of tractability, I assume that outside option is a constant value, especially constant to the wealth level $w$ of an entrepreneur. A more realistic setup would be such that $V^{0}(w) = v^{0} + \theta w$. Intuitively, linear outside option means that an entrepreneur can sell her wealth holdings at price $\theta$ when she closes her firm. Mathematically, outside option plays a similar role to operational cost. However, analytical solution does not exist when $V^{0}(w)$ is linear or when operational cost is included (even a fixed cost). The decision rule for an entrepreneur with initial wealth $w_{0}$ can only be characterized numerically, for example Figure 3. The shape of continuation values, especially the non-concavity, is preserved under this more realistic setting, hence the same mechanism in the simple model still works.
3 Full Model

3.1 Setup

Time is discrete, with infinite horizon. At the beginning of each period, an incumbent entrepreneur decides whether her firm will stay in business for this period based on her current wealth level $w$ and average profitability $A$. If she chooses to exit, she gets $V^0(w, A)$ immediately. If she chooses to stay, then she decides investment $k$ and hires labor $l$ given labor cost $c$ per unit, and consume the rest of her wealth; after this, she in turn chooses between the safe technology and the risky one. Technology choice, investment and employment jointly determine the incumbent’s next period wealth.

$$V(w, A) = \max \left\{ V^0(w, A), V^1(w, A) \right\}$$

where the value of exiting is defined as follows:

$$V^0(w, A) = v^0 + \theta w,$$

and the value of staying:

$$V^1(w, A) = \max_{k, l} \left\{ w - k - cl + \beta \left\{ E \left[ V\left( w^{A'}, A' \right) | A \right] \right\}, E \left[ p' V'(w^y, A') + (1 - p') V (0, A') | A \right] \right\}$$
subject to

\[ \begin{align*}
    k + cl &\leq w, \\
    w^{A'} &= A'k^\alpha l^\nu, \\
    w^y &= yk^\alpha l^\nu, \\
    A' &\sim \pi (A'|A), \\
    k &\geq 0, l \geq 0.
\end{align*} \]

As is discussed in the simple example, the optimal choice of labor hiring is always proportional to the optional investment level. Therefore, it is convenient to re-write the value of staying such that \( k \) is the only control variable.

\[
V^1(w, A) = \max_k \left\{ w - \frac{a}{\alpha} k + \beta \left\{ E \left( V \left( w^{A'}, A' \right) | A \right) \right\}, E \left[ p' V (w^y, A') + (1 - p') V (0, A') | A \right] \right\}
\]

subject to

\[ \begin{align*}
    0 &\leq k \leq \frac{a}{\alpha} w, \\
    w^{A'} &= \tilde{A}'k^\alpha, \\
    w^y &= \tilde{y}k^\alpha, \\
    A' &\sim \pi (A'|A);
\end{align*} \]

where \( a = \alpha + \nu \), and \( \tilde{Z} = Z \left( \frac{u}{\alpha c} \right)^\nu \), as defined in simple model.

### 3.2 Steady State Equilibrium

[To be written]

### 3.3 Calibration

[To be written]

### 4 Conclusion

[To be written]
References


