Health Insurance, Treatment Plan, and
Delegation to Altruistic Physician

Ting Liu
Department of Economics
Michigan State University
110 Marshall-Adams Hall
East Lansing, MI 48824
tingliu@msu.edu

Ching-to Albert Ma
Department of Economics
Boston University
270 Bay State Road
Boston, MA 02215
ma@bu.edu

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Abstract

We study delegating a consumer’s treatment plan decisions to an altruistic physician. The physician’s degree of altruism is his private information. The consumer’s illness severity will be learned by the physician, and also become his private information. Treatments are discrete choices, and can be combined to form treatment plans. We distinguish between two commitment regimes. In the first, the physician commits to treatment decisions at the time a payment contract is accepted. In the second, the physician does not commit to treatment decisions at that time, and can wait until he learns the patient’s illness to do so. In the commitment game, the first best is implemented by a single payment contract to all types of altruistic physician. In the noncommitment game, the first best is not implementable. All but the most altruistic physician type earn positive profits, and treatment decisions are distorted from the first best.

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1 Introduction

Modern medicine is complex, and offers many treatment options. Currently, medicine is about a plan. If one treatment does not solve the patient’s problem, the physician has a plan to try an alternative, and so on. Given the complexity of treatment plans, patients have to rely on the physician. In this paper, we study health insurance, treatment plans, and delegation of treatment plan decisions to physicians.

A number of questions naturally come to mind. First, how are different treatments ordered? Should a less effective and less expensive treatment be used before a more effective and more expensive treatment? Should it be the other way round? Or, if a treatment fails, should a different one be used at all? Second, when physicians may not act in the patient’s best interest, how is this divergence in preferences mitigated? Must delegation lead to efficiency loss? Third, how are insurance premiums affected?

We choose to model medical treatments as discrete procedures. This is in contrast to many models in the literature where “health care quantity” is a continuous variable. By considering discrete procedures, we can study treatment plans, which consist of one procedure after another. A treatment in our model succeeds in eliminating a patient’s illness disutility with some probability. A low-cost treatment has a lower success probability; a high-cost treatment has a higher success probability. Treatments can be combined. If a low-cost treatment fails to eradicate the illness, a high-cost treatment may be used. Again, this is in contrast to many models where health care quantities are supplied only once. In our structural model, we can determine whether or not some treatment plans are efficient.

Delegation of decisions to physicians is widely practiced in medicine. For a long time economists interested in the health market have recognized that physicians are altruistic. Papers in the literature often use the term physician agency for this delegation. Physician agency would not create any issue if the physician always acts in the patient’s best interest. This is unlikely, so we let the physician be an imperfect agent for the patient. His preferences are a weighted sum of his profit and the patient’s utility; the physician’s

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behavior will be guided by partial altruism. Many papers in the literature have assumed that the degree of altruism is given and known. We go beyond the fixed altruism assumption. We let there be many physician types with different degrees of altruism, this being the physician’s private information.

The altruistic physician may trade off his own profit against the consumer’s utility. Nevertheless, we insist that the physician must earn some minimum profit. Sustaining an unbounded financial loss in the long run is simply infeasible for any market participant. We normalize this minimum profit at zero, so in effect we will impose a nonnegative expected profit constraint.

The fundamental problem in the health market is the missing information about health status. Recognizing this, we let the physician learn the patient’s illness severity before he delivers treatments, and this is also his private information. The problem we address, therefore, is delegating decisions to a physician who possesses private information about his own degree of altruism, and the patient’s illness severity. We study how an insurer who aims to maximize consumer surplus will solve these problems.

Information asymmetry often generates inefficiencies. One might suspect that with two dimensions of missing information, delegating the decision to the physician must lead to distortions from full information allocations. Surprisingly, we show that the first best can be achieved if and only if the physician commits to making treatment plan decisions when he accepts a payment contract. Despite the problem of missing information about the physician’s preferences, as well as the consumer’s illness severity, the first best may still be implemented.

We derive equilibria for two extensive-form games. The key difference in these games is when treatment plans are decided. Both games have four stages, and the first two are identical. In Stage 1, an insurer offers an insurance contract to the consumer, and a payment contract to the physician. In Stage 2, nature determines the physician’s degree of altruism and the consumer’s illness severity; the altruism parameter is now made known to the physician. In Stage 3, the physician and the consumer decide whether to accept the contract. Under treatment plan commitment, the physician also decides on treatment plans now. In Stage 4, the physician learns the consumer’s illness severity. Under treatment plan commitment, the physician continues

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2 All the papers in footnote 1 use the known altruism assumption except Choné and Ma (2010) and Jack (2005).
to implement the treatment plan decided earlier. Under treatment plan noncommitment, the physician now decides on which treatment to prescribe.

Both extensive forms have in common that at the contracting Stage 3, the physician already has private information about his altruistic preferences, and he will learn the consumer’s illness severity only after accepting a payment contract. However, in Stage 3, under treatment plan commitment, he decides how he will treat a patient, contingent on the information he will receive, contemporaneously with accepting the payment contract. Under treatment plan noncommitment, the physician does not make treatment decisions when accepting the payment contract, and delays that decision until after he has learned the consumer’s illness severity in Stage 4.

The first-best treatment plan prescribes a conservative approach under a cost convexity assumption, which says that the higher the success probability is, the higher the cost per unit success probability. If the severity is low, then no treatment is used; if it is of medium value, a low-cost treatment will be used; if it is high, then the low-cost treatment will be used, followed by the high-cost treatment if necessary. The high-cost treatment followed by the low-cost, or the high-cost treatment alone, are shown to be inefficient.

If the physician’s altruism is common knowledge, the first best can be implemented. Here, the only missing information is the consumer’s illness severity. How is the first best implemented? Each treatment offers some benefits to the consumer. The altruistic physician also values this benefit, albeit partially. To get the physician to perform the benefit-cost calculus as in the first best, the insurer lets the physician bear an appropriate fraction of the cost. Intuitively, if the physician puts a 50% weight on consumer’s utility, simply let him bear 50% of the cost (adjusted by a factor which translates between cost in monetary units and benefits in utilities). Fully internalizing the social costs and benefits (all multiplied by the altruism parameter), the physician’s incentive is aligned to the first best. A lump-sum transfer equal to 50% of the expected cost in the first best allows the physician to break even.

The problem is that the exact degree of altruism is unknown! Can cost sharing and delegation achieve the first best? Our surprising result is that under treatment plan commitment, the first best is implemented even when the physician’s degree of altruism is unknown. Furthermore, it is implemented by a single cost
share. Complicated menus of cost shares and transfers are not needed. How is this possible?

Under treatment plan commitment, the insurer offers the cost share corresponding to the least altruistic physician. If the degree of altruism can be 10%, 50% or 90%, the insurer offers a contract with a 10% cost share and a transfer equal to 10% of the expected first-best cost. Clearly, the physician whose altruism is 10% can accept this contract: implementing the first best is a best response and allows him to break even. What about the physician types whose degrees of altruism are 50% or 90%? If they take the 10% cost share, and implement the first best, they also break even. Can they do better?

They would have liked to offer more generous treatments because they are more altruistic than the 10% altruistic physician. But then if they had done so, they would not be able to break even. The transfer is so low—only 10% of the cost corresponding to the first best—that more generous treatment plans would put them in the red. Their nonnegative expected profit constraint is so binding that they have no other choice than following the strategy of the least altruistic physician. Again, more generous treatment plans are simply not feasible for more altruistic physicians—because they have to break even.

When treatment plan commitment is infeasible, a physician with 50% or 90% degrees of altruism will reject a 10% cost-share contract. Because they only make treatment decisions after having accepted the contract, they rationally anticipate offering patient treatments more generous than the first best. The low transfer corresponding to the 10% cost share contract would not allow them to break even, so they reject that contract.

If the insurer has to retain a physician with high degrees of altruism, contracts with higher cost shares will have to be offered. In fact, a menu of incentive compatible payment contracts will be offered. Information rent will have to be given up: all but the most altruistic physician will earn strictly positive profits. To limit information rent, distortions from first-best treatment plans will have to be implemented, and the insurance premium for the consumer may be higher.

Our results draw a contrast between equilibria under commitment and noncommitment of treatment plans. Under commitment, treatment plans are chosen at the time of contract acceptance, at which time the nonnegative profit constraint is still relevant. Under noncommitment, the treatment plans are chosen
after contract acceptance, at which time the nonnegative profit constraint is irrelevant. The economics literature has demonstrated in many contexts that commitment is very important. Typically a party can use commitment to his advantage, the classic example being Stackelberg and Cournot oligopoly games. We show, by contrast, how an insurer can exploit a physician who can commit. A physician earns profits if and only if he cannot commit to a treatment plan at the time of contract acceptance.

Either the commitment or noncommitment assumption may be more appropriate in a particular scenario. In the broad economics literature, both assumptions have been made. For the health market, physician practice style is influenced by education, social norm, economic climate, legal system, and culture. It is beyond the scope of our analysis to study the emergence or decay of practice commitment. Our analysis does point out the social value of a specific way medicine is practiced. The normative implication is that committing to treatment decisions when the bottom line is still relevant should be encouraged. An insurer may foster treatment plan commitment. For example, the insurer can ask the physician to announce a treatment plan when accepting the payment contract and impose a punishment if the physician is found, upon an audit, to have deviated from the announced treatment plan.

The literature on physician payment is large. An earlier survey is McGuire (2000), and a more recent one is Léger (2008). Despite the prevalence of multiple treatment options, most existing works either do not model treatment plans (Pauly (1968), Zechhauser (1970), Choné and Ma (2010)), or allow patients to take only one treatment (Ma and Riordan (2002)). One exception is Chernew, Encinosa and Hirth (2000) who allow the patient to choose one treatment out of many options. Different from all these works, our model has multiple treatment options and considers a treatment sequence. To the best of our knowledge, this is the first attempt to do that.

This paper uses the assumption that economic agents have nonmonetary motives. This, by now, is quite common, as evidenced in recent papers by Akerlof and Kranton (2005), Bénabou and Tirole (2003), Besley and Ghatak (2005), Delfgaauw and Dur (2007, 2008), Dixit (2002, 2005), Francois (2000), Murdock (2002), and Prendergast (2007, 2008). In our model, the agent is the physician who cares about his own financial gains and the patient’s welfare. Our paper differs from these works in that the physician’s degree of altruism
is unknown (see also footnotes 1 and 2 above).

Unknown altruism in the health market has been considered before by Jack (2005) and Choné and Ma (2010). Nevertheless, our paper differs in many ways. Jack’s model considers choices of noncontractible quality by a provider, and lets the physician suffer some financial losses. We do not consider quality, and impose a nonnegative expected profit constraint. In Choné and Ma, health care quantities are contractible, and the physician possesses private information about both altruism and patient illness severity before accepting a contract. In Jack (2005) and Choné and Ma (2010), there are no equilibria in which the first best is implemented.

Allard, Jelovac and Léger (2008) focus on a partially altruistic general practitioner’s incentive to refer patients to a specialist under various payment schemes. In their model the altruism parameter is known, and the patient may receive a treatment from a general practitioner, or be referred to a specialist for a different treatment. Their focus is on referral incentives when the illness severity may change over time.

The rest of the paper is organized as follows. Section 2 presents the model and the first best. Section 3 studies the two delegation games. Section 4 discusses related issues and policy implications. Section 5 draws some conclusions. All proofs are in an Appendix.

2 The model and the first best

A risk-averse consumer has income \( Y \) and suffers from an illness. The loss due to illness is described by a random variable \( \ell \) on a support \([0, \overline{\ell}]\), with distribution and density functions \( F(\ell) \) and \( f(\ell) > 0 \), respectively. We assume that the upper support of the illness loss, \( \overline{\ell} \), is sufficiently large. We let the consumer’s utility function be separable in income and the loss from illness, and measure the disutility of illness loss by the loss, so the consumer’s utility is \( U(Y) - \ell \) when \( \ell \) is the loss due to illness. The function \( U \) is strictly increasing, strictly concave, and the marginal utility at zero income is infinite \( (U'(x) \to \infty \text{ as } x \to 0^+) \). Unlike most other models, we do not set up a probability of the consumer falling ill, where the loss occurs contingent on illness. Our model is slightly more general because we allow a large density around \( \ell = 0 \), so it can approximate models with a fixed probability of falling ill.
The consumer’s loss due to illness can be recovered by taking medical treatments. We assume that there are two treatments; at the end of this section we will discuss when more treatments are available. A treatment either recovers the loss $\ell$ or not at all, and is defined by the probability of success and the cost. Treatment can be taken sequentially, so if a treatment does not succeed, a second treatment can be used. We assume that when a treatment fails once, it will fail again. In other words, the effectiveness of a treatment is perfectly correlated over trials. Given the binary structure, a treatment will never be used twice.

We call the two treatments, Treatment 1 and Treatment 2. Treatment 1 succeeds with probability $\theta_1$ and costs $c_1$. Treatment 2 succeeds with probability $\theta_2$ and costs $c_2$. These four parameters are strictly positive. Treatment 2 is more effective than Treatment 1 but also costs more, so we have $\theta_1 < \theta_2$ and $c_1 < c_2$. We make an assumption on the relative effectiveness of the treatments:

**Assumption 1 (Cost Convexity)** $\frac{c_1}{\theta_1} < \frac{c_2}{\theta_2}$.

Assumption 1 says that the cost per unit of success probability of Treatment 2 is higher than Treatment 1. This is a sort of convexity assumption on treatment costs; the cost per unit of success probability increases with the success probability. We will discuss what will happen if Assumption 1 is violated.

In this paper we consider *Treatment Protocols*. A treatment protocol describes a sequence of treatments. There are five treatment protocols:

**Protocol 0:** Do not use any treatment.

**Protocol 1:** Use Treatment 1 only.

**Protocol 2:** Use Treatment 2 only.

**Protocol 3:** Use Treatment 1, and then Treatment 2 if Treatment 1 fails.

**Protocol 4:** Use Treatment 2, and then Treatment 1 if Treatment 2 fails.

Again, because we have assumed that a treatment outcome is perfectly correlated across trials, Treatment Protocols do not include multiple trials of the same treatment. The *ex ante* success probabilities of Protocols 3
and 4 are, respectively, $\theta_3 \equiv \theta_1 + (1 - \theta_1)\theta_2$ and $\theta_4 \equiv \theta_2 + (1 - \theta_2)\theta_1$. These *ex ante* success probabilities are the same because each of Protocols 3 and 4 allows the consumer to try both treatments; each also offers a higher success probability than either Protocol 1 or Protocol 2.\(^3\) We also compute the expected costs of these Protocols. They are, respectively, $c_3 \equiv c_1 + (1 - \theta_1)c_2$ and $c_4 \equiv c_2 + (1 - \theta_2)c_1$. By Assumption 1, Protocol 4 costs more than Protocol 3: $c_4 - c_3 = c_2\theta_1 - c_1\theta_2 > 0$.

Without any insurance, the consumer will decide on the treatment protocol after she learns her illness loss. For low values of $\ell$, she may not get any treatment; for high values, she may. The consumer faces fluctuations in income since she has to bear treatment costs. The consumer can insure herself against income fluctuations due to illness by purchasing an insurance contract in a competitive insurance market. Insurers are risk neutral, and they offer insurance contracts to maximize the consumer’s expected utility subject to a breakeven constraint.\(^4\)

In the first best, the loss due to illness is verifiable; in the second best, it is not. Throughout the paper we assume that treatment costs can be verified *ex post*, so that physician payment can be based on incurred costs. We present the first best in the next subsection. There are two models for the second best, which will be presented in Section 3.

### 2.1 First best

In the first best, illness loss $\ell$ is verifiable. An insurance contract can be made contingent on the value of $\ell$. Due to risk aversion, the first best shields the consumer from all risks due to treatment costs. A first-best contract specifies a premium $P$ and four treatment protocol functions $\tau_i : [0, \ell] \rightarrow [0, 1], \; i = 1, 2, 3, 4$. The consumer pays $P$ before the realization of $\ell$, and will not incur any payment after $\ell$ is realized and when treatment is used. The function $\tau_i, \; i = 1, 2, 3, 4$, specifies the probability that Protocol $i$ is to be used when the consumer’s loss is $\ell$. We have used the nontreatment Protocol 0 as default.

\(^3\)We abstract from time delays in treatments. We can certainly build into our model such delays. For example, if a treatment is tried second, the success probability is reduced by a fraction. We do not consider this more complex model here.

\(^4\)Also, the insurer may be a public regulator.
The first-best contract \((P, \tau_1, \tau_2, \tau_3, \tau_4)\) maximizes the consumer’s expected utility

\[
\int_0^\ell [U(Y - P) - \ell + \sum_{i=1}^4 \tau_i(\ell)\theta_i\ell]dF(\ell)
\]

subject to the breakeven constraint

\[
P = \int_0^\ell \sum_{i=1}^4 \tau_i(\ell)c_i dF(\ell)
\]

and the boundary conditions

\[
\sum_{i=1}^4 \tau_i(\ell) \leq 1 \quad \text{and} \quad 0 \leq \tau_i(\ell) \leq 1,
\]

for each \(\ell \in [0, \ell]\) and \(i = 1, 2, 3, 4\). The utility function in (1) consists of the utility from the income less the premium, the utility loss \(\ell\), as well as the recovery prospects from the four treatment protocols. The breakeven constraint (2) ensures that any insurance firm offering the contract will make zero expected profit. The remaining constraints in (3) make sure that the treatment protocol probabilities are consistent.

First, we rank the relative cost effectiveness of the treatment protocols:

**Lemma 1** Under Assumption 1, \(\frac{c_1}{\theta_1} < \frac{c_3}{\theta_3} < \frac{c_4}{\theta_4} < \frac{c_2}{\theta_2}\).

According to Lemma 1, in terms of cost per unit of success probability, the ranking, in ascending order, is Protocol 1, Protocol 3, Protocol 4, and Protocol 2. Now, \(\theta_3 = \theta_4 > \theta_2\), so both in terms of success probability and cost per unit of success probability, Protocols 2 and 4 are dominated by Protocol 3. In other words, Protocols 2 and 4 are less efficient than Protocol 3.

**Proposition 1** In the first best, the consumer pays a premium \(P^*\), and receives no treatment if her loss is lower than \(\ell^*\), Protocol 1 if her loss is between \(\ell^*\) and \(\ell^{**}\), and Protocol 3 if her loss is higher than \(\ell^{**}\), where \(\ell^* \equiv U'(Y-P^*)\frac{c_1}{\theta_1} < U'(Y-P^*)\frac{c_2}{\theta_2} \equiv \ell^{**}\). The premium is given by \(P^* = c_1[1-F(\ell^*)] + (1-\theta_1)c_2[1-F(\ell^{**})]\).

Proposition 1 presents two principles in the first best. First, the consumer is risk averse, so financial risks due to illness will be borne by the insurer. The premium is the consumer’s only payment. Second, the choice of treatment follows a cost-effectiveness principle. By Lemma 1, Protocols 2 and 4 are inefficient, so they are never used. If the illness loss is very low, less than \(\ell^*\), it is not cost effective to use any treatment.
because the benefit $\ell \theta_i$ is lower than the cost adjusted by the marginal utility of income $\lambda c_i$, where $\lambda$ is the Lagrangean multiplier and equals $U'(Y - P^*)$.

When the illness loss is higher than $\ell^*$, treatment should be used. Protocol 1 yields a net benefit of $\ell \theta_1 - \lambda c_1$, while Protocol 3 yields $\ell \theta_3 - \lambda c_3$. The difference between the net benefit of Protocols 1 and 3 is $(\theta_3 - \theta_1)\ell - \lambda(c_3 - c_1)$. Protocol 3 is simply Protocol 1 with Treatment 2 as an option, so the incremental success probability $\theta_3 - \theta_1$ is just $\theta_2$, and the incremental cost $c_3 - c_1$ is $c_2$. The incremental benefit $(\theta_3 - \theta_1)\ell$ is increasing in $\ell$, so when $\ell$ is higher than $\ell^{**}$, the more expensive Protocol 3 is cost effective, while for $\ell$ lower than $\ell^{**}$, the less expensive Protocol 1 is cost effective.

We have assumed that there are only two treatments available. Proposition 1 can be extended to an arbitrary number of treatments under Cost Convexity. For example, when there are three treatments, we can construct many treatment protocols by various treatment sequences. However, only three are efficient. These three are (i) use Treatment 1 only; (ii) use Treatment 1, and if it fails use Treatment 2; and (iii) begin with Treatment 1, if it fails use Treatment 2, and if that also fails, use Treatment 3. The intuition behind the inefficiency of protocols other than those in (i), (ii), and (iii) mimics that in Lemma 1. For example, the protocol of Treatment 2 and then Treatment 3 upon failure of Treatment 2 is dominated by the protocol in (iii). Adding Treatment 1 before Treatment 2 raises the total probability of success, and reduces the expected cost due to Cost Convexity, because Treatment 1 has the lowest cost-success probability ratio. Retaining the two-treatment assumption saves on notation, while relaxing it would not lead to qualitatively new results.

We briefly comment on the case when the Cost Convexity assumption is violated. In that case, we have $\frac{c_1}{\theta_1} > \frac{c_2}{\theta_2}$. The ranking of cost per unit of success probability becomes $\frac{c_2}{\theta_2} < \frac{c_4}{\theta_4} < \frac{c_3}{\theta_3} < \frac{c_1}{\theta_1}$, so that Protocols 3 and 1 will be inefficient. Proposition 1 will be modified: Protocol 2 will be used for intermediate values of $\ell$, while Protocol 4 for high values.

3 Altruistic physician and delegation

Now we let illness loss be nonverifiable. The consumer is under the care of a partially altruistic physician. An insurance company establishes a payment contract with the physician, and an insurance contract with
the consumer. The physician is delegated with making treatment decisions for the consumer.

The insurance contract for the consumer consists of a premium $P$. We focus on physician payment and delegation, so we assume that the patient does not bear any financial risks *ex post*. In fact, this is what the first best prescribes, and we will see that the first best may be implemented. The payment contract for the physician is a two-part tariff, $(S, T)$, where $S$ is the physician’s share of the incurred treatment cost, and $T$ is a lump-sum or capitation payment. The physician learns about the illness loss $\ell$ only after the payment contract has been accepted. This is a natural assumption in an insurance model because at the time the insurer offers contracts, the consumer is not yet sick.

When the consumer suffers a loss $\ell$ and is treated by Protocol $i$, her expected payoff is $U(Y - P) - \ell + \theta_i \ell$. The physician learns the consumer’s illness loss, and this becomes his private information. The physician is risk neutral, and partially altruistic to the consumer. When the physician treats the consumer with this Protocol, his expected payoff consists of profit and the consumer’s utility: $T - Sc_i + \alpha[U(Y - P) - \ell + \theta_i \ell]$. Both $S$ and $T$ are nonnegative, but we do not restrict $S$ from being less than 1. The profit from using Protocol $i$ is $T - Sc_i$; he receives the transfer $T$, and bears a cost $Sc_i$, with the balance of the cost paid for by the insurer. The parameter $\alpha$ measures the strength of the consumer’s utility in the physician’s preferences.

The altruism parameter $\alpha$ is a random variable, drawn on a strictly positive support $[\underline{\alpha}, \overline{\alpha}]$, with distribution and density functions, respectively, $G(\alpha)$ and $g(\alpha)$. We assume the hazard rate $\frac{G(\alpha)}{g(\alpha)}$ is increasing in $\alpha$. The physician knows $\alpha$, and this is his private information. We use the term a type-$\alpha$ physician for a physician with altruism parameter $\alpha$. We assume that $F$ and $G$ are independent.

A higher value of $\alpha$ indicates a physician who cares more about the patient’s welfare. The strength of the physician’s trade-off between profit and patient utility is captured by the altruism parameter $\alpha$. In making a decision based on this trade-off, the physician must respect an *ex ante* minimum profit constraint. This is a natural assumption; a minimum profit seems necessary for any market participant to continue with his market activity. As a normalization, we set this minimum profit to 0; we could allow the physician’s minimum profit to be positive, and the analysis remains the same.\footnote{We do require that the minimum profit is independent of the altruism parameter. One might have assumed that a more altruistic physician has a lower minimum profit requirement. This assumption requires different techniques}
3.1 First best and known altruism

In this subsection, we assume that the altruism parameter $\alpha$ is common knowledge, and show how the first best can be implemented by delegation and cost sharing. The physician, with known altruism parameter $\alpha$, is paid a lump-sum $T(\alpha)$, and bears a cost of $S(\alpha)c_i$ when he uses Protocol $i$ for the patient. Subject to the payment scheme, the physician makes treatment decisions for the patient.6

Suppose that, on observing the illness loss $\ell$, the physician uses treatment protocol $i$ with probability $\tau_i(\ell)$. His expected utility is

$$
\int_{0}^{\bar{\ell}} \left\{ T(\alpha) - S(\alpha) \sum_{i=1}^{4} \tau_i(\ell)c_i + \alpha[U(Y - P) - \ell + \sum_{i=1}^{4} \tau_i(\ell)\theta_i\ell] \right\} \, dF(\ell).
$$

(4)

He chooses $\tau_i$ to maximize (4) subject to a nonnegative expected profit constraint

$$
\int_{0}^{\bar{\ell}} \left\{ T(\alpha) - S(\alpha) \sum_{i=1}^{4} \tau_i(\ell)c_i \right\} \, dF(\ell) \geq 0.
$$

(5)

For a payment scheme that implements the first best when $\alpha$ is known, for each $\alpha \in [\alpha_l, \alpha_u]$ we set

$$
S(\alpha) \equiv \lambda \alpha
$$

(6)

$$
T(\alpha) \equiv \lambda \alpha \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^{**}}^{\bar{\ell}} c_3 \, dF(\ell) \right],
$$

(7)

where $\lambda = U'(Y - P^*)$, and $P^*$, $\ell^*$, and $\ell^{**}$ are the first-best premium and threshold loss levels defined in Proposition 1.

**Lemma 2** Given $S(\alpha)$ and $T(\alpha)$ defined in (6) and (7), the delegation scheme implements the first best.

The cost share $S(\alpha) \equiv \lambda \alpha$ in Lemma 2 makes the physician internalize the consumer’s treatment cost and benefit. The physician is partially altruistic, and values the patient’s benefit according to $\alpha \theta_i \ell$. To align his preferences with the first best, he will be made to bear the cost at $\lambda \alpha c_i$, where $\lambda$, the marginal utility of income at first best, adjusts for the difference in the measurement between benefits (in utility) and cost (in for simplifying the incentive constraints. We do not pursue it here.

6Our result is consistent with the general principle that the first best is implemented when the agent only acquires private information after contracting. We do restrict contracts to be two-part tariffs. Furthermore, the agent is partially altruistic, which is not the case in the literature.
money). This is exactly what \( S(\alpha) = \lambda \alpha \) does. Under this cost share, the physician’s expected utility in (4) becomes

\[
\int_0^\infty \left\{ T(\alpha) - S(\alpha) \sum_{i=1}^4 \tau_i(\ell) c_i + \alpha[U(Y - P) - \ell + \sum_{i=1}^4 \tau_i(\ell) \theta_i \ell] \right\} \, dF(\ell)
\]

\[
= \int_0^\infty \left\{ \alpha \left[ \sum_{i=1}^4 \tau_i(\ell) \{ \theta_i \ell - \lambda c_i \} \right] + T(\alpha) + \alpha[U(Y - P) - \ell] \right\} \, dF(\ell),
\]

so the term inside the big square brackets is the benefit less cost. The transfer \( T(\alpha) \) ensures that the physician makes a zero expected profit. The higher the value of \( \alpha \), the more altruistic is the physician, and he bears more costs \textit{ex post} but receives a larger \textit{ex ante} transfer.

The physician’s behavior for the maximization of (4) subject to (5) assumes that he chooses the treatment protocols at the time of contract acceptance and before he observes the illness severity. This assumption is only made for convenience, and unnecessary. When \( S(\alpha) \) and \( T(\alpha) \) are given by (6) and (7), the physician can also make the treatment decision \textit{after} he observes \( \ell \). The treatment decisions will be exactly the same. Commitment is not an issue when altruism is known.

Finally, when Protocol 3 is executed, the physician will find it optimal to use Treatment 2 when Treatment 1 fails. When \( \ell > \ell^{**} \) and Treatment 1 fails, the physician’s decision to continue with Treatment 2 gives him a utility \( \alpha \theta_2 \ell - \alpha \lambda c_2 > 0 \) by Proposition 1, so it is better than refusing to provide Treatment 2. In other words, the physician’s decision is fully time consistent.

### 3.2 Unknown altruism

In this subsection, we study delegation games with and without treatment plan commitment. In both games, the altruistic parameter \( \alpha \) is the physician’s private information at the time of contract, and illness severity \( \ell \) will be learned by the physician later. Each extensive form has four stages, and the first two stages are identical:

**Stage 1:** An insurer offers an insurance contract to the consumer and a payment contract to the physician.

**Stage 2:** Nature draws \( \alpha \) and \( \ell \) from the respective distributions. The physician learns \( \alpha \).

Then the next two stages differ:
Delegation with treatment plan commitment

Stage 3: The physician decides whether to accept the payment contract, and the consumer decides whether to accept the insurance contract. The game ends if neither party accepts. If the physician accepts the contract, he also decides on how he will prescribe treatment protocols depending on illness loss.

Stage 4: The value of $\ell$ now becomes known to the physician, and he carries out treatment protocols according to the prescription rule decided in Stage 3. The physician will be paid according to the payment contract.

Delegation without treatment plan commitment

Stage 3: The physician decides whether to accept the payment contract, and the consumer decides whether to accept the insurance contract. The game ends if either party does not accept. If the physician accepts the contract, he picks an item from the menu of payment contracts.

Stage 4: The value of $\ell$ now becomes known to the physician, and he decides on treatment protocols. The physician will be paid according to the payment contract that he has selected in Stage 3.

The key difference between the two extensive forms is the timing of treatment decisions. Under delegation with treatment plan commitment, in Stage 3 the physician formulates a treatment protocol plan for each illness loss that may be observed later. In Stage 4, he simply implements the plan decided earlier. Under delegation without treatment plan commitment, the physician makes his treatment decision after he has accepted the contract. In Stage 3, he picks an item from a contract menu, but makes no commitment about his treatment decision. Given the chosen contract, the physician makes treatment decisions after he learns the illness loss that will be observed in Stage 4.

In each game, the physician accepts the contract if his expected profit is nonnegative. With treatment plan commitment, his acceptance and treatment decisions are made together. Without treatment plan commitment, for each item in the menu he must anticipate his equilibrium treatment decisions in Stage 4.

\textsuperscript{7}In this extensive form, the insurer offers a single payment contract, rather than a menu, to the physician. This is without loss of generality. We could allow for a menu, but the equilibria remain the same. See the discussion below.
Then he will have to assess the expected profit resulting from these equilibrium treatment decisions. To accept a contract, the expected profit from the continuation equilibrium must be nonnegative.

In each delegation game, the insurer chooses contracts to maximize the consumer’s expected utility, so we assume that the consumer will accept the contract. We have assumed that the consumer does not know the physician’s types. One might wonder if there would be an incentive for a consumer to seek out a more altruistic physician. We will discuss this issue after characterizing the solutions for the two delegation games.

3.2.1 Equilibria in delegation with treatment plan commitment

The physician’s degree of altruism, $\alpha$, is unknown, and follows distribution $G(\alpha)$ on a strictly positive support $[\underline{\alpha}, \overline{\alpha}]$. By Lemma 2, if $(S(\alpha), T(\alpha))$ defined in (6) and (7) were offered to a type-$\alpha$ physician, his delegated treatment decision would coincide with the first best. But $\alpha$ is now unknown, and a type-$\alpha$ physician may mimic another type. Our next result shows that, in fact, each type of physician in the delegation with commitment game can be made to implement the first best. This is achieved by a very simple payment contract, namely $(S(\alpha), T(\alpha))$, again defined in (6) and (7). This contract is the one that implements the first best if the physician’s type is known to be the least altruistic type $\underline{\alpha}$.

A type-$\alpha$ physician’s best response against $(S(\alpha), T(\alpha))$ is to select $\tau_i(\ell)$ to maximize

$$
\int_0^{\tau} \left\{ T(\alpha) - S(\alpha) \sum_{i=1}^{4} \tau_i(\ell)c_i + \alpha[U(Y - P) - E(\ell) + \sum_{i=1}^{4} \tau_i(\ell)\theta_i] \right\} \, dF(\ell)
$$

subject to

$$
\int_0^{\tau} \left\{ T(\alpha) - S(\alpha) \sum_{i=1}^{4} \tau_i(\ell)c_i \right\} \, dF(\ell) \geq 0.
$$

A type-$\alpha$ physician’s choice of treatment decision in Stage 3 is made contingent on possible illness loss. Anticipating that he will commit to this treatment plan after observing the illness severity in Stage 4, the physician decides whether to accept the payment contract.

Lemma 3 When given contract $(S(\alpha), T(\alpha))$ defined in (6) and (7), a type-$\alpha$ physician, $\alpha > \underline{\alpha}$, chooses the first-best treatment thresholds $l^*$ and $l^{**}$.

Lemma 3 reports a surprising result. A single payment contract implements the first best for all types
of the altruistic physician. This contract is the first-best contract for the least altruistic physician when the altruism parameter is known, but it also implements the first best when more altruistic physicians are delegated to make treatment decisions. The physician’s private information about both $\alpha$ and $\ell$ can be circumvented.

The result stems from treatment plan commitment and the interaction between altruism and the nonnegative profit requirement. Under the payment contract $(S(\alpha), T(\alpha))$, the best response of the type-$\alpha$ physician is the first-best treatment protocols. His incentives have been aligned with the first best.

Now consider a more altruistic type-$\alpha$ physician. He cares more about the consumer’s utility than type-$\alpha$, so he would like to be more generous, offering Protocol 1 at $\ell < \ell^*$, and Protocol 3 at $\ell < \ell^{**}$. Indeed, the first-order derivative of (8) with respect to $\tau_i$ is $\alpha \theta_i \left[ \ell - \frac{\alpha c_i}{\alpha \theta_i} \right]$, which is greater than $\alpha \theta_i \left[ \ell - \frac{c_i}{\theta_i} \right]$, the corresponding first-order derivative in the first best. The capitation payment, however, is $T(\alpha)$, which only compensates for the cost share $S(\alpha)$ when treatments are at the first best. The type-$\alpha$ suffers a loss if he follows a treatment plan more generous than the first best. The binding nonnegative profit constraint therefore stops the type-$\alpha$ physician from being more generous than a type-$\alpha$ physician. Since he is able to commit to a treatment plan, it is a best response for the type-$\alpha$ physician to accept the contract $(S(\alpha), T(\alpha))$, and to implement the first best.

We now argue that the insurer must only offer a single contract $(S(\alpha), T(\alpha))$. In fact, the full menu of contracts $(S(\alpha), T(\alpha))$ defined in (6) and (7) will not implement the first best for all types of the physician. Suppose the full menu of contracts is offered. From Lemma 2, if a type-$\alpha$ physician selects $(S(\alpha), T(\alpha))$, he will choose the first-best treatment protocols and break even. However, the type-$\alpha$ physician can do better by exaggerating $\alpha$ and choosing a contract meant for type-$\alpha'$, $\alpha' > \alpha$. Under $(S(\alpha'), T(\alpha'))$, he can still implement the first best and break even, but will gain by being slightly less generous than offering first-best treatments. This deviation will result in a second-order loss in the consumer’s expected utility but a first-order gain in the profit because $T(\alpha') > T(\alpha)$. To summarize, we present (proof omitted)

---

8Indeed, the proof shows that the multiplier for the nonnegative profit constraint (9) becomes larger, the larger $\alpha$ is. Note also that the breakeven constraint (31) in the proof of Lemma 2 does not bind.
**Proposition 2** In the equilibrium under delegation with treatment plan commitment, the insurer offers a single payment contract \((S(\alpha), T(\alpha))\). In equilibrium, each physician type accepts the contract and delivers the first-best treatment protocols to the consumer.

The key to the first-best result stems from the requirement that treatment plans are made when nonnegative expected profit consideration is still relevant. We could consider an alternative extensive form where the physician decides on treatment plans after he has accepted a contract, but before he observes \(\ell\) (but fully anticipating that he will). This kind of commitment has no bite, and the equilibrium will be exactly the same as if commitment were impossible (as in the next subsection). This is because once the contract \((S(\alpha), T(\alpha))\) has been accepted, the treatment plan decision will be determined only by the cost share \(S(\alpha)\) while the transfer \(T(\alpha)\) is already received.

### 3.2.2 Equilibria in delegation without treatment plan commitment

We begin with the physician’s treatment protocol decisions in Stage 4. Suppose that a type-\(\alpha\) physician has accepted a payment contract \((S(\alpha'), T(\alpha'))\) in Stage 3, and learns that the consumer’s illness loss is \(\ell\). His decision is only affected by the cost sharing parameter \(S(\alpha')\), not the transfer \(T(\alpha')\). Given \(\ell\) and \(S(\alpha')\), his payoff from choosing Protocol \(i\) with probability \(\tau_i\) is

\[-S(\alpha') \sum_{i=1}^{4} \tau_i c_i + \alpha \left[ U(Y - P) - \ell + \sum_{i=1}^{4} \tau_i \theta_i \ell \right].\]

The first-order derivative with respect to \(\tau_i\) is \(\alpha \theta_i \ell - S(\alpha') c_i\). As in the earlier analysis, the equilibrium treatment is characterized by two thresholds \(\hat{l}(\alpha'; \alpha)\) and \(\tilde{l}(\alpha'; \alpha)\). The physician will never use the inefficient protocols. A consumer with \(\ell\) smaller than \(\hat{l}(\alpha'; \alpha)\) receives no treatment; \(\ell\) between \(\hat{l}(\alpha'; \alpha)\) and \(\tilde{l}(\alpha'; \alpha)\), Protocol 1; \(\ell\) larger than \(\tilde{l}(\alpha'; \alpha)\), Protocol 3. The equilibrium in Stage 4 is completely characterized by the thresholds

\[
\hat{l}(\alpha'; \alpha) = \frac{S(\alpha') c_1}{\alpha \theta_1} \quad \text{and} \quad \tilde{l}(\alpha'; \alpha) = \frac{S(\alpha') c_2}{\alpha \theta_2}.
\]

In contrast to delegation with treatment plan commitment, the equilibrium treatment decisions are to be made without any reference to the nonnegative profit requirement. In Stage 4, the physician does not have the option of rejecting a payment contract; he has already agreed to the payment contract in Stage 3. The requirement of making a nonnegative expected profit has no bite here.
Next we study the physician’s equilibrium choice of a payment contract in Stage 3. Suppose that the menu \{(S(\alpha), T(\alpha))\} has been offered to the physician in Stage 2. By the revelation principle, we consider equilibria in which a type-\(\alpha\) physician selects contract \((S(\alpha), T(\alpha))\), given the continuation equilibrium in (10). Define a type-\(\alpha\) physician’s expected payoff from selecting contract \((S(\alpha'), T(\alpha'))\) by

\[
V(\alpha'; \alpha) \equiv T(\alpha') - S(\alpha') \left[ \int_{I(\alpha'; \alpha)} \hat{b}(\alpha') \, \text{d}F(l) + \int_{I(\alpha'; \alpha)} \hat{c}_3 \, \text{d}F(l) \right] + \alpha \left[ U(Y - P) - E(l) + \int_{I(\alpha)} \hat{b}(\alpha) \, \text{d}F(l) + \int_{I(\alpha)} \theta_1 \, \text{d}F(l) + \int_{I(\alpha)} \theta_3 \, \text{d}F(l) \right],
\]

where the thresholds \(\hat{I}(\alpha', \alpha)\) and \(\hat{I}(\alpha'; \alpha)\) are given by the continuation equilibrium (10). A menu of contracts is said to be incentive compatible if \(V(\alpha; \alpha) \geq V(\alpha'; \alpha)\) for all \(\alpha'\) and \(\alpha\). Given a menu \((S(\alpha), T(\alpha))\), define the type-\(\alpha\) physician’s maximum payoff by \(W(\alpha) \equiv \max_{\alpha'} V(\alpha'; \alpha)\). To save on notation, we write \(\hat{I}(\alpha; \alpha)\) and \(\hat{I}(\alpha; \alpha)\) as \(\hat{I}(\alpha)\) and \(\hat{I}(\alpha)\), respectively.

**Lemma 4** A menu of contracts \(\{(S(\alpha), T(\alpha))\}, \alpha \in [\underline{\alpha}, \bar{\alpha}]\), is incentive compatible only if \(W\) is convex,

\[
W'(\alpha) = U(Y - P) - E(l) + \int_{I(\alpha)} \hat{b}(\alpha) \, \text{d}F(l) + \int_{I(\alpha)} \theta_1 \, \text{d}F(l) + \int_{I(\alpha)} \theta_3 \, \text{d}F(l),
\]

and both \(\hat{I}(\alpha)\) and \(\hat{I}(\alpha)\) are decreasing in \(\alpha\).

According to Lemma 4, incentive compatibility requires that the physician’s equilibrium utility be convex in the altruism parameter. The physician’s equilibrium payoff, \(W(\alpha)\), must rise at an increasing rate.\(^9\) Furthermore, it says that the equilibrium thresholds must be decreasing so that a more altruistic physician prescribes more treatments. We write the continuation equilibrium condition (10) as

\[
\hat{I}(\alpha) = \frac{S(\alpha)}{\alpha} \frac{c_1}{\theta_1} \quad \text{and} \quad \hat{I}(\alpha) = \frac{S(\alpha)}{\alpha} \frac{c_2}{\theta_2},
\]

so incentive compatibility requires the cost share to altruism parameter ratio, \(S(\alpha)/\alpha\), be decreasing.

Next, we analyze the physician’s nonnegative profit constraint. By selecting \((S(\alpha), T(\alpha))\), a type-\(\alpha\) physician’s expected profit is

\[
\pi(\alpha) \equiv T(\alpha) - S(\alpha) \left[ \int_{\hat{I}(\alpha)} \hat{b}(\alpha) \, \text{d}F(l) + \int_{\hat{I}(\alpha)} \hat{c}_3 \, \text{d}F(l) \right].
\]

\(^9\)Because \(U\) is a utility function of income, its sign can be positive or negative; hence, \(W\) and \(W'\) can be positive or negative. Indeed, signs of \(W\) and \(W'\) are irrelevant for incentive compatibility.
Substituting this expression into $W(\alpha) = V(\alpha; \alpha)$, we have $W'(\alpha) = \pi(\alpha) + \alpha W''(\alpha)$, or

$$\pi(\alpha) = W(\alpha) - \alpha W''(\alpha).$$

(13)

Differentiating both sides of this equation, we have $\pi'(\alpha) = -\alpha W'''(\alpha)$. The convexity of $W(\alpha)$ implies that $\pi(\alpha)$ is decreasing. The physician’s nonnegative profit constraints are therefore simplified to $\pi(\overline{\alpha}) \geq 0$. In other words, if the most altruistic physician breaks even, so do all other physician types.

**Lemma 5** Incentive compatibility is equivalent to $S(\alpha)/\alpha$ being decreasing, and hence $\hat{l}(\alpha)$ and $\hat{l}(\alpha)$ decreasing. Nonnegative expected profit for the physician is equivalent to $\pi(\overline{\alpha}) \geq 0$.

We continue with the derivation of the equilibrium contract menu. The insurer must break even given the continuation equilibrium after Stage 1. The total expected expenditure by the insurer equals the expected profit and treatment cost, averaged over all physician types. Hence, the premium $P$ satisfies

$$P = \int_{\overline{\alpha}}^{\alpha} \pi(\alpha) dG(\alpha) + \int_{\overline{\alpha}}^{\alpha} \left[ \int_{\hat{l}(\alpha)}^{\overline{l}(\alpha)} c_1 dF(l) + \int_{\overline{l}(\alpha)}^{T} c_3 dF(l) \right] dG(\alpha).$$

(14)

>From $W(\alpha) = W(\overline{\alpha}) - \int_{\alpha}^{\overline{\alpha}} W'(x) dx$, we can substitute for $W$ in the expression for $\pi$ in (13):

$$\pi(\alpha) = W(\overline{\alpha}) - \int_{\alpha}^{\overline{\alpha}} W'(x) dx - \alpha W'(\alpha).$$

Then we use (11) in Lemma 4 to replace $W'(x)$. After integration by parts, we can substitute for $\pi(\alpha)$ and express (14) as

$$P = \int_{\overline{\alpha}}^{\alpha} \left[ \int_{\hat{l}(\alpha)}^{\overline{l}(\alpha)} c_1 dF(l) + \int_{\overline{l}(\alpha)}^{T} c_3 dF(l) \right] dG(\alpha) + W(\overline{\alpha})$$

$$- \int_{\overline{\alpha}}^{\alpha} \left\{ \left( \frac{G(\alpha)}{g(\alpha)} + \alpha \right) \left( U(Y - P) - E(l) + \int_{\hat{l}(\alpha)}^{\overline{l}(\alpha)} \theta_1 l dF(l) + \int_{\overline{l}(\alpha)}^{T} \theta_3 l dF(l) \right) \right\} dG(\alpha).$$

(15)

The premium for the patient includes treatment costs and the physician’s utility, which consists of the base utility $W(\overline{\alpha})$ less the consumer’s utility multiplied by the physician’s altruism parameter adjusted by the hazard rate $(G(\alpha)/g(\alpha) + \alpha)$.

>From (13), we have

$$\pi(\overline{\alpha}) = W(\overline{\alpha}) - \overline{\alpha} W'(\overline{\alpha})$$

$$= W(\overline{\alpha}) - \overline{\alpha} \left[ U(Y - P) - E(l) + \int_{\hat{l}(\overline{\alpha})}^{\overline{l}(\alpha)} \theta_1 l dF(l) + \int_{\overline{l}(\alpha)}^{T} \theta_3 l dF(l) \right].$$
so \( \pi(\bar{w}) \geq 0 \) if and only if

\[
W(\bar{w}) \geq \bar{w} \left[ U(Y - P) - E(l) + \int_{\bar{l}(\bar{w})}^{\bar{l}(\bar{w})} \theta_1 dF(l) + \int_{\bar{l}(\bar{w})}^{\bar{l}(\bar{w})} \theta_3 dF(l) \right].
\] (16)

The equilibrium in Stage 4 also requires (12), which says that \( \hat{l}(\alpha) \) and \( \hat{l}(\alpha) \) follow a fixed ratio; this will be shown to be satisfied, so we will ignore this requirement for now.

The equilibrium allocation implemented by the insurer is the solution to the following program: choose \( P \), \( W(\bar{w}) \), \( \hat{l}(\alpha) \), and \( \hat{l}(\alpha) \) to maximize the consumer’s expected utility

\[
U(Y - P) - E(l) + \int_{\alpha}^{\bar{w}} \left( \int_{\hat{l}(\alpha)}^{\hat{l}(\alpha)} \theta_1 dF(l) + \int_{\hat{l}(\alpha)}^{\bar{l}(\alpha)} \theta_3 dF(l) \right) dG(\alpha)
\]

subject to the breakeven constraint (15), the physician nonnegative profit constraint (16), and \( \hat{l}(\alpha) \), \( \hat{l}(\alpha) \) both decreasing. Let \( \mu \) denote the multiplier for the insurer’s breakeven constraint (15). We present the characterization of the solution:

**Proposition 3** Under treatment plan noncommitment, the equilibrium thresholds and premium, \( \hat{l}(\alpha) \), \( \hat{l}(\alpha) \), and \( P \) are given by

\[
\hat{l}(\alpha) = \frac{\mu c_1}{\theta_1} \left[ 1 + \left( \frac{G(\alpha)}{g(\alpha) + \alpha} \right) \mu \right]^{-1},
\] (17)

\[
\hat{l}(\alpha) = \frac{\mu c_2}{\theta_2} \left[ 1 + \left( \frac{G(\alpha)}{g(\alpha) + \alpha} \right) \mu \right]^{-1},
\] (18)

\[
\frac{1}{U'(Y - P)} = \frac{1}{\mu} + \int_{\alpha}^{\bar{w}} \left( \frac{G(\alpha)}{g(\alpha) + \alpha - \bar{w}} \right) dG(\alpha).
\] (19)

The type-\( \bar{w} \) physician earns zero profit, and \( W(\bar{w}) \) is given by (16) as an equality; all other physician types earn strictly positive profits.

> From the equilibrium thresholds in Proposition 3 and equation (12), we can find the cost share and transfer functions for the implementation. The cost share function is

\[
S(\alpha) = \alpha \mu \left[ 1 + \mu \left( \frac{G(\alpha)}{g(\alpha) + \alpha} \right) \right]^{-1}
\] (20)

and the transfer function is

\[
T(\alpha) = W(\alpha) - \alpha U'(\alpha) + S(\alpha) \left[ \int_{\hat{l}(\alpha)}^{\hat{l}(\alpha)} c_1 dF(l) + \int_{\hat{l}(\alpha)}^{\bar{l}(\alpha)} c_3 dF(l) \right],
\] (21)
where \( W'(\alpha) \) is determined by equation (11) and \( W(\alpha) \) is obtained by integrating \( W'(\alpha) \). The physician’s implementation of Protocol 3 is time consistent. If \( \ell > \tilde{l}(\alpha) \), his utility from continuing with Treatment 2 for the consumer is \( \alpha \ell \theta_2 - S(\alpha) \ell_2 \). From (20), this is \( \alpha \ell \theta_2 - \alpha \mu \left[ 1 + \mu \left( \frac{G(\alpha)}{g(\alpha)} + \alpha \right) \right]^{-1} c_2 \), which is strictly positive by Proposition 3.

The determination of the equilibrium thresholds includes the term \( \frac{G(\alpha)}{g(\alpha)} + \alpha \), and this is the key difference from the first best in Proposition 1. The first-best thresholds are determined by a straightforward cost-effectiveness principle. This has to be modified due to the missing information about the physician’s degree of altruism. From (6) and (7) in Lemma 2 for the implementation of the first best when altruism is known, the cost share and transfer should increase proportionally with respect to \( \alpha \) if \( \alpha \) were known. This creates an incentive problem when \( \alpha \) is unknown. A less altruistic physician benefits by claiming to be more altruistic, obtaining a higher transfer and cutting back on treatments.

The cost share and transfer in Proposition 3 must increase less than proportionally with \( \alpha \) to deter a physician from exaggerating his degree of altruism. More altruistic physicians provide more treatments at the cost of receiving less transfers. The equilibrium cost shares and transfers involve the hazard rate, \( \frac{G(\alpha)}{g(\alpha)} \), a standard, Myerson “virtual valuation” adjustment due to private information. Furthermore, treatment benefits are valued by physicians, so the adjustment also includes the term \( \alpha \) in addition to the virtual valuation component.

While the virtual valuation adjustment is common in models of asymmetric information, Proposition 3 also reports a less common adjustment. The premium \( P \) depends on the difference between the highest altruism parameter \( \bar{\alpha} \) and the mean of the altruism parameter \( E(\alpha) \). In fact, by equation (19), the premium increases in \( \bar{\alpha} - E(\alpha) \). This is because incentive compatibility requires the profit function \( \pi(\alpha) \) to be decreasing. Hence, all but the most altruistic physician make a positive profit from the optimal contract. Since the insurer must break even, the physician’s profit is paid for by consumers through the premium. When \( \bar{\alpha} - E(\alpha) \) is larger, the physician will gain a larger profit on average. As a result, consumers have to bear a higher premium.\(^{10}\)

\(^{10}\) As the uncertainty with respect to \( \alpha \) decreases, in the sense that the support of \( \alpha \) shrinks towards a single point, the solution described in Proposition 3 converges to the first best. To see this, observe that as the support shrinks,
Comparing equilibrium thresholds and the premium in Proposition 3 with the first best is not straightforward. The first best is independent of the distribution of $\alpha$, but the functions $\hat{l}$ and $\hat{l}$ have ranges that depend on the distribution as well as the support of $\alpha$. Nevertheless, we provide a sufficient condition for the premium to exceed the first best:

**Corollary 1** Suppose that

$$\int_{\alpha}^{\overline{\alpha}} \left( \frac{G(\alpha)}{g(\alpha)} + \alpha - \overline{\alpha} \right) dG(\alpha) \leq 0. \tag{22}$$

The equilibrium premium $P$ is higher than the first best premium $P^*$. 

The condition in the Corollary involves the hazard rate and the difference between the expected value of $\alpha$ and the upper support $\overline{\alpha}$. If the upper support is large, the condition is more easily satisfied. As we have discussed previously, all physician types earn profits, and the larger the upper support is, the higher the profit. Corollary 1 gives a sufficient condition. The equilibrium premium may be higher even when the condition is not satisfied.

The comparison between the thresholds in Proposition 3 and the first best is again not straightforward. One expects that for low values of $\alpha$, equilibrium thresholds will be higher than first best, while for high values of $\alpha$, they will be lower. That is, less altruistic physicians provide treatments less than the first best, and the opposite for more altruistic physicians. We can verify this with an example. Let the utility function be $U(Y) = \ln Y$, so $U'(Y) = 1/Y$. Suppose that $l$ is uniformly distributed on $[0, 1]$ while $\alpha$ is uniformly distributed on $[\underline{\alpha}, \overline{\alpha} + 1]$, $\underline{\alpha} > 0$. By Proposition 1, the first-best thresholds are

$$l^* = \frac{1}{Y - P^*} \frac{c_1}{\theta_1} \quad \text{and} \quad l^{**} = \frac{1}{Y - P^*} \frac{c_2}{\theta_2}. \tag{23}$$

From the uniform distribution of $\alpha$ we have

$$\int_{\underline{\alpha}}^{\overline{\alpha}} \left( \frac{G(\alpha)}{g(\alpha)} + \alpha - \overline{\alpha} \right) dG(\alpha) = \int_{\underline{\alpha}}^{\overline{\alpha} + 1} (2\alpha - 2\overline{\alpha} - 1) d\alpha = 0,$$

the density $g$ tends to infinity, so that the virtual valuation adjustment $G/g$ tends to 0. Furthermore, $E(\alpha)$ tends to $\overline{\alpha}$ and the monotonicity requirement on $\hat{l}$ and $\hat{l}$ is no longer needed.
which satisfies the condition in Corollary 1. Equation (19) reduces to $\mu = 1/(Y - P)$. The equilibrium thresholds in Proposition 3 are

$$\hat{\theta}(\alpha) = \frac{1}{Y - P} \frac{c_1}{\theta} \left[ 1 + \frac{2\alpha - \alpha}{Y - P} \right]^{-1} \quad \text{and} \quad \hat{\theta}(\alpha) = \frac{1}{Y - P} \frac{c_2}{\theta} \left[ 1 + \frac{2\alpha - \alpha}{Y - P} \right]^{-1}. \quad (24)$$

By Corollary 1, the premium $P$ is larger than the first-best premium $P^*$. From (23) and (24), $\hat{\theta}(\alpha)$ and $\hat{\theta}(\alpha)$ are larger than the first-best thresholds for $\alpha < \frac{P - P^* + \alpha}{2}$, and are smaller than or equal to the first-best thresholds otherwise. If the difference $P - P^*$ is between $\alpha$ and $\alpha + 2$, there exists a type-\(\alpha\) physician delivering first-best treatments. Physicians less altruistic than type-\(\alpha\) will provide less treatment than the first best whereas physicians more altruistic than type-\(\alpha\) will provide more.

4 Discussions and policies

4.1 Commitment

The contrast between equilibria in Propositions 2 and 3 is striking. Under treatment plan commitment, the first best is achieved by a single contract for all physician types. Without treatment plan commitment, the equilibrium results in profit for the physician and treatment distortions, and each physician type chooses a specific contract. This contrast highlights the social value of treatment plan commitment.

The policy implication is that the insurer should encourage physicians to think about their treatment plans when accepting the contract and make the plan transparent so that they can be held accountable for it. The insurer may come up with mechanisms to enforce treatment plan commitment. For example, the insurer can require the physician to announce his treatment plan when accepting the payment contract, and check against the announcement with an audit. Quality assurance programs in managed care plans, which seek to review physician decisions, are examples of monitoring.

One often thinks that commitment is a powerful ability, and this has been shown time and again in economic models. Yet, in our model, a physician earns a zero profit when he is able to commit to a treatment plan, but a positive profit when he is unable to do so. In other words, a physician’s commitment ability is being exploited by the insurer. We do not suggest that physicians have an incentive to give up treatment plan commitment. In fact, physicians in our model are altruistic and their preferences are not
based on profits alone.

4.2 Searching for altruistic physicians

In Proposition 3, a physician provides more treatments when he is more altruistic. Therefore, \textit{ex post}, a consumer prefers to be treated by a more altruistic physician. In Lemma 5, a physician reveals his type by selecting an item from the full cost-share-transfer menu. Typically, however, consumers may not be aware of the financial arrangement between the insurer and the physician, so a physician’s altruism information may not be inferred.

In repeated interactions, without treatment plan commitment, consumers’ incentive to search must exist. In our setup, after an initial treatment episode, if a consumer knows the illness severity, then she can update her belief about the physician’s altruism. For example, suppose that the severity is moderate, but the physician does not recommend Treatment 2 after Treatment 1 has failed. Then the consumer will infer that the physician is not very altruistic.

Searching for more altruistic physicians is irrelevant when treatment plan commitment is possible. In the first-best equilibrium in Proposition 2, all physician types provide the same treatment. When search is relevant, it is associated with inefficiency and likely higher premium due to the lack of commitment in Proposition 3. Search likely exacerbates inefficiency. To attract consumers, physicians may offer more treatments even when their own degree of altruism is low. Clustering of consumers among altruistic physicians likely increases premium, too. A policy implication is that inefficient search can be avoided if treatment plan commitment is possible.

4.3 Selecting physicians

Physicians earn profits when there is a lack of treatment plan commitment. Although more altruistic physicians earn less profit, the total profit is due to altruistic physicians’ generous treatments. A way to limit profit is to reject some physician types. We have assumed that all types in \([\alpha, \overline{\alpha}]\) must earn nonnegative profits. It is possible to relax this by allowing the insurer to retain only those with \(\alpha\) between \(\alpha\) and \(\overline{\alpha} < \overline{\alpha}\). This can be implemented by reducing the transfer function \(T\) in (21) (say, by a constant). Those physicians
with $\alpha$ larger than $\alpha^*$ will not accept any contract. All those who accept will make less profits, and the distortion can be reduced.

The cost of rejecting highly altruistic physician types comes in the form of rationing. We have considered contracts for one consumer and one physician. We implicitly have assumed that the aggregate supply equals aggregate demand. Rejecting some physician types reduces the physician supply. Even in a competitive insurance market, the premium may have to increase; otherwise, nonprice rationing results.

4.4 Comparison with classical moral hazard

In the classical moral hazard model, the consumer makes all the treatment decisions, and will be made to bear partial treatment costs, but is free to choose the order of treatments. We think that this will perform poorly. First, as we have argued before, this scenario is unrealistic. Consumers currently must rely on physicians for treatments; the complexity of modern medicine rules out independent consumer decisions. Second, consumer cost sharing may lead to more inefficient decisions. For example, if the consumer is to bear 20% of treatment costs, she may find Protocols 2 and 4 attractive, even though these are socially inefficient. In addition, this will impose financial risks to patients. An equilibrium in the classical, consumer cost-share model perform worse than the equilibrium in Proposition 2. Fostering treatment plan commitment should perform better than mitigating moral hazard through consumer cost sharing.

5 Concluding remarks

Our model consists of two elements that are missing in the classical optimal insurance model. Treatments can be combined, and physicians are altruistic, with different degrees of altruism. We develop new principles from this setup. First, we show that some treatment combinations are inefficient. Second, we consider delegating treatment decisions to physicians. While inefficient treatment sequences can be avoided by delegation, the first best is available only when a physician can commit to treatment plans at the time of contract acceptance. We offer various policy implications.

Treatment sequences involve a time dimension, and it is natural that commitment plays a key role in the analysis. The physician committing to using particular sequences may result in time-inconsistent decisions.
But such commitment has social value; it reduces premium and inefficient search.

The treatment technology is richer than the usual, static health care quantity approach. This lets us rule out some treatment combinations as inefficient. However, our main results for delegation under treatment plan commitment and noncommitment should hold without any modification if the physician is choosing a quantity of services.

We do acknowledge that our model abstracts from learning. Two issues naturally arise when learning is important. First, the likelihood of treatment success may itself be uncertain. A first treatment is often an experimentation for the physician to learn about treatment efficacy. The failure of a treatment may then update the likelihood that other treatments may be successful. Second, illness severity may be uncertain. A first treatment may reveal that the illness is more or less severe than initially thought. This new information will impact subsequent treatments. These issues are for further research.
Appendix

Proof of Lemma 1: Let \( k_1 = \frac{c_1}{\theta_1} \) and \( k_2 = \frac{c_2}{\theta_2} \). By Assumption 1, \( k_1 < k_2 \). From the definitions of \( \theta_3 \) and \( c_3 \), we substitute \( c_1 \) and \( c_2 \) by \( k_1 \theta_1 \) and \( k_2 \theta_2 \), respectively, and obtain

\[
\frac{c_3}{\theta_3} = \left[ \frac{\theta_1}{\theta_1 + (1 - \theta_1)\theta_2} \right] k_1 + \left[ \frac{(1 - \theta_1)\theta_2}{\theta_1 + (1 - \theta_1)\theta_2} \right] k_2,
\]

which is a weighted average of \( k_1 \) and \( k_2 \), so \( \frac{c_1}{\theta_1} < \frac{c_3}{\theta_3} < \frac{c_2}{\theta_2} \).

Because \( \theta_3 = \theta_4 \) and \( c_3 < c_4 \) by Assumption 1, we have \( \frac{c_3}{\theta_3} < \frac{c_4}{\theta_4} \). It remains to show that \( \frac{c_4}{\theta_4} < \frac{c_2}{\theta_2} \). By Assumption 1, \( c_1 \theta_2 < c_2 \theta_1 \). To both sides of this inequality we multiply by \( (1 - \theta_2) \) and then add \( c_2 \theta_2 \). This results in \( (c_2 + (1 - \theta_2)c_1)\theta_2 < c_2(\theta_2 + (1 - \theta_2)\theta_1) \). Since \( c_4 = c_2 + (1 - \theta_2)c_1 \) and \( \theta_4 = \theta_2 + (1 - \theta_2)\theta_1 \), we have \( c_4\theta_2 < c_2\theta_4 \), so \( \frac{c_4}{\theta_4} < \frac{c_2}{\theta_2} \).

Proof of Proposition 1: Omit the boundary conditions. Use pointwise optimization, and form the Lagrangian for \( \ell \):

\[
L = \int_0^\ell \left[ U(Y - P) - \ell + \sum_{i=1}^4 \tau_i(\ell)\theta_i\ell \right] dF(\ell) + \lambda \left( P - \int_0^\ell \sum_{i=1}^4 \tau_i(\ell)c_i dF(\ell) \right),
\]

where \( \lambda > 0 \) is the multiplier of the premium constraint. The first-order derivatives are

\[
\frac{\partial L}{\partial P} = -U'(Y - P) + \lambda
\]

\[
\frac{\partial L}{\partial \tau_i} = f(\ell) (\theta_i\ell - \lambda c_i) = f(\ell) \theta_i \left( \ell - \frac{\lambda c_i}{\theta_i} \right), \quad i = 1, 2, 3, 4.
\]

The derivatives in (26) are independent of \( \tau_i \), so at each \( \ell \), the Protocol with the highest positive value of \( \frac{\partial L}{\partial \tau_i} \) among \( i = 1, 2, 3, 4 \) will be used. If all the derivatives are negative, then no treatment will be used.

First, \( \tau_2(\ell) = \tau_4(\ell) = 0 \) for all \( \ell \); the consumer never uses Protocols 2 and 4. Because \( \frac{c_3}{\theta_3} < \frac{c_2}{\theta_2} \) by Lemma 1 and \( \theta_2 < \theta_3 \), \( \frac{\partial L}{\partial \tau_2} < \frac{\partial L}{\partial \tau_3} \), \( \forall \ell \). Therefore, we must have \( \tau_2(\ell) = 0, \forall \ell \). Because \( \frac{c_3}{\theta_3} < \frac{c_4}{\theta_4} \) by Lemma 1 and \( \theta_3 = \theta_4 \), \( \frac{\partial L}{\partial \tau_4} < \frac{\partial L}{\partial \tau_3} \), \( \forall \ell \). Therefore, we must have \( \tau_4(\ell) = 0, \forall \ell \).

By Lemma 1, when \( \ell < \lambda \frac{c_1}{\theta_1} \), the first-order derivatives \( \frac{\partial L}{\partial \tau_i} \) are all negative. Define \( \ell^* = \lambda \frac{c_1}{\theta_1} \). From Lemma 1, when \( \ell < \ell^* \), \( \tau_i(\ell) = 0 \), \( i = 1, 2, 3, 4 \). Hence, the consumer does not use any treatment when \( \ell < \ell^* \).
Next, from (26), we have
\[
\frac{\partial L}{\partial \tau_3} - \frac{\partial L}{\partial \tau_1} = \left[(\theta_3 - \theta_1)\ell - \lambda(c_3 - c_1)\right]f(l) = (1 - \theta_1)[\theta_2\ell - \lambda c_2]f(l) \quad (27)
\]
Now define \(\ell^{**} = \frac{\lambda c_2}{\theta_2}\). (Because we assume that \(\ell\) is sufficiently large, we have \(\ell^{**} < \ell\), and it is well-defined.) The expression in (27) is positive if and only if \(\ell > \ell^{**}\). Both \(\frac{\partial L}{\partial \tau_3}\) and \(\frac{\partial L}{\partial \tau_1}\) are positive when \(\ell > \ell^{*}\). Together, we have \(\tau_1(\ell) = 1\) when \(\ell^{*} \leq \ell < \ell^{**}\), and \(\tau_3(\ell) = 1\) when \(\ell^{**} < \ell < \ell^{*}\).

Setting the first-order derivative (25) to 0, we have \(\lambda = U'(Y - P)\), so the values of \(\ell^{*}\) and \(\ell^{**}\) are those in the Proposition. Finally, the premium \(P^*\) is \([F(\ell^{**}) - F(\ell^{*})]c_1 + [1 - F(\ell^{**})]c_3\), which simplifies to
\[
P^* = c_1[1 - F(\ell^{*})] + (1 - \theta_1)c_2[1 - F(\ell^{**})]. \quad (28)
\]
There is a unique solution for \(P\) between 0 and \(Y\). Let \(g(P)\) denote the right-hand side of (28), where \(\ell^{*}\) and \(\ell^{**}\) are now regarded as functions of \(P\). Since \(U'(Y - P)\) increases in \(P\), \(\ell^{*}\) and \(\ell^{**}\) increase in \(P\). The function \(g(P)\) is decreasing in \(P\). The function \(g(P)\) reaches the maximum at \(P = 0\), and \(g(0) = c_1\left[1 - F\left(\frac{U'(Y)c_1}{\theta_1}\right)\right] + (1 - \theta_1)c_2\left[1 - F\left(\frac{U'(Y)c_2}{\theta_2}\right)\right] > 0\). The function \(g(P)\) reaches the minimum at \(P = Y\), and \(g(Y) = 0\) because \(U'(0) = +\infty\). We conclude that there is a unique solution for \(P = g(P)\).

**Proof of Lemma 2:** First, given the contract \((S(\alpha), T(\alpha))\), the type-\(\alpha\) physician chooses treatment protocols \(\tau_i(\ell), i = 1, 2, 3, 4\), to maximize his expected utility
\[
\int_0^\infty \left\{T(\alpha) - S(\alpha) \sum_{i=1}^4 \tau_i(\ell)c_i + \alpha[U(Y - P) - \ell + \sum_{i=1}^4 \tau_i(\ell)\theta_i \ell] \right\} dF(\ell). \quad (29)
\]
The first-order derivative of (29) with respect to \(\tau_i(\ell)\) is
\[
\alpha f(l)\theta_i \left[\ell - \frac{S(\alpha)c_i}{a\theta_i}\right] = \alpha f(l)\theta_i \left(\ell - \frac{\lambda c_i}{\theta_i}\right) \quad (30)
\]
upon substitution \(S(\alpha)\) by \(\lambda_\alpha\). The first-order derivative (30) is the first-order derivative (26) for the first best multiplied by \(\alpha\), a constant. We conclude that the type-\(\alpha\) physician’s optimal treatment decision is first best.
Given the contract, the physician’s expected profit from his optimal, first-best treatment decision is

\[ T(\alpha) - S(\alpha) \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^*}^{\bar{\ell}} c_3 \, dF(\ell) \right] \]

\[ = \lambda \alpha \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^*}^{\bar{\ell}} c_3 \, dF(\ell) \right] - \lambda \alpha \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^*}^{\bar{\ell}} c_3 \, dF(\ell) \right] = 0, \]

so constraint (5) is satisfied.

It remains to show that the insurer breaks even. The insurer receives the first best premium \( P^* \) from the consumer. He pays the physician the transfer \( T(\alpha) \), and \( 1 - S(\alpha) \) share of the cost to the physician. The insurer’s expected profit is therefore

\[ P^* - T(\alpha) - (1 - S(\alpha)) \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^*}^{\bar{\ell}} c_3 \, dF(\ell) \right] \]

\[ = P^* - \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^*}^{\bar{\ell}} c_3 \, dF(\ell) \right] - \left\{ T(\alpha) - S(\alpha) \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^*}^{\bar{\ell}} c_3 \, dF(\ell) \right] \right\}. \] (31)

The insurer breaks even in the first-best contract, so \( P^* - \left[ \int_{\ell^*}^{\ell^{**}} c_1 \, dF(\ell) + \int_{\ell^*}^{\bar{\ell}} c_3 \, dF(\ell) \right] = 0. \) The term inside the big curly brackets in (31) is the physician’s profit and has been shown to be zero. Hence, the insurer makes zero expected profit. ■

**Proof of Lemma 3:** The Lagrangian for the constraint optimization program maximizing (8) subject to (9) is

\[ L = \int_{0}^{\bar{\ell}} \left\{ (1 + \varphi) \left[ T(\alpha) - S(\alpha) \sum_{i=1}^{4} \tau_i(\ell)c_i \right] + \alpha \left( U(Y - P) - \mathbb{E}(\ell) + \sum_{i=1}^{4} \tau_i(\ell)\theta_i \right) \right\} dF(\ell), \]

where \( \varphi \geq 0 \) is the multiplier for the nonnegative expected profit constraint. From pointwise optimization, the first-order derivative with respect to \( \tau_i(\ell) \) is:

\[ \frac{\partial L}{\partial \tau_i} = \alpha f(l)\theta_i \left[ l - \frac{\lambda c_1(1 + \varphi)}{\alpha \theta_i} \right]. \] (32)

after substitution by \( S(\alpha) = \lambda \alpha \). Define \( \ell' \equiv \lambda \frac{c_1}{\theta_1} \left[ \frac{\alpha(1 + \varphi)}{\alpha} \right] \) and \( \ell'' = \lambda \frac{c_2}{\theta_2} \left[ \frac{\alpha(1 + \varphi)}{\alpha} \right] \). From the proof of Proposition 1, the physician will not prescribe any treatment if \( l \leq \ell' \), will use treatment Protocol 1 if \( l' < l < \ell'' \) and treatment Protocol 3 for \( \ell'' < l \).

Next, we show that the Lagrangian multiplier \( \varphi \) must equal \( \frac{\alpha}{\lambda} - 1 \) for a type-\( \alpha \) physician. When \( \varphi = \frac{\alpha}{\lambda} - 1 \), the loss thresholds \( \ell' \) and \( \ell'' \) are identical to the first-best levels, \( \ell^* \) and \( \ell^{**} \), respectively, so the first best is optimal. It remains to show that \( \varphi = \frac{\alpha}{\lambda} - 1 \), and we do that by contradiction.
Suppose that \( \varphi < \frac{\alpha}{2} - 1 \). Then the loss thresholds satisfy \( l' < l^* \) and \( l'' < l^{**} \). The difference between the physician’s expected profit from choosing thresholds \( l' \) and \( l'' \) and that from choosing the first-best thresholds \( l^* \) and \( l^{**} \) is

\[
\left\{ T(\alpha) - S(\alpha) \left[ \int_{l'}^{l^*} c_1 dF(\ell) + \int_{l'}^{l^*} c_3 dF(\ell) \right] \right\} - \left\{ T(\alpha) - S(\alpha) \left[ \int_{l^*}^{l^{**}} c_1 dF(\ell) + \int_{l^*}^{l^{**}} c_3 dF(\ell) \right] \right\}
\]

\[
= S(\alpha) \left\{ \int_{l^*}^{l^{**}} (1 - \theta_1)c_2 dF(\ell) + \int_{l^*}^{l^{**}} c_1 dF(\ell) \right\} < 0.
\]

Given that under \((S(\alpha), T(\alpha))\) the expected profit from the first-best treatments (the second term, in curly brackets, on the first line) is 0, the physician’s expected profit from choosing thresholds \( l' \) and \( l'' \) is negative. This violates the nonnegative expected profit constraint, and contradicts the assumption that \( \varphi < \frac{\alpha}{2} - 1 \). Hence, we conclude that \( \varphi \geq \frac{\alpha}{2} - 1 \).

Next, suppose that \( \varphi > \frac{\alpha}{2} - 1 \). Then the loss thresholds satisfy \( l' > l^* \) and \( l'' > l^{**} \). The difference between the physician’s expected profit from choosing thresholds \( l' \) and \( l'' \) and that from choosing the first-best thresholds is

\[
\left\{ T(\alpha) - S(\alpha) \left[ \int_{l^*}^{l''} c_1 dF(\ell) + \int_{l^*}^{l''} c_3 dF(\ell) \right] \right\} - \left\{ T(\alpha) - S(\alpha) \left[ \int_{l^*}^{l^*} c_1 dF(\ell) + \int_{l^*}^{l^*} c_3 dF(\ell) \right] \right\}
\]

\[
= S(\alpha) \left\{ \int_{l^*}^{l''} (1 - \theta_1)c_2 dF(\ell) + \int_{l^*}^{l''} c_1 dF(\ell) \right\} > 0.
\]

Again, given that under \((S(\alpha), T(\alpha))\) the expected profit from the first-best treatments is 0, the physician earns a strictly positive expected profit. Hence, the nonnegative expected profit constraint does not bind, and the multiplier \( \varphi \) must be zero. This contradicts the assumption that \( \varphi > \frac{\alpha}{2} - 1 > 0 \). Hence we conclude that \( \varphi \leq \frac{\alpha}{2} - 1 \). In sum, we have \( \varphi = \frac{\alpha}{2} - 1 \). ■

**Proof of Lemma 4:** Because \( W(\alpha) \) is the upper bound of affine functions of \( \alpha \), it is convex (Rockafellar, 1972, Theorem 5.5), and therefore almost everywhere differentiable (Rockafellar, 1972, Theorem 25.5). Incentive compatibility implies \( V(\alpha, \alpha) = W(\alpha) \). By the envelope theorem,

\[
W'(\alpha) = \frac{\partial V(\alpha'; \alpha)}{\partial \alpha} \bigg|_{\alpha' = \alpha} = U(Y - P) - E(l) + \int_{\bar{l}(\alpha)}^{\bar{l}(\alpha)} \theta_1 l dF(l) + \int_{\bar{l}(\alpha)}^{\bar{l}(\alpha)} \theta_3 l dF(l).
\]

and we obtain the expression in the Lemma.
Next, rewrite $W'(\alpha)$ as

$$U(Y - P) - E(l) + \int_{\tilde{l}(\alpha)}^{T} \theta_1 l dF(l) + \int_{\tilde{l}(\alpha)}^{T} (1 - \theta_1) \theta_2 l dF(l).$$ (33)

Because $\frac{d\tilde{l}(\alpha)}{d\alpha} = \frac{d(S(\alpha)}{d\alpha} \quad$ and $\frac{d\tilde{l}(\alpha)}{d\alpha} = \frac{d(S(\alpha)}{d\alpha} \quad$ share the same sign as that of $\frac{d(S(\alpha)}{d\alpha}$. If $\tilde{l}(\alpha)$ and $\hat{l}(\alpha)$ were increasing at some $\alpha$, then from (33) $W'(\alpha)$ would be decreasing at $\alpha$. This contradicts incentive compatibility. We conclude that $\tilde{l}(\alpha)$ and $\hat{l}(\alpha)$ must be decreasing. ■

**Proof of Lemma 5:** We only need to show that any menu satisfying $S(\alpha)/\alpha$ decreasing and $\pi(\overline{r}) \geq 0$ implies incentive compatibility and nonnegative expected profit. We start with a given cost share rule $S(\alpha)$ with $S(\alpha)/\alpha$ decreasing. From Lemma 4 and the equilibrium condition for Stage 4 in (12), we have the thresholds $\hat{l}(\alpha)$ and $\tilde{l}(\alpha)$ being decreasing. We can construct $T(\alpha)$ so that $(S(\alpha), T(\alpha)), \alpha \in [\underline{\alpha}, \overline{\alpha}]$, is incentive compatible. First, we set $\hat{l}(\alpha)$ and $\tilde{l}(\alpha)$ by (12) for the continuation equilibrium in Stage 4. Second, we use (11) to construct a function $W'(\alpha)$. Setting a value for $W(\overline{r})$, we integrate $W'(\alpha)$ to obtain $W(\alpha)$.

Third, we set

$$T(\alpha) = W(\alpha) - \alpha W'(\alpha) + S(\alpha) \left[ \int_{\tilde{l}(\alpha)}^{\tilde{l}(\alpha)} c_1 dF(l) + \int_{\hat{l}(\alpha)}^{T} c_3 dF(l) \right].$$ (34)

It is straightforward to check that $S(\alpha)$ and the $T(\alpha)$ in (34) satisfy incentive compatibility. Finally, we can choose $W(\overline{r})$ so that $\pi(\overline{r}) \geq 0$. ■

**Proof of Proposition 3:** From the Lagrangian function $L$, where $\mu$ and $\gamma$ are the multipliers for constraint (15) and (16), respectively.

$$L = U(Y - P) - E(l) + \int_{\hat{l}(\alpha)}^{\tilde{l}(\alpha)} \theta_1 l dF(l) + \int_{\hat{l}(\alpha)}^{T} \theta_3 l dF(l) + \mu \left\{ P - W(\overline{r}) - \int_{\hat{l}(\alpha)}^{\tilde{l}(\alpha)} c_1 dF(l) - \int_{\hat{l}(\alpha)}^{T} c_3 dF(l) \right\}$$

$$+ \mu \left( \frac{G'(\alpha)}{g'(\alpha)} + \alpha \right) \left( U(Y - P) - E(l) + \int_{\hat{l}(\alpha)}^{\tilde{l}(\alpha)} \theta_1 l dF(l) + \int_{\hat{l}(\alpha)}^{T} \theta_3 l dF(l) \right)$$

$$+ \gamma \left\{ W(\overline{r}) - \overline{r} \left[ U(Y - P) - E(l) + \int_{\hat{l}(\alpha)}^{\tilde{l}(\alpha)} \theta_1 l dF(l) + \int_{\hat{l}(\alpha)}^{T} \theta_3 l dF(l) \right] \right\}.$$

We use pointwise optimization for $\hat{l}(\alpha)$, $\tilde{l}(\alpha)$, and take the derivatives of the Lagrangian function with respect to them at $\alpha$. To simplify, we drop constant terms in the derivatives $\frac{\partial L}{\partial l}$ and $\frac{\partial L}{\partial \hat{l}}$. These (simplified)
derivatives are in expressions (35) - (38). The derivatives of the Lagrangian function with respect to \( P \) and \( W(\bar{\alpha}) \) are in (39) and (40).

\[
\frac{\partial L}{\partial \alpha < \bar{\alpha}} = -\theta_1 \hat{\lambda} + \mu c_1 - \mu \left( \frac{G(\alpha)}{g(\alpha)} + \alpha \right) \theta_1 \hat{\lambda} \quad (35)
\]

\[
\frac{\partial L}{\partial \alpha = \bar{\alpha}} = -\theta_1 \hat{\lambda} + \mu c_1 - \mu \left( \frac{G(\bar{\alpha})}{g(\bar{\alpha})} + \bar{\alpha} \right) \theta_1 \hat{\lambda} + \gamma \bar{\alpha} \hat{\lambda} \quad (36)
\]

\[
\frac{\partial L}{\partial \alpha > \bar{\alpha}} = -\theta_2 \hat{\lambda} + \mu c_2 - \mu \left( \frac{G(\alpha)}{g(\alpha)} + \alpha \right) \theta_2 \hat{\lambda} \quad (37)
\]

\[
\frac{\partial L}{\partial \alpha = \bar{\alpha}} = -\theta_2 \hat{\lambda} + \mu c_2 - \mu \left( \frac{G(\bar{\alpha})}{g(\bar{\alpha})} + \bar{\alpha} \right) \theta_2 \hat{\lambda} + \gamma \bar{\alpha} \hat{\lambda} \quad (38)
\]

\[
\frac{\partial L}{\partial P} = -U'(Y - P) + \mu \left[ 1 - U'(Y - P) \int_\alpha^{\bar{\alpha}} \left( \frac{G(\alpha)}{g(\alpha)} + \alpha \right) dG(\alpha) \right] + \gamma \bar{\alpha} U'(Y - P) \quad (39)
\]

\[
\frac{\partial L}{\partial W(\bar{\alpha})} = -\mu + \gamma \quad (40)
\]

> From (40), we have \( \mu = \gamma \). Substituting \( \gamma \) by \( \mu \) in (35), (37), and (39) and setting them to zero, we obtain (17), (18), and (19) in the Proposition.

The first-order conditions for \( \hat{\lambda} \) and \( \hat{\lambda} \) at \( \alpha = \bar{\alpha} \) are

\[
\hat{\lambda}(\bar{\alpha}) = \mu \frac{c_1}{\theta_1} \left[ 1 + \frac{G(\bar{\alpha})}{g^{(\bar{\alpha})}} \right]^{-1} \quad (41)
\]

\[
\hat{\lambda}(\bar{\alpha}) = \mu \frac{c_2}{\theta_2} \left[ 1 + \frac{G(\bar{\alpha})}{g^{(\bar{\alpha})}} \right]^{-1} \quad (42)
\]

The limit of \( \hat{\lambda}(\alpha) \) as \( \alpha \) converges to \( \bar{\alpha} \) from below is \( \frac{c_1}{\theta_1} \left[ 1 + \left( \frac{G(\bar{\alpha})}{g^{(\bar{\alpha})}} + \bar{\alpha} \right) \right]^{-1} \). Clearly, \( \lim_{\alpha \rightarrow \bar{\alpha}^{-}} \hat{\lambda}(\alpha) < \hat{\lambda}(\bar{\alpha}) \).

Because incentive compatibility requires \( \hat{\lambda}(\alpha) \) to be decreasing, the monotonicity constraint must bind at \( \hat{\lambda}(\bar{\alpha}) \), so \( \hat{\lambda}(\bar{\alpha}) = \lim_{\alpha \rightarrow \bar{\alpha}^{-}} \hat{\lambda}(\alpha) \). By the same argument, we have \( \hat{\lambda}(\bar{\alpha}) = \lim_{\alpha \rightarrow \bar{\alpha}^{-}} \hat{\lambda}(\alpha) \).

By assumption, the hazard rate \( \frac{G(\alpha)}{g(\alpha)} \) is increasing, so \( \frac{G(\alpha)}{g(\alpha)} + \alpha \) is increasing. Hence, \( \hat{\lambda}(\alpha) \) and \( \hat{\lambda}(\alpha) \) are decreasing in \( \alpha \). Finally, from (17) and (18), the ratio of \( \hat{\lambda}(\alpha) \) to \( \hat{\lambda}(\alpha) \) is a constant, so the equilibrium condition in Stage 4, (12), is satisfied. 

**Proof of Corollary 1:** Suppose that (22) is satisfied but that \( P \leq P^* \). Then

\[
\frac{1}{U'(Y - P)} \geq \frac{1}{U'(Y - P^*)} = \frac{c_1}{\theta_1} \frac{1}{\bar{\alpha}} \quad (41)
\]

where the equality follows from Proposition 1. From (19), we have

\[
\frac{1}{\mu} + \int_\alpha^{\bar{\alpha}} \left( \frac{G(\alpha)}{g(\alpha)} + \alpha - \bar{\alpha} \right) dG(\alpha) = \frac{1}{U'(Y - P)} \geq \frac{1}{U'(Y - P^*)} = \frac{c_1}{\theta_1} \frac{1}{\bar{\alpha}} \quad (41)
\]
so (22) implies that

\[
\frac{1}{\mu} \geq \frac{c_1}{\theta_1 \ell^*}.
\]  

(43)

By (17), we have

\[
\frac{c_1}{\theta_1 \ell(\alpha)} = \frac{1}{\mu} \left[ 1 + \left( \frac{G(\alpha)}{g(\alpha)} + \alpha \right) \mu \right] \geq \frac{c_1}{\theta_1 \ell^*} \left[ 1 + \left( \frac{G(\alpha)}{g(\alpha)} + \alpha \right) \mu \right] > \frac{c_1}{\theta_1 \ell^*},
\]

where the weak inequality is due to (43), and the strict inequality follows from the term inside the square brackets of (44) being strictly positive. Therefore, \( \hat{\ell}(\alpha) < \ell^* \) for all \( \alpha \). Repeating the same argument, we have \( \hat{\ell}(\alpha) < \ell^{**} \) for all \( \alpha \). The consumer receives more treatments and the physician receives profits. This therefore implies that \( P > P^* \), which is a contradiction. ■
References


