Temporal Resolution of Uncertainty and Recursive Models of Ambiguity Aversion

Tomasz Strzalecki
Harvard University
Preference for Earlier Resolution of Uncertainty
Persistence/Long Term Risk

1

1

0

0

1

1

0

0

1

1

0

0

0

0
Setting
Consumption Plan

\[ h_0 \rightarrow h_1(s^1) \rightarrow h_2(s^2) \]

\[ t = 0 \quad t = 1 \quad t = 2 = T \]
Evaluation

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta \int_S V_{t+1}((s^t, s_{t+1}), h) \, dp(s_{t+1} | s^t) \]
Evaluation

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta \int_S V_{t+1}((s^t, s_{t+1}), h) \, dp(s_{t+1} | s^t) \]

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta \int_S V_{t+1}((s^t, s_{t+1}), h) \, dp(s_{t+1}) \]
Preference for Earlier Resolution of Uncertainty
\[ V_t(s^t, h) = u(h_t(s^t)) + \beta \int_S V_{t+1}((s^t, s_{t+1}), h) \, dp(s_{t+1}) \]
Evaluation

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta \int_{s} V_{t+1}(s^t, s_{t+1}, h) \, dp(s_{t+1}) \]

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta I\left(V_{t+1}(s^t, \cdot, h)\right) \]

where \( I : \mathbb{R}^S \rightarrow \mathbb{R} \)
Evaluation

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta \int_S V_{t+1}((s^t, s^{t+1}), h) \, dp(s^{t+1}) \]

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta I \left( V_{t+1}((s^t, \cdot), h) \right) \]

where \( I : \mathbb{R}^S \rightarrow \mathbb{R} \)

IID Ambiguity (Epstein and Schneider, 2003)
Aggregator

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta I(V_{t+1}((s^t, \cdot), h)) \]
Aggregator

\[ V_t(s^t, h) = u(h_t(s^t)) + \beta I(V_{t+1}((s^t, \cdot), h)) \]

\[ V_t(\omega, h) = W \left( h_t(s^t), I(V_{t+1}((\omega^t, \cdot), h)) \right) \]

where \( W : X \times \mathbb{R} \to \mathbb{R} \)
Ambiguity Aversion

$I : \mathbb{R}^S \rightarrow \mathbb{R}$ is:

- continuous (supnorm)
- monotonic $(\forall s \in S \; \xi(s) \geq \zeta(s) \Rightarrow I(\xi) \geq I(\zeta))$
- normalized $(\forall r \in R \; I(r) = r)$
- quasiconcave

Cerreia-Vioglio, Maccheroni, Marinacci, and Montrucchio (2008)
Ambiguity Aversion

1. Expected utility preferences: \( I(\xi) = \int \xi \, dp \)
2. Maxmin expected utility preferences: \( I(\xi) = \min_{p \in C} \int \xi \, dp \)
3. Second order expected utility preferences: \( I(\xi) = \phi^{-1} \left( \int \phi(\xi) \, dp \right) \)
4. Smooth ambiguity preferences: \( I(\xi) = \phi^{-1} \left( \int_{\Delta(\Sigma)} \phi(\int \xi \, dp) \, d\mu(p) \right) \)
5. Variational preferences: \( I(\xi) = \min_{p \in \Delta(\Sigma)} \int \xi \, dp + c(p) \)
6. Multiplier preferences: \( I(\xi) = \min_{p \in \Delta(\Sigma)} \int \xi \, dp + \theta R(p \parallel q) \)
7. Constraint preferences: \( I(\xi) = \min_{\{p \in \Delta(\Sigma) \mid R(p \parallel q) \leq \eta\}} \int \xi \, dp \)
8. Confidence preferences: \( I(\xi) = \min_{\{p \in \Delta(\Sigma) \mid \varphi(p) \geq \alpha\}} \frac{1}{\varphi(p)} \int \xi \, dp \)
Applications to macroeconomics and finance


Results
Main Message

What we assume about $I$ will have impact on Preference for Earlier Resolution of Uncertainty.
ambiguity
averse
preferences
ambiguity
averse
preferences
There exists $A > 0$ such that $-\frac{\phi''(x)}{\phi(x)} \in [\beta A, A]$ for all $x \in \mathbb{R}$
There exists $A > 0$ such that $-\frac{\phi''(x)}{\phi(x)} \in [\beta A, A]$ for all $x \in \mathbb{R}$. 
Persistence
Duffie and Epstein (1992). Suppose that a fair coin is flipped iid every day. Do you prefer to get 0 or 1 every day, depending on the coin toss that day? Or either 0 forever or 1 forever, depending on the first coin toss?
Persistence

```
1
/ \
1 0
/ \
0 1
/ \
0 0
```

```
1
/ \
1 0
/ \
0 1
/ \
1 1
/ \
0 0
```
Two features of Kreps-Porteus

Preference for earlier resolution of uncertainty: Epstein–Zin, Weil, Tallarini, etc.

Persistence (long run risk): Bansal–Yaron, Hansen–Heaton–Li
Theorem 6. A family of discounted variational preferences, 
\[ I(\xi) = \min_{p \in \Delta(\Sigma)} \int \xi \, dp + c(p), \] 
always satisfies preference for iid. Indifference to iid is satisfied if and only if 
\[ I(\xi) = \min_{p \in C} \int \xi \, dp. \]
Aggregator
\[ V_t(s^t, h) = u(h_t(s^t)) + \beta I(V_{t+1}((s^t, \cdot), h)) \]
$V_t(s^t, h) = u(h_t(s^t)) + \beta I(V_{t+1}((s^t, \cdot), h))$

$V_t(\omega, h) = W\left(h_t(s^t), I(V_{t+1}((\omega^t, \cdot), h))\right)$

where $W : X \times \mathbb{R} \rightarrow \mathbb{R}$
Question

$\left( I^{MEU}, W^{disc} \right)$
Question

\((I, W^{disc})\)

\((I^{MEU}, W^{disc})\)
Question

\[(I, W^{\text{disc}})\]

\[(I^{\text{MEU}}, W^{\text{disc}})\]

\[(I^{\text{MEU}}, W)\]
Question

Are they the same?
Question

\[(I, W^{disc})\]

\[(I^{MEU}, W^{disc})\]

\[\text{Same iff } I(\xi) = \min_{p \in C} \phi^{-1}\left( \int \phi(\xi) \, dp \right)\]
Theorem 7. Suppose that \( \{ \sim_{t,\omega} \}_{(t,\omega) \in \mathcal{T} \times \Omega} \) is a family of recursive uncertainty averse preferences with \((W^{\text{disc}}, I, u)\) and there exists an essential event \( E \in \Sigma \).

It has a recursive uncertainty averse representation with \((W, I^{\text{MEU}}, \nu)\) if and only if \( I(\xi) = \min_{p \in C} \phi^{-1} \left( \int \phi(\xi) \, dp \right) \) for some strictly increasing and concave function \( \phi : \mathbb{R} \to \mathbb{R} \) and some convex and weak*-closed set \( C \subseteq \Delta(\Sigma) \).

In this case \( I^{\text{MEU}}(\xi) = \min_{p \in C} \int \xi \, dp \) and \( W(x, d) = \phi \left( u(x) + \beta \phi^{-1}(d) \right) \) for all \( x \in X \) and all \( d \in \text{Range}(\phi) \).
Risk
Similar things happening with risk instead of ambiguity

Chew and Epstein (1989), Segal (1990), Grant, Kajii, and Polak (2000), and Dillenberger (2008)

But different setup, so the results are different
Conclusion
Conclusion

Interdependence of ambiguity and timing

MEU—only case of indifference
Thank you


