Partially Binding Platforms and the Advantages of Being an Extreme Candidate

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Abstract

This paper develops a model in which platforms are partially binding: a candidate who implements a policy different from his platform must pay a cost of betrayal that increases with the size of the discrepancy. I also suppose that voters are uncertain about candidate preferences for policies. If voters believe that a candidate is likely to be extreme, there exists a semi-separating equilibrium: an extreme candidate mimics a moderate candidate with some probability, and with the remaining probability, he announces a platform that is more moderate than a moderate candidate’s platform. Although an extreme candidate will implement a more extreme policy than a moderate candidate in equilibrium, partial pooling ensures that voters prefer an extreme candidate who does not pretend to be moderate over an uncertain candidate announcing a moderate candidate’s platform. As a result, an extreme candidate may have a higher probability of winning than a moderate one.

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1 Introduction

Before an election, candidates announce platforms, and the winner implements policy after the election. Politicians usually betray their platforms, but if the winner betrays his platform, such betrayal should be costly. For example, in 1988, George H. W. Bush promised “read my lips, no new taxes,” but he increased taxes after he became President. The media and voters visibly noted this betrayal, and he lost the 1992 Presidential election. Based on this “cost of betrayal” and his platform, the winner decides on a policy after election.

To my knowledge, this paper is the first to introduce these characteristics of platforms into the two-candidate competition model. Most past studies use one of two polar assumptions about platforms. First, models with Completely Binding Platforms suppose that a politician cannot implement any policy other than the platform. Second, models with Nonbinding Platforms suppose that a politician can implement any policy freely without any cost. Neither model captures how, for example, Bush betrayed his platform, and then was punished for doing so by the electorate. As Persson and Tabellini (2000) indicate, “it is thus somewhat schizophrenic to study either extreme: where platforms have no meaning or where they are all that matter. To bridge the two models is an important challenge (p. 483).”

In this paper, I build a model with partially binding platforms that incorporates the two settings described above as extreme cases. My model with Partially Binding Platforms supposes that a candidate can choose any policy, but that betrayal is costly, and the “cost of betrayal” increases with the degree of betrayal. If politicians betray platforms, the people and the media criticize the politicians, who must answer to their complaints, their approval ratings may fall, and the possibility of losing the next election may increase. A stronger party or the Congress also may discipline politicians. In addition to the cost of betrayal, I introduce asymmetric information by assuming that candidate policy preferences are private.

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1Campbell (2008) indicates that “President George H. W. Bush lost in 1992 partly because he reneged on his “no new taxes” pledge of the 1988 campaign” (p. 104).

2I also study partially binding platforms in Asako (2009a, 2009b).

3This is the case in electoral competition models in the Downsian tradition (Downs (1957) and Wittman (1973)). For example, see Roemer (2001).

4For example, this approach is taken in citizen-candidate models, such as Besley and Coate (1997) and Osborne and Slivinski (1996) and retrospective voting models such as Barro (1973) and Ferejohn (1986).

5Some papers show the relationship between the media and the credible commitment of politicians. Reinikka and Svensson (2005) shows that newspaper campaigns reduce corruption in Uganda. Djakov et al. (2003) empirically show that policy making is distorted if the media is owned by the government.

information. Politician preferences may change depending on surrounding conditions or the particularly important issues in an election. In particular, when candidates are not famous, it is difficult to know their preferred policies.

The striking result is that an extreme candidate may have a higher probability of winning compared to a moderate candidate. Moreover, in equilibrium, an extreme candidate will implement a policy farther from the median policy than would a moderate candidate, so partially binding platforms can induce *ex post* inefficiency.

The model supposes a two-candidate political competition in one-dimensional policy space. One candidate’s ideal policy is to the left of the median voter’s ideal policy while the other candidate’s ideal policy is to the right. Each candidate is one of two types, moderate or extreme, and a moderate type’s ideal policy is closer to the median policy than an extreme type. A candidate knows his own type, but voters and the opposition do not. Before the election, candidates announce platforms. The winner decides on the policy to be implemented based on his platform and the cost of betrayal.\(^7\)

If voters and the opposition believe *ex ante* that a candidate is likely to be an extreme type, a semi-separating equilibrium exists. In a semi-separating equilibrium, a moderate type chooses a pure strategy, while an extreme type chooses a mixed strategy. With some probability, the extreme type announces the same platform as the moderate type, but with the remaining probability, the extreme type compromises more by announcing a more moderate platform than a moderate type’s platform. In the later case, he reveals his type to voters, and his implemented policy approaches the median policy. An extreme type will always implement a policy that is further from the median policy than a moderate type. However, if one candidate announces a moderate type’s platform, and the other candidate is an extreme type who compromises more, the majority of voters will elect the extreme type who compromised.

Because an extreme type pretends to be moderate with some probability, voters cannot distinguish which type announces the moderate type’s platform. On the other hand, voters can recognize an extreme type who compromises more. In equilibrium, a majority of voters chooses an extreme type who compromises in order to avoid electing an extreme type who mimics a moderate type since an extreme type who pretends to be moderate will implement a very extreme policy. Voters do this even at the cost of preventing the election of a moderate type.

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\(^7\) An increase in the possibility of losing the next election is a problem for dynamic electoral situations, but, in order to simplify, it is included in the cost of betrayal as a current term. Moreover, I suppose that there exist other types of costs of betrayal that are not dynamic problems discussed above, such as party discipline. Thus, I consider the cost of betrayal as a current term.
type. An extreme type has a stronger incentive to prevent the opposition from winning because an extreme type’s ideal policy is further from the opposition’s policy than a moderate type. Thus, an extreme type has an incentive to announce a more moderate platform in order to increase the probability of winning, while a moderate type accepts a lower probability of winning because the opposition’s policy is closer to his own ideal policy.

Partially binding platforms and asymmetric information are critical to finding the above described equilibrium. If platforms are nonbinding, an extreme type must implement his ideal policy, so he cannot use the above strategy. Indeed, Banks (1990) and Callander and Wilkie (2007) show that a moderate type may defeat an extreme type under nonbinding platforms even if there is asymmetric information about the candidates’ ideal policies. If platforms are completely binding, the ideal policy is irrelevant, so both types of candidates commit to implementing the median policy, as in the basic Downsian model. Asako (2009b) shows that, under partially binding platforms with complete information, a moderate type may defeat an extreme type.

If and only if a semi-separating equilibrium does not exist that is, if voters and the opposition believe ex ante that a candidate is likely to be a moderate type, then a pooling equilibrium exists. Because voters believe that a candidate who announces a moderate type’s platform will implement a moderate policy, an extreme type needs to compromise greatly to increase his chance of winning. Such a large compromise reduces his expected utility, so an extreme type prefers mimicking a moderate type.

Several papers discuss similar ideas of partially binding platforms. Harrington (1993) and Aragones et al. (2007) show that, in a repeated game, it is possible for two candidates to never betray their platforms that is, nonbinding platforms can be completely binding in equilibrium. In Austen-Smith and Banks (1989), both candidates prefer not to make an

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8There are some examples in which a party that was believed to be extreme compromised and won against an uncertain-type opposition. In Turkey, broadly speaking, the Justice and Development Party (AKP) famous for advocating political Islam, and because most voters who support secularism, they were afraid that the AKP would implement extreme Islamic policies. In 2007, the AKP compromised a lot and promised to support secular policies. On the other hand, the Turkish Military, which supports secular parties, threatened a coup, so, citizens started to be uncertain about secular parties’ preference. As a result, the AKP won the 2007 elections. Similarly, in the U.K., the Labour Party, which famous for their support for socialist policies, compromised and promised the “Third Way” and free-market policies in 1997. On the other hand, voters were uncertain about the Conservative Party preference because of intra-party infighting between factions. As a result, the Labour Party won the 1997 election.

9In Asako (2009b), when candidates have linear utility functions, the outcome is a tie. In Banks (1990), if the signaling cost is sufficiently low, the outcome is also a tie. If there is uncertainty about the median policy for candidates with completely binding platforms, a moderate type has a higher probability of winning (Alesina and Rosenthal (1995)). Harrington (1992) and Kartik and McAfee (2007) also show that a moderate type has an advantage.
effort. However, they need to commit a positive effort level to win, and, if they betray the platform, the probability of winning in the next election decreases. Their model is based on a retrospective voting model, and candidates do not care about policy (office-motivated). Grossman and Helpman (2005, 2006) develop a legislative model in which the preferences of the decision maker are different before and after an election. Before the election, office-motivated parties announce platforms, and the victorious legislators who care about policies (policy-motivated) decide policy. If legislators betray party platform, the party punishes them. On the other hand, my model is based on the prospective and two-candidate competition model, and considers that candidates who are policy-motivated decide on both a platform and a policy. In addition, these past studies consider the case of complete information while my paper examines the case of asymmetric information concerning candidate policy preferences.

Some earlier papers analyze the signaling aspect of campaign platforms. In particular, Banks (1990) and Callander and Wilkie (2007) assume that the platform is a signal of an implemented policy. They consider nonbinding platforms, and, because there is a “cost to lie,” a candidate does not want to announce a platform that is far from his ideal policy. In Banks (1990) and Callander and Wilkie (2007), candidates implement their own ideal policies automatically. However, if there is such a cost, a rational candidate should want to adjust the implemented policy rather than automatically implement the ideal policy to reduce the cost. A politician’s decision after the election should also be rational. I allow for partially binding platforms and examine rational choices as results of both platforms and policy\textsuperscript{10}.

Section 2 presents the model, Section 3 analyzes political equilibria, and Section 4 concludes. All proofs are presented in an appendix.

2 Setting

The policy space is \( \mathbb{R} \). There is a continuum of voters, and their ideal policies are distributed on some interval of \( \mathbb{R} \). The distribution function is continuous and strictly increasing, so there is a unique median voter’s ideal policy, \( x_m \). There are two candidates, \( L \) and \( R \), and each candidate is one of two types, moderate or extreme. Denote \( x_i^M \) and \( x_i^E \) as the ideal policies

\textsuperscript{10}Harrington (1992) indicates that, if there is a chance that the winner must implement the voters’ ideal policy, candidates may reveal their true ideal policy. Some other papers assume that a completely binding platform is a signal for something other than ideal policies, such as the functioning of the economy (Schulz, 1996), the candidate’s political motivation (Callander, 2007) and the candidate’s degree of honesty (Kartik and McAfee, 2007).
for the moderate and extreme types where \( i = L \) or \( R \), and \( x_L^E < x_L^M < x_m < x_R^M < x_R^E \). Superscripts \( M \) and \( E \) mean a moderate and extreme types, respectively, and the moderate type’s ideal policy is closer to the median policy. Assume \( x_m - x_L^t = x_R^t - x_m \) for \( t = M \) or \( E \), that is, the ideal policies of the same type are equidistance from the median policy. A candidate knows the own type, but voters and the opposition have uncertainty about the candidate’s type. For both candidates, \( p^M \) is the prior probability that the candidate is a moderate type, and the prior probability that the candidate is an extreme type is \( p^E = 1 - p^M \).

After the types of candidates are decided, each candidate announces a platform, denoted by \( z_i^t \in \mathbb{R} \) where \( i = L \) or \( R \) and \( t = M \) or \( E \). Based on these platforms, voters decide on a winner according to a majority voting rule. In the last period, the winning candidate decides the implemented policy, denoted by \( \chi_i^t \) where \( i = L \) or \( R \) and \( t = M \) or \( E \).

If the implemented policy is different from the candidate’s ideal policy, the candidate experiences a disutility. This disutility is represented by \(-v(|\chi - x_i^t|)\) where \( i = L \) or \( R \), \( t = M \) or \( E \), and \( \chi \) is the policy implemented by the winner. Assume that \( v(.) \) satisfies \( v(0) = 0, v'(0) = 0, v'(d) > 0 \) and \( v''(d) > 0 \) when \( d > 0 \).

If the implemented policy is not the same as the platform, the winning candidate needs to pay some costs. The function describing a cost of betrayal is \( c(|z_i - \chi|) \). Assume that \( c(.) \) satisfies \( c(0) = 0, c'(0) = 0, c'(d) > 0 \) and \( c''(d) > 0 \) when \( d > 0 \).

Moreover, I assume throughout that \( \frac{c'(d)}{c(d)} \) and \( \frac{v'(d)}{v(d)} \) decreases as \( d \) increases, and one or all of them is strictly decreasing. This assumption means that the relative marginal cost and disutility decrease as \( |z_i - \chi| (|x_i - \chi|) \) increases. For example, if the function is monomial, this assumption holds, and many polynomial functions satisfy them\(^{11}\).

In the last period, the winning candidate chooses a policy that maximizes \(-v(|\chi - x_i^t|) - c(|z_i - \chi|)\). Denote \( \chi_i^t(z_i) = \arg \max_{\chi} -v(|\chi - x_i^t|) - c(|z_i - \chi|) \).

Upon observing a platform, the utility of voter \( n \) when candidate \( i \) with type \( t \) wins is \(-u(|\chi_i^t(z_i) - x_n|)\). Assume that \( u(.) \) satisfies \( u'(|\chi_i^t(z_i) - x_n|) > 0 \) when \( |\chi_i^t(z_i) - x_n| > 0 \). Let \( p_i(t|z_i) \) denote the voters’ revised beliefs that candidate \( i \) is type \( t \) upon observing the platform, \( z_i \). The expected utility of voter \( n \) when an winner is candidate \( i \) who promises \( z_i \) is \( U_n(z_i) = -p_i(M|z_i)u(|\chi_i^M(z_i) - x_n|) - p_i(E|z_i)u(|\chi_i^E(z_i) - x_n|) \).

Denote \( Prob_i^j(win|z_j, z_i) \) as the probability of winning of type-\( t \) candidate \( i \) given \( z_j \) and \( z_i \). Let \( F_i^j(.) \) denote the distribution function of the mixed strategy chosen by a candidate \( i \) of type \( t \). The expected utility of the type-\( t \) candidate \( i \) who promises \( z_i \) (pure strategy) in

\(^{11}\)The special case without this assumption is discussed in Appendix B
the first period is:

\[
V^i_t((F^M_j(z_j), F^E_j(z_j)), z_i) = \sum_{s=M,E} p^s \int z_j \text{Prob}_i(win|z_j, z_i) dF^s_j(z_j) \left[ -v(|\chi^i_t(z_i) - x_i|) - c(|z_i - \chi^i_t(z_i)|) \right]
\]

\[
- \sum_{s=M,E} p^s \int z_j (1 - \text{Prob}_i(win|z_j, z_i)) v(|\chi^s_j(z_j) - x_i|) dF^s_j(z_j),
\]

(1)

where \(i,j = L,R\) and \(t = M,E\). The first term is when the candidate wins over each type of the opposition. The second term is when the candidate loses to each type of the opposition. In summary, the timing of events and a political equilibrium are as follows.

1. Nature decides each candidate’s type, and a candidate knows own type.

2. The candidates announce their platforms.

3. Voters vote.

4. The winning candidate chooses which policy to implement.

**Definition 1.** A political equilibrium is a perfect Bayesian equilibrium in the game played by two candidates. A political equilibrium has a distribution function \(F^i_t(.)\), the implemented policy \(\chi^i_t(z_i)\) and the voters’ belief \(p_i(t|z_i)\) where \(i = L, R\) and \(t = M, E\) such that:

For all \(z_i\) in the support of \(F^i_t(.)\):

\[V^i_t((F^M_j(z_j), F^E_j(z_j)), z_i) \geq V^i_t((F^M_j(z_j), F^E_j(z_j)), z_i') \forall z_i'.\]

The voters’ posterior beliefs conditional on the platforms \(p_i(t|z_i)\) must satisfy Bayes’ rule whenever \(z_i\) is in the support of \(F^i_t(.)\).

\[\chi^t_i(z_i) = \text{argmax}_\chi - v(|\chi - x^t_i|) - c(|z_i - \chi|).\]

### 3 Political Equilibrium

All voters have a same belief about a candidate’s type. When one voter \(n\) prefers candidate \(R\) to \(L\), other voters, who are to the right of voter \(n\) (\(x_n\)), also prefer \(R\) because the position is further right, and the belief is the same. Therefore, if the candidate chooses any platform that is more attractive for the median voter than the opposition’s platform, this candidate is certain to win.
In the last period, the winning candidate implements the policy that maximizes the utility after a win, \(-v(|\chi_i(z_i) - x_i|) - c(|z_i - \chi_i(z_i)|)\).

**Lemma 1.** The implemented policy, \(\chi_i^t(z)\) satisfies \(v'(|\chi_i^t(z)| - x_i|) = c'(|z - \chi_i^t(z)|)\).

### 3.1 Separating Equilibrium

This section shows that a separating equilibrium does not exist. If the separating equilibrium exists, for all types, the utility of \(i\) when the candidate \(i\) wins \((-v(|x_i^t - \chi_i^t(z_i^t)|) - c(|z_i^t - \chi_i^t(z_i^t)|))\) should be the same as the utility of \(i\) when a same-type opposition wins \((-v(|x_i^t - \chi_j^t(z_j^t)|))\). If the utility when the candidate wins is higher than the utility when a same-type opposition wins, this candidate has an incentive to compromise more and win at least over the same-type opposition. If the utility when the candidate wins is lower than the utility when a same-type opposition wins, the candidate has an incentive to lose at least against the same-type opposition.

**Lemma 2.** If a separating equilibrium exists, the utility when the candidate wins \((-v(|x_i^t - \chi_i^t(z_i^t)|) - c(|z_i^t - \chi_i^t(z_i^t)|))\) and the utility when a same-type opposition wins \((-v(|x_i^t - \chi_j^t(z_j^t)|))\) are the same, and platforms of both candidates are symmetric, \(z_R^t - x_m = x_m - z_L^t\).

Denote \(\hat{z}^t\) as the platform under which both the utilities are the same for a type-\(t\) candidate, that is, \(-v(|x_i^t - \chi_i^t(\hat{z}_i^t)|) = -v(|x_i^t - \chi_i^t(\hat{z}_i^t)|) - c(|\hat{z}_i^t - \chi_i^t(\hat{z}_i^t)|)\) where \(t = M,E\). Such platforms satisfy the following Lemma\(^\text{12}\).

**Lemma 3.** Suppose that the utility when the candidate wins and the utility when a same-type opposition wins are the same for both types. Then, an extreme type announces more moderate platform \((\hat{z}_R^M - \hat{z}_L^M > \hat{z}_R^E - \hat{z}_L^E)\) but will implement more extreme policy than a moderate type \((\chi_R^E(\hat{z}_R^E) - \chi_R^M(\hat{z}_R^M) = \chi_L^M(\hat{z}_L^M) - \chi_R^M(\hat{z}_R^M))\). In addition, the difference between the utilities when the candidate wins and a same-type opposition wins \((-v(|x_i^t - \chi_i^t(z_i^t)|) - c(|z_i^t - \chi_i^t(z_i^t)|) + v(|x_i^t - \chi_j^t(z_j^t)|))\) is higher for an extreme type.

After an election, an extreme type will betray the platform more severely and pay the higher cost of betrayal, so the implemented policy will be more extreme. However, why does an extreme type promises more moderate platform? The most important point is that \(-v(|x_i^t - \chi_i^t(z_i^t)|) - c(|z_i^t - \chi_i^t(z_i^t)|) + v(|x_i^t - \chi_j^t(z_j^t)|)\) is higher for an extreme type. It means that, before an election, an extreme type finds especially costly for the opposition to win

\(^12\)This Lemma is based on Proposition 4 of Asako (2009a).
For a candidate of type $t$, the platform is $\hat{z}_t^R$, the implemented policy is $\chi^R_\hat{z}_t^R$, and the ideal policy is $x_t^R$. An extreme type has an incentive to pretend to be moderate by choosing the moderate type’s platform $\hat{z}_t^M$ because the probability of winning increases, and the implemented policy approaches the ideal policy (the implemented policy is $\chi^E_\hat{z}_t^M$ when an extreme type announces $\hat{z}_t^M$).

more than a moderate type does. The ideal policy of an extreme type is further from the median policy than that of a moderate one, which means that an extreme type’s ideal policy is also further from the opposition’s implemented policy. Thus, an extreme type has a higher disutility, so tries to commit to a more moderate policy to prevent letting the opposition win.

Does a separating equilibrium exist? The answer is no. Under the separating case, an extreme type always has an incentive to pretend to be moderate by choosing $\hat{z}_t^M$.

**Proposition 1.** A separating equilibrium does not exist.

The proof is straight-forward. From Lemma 2, the utility when the candidate wins should be the same as the utility when a same-type opposition wins if a separating equilibrium exists. According to Lemma 3, the moderate type has a higher probability of winning since the implemented policy will be more moderate, but the moderate type’s platform is more extreme than the extreme type’s platform. For example, $\hat{z}_t^E$ is more moderate than $\hat{z}_t^M$ as Figure 1 shows using the case of $R$. Therefore, if an extreme type deviates to a moderate type’s platform (from $\hat{z}_t^E$ to $\hat{z}_t^M$), an extreme type can gain a higher probability of winning. With this deviation, a future implemented policy moves from $\chi^E_\hat{z}_t^E$ to $\chi^E_\hat{z}_t^M$ in Figure 1, so an extreme type can implement a policy closer to his ideal policy, and the disutility after a win decreases. Moreover, the cost of betrayal also decreases. Thus, an extreme type can increase his or her expected utility from this deviation$^{13}$.

$^{13}$There may exist a separating equilibrium in a special case. See Appendix B.
3.2 Pooling Equilibrium

In a pooling equilibrium, the expected utility of candidate $i$ when an opposition wins is

$$-p^M v(|x_i^t - \chi_j^M(z_j)|) - (1 - p^M) v(|x_i^t - \chi_j^E(z_j)|)$$

where $i,j = L,R, i \neq j$ and $t = M,E$ given $z_j$. The utility of $i$ when the candidate $i$ of type-$t$ wins is

$$-v(|x_i^t - \chi_i^t(z_i)|) - c(|z_i^t - \chi_i^t(z_i^t)|).$$

Denote $z_i^{t*}$ as the platform under which both (expected) utilities are the same for a type-$t$ candidate where $t = M,E$ given the opposition’s pooling platform, $z_j$, that is,

$$-p^M v(|x_i^t - \chi_j^M(z_j)|) - (1 - p^M) v(|x_i^t - \chi_j^E(z_j)|) = -v(|x_i^t - \chi_i^t(z_i^t)|) - c(|z_i^{t*} - \chi_i(z_i^{t*})|).$$

Is an extreme type’s platform, $z_i^{E*}$, still more moderate than a moderate type’s one, $z_i^{M*}$? To show it, Lemma 3 needs to be generalized. Suppose that the both-type opposition announces the platform, $z_j$, and the probability that the opposition who announces $z_j$ is a moderate type is $q \in [0,1]$. When the (expected) utilities when the candidate wins and the opposition wins are same, a moderate type’s implemented policy is more moderate than an extreme type’s one while an extreme type’s platform is more moderate than a moderate type’s one.

**Corollary 1.** Suppose that the opposition announces the same platform regardless of types. For any probability that the opposition who announces $z_j$ is a moderate type, $q \in [0,1]$. If the (expected) utilities when the candidate wins and the opposition wins are same, an extreme type announces more moderate platform ($\hat{M}_i^{E*} - \hat{L}_i^{E*} > \hat{M}_i^{M*} - \hat{L}_i^{M*}$) but will implement more extreme policy than a moderate type ($\chi_i^{E*}(\hat{E}_i^{E*}) - \chi_i^{E*}(\hat{E}_i^{M*}) > \chi_i^{M*}(\hat{M}_i^{E*}) - \chi_i^{M*}(\hat{M}_i^{M*})$). In addition, the difference of the (expected) utilities when the candidate wins and the opposition wins, $-v(|x_i^t - \chi_i^t(z_i)|) - c(|z_i^t - \chi_i^t(z_i)|) + q v(|x_i^t - \chi_j^M(z_j)|) + (1 - q) v(|x_i^t - \chi_j^E(z_j)|)$, is higher for an extreme type.

A pooling equilibrium is the case of Corollary 1 with $q = p^M$. The platform of a moderate type, $z_i^{M*}$ is more extreme than the platform of an extreme type, $z_i^{E*}$.

At the pooling equilibrium, for a moderate type, the utility when the candidate wins and the expected utility when the opposition wins should be the same for a similar reason as for Lemma 2. If not, both types have an incentive to deviate to approach $x_m$.

**Lemma 4.** If a pooling equilibrium exists and $p(M|z_i) = p^M$ for all $z_i$, the utility when the candidate wins and the expected utility when the opposition wins are the same for a moderate type, and platforms of both candidates are symmetric, $z_i^{M*} - x_m = x_m - z_i^{M*}$.

In the next part, I make the following assumption to avoid showing the complicated condition about an off-path belief. In the off-path, if the platform is more moderate than
$z_i^{M*}$, voters believe that a candidate must be an extreme type because a moderate type has no incentive to compromise more than $z_i^{M*}$, but an extreme type may have such an incentive as Corollary 1 shows. If the platform is more extreme than $z_i^{M*}$, the off-path belief is the same as the prior belief since both types have an incentive to approach the median policy.

**Assumption 1.** If the platform is more moderate than $z_i^{M*}$, $p_i(M|z_i) = 0$. If the platform is more extreme than $z_i^{M*}$, $p_i(M|z_i) = p^M$.

In a pooling equilibrium, an extreme type chooses the same platform as, and mimics, a moderate type. If the prior belief that a candidate is a moderate one, $p^M$, is sufficiently high, a pooling equilibrium exists. First, when an extreme type chooses $z_i^{M*}$, this candidate has no incentive to deviate from any platform that is more extreme than $z_i^{M*}$. From Assumption 1, the belief for this candidate is still $p^M$ from this deviation, so this candidate will be certain to lose, and the expected utility decreases.

Next, I show whether or not an extreme type deviates from $z_i^{M*}$ to any platform which is more moderate than $z_i^{M*}$. Voters do not know the type in a pooling equilibrium, so the expected utility of voters is the weighted average of the utility between a moderate type and an extreme type. On the other hand, if an extreme type deviates, the expected utility of voters from choosing such an extreme type is the utility to choose an extreme type from Assumption 1. If an extreme type deviates in this way, such an extreme type’s implemented policy approaches the median policy compared to an extreme type in a pooling equilibrium.

If an extreme type’s platform (so implemented policy) approaches the median policy sufficiently, this extreme type can win over an uncertain opposition which chooses a pooling-equilibrium’s platform. Denote:

$$Z_i = \{z_i| - p^M u(|\chi_i^M(z_i^{M*}) - x_m|) - p^E u(|\chi_i^E(z_i^{M*}) - x_m|) < -u(|\chi_i^E(z_i) - x_m|)\}.$$ (2)

That is, $Z_i$ is a set of platforms of an extreme candidate $i$, which the median voter prefers to the platform, $z_i^{M*}$. Denote $z_i'$ as the supremum of platforms in $Z_i$ for $R$. For $L$, it is the infimum value. The set, $Z_i$ and $z_i'$ are showed in Figure 2.

The expected utility when an extreme type stays in a pooling equilibrium is:

$$V_i^E((z_j^{M*}, z_j^{M*}), z_i^{M*}) = \frac{1}{2} \left[ -p^M v(|\chi_j^M(z_j^{M*}) - x_i^E|) - (1 - p^M) v(|\chi_j^E(z_j^{M*}) - x_i^E|) \right. \\
- v(|\chi_i^E(z_i^{M*}) - x_i^E|) - c(|\chi_i^E(z_i^{M*}) - z_i^{M*}|).$$ (3)
The platform is $z_R$, the implemented policy is $x_R^t$, and the ideal policy is $x_R^t(z_R^M)$.

Let denote $E(pM) = pM x_R^M(z_R^M) + (1 - pM) x_R^E(z_R^M)$ as the expected policy implemented by a party announcing $z_R^M$. The supremum of platforms in $Z_R$ is $z_R'$. If an extreme type’s promise is smaller than $z_R'$, such extreme type is more attractive than a candidate with for the median voter.

The expected utility when the extreme type deviates to $z_i'$ is:

$$V_i^E((z_j^M, z_j^M), z_i') = -v(|x_i^E(z_i') - x_i^E|) - c(|x_i^E(z_i') - z_i'|).$$ (4)

Note that if a candidate approaches the median policy, the cost of betrayal and the disutility after a win increase. Thus, an extreme type can increase the expected utility from this deviation\(^{14}\) if:

$$V_i^E((z_j^M, z_j^M), z_i^M) < V_i^E((z_j^M, z_j^M), z_i'),$$ (5)

From the same reasons as a deviation to any platform which is more extreme than $z_i^M$, an extreme type never deviate to $z_i \in (z_i', z_i^M)$ since the probability of winning becomes zero. If (5) does not hold, an extreme type also does not deviate to $z_i'$. If (5) holds, a pooling equilibrium does not exist since an extreme type deviates to $z_i'$.

**Proposition 2.** Suppose Assumption 1. If and only if (5) does not hold, then a pooling equilibrium under which all types announce $z_i^M$ exists.

When $z_i'$ is closer to $z_i^M$, then the inequality (5) tends to hold. The value of $z_i'$ is decided by the median voters’ utility. Suppose that voters have a linear disutility function. Suppose also that $L$ chooses $z_L^M$ as a pooling equilibrium, and $R$ is an extreme type. If $p^M$ is high, the extreme-type $R$ needs to compromise greatly in order to win because the expected policy to be implemented by $L$ with $z_L^M$ is closer to the implemented policy when $L$ is moderate,  

\(^{14}\)If a platform becomes more extreme, (4) increases, so it is sufficient to check the extreme bound of $Z_i$, $z_i'$, to know an incentive to deviate.
\( \chi^M_L(z^*_L) \). That is, in Figure 2, \( z'_R \) is very far from the ideal policy, \( x^E_R \), so this compromise decreases the expected utility of the extreme-type \( R \). As a result, the extreme-type \( R \) does not deviate from a pooling equilibrium. However, if \( p^M \) is sufficiently low, the expected policy to be implemented by \( L \) is closer to the implemented policy when \( L \) is extreme, \( \chi^E_L(z^*_L) \), so, if \( R \) compromises slightly, the implemented policy of \( R \) becomes better for the median voter. In Figure 2, \( z'_R \) is closer to \( z^*_R \). Candidate \( R \) may be able to increase the expected utility because the probability of winning increases without a great increase of the cost of betrayal and the disutility after a win. From these reasons, if \( p^M \) is sufficiently low, the extreme type will deviate, and a pooling equilibrium does not exist.

In addition, if voters have a strictly convex disutility function \((u(.)\)), they care about the expected utility instead of the expected policy, and the expected utility to choose a candidate in a pooling equilibrium becomes lower than the linear utility function’s case, so an extreme type will deviate in broader cases.

### 3.3 Semi-separating Equilibrium

#### 3.3.1 The Definition

If (5) holds, a semi-separating equilibrium exists. In a semi-separating equilibrium, a moderate type chooses a pure strategy, \( z^*_i \). The value of \( z^*_i \) satisfies the following equation.

\[
- \frac{p^M}{p^M + \sigma^M(1 - p^M)}v(|\chi^M_j(z^*_j) - x^M_i|) - \frac{\sigma^M(1 - p^M)}{p^M + \sigma^M(1 - p^M)}v(|\chi^E_j(z^*_j) - x^M_i|) = -v(|\chi^M_i(z^*_i) - x^M_i|) - c(|\chi^M_i(z^*_i) - z^*_L|) \tag{6}
\]

where \( \sigma^M \) is the probability that an extreme type (opposition) pretends to be moderate by announcing \( z^*_j \). The left-hand side is the utility when the opposition promising \( z^*_j \) wins, and the right-hand side is the utility when the candidate wins. That is, a moderate type is indifferent between winning and losing over the opposition announcing \( z^*_j \).

**Definition 2.** Given \( \sigma^M \). Platforms \( z^*_i \) and \( z^*_j \) satisfy (6), and these platforms are symmetric, \( z^*_R - x_m = x_m - z^*_L \).

An extreme type chooses a mixed strategy. If this mixed strategy does not include \( z^*_i \), this is a separating case, and such a case cannot be equilibrium (from the same as Section 3.1). Thus, one platform in a mixed strategy should be \( z^*_i \). To have a mixed strategy, in the other case, an extreme type chooses any platform that is more moderate than \( z^*_i \) and reveals his own type to voters.
There are two types of semi-separating equilibrium. The first is called a continuous semi-separating equilibrium, and the second is a two-policy semi-separating equilibrium. In both types of semi-separating equilibrium, an extreme type chooses $z_i^*$ with the probability $\sigma^M$, and, with the probability $1 - \sigma^M$, an extreme type reveals the type. In a continuous semi-separating equilibrium, an extreme type also has a distribution function, $F(\cdot)$, with support $[\bar{z}_L, z_L]$ for $L$ and $[\bar{z}_R, z_R]$ for $R$. To be concrete, the distribution is $(1 - \sigma^M)F(\cdot)$. In a two-policy semi-separating equilibrium, an extreme type chooses one platform, $\bar{z}_i$, with the probability $1 - \sigma^M$. In a two-policy semi-separating equilibrium, a mixed strategy includes only two policies, $z_i^*$ and $\bar{z}_i$, while a continuous semi-separating equilibrium includes $z_i^*$ and a continuous support.

**Definition 3.** A continuous semi-separating equilibrium is a collection $(z_i^*, \sigma^M, F(\cdot), \Pi)$ and a two-policy semi-separating equilibrium is a collection $(z_i^*, \sigma^M, \bar{z}_i, \Pi)$ where $z_i^*$ is a pure strategy chosen by a moderate type satisfying Definition 2, $\sigma^M$ is the probability of choosing $z_i^*$ in an extreme type’s mixed strategy, $F(\cdot)$ is a distribution function with the support of $[\bar{z}_L, z_L]$ for $L$ and $[\bar{z}_R, z_R]$ for $R$, and $\Pi$ is a scalar, such that (a.1) $\Pi = V^E_i(z_i) = V^E_i(z_i^*)$ for all $z_i$ in support of $F(\cdot)$ in a continuous semi-separating equilibrium; (a.2) $\Pi = V^E_i(z_i^*) = V^E_i(\bar{z}_i)$ in a two-policy semi-separating equilibrium, and (b) Definition 1 holds. All variables of both candidates are symmetric.

The value of the expected utility is $\Pi$, and $V^t_i(\cdot)$ is the expected utility given that an opposition takes an equilibrium strategy which is symmetric with the candidate where $i = L, R$ and $t = M, E$. Condition (a) implies that an extreme type is indifferent to all platforms in a mixed strategy, whereas (b) implies there is no incentive to change the platform for either type and voters’ belief is based on Bayes’ rule. Figure 3 summarizes the above definition. I also use a similar assumption to Assumption 1.

**Assumption 2.** If the platform is more moderate than $z_i^*$, then $p_i(M|z_i) = 0$. If the platform is more extreme than $z_i^*$, then $p_i(M|z_i) = p^M$.

If the platform is more extreme than $z_i^*$, a moderate and extreme types have an incentive to approach the median policy more, so $p_i(M|z_i) = p^M$. If the platform is more moderate than $z_i^*$, a moderate type has an incentive to lose to an opposition that chooses a symmetric platform while an extreme type may not have from Corollary 1\(^{15}\), so $p_i(M|z_i) = 0$. Note that Assumption 2 is the more general definition of Assumption 1 to include the possibility that an extreme type chooses a mixed strategy. When $\sigma^M = 1$, it is same as Assumption 1.

\(^{15}\) A semi-separating equilibrium is the case of Corollary 1 with $q = \frac{p^M}{p^M + \sigma^M(1 - p^M)}$.  

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Figure 3: Semi-separating Equilibrium

The platform is $z_R$, and the implemented policy is $\chi^I_R(z_R)$. A moderate type’s platform is $z^*_R$. Voters prefer to choose an extreme type who compromises rather than a candidate announcing $z^*_R$ when an extreme type announces $z_R$ or more moderate platform. A moderate type’s implemented policy, $\chi^M_R(z^*_R)$, is more moderate than any extreme type’s implemented policy. If an extreme type pretends to be moderate, this extreme type’s implemented policy, $\chi^E_R(z^*_R)$, is more extreme than an implemented policy when an extreme type compromises, $[\chi^E_R(z_R), \chi^E_R(z_R)]$. 

(a) A two policy semi separating equilibrium

(b) A continuous semi separating equilibrium
Right now, I suppose that the positions of platforms, $\sigma^M$ and $F(.)$ are symmetric for both candidates. It means that both candidates’ platforms and $F(.)$ are symmetric about the median policy, and both candidates have a same value of $\sigma^M$. However, I will show that they are always symmetric in equilibrium with Assumption 2.

### 3.3.2 The Equilibrium

To determine the mixed strategy, I build on techniques introduced by Burdett and Judd (1983). They consider price competition and show that firms randomize over different prices when there is a possibility that consumers will observe only one price. Just as Burdett and Judd (1983) show that firms are indifferent over a range of prices, I show that an extreme type is indifferent over a range of platforms.

To have a mixed strategy as equilibrium, an extreme type needs to be indifferent among all platforms in the support of the mixed strategy. When an extreme type chooses any platform that is more moderate than $z^*_i$, the disutility and the cost of betrayal after a win are higher than the case under which an extreme type chooses $z^*_i$. Thus, an extreme type who chooses a more moderate platform than $z^*_i$ needs to win over an opposition who chooses $z^*_j$. Denote:

$$Z'_i(\sigma^M) = \{z_i | -\frac{p^M}{p^M + \sigma^M(1-p^M)}u(|\chi^M_i(z^*_i) - x_m|) - \frac{\sigma^M(1-p^M)}{p^M + \sigma^M(1-p^M)}u(|\chi^E_i(z^*_i) - x_m|) < -u(|\chi^E_i(z_i) - x_m|)\}.$$  \hspace{1cm} (7)

That is, $Z'_i(\sigma^M)$ is a set of promises of an extreme type $i$, which the median voter prefers to the platform, $z^*_i$, given $\sigma^M$. Define $\bar{z}_i$ as follows.

**Definition 4.** The value of $\bar{z}_R$ is supremum of $Z'_R$, and $\bar{z}_L$ is infimum of $Z'_L$.

The probability that a candidate announcing $z^*_i$ is a moderate type increases as $\sigma^M$ decreases, so voters’ utility to choose a candidate announcing $z^*_i$ increases. Therefore, when $\sigma^M$ decreases, $\bar{z}_i$ becomes more moderate.

In the following part, I discuss a two-policy semi-separating equilibrium at first, and then move to discuss a continuous semi-separating equilibrium. Without loss of generality, let us focus on $R$’s expected utility and choices.

In a two-policy semi-separating equilibrium, an extreme type announces $\bar{z}_R$ with the probability $1 - \sigma^M$. To show it, conversely, suppose that an extreme type announces a platform which is lower (more moderate) than $\bar{z}_R$ instead of $\bar{z}_R$. Then, this extreme type
has an incentive to deviate to $\bar{z}_R$ since the probability of winning is unchanged, and the cost of betrayal and the disutility after a win decreases with such a deviation. If a platform is higher than $\bar{z}_R$, an extreme type cannot win over even an opposition announcing $z^*_L$. Thus, an extreme type will announce $\bar{z}_R$ with $1 - \sigma^M$ in equilibrium. Additionally, an extreme type announces $z^*_R$ with the probability $\sigma^M$.

When an extreme type announces $z^*_R$, the candidate ties with an opposition announcing $z^*_L$, but loses to the other oppositions. Thus, the expected utility when an extreme type chooses $z^*_R$ to pretend to be moderate is as follows.

$$V^E_R(z^*_R) = \frac{1}{2} \left[ -p^M v(|\chi^E_R(z^*_L) - x^E_R|) - \sigma^M (1 - p^M) v(|\chi^E_L(z^*_L) - x^E_R|) ight]$$

$$- (p^M + \sigma^M (1 - p^M)) \left[ v(|\chi^E_R(z^*_R) - x^E_R|) + c(|\chi^E_L(z^*_R) - z^*_L|) \right]$$

$$- (1 - \sigma^M) v(|\chi^E_L(\bar{z}_L) - x^E_R|),$$

(8)

When an extreme type announces $\bar{z}_R$, he wins over an opposition announcing $z^*_L$, but ties with an opposition announcing $\bar{z}_L$. Thus, the expected utility when an extreme type chooses $\bar{z}_R$ is as follows:

$$V^E_R(\bar{z}_R) = (p^M + \sigma^M (1 - p^M)) \left[ - v(|\chi^E_R(\bar{z}_R) - x^E_R|) - c(|\chi^E_L(\bar{z}_R) - \bar{z}_L|) \right]$$

$$- \frac{1}{2} (1 - \sigma^M)(1 - p^M) \left[ v(|\chi^E_L(\bar{z}_L) - x^E_R|) + v(|\chi^E_L(\bar{z}_R) - x^E_R|) + c(|\chi^E_R(\bar{z}_R) - \bar{z}_L|) \right].$$

(9)

In a two-policy semi-separating equilibrium, $\Pi$ and $\sigma^M$ are determined by:

$$V^E_i(z^*_i) = V^E_i(\bar{z}_i) = \Pi.$$  

(10)

Definition 4 determines $\bar{z}_R$, and $\Pi$ and $\sigma^M$ are determined by (10). If (5) does not hold, a semi-separating equilibrium does not exist. The probability of being moderate at $z^*_R$ increases as $\sigma^M$ decreases, so $V^E_R(z^*_R)$ increases as $\sigma^M$ decreases because the probability that the opposition will implement a moderate type’s policy increases. On the other hand, as $\sigma^M$ decreases, $\bar{z}_R$ becomes smaller from Definition 4, so $V^E_R(\bar{z}_R)$ decreases as $\sigma^M$ decreases. If (5) does not hold, $V^E_R(z^*_R)$ is higher than $V^E_R(\bar{z}_R)$ when $\sigma^M = 1$; this means that, for all $\sigma^M$, $V^E_R(z^*_R)$ is higher than $V^E_R(\bar{z}_R)$. If (5) holds, $V^E_R(z^*_R) < V^E_R(\bar{z}_R)$ at $\sigma^M = 1$. When $\sigma^M$ converges to zero, the situation also converges to a completely separating case. In a separating case, an extreme type has an incentive to deviate to mimic a moderate type; this means that $V^E_R(z^*_R)$ is higher than $V^E_R(\bar{z}_R)$ when $\sigma^M$ is closer to zero. All functions are continuous, so it means that there exists $\sigma^M$ under which $V^E_R(z^*_R) = V^E_R(\bar{z}_R)$.
Consider $\sigma^M$ under which $V^E_R(z^*_R) = V^E_R(\bar{z}_R)$. Under such $\bar{z}_R$, if:

$$-v(|\chi^E_R(\bar{z}_R) - x^E_R|) - c(|\chi^E_R(\bar{z}_R) - \bar{z}_R|) \leq -v(|\chi^E_L(\bar{z}_L) - x^E_R|),$$  \hspace{1cm} (11)

then an extreme type has no incentive to compromise to win over an extreme-type opposition announcing $\bar{z}_L$. Therefore, a two-policy semi-separating equilibrium exists. An extreme type with $\bar{z}_R$ does not want to deviate to lose because that means the candidate loses not only to an opposition with $\bar{z}_L$ but also to an opposition with $z^*_L$, so such deviation decreases the expected utility.

If (11) does not hold, an extreme type still has an incentive to compromise more than $\bar{z}_R$ to win over an extreme-type opposition announcing $\bar{z}_L$, so a continuous semi-separating equilibrium exists instead of a two-policy semi-separating equilibrium. When an extreme type announces $z^*_R$, the expected utility is:

$$V^E_R(z^*_R) = \frac{1}{2} [-p^M v(|\chi^M_L(z^*_L) - x^E_R|) - \sigma^M (1 - p^M) v(|\chi^E_L(z^*_L) - x^E_R|) - (p^M + \sigma^M (1 - p^M)) v(|\chi^E_R(z^*_R) - x^E_R|) + c(|\chi^E_R(z^*_R) - \bar{z}_R|)]$$

$$- (1 - \sigma^M)(1 - p^M) \int_{\bar{z}_L}^{\bar{z}_R} v(|\chi^E_L(z_L) - x^E_R|) dF(z_L).$$ \hspace{1cm} (12)

The platform $\bar{z}_R$ is used as the highest (extreme) bound of the support of $F(.)$ in a continuous semi-separating equilibrium. When an extreme type announces $\bar{z}_R$, such an extreme type wins over an opposition announcing $z^*_L$, but loses to the other extreme-type opposition who compromises. Thus, the expected utility when an extreme type chooses $\bar{z}_R$ is as follows:

$$V^E_R(\bar{z}_R) = (p^M + \sigma^M (1 - p^M)) [-v(|\chi^E_R(\bar{z}_R) - x^E_R|) - c(|\chi^E_R(\bar{z}_R) - \bar{z}_R|)]$$

$$- (1 - \sigma^M)(1 - p^M) \int_{\bar{z}_L}^{\bar{z}_R} v(|\chi^E_L(z_L) - x^E_R|) dF(z_L).$$ \hspace{1cm} (13)

In a continuous semi-separating equilibrium, $\Pi$ and $\sigma^M$ are also determined by (10), and from the same reasons as a two-policy semi-separating equilibrium, there exists $\sigma^M$ under which $V^E_R(z^*_R) = V^E_R(\bar{z}_R)$. In a continuous semi-separating equilibrium, an extreme type has the distribution $F(.)$ on some policies. The distribution function, $F(.)$, satisfies the following lemma.

\[16\text{Actually, the platform } \bar{z}_R \text{ in a continuous semi-separating equilibrium slightly lower than one in a two-policy semi-separating equilibrium. See Appendix A.6.6.}\]
Lemma 5. Suppose that a continuous semi-separating equilibrium exists. In such equilibrium, \( F(.) \) is continuous with connected support.

Because of this distribution, for \( R \), when a platform becomes smaller continuously in support, the probability of winning increases continuously too while the cost of betrayal and the disutility from a win increases. Therefore, there exist combinations of \( z_R \) and \( F(.) \) under which an extreme type is indifferent among any platform in the connected policies. This platform, \( z_R \), is another lowest (moderate) bound of the support of \( F(.) \). From Lemma 5, \( F(z_L) = 0 \), so, when an extreme type chooses \( z_R \), the probability of winning is one. The expected utility when an extreme type chooses \( z_R \) is as follows:

\[
V^E_R(z_R) = -v(|\chi^E_R(z_R) - x^E_R|) - c(|\chi^E_R(z_R) - z_R|).
\]

The expected utility when an extreme type chooses any \( z'_R \in (z_R, z_R) \) is:

\[
V^E_R(z'_R) = (1 - (1 - \sigma^M)(1 - p^M)F(z'_R))\left[-v(|\chi^E_R(z'_R) - x^E_R|) - c(|\chi^E_R(z'_R) - z'_R|)\right] \\
- (1 - \sigma^M)(1 - p^M) \int_{z'_L}^{\tilde{z}_L} v(|\chi^E_L(z_L) - x^E_R|)dF(z_L).
\]

An extreme type should be indifferent among all platforms in the support of a mixed strategy, so \( z_i \) and \( F(.) \) are decided by:

\[
V^E_i(z_i) = V^E_i(z'_i) = \Pi.
\]

Given this behavior of the extreme type, does the moderate type want to deviate by approaching the median policy? A moderate type has no incentive to win over an extreme-type opposition who compromises because such an extreme-type opposition compromises sufficiently and will implement a policy that is closer to a moderate type’s ideal policy.

In addition, as Figure 3 shows, an extreme type implements more extreme policy than a moderate type regardless of a platform in the mixed strategy, and an extreme type who compromises will implement a more moderate policy than an extreme type who pretends to be moderate.

Proposition 3. Suppose Assumption 2. If and only if the equation (5) holds, the one of the following semi-separating equilibria exists as a political equilibrium.

1. A continuous semi-separating equilibrium, \( (z^*_i, \sigma^M, F(.), \Pi) \), defined by Definition 3 where \( F(.) \) is continuous with connected support, \( z_i \) and \( \sigma^M \) satisfy Definition 4 and
(10), and (16) defines $\bar{z}_i$ and $F(.)$.

2. A two-policy semi-separating equilibrium, $(z^*_i, \bar{z}_i, \sigma^M, \Pi)$, defined by Definition 3 where $\bar{z}_i$ satisfies Definition 4, and (10) defines $\Pi$ and $\sigma^M$.

In a continuous semi-separating equilibrium, $\chi^E_i(z_i)$ is more extreme than $\chi^M_i(z^*_i)$. In a two-policy semi-separating equilibrium, $\chi^E_i(\bar{z}_i)$ is more extreme than $\chi^M_i(z^*_i)$. Thus, any implemented policy of an extreme type will be more extreme than the moderate type’s implemented policy. All variables of both candidates are symmetric.

In both types of semi-separating equilibria, an extreme type has a positive probability of choosing a moderate type’s platform, $z^*_i$, and induces voters to be still uncertain about the candidate’s type with $z^*_i$. The majority of voters prefers an extreme type who compromises to an uncertain type promising $z^*_i$ to avoid choosing an extreme type who pretends to be moderate. Therefore, an extreme type who compromises wins over a moderate type and an extreme type who mimics a moderate type.

Finally, from Proposition 2 and 3, there always exists a political equilibrium. According to (5), there is a cut-off point of the prior probability of being moderate, $\bar{p}^M$, under which $V^E_i((z^*_j, z^*_j, z^*_i)) = V^E_i((z^*_j, z^*_j, z^*_i))$, and if the prior probability of being moderate, $p^M$, is higher than $\bar{p}^M$, only a pooling equilibrium exists. If $p^M < \bar{p}^M$, only semi-separating equilibria exist. The following proposition summarizes the above points.

**Proposition 4.** Suppose Assumption 2. A political equilibrium exists. There exists a cut-off point, $\bar{p}^M$ such that if $p^M \geq \bar{p}^M$, only a pooling equilibrium exists, and if $p^M < \bar{p}^M$, only semi-separating equilibria exist.

Suppose that, after period 1, nature decides that one candidate is a moderate type, and another candidate is an extreme type. *Ex post*, the optimal candidate is a moderate type. In a pooling equilibrium, a campaign platform has no meaning for choosing the optimal candidate. The expected probability of getting a moderate type is completely the same as the prior belief, $p^M$, *ex ante*. In a semi-separating equilibrium, for the majority of voters, the first best is a moderate type, the second best is an extreme type who compromises, and the worst one is an extreme type who mimics a moderate type. An extreme type has a higher expected probability of winning than a moderate type, so the probability of choosing an extreme type is higher than the prior belief. Moreover, there still exists a probability to

\[17\] There may exist several semi-separating equilibria since, in a continuous semi-separating equilibrium, both $\bar{z}_i$ and $F(.)$ are decided by one equation. However, all semi-separating equilibria have the same characteristics discussed in the above.
choose an extreme type pretending to be moderate. Under nonbinding or completely binding platforms, an extreme type never has a higher probability of winning than a moderate type.

**Corollary 2.** Partially binding platforms with an incomplete information leads to an *ex post* inefficient aggregation of preferences.

## 4 Conclusion

This paper examines the effects of partially binding platforms in electoral competition. When there is asymmetric information, voters cannot always determine a candidate’s political preferences. If the probability that a candidate is moderate is sufficiently high, there exists a pooling equilibrium under which an extreme candidate pretends to be moderate. If not, there exists a semi-separating equilibrium: an extreme candidate pretends to be moderate with some probability and, with the remaining probability, reveals their own preferences and approaches the median voter’s ideal policy. When one candidate is moderate, and another candidate is extreme, an extreme candidate who reveals his preference type will defeat an uncertain candidate even though the extreme candidate will implement more extreme policy than a moderate candidate. Therefore, if platforms are partially binding, and there exists asymmetric information about the candidates’ ideal policies, then an extreme candidate wins with a higher expected probability than a moderate candidate. Partially binding platforms with an incomplete informational environment leads to an *ex post* inefficient aggregation of preferences.

This paper suggests a number of questions for future studies. One possible area of research is to endogenize the cost of betrayal. In this paper, the cost of betrayal only depends on the degree of betrayal, but it may be decided endogenously. For example, one kind of costs of betrayal is a decrease in the probability of winning the next election. In order to analyze such reputational costs, a dynamic model with two periods should be analyzed. Second, candidates may have different characteristics, such as different prior beliefs that a candidate is moderate, so this also should be an important issue to further analyze. Third, models of political competition are applied in many other topics, but most models consider completely binding or nonbinding platforms. As this paper shows, partially binding platforms induce many different predictions, so applying the model of partially binding platforms should also be the subject of interesting future research.
A Proofs

A.1 Lemma 2

I use a contradiction, i.e., if the utility when the candidate wins is not the same as the utility when a same-type opposition wins, it is not a separating equilibrium. The following parts assume that the platforms are symmetric, and candidates tie among the same-type candidates. If platforms are not symmetric, one of two candidates will win with certainty over a same-type opposition. The winner prefers to approach the own ideal policy, $x_i^t$, and still wins over the same-type opposition since the policy space is continuous.

If $-v(\chi_1^t(z_1^t) - x_i^t) - c(z_1^t - \chi_1^t(z_1^t)) > -v(\chi_1^t(z_2^t) - x_i^t)$, the candidate deviates to move slightly to any platform that is better for the median voter and be certain to win over a same-type opposition. Consider that the deviation to approach $x_m$ is minor. Before this deviation, the expected utility for an extreme type is $\frac{1}{2}(1 - p^M)[-v(\chi_i^E(z_i^E) - x_i^E)] - c(z_i^E - \chi_i^E(z_i^E))] + \frac{1}{2}(1 - p^M)[-v(\chi_j^E(z_j^E) - x_i^E)] + p^M V(z_j^M, z_i^E)$. The final term, $V(z_j^M, z_i^E)$, is the expected utility when the opposition is a moderate type. After the deviation, the expected utility for the extreme type is slightly lower than $\frac{1}{2}(1 - p^M)[-v(\chi_i^E(z_i^E) - x_i^E)] - c(z_i^E - \chi_i^E(z_i^E))] + p^M V(z_j^M, z_i^E)$. This extreme type can increase the utility by slightly less than $\frac{1}{2}(1 - p^M)[-v(\chi_i^E(z_i^E) - x_i^E)] - c(z_i^E - \chi_i^E(z_i^E))] + p^M V(z_j^M, z_i^E)$. This extreme type can increase the utility by slightly less than $-v(\chi_i^t(z_i^t) - x_i^t) - c(z_i^t - \chi_i^t(z_i^t)) > -v(\chi_j^t(z_j^t) - x_i^t)$.

If $-v(\chi_1^t(z_1^t) - x_i^t) - c(z_1^t - \chi_1^t(z_1^t)) < -v(\chi_1^t(z_1^t) - x_i^t)$, the candidate deviates to announce any platform that is worse for the median voter and lose to a same-type opposition. The expected utility before this deviation is the same as above. After the deviation, the expected utility for the extreme type is $\frac{1}{2}(1 - p^M)[-v(\chi_j^E(z_j^E) - x_i^E)] + p^M V(z_j^M, z_i^E)$. This extreme type can increase the utility by $\frac{1}{2}(1 - p^M)[-v(\chi_j^E(z_j^E) - x_i^E)] + c(z_i^E - \chi_i^E(z_i^E))] - v(\chi_j^E(z_j^E) - x_i^E)]$ from this deviation since $-v(\chi_1^t(z_1^t) - x_i^t) - c(z_1^t - \chi_1^t(z_1^t)) < -v(\chi_1^t(z_1^t) - x_i^t)$. From the same reasons, a moderate type also deviates. □

A.2 Lemma 3

To prove it, I show that as $x_R - x_L$ increases, $z_R - z_L$ decreases and $\chi_R(z_R) - \chi_L(z_L)$ increases if the utilities when the candidate wins and the opposition wins are the same.

Consider a case of complete information and $R$ without loss of generality. First, assume that the utilities when the candidate wins and when the opposition wins are the same for
both candidates, and platforms are symmetric. This means that:

\[ v(x_R + \chi_R - 2x_m) - v(x_R - \chi_R) = c(\chi_R - z_R(\chi_R)). \] (17)

Denote \( z_R(\chi_R) = \chi_R^{-1}(\chi_R) \), which means the platform committing him to \( \chi_R \). In addition, \( x_R - \chi_L = (x_R - x_m) + (\chi_R - x_m) = x_R + \chi_R - 2x_m \) because the platforms are symmetric. Then, differentiate both sides of (17) by \( x_R \). The differential of the left-hand side by \( x_R \) is

\[ v'(x_R + \chi_R - 2x_m) - v'(x_R - \chi_R) + \frac{\partial \chi_R}{\partial x_R} v'(x_R + \chi_R - 2x_m) + v'(x_R - \chi_R)). \]

The differential of the right-hand side by \( x_R \) is \( c'(\chi_R - z_R(\chi_R)) - \frac{\partial z_R(\chi_R)}{\partial x_R} - \frac{\partial z_R(\chi_R)}{\partial x_R} \). Both of these differentials should be the same. From Lemma 1, \( v'(x_R - \chi_R) = c'(\chi_R - z_R(\chi_R)) \), so the condition becomes:

\[ v'(x_R + \chi_R - 2x_m) - v'(x_R - \chi_R) + \frac{\partial \chi_R}{\partial x_R} v'(x_R + \chi_R - 2x_m) = -v'(x_R - \chi_R)
\left(\frac{\partial z_R(\chi_R)}{\partial x_R} + \frac{\partial z_R(\chi_R)}{\partial x_R}\right). \] (18)

Suppose Lemma 1. Fix \( \chi_R \) and differentiate \( v'(x_R - \chi_R) = c'(\chi_R - z_R(\chi_R)) \) by \( x_R \), then

\[ \frac{\partial z_R(\chi_R)}{\partial x_R} = -\frac{v''(x_R - \chi_R)c'(\chi_R - z_R(\chi_R))}{c'(\chi_R - z_R(\chi_R))} < 0. \]

Substitute it into (18), so it becomes:

\[ \frac{\partial \chi_R}{\partial x_R} = \frac{v''(x_R - \chi_R)c'(\chi_R - z_R(\chi_R)) - (v'(x_R - \chi_R) - v'(x_R - \chi_R))}{v'(x_R - \chi_R) + v'(x_R - \chi_R)\frac{\partial z_R(\chi_R)}{\partial x_R}}. \] (19)

If (19) is positive, an extreme type will implement a more extreme policy than a moderate type. From the same way as deriving \( \frac{\partial z_R(\chi_R)}{\partial x_R} \), \( \frac{\partial z_R(\chi_R)}{\partial x_R} = 1 + \frac{v''(x_R - \chi_R)c'(\chi_R - z_R(\chi_R))}{v'(x_R - \chi_R)c'(\chi_R - z_R(\chi_R))} \) > 0. To prove that (19) is positive, it is sufficient to show that the numerator of (19) is positive. In other words:

\[ \frac{v'(x_R - \chi_R) - v'(x_R - \chi_R)}{v''(x_R - \chi_R)} < \frac{c'(\chi_R - z_R(\chi_R))}{c'(\chi_R - z_R(\chi_R))}. \] (20)

Note that, from (17) and Lemma 1:

\[ \frac{v(x_R - \chi_L) - v(x_R - \chi_R)}{v'(x_R - \chi_R)} = \frac{c(\chi_R - z_R(\chi_R))}{c'(\chi_R - z_R(\chi_R))}. \] (21)

Since \( \frac{c'(x)}{c''(x)} \) strictly decreases as \( d \) increases, \( \frac{c'(\chi_R - z_R(\chi_R))}{c''(\chi_R - z_R(\chi_R))} > \frac{c(\chi_R - z_R(\chi_R))}{c'(\chi_R - z_R(\chi_R))} \). The right-hand side of (20) is higher than the left-hand side of (21). If \( \frac{v'(x_R - \chi_L) - v'(x_R - \chi_R)}{v''(x_R - \chi_R)} < \frac{v(x_R - \chi_L) - v(x_R - \chi_R)}{v'(x_R - \chi_R)} \), (20) holds. This equation can be changed to \( \frac{v''(x_R - \chi_R)}{v'(x_R - \chi_R)} - \frac{v''(x_R - \chi_R)}{v'(x_R - \chi_R)} = \frac{v'(x_R - \chi_R) - v'(x_R - \chi_R)}{v'(x_R - \chi_R)} \).
Assume that $v'(x_R - \chi_R) < \frac{v''(x_R - \chi_R)}{v'(x_R - \chi_R)}$. Since $\frac{v'(d)}{v(d)}$ strictly decreases as $d$ increases, the right-hand side is positive. If $x_R - \chi_L = x_R - \chi_R$, both sides are the same. If $x_R - \chi_L$ increases, the left-hand side decreases. The reason is as follows. Differentiate the left-hand side with respect to $x_R - \chi_L$, then

$$\frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)} - \frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)}.$$ 

This value is negative because $\frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)} < \frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)}$ when $x_R - \chi_L > x_R - \chi_R$ and $v''(.) > 0$. As a result, the left-hand side of (20) is lower than the left-hand side of (21), so (20) holds. This result can be derived even if only $c'(d)$ or $\frac{v'(d)}{v(d)}$ strictly decreases as $d$ increases. Thus, (19) is positive. It also means that $v(\{x_i^t - \chi_i^t(z_i^t)\}) - \frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)}c(\chi_i^t(z_i^t) - z_i^t)$ is higher for an extreme type.

To determine the effect on platforms, it is sufficient to know the sign of $\frac{\partial z_R(x_R)}{\partial x_R} + \frac{\partial z_R(x_R)}{\partial x_R} \frac{\partial x_R}{\partial x_R}$. From the above, it is:

$$\frac{v''(x_R - \chi_R)}{v'(x_R - \chi_R)}c'(x_R - z_R(x_R)) + \frac{v''(x_R - \chi_R)c'(x_R - z_R(x_R))}{v'(x_R - \chi_R)} - \frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)}c'(x_R - z_R(x_R)) - (\frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)}) - \frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)} \frac{\partial z_R(x_R)}{\partial x_R} \frac{\partial x_R}{\partial x_R}.$$

Assume that $v'(x_R - \chi_L)$ in the denominator is zero. Then,

$$\frac{v''(x_R - \chi_R)}{v'(x_R - \chi_R)}c'(x_R - z_R(x_R)) + \frac{v''(x_R - \chi_R)c'(x_R - z_R(x_R))}{v'(x_R - \chi_R)} - (\frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)}) = - \frac{v'(x_R - \chi_L)}{v'(x_R - \chi_R)} < 0.$$

Even though $v'(x_R - \chi_L)$ in the denominator is positive, the value is still negative because the positive part of (22) is still lower than the negative part. As a result, a more extreme type promises a more moderate platform. □

### A.3 Corollary 1

The way to prove it is same as Lemma 3. The only difference is (17) is replaced by $-v(|x_i^t - \chi_i^t(z_i^t)|) - c(|z_i - \chi_i^t(z_i^t)|) - [q v(|x_j^t - \chi_j^M(z_j)|) - (1 - q)v(|x_i^t - \chi_j^E(z_j)|)]$. □

### A.4 Lemma 4

The following parts assume that the platforms are symmetric, and candidates tie among the candidates. If platforms are not symmetric, one of two candidates will win with certainty. The winner prefers to approach the own ideal policy, $x_i^t$, and still wins over the opposition since the policy space is continuous. If the utility when the candidate wins is higher than the expected utility when an opposition wins for a moderate type, a candidate has an incentive to deviate to approach $x_m$ regardless of the type. If a candidate approaches $x_m$, this candidate
can be certain to win over the opposition because $p(M|z_i) = p^M$ for all $z_i$, and the candidate can increase the expected utility, so any candidate has an incentive to deviate. If the utility when the candidate wins is lower than the expected utility when an opposition wins for a moderate type, a moderate type will deviate to lose. □

A.5 Lemma 5

If $F(.)$ has a discontinuity at some policy, say $z'_j$, i.e., $F(z'_j+) > F(z'_j-)$, there is a strictly positive probability that an opposition also chooses $z'_j$ (the probability density function is $f(z'_j) > 0$). If this candidate compromises infinitesimally, it increases the probability of winning by $\frac{1}{2}f(z'_j) > 0$. On the other hand, as this compromise is minor, the expected utility changes by slightly less than $\frac{1}{2}f(z'_j)[-v(|x_i - \chi^E(z'_j)|) - c(|z'_j - \chi^E(z'_j)|)] - (-v(|x_i - \chi^E(z'_j)|))$, and it is positive (or negative). This implies that if $F(.)$ has a discontinuity, it cannot be part of a continuous semi-separating equilibrium.

Assume $F(.)$ is constant on some region $[z_1, z_2]$ in the convex hull of the support. In this case, if a candidate chooses $z_1$, this candidate has an incentive to deviate to $z_2$ because the probability of winning does not change, but the implemented policy will approach the own ideal policy, so the expected utility increases. Thus, the support of $F(.)$ must be connected. □

A.6 Proposition 3

A.6.1 Define $\sigma^M$ and $\Pi$

First, a continuous semi-separating equilibrium is discussed. The value of $\sigma^M$ is decided at the point under which the extreme type’s expected utilities under $z^*_i$ and $\bar{z}_i$ are the same, that is, $V^E_i(z^*_i) = V^E_i(\bar{z}_i)$ defined by (12) and (13).

$$
\frac{1}{2} \left[ - \frac{p^M}{p^M + \sigma^M(1-p^M)} v(|x^M_j(z^*_j) - x^E_i|) - \frac{\sigma^M(1-p^M)}{p^M + \sigma^M(1-p^M)} v(|x^E_j(z^*_j) - x^E_i|) 
- v(|x^E_i(z^*_i) - x^E_i|) - c(|x^E_i(z^*_i) - z^*_i|) \right] 
= -v(|x^E_i(\bar{z}_i) - x^E_i|) - c(|x^E_i(\bar{z}_i) - \bar{z}_i|) 
$$

When $\sigma^M = 1$, the left-hand side is lower than the right-hand side because (5) holds. When $\sigma^M$ goes to 0, if the left-hand side is higher than the right-hand side, the value of $\sigma^M \in (0,1)$ under which an extreme type is indifferent between $z^*_i$ and $\bar{z}_i$ exists. The following condition means that the left-hand side is higher than the right-hand side of (23) when $\sigma^M$
goes to zero.

\[-\frac{1}{2}[v(|\chi^E_i(z^*_i) - x_i^E|) + v(|\chi^E_i(z^*_j) - x_i^E|) + c(|\chi^E_i(z^*_i) - z^*_i|)]
\geq -v(|\chi^E_i(\bar{z}_i) - x_i^E|) - c(|\chi^E_i(\bar{z}_i) - \bar{z}_i|). \quad (24)\]

First, \(-v(|\chi^E_i(z^*_i) - x_i^E|) - c(|\chi^E_i(z^*_j) - z^*_i|) > -v(|\chi^E_i(\bar{z}_i) - x_i^E|) - c(|\chi^E_i(\bar{z}_i) - \bar{z}_i|)\) since \(\bar{z}_i\) is more moderate than \(z^*_i\). Second, \(\bar{z}_i\) is the platform under which an extreme type can win over a moderate type who announces \(z^*_i\). When \(\sigma^M\) goes to zero, voters guess that a candidate announcing \(z^*_i\) is a moderate type. It means that, from the definition of \(\bar{z}_i\), an extreme type's implemented policy, \(\chi^E_i(\bar{z}_i)\), needs to be more moderate than a moderate type's implemented policy, \(\chi^M_i(z^*_i)\). From Corollary 1, a moderate type has an incentive to compromise about the implemented policy more than an extreme type, and a moderate type is indifference to win or lose at \(z^*_i\). It means that \(-v(|\chi^E_i(z^*_i) - x_i^E|) > -v(|\chi^E_i(\bar{z}_i) - x_i^E|) - c(|\chi^E_i(\bar{z}_i) - \bar{z}_i|)\). As a result, (24) holds, so if (5) holds, \(\sigma^M\) under which an extreme type is indifferent between \(\bar{z}_i\) and \(z^*_i\) exists. The value of \(\Pi\) is the same as \(V^E_i(z^*_i) = V^E_i(\bar{z}_i)\). If (5) does not hold, there does not exist such \(\sigma^M\) since \(V^E_i(z^*_i) > V^E_i(\bar{z}_i)\) for all \(\sigma^M\).

### A.6.2 The Other Bound of Support for the \(F(.)\)

At \(\bar{z}_i\), the expected utility is \(V^E_i(\bar{z}_i)\) defined by (14). If:

\[-v(|\chi^E_i(\bar{z}_i) - x_i^E|) - c(|\chi^E_i(\bar{z}_i) - \bar{z}_i|) > -v(|\chi^E_i(z_j) - x_i^E|), \quad (25)\]

\(V^E_i(\bar{z}_i)\) is higher than \(V^E_i(\bar{z}_i)\) when \(\bar{z}_i = \bar{z}_i\), so \(\bar{z}_i \neq \bar{z}_i\) in equilibrium, and it means that a continuous semi-separating equilibrium exists. If (25) does not hold, the extreme bound and the moderate bound are equivalent (a two-policy semi-separating equilibrium). Suppose that (25) holds. In equilibrium, \(V^E_i(\bar{z}_i)\) and \(V^E_i(\bar{z}_i)\) should be same, so \(\bar{z}_i\) and \(F(.)\) should satisfy the following equation. Suppose \(R\) without loss of generality.

\[-v(|\chi^E_R(\bar{z}_R) - x_R^E|) - c(|\chi^E_R(\bar{z}_R) - \bar{z}_R|) = (p^M + \sigma^M(1 - p^M))[-v(|\chi^E_R(\bar{z}_R) - x_R^E|) - c(|\chi^E_R(\bar{z}_R) - \bar{z}_R|)]
- (1 - \sigma^M)(1 - p^M) \int_{\bar{z}_L}^{\bar{z}_R} v(|\chi^E_L(z_L) - x_R^E|)dF(z_L). \quad (26)\]

I assume that two candidates’ positions are symmetric, so when \(\bar{z}_R\) decreases, \(\bar{z}_L\) increases. Then, \(V^E_R(\bar{z}_R)\) increases because \(\int_{\bar{z}_L}^{\bar{z}_R} v(|\chi^E_L(z_L) - x_R^E|)dF(z_L)\) decreases while (14) decreases.
and \( F(.) \) also adjusts the value of \( \int_{\tilde{z}_L}^{\tilde{z}_R} v(|\chi_i^E(z_L) - x_{R_i}^L|)dF(z_L) \). Thus, there exist combinations of \( \tilde{z}_i \) and \( F(.) \) which satisfy (26).

Denote \( \tilde{z}_i^E \) such that \(-v(|\chi_i^E(\tilde{z}_i^E) - x_i^E|) = -v(|\chi_i^E(\tilde{z}_i) - x_i^E|) - c(|\chi_i^E(\tilde{z}_i^E) - \tilde{z}_i^E|)\). The moderate bound, \( \tilde{z}_j \), should be more extreme than \( \tilde{z}_i^E \). If \( \tilde{z}_j \) is more moderate than \( \tilde{z}_i^E \), it means \(-v(|\chi_j^E(\tilde{z}_j) - x_j^E|) > -v(|\chi_j^E(\tilde{z}_j) - x_j^E|) - c(|\chi_j^E(\tilde{z}_j) - \tilde{z}_j|)\). Thus, an extreme type with \( \tilde{z}_i \) has an incentive to lose against an extreme-type opposition with a platform close to \( \tilde{z}_j \). Any platform in the support of \( F(.) \), say \( \hat{z}_i \), need to satisfies \(-v(|\chi_i^E(\hat{z}_i) - x_i^E|) > -v(|\chi_i^E(z_i^E) - x_i^E|) - c(|\chi_i^E(\hat{z}_i) - \hat{z}_i|)\) to avoid to deviate to lose. Therefore, \( \chi_i^E(\tilde{z}_i) \) is more extreme than \( \chi_i^E(z_i^E) \) since \( \chi_i^E(\hat{z}_i) \) is more extreme than \( \chi_i^E(z_i^E) \).

A.6.3 Define \( F(.) \)

Suppose the expected utility of \( R \) without loss of generality. Denote \( X(z'_L) = \int_{\tilde{z}_L}^{\tilde{z}_R} v(|\chi_i^E(z_L) - x_{R_i}^E|)dF(z_L) \). For any \( z'_R \in (\tilde{z}_R, \bar{z}_R) \), the expected utility should be the same as \( \Pi \). It means:

\[
F_X(z'_R) = \frac{\Pi + v(|\chi_i^E(z'_R) - x_{R_i}^E|) + c(|\chi_i^E(z'_R) - z'_R|) + (1 - \sigma^M)X(z'_L)}{(1 - \sigma^M)(1 - p^M)(v(|\chi_i^E(z'_R) - x_{R_i}^E|) + c(|\chi_i^E(z'_R) - z'_R|))}. \tag{27}
\]

The distribution function, \( F_X(.) \), is defined by (27) for any platform in the support of \( F(.) \) given \( X(z'_L) \). When \( F_X(z'_R) = 0 \), it is \( \Pi + v(|\chi_i^E(z'_R) - x_{R_i}^E|) + c(|\chi_i^E(z'_R) - z'_R|) + (1 - p^M)X(z'_L) = 0 \). This equation holds if and only if \( z'_R = \tilde{z}_R \) and \( X(z'_L) = 0 \) in order to have \( \Pi = V^E_R(\tilde{z}_R) \). If and only if \( z'_L = \bar{z}_L \), \( X(z'_L) = 0 \), so, when \( z'_R \) and \( z'_L \) goes to \( z_R \) and \( z_L \), \( F(z'_R) \) goes to zero.

When \( F(z'_R) = 1 \), it is \( \Pi = (p^M + \sigma^M(1 - p^M))(-v(|\chi_i^E(z'_R) - x_{R_i}^E|) - c(|\chi_i^E(z'_R) - z'_R|) - (1 - \sigma^M)(1 - p^M)X(z'_L) \). This equation holds if and only if \( z'_R = \bar{z}_R \) and \( X(z'_L) = \int_{\tilde{z}_L}^{\tilde{z}_R} v(|\chi_i^E(z_L) - x_{R_i}^E|)dF(z_L) \) in order to have \( \Pi = V^E_R(\bar{z}_R) \). It means that when \( z'_R \) and \( z'_L \) goes to \( z_R \) and \( z_L \), \( F(z'_R) \) goes to one.

When \( z'_L \) satisfies \( |z'_L - x_m| = |z'_R - x_m| \), that is, \( F(.) \) is symmetric for both candidates, the value of \( X(z'_L) \) increases continuously (because \( F(.) \) is continuous with connected support from Lemma 5) as \( z'_R (z'_L) \) becomes more extreme, so if the platform moves from \( z_R \) to \( \bar{z}_R \), \( F(z'_R) \) increases from zero to one. Thus, if \( F(.) \) is symmetric for both candidates, \( F_i(.) \) can be defined for \( i = L, R \).
A.6.4 An Extreme Type does not Deviate

An extreme type does not deviate to a more moderate platform than $z_i$ since the probability of winning is still one, but the cost of betray and the disutility after a win increases.

If an extreme type deviates any platform that is more extreme than $z_i^*$ or between $z_i^*$ and $\bar{z}_i$. From Assumption 1, this candidate is certain to lose, so the expected utility is:

$$-p^M v(|\chi_j^M(z_j^*) - x_i^E|) - \sigma^M (1 - p^M) v(|\chi_j^E(z_j^*) - x_i^E|)$$

$$- (1 - \sigma^M)(1 - p^M) \int v(|\chi_j^E(z_j) - x_i^E|) dF(z_j). \quad (28)$$

Subtract (28) from $V^E_i(z_i^*)$, then:

$$-v(|\chi_j^E(z_i^*) - x_i^E|) - c(|\chi_j^E(z_i^*) - z_i^*|) + \frac{p^M}{p^M + \sigma^M (1 - p^M)} v(|\chi_j^M(z_j^*) - x_i^E|)$$

$$+ \frac{\sigma^M (1 - p^M)}{p^M + \sigma^M (1 - p^M)} v(|\chi_j^E(z_j^*) - x_i^E|). \quad (29)$$

A moderate type is indifferent to win and lose at $z_i^*$, that is, (6) holds. Thus, from Corollary 1, the value of (29) is positive, and this deviation decreases the expected utility. Note that, in this case, $q = \frac{p^M}{p^M + \sigma^M (1 - p^M)}$.

A.6.5 A Moderate Type does not Deviate

suppose $R$ without loss of generality. When a moderate type chooses $z_R^*$, the expected utility is as follows:

$$\frac{1}{2} [-p^M v(|\chi_L^M(z_L^*) - x_R^M|) - \sigma^M (1 - p^M) v(|\chi_L^E(z_L^*) - x_R^M|)$$

$$- (p^M + \sigma^M (1 - p^M))[v(|\chi_R^M(z_R^*) - x_R^M|) + c(|\chi_R^M(z_R^*) - z_R^*|)]$$

$$- (1 - \sigma^M)(1 - p^M) \int_{\bar{z}_L}^{\bar{z}_L} v(|\chi_L^E(z_L) - x_R^M|) dF(z_L). \quad (30)$$

Since a moderate type is indifferent between winning and losing at $z_R^*$, a moderate type is indifference to deviate or not to any platform that is more extreme than $z_R^*$ or between $z_R^*$ and $\bar{z}_R$. The second possible deviation is to deviate to any platform in $z_R' \in [\bar{z}_R, \bar{z}_R]$. For an
Differentiate the left-hand side with respect to \( p \). From (23), the right-hand side of (32) is \((1 - p)\). Differentiate \( z \) with respect to \( x \):

\[
(p^M + \sigma^M (1 - p^M))(v(|\chi_R^E(z'_R) - x^E_R|) + c(|\chi_R^E(z'_R) - z'_R|))
- \frac{1}{2} \left[ p^M v(|\chi_L^M(z'_L) - x^M_L|) + \sigma^M (1 - p^M) v(|\chi_L^E(z'_L) - x^E_L|) \right]
+ \left( p^M + \sigma^M (1 - p^M) \right) \left( v(|\chi_R^E(z'_R) - x^E_R|) + c(|\chi_R^E(z'_R) - z'_R|) \right)
= (1 - \sigma^M) (1 - p^M) \int_{z'_L}^{z'_R} v(|\chi_L^E(z'_L) - x^M_R|) dF(z_L)
- (1 - \sigma^M) (1 - p^M) (1 - F(z'_R)) \left[ v(|\chi_R^E(z'_R) - x^E_R|) + c(|\chi_R^E(z'_R) - z'_R|) \right].
\tag{31}
\]

A moderate type has no incentive to deviate to \( z'_R \) if:

\[
(p^M + \sigma^M (1 - p^M))(v(|\chi_R^M(z'_R) - x^M_R|) + c(|\chi_R^M(z'_R) - z'_R|))
- \frac{1}{2} \left[ p^M v(|\chi_L^M(z'_L) - x^M_R|) + \sigma^M (1 - p^M) v(|\chi_L^E(z'_L) - x^E_L|) \right]
+ \left( p^M + \sigma^M (1 - p^M) \right) \left( v(|\chi_R^M(z'_R) - x^M_R|) + c(|\chi_R^M(z'_R) - z'_R|) \right)
\geq (1 - \sigma^M) (1 - p^M) \int_{z'_L}^{z'_R} v(|\chi_L^E(z'_L) - x^M_R|) dF(z_L)
- (1 - \sigma^M) (1 - p^M) (1 - F(z'_R)) \left[ v(|\chi_R^M(z'_R) - x^M_R|) + c(|\chi_R^M(z'_R) - z'_R|) \right].
\tag{32}
\]

Ignore \((1 - \sigma^M) (1 - p^M)\). Differentiate the right-hand side of the above equations with respect to \( x_R^t \), and obtain \( \int_{z'_L}^{z'_R} v'(|x_R^t - \chi_L^E(z'_L)|) dF(z_L) - (1 - F(z'_R)) v'(|x_R^t - \chi_R^E(z'_R)|) \). This is positive because the opposition’s implemented policy is further from the ideal policy compared with \( z'_R \), so the right-hand side of (31) is higher than the right-hand side of (32). From (23), at \( z'_R = \bar{z}_R \), the left-hand side of (31) is zero. From (6), the left-hand side of (32) is \((p^M + \sigma^M (1 - p^M))(v(|\chi_R^M(z'_R) - x^M_R|) + c(|\chi_R^M(z'_R) - z'_R|)) - (p^M + \sigma^M (1 - p^M))(v(|\chi_R^M(z_R^*) - x^M_R|) + c(|\chi_R^M(z_R^*) - z_R^*|)) \), so it is positive since \( z_R^* \) is smaller than \( z'_R \). Differentiate the left-hand side with respect to \( z_R^t \). Note that \( \sigma^M \) and \( z_R^* \) is already decided, so only \( z_R^t \) changes. Then, \((p^M + \sigma^M (1 - p^M)) [-v'(|x_R^t - \chi_R^E(z'_R)|)] \frac{\partial \chi_R^E(z'_R)}{\partial z_R^t} + c'(\chi_R^E(z'_R) - z'_R)) \frac{\partial \chi_R^E(z'_R)}{\partial z_R^t} - c'(\chi_R^E(z'_R) - z'_R))\). Ignore \( p^M + \sigma^M (1 - p^M) \). From Lemma 1, it is negative, that is, \(-v'(|x_R^t - \chi_R^E(z'_R)|) < 0 \). This implies that if \( z'_R \) becomes smaller, then the left-hand sides of both equations increase. The next problem is the degree of an increase. Differentiate \(-v'(\chi_R^E(z'_R) - x^M_R)) \) with respect to \( x_R^t \), then:

\[
-v''(|x_R^t - \chi_R^E(z'_R)|)(1 - \frac{\partial \chi_R^E(z'_R)}{\partial x_R^t}).
\tag{33}
\]
Differentiate (23) with respect to $x'_R$, then $0 < \frac{\partial \chi'_R(z'_R)}{\partial x'_R} = \frac{v''c'}{v'u'c' + c'u'} < 1$. Thus, the value of (33) is negative. This implies that if $x'_R$ is more extreme, the increase of the left-hand side is lower when $z'_R$ becomes smaller. At $z'_R = \bar{z}_R$, the left-hand side of (31) is lower than the right-hand side of (32). If $z'_R$ becomes more moderate, both left-hand sides increase, but an increase of (32) is higher than an increase of (31). As a result, for all $z'_R$, the left-hand side of (31) is lower than the right-hand side of (32), so (32) is satisfied.

Finally, since a moderate has no incentive to deviate to $\bar{z}_R$, a moderate type does not deviate to any policy that is more moderate than $\bar{z}_R$.

### A.6.6 A Two-Policy Semi-separating Equilibrium

When (25) does not hold, a two-policy semi-separating equilibrium exists. When an extreme type chooses $z^*_i$, the expected utility is $V^E_i(z^*_i)$ defined by (8). The expected utility when the candidate chooses $\bar{z}_i$ is $V^E_i(\bar{z}_i)$ defined by (9). When $\sigma^M = 1$, $V^E_i(\bar{z}_i)$ is higher than $V^E_i(z^*_i)$ since it is assumed that (5) holds. Denote $\bar{\sigma}^M$, which satisfies (23). If (25) does not hold, then $V^E_i(\bar{z}_i)$ is lower than $V^E_i(z^*_i)$ at $\bar{\sigma}^M$. When $\sigma^M$ increases continuously from $\bar{\sigma}^M$, $V^E_i(\bar{z}_i)$ increases and $V^E_i(z^*_i)$ decreases continuously too, so there exists $\sigma^M$ under which $V^E_i(\bar{z}_i) = V^E_i(z^*_i)$, and such $\sigma^M$ should be higher than $\bar{\sigma}^M$.

The platform $\bar{z}_i$ should be such that $\chi^E_i(\bar{z}_i)$ is between $\chi^M_i(z^*_i)$ and $\chi^E_i(z^*_i)$ if $p^M > 0$ and $\sigma^M > 0$ since, in the region, there exists a policy which voters prefer to the expected implemented policy of a candidate with $z^*_i$. Thus, $\chi^E_i(\bar{z}_i)$ is more extreme than $\chi^M_i(z^*_i)$.

An extreme type does not deviate from the same reason as in Section A.6.4. If an extreme type deviates to any platform that is more moderate than $\bar{z}_i$, the expected utility changes by $\frac{(1 - \sigma^M)(1 - p^M)}{2} [v(|\chi^E_j(\bar{z}_j) - x^E_j|) - v(|\chi^M_i(\bar{z}_i) - x^M_i|) - c(|\chi^E_i(\bar{z}_i) - \bar{z}_i|)]$. This is negative because (25) does not hold.

A moderate type does not deviate to any policy that is more extreme than $\bar{z}_i$ for the same reason as in Section A.6.5. A moderate type does not deviate to $\bar{z}_i$ if:

$$
(p^M + \sigma^M(1 - p^M))[v(|\chi^M_i(\bar{z}_i) - x^M_i|) + c(|\chi^M_i(\bar{z}_i) - \bar{z}_i|)]
- v(|\chi^M_i(z^*_i) - x^M_i|) - c(|\chi^M_i(z^*_i) - z^*_i|)]
- (1 - \sigma^M)(1 - p^M)\frac{1}{2} [v(|\chi^E_j(\bar{z}_j) - x^M_i|)
- v(|\chi^M_i(\bar{z}_i) - x^M_i|) - c(|\chi^M_i(\bar{z}_i) - \bar{z}_i|)] > 0.
$$

(34)
Since $\bar{z}_i$ is more moderate than $z_i^*$, $v(\chi^M_i(\bar{z}_i) - x_i^M) + c(|\chi^M_i(z_i) - \bar{z}_i|) - v(\chi^M_i(z_i^*) - x_i^M) - c(|\chi^M_i(z_i^*) - z_i^*|)$ is positive. For an extreme type, $v(\lambda^E_i(\bar{z}_i) - x_i^E) - v(\lambda^E_i(z_i^*) - x_i^E) - c(|\chi^E_i(\bar{z}_i) - z_i|) - v(\lambda^E_i(\bar{z}_i) - \bar{z}_i) - c(|\chi^E_i(z_i^*) - z_i^*|)$ is negative because (25) does not hold. From Corollary 1, the value of it for a moderate type is lower, so $v(\lambda^E_j(\bar{z}_j) - x_i^M) - v(\lambda^M_i(\bar{z}_i) - x_i^M) - c(|\chi^M_i(\bar{z}_i) - z_i|)$ is negative for a moderate type too. As a result, (34) is satisfied. A moderate type also has no incentive to deviate to any policy that is more moderate than $\bar{z}_i$ because $v(\lambda^E_j(\bar{z}_j) - x_i^M) - v(\lambda^M_i(\bar{z}_i) - x_i^M) - c(|\chi^M_i(\bar{z}_i) - \bar{z}_i|)$ is negative.

A.6.7 Asymmetric Equilibrium

Does there exist an asymmetric equilibrium in which candidates choose asymmetric platforms or different values of $\sigma_i^M$ or $F(.)$? First, suppose that the support of $F(.)$ is asymmetric. Then, the probability of winning is constant some regions of the support for at least one candidate, and it cannot be equilibrium from the same reason as Lemma 5. As I showed in Section A.6.3, $F(.)$ must be symmetric in equilibrium when the support is symmetric. Second, suppose that moderate type’s platforms are asymmetric. Note that a moderate type should be indifferent to win or lose in equilibrium, so assume that a moderate type is indifferent to win or lose. Note also that according to Definition 2, $z_i^*$ and $z_j^*$ are symmetric, and Assumption 2 defines an off-path belief based on such $z_i^*$. If a moderate type announces a more extreme platform than $z_i^*$, an extreme type does not have an incentive to announce it and will announce a slightly more moderate platform than a moderate type’s choice. The off-path belief is still $p^M$, so the probability of winning increases, and an extreme type still has an incentive to compromise when a moderate type is indifferent to win and lose from Corollary 1. Assume that a moderate type does not announce a more extreme platform than $z_i^*$. According to Definition 2, a moderate type is indifferent to win or lose at $z_i^*$, so if a moderate type compromises more than $z_i^*$ when a moderate-type opposition announces $z_i^*$ or more moderate platform, it means that such a moderate type will deviate to lose from Corollary 1. Therefore, asymmetric equilibrium does not exists with Assumption 2. □

A.7 Proposition 4

I already show that if and only if the equation (5) holds, semi-separating equilibria exist, and, if and only if the equation (5) holds, a pooling equilibrium exists. Therefore, a political equilibrium exists, and there is a cut-off point, $\bar{p}^M$ such that if and only if $p^M \geq \bar{p}^M$, a pooling equilibrium exists, and if and only if $p^M < \bar{p}^M$, semi-separating equilibria exist.

A moderate type never choose a mixed strategy. If a moderate type’s mixed strategy is
discrete, and a moderate type announces a more extreme platform than $z_i^*$ (or $z_i^{M*}$), then he has an incentive to deviate to compromise slightly since the probability of winning increases discretely but the cost of betrayal and the disutility increases slightly. If all strategies in a mixed strategy is more moderate than $z_i^*$ (or $z_i^{M*}$), a moderate type deviates to lose. Suppose that a mixed strategy is distributed on a continuous policy space, and the probability of winning is zero against a moderate opposition when a moderate type announces the most extreme platform in his mixed strategy. A moderate type will deviate from such a platform with zero probability of winning if a moderate type has an incentive to win. A moderate type deviates to lose from platforms in a mixed strategy if a moderate type does not have an incentive to win. In order to have a positive probability of winning at the most extreme platform in a mixed strategy, the probability of winning should be same in some platforms in a mixed strategy including the most extreme one, but, in this case, a moderate type announcing such platforms will deviate to the most extreme platform since the probability of winning does not change, but the cost of betrayal and the disutility decreases. For any extreme type’s strategy and $\sigma^M$, in equilibrium, a moderate type never chooses any platform except $z_i^*$ under Assumption 2. If a moderate type announces more moderate platform than $z_i^*$, a moderate type has an incentive to lose. If a moderate type announces more extreme platform than $z_i^*$, a moderate type has an incentive to deviate to compromise and wins with a higher probability. From Assumption 2, a moderate candidates’ platforms are symmetric in a semi-separating equilibrium. If they are asymmetric, it means that at least one of two candidates’ platform is more extreme or moderate than $z_i^*$, and they such a moderate type has an incentive to deviate. From Lemma 4, both candidates’ positions are symmetric in a pooling equilibrium.

Suppose that a moderate type chooses $z_i^*$. An extreme type never chooses any other strategy except a pooling or semi-separating equilibrium. When an extreme type announces any platform between $\bar{z}_i$ and $z_i^*$ or more extreme than $z_i^*$, an extreme type will deviate to $z_i^*$ to increase the probability of winning. If an extreme type announces more moderate platform than $\bar{z}_i$ (or $\bar{z}_i$), an extreme type will deviates to $\bar{z}_i$ to decrease the cost of betrayal and the disutility with the same probability of winning.

Finally, from Proposition 1, a separating equilibrium does not exist. Therefore, only a pooling or semi-separating equilibrium exists. □
B A Separating Equilibrium in a Special Case

This appendix briefly explains one special case in which a separating equilibrium may exist. This paper assumes that \( \frac{c'(d)}{c(d)} \) and \( \frac{v'(d)}{v(d)} \) decreases as \( d \) increases, and one or all of them is strictly decreasing. If this assumption is not satisfied, a separating equilibrium may exist, and an extreme type always wins over a moderate type. This assumption is critical to derive Lemma 3.

To be precise, the critical condition to derive Lemma 3 is (20). If the above assumption is satisfied, then (20) is also satisfied. If (20) is not satisfied, \( \chi_E^E(\hat{z}_E^E) - \chi_E^L(\hat{z}_E^L) < \chi_M^M(\hat{z}_M^M) - \chi_M^L(\hat{z}_M^L) \) if the utilities when the candidate wins and when the same-type opposition wins are the same in a separating equilibrium. In words, an extreme type will implement a more moderate policy than a moderate type does. Note that an extreme type’s platform is still more moderate than a moderate type. The median voter prefers to choose an extreme type. A moderate type does not have an incentive to pretend to be extreme since a moderate type’s optimal platform is more extreme than an extreme type’s one. Thus, it is a separating equilibrium under which an extreme type defeats a moderate type.

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