Mis-match, Re-match and Investment

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Matching Matters

Private and social payoffs to many activities depend not only on one’s own characteristics but also on those of one’s partners:

- Production
- School
- Marriage

Often characteristics endogenous, result of investments. Payoffs from future match will affect investment incentives:

- Production – skill acquisition
- School – parents, early education
- Marriage – charm school, health clubs
Sharing the Surplus

Efficient matching and investment depends on partners’ ability to share the fruits of joint production flexibly

- Assures that partners get the appropriate “marginal product”
- Investment returns correctly reflect scarcity

In many situations, sharing is problematic and payoffs instead are rigid (NTU). This project

- investigates consequences of NTU for nature of matching and investment
- provides efficiency rationale for affirmative action-type policies (AR)
- looks at effects of various policies
Associational Redistribution

- AR policies, such as affirmative action, can be understood as correcting mismatch (e.g. in form of too much segregation).
- Mismatch may lead to persistent inequality, exclusion, low productivity of some groups.
- Motivation appears in part to encourage investment, especially by the “disadvantaged”.

Sources of mismatch:

- Search costs,
- statistical discrimination, self-confirming beliefs (Coate-Loury, 1993),
- NTU (this paper).
Nontransferable Utility

Compensating partners may be costly in terms of joint surplus:

- incentive problems within relationships (moral hazard in teams, incomplete contracts)
- commitment problems: surplus sharing determined by ex-post bargaining
- reputational payoffs
- liquidity constraints
- risk aversion
- “behavioral” considerations, e.g., inequity aversion, envy.
A Basic Example: Effect of NTU on Matching

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- Payoffs $2w > W > w$, (i.e. $2W - 2w < 2w - 0$)
- $(h, \ell)$ firms form in equilibrium
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- Payoffs $2w > W > w$, (i.e. $2W - 2w < 2w - 0$)
- $(h, \ell)$ firms form in equilibrium

- Payoffs $W > w$,
- $(h, h)$ and $(\ell, \ell)$ firms form.
Effect of NTU on Market’s Scarcity Signals

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*More inequality:* TU NTU
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*More inequality:*
- TU
- NTU

*Larger return on skill:*
- TU
- NTU
Matching and Investment

If (1) TU and (2) no asymmetric info efficient equilibria exist (Cole-Mailath-Postlewaite, 2001; Felli-Roberts, 2002)
Matching and Investment

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Efficiency
Of the Match, By the Match, For the Match

Of (Mismatch due to NTU)

By (Pareto frontier $\neq$ iso-surplus)

For (Distorted investments)
Efficiency

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Labor Market and Production

- Continuum of agents with unit measure.
- Agents have type $a \in \{h, \ell\}$; $q$ is fraction of type $h$.
- Production in firms of size two: total output $y(a, a')$:

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Diversity gain: $2w > W > w$
$W < 1$ (normalization)

- Agent’s utility: income.
- **Stable matches**, match $m(a)$, payoff $u(a)$: no pair $\langle a, a' \rangle$, $a' \neq m(a)$ can obtain more than $u(a)$ and $u(a')$. 
**TU Benchmark**

Stable matches are always integrated:
- firms are of the form \( \langle h, \ell \rangle \) as much as possible,
- some of the majority type segregate,
- output is maximized.

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<tr>
<th>Wages ( w(\ell), w(h) )</th>
<th>Output</th>
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<td>( q &lt; \frac{1}{2} )</td>
<td>0, 2w</td>
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<tr>
<td>( q &gt; \frac{1}{2} )</td>
<td>2w − ( W ), ( W )</td>
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NTU – Laissez-faire

Strict NTU: output is shared equally within matches (can be relaxed). Individual payoffs are

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\[ W > w \Rightarrow \text{unique stable match is segregation, yields measures } q/2 \text{ of } (h, h) \]
and \((1-q)/2 \text{ of } (l, l) \text{ firms.}

Welfare?

- Total output is \(qW\): less than under TU, too much segregation.
- Policy: Rematching (AR) may increase output.
Achievement Based AR

- Match $h$ and $\ell$ where possible; remaining agents match in homogeneous firms.
- Payments are $w$ for agents in integrated ($\langle h, \ell \rangle$) and 0 (or $W$) for those in segregated ($\langle \ell, \ell \rangle, \langle h, h \rangle$) firms.
- Since the match replicates the TU match, aggregate surplus increases.
- This illustrates a generic justification for AR.
- Examples: workfare, Work Opportunity Tax Credit, certain European active labor market programs.
- Such policies have been criticized as bad for incentives; this is possible if $h$ and $\ell$ are endogenous.
Investments

- Suppose $\pi \in [0, 1]$ of the population can invest.
- Investing $e$, an agent incurs utility cost $e^2/2$ and has probability $e$ (independent across agents) of achieving education $h$ (with $1 - e$ achievement is $\ell$).

TU Benchmark

- Agents rationally expect payoffs from stable match, taking $q$ as given; in symmetric equilibrium $q = \pi e$.
- Agents maximize $ew(h) + (1 - e)w(\ell) - \frac{1}{2}e^2$.
- TU Equilibrium:
  - if $2w\pi < \frac{1}{2}$: $q < \frac{1}{2}$, $w(h) = 2w$, $w(\ell) = 0$ and $e = 2w$,
  - if $2(W - w)\pi > \frac{1}{2}$: $q > \frac{1}{2}$, $w(h) = W$, $w(\ell) = 2w - W$, and $e = 2(W - w)$,
  - else $q = \frac{1}{2}$, $w(h) = w + \frac{1}{4\pi}$, $w(\ell) = w - \frac{1}{4\pi}$ and $e = \frac{1}{2\pi}$. 
Laissez-faire: NTU

- Individual optimization when labor market segregates:
  \[ \max_e e W - \frac{1}{2} e^2. \]

- Optimal choice satisfies \( e^{LF} = W \).

Investment:

- \textit{Overinvestment} relative to TU if \( \pi W > \frac{1}{2} \) (since \( W > 2(W - w) \)).
- \textit{Underinvestment} if \( \pi W < \frac{1}{2} \).
Achievement Based Associational Redistribution

- Suppose $q < 1/2$ (the only equilibrium).

$$
\max_{e} e w + (1 - e) \frac{q}{1 - q} w - e^2 / 2
\Rightarrow e^A = \frac{1 - 2q}{1 - q} w < w < e^{LF}
$$

- Also: $e^A < e^{TU}$, $e^A$ decreases with $\pi$ (insurance effect).

Two effects:
- Dynamic: investment incentives decrease due to “insurance”.
- Static: improved matching though re-matching $h$ to $\ell$. 
Background

- Ramsey taxation logic: AR policy ought to aim at less elastic characteristics.
- Now agents have background $b \in \{p, u\}$, $\pi$ is fraction of $p$.
- Background can affect *investment* or *production* stage.
- Suppose for now it affects investment in that $p$ can invest, $u$ agents cannot.
- Background based policy matches $p$ with $u$ wherever possible; leaves agents free to match as they wish by achievement.
Background Based Policy – *Investment Relevant Case*

- Background is a good predictor of achievement (*u* are likelier to have outcome *ℓ* than *p* agents).
- Hence, a background based policy achieves a degree of efficient sorting by achievement (for low *π* it does as well as possible!).
- Investment incentives?
Background Based Policy – *Investment Relevant Case*

- Background is a good predictor of achievement (*u* are likelier to have outcome *l* than *p* agents).
- Hence, a background based policy achieves a degree of efficient sorting by achievement (for low *π* it does as well as possible!).
- Investment incentives?

\[ e^{LF} > e^B > e^A \]

- Reason: background based policy has no insurance effect for *p* agents.
Comparing A and B

(a) $\pi < 1/2$

(b) $\pi > 1/2$

Figure: Laissez-faire, Achievement, and Background Based Policies
Background Affects Output

Individual background matters for interaction in firms, schools, etc.

Social externalities through background may take place

- **at investment stage**: peer effects in schools $\langle b, b' \rangle$ (see paper).
- **at production stage**: firm output depends also on background $\hat{y}(a, a', b, b')$, suppose this is the case in the following.
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Individual background matters for interaction in firms, schools, etc.

Social externalities through background may take place

- at investment stage: peer effects in schools \( \langle b, b' \rangle \) (see paper).
- at production stage: firm output depends also on background \( \hat{y}(a, a', b, b') \), suppose this is the case in the following.
- Now background-dependent cost \( g(b, b') \) incurred at production, so that output is \( \hat{y} = y(a, a') - g(b, b') \) with:

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<th>( u )</th>
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Let \( F > w > f, \ F > 2f, \) and \( W - w > F - f \), i.e. convex cost (DD again) and education matters more than background.

NTU: equal sharing.
Investment Incentives?

- TU allocation: exhausts all possible \((hu, lp)\) and \((lu, hp)\) matches, investments depend on marginal social benefit (that is, on scarcity and technology).
- Laissez-faire under NTU: all types segregate, investments are \(W\) for \(p\), and \(W - F\) for \(u\).
- Since wages are independent of scarcity, for both \(W - w\) and \(\pi\) high, simultaneous overinvestment at the top, underinvestment at the bottom for \(W - w\) small and \(\pi\) high, universal underinvestment.
**AR Policy**

- Achievement based policy: match $h$ and $\ell$, allow segregation by background
- Background based policy: match $u$ and $p$, allow segregation in achievement

**Proposition** Agents invest

- $e_p^{LF} = W$ and $e_u^{LF} = W - F$
- $e_p^A \leq w$ and $e_u^A \leq w - f < W - F$
- $W > e_p^B > W - F$ and $W > e_u^B > W - F$

- Encouragement effect of affirmative action (c.f Coate-Loury 1993) for $u$ – real return to investment increased by policy
- For $p$'s incentives are reduced: the group not favored by the policy has reduced incentives, but this may be desirable
Conclusion

- Limits to redistribution of surplus within firms/schools/clubs lead to mismatch that may take the form of segregation.
- This provides independent economic rationale for AR.
- Inefficiency takes the form of overinvestment at the top, underinvestment at the bottom, or universal discouragement.
- AR as response to this has to balance (static) diversity gains and (dynamic) incentive losses.
- In the paper:
  - Endogenous matching at the investment stage (school choice),
  - allows AR at each stage (potential tension between school and labor stage intervention).
  - allows dynamic policy that conditions labor market intervention on school choice (for instance Texas Top Ten Percent Law)
- "Optimal Policy"?