Ambiguity, Information Quality and Credit Risk

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June 5, 2009
Motivation

- Observe large increases in CDS spreads in July/August 2007
Motivation

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Motivation
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[Graph showing the 10-year spread for BAC, Citi, GS, JPM, and LEH from Jan 2006 to Jan 2009.]

[Graph showing the 10-year spread for BNP Paribas, Bear Stearns, and Lehman Brothers from Oct 2007 to Apr 2009.]

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Motivation

- Observe large increases in CDS spreads in July/August 2007
- Break-down of CDX pricing after July/August 2007
Main idea

Compare two competing explanations

- Deterioration in information → increase in ambiguity about signal quality (Model 1)
- Increase in ambiguity about fundamentals (Model 2)

Model both through model misspecification
Theoretical

- Worst-case scenario →
  - High signal quality when observing a bad signal; low signal quality when observing a good signal
  - Lowest admissible growth rate for asset value
- Higher CDS (and credit) spreads
Results Preview

Calibration

- Initial (July/August 2007) increase due to increase in ambiguity about fundamentals
- Post- Bear Stearns increase due to increase in ambiguity about signal quality

- **Asset value**: \( A_t = e^{Z_t} \);

\[ dZ_t = m dt + \sigma dW_t, \quad t \geq 0 \]

- \( A_B \): exogenous default boundary; \( a = \log A_B \)

- Information set: \( \mathcal{F}_t = \sigma \{ Z_s; \ s \leq t \} \)
Asymmetric Information

- Observe imperfect signals \( \hat{A} \) at discrete dates \( t_1, \ldots, t_n \);
  \[
  Y(t) \equiv \log \hat{A}_t = Z(t) + U(t)
  \]
- \( U_t \sim N(\bar{u}, \sigma_u^2), \bar{u} = -\frac{\sigma_u^2}{2} \)
- Information set: \( G_t = \sigma \{ Y(t_1), \ldots, Y(t_n), 1_{\tau \leq s} : 0 \leq s \leq t \} \)
- Conditional distribution:
  \[
  g(z|y, z_0, t) \propto e^{-J(\tilde{y}, \tilde{z}, \tilde{z}_0)} \left[ 1 - \exp\left( \frac{-2\tilde{z}_0\tilde{z}}{\sigma^2 t} \right) \right]
  \]
Default-swap spreads

**Notation**

- **$T$**: maturity date of the swap
- **$X$**: payment if default occurs before maturity date $T$
- **$c(t, T)$**: CDS spread at time $t$ for a swap with maturity $T$
- **$\Upsilon(T - t, A_t, A_B)$**:
  \[ \mathbb{E} \left[ e^{-r(T-t)}1_{\tau \leq T} | \mathcal{F}_t \right] \]
- **$P(s)$**:
  \[ \mathbb{E} \left[ 1_{\tau > s} | \mathcal{F}_t \right] \]

**CDS calculation**

\[ c(t, T) = \frac{2X \mathbb{E} \left[ \Upsilon(T - t, A_t, A_B) | \mathcal{G}_t \right]}{\sum_{i=1}^{k} e^{-\frac{r_i}{2}} \mathbb{E} \left[ P \left( t + \frac{1}{2}i \right) | \mathcal{G}_t \right]} . \]
CDS spreads: Black and Cox [1976] model

Black and Cox [1976] model

CDS spread (in bps)

Student Version of MATLAB

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CDS spreads: Duffie and Lando [2001] model
Ambiguity Aversion

- **Utility:**

\[
U(C_t, t) = \min \mathbb{E} \left[ \int_{0}^{+\infty} e^{-ru} C_{t+u} du \mid G_t \right].
\]

- **CDS spreads:**

\[
c(t, T) = \max \frac{2X \mathbb{E}^a [\gamma(T - t, A_t, A_B) \mid G_t]}{\sum_{i=1}^{k} e^{-\frac{r_i}{2} \mathbb{E}^a [P(t + \frac{1}{2}i) \mid G_t]}}.
\]
Ambiguity about information quality

Assume: $\sigma^2_u \in [\underline{\sigma}^2_u, \overline{\sigma}^2_u]$. CDS spreads:

(1) $$c(t, T) = \max_{\sigma^2_u \in [\underline{\sigma}^2_u, \overline{\sigma}^2_u]} \frac{2X \mathbb{E}\sigma^2_u [\gamma(T - t, A_t, A_B) | G_t]}{\sum_{i=1}^{k} e^{-\frac{r_i}{2} \mathbb{E}\sigma^2_u [P(t + \frac{1}{2} i) | G_t]}}.$$ 

Proposition 2.1

The worst-case likelihood solving the problem 1 is given by:

(2) $$\tilde{\sigma}^2_u = \begin{cases} \overline{\sigma}^2_u, & y - \overline{u} > a \\ \underline{\sigma}^2_u, & y - \overline{u} < a \end{cases}$$
Optimal signal quality

**Intuition**

- $y - \bar{u} - a$: expected distance to default
- if positive, investors treat signal as imprecise
- if negative, investors treat signal as very precise
Ambiguity about fundamentals

Assume asset dynamics misspecified

- $dW_t = h_t dt + dW_t^h$

- Admissible disturbances:

  $$\Xi(m) \equiv \left\{ h(m) : \frac{1}{2} h^2(m, t) \leq \eta \ \forall \ t \geq 0 \right\}$$

Proposition 2.2

The worst-case likelihood solving the default spread problem is given by:

$$h^*(m, t) = -\sqrt{2\eta}.$$
Optimal Disturbance

**Intuition**

- Implies investors evaluate under the lowest possible drift rate for assets
- Since $m > 0$, implies that firm less likely to recover from a negative shock

Will use $\eta(m) = \eta m^2$
Numerical examples

Thought experiment

Instead of using the full history, update $z_0$ to conditional mean, period - by - period

Base Calibration

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<th>$\alpha$</th>
<th>$\sigma$</th>
<th>$\sigma_u^2$</th>
<th>$C$</th>
<th>$A_B$</th>
<th>$D$</th>
<th>$X$</th>
<th>$C/D$</th>
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<td>1%</td>
<td>5%</td>
<td>30%</td>
<td>5%</td>
<td>5%</td>
<td>8</td>
<td>78</td>
<td>129.4</td>
<td>97.38%</td>
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</table>
Two histories

- Good-good-good-good
- Good-neutral-bad-neutral
CDS spreads

Duffie and Lando [2001]

Signal ambiguity

Dynamics ambiguity

Filter

\[ \tau \]

\[ \text{CDS spread (bps)} \]

\[ \text{period} \]

\[ z_t \]

\[ \text{Filter} \]

\[ \text{DL01} \]

\[ \text{Signal} \]

\[ \text{Asset} \]

Student Version of MATLAB
CDS spreads

Duffie and Lando [2001]

Signal ambiguity

Dynamics ambiguity

Filter

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Calibrating to Financial Institutions

- Use balance sheet data of financial institutions to calibrate base model parameters (Q1 1980-Q2 2007)
- Calibrate initial $\sigma_u$, $\eta$ to match the CDS spreads July 2007
- Look at what $\sigma_u$, $\eta$ are needed to match CDS spreads August 2007, March 2008
## Parameter Calibration

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<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\delta$</th>
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<th>Recovery</th>
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<td>12.09</td>
<td>3.81</td>
<td>124.73</td>
<td>1.52</td>
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<td>3.54</td>
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<td>Citi Corp</td>
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<td>5.16</td>
<td>15.33</td>
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<td>397.03</td>
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<td>Wells Fargo</td>
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<td>10.34</td>
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9.75% recovery for senior debt in Lehman Brothers liquidation (source: Business Week)
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<td>03/28/2008</td>
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Conclusion

Alternative Explanations

- Increase in doubts about the validity of credit ratings
- Negative shocks to liquidity