Retrading in an Adverse Selection Economy

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Introduction

Basic Model

Decentralizing Incentive Efficient Allocations

Anonymous Trading

Parametric example

Conclusion
Introduction

- Rothschild-Stiglitz insurance economy with adverse selection
- Typical assumptions: contracts are exclusive and trading among agents prohibited
- Prescott - Townsend:
  - Pareto optima of adverse selection economies hard to support as a CE
  - Problems decentralizing using a price system
  - Externalities in consumption
- Alternate mechanism - anonymous mechanism - allows retrading and nonexclusivity
Basic Model

- Countable infinity of agents; 2 types \( a \) and \( b \) with fraction \( f_\eta \), \( \eta = a, b \)
- Endowment economy with perishable good \( 0 < \theta_1 < \theta_2 \)
- Conditional probability \( g_{\eta i} \) for \( \eta = a, b \) and \( i = 1, 2 \)
- Assume \( g_{a2} > g_{b2} \) so \( a \) is low risk while \( b \) is high risk
- Total endowment \( \bar{\theta} = f_a(g_{a1}\theta_1 + g_{a2}\theta_2) + f_b(g_{b1}\theta_1 + g_{b2}\theta_2) \)
- Let \( \bar{\theta}_\eta \) denote expected endowment of type \( \eta \)
Preferences - $\sum_i g_{\eta i} U(c_i)$

Net trade $X_\eta = (x_{\eta 1}, x_{\eta 2})$

Set of individually feasible net trades

$$X = \{x_\eta = (x_{\eta 1}, x_{\eta 2}) \mid \theta_i + x_{\eta i} \geq 0, i = 1, 2\}$$

Feasible in the aggregate if

$$F = \left\{(x_a, x_b) \in X^2 \mid 0 \geq \sum_\eta \sum_i f_{\eta i} g_{\eta i} x_{\eta i}\right\}.$$ \(1\)

The pair of net trades $x \in F$ is incentive compatible if

$$\sum_i g_{\eta i} U(\theta_i + x_{\eta i}) \geq \sum_i g_{\eta i} U(\theta_i + x_{hi}), \quad h \neq \eta, \quad \eta = a, b.$$ 

Let $I \subset F$ denote the set of feasible net trades that are incentive compatible.
Standard approach to decentralization

Following Bisin and Gottardi (JPE)

- Trade takes place between agents and firms; agents cannot trade among themselves
- Exclusivity in markets - enter one market only
- Quantity restrictions - a contract is a fixed bundle of contingent claims
- Set of contracts offered satisfy incentive compatibility constraints
- Contracts are exclusive
- Prices are "fair" - so contingent claim price is \( g_{\eta_i} \) in market \( \eta \) contingent on \( \theta_i \)
Retrading

- Incentive efficient allocation: one set agents is quantity constrained
- Allow retraining after agents self select into market but before $\theta$ realized
- Assume low risk are constrained $x_a^*$ and high risk have certain $c(b)$
- Retrade such that

\[ \hat{q}_a x_a^* \geq \hat{q}_a x, \]  

(2)

- Achieve constant consumption $c(a)$, but $c(a)$ doesn't satisfy incentive compatibility constraint
If \( c(a) > c(b) \), type \( b \) agents will anticipate formation of side markets or retrading and misrepresent type.

Subsequent retrading opportunities change the information revealed by an agent.

Krasa [1999] Economic Theory:

If all agents self-select into market \( A \), \( c(a) \) is no longer feasible.
Anonymous Trading

- Agents trade claims contingent on endowment
- Face identical budget constraint

\[ B(q) \equiv \{ x \in X \mid qx_1 + x_2 = 0 \} \]

- Define \( R_\eta \)

\[ R_\eta \equiv \frac{g_{\eta 1}}{g_{\eta 2}}, \quad \eta = a, b. \]

Since the agents have been labeled so that \( b \) is the “high risk” type, \( R_b > R_a \).

- For any \( q > 0 \), define the net demand functions

\[ \xi_\eta(q) \equiv \arg \max_{x \in B(q)} \sum_i g_{\eta i} U(\theta_i + x_i), \quad \eta = a, b. \]
The solution satisfies

\[ q = R_\eta \frac{U'[\theta_1 + \xi_{\eta_1}(q)]}{U'[\theta_2 + \xi_{\eta_2}(q)]} \]  

Define autarky price

\[ Q_\eta \equiv R_\eta \frac{U'(\theta_1)}{U'(\theta_2)}, \quad \eta = a, b. \]

Since \( \theta_1 < \theta_2 \), it follows that \( Q_\eta > R_\eta, \eta = a, b \) and since \( R_b > R_a \) it follows that \( Q_b > Q_a \). Hence, there are two cases,

\[ R_a < Q_a \leq R_b < Q_b, \]  

(4) \[ R_a < R_b \leq Q_a < Q_b. \]  

(5)
Properties of the demand functions $\xi_\eta$, $\eta = a, b$

(i) An agent of type $\eta$ insure (does not insure, takes on risk) if and only if $q < Q_\eta$ ($q = Q_\eta$, $q > Q_\eta$).

\[
\xi_{\eta 1}(q) \geq 0 \quad \text{as} \quad q \geq Q_\eta.
\]

(ii) An agent of type $\eta$ insure partially (fully, more than fully) if and only if $q > R_\eta$ ($q = R_\eta$, $q < R_\eta$).

\[
\theta_1 + \xi_{\eta 1}(q) \leq \theta_2 + \xi_{\eta 2}(q) \quad \text{as} \quad q \geq R_\eta.
\]

An immediate consequence of (i) and (ii) is

(iii) If $Q_a < R_b$, then full consumption insurance for high risk (type $b$) agents implies that low risk type $a$ agents take on risk. That is,

\[
Q_a < R_b \implies \xi_{a 1}(R_b) < 0.
\]
An Anonymous Trading Example

Decentralizing Incentive Efficient Allocations

Excess Demand

Relative price $q$

$a$ partially insures

"b" over insures

"b" partially insures

$a$ takes on risk

"b" partially insures

Excess Demand

Relative price $q$
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Anonymous Equilibrium

**Definition:** An *anonymous equilibrium* is a price $q^e > 0$ and net trades $(x_a^e, x_b^e) \in F$, such that

$$x_\eta^e = \xi_\eta(q^e), \quad \eta = a, b.$$

Define

$$\Xi_\eta(q) \equiv f_\eta \sum_i g_{\eta i} \xi_{\eta i}(q), \quad \eta = a, b$$

and the sum

$$\Xi(q) \equiv \sum_\eta \Xi_\eta(q).$$
\[ \Xi(q) = \sum_{\eta} f_{\eta} [g_{\eta 1} \xi_{\eta 1}(q) - g_{\eta 2} q \xi_{\eta 1}(q)] \quad (6) \]

\[ = \sum_{\eta} f_{\eta} g_{\eta 2} \xi_{\eta 1}(q) [R_{\eta} - q]. \quad (7) \]

The following preliminary result bounds the price ratio in any equilibrium.

**Lemma**

*Under assumptions (1)–(2), in any equilibrium \( R_a < q < Q_b \).*
Define $\bar{R}$ as the ratio of unconditional probabilities,

$$\bar{R} = \frac{f_aga_1 + f_bgb_1}{f_aga_2 + f_bgb_2}$$

so $R_a < \bar{R} < R_b$.

The next result strengthens the lower bound in Lemma 1.

**Lemma**

*Under assumptions (1)–(2), in any equilibrium $q > \bar{R}$.***
Theorem:
Under assumptions (1)–(2),
(i) if \( Q_a > R_b \), there exists at least one equilibrium with \( q \in (\bar{R}, R_b) \) and at least one with \( q \in (Q_a, Q_b) \), and there are no equilibria with \( q \in [R_b, Q_a] \);
(ii) if \( R_b = Q_a \), then \( q = R_b \) is an equilibrium;
(iii) if \( Q_a < R_b \) then in any equilibrium \( q \in (\bar{R}, Q_a) \cup (R_b, Q_b) \).
Preferences are logarithmic, $U(c) = \ln(c)$. The first-order condition is

$$q = R_\eta \left[ \frac{\theta_2 - \xi_{\eta_1}(q)q}{\theta_1 + \xi_{\eta_1}(q)} \right],$$  

so that $\xi_{\eta_1}(q)$ is

$$\xi_{\eta_1}(q) = \frac{R_\eta \theta_2 - q\theta_1}{q(1 + R_\eta)}.$$  

Substitute $\xi_{\eta_1}(q)$, $\eta = a, b$ into the market-clearing condition results in a quadratic equation in $q$. 

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<table>
<thead>
<tr>
<th>Allocation</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full insurance:</td>
<td>1.723</td>
</tr>
<tr>
<td>Anonymous Equil.: $(q = 1.44)$</td>
<td>1.57</td>
</tr>
<tr>
<td>Anonymous Equil.: $(q = 2.882)$</td>
<td>1.526</td>
</tr>
<tr>
<td>Rothschild-Stiglitz:</td>
<td>1.622</td>
</tr>
<tr>
<td>$EU^{ie}$</td>
<td>1.622</td>
</tr>
<tr>
<td>Autarky</td>
<td>1.525</td>
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<tr>
<td>Pooled allocation:</td>
<td>1.603</td>
</tr>
<tr>
<td>Type a</td>
<td>1.355</td>
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<tr>
<td>Type b</td>
<td>1.521</td>
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<td></td>
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<td>1.109</td>
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<td></td>
<td>1.603</td>
</tr>
</tbody>
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Table I: Comparison of Expected Utility for Several Allocations (Case I), with $\theta_1 = 2$ and $\theta_2 = 8$, $g_{a1} = 0.4$ and $g_{b1} = 0.7$, and $f_a = 0.65$. $R_a = 0.667$, $R_b = 2.333$, $Q_a = 2.667$, $Q_b = 9.333$, and $\bar{R} = 1.02$, $\bar{\theta} = 4.97$, $\bar{\theta}_a = 5.6$, and $\bar{\theta}_b = 3.8$. 

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Conclusion

- If agents can retrade, then affects information they reveal
- Risk sharing in the form of side markets will result in the anonymous equilibrium
- Markets are not separated by type, contracts are not exclusive
- Some agents are under-insured while others over insure
- Under what conditions is the anonymous mechanism coalitionally incentive compatible?