Dynamics of the price distribution in a general model of state-dependent pricing

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Motivation

- **Price rigidity**
  - Essential element of modern monetary macro models
  - But how to model price stickiness remains controversial

- **Calvo (1983)-Yun (1996): constant probability of adjustment**
  - Analytical tractability: current workhorse model for monetary policy analysis (underlying NAWM, SIGMA, GEM)
  - Lucas critique: the probability of adjustment should depend on economic conditions
  - E.g. Calvo model inappropriate for studying the optimal rate of inflation

- **Golosov-Lucas (2007): fixed menu costs as microfoundation**
  - Introduce firm-level productivity shocks
  - Calibrate the menu cost to replicate the average (absolute) size of price changes in US micro data
  - **Real effects** of nominal shocks are much weaker and less persistent than in the Calvo-Yun model
Goal of this paper

- Study the dynamics of a flexible and tractable state-dependent pricing framework which nests Calvo-Yun, GL, and other popular models

- Estimate the model to match price adjustments in US data
  - GL model is rejected because it is unable to generate small price changes (in the data roughly 30% of all price changes are less than 5%)
  - The model preferred by the data is closer to Calvo than to GL, even though the probability of adjustment depends on the value of adjustment

- Simulate distributional dynamics with the method of Reiter (2008)
  - Infinite dimensional problem in general. Solution:
    - Projection of the aggregate steady-state on a finite price-productivity grid
    - Linearization of the responses to aggregate shocks around grid points

- Plot impulse-response functions for all three models
  - In the estimated model, the real effects of monetary policy are much stronger than in GL case (but somewhat weaker than in Calvo-Yun case)
  - Near-neutrality of monetary policy in Golosov-Lucas case related to absence of small price changes
Related literature

- Partial equilibrium

- General equilibrium without idiosyncratic shocks
  - Dotsey-King-Wolman (1999)

- Strong restrictions on idiosyncratic process
  - Gertler-Leahy (2006)

- Distributional dynamics with idiosyncratic shocks
  - Golosov-Lucas (2007) fixed menu cost model
    - Strong simplifying assumptions for dynamics: constant consumption
    - Study only i.i.d. money growth shocks
  - Midrigan (2006) fixed menu cost model
    - Different model: multi-product firms, leptokurtic shocks
    - Krusell-Smith method for dynamics: assume average price is a sufficient statistic for firms’ pricing decision
  - Dotsey-King-Wolman (2008) stochastic menu cost model
    - Two types of firms: flexible and sticky price
    - Two types of shocks: normal and “extreme”
    - We fit a much finer histogram of price changes with less free parameters
1. Introduction

2. Main features of the model

3. Nesting alternative models

4. Estimation

5. Results
   - Steady-state
   - Dynamics

6. Conclusions
Main features of the model

- **Main assumption:** probability of a price change is a (non-decreasing) function of the firm’s gains from price adjustment:

\[
\lambda = \lambda(L), \quad \text{where} \quad L = V^*(A) - V(P, A)
\]

- Necessary conditions: \( 0 \leq \lambda(L) \leq 1, \quad \frac{\partial \lambda(L)}{\partial L} \geq 0 \)

- Possible interpretations:
  - stochastic menu costs
  - bounded rationality/information: mistakes happen, but big mistakes are less likely

- In particular, we assume

\[
\lambda(L) = \frac{\bar{\lambda}}{(1-\bar{\lambda})(\alpha / L)^{\xi} + \bar{\lambda}} \quad \alpha \geq 0, \; \xi \geq 0, \; 0 \leq \bar{\lambda} \leq 1
\]

- Firm-level productivity shocks

- The rest: standard New Keynesian model (e.g. Woodford, 2003)
  - Standard CES preferences, linear technology, Dixit-Stiglitz monopolistic competition
  - Central bank: AR(1) money growth shocks or a Taylor-type interest rate rule
Bellman equation:

\[ V_t = U_t + \beta E_t \left( \frac{p_t c_t}{p_{t+1} c_{t+1}} R_{t+1} \right)' \ast (V_{t+1} + \Lambda_{t+1} \ast D_{t+1}) \ast S \]

Distributional dynamics:

- **Beginning of period:**

\[ \tilde{\Psi}_t = R_t \ast \Psi_{t-1} \ast S' \]

- **Time of production:**

\[ \Psi_t = (1 - \Lambda_t) \ast \tilde{\Psi}_t + P_t \ast (1 \ast (\Lambda_t \ast \tilde{\Psi}_t)) \]

Upper case variables are \( n^p \)-by-\( n^a \) matrices. For example:

\[ v_t^{jk} \equiv v(p^j, a^k, \Omega_t) \]

\[ d_t^{jk} \equiv \max_p v(p, a^k, \Omega_t) - v_t^{jk} \]

\[ \lambda_t^{jk} \equiv \lambda(d_t^{jk} / w_t) \]
Bellman equation:

$$V_t = U_t + \beta E_t \left( \frac{p_t c_t^\gamma}{p_{t+1} c_{t+1}^\gamma} R_{t+1}' * (V_{t+1} + \Lambda_{t+1} \cdot D_{t+1}) \cdot S \right)$$

Distributional dynamics:
- Beginning of period:
  $$\tilde{\Psi}_t = R_t \cdot \Psi_{t-1} \cdot S'$$
- Time of production:
  $$\Psi_t = (1 - \Lambda_t) \cdot \tilde{\Psi}_t + P_t \cdot (1 \cdot (\Lambda_t \cdot \tilde{\Psi}_t))$$
- Remaining equations:
  $$c_t^{-\gamma} = \chi / w_t$$
  $$\nu p_t = c_t^{-\gamma} (1 - R_t^{-1})$$
  $$R_t^{-1} = \beta E_t \left( \frac{p_t c_t^\gamma}{p_{t+1} c_{t+1}^\gamma} \right)$$
  $$N_t = \sum_j \sum_k \Psi_t^{jk} \left( \frac{p_j}{p_t} \right)^{-\epsilon} c_t \div a_k$$
  $$p_t^{1-\epsilon} = \sum_j \sum_k \Psi_t^{jk} (p_j)^{1-\epsilon}$$
Bellman and distributional dynamics involve $n^p$-by-$n^a$ matrices:

- **Value function:**
  \[ v_{t}^{jk} \equiv v(p^j, a^k, \Omega_t) \]

- **Value of adjusting:**
  \[ d_{t}^{jk} \equiv \max_p v(p, a^k, \Omega_t) - v_{t}^{jk} \]

- **Probability of adjusting:**
  \[ \lambda_{t}^{jk} \equiv \lambda(d_{t}^{jk} / w_t) \]

- **Current profits:**
  \[ u_{t}^{jk} = (p^j - w_t / a^k) c_t \rho^\epsilon (p^j)^{-\epsilon} \]

- **Beginning-of-period distribution:**
  \[ \Psi_{t}^{jk} \equiv prob(\tilde{p}_{it} = p^j, a_{it} = a^k) \]

- **Distribution at time of production:**
  \[ \Psi_{t}^{jk} \equiv prob(p_{it} = p^j, a_{it} = a^k) \]
Nesting alternative models

- Calvo-Yun ($\xi = 0$)
  \[ \lambda(L) = \frac{\bar{\lambda}}{(1 - \bar{\lambda})(\alpha/\bar{L})^\xi + \bar{\lambda}} \xrightarrow{\xi \to 0} \bar{\lambda} \]

- Golosov-Lucas ($\xi = \infty$)
  \[ \lambda(L) \xrightarrow{\xi \to \infty} \begin{cases} 0, & L < \alpha \\ 1, & L \geq \alpha \end{cases} \]

- Costain-Nakov $\xi \in (0, \infty)$

- Other nested models: DKW, Woodford (2008)
**Fixed parameters** (as in GL)
- Quarterly discount factor: $\beta = 0.99$
- Risk aversion: $\gamma = 2$
- Disutility of labor: $\chi = 6$
- Money demand: $\nu = 1$
- Elasticity of substitution among goods: $\varepsilon = 7$
- Money growth factor: $\mu = 1$ (inflation in dataset $\approx 0\%$)

**Estimated parameters:** $\rho, \sigma_\varepsilon, \lambda, \alpha, \xi$
- Idiosyncratic productivity process: $\log A_{t+1} = \rho \log A_t + \varepsilon_{t+1}^A$
- Adjustment function: $\lambda(L) = \lambda / (\lambda + (1 - \lambda)(\alpha / L)^{\xi})$

**Data:** AC Nielsen (US), Midrigan (2006)

**Estimation method:** minimize objective function with two terms:
- the histogram of price changes of the model vs. the data (25 bins)
- the mean frequency of price changes of the model vs. the data
Price changes: models vs. evidence

Actual and simulated distribution of price changes

- AC Nielsen
- Model MC

Actual and simulated distribution of price changes

- AC Nielsen
- Model Calvo

Actual and simulated distribution of price changes

- AC Nielsen
- Model SDSP
Price changes: models vs. evidence

Actual and simulated distribution of price changes

AC Nielsen
Model Woodford

Size of price changes
Density of price changes

Actual and simulated distribution of price changes

AC Nielsen
Model Midrigan

Size of price changes
Density of price changes

Top 3 CPI areas
Model KK-GL67

Size of standardized price changes
Density of price changes

AC Nielsen
Model Woodford

Size of price changes
Density of price changes

AC Nielsen
Model Midrigan

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Density of price changes
### Price changes: models vs. evidence

<table>
<thead>
<tr>
<th></th>
<th>GL</th>
<th>Calvo</th>
<th>CN</th>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly frequency of changes</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>NS07</td>
</tr>
<tr>
<td>Mean absolute price change</td>
<td>18.3</td>
<td>6.4</td>
<td>10.1</td>
<td>10.5</td>
<td>VM08</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>18.8</td>
<td>8.2</td>
<td>12.1</td>
<td>13.2</td>
<td>VM08</td>
</tr>
<tr>
<td>Changes less than 5%</td>
<td>0%</td>
<td>49.7%</td>
<td>25.2</td>
<td>25%</td>
<td>VM08</td>
</tr>
<tr>
<td>Measure of state-dependence</td>
<td>1</td>
<td>0</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The probability of adjustment

\[ \lambda(L) = \frac{\lambda}{(1-\lambda)(\alpha / L)^{\xi} + \lambda} \]

\( (\sigma^2, \rho, \lambda, \alpha, \xi) = (0.005, 0.881, 0.109, 0.031, 0.290) \)
Stationary distribution and policy

**Density of firms after mc shock and inflation**

- Log relative price
- Log (inverse) productivity

**Stationary density of firms**

- Log relative price
- Log (inverse) productivity

**Optimal price policy**

- Log target relative price
- Log (inverse) productivity

**Density of adjusting firms**

- Log relative price
- Log (inverse) productivity
In many contexts individual shocks are bigger than aggregate shocks

- Aggregate inflation volatility: 0.25% quarterly
- Average monthly absolute price change in microdata: 10.5% (AC Nielsen)

Computational implications (Reiter, 2008, *JEDC forthcoming*):

- **Individual choice** requires **nonlinear solution**
- **Aggregate dynamics** can be **linearized**

**Step 1: Nonlinear solution of the aggregate steady state**

- Solve steady-state problem by projection on a **finite price-productivity grid**

**Step 2: Linearize the aggregate responses around the grid points**

- Viewed **point-by-point**, the Bellman equation is just a set of first-order difference equations
- Viewed **point-by-point**, distributional dynamics are just a set of first-order difference equations
- **Many** equations but standard QZ solution approach applicable (e.g. Klein)
Uncorrelated money growth shock
Inflation decompositions

- **Inflation identity**

  Desired price change

  \[ \pi_t = \int \int x_i \lambda_t \Psi_t \]

  Distribution after shock

  Probability of adjustment

- **Klenow and Kryvtsov’s decomposition**
  - **Intensive and extensive margin**

  \[ \pi_t = \int \int x_i \lambda_t \Psi_t = \int \int x_i \lambda_t \Psi_t \int \lambda \Psi_t = av_i fr_t \]

  \[ \frac{\partial \pi_t}{\partial \mu_t} = \frac{\partial av_i}{\partial \mu_t} fr_t + \frac{\partial fr_i}{\partial \mu_t} av + h.o.t \]

  Intensive margin

  Extensive margin
Inflation decompositions

- Inflation identity
  - Desired price change
    \[ \pi_t = \int \int x_t \lambda_t \Psi_t \]
    \[ x_t^{jk} = \log \left( \frac{p_t^* (a^k)}{p^j} \right) \]
  - Distribution after shock
  - Probability of adjustment

- Our decomposition
  - Intensive margin, extensive margin, and selection effect
    \[ \pi_t = \int \int x_t \tilde{\Psi}_t \int \lambda_t \tilde{\Psi}_t + \int \int x_t (\lambda_t - \int \lambda_t \tilde{\Psi}_t) \tilde{\Psi}_t = av_t^* fr_t + sel_t \]
    \[ \frac{\partial \pi_t}{\partial \mu_t} = \frac{\partial \int \int x_t \tilde{\Psi}_t}{\partial \mu_t} \int \lambda \tilde{\Psi} + \frac{\partial \int \lambda_t \tilde{\Psi}_t}{\partial \mu_t} \int \int x \tilde{\Psi} + \frac{\partial \int \int x_t (\lambda_t - \int \lambda_t \tilde{\Psi}_t) \tilde{\Psi}_t}{\partial \mu_t} + h.o.t. \]
Fixed menu costs imply a **strong selection effect**: Firms that adjust are far from their optimal price.

**Shock redistributes mass from price decreases to price increases**
- Large change in average price adjustment, $\Delta \bar{a}_t$
- Even if small change in average desired price adjustment, $\Delta \bar{a}_t^*$

**Depends on firms jumping from** $\lambda_{jk}^k = 0$ to $\lambda_{jk}^k = 1$
- Such strong state dependence rejected by estimate of $\lambda(L)$
- In estimated flexible model, many adjusters are near optimal price

- Selection effect smaller (1/3 of change in inflation)
  - Average adjustment similar to average desired adjustment: $\Delta av_t \approx \Delta av_t^*$

- Shock falls less on inflation, and more on output
Correlated money growth shock
A “Phillips curve” regression

$$\Pi_t = \alpha_1 + \alpha_2 \mu_t + \varepsilon_t^1$$

$$\log(C_t) = \beta_1 + \beta_2 \log(\Pi_t) + \varepsilon_t^2$$

$$\phi(\mu_t = 0) \quad \mu_t - \mu = \phi_\mu (\mu_t - \mu) + \varepsilon_t, \quad \mu_t = M_t / M_{t-1}$$

<table>
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<tr>
<th>Uncorrelated money growth shocks</th>
<th>GL</th>
<th>Calvo</th>
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<tbody>
<tr>
<td>Std dev money growth shock (x100)</td>
<td>0.52</td>
<td>1.05</td>
<td>0.81</td>
</tr>
<tr>
<td>Std dev quarterly inflation (x100)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>% explained by nominal shock</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Std dev quarterly output growth (x100)</td>
<td>0.13</td>
<td>0.72</td>
<td>0.49</td>
</tr>
<tr>
<td>% explained by nominal shock</td>
<td>26%</td>
<td>142%</td>
<td>96%</td>
</tr>
<tr>
<td>Slope of the Phillips curve ($\beta_2$)</td>
<td>0.40</td>
<td>4.64</td>
<td>2.59</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.01</td>
<td>0.25</td>
<td>0.09</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.70</td>
<td>0.19</td>
<td>0.32</td>
</tr>
</tbody>
</table>
A “Phillips curve” regression

\[ \Pi_t = \alpha_1 + \alpha_2 \mu_t + \varepsilon_t^1 \]

\[ \log(C_t) = \beta_1 + \beta_2 \log(\Pi_t) + \varepsilon_t^2 \]

\[(\phi_\mu = 0.8) \quad \mu_t - \mu = \phi_\mu (\mu_t - \mu) + \varepsilon_t, \quad \mu_t = M_t / M_{t-1} \]

<table>
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<th>Correlated money growth shocks</th>
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<th>Calvo</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Std dev money growth shock (x100)</td>
<td>0.11</td>
<td>0.21</td>
<td>0.16</td>
</tr>
</tbody>
</table>

| Std dev quarterly inflation (x100) | 0.25 | 0.25  | 0.25 |
| % explained by nominal shock | 100% | 100%  | 100% |

| Std dev quarterly output growth (x100) | 0.51 | 0.67  | 0.47 |
| % explained by nominal shock | 29%  | 131%  | 91%  |

| Slope of the Phillips curve (\beta_2) | 0.48 | 4.58  | 2.40 |
| Standard error | 0.01 | 0.05  | 0.00 |
| R^2 | 0.84 | 0.86  | 0.99 |
Interest shock with Taylor rule
Tech shock with Taylor rule
Near-permanent disinflation

- Shock process
- Inflation
- Nominal interest rate
- Extensive margin
- Selection effect
- Intensive margin
- Price dispersion
- Real interest rate
- Real money holdings
- Real wage
- Consumption
- Labor

Graphs showing the evolution of various economic indicators over time.
Conclusions

- Reiter’s (2008) method makes state-dependent pricing DSGE models an accessible and promising area for future research.

- When fitting a flexible SDP model to US micro data, we find that nominal shocks have strong real effects, close to the Calvo model.

- Near-neutrality of the GL model is related to an extreme selection effect:
  - Both the excessive selection effect and the lack of small price changes reflect an unrealistic degree of state dependence of fixed menu cost models with large idiosyncratic shocks.

- Other findings:
  - A positive aggregate productivity shock causes labor to fall (like standard NK).
  - Price dispersion is quantitatively important (unlike standard Calvo model).
  - Near permanent disinflation causes a contraction (unlike standard Calvo).