A DSGE model of the term structure with regime shifts

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Motivation (I)

long run interest rate - short run interest = inflation expectations + risk premium, difficult to disentangle
Motivation (II)

- Including yields as sources of information is also a powerful test of macro-models.
- Linearised models, however, appear to be inconsistent with yields at a basic level. They imply that:
  - the unconditional slope of the term structure should be zero
  - any change in the slope of the yield curve should reflect changes in expectations
Motivation III: why a DSGE model?

- To understand the relationship between interest rates, monetary policy and macroeconomic fundamentals
- Traditional microfounded models do not match basic features of yields: puzzles on sign/size of slope; variance of long yields.
- Inspiration from results of non-microfounded macro-finance literature (Ang Piazzesi 2003). Yields dynamics can be explained in terms of macro dynamics
In this paper

- We explore ability of a small microfounded model to match both macroeconomic and term structure data

- Key features of the model:
  - solved and estimated up to a second-order approximation to allow for non-zero term-premia;
  - heteroskedastic shocks (MS) to:
    - accommodate established features of macro data (e.g. monetary experiment)
    - generate occasional boosts in structural variances
    - produce time-variation in risk premia
Considerable support (e.g. residuals diagnostics) for a specification with heteroskedastic shocks. This is the case for both linear and nonlinear specifications.

Estimated regimes have intuitively appealing features: "monetary experiment", Great moderation, cyclical features.

The quadratic model with regime switches generates considerable variation in risk premia.

Needed "exotic" preferences.
A consumption-based model

Notation

Stochastic discount factor

\[ Q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \]

Short rate: \( I_t \)

\[ I_t^{-1} = E_t Q_{t,t+1} \]

State variables: \( z_t \)

\[ z_{t+1} = \rho z_t + \sigma u_{t+1} \]

Market price of risk: \( \zeta_t \)
Risk premia

- Linear approximation $\xi_t \equiv 0$
- Quadratic approximation + constant variance: $\xi_t \equiv -\phi \sigma$
- Quadratic approximation + heteroskedasticity: $\xi_t \equiv -\phi \tilde{\sigma}_t$
Related Literature

- Theoretical DSGE models: Hördahl, Tristani and Vestin (2008), Ravenna and Seppala (2007a, b), Rudebusch, Sack and Swanson (2007); Rudebusch and Swanson (2007, 2008)
A DSGE model

- Temporary utility
  \[ u_t = (C_t - hC_{t-1}) \cdot v(N_t) \]

- Intertemporal aggregator (Epstein and Zin, 1989)
  \[ U\left[u_t, \left(E_t V_{t+1}^{1-\gamma}\right)\right] = \left\{ (1 - \beta) u_t^{1-\delta} + \beta \left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\delta}{1-\gamma}} \right\}^{\frac{1}{1-\delta}} \]

- \(\gamma = RRA\) and \(\delta^{-1}\) = elasticity of mg utility (when \(\gamma/\delta = 1\) then we have power utility)

- Stochastic discount factor
  \[ Q_{t,t+1} = \beta \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \frac{1}{\pi_{t+1}} \left( \frac{E_t J_{t+1}^{1-\gamma}}{J_{t+1}} \right)^{\gamma-\delta} \]
Consumption process

- Obtained endogenously as
  \[
  \hat{c}_t = F_c \hat{z}_t + \frac{1}{2} \hat{z}_t' E_c \hat{z}_t + k_{c,s} \sigma^2
  \]
  where \( z'_t = [(x_t)', (s_t)']' \),

- Exact structure endogenously from macro model with technology
  \[
  Y_t = A_t L_t^\alpha
  \]

- Aggregate resource constraint
  \[
  Y_t = C_t + G_t
  \]

- Sticky prices (Calvo pricing or quadratic adjustment + partial indexation)

- Monetary policy rule
  \[
  i_t = \text{const} + \psi_{\Pi} (\pi_t - \pi_t^*) + \psi_Y (y_t - y) + \rho_i i_{t-1} + \varepsilon_{i,t+1}
  \]
Shocks

- Cost push shock with constant variance
  \[ \varepsilon_{t+1}^\mu \sim N(0, \sigma_\mu) \]

- Shocks potentially with MS variances:
  \[ \varepsilon_{t+1}^A \sim N(0, \sigma_{a,Y,t}) \]
  \[ \varepsilon_{i,t+1} \sim N(0, \sigma_{i,s_l,t}) \]
  \[ \varepsilon_{G,t+1} \sim N(0, \sigma_{c,s_G,t}) \]

  plus

  \[ \Pi_t^* = \Pi_{i,L}^*, \Pi_{i,H}^* \]

- Intuition: "great moderation", "monetarist experiment", cyclical volatility

- Schorfheide (2005), Sims and Zha (2006).
Content of the DSGE model

- **y**, control variables (output, inflation, interest rates at different maturities)
- **x**, state variables (exogenous or predetermined): lagged ys, technology, demand shock, inflation target, monetary policy shock
- **s_t** Markov Switching processes
- innovations
- measurement errors
Solution of the DSGE model

- **Structural form**
  \[
  E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0
  \]

- **Reduced form**
  \[
  y_t = g(x_t, \sigma) \\
  x_{t+1} = h(x_t, \sigma) + \zeta(x_t) \sigma w_{t+1}
  \]

- unknown functions to be approximated
- Taylor expansion around non stochastic steady state: linear, quadratic, or higher order.
The approximate solution

\[
\hat{\lambda}_t = \mathbf{F}_\lambda \hat{\mathbf{z}}_t + \frac{1}{2} \hat{\mathbf{z}}_t'E_\lambda \hat{\mathbf{z}}_t + k_{\lambda,s}\sigma^2
\]

\[
\hat{\pi}_t = \mathbf{F}_\pi \hat{\mathbf{z}}_t + \frac{1}{2} \hat{\mathbf{z}}_t'E_\pi \hat{\mathbf{z}}_t + k_{\pi,s}\sigma^2
\]

Given price of a bond of maturity \( n \), \( \hat{b}_{t,n} \). Expected excess holding period return is

\[
\hat{hpr}_{t,n} - \hat{i}_t = \sigma^2 \mathbf{b}_{n-1,z} \mathbf{\zeta}'_{t} \mathbf{\zeta}_t \left( \mathbf{F}'_{\pi} - \mathbf{F}'_{\lambda} \right)
\]

where \( \mathbf{\zeta}'_{t} \mathbf{\zeta}_t \) is the state-dependent conditional variance-covariance matrix of vector \( \mathbf{z}_t \)
Risk premia

- "Prices of risk" change across regimes

\[ \xi_t \equiv \sigma \zeta_t' \left( F_{\pi}' - F_{\lambda}' \right) \]

- Partition \( \zeta_t' = \left[ (\xi_t^x)', (\xi_t^s)' \right]' \) to write regime-switching prices of risk

\[ \xi_t^x = \sigma \left( \zeta_t^x \right)' \left[ (F_{\pi}^x)' - (F_{\lambda}^x)' \right] \]

prices of regime-switching risk

\[ \xi_t^s = \sigma \left( \zeta_t^s \right)' \left[ (F_{\pi}^s)' - (F_{\lambda}^s)' \right] \]
A nonlinear- non Gaussian state space

• define $z_t = [x_t', s_t']'$ where $x_t$ continuous and $s_t$ discrete
• Convenient to rewrite the model as

$$
\begin{align*}
    y_{t+1}^o &= c_s + C_{1,s}x_{t+1} + C_2vech(x_{t+1}x_{t+1}') + Dv_{t+1} \\
    x_{t+1} &= a_s + A_{1,s}x_t + A_2vech(x_tx_t') + B_s w_{t+1} \\
    s_t &\sim \text{Markov switching}
\end{align*}
$$

• Time-variation in intercept and slope coefficients
Bayesian inference (I)

- use sequential MC to get likelihood to use in Metropolis-Hastings (not the only way); alternatives
Bayesian inference (II): a simpler alternative

- Assuming as many structural and measurement shocks as observables,
  - conditional on a state $s$, invert observation equation to back-out shocks
  - apply Hamilton’s filter to integrate out discrete latent variables
- Procedure immediately applicable to the linear case

$$y_{t+1}^o = C_1 x_{t+1} + D w_{t+1}$$

- In the quadratic case we invert measurement equation numerically
Data

- Quarterly US data: 1966:q1 to 2006:q2
- GDP, GDP deflator, 3-month nominal interest rate and yields on 3-year and 10-year zero-coupon bonds
- "Measurement errors" characterize the 10-year yield
## Parameter estimates

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<tr>
<th>Parameter</th>
<th>post mean</th>
<th>post sd</th>
<th>prior mean</th>
<th>prior sd</th>
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<td>$\sigma_{I,1}$</td>
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<td>0.0009</td>
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</table>
Fit
Holding period returns

![Filtered premia graph](chart1)

![Smoothed premia graph](chart2)
Conclusions

- We find considerable support for a macro-yield curve model with regime switching variances:
  - different regimes help capturing residuals’ heteroskedasticity
  - and to generate time-variability in risk premia
  - estimated regimes are intuitively appealing

- To do: more systematic comparison of various models, analysis of yield premia
THANK YOU