What Do Asset Returns Imply About the Gains from International Risk Sharing?

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Important Question in International Finance =>
Gains from International Risk Sharing

Important Differences in Literature:

1. Consumption-Based Estimates of Gains => Typically small
   - Cole and Obstfeld (1991): approx 0.1% of consumption
   - Lewis (1999) and others: up to 1-2%

2. Asset Market-Based Estimates of Gains => Typically large
   - Obstfeld (1994 AER): approx 80% - 200%
   - Lewis (2000) and others: With equity returns approx 50-80%

Problem: Consumption growth is low and smooth, while equity returns are high and volatile (Equity Premium Puzzle) => Large disparity between in welfare gains measured from consumption versus asset prices
Progress to resolve equity premium puzzle for the US:

1. "Long Run Risk" (Bansal-Yaron 2004)
Progress to resolve equity premium puzzle for the US:

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   - Small persistent component of consumption variability
Recent Resolution in Domestic Literature

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Asset Returns & Gains from Intl Risk Sharing
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1. "Long Run Risk" (Bansal-Yaron 2004)
   - Small persistent component of consumption variability
   - Stochastic volatility

2. "Habit Persistence" (Campbell-Cochrane 2000)
   - Surplus Ratio engineered to match asset pricing data
This paper

- Address disconnect between international consumption-based gains and asset return moments
  - Use benchmark international risk sharing framework: Welfare gains of going from Autarky to Open Economy with Complete Markets
  - Combine with consumption-based asset pricing model, for now Long Run Risk, that can account for asset return puzzles
  - Match first and second moments of returns for set of 7 industrialized countries
  - Empirically evaluate model < still working on >
A. Long Run Risk in International Data
Outline

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B. Sketch the framework of the Model
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   1. Autarky
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   2. Open Economy with Complete Markets
C. Benchmark International Model with Two Countries
D. Preliminary Welfare Gains based upon International Data
E. Conclude and Further Research
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A. Long Run Risk in International Data

Equity Premium in International Data

**Table: Summary Statistics (in Annual %)**

<table>
<thead>
<tr>
<th></th>
<th>AUS</th>
<th>CAN</th>
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<th>GER</th>
<th>JAP</th>
<th>UK</th>
<th>US</th>
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<tbody>
<tr>
<td><strong>Asset Pricing Data: Campbell 1970-1999</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$E(R_m)$</td>
<td>3.55</td>
<td>5.43</td>
<td>8.73</td>
<td>7.73</td>
<td>4.96</td>
<td>7.92</td>
<td>6.93</td>
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<tr>
<td>$\sigma(R_m)$</td>
<td>22.60</td>
<td>17.28</td>
<td>22.51</td>
<td>19.81</td>
<td>21.77</td>
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<td>$E(R_f)$</td>
<td>2.06</td>
<td>2.69</td>
<td>2.42</td>
<td>2.61</td>
<td>1.24</td>
<td>1.28</td>
<td>1.46</td>
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<tr>
<td>$\sigma(R_f)$</td>
<td>2.49</td>
<td>1.77</td>
<td>1.69</td>
<td>1.32</td>
<td>2.17</td>
<td>2.92</td>
<td>1.53</td>
</tr>
<tr>
<td>$E(R_m - R_f)$</td>
<td><strong>1.49</strong></td>
<td>2.74</td>
<td>6.31</td>
<td>5.12</td>
<td>3.72</td>
<td>6.65</td>
<td><strong>5.47</strong></td>
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</table>

<table>
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</thead>
<tbody>
<tr>
<td><strong>Log Consumption Growth Data: PWT 1950-2000</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(g_c)$</td>
<td>2.17</td>
<td>1.90</td>
<td>3.12</td>
<td>2.85</td>
<td>4.90</td>
<td>2.17</td>
<td>2.29</td>
</tr>
<tr>
<td>$\sigma(g_c)$</td>
<td>3.51</td>
<td>2.05</td>
<td>3.28</td>
<td>3.86</td>
<td>3.35</td>
<td>1.86</td>
<td><strong>1.89</strong></td>
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<tr>
<td>$\rho_1(g_c)$</td>
<td>-0.074</td>
<td>0.236</td>
<td>0.110</td>
<td>0.164</td>
<td>0.552</td>
<td>0.323</td>
<td>0.188</td>
</tr>
</tbody>
</table>

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Asset Returns & Gains from Intl Risk Sharing
A. Long Run Risk in International Data

CRRA preferences with iid consumption growth can not match observed asset pricing data (Mehra and Prescott)

Table: Asset Pricing and Long Run Risk for the US

<table>
<thead>
<tr>
<th></th>
<th>Mehra/Prescott</th>
<th>Bansal/Yaron</th>
<th>Lewis/Liu</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma(g_c)$</td>
<td>3.6</td>
<td>n/a</td>
<td>2.93</td>
</tr>
<tr>
<td>$\rho_1(g_c)$</td>
<td>-0.14</td>
<td>n/a</td>
<td>0.49</td>
</tr>
<tr>
<td>$\sigma(g_d)$</td>
<td>n/a</td>
<td>n/a</td>
<td>11.49</td>
</tr>
<tr>
<td>$\rho_1(g_d)$</td>
<td>n/a</td>
<td>n/a</td>
<td>0.21</td>
</tr>
<tr>
<td>$E[R_m - R_f]$</td>
<td>6.18</td>
<td>1.42</td>
<td>6.33</td>
</tr>
<tr>
<td>$\sigma(R_m)$</td>
<td>n/a</td>
<td>n/a</td>
<td>19.42</td>
</tr>
<tr>
<td>$E(R_f)$</td>
<td>0.80</td>
<td>12.71</td>
<td>0.86</td>
</tr>
<tr>
<td>$\sigma(R_f)$</td>
<td>n/a</td>
<td>n/a</td>
<td>0.97</td>
</tr>
</tbody>
</table>
B. Model

Basic Framework:
- For each country $j$, there’s a representative agent with Epstein-Zin-Weil Preferences: with Risk Aversion $\gamma$, IES $\psi$, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$

$$U(C^j_t, E_t[U(C^j_{t+1})]) = \left\{ (1-\delta)(C^j_t)^{\frac{1-\gamma}{\theta}} + \delta \left( E_t \left[ U(C^j_{t+1}, E_{t+1}[U(C^j_{t+1})])^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}$$

- The agent is endowed with a log consumption growth process with a small, but persistent, Long Run Risk component

$$g^j_{c,t+1} = \mu^j + \chi^j_t + \sigma^j \eta^j_{t+1}$$
$$x^j_{t+1} = \rho x^j_t + \sigma^j \phi^j e^j_{t+1}$$
$$g^j_{d,t+1} = \mu^j_d + \phi^j x^j_t + \phi^d \sigma^j u^j_{t+1}$$

where $\eta^j_{t+1}, u^j_{t+1}, e^j_{t+1} \sim N.i.i.d.(0,1)$
B. Solution in Autarky Model

Solution Method for Long Run Risk Model:

- Conjecture a form for $\log\left(\frac{P}{C}\right) = \log(Z) = z : z^j_t = A_0^j + A_1^j x^j_t$
- Use Campbell-Shiller approximation for returns
  
  $$r^j_{t+1} = k^j_0 + k^j_1 z^j_{t+1} - z^j_t + g^j_{c,t+1}$$

- Substitute into the Euler equation for country j for any return $\ell$:
  
  $$E_t \left\{ \delta^\theta \left( C^j_{t+1} / C^j_t \right) \left( -\frac{\theta}{\psi} \right) \left( R^j_{t+1} \right)^{\theta-1} R^\ast_{t+1} \right\} = 1 \tag{1}$$
  
- Using the two assumptions in the Euler equation implies solutions for $A_0^j, A_1^j$ in terms of preference and consumption process parameters
- With solution to $z_{c,t} = \log(P_t / C_t)$, compute the value function:
  
  $$V_t(C_t, W_t) = \left[ (1 - \delta) - \psi \left( \frac{C_t}{W_t} \right) \right]^{1-\psi} W_t$$

  $$= (1 - \delta)^{\frac{-\psi}{1-\psi}} \left( 1 + Z_{c,t} \right)^{\frac{\psi}{\psi-1}} C_t$$
B. Model: Open Economy with Complete Markets

- Each country sells off rights to own endowment stream valued at world markets, $\bar{p}_t^{j+1}$, and buys shares of other country’s endowment streams at $\bar{p}_t^{l+1}$

In Equilibrium:

- Each country will want to hold same shares, ie. buy into a World Mutual Fund at the price $\bar{p}_t^w$
- Consume a portion of the World Mutual Fund output $= \frac{\bar{p}_t^{j+1}}{\bar{p}_t^w}$
- All endowment streams are priced by a World Rep Agent, and must satisfy the Euler equation:

$$E_t \left\{ \delta^\theta \left( \frac{C_t^w}{C_t} \right) \left( -\frac{\theta}{\psi} \right) \left( \bar{R}_t^w P \right)^{(\theta-1)} \bar{R}_t^\ell \right\} = 1$$

(2)
Following similar steps to the Autarky case, conjecture and solve for
\[ \log(\bar{Z}^j) = \bar{z}^j: \bar{z}^j_t = A_0^j + A_1^j x^j_t \]

Use model to rewrite value function in terms of price to consumption ratio as before, but in open economy, \( \bar{Z}^j_{c,t} \)

Solution to the Bellman Equation under open markets is:
\[ V_t(C^j_t, W^j_t) = (1 - \delta)^{\frac{-\psi}{1-\psi}} (1 + Z^w_{c,t})^{\frac{\psi}{\psi-1}} w^j Y^w_t \]  \hspace{1cm} (3)

where \( w^j = \frac{P^j_{t+1}}{P^{iw}_{t+1}} \)
With the solutions for $z^A_t$, $\bar{z}^w_t$, $\bar{z}^j_t$, we can compute welfare gains using:

\[
(1 + \Delta) = \frac{V_0(C^j, W^j)}{V_0(C^j, W^j)} = \left\{ \frac{1 + Z^w_{c,t}}{1 + Z^j_{c,t}} \right\} \left\{ \frac{C^j}{C^j} \right\} \\
= \left\{ \frac{1 + \exp(A^w_0)}{1 + \exp(A^j_0)} \right\} \left( \frac{\bar{Y}^w_0}{\bar{Y}^j_0} \right)
\]

where $w^j = (\bar{P}^j_{t+1} / \bar{P}^w_{t+1})$ and $\bar{Y}^w_0$ is Aggregate Per Capita World Endowment

Two Alternatives to Price Weighted Gains:

- Equally weighted gains: $\Delta$ when $w^j = w^i$, $\forall i, j$

- Reservation gains: $\Delta^j$ when all other country $i(s)$ consume $w^j$ such that $\Delta^i = 0$
C. Welfare Gains with Two Country Example

Assume: \( \sigma(\eta_t^j, \eta_t^i) = 0.5, \sigma(e_t^j, e_t^i) = 0, \beta = 0.987, \gamma = 10, \psi = 1.5, \mu = 0.15, \sigma = 0.78, \varphi_e = 0.044, \rho = 0.979 \)

\[
\begin{align*}
    g_{c,t+1}^j &= \mu^j + x_t^j + \sigma^j \eta_{t+1}^j \\
    x_{t+1}^j &= \rho x_t^j + \sigma^j \varphi_e e_{t+1}^j
\end{align*}
\]

**Table:** Two Country Welfare Gains w/ Individual LRR (in Annual %)

<table>
<thead>
<tr>
<th>Equal Wgt</th>
<th>Reserve 1</th>
<th>Reserve 2</th>
<th>Price Wgt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gain</td>
<td>Alloc</td>
<td>Gain</td>
</tr>
<tr>
<td><strong>Symmetric:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country 1</td>
<td>66.7%</td>
<td>1.00</td>
<td>0.0%</td>
</tr>
<tr>
<td>Country 2</td>
<td>66.7%</td>
<td>1.00</td>
<td>233.4%</td>
</tr>
<tr>
<td><strong>Different ( \sigma ): ( \sigma_2 = 1.10 \times \sigma_1 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country 1</td>
<td>55.2%</td>
<td>1.00</td>
<td>0%</td>
</tr>
<tr>
<td>Country 2</td>
<td>90.9%</td>
<td>1.00</td>
<td>158.9%</td>
</tr>
<tr>
<td><strong>Different ( \mu ): ( \mu_2 = 1.10 \times \mu_1 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country 1</td>
<td>78.8%</td>
<td>1.00</td>
<td>0%</td>
</tr>
<tr>
<td>Country 2</td>
<td>59.1%</td>
<td>1.00</td>
<td>129.2%</td>
</tr>
</tbody>
</table>

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### Table: Estimated Monthly Parameters by Country w/o SV (in %)

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<th>JAP</th>
<th>UK</th>
<th>US</th>
<th>BY-US</th>
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<tbody>
<tr>
<td><strong>Calibrated</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.181</td>
<td>0.158</td>
<td>0.260</td>
<td>0.238</td>
<td>0.408</td>
<td>0.181</td>
<td>0.191</td>
<td>0.150</td>
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<tr>
<td>$\mu_g$</td>
<td>0.053</td>
<td>-0.035</td>
<td>-0.036</td>
<td>0.021</td>
<td>-0.191</td>
<td>0.062</td>
<td>0.124</td>
<td>0.150</td>
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<tr>
<td><strong>Individual LRR</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\sigma_j$</td>
<td>1.050</td>
<td>0.570</td>
<td>0.970</td>
<td>1.430</td>
<td>1.600</td>
<td>0.720</td>
<td>0.490</td>
<td>0.780</td>
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<tr>
<td>$\rho$</td>
<td>0.979</td>
<td>0.979</td>
<td>0.979</td>
<td>0.979</td>
<td>0.979</td>
<td>0.979</td>
<td>0.979</td>
<td>0.979</td>
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<tr>
<td>$\varphi_e$</td>
<td>0.034</td>
<td>0.051</td>
<td>0.051</td>
<td>0.021</td>
<td>0.045</td>
<td>0.119</td>
<td>0.200</td>
<td>0.044</td>
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<tr>
<td>$\varphi_e \times \sigma_j$</td>
<td>0.035</td>
<td>0.029</td>
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<td>0.073</td>
<td>0.086</td>
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<td>$\phi$</td>
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<td>3.50</td>
<td>3.50</td>
<td>3.50</td>
<td>3.50</td>
<td>3.00</td>
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<td>$\varphi_d$</td>
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<td>2.9</td>
<td>1.6</td>
<td>4.5</td>
<td>3.8</td>
<td>4.5</td>
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*Preference Parameters used: $\beta = 0.987$, $\gamma = 10$, $\psi = 1.5$*
### Mult-Country Welfare Gains

#### Table: Multi-Country Welfare Gains

<table>
<thead>
<tr>
<th>Country</th>
<th>Eq Wtd Gains</th>
<th>Reserve Share</th>
<th>P/C Ranking</th>
<th>Gains</th>
<th>Pr Wtd Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>364.8%</td>
<td>0.22</td>
<td>3</td>
<td>0.0%</td>
<td>n/a</td>
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<tr>
<td>CAN</td>
<td>211.0%</td>
<td>0.32</td>
<td>2</td>
<td>0.0%</td>
<td>n/a</td>
</tr>
<tr>
<td>FRA</td>
<td>440.1%</td>
<td>0.19</td>
<td>4</td>
<td>0.0%</td>
<td>n/a</td>
</tr>
<tr>
<td>GER</td>
<td>214.4%</td>
<td>0.32</td>
<td>1</td>
<td>1864.0%</td>
<td>n/a</td>
</tr>
<tr>
<td>JPN</td>
<td>1382.9%</td>
<td>0.07</td>
<td>5</td>
<td>0.0%</td>
<td>n/a</td>
</tr>
<tr>
<td>UK</td>
<td>4646.1%</td>
<td>0.02</td>
<td>6</td>
<td>0.0%</td>
<td>n/a</td>
</tr>
<tr>
<td>US</td>
<td>7039.1%</td>
<td>0.01</td>
<td>7</td>
<td>0.0%</td>
<td>n/a</td>
</tr>
</tbody>
</table>

---

\( ^a \) Preference Parameters used: \( \beta = 0.987, \gamma = 10, \psi = 1.5 \)

\( ^b \) Model Parameters used: data correlation on \( \eta_t \) and estimated parameters

\( ^c \) To Be Completed
E: Conclusion and Further Research

- We begin bridging the disconnect between consumption-based and asset pricing based measures of international risk-sharing gains.
- We analyze the “Long Run Risk” model in a benchmark international risk sharing framework.
- We show:
  - Ability to diversify Long Run Risk implies large welfare gains.
  - Price effects are large in the presence of Long Run Risk.
  - RA, rather than IES, is a larger effect on welfare gains.
- Still to do:
  - Stochastic Volatility.
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We show:

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Still to do:

- Stochastic Volatility
- Incomplete Markets
- Campbell-Cochrane Habit Model