Uncertainty Driven Business Cycle

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Recently, uncertainty facing economic agents raised substantially
- Especially in current business cycle. cf. Great Moderation

Not many business cycle frameworks to allow us to assess the role
- irreversibility literature: comparative statics, partial equilibrium

This paper builds upon irreversible investment literature
Extends it into a general equilibrium with time-varying uncertainty
- a generalization of Dixit (1989) and Caballero and Pindyck (1996)

Uncertainty is related to business fluctuation by its implication for
- incentives to create and destroy firms (i.e., the value of marginal firm)
  - and the influence of resulting firm dynamics on capital accumulation
Two Empirical Regularities

- **Counter cyclicality of idiosyncratic uncertainty**
  - -0.41~0.51 between the dispersion of stock returns and aggregate output growth: Campbell et al (2001) and Bloom et al (2007)
  - -0.67, between dispersion of 4-digits SIC TFP growth and aggregate output growth: Eisfeldt and Rampini (2006)

- **Procyclical net business formation**
  - correlation with output 0.73 for US and 0.60 for France: Chatterjee and Cooper (1993), Devereux et al (1996), Bergin and Corsetti (2005), Dos Santos and Dufourt (2006) and Jaimovich (2007)

- Logical conclusion: negative correlation between idiosyncratic uncertainty and net business formation.
Findings

- Endogenous entry/exit model with time-varying uncertainty shock
- Replicates key unconditional moments of aggregate quantities
- Generates realistic comovement among
  - (i) measured TFP
  - (iii) labor productivity
  - (iii) uncertainty measure
  - (iv) net business formation
- Firm dynamics together with time-varying uncertainty provides
  - powerful internal propagation mechanism
  - hump-shaped responses for all endogenous variables
  - strong forecastability for aggregate quantities such as documented by Rotemberg and Woodford (1996)
A representative household
- work for competitive wages, accumulate capital and invest in shares
- sources of incomes: wages, rental incomes and dividends
- capital accumulation is subject to adjustment frictions

A continuum of monopolistically competitive firms
- the economy consists of active and inactive firms
- production requires initial start-up costs $\gamma_S$, sunk upon entry
- fixed operation costs, $\gamma_F$ (mothballing not allowed)
- hence, the endogenous entry/exit problem

A competitive final goods industry
- CES aggregator with returns to specialization,
  (Dixit and Stiglitz (1977) and Benassy (1996))
Two equilibrium concepts

- Free entry allocation:
  - a firm makes its own entry/exit decision to maximize its value
  - entry/exit continue until firm creation/destruction generates zero profit
  - underlying assumption: a competitive equilibrium provides the first best

- Constrained optimum allocation:
  - all entry/exit decisions are made by a capital planner
  - to maximize the total value of firms
  - subject to mark-up pricing rule of monopolistic competition

- Are they equivalent? A few non-neoclassical elements such as
  - monopolistic competition, non-convex costs and returns to specialization
  - may create discrepancy between the two
The technology: a CRS Cobb-Douglas,

\[ y(i) = m(i)z(i)^\nu k(i)^\alpha n(i)^{1-\alpha}, \quad m(i) \in \{0,1\}, \quad i \in [0,1] \]

- \( m(i) \): an indicator of activity status
- \( \nu \) makes profit linear in \( z(i) \)
- idiosyncratic shock follows \( \log z(i) \sim N(-0.5\sigma^2, \sigma^2) \)

Uncertainty(\( \sigma \)): a stationary Markov process

\[ \log \sigma = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{-1} + \varepsilon, \quad \varepsilon \sim N(0, \Sigma) \]


No first moment shock (no aggregate TFP shock)

\[ E(z|\sigma) = \exp(-0.5\sigma^2 + 0.5\sigma^2) = 1 = E(z) \]
Profit and externality

- Final goods producer maximizes \( PY - \int_0^M p(i)y(i)di \) s.t

\[
Y = M^{1+v-1/\theta} \left[ \int_0^M y(i)^\theta di \right]^{1/\theta} \quad 0 < \theta < 1
\]

- \( M \in [0, 1] \): the total measure of active firms.
- Returns to specialization (Benassy(1996)): \( 1 + \nu \)

- Monopolistic competitor’s profit

\[
\pi(i) = m(i) (1 - \theta) \theta^{\frac{\alpha}{1-\theta}} \left[ z(i)^\nu \left( \frac{\alpha}{r} \right)^\alpha \left( \frac{1 - \alpha}{w} \right)^{1-\alpha} \right]^{\frac{\theta}{1-\theta}} Y M^{\xi_{\theta-1/\theta}}
\]

- when \( \nu = (1-\theta)/\theta \), the profit is linear in \( z(i) \)
- expected profit is not affected by \( \sigma \).
- when \( 1 + \nu = 1/\theta \), \( \xi = 1 + \nu - 1/\theta \) becomes zero
- profit not affected by the degree of competition \( (M) \)
Total value maximization under Constrained Optimum

- Venture capitalist decides \( m(i) = \{0, 1\}, \ i \in [0, 1] \)
  - to maximize \( \bar{V}(s) = \int V(i,s)di, \) where \( s \equiv [\sigma, K, M_{-1}] \)
- Monotonicity of profit implies reservation property:
  - policy rules are summarized by entry/exit thresholds \( z(s) = [z_0(s) \ z_1(s)] \)
- Recursive representation:
  \[
  \bar{V}(s) = \max_{z_0, z_1} \left\{ \Pi(z) - \Gamma_S(z) - \Gamma_F(z) + \int \Lambda(s, s') \bar{V}(s')dQ(\sigma'|\sigma) \right\}
  \]
  - \( \Pi, \Gamma_S \) and \( \Gamma_F \) are aggregate profits, sunk and fixed costs
  - maximization is subject to the law of motion for the stock of firms
  \[
  M(s) = [1 - F(z_1(s)|\sigma)]M(s_{-1}) + [1 - M(s_{-1})][1 - F(z_0(s)|\sigma)]
  \]
Efficiency: two FOCs and one envelope condition.

- **Two FOCs for** $z_0(s)$ and $z_1(s)$:

\[ \gamma_s + \gamma_F - \left[ \pi(z_0(s)) + \frac{\partial \Pi(s)}{\partial M(s)} \right] = q^M(s) \text{ and} \]
\[ \gamma_F - \left[ \pi(z_1(s)) + \frac{\partial \Pi(s)}{\partial M(s)} \right] = q^M(s) \text{ where} \]
\[
q^M(s) \equiv \int \Lambda(s, s') \frac{\partial V(s')}{\partial M(s)} dQ(\sigma'|\sigma) \\
\text{value of marginal firm (beachhead)}
\]

- **Envelope condition:**

\[ q^M(s) = \int \Lambda(s, s') \left[ \frac{d\Pi(s')}{dM(s)} - \frac{\partial \Gamma(s')}{\partial M(s)} + \frac{\partial M(s')}{\partial M(s)} q^M(s') \right] dQ(\sigma'|\sigma) \]

where \[ \frac{d\Pi(s')}{dM(s)} = \frac{\partial \Pi(s')}{\partial M(s)} + \frac{\partial M(s')}{\partial M(s)} \frac{\partial \Pi(s')}{\partial M(s')} \], \[ \Gamma(s') \equiv \Gamma_F(s') + \Gamma_S(s') \]
**Individual value maximization under Free Entry**

- **Optimal stopping problem:**
  \[ V(z, s^*, m_{-1}) = \max \{ W(z, s^*, m_{-1}, 1), W(z, s^*, m_{-1}, 0) \} \]
  where
  \[ W(z, s^*, m_{-1}, m) = m \left[ \pi(z, s^*) - (1 - m_{-1}) \gamma_S - \gamma_F \right] + \int \int \Lambda(s^*, s'^*) V(z', s'^*, m) dF(z'|\sigma') dQ(\sigma'|\sigma) \]

- **Value matching:**
  \[ W(z_{m_{-1}}^*(s^*), s^*, m_{-1}, 1) = W(z_{m_{-1}}^*(s^*), s^*, m_{-1}, 0) \]

- **Equivalent to**
  \[ \gamma_S + \gamma_F - \pi(z_0^*(s^*)) = J(s^*) \text{ and } \gamma_F - \pi(z_1^*(s^*)) = J(s^*) \] where
  \[ J(s^*) \equiv \int \int \Lambda(s^*, s'^*) \left[ V(z', s'^*, 1) - V(z', s'^*, 0) \right] dF(z'|\sigma') dQ(\sigma'|\sigma) \]
(Non)equivalency of two allocations

- By manipulating the definition of $J(s)$ and threshold shocks,

$$J(s^*) = \int \Lambda(s^*, s^{*'}) \left[ \frac{\partial \Pi(s^{*'})}{\partial M(s^*)} - \frac{\partial \Gamma(s^{*'})}{\partial M(s^*)} + \frac{\partial M(s^{*'})}{\partial M(s^*)} J(s^{*'}) \right] dQ(\sigma'|\sigma)$$

- If $d\Pi(s')/dM(s) = \partial \Pi(s')/\partial M(s)$, two allocations are equivalent, i.e.,

$$q^M(s) = J(s^*), \ z_0(s) = z_0^*(s^*) \text{ and } z_1(s) = z_1^*(s^*)$$

- Total effects on profits:

$$\frac{d\Pi(s')}{dM(s)} = \frac{\partial \Pi(s')}{\partial M(s)} + \frac{\partial M(s')}{\partial M(s)} \frac{\partial \Pi(s')}{\partial M(s')}$$

  - direct

  - indirect effects

- Optimality of myopic equilibrium (Leahy(1996))
  - requires absence of externality ($1 + v = 1/\theta, \ \partial \Pi(s')/\partial M(s') = 0$)
  - if $1 + v \geq 1/\theta, \ \partial \Pi(s')/\partial M(s') \geq 0$, “business enhancing (stealing)”
Quality-adjusted firm measure and TFP

- Measured TFP fluctuates over the business cycle without TFP shocks

\[
Y(s) = \left[ M_1(s) \frac{1 - \Phi(\mu_1(s) - \sigma)}{1 - \Phi(\mu_1(s))} + M_0(s) \frac{1 - \Phi(\mu_0(s) - \sigma)}{1 - \Phi(\mu_0(s))} \right] \times y(\bar{z}, s)
\]

≡ Ξ(s), a quality adjusted firm measure

- True source of TFP: capacity utilization, \( \Xi(s) = Y(s) / y(\bar{z}, s) \)
  - composition and productivity differentials of incumbents and entrants
  - \( \Xi(s) \) is a quality adjusted measure of firms. cf. \( M(s) = M_1(s) + M_0(s) \)

- Capital market clears \( K^D(s) = K(s) \), where \( K^D(s) = \Xi(s) k^D(\bar{z}, s) \)

- Household’s optimization leads to dynamic supply condition:

\[
q^K(s) = \int \Lambda(s, s') \left\{ r(s') - \frac{\partial \lambda(I(s'), K(s))}{\partial K(s)} \right\} dQ(\sigma' | \sigma) + (1 - \delta)q^K(s')
\]
Calibration

- **Standard parameters:**
  - household preferences: \( \log c + \zeta (1 - n) \) with \( \beta = 0.98 \)
  - elasticity of substitutability in CES aggregator (\( \theta \)), \( 3/4 \)
  - technology: capital share (\( \alpha \)), 0.3, depreciation rate (\( \delta \)), 0.02
  - capital adjustment friction: \( \lambda / 2 I/K - \delta)^2 K \) with \( \lambda = 3.50 \)

- **Non-standard parameters:**
  - steady state dispersion (\( \bar{\sigma} \)), 0.4 (cf. in literature 0.1~0.6)
  - uncertainty process, \( \rho_\sigma = 0.85 \) and \( \Sigma_\epsilon = 0.2 \) (Bloom(2007))
  - long run exit rate = \( \Phi (\bar{\mu}_1) = 0.025 \) (Dunne et al(1988))
  - sunk-cost-to-capital (\( \gamma_S/\bar{k}^D \)), 3.7% (D’eramoso(2006))
  - euler equation for \( q^M \), together with \( \gamma_S/\bar{k}^D \), pins down \( \gamma_F/\bar{k}^D \)
  - use the steady state capital market clearing condition for \( \bar{\mu}_0 \).
  - \( \bar{\mu}_0 \) and \( \bar{\mu}_1 \) determine \( \tilde{M} \)
Impact of uncertainty shock

- Increase in uncertainty decreases the value of marginal firm ($q^M = J$)
- Equilibrium number of firm declines, causing weaker demand for capital
- Investment, hours and output show strong hump-shaped responses

\[ M(s) = \left[ \Phi(\mu_0(s)) - \Phi(\mu_1(s)) \right] M(s-1) + 1 - \Phi(\mu_0(s)) \]

Time Varying AR(1) Coefficient

Shock to Turn-over
Role of capital adjustment friction

- Hump-shaped cycle does not depend on the adjustment friction
- capital adjustment friction helps matching moments of the data
- Neither the firm dynamics nor measured TFP are affected by the adjustment friction
Unconditional moments

- Closely matches the volatilities of key macroeconomic aggregates
- Generates similar degrees of comovements to those in the data
  - Too sensitive entry/exit: too much correlation between $M$ and $Y$
- Higher(15%) sunk cost parameterization: a stronger hysteresis
  - $\Phi(\bar{\mu}_0) - \Phi(\bar{\mu}_1)$ in the steady state is increased from 0.56 to 0.94
  - Larger fluctuation in resources used for firm turn-over
  - Implying greater volatilities and weaker comovements

Table 1. Volatilities and Comovements

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviations</th>
<th>Comovement with Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S.</td>
<td>M(3.7%)</td>
</tr>
<tr>
<td>Output</td>
<td>1.57</td>
<td>1.67</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.18</td>
<td>1.00</td>
</tr>
<tr>
<td>Investment</td>
<td>7.02</td>
<td>7.29</td>
</tr>
<tr>
<td>Hours</td>
<td>1.52</td>
<td>1.09</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>-0.67</td>
<td>-0.76</td>
</tr>
</tbody>
</table>
In the data, about 50 ~ 60% of output variance is forecastable.

Due to the rich propagation, the model generates strong forecastability.

- for the short run, the propagation is somewhat too strong.
- forecastability tends to increase with the size of sunk entry cost.

Table 2. Forecastable movements in output: data vs. model.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>24</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forecastable Change in Output</td>
<td>0.006</td>
<td>0.011</td>
<td>0.019</td>
<td>0.030</td>
<td>0.032</td>
<td>0.031</td>
<td>0.021</td>
</tr>
<tr>
<td>Actual Change in Output</td>
<td>0.011</td>
<td>0.018</td>
<td>0.027</td>
<td>0.038</td>
<td>0.045</td>
<td>0.055</td>
<td>0.032</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.570</td>
<td>0.600</td>
<td>0.679</td>
<td>0.778</td>
<td>0.717</td>
<td>0.556</td>
<td>0.659</td>
</tr>
<tr>
<td>M(3.7%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forecastable Change in Output</td>
<td>0.007</td>
<td>0.010</td>
<td>0.014</td>
<td>0.020</td>
<td>0.024</td>
<td>0.029</td>
<td>0.017</td>
</tr>
<tr>
<td>Actual Change in Output</td>
<td>0.008</td>
<td>0.015</td>
<td>0.025</td>
<td>0.034</td>
<td>0.039</td>
<td>0.044</td>
<td>0.028</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.850</td>
<td>0.664</td>
<td>0.542</td>
<td>0.572</td>
<td>0.616</td>
<td>0.656</td>
<td>0.650</td>
</tr>
<tr>
<td>M(15.0%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forecastable Change in Output</td>
<td>0.016</td>
<td>0.025</td>
<td>0.032</td>
<td>0.034</td>
<td>0.036</td>
<td>0.040</td>
<td>0.030</td>
</tr>
<tr>
<td>Actual Change in Output</td>
<td>0.019</td>
<td>0.027</td>
<td>0.037</td>
<td>0.048</td>
<td>0.053</td>
<td>0.059</td>
<td>0.041</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.817</td>
<td>0.909</td>
<td>0.855</td>
<td>0.709</td>
<td>0.667</td>
<td>0.665</td>
<td>0.770</td>
</tr>
</tbody>
</table>
Does uncertainty lead to a boom or a recession?

- Crucially depends on the degree of returns to specialization
  - with $1 + v < 1.2$ or equivalently $\xi \theta / (1 - \theta) < -0.4$
  - uncertainty can lead to a boom owing to reduced competition
  - causing a negative correlation between output and firm dynamics
- However, the parameter choice behind this result is implausible
For both allocations, externality tends to increase volatility
  - externality: additional propagation regardless of its internalization

In a relative sense, FE dampens (amplifies) fluctuations under negative (positive) externality
  - no substantial differences in comovement properties

Table 3. Externality and relative (de)stabilization of fluctuations

<table>
<thead>
<tr>
<th></th>
<th>Profit Elasticity=-0.3</th>
<th>Profit Elasticity=0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FE</td>
<td>CO</td>
</tr>
<tr>
<td>Output</td>
<td>2.33</td>
<td>2.67</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.05</td>
<td>1.38</td>
</tr>
<tr>
<td>Investment</td>
<td>11.56</td>
<td>13.88</td>
</tr>
<tr>
<td>Hours</td>
<td>1.62</td>
<td>1.49</td>
</tr>
</tbody>
</table>
Current analysis shows that a simple model with endogenous firm creation/destruction with uncertainty shock can go a long way.

Uncertainty as a source of business cycle and firm dynamics as a propagation should be explored more.

Introducing financial friction would be very useful given that uncertainty has a direct implication for bond spreads.

This research builds on a notion that dispersion causes aggregate cycle.

Reverse causality is also possible such as in Eisfeldt and Rampini’s work.