Revisiting Cross-Country Correlation Anomalies

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Outline of the Presentation

I. Introduction

II. Preview of our Results

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I. Introduction

• In the data, $\text{corr}(n, n^*)$ and $\text{corr}(I, I^*)$ are small and positive.

[Backus-Kehoe-Kydland (Cooley 1995), Ambler-Cardia-Zimmerman (JME 2004), Baxter-Farr (JIE 2005)]

• Seminal work by Backus-Kehoe-Kydland (BKK) (JPE 1992) shows that their international RBC (IRBC) model tends to deliver negative cross-country correlations in $Y$, $N$, and $I$ and too high a positive correlation in $C$. 
• Producing $corr(I, I^*) > 0$ is challenging.


⇒ Weakness: generally increase $corr(I, I^*)$ by reducing variance of $I$

• Figure: Tradeoff between $corr(I, I^*)$ and $\sigma_I/\sigma_Y$ in standard IRBC model
What do we do?

- Our goal: A model with plausible mechanisms that delivers
  - $0 < \text{corr}(n, n^\ast)$
  - $0 < \text{corr}(I, I^\ast)$
  - while still producing
    - $0 < \text{corr}(C, C^\ast)$
    - $0 < \text{corr}(Y, Y^\ast)$
    - $\sigma_I/\sigma_Y$ identical to the data

- Our Model: We modify a standard IRBC model with incomplete asset markets à la Baxter-Crucini in two ways
  i. Use GHH preferences (Greenwood, Hercowitz and Huffman (1988])
  ii. Add internal learning by doing (LBD) [Cooper and Johri (2002)]
Why Learning by Doing?


- Empirical evidence on LBD is pervasive. All sectors. Over 100 years of evidence.


• Previous work on business cycle models provide support for LBD
  – Results show LBD is a strong propagation mechanism.
  – Implied dynamic labour supply greatly improves fit of model to data.

• Internal LBD implies additional dynamics in hours and investment decisions which help raise $\text{corr}(I, I^*)$. 
II. Preview of Our Results

- Framework: We use GHH (1988) preferences and learning by doing (LBD) in an incomplete asset markets IRBC model à la Baxter-Crucini.
  - GHH preferences make $\text{corr}(n, n^*)$ positive and $\text{corr}(Y, Y^*)$ bigger.
  - LBD makes $\text{corr}(I, I^*)$ positive...
  - ...while still matching $\sigma_I/\sigma_Y$ and producing a counter-cyclical trade balance.
  - LBD’s ability to raise $\text{corr}(I, I^*)$ is robust.
III. Model

• Two countries.

• One good (traded).

• Large number of identical agents in each country.

• Only one type of financial assets: non-contingent one-period real bonds.

• The two countries are symmetrical except for country-specific shocks.

• Capital is internationally mobile but labour is not.

• Home country’s representative agent’s problem:

\[
\max U = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad 0 < \beta < 1,
\]
where preferences are:

$$\left[ \frac{C_t - \psi n^\nu_t}{1 - \sigma} \right]^{1-\sigma}, \quad \psi > 0, \ \nu > 1, \ \sigma > 0$$

subject to

$$C_t + I_t + P_t B_{t+1} = Y_t + B_t.$$

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t, \quad 0 \leq \phi, \ 0 < \delta < 1$$

$$Y_t = A_t H_\varepsilon^\varepsilon K_t^\theta n_t^\alpha, \quad 0 < \varepsilon, \ \theta, \ \alpha < 1,$$

$$H_{t+1} = H_t^\gamma Y_t^\eta, \quad 0 < \gamma < 1, \ 0 < \eta \leq 1$$

Stochastic process of productivity shocks:

$$\begin{bmatrix} \ln A_{t+1}^* \\ \ln A_t^* \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \ln A_t \\ \ln A_t^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+1}^* \end{bmatrix}$$
For stationarity reasons, we impose
\[ \frac{1}{P_t} = \frac{1}{P^*_t} e^{-\chi [B_{t+1} - \bar{B}]}, \quad \chi > 0. \]

IV. Parameter Values

- Period=quarter
- Coefficient of relative risk aversion: \( \sigma = 2 \)
- Preference parameter:
  - Discount factor: \( \beta = 0.993 \)
  - \( \psi = 6.174 \) set so \( \bar{n} = 1/3 \)
  - With GHH preferences, \( \nu = 3 \) implies a Frisch labour supply elasticity of 0.5.
• Steady-state HC net foreign assets: $\bar{b} = 0$

• We are using estimates from Johri and Letendre (2007) for parameters of the production function and accumulation equations for $H$ and $K$:

$-K_{t+1} = (1 - \delta)K_t + I_t: \quad \delta = 0.02$

$-H_{t+1} = H_t^\gamma Y_t^\eta: \quad \gamma = 0.95, \eta = 0.05 \quad [\text{CRS}]$

$-Y_t = A_t H_t^\epsilon K_t^\theta n_t^\alpha: \quad \theta = 0.21, \alpha = 0.55 \text{ and } \epsilon = 0.24 \quad [\text{CRS}]$

• Cross-country correlation of shocks=0.323

• Shock variance $\sigma_\varepsilon^2$ set to match observed variance of $Y$

• For no-LBD and LBD models, the capital adjustment cost parameter $\phi$ is set to have $\sigma_I/\sigma_Y = 3$. no-LBD: $\phi = 1.73$; LBD: $\phi = 2.76$
- Shocks persistence ($\rho$):
  
  - Recall production function in LBD model: $Y_t = A_t H_t^\varepsilon K_t^\theta n_t^\alpha$
  
  - Solow residuals (SR) in the LBD model:
    \[
    \ln(SR_t) \equiv \ln(Y_t) - \alpha \ln(K_t) - \theta \ln(n_t) = \ln(A_t) + \varepsilon \ln(H_t)
    \]
  
  - Whereas the SR in the no-LBD model:
    \[
    \ln(SR_t) \equiv \ln(Y_t) - \alpha \ln(K_t) - \theta \ln(n_t) = \ln(A_t)
    \]

  no-LBD: $\rho = 0.95$, \quad LBD: $\rho = 0.945$. 
V. Analysis

• Look at responses of variables to

\[
\begin{bmatrix}
\epsilon_t \\
\epsilon^*_t
\end{bmatrix}_{t=1}^{\infty} = \begin{bmatrix}
0.01 & 0 & 0 & 0 & \ldots \\
0.01 \times corr(\epsilon, \epsilon^*) & 0 & 0 & 0 & \ldots
\end{bmatrix}.
\]

Where \( corr(\epsilon, \epsilon^*) = 0.323 \) or zero.

• Figures:

  – First figure: \( corr(\epsilon, \epsilon^*) = 0.323 \) as in simulations.

    In the period of the shock, \( *I \) increase in the LBD model but not in the model without LBD.

  – Second figure: \( corr(\epsilon, \epsilon^*) = 0 \).

    In the period of the shock, \( *I \) falls much less and turns positive sooner in the LBD model.
Investment responses to correlated shocks

% dev. from steady state

Time

-0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0

0 2 4 6 8 10 12 14 16 18 20
Investment responses to uncorrelated shocks

% dev. from steady state

Time

I
I *
I LBD
I* LBD
• How does LBD change the way investment responds to shocks?

– Start with the no-LBD model and abstract from adjustment costs and cross-country correlations in shocks. The Euler equations are

\[
1 = E_t \beta \frac{\lambda_1 t + 1}{\lambda_1 t} MRS_{t,t+1} \left\{ \frac{\theta A_{t+1} N_{t+1}^\alpha}{[(1 - \delta)K_t + I_t]^{1-\theta}} + 1 - \delta \right\}, \quad P_t = E_t \beta \frac{\lambda_1 t + 1}{\lambda_1 t}
\]

\[
1 = E_t \beta \frac{\lambda_1^* t + 1}{\lambda_1^* t} MRS_{t,t+1}^* \left\{ \frac{\theta A^* N^*_{t+1}^\alpha}{[(1 - \delta)K_t^* + I_t^*]^{1-\theta}} + 1 - \delta \right\}, \quad P_t^* = E_t \beta \frac{\lambda_1^* t + 1}{\lambda_1^* t}
\]

– HC: \( A \uparrow \) and \( MRS \downarrow \) a bit \( \rightarrow I \) has to increase.

– FC: \( A \uparrow \rightarrow \Delta A^* = 0, \Delta N^* = 0 \) and \( MRS^* \downarrow \rightarrow I^* \) has to decrease.

– Why \( \downarrow MRS^* \)? HC borrows \( \Leftrightarrow \uparrow \) supply of bonds \( \rightarrow \) bond price \( \downarrow \).
– Now the **LBD model**.

– In the HC we have

\[
1 = E_{t/\beta} \left\{ \frac{\lambda_{1t+1}}{\lambda_{1t}} \left( \frac{\theta A_{t+1} N_{t+1}^{\alpha} H_{t+1}^{\varepsilon}}{[(1 - \delta) K_t + I_t]^{1 - \theta}} + 1 - \delta \right) + \frac{\lambda_{3t+1}}{\lambda_{1t}} \frac{H_{t+2}}{\left( \frac{\partial H_{t+2}}{\partial K_{t+1}} \right)} \theta \eta \left[ (1 - \delta) K_t + I_t \right] \right\}
\]

\[
\left[ \frac{H_{t+2}^{1/\eta}}{H_{t+1}^{\gamma + \varepsilon} A_{t+1} \eta_{t+1}^{\alpha}} \right]^{1/\theta} = K_{t+1} = (1 - \delta) K_t + I_t.
\]

The equation above shows a negative relationship between \( A_{t+1} \) and \( I_t \) that is not build into the no-LBD model. This tempers the HC’s desire to increase investment [think of a wealth effect].

→ HC supply less bonds so the bond price (and MRS) falls less than in no-LBD model.
In the FC, the $K$ Euler equation is

$$1 = E_t^\beta \left\{ \frac{\lambda_{1t+1}^*}{\lambda_{1t}^*} \left( \frac{\theta A_* N_{t+1}^* \alpha H_{t+1}^* \varepsilon}{[(1 - \delta) K_t^* + I_t^*]^{1-\theta}} + 1 - \delta \right) + \frac{\lambda_{3t+1}^*}{\lambda_{1t}^*} \frac{\theta \eta}{\left( \frac{A_* H_{t+1}^*}{\eta n_{t+1}^* \alpha} \right)^\eta} \right\}$$

Mechanically, $I^*$ falls less in the presence of LBD because

1. Bond price (and $MRS^*$) falls less

2. $I^*$ appears at the denominator of both terms on the right side

Intuitively: Reducing investment in period $t$ does not only reduce future $K$ but also future $H$.

It is therefore more costly to reduce $I^*$ in the presence of LBD.
<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>same param.</th>
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<td>0.48</td>
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<td>Employment</td>
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<td>Investment</td>
<td>2.78</td>
<td>3</td>
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<td>2.55</td>
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<td><strong>Intl Correl.</strong></td>
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<td>Output</td>
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<td>0.31</td>
<td>0.29</td>
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<td>Consumption</td>
<td>0.38</td>
<td>0.43</td>
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<tr>
<td>Employment</td>
<td>0.39</td>
<td>0.32</td>
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<td>0.30</td>
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</tr>
<tr>
<td>Investment</td>
<td>0.33</td>
<td>0.15</td>
<td>-0.38</td>
<td>-0.15</td>
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</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
<td></td>
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<tr>
<td>$autocorr(I)$</td>
<td>0.91</td>
<td>0.67</td>
<td>0.65</td>
<td>0.66</td>
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<tr>
<td>$corr(TB/Y, Y)$</td>
<td>-0.32</td>
<td>-0.16</td>
<td>-0.46</td>
<td>-0.48</td>
<td></td>
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<tr>
<td>$SD(TB/Y)$</td>
<td>0.62</td>
<td>0.15</td>
<td>0.87</td>
<td></td>
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</tr>
</tbody>
</table>
Sensitivity Analysis for LBD Model

-Asset markets: LBD still raises $corr(I, I^*)$ when asset markets are complete.

-Change in parameters

\[
\begin{align*}
\beta & \quad \sigma & \quad (\theta, \varepsilon) & \quad (\gamma, \eta) \\
0.984 & \quad 3 & \quad (0.25, 0.2) & \quad (0.8, 0.2)
\end{align*}
\]

$corr(I, I^*) \quad 0.17 \quad 0.16 \quad -0.06 \quad -0.06$

-When we lower $\rho$ to 0.9, the LBD model still delivers $corr(I, I^*) > 0$. When we raise $\rho$ (not changing anything else) $corr(I, I^*)$ is negative in the LBD model but it is still the case that LBD raises $corr(I, I^*)$.

-When we raise international spillovers to $\nu = 0.04$, $corr(I, I^*)$ is negative in the LBD model but it is still the case that LBD raises $corr(I, I^*)$. 
VI. Concluding Remarks

• Adding LBD to the IRBC model raises $corr(I, I^*)$.

• LBD’s ability to raise $corr(I, I^*)$ is robust.

• Adding GHH and LBD to the model: $corr(I, I^*) > 0$ and $corr(N, N^*) > 0$.

• LBD model: Investment and hours correlations are in the right ballpark.

• Model with LBD still matches $\sigma_I/\sigma_Y$.

• Model with LBD produces a counter-cyclical $TB/Y$. 
<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>LBD</th>
<th>no LBD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard Deviations (SD)</strong></td>
<td></td>
<td></td>
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<tr>
<td>$SD(C)/SD(Y)$</td>
<td>0.95</td>
<td>0.79</td>
<td>0.77</td>
</tr>
<tr>
<td>$SD(n)/SD(Y)$</td>
<td>0.78</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>$SD(I)/SD(Y)$</td>
<td>2.66</td>
<td>2.66</td>
<td>2.66</td>
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<tr>
<td><strong>Cross-Country Correlations</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>0.63</td>
<td>0.38</td>
<td>0.24</td>
</tr>
<tr>
<td>$C$</td>
<td>0.42</td>
<td>0.35</td>
<td>0.30</td>
</tr>
<tr>
<td>$n$</td>
<td>0.27</td>
<td>0.42</td>
<td>0.24</td>
</tr>
<tr>
<td>$I$</td>
<td>0.10</td>
<td>0.07</td>
<td>-0.53</td>
</tr>
<tr>
<td>$SR$</td>
<td></td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td><strong>Autocorrelations</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$I$</td>
<td>0.94</td>
<td>0.86</td>
<td>0.69</td>
</tr>
</tbody>
</table>
How do GHH preferences make $corr(n, n^*) > 0$?

- By “turning off” the wealth effect.

- Hours FOC in percentage deviation: $\bar{Y}_t = \nu \bar{N}_t$

Substitute into production function in percentage deviations:

$$\bar{N}_t = \frac{1}{\nu - \alpha}[\bar{A}_t + \theta \bar{K}_t]$$

so hours follow the shock closely in both countries.